Workshop on *Bayesian Networks and Argumentation in Evidence Analysis*

Basic probabilistic issues in the Sciences and in Forensics (hopefully) clarified by a Toy Experiment modelled by a BN

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"Probability is the very guide of life"
(*Digest* of Cicero's thought)

"Probability is good sense reduced to a calculus"
(S. Laplace)

Short presentation

- Experimental particle physicist

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- but not a *former physicist* doing forensic physics. …

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More on my web page.

What is measurement?

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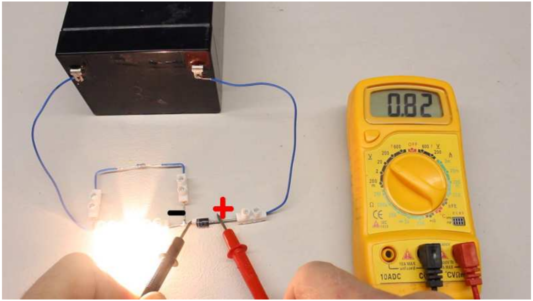

What is measurement?

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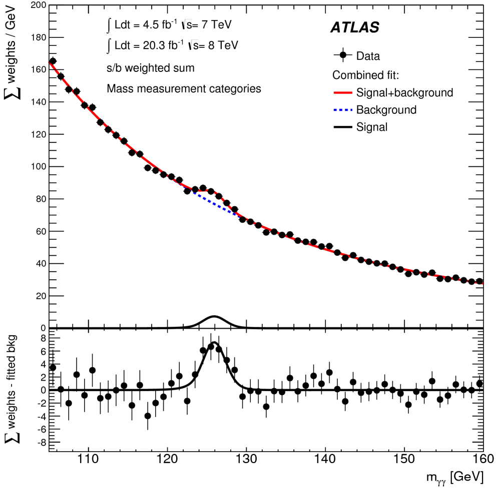
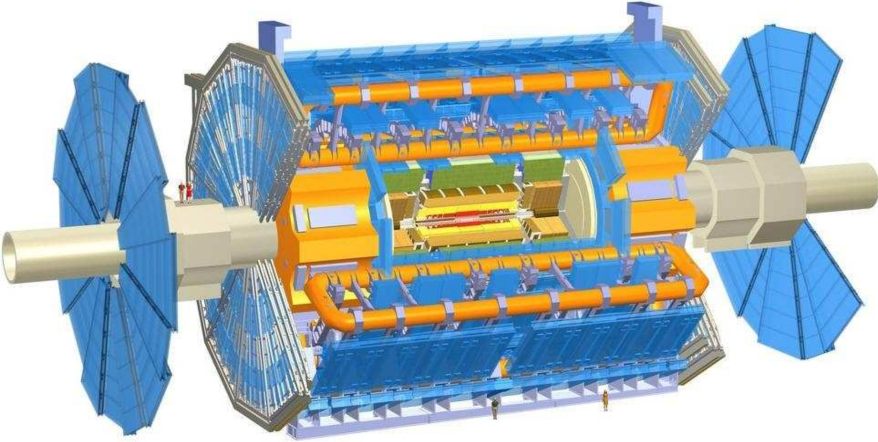
Higgs $\rightarrow \gamma\gamma$ (2012)



Two-photon *invariant mass*

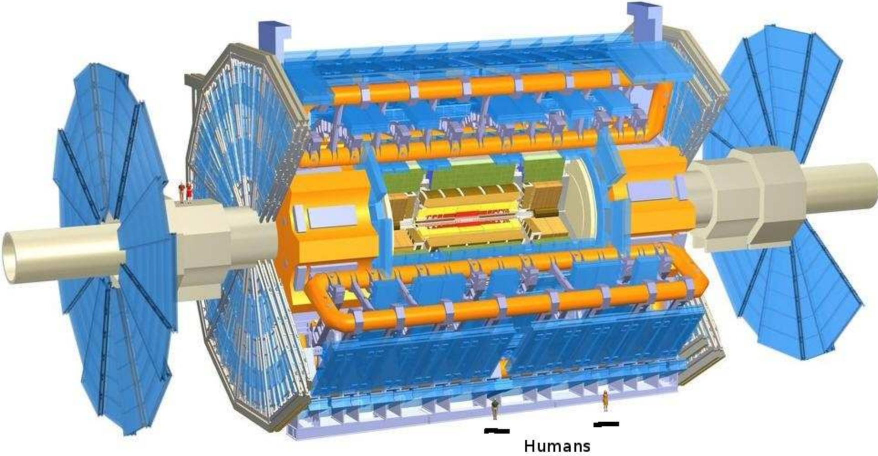
What is measurement?

ATLAS Experiment at LHC (CERN, Geneva)



What is measurement?

ATLAS Experiment at LHC [length: 46 m; $\varnothing$ 25 m]



$\approx$ 3000 km cables

$\approx$ 7000 tonnes

$\approx$ 100 millions electronic channels

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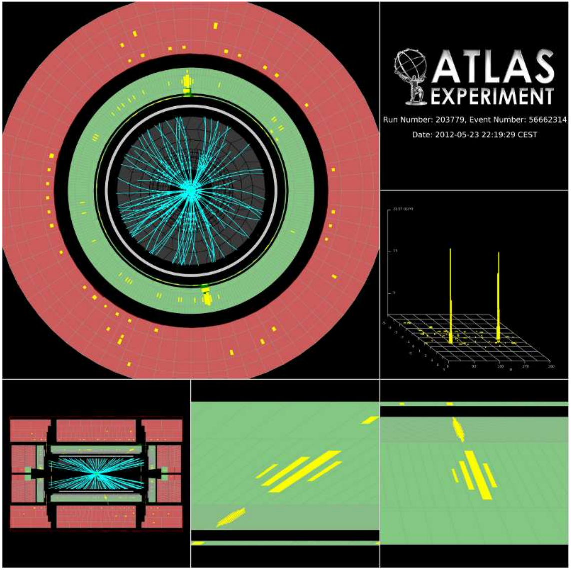

Two flashes of 'light' (2 γ's) in a 'noisy' environment.

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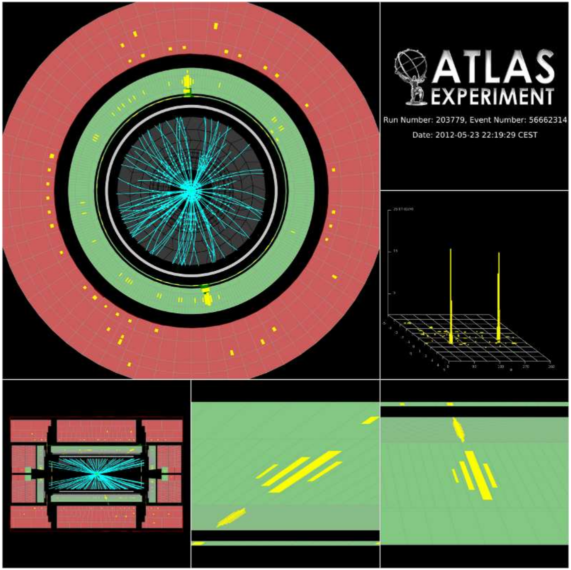

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Higgs $\to \gamma\gamma$?

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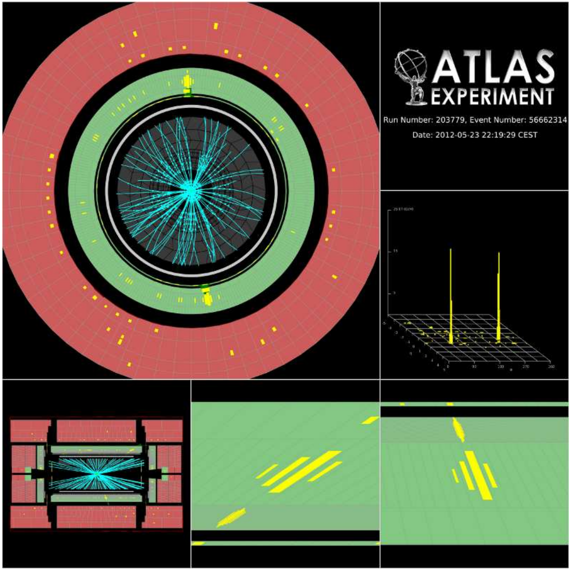
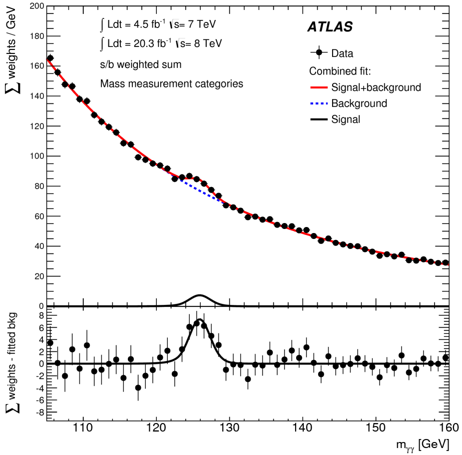


Two flashes of 'light' (2 γ's) in a 'noisy' environment.

Higgs $\rightarrow \gamma\gamma$? Probably not...

What is measurement?

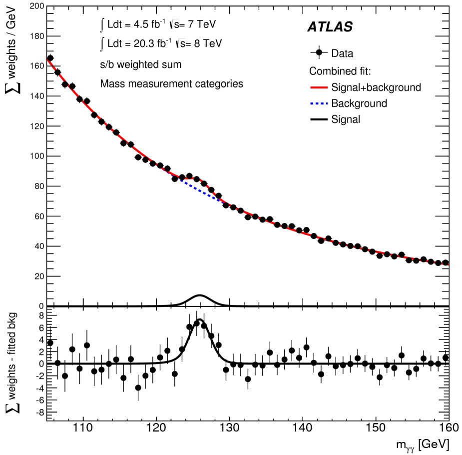
Higgs $\to \gamma\gamma$



$\Rightarrow \left\{ \begin{array}{l} \text{Mass value} \\ \text{Production rate} \end{array} \right.$

What is measurement?

Higgs $\to \gamma\gamma$



$\Rightarrow \begin{cases} \text{Mass value} \\ \text{Production rate} \end{cases}$

(with uncertainties)

What is measurement?

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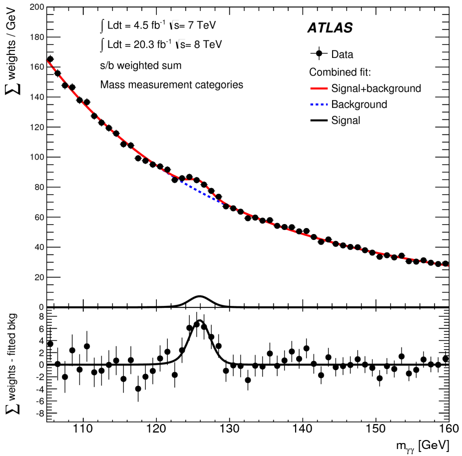


$\Rightarrow$ Mass value
Production rate
(with uncertainties)

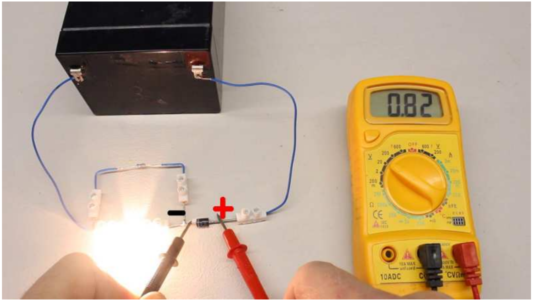
Quite indirect measurements of something we do not "see"!

Can we "see" physics quantities?

But, can we <u>see</u> our mass?

Can we "see" physics quantities?

. . . or a voltage?



Can we "see" physics quantities?

. . . or our blood pressure?

Can we “see” physics quantities?

Certainly not!

Can we “see” physics quantities?

Certainly not!

. . . although for some quantities we can have

a ‘vivid impression’ (in the David Hume’s sense)

Measuring a mass on a scale

Equilibrium:

$$mg - k\Delta x = 0$$
$$\Delta x \rightarrow \theta \rightarrow \text{scale reading}$$

(with 'g' *gravitational acceleration;* 'k' spring constant.)

Measuring a mass on a scale

Equilibrium:

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From the reading to the value of the mass:

$$\text{scale reading} \xrightarrow[\text{given } g,\ k,\ \text{“etc.”}\ldots]{} m$$

Measuring a mass on a balance

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Dependence on 'g': $$g \stackrel{?}{=} \frac{GM_{\oplus}}{R_{\oplus}^2}$$

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Dependence on 'g': $$g \stackrel{?}{=} \frac{GM_{\oplus}}{R_{\oplus}^2}$$

- Position is usually not at "$R_{\oplus}$" from the Earth center;
- Earth not spherical...
- ... not even ellipsoidal...
- ... and not even homogeneous.
- Moreover we have to consider centrifugal effects
- ... and even the effect from the Moon

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But think about it!

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- temperature
- non linearity
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Δx $\rightarrow \theta \rightarrow$ scale reading:

- left to your imagination...

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$+$ **random effects:**

- stopping position of damped oscillation;
- variability of all quantities of influence (in the ISO-GUM sense);
- reading of analog scale.

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$\Rightarrow$ m??

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A number, outside a contest, and denuted of all contextual information provides little (or zero) knowledge:

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[$^{(*)}$ Performing an experiment is just a subclass of 'questioning']

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$\rightarrow$ Monty Hall problem and variations;

$\rightarrow$ Three prisoners problem.

  $\rightarrow$ Very relevant in Forensics!

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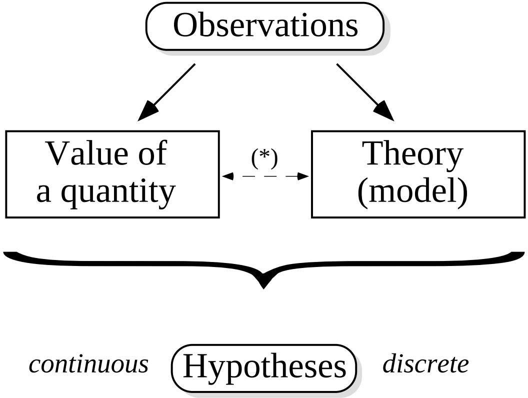
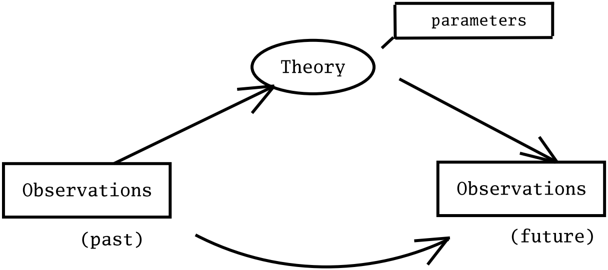
Learning from data



(*) A quantity might be meaningful only within a theory/model

From past to future



Our task:

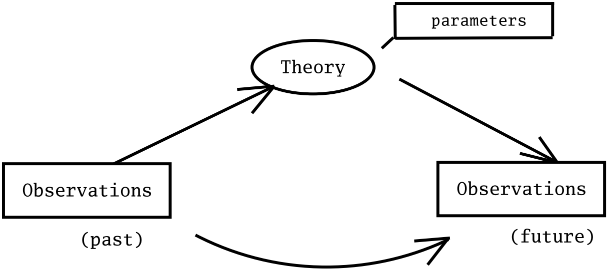
- Describe/understand the 'physical' world

  $\Rightarrow$ inference of laws and their parameters

- Predict observations

  $\Rightarrow$ forecasting

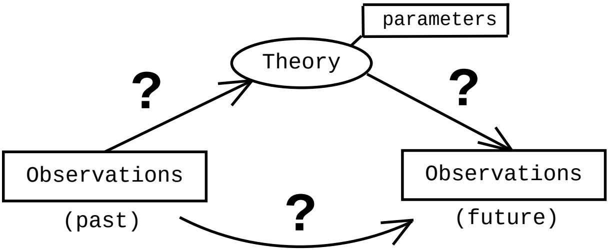
From past to future



⇒ Uncertainty:

1. Given the past observations, in general we are not sure about the theory parameters (and/or the theory itself)
2. Even if we were sure about theory and parameters, there could be internal (e.g. Q.M.) or external effects (initial/boundary conditions, 'errors', etc) that make the forecasting uncertain.

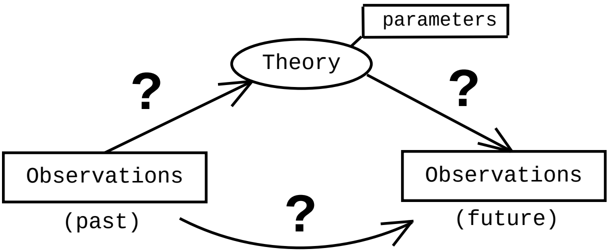
From past to future



⇒ Decision

- What is be best action ('experiment') to take in order 'to be confident' that what "we would like" will occur?
(Non trivial decision issues always assume uncertainty about future outcomes.)
- Before tackling problems of decision we need to learn to reason about uncertainty, possibly in a quantitative way.

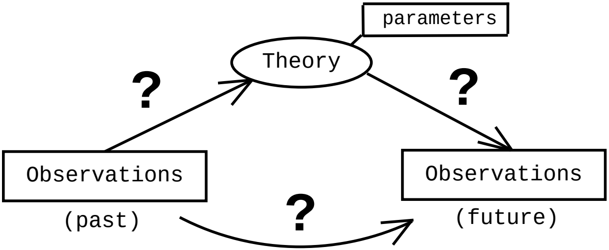
From past to future



Deep reason of uncertainty

Theory	— ? ⟶	Future observations
Past observations	— ? ⟶	Theory
Theory	— ? ⟶	Future observations

From past to future



Deep reason of uncertainty

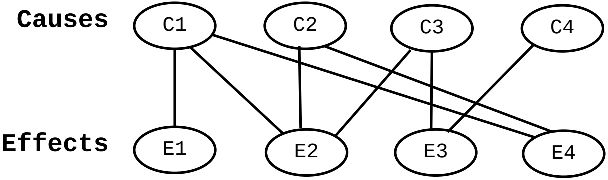
Theory	— ? ⟶	Future observations
Past observations	— ? ⟶	Theory
Theory	— ? ⟶	Future observations

$\Longrightarrow$ **Uncertainty about causal connections**

CAUSE $\Longleftrightarrow$ **EFFECT**

Causes → effects

The same *apparent* cause might produce several,different effects



Given an observed effect, we are not sure about the exact cause that has produced it.

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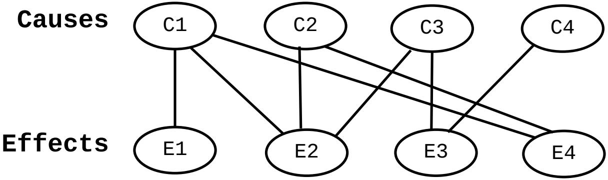

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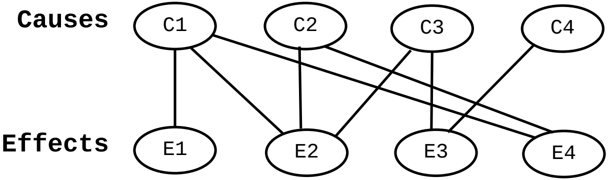

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$$\mathbf{E_2} \Rightarrow \{C_1,\ C_2,\ C_3\}?$$

The "essential problem" of the Sciences

"Now, these problems are classified as *probability of causes*, and are most interesting of all their scientific applications. I play at *écarté* with a gentleman whom I know to be perfectly honest. What is the chance that he turns up the king? It is $1/8$. This is a problem of the probability of effects.

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I play with a gentleman whom I do not know. He has dealt ten times, and he has turned the king up six times. What is the chance that he is a sharper? This is a problem in the probability of causes. It may be said that it is the essential problem of the experimental method.”

(H. Poincaré – *Science and Hypothesis*)

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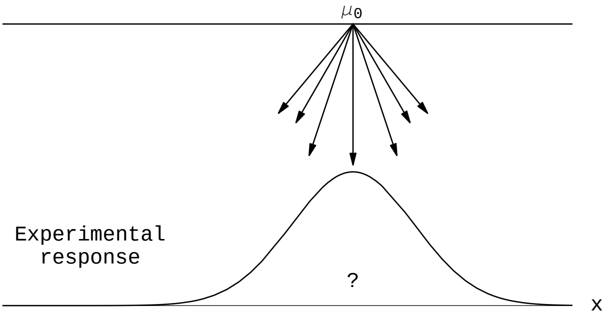
> (H. Poincaré – *Science and Hypothesis*)

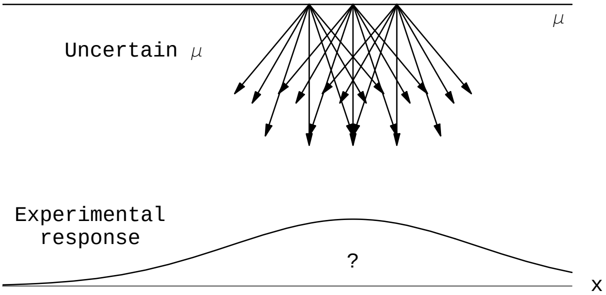
Why we (or most of us) have not been taught how to tackle this kind of problems?

From 'true value' to observations



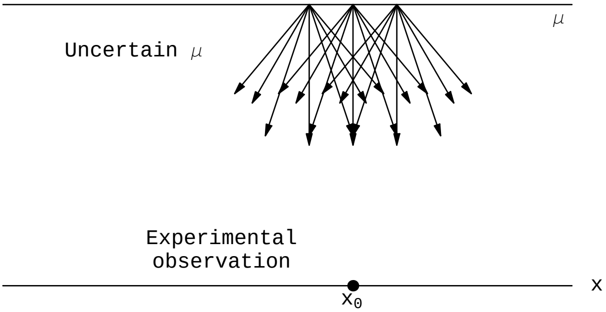
Given μ (exactly known) we are uncertain about x

From ‘true value’ to observations



Uncertainty about μ makes us more uncertain about x

. . . and back: Inferring a true value



The observed data is <u>certain</u>: $\rightarrow$ 'true value' uncertain.

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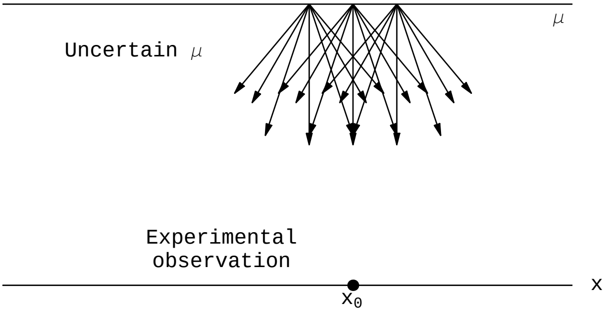

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"data uncertainty" ?

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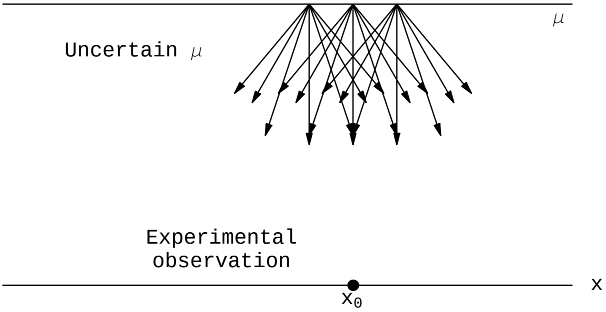

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"data uncertainty" ? Data corrupted?

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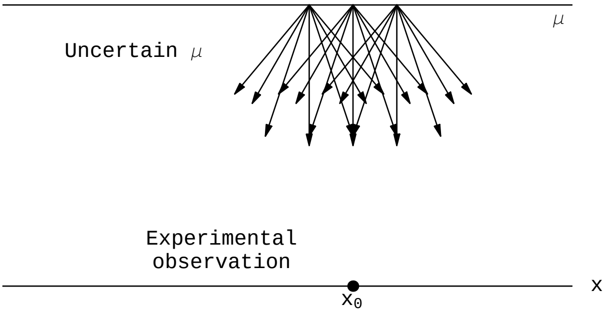
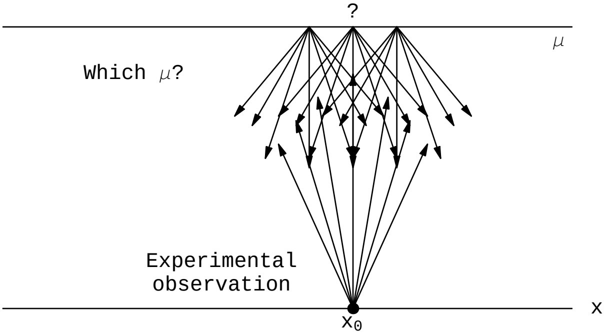


The observed data is <u>certain</u>: $\rightarrow$ 'true value' uncertain.

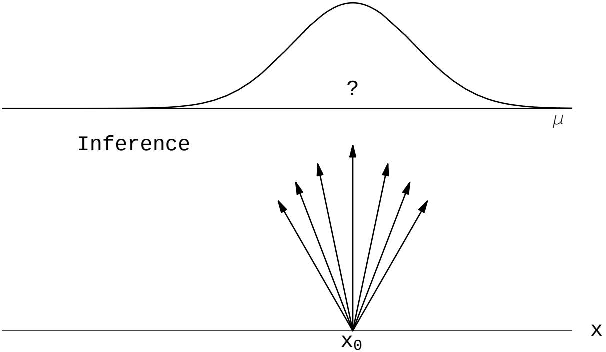
"data uncertainty" ? Data corrupted?
Even if the data were corrupted, the <u>data</u> were the corrupted data**!!** . . .

. . . and back: Inferring a true value



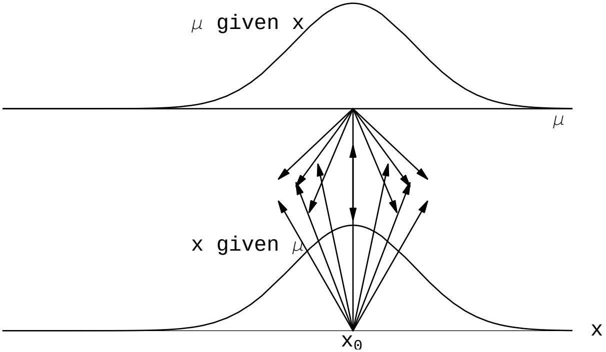
Where does the observed value of x comes from?

... and back: Inferring a true value



We are now uncertain about μ, given x.

. . . and back: Inferring a true value



Note the symmetry in reasoning.

A very simple experiment

Let's make an experiment

A very simple experiment

Let's make an experiment

- Here
- Now

A very simple experiment

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For simplicity

- μ can assume only six possibilities:

$$\mathbf{0}, \mathbf{1}, \ldots, \mathbf{5}$$

- x is binary:

$$\mathbf{0}, \mathbf{1}$$

$$[\,(1,2);\ \text{Black/White};\ \text{Yes/Not};\ \ldots\,]$$

A very simple experiment

Let's make an experiment

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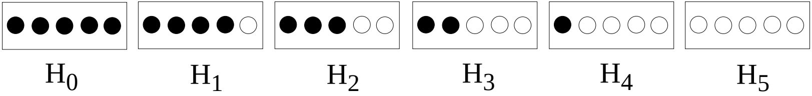
[(1, 2); Black/White; Yes/Not; ...]

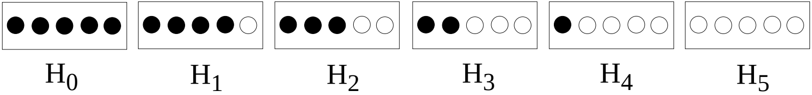
⇒ Later we shall make μ continuous.

Which box? Which ball?



Let us take at random one of the boxes.

Which box? Which ball?



Let us take at random one of the boxes.

We are in a state of uncertainty concerning several *events*, the most important of which correspond to the following questions:

(a) Which box have we chosen, H_0, H_1, ..., H_5?

(b) If we extract randomly a ball from the chosen box, will we observe a white ($E_W \equiv E_1$) or black ($E_B \equiv E_2$) ball?

Our certainties:

$$\begin{aligned} \cup_{j=0}^{5} H_j &= \Omega \\ \cup_{i=1}^{2} E_i &= \Omega\,. \end{aligned}$$

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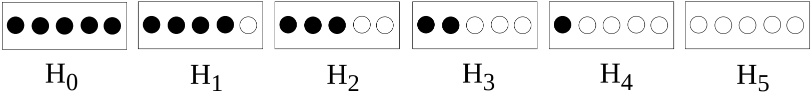

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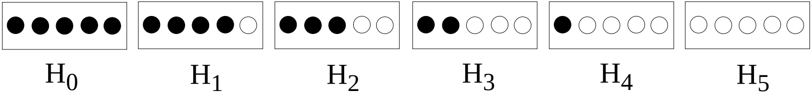

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(Ellsberg's paradox)

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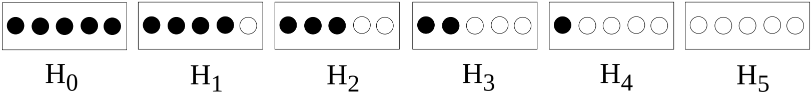
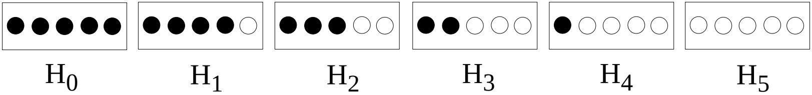


- What happens after we have extracted one ball and looked its color?
  - Intuitively feel *how to* roughly *change* our opinion about
    - the possible cause
    - a future observation

Which box? Which ball?



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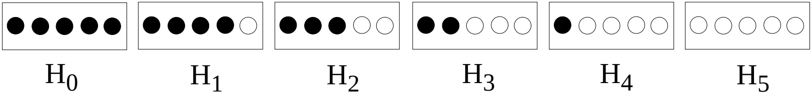

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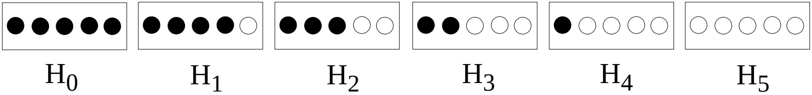

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- And after a sequence of extractions?
  - Imagine we observe W, W, W, W, ...

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[$^{(*)}$And senses (+ memory & 'information process') are notoriously fallacious!]

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Certainly <u>not</u> *in* the box!

Subjective nature of probability

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$\Rightarrow$ Probability is always conditional probability.

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$\Rightarrow$ How much we believe something

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$\rightarrow$ All 'ideas' our mind can conceive

Ideas, belief and probability

First deep analysis which goes to the roots of Human Understanding

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> *"If we were not ignorant there would be no probability, there could only be certainty. But our ignorance cannot be absolute, for then there would be no longer any probability at all. Thus the problems of probability may be classed according to the greater or less depth of our ignorance."*
>
> *(Poincaré)*

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⇒ Famous position of Laplace about intrinsic determinism of the world.

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There is certainly a probability, which arises from a superiority of chances on any side; and according as this superiority increases, and surpasses the opposite chances, the probability receives a proportionable increase, and begets still a higher degree of belief or assent to that side, in which we discover the superiority.

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Die with two kinds of marks → box of known composition of Black and White balls

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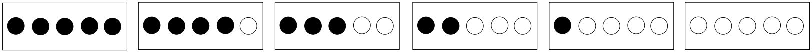

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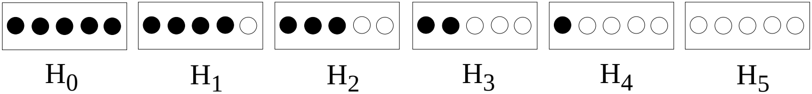

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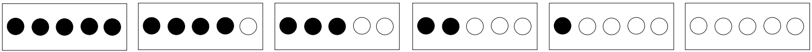

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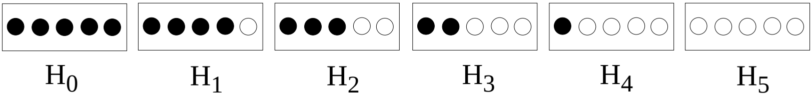

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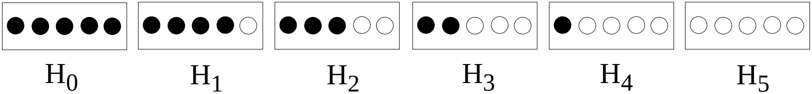

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(Note how this famous formula can be read as probabilities of probabilities!)

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- But we can model how p changes with time, and infer its value (with uncertainty) $\forall\ t$.

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  [For example the *infamous* L-word, “introduced by R. A. Fisher with the object of *avoiding* the use of Bayes’ theorem” (I.J. Good) is recognized as a major source of misunderstandings and of serious errors in physics and elsewhere and therefore in my opinion should be banned/restricted, together with LR’s.]
- Recent talk at MaxEnt 2016 in July, “Probability, propensity and probabilities of propensities”, although I do not like the word ‘propensity’ ... and Popper.
- Suggestions are welcome (‘bent’?), but what is important to use for ‘physical probability’ a name different for ‘probability’ (if we want to have a single noun to indicate it), reserving probability for ‘degree of belief’, as it has been historically (Cicero, Hume, Laplace, Gauss, ...) and in normal language.

[For the same reason I prefer “Bayes factor” (BF), or perhaps even “Bayes-Turing factor” (BTF), to LR.]

Laplace's "Bayes Theorem"

"The greater the probability of an observed event given any one of a number of causes to which that event may be attributed, the greater the likelihood of that cause {given that event}.

$$P(C_i \mid E) \propto P(E \mid C_i)$$

Laplace's "Bayes Theorem"

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$$P(C_i \mid E) = \frac{P(E \mid C_i)}{\sum_j P(E \mid C_j)}$$

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$$P(C_i \,|\, E) = \frac{P(E \,|\, C_i)\, P(C_i)}{\sum_j P(E \,|\, C_j)\, P(C_j)}$$

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$$P(C_i \mid E) = \frac{P(E \mid C_i)\, P(C_i)}{P(E)}$$

(*Philosophical Essai on Probabilities*)

Laplace's "Bayes Theorem"

$$P(C_i \mid E) \;=\; \frac{P(E \mid C_i)\, P(C_i)}{\sum_j P(E \mid C_j)\, P(C_j)}$$

"This is the fundamental principle $^{(*)}$ of that branch of the analysis of chance that consists of reasoning a posteriori from events to causes"

(*) In his "Philosophical essay" Laplace calls 'principles' the 'fundamental rules'.

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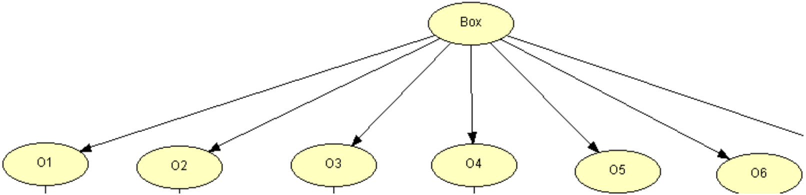
Most convenient way to remember Bayes theorem

Cause-effect representation

box content $\rightarrow$ observed color



$$P(B^{(1)} \,|\, H_j), \quad P(B^{(2)} \,|\, H_j), \,\ldots$$

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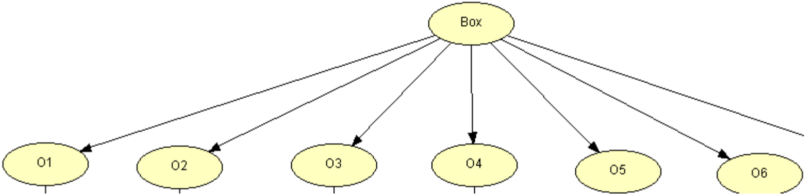
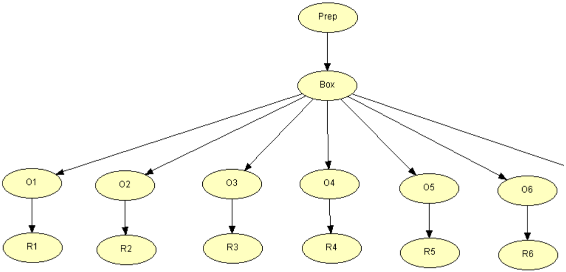


An effect might be the cause of another effect $\Longrightarrow$

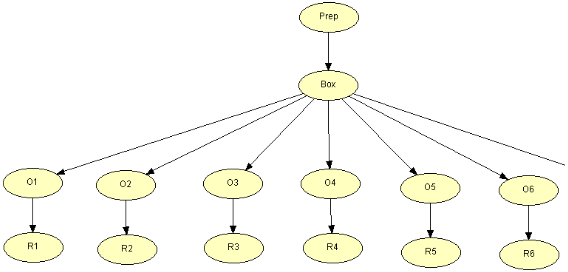
A network of causes and effects



A network of causes and effects

Preparation 'node' models prior knowledge about Box.

$\Rightarrow P(H_j \,|\, \text{Prep}_k)$



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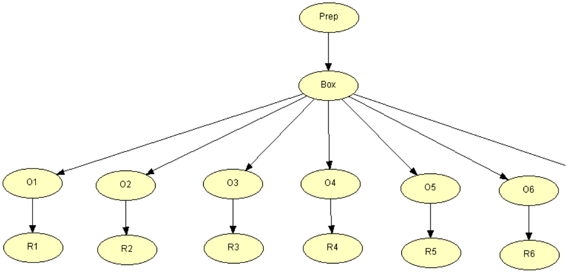

R_i model extra uncertainty in cascade.

$\Rightarrow P(W_R \,|\, W)$, $P(B_R \,|\, W)$, etc.

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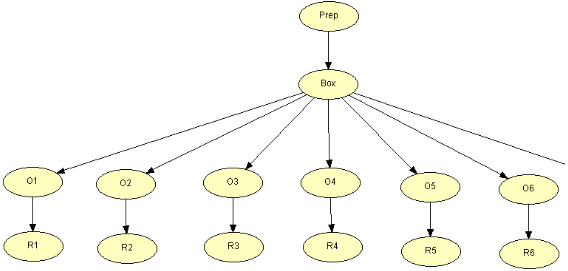

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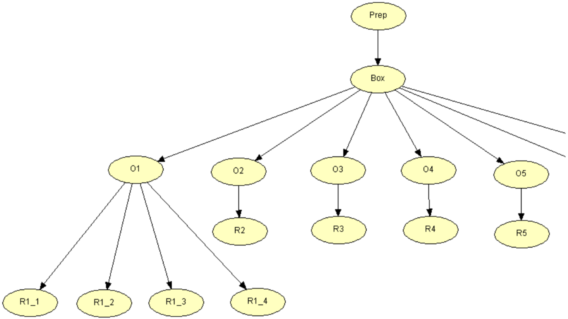
We shall also include multi-reporters and systematic effects

Multi-reporters

Multiple 'testimonies' of the same empirical fact.



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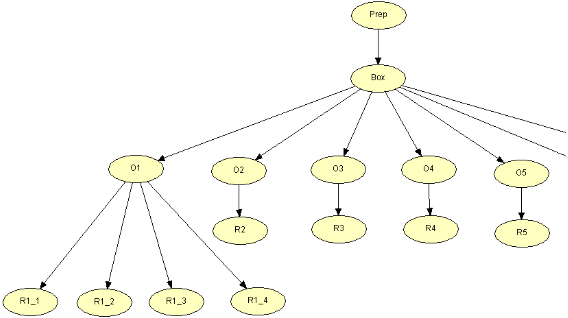
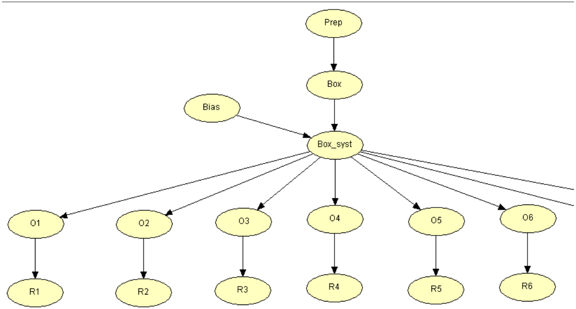


$\Rightarrow$ Our belief on O_1 being Black or White will depend on the consistencies of the ‘testimonies’

Systematic effects

The box content could be biased...



Systematic effects

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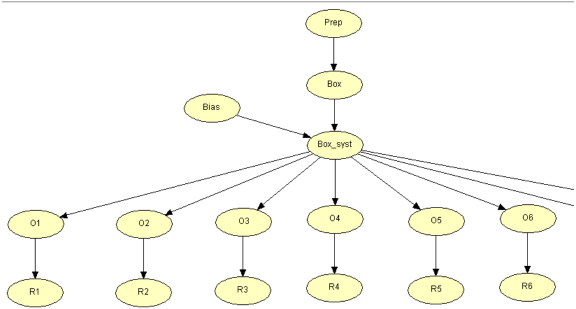

. . . if one or more balls of either color might be added to the original box content

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⇒ Nowadays, thanks to progresses in mathematics and computing, drawing the problem as a 'belief network' is more than 1/2 step towards its solution!

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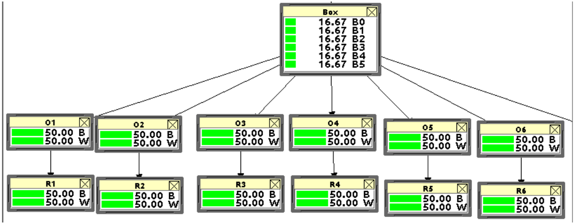
  - Many actors of the legal world might help to validate the general structure, or the local links
  - Although intuition can be misleading in propagation of probabilities.
  - Anyone can add 7 and 7 (and understand that perhaps the first 7 could also be 6; and the second 7 could be 6 or 8). But adding 35783 times 7 is an operation we delegate to a pocket calculator.
  - A similar role should have BN's in combining pieces of evidence, with professional support by experts.

Propagating the evidence in a simple BN

Let's play!

Six Boxes with reported evidence



For sake of simplicity symmetric probabilities of the *reported* color given the *outcome* of the extraction

$$
\begin{aligned}
P(R_i = W \mid O_i = W) &= 5/6 \approx 83\% \\
P(R_i = B \mid O_i = W) &= 1/6 \approx 17\%
\end{aligned}
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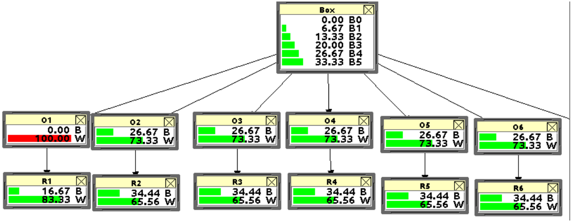
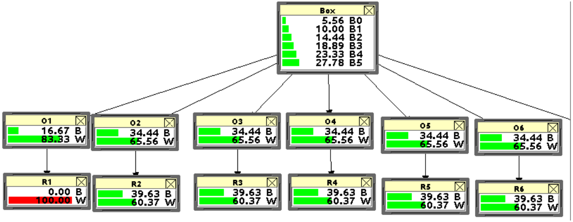
Six Boxes with reported evidence

Effect of the testimony: R_1



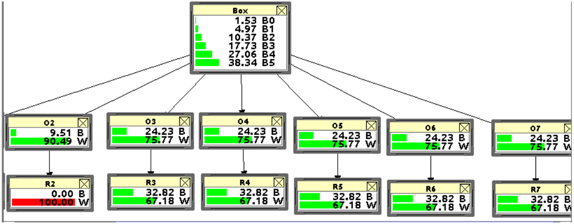


→ B_0 no longer *falsified*

→ We believe 5/6 (83.3%) that the ball was really white.

Six Boxes with reported evidence

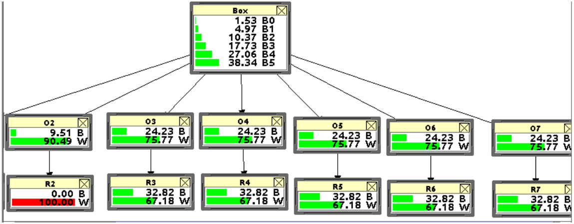
Effect of the testimony: R_1 followed by R_2



→ We believe more the testimony of the second report (90.5% Vs 83.3%)

Six Boxes with reported evidence

Effect of the testimony: R_1 followed by R_2



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???

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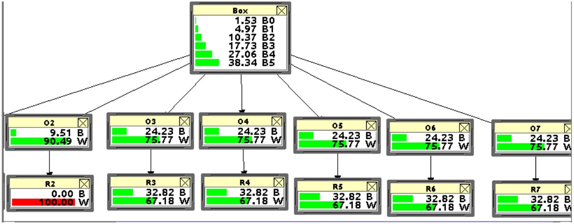


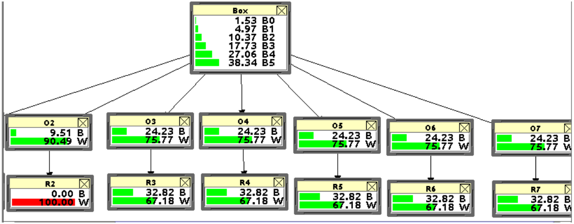
$\rightarrow$ We believe more the testimony of the second report (90.5% Vs 83.3%)

???

- From the previous slide we can see that indeed, after the *first* testimony, ourexpectation of White in the second extraction has increased to $\approx$ 66%, and this value acts as *prior* in the second inference.

Six Boxes with reported evidence

Effect of the testimony: R_1 followed by R_2



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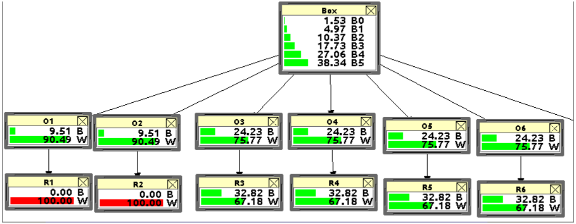
???

- From the previous slide we can see that indeed, after the *first* testimony, ourexpectation of White in the second extraction has increased to $\approx 66\%$, and this value acts as *prior* in the second inference.
- But how credible is *now* the hypothesis that the ball of the *first* extraction was really White?

Six Boxes with reported evidence

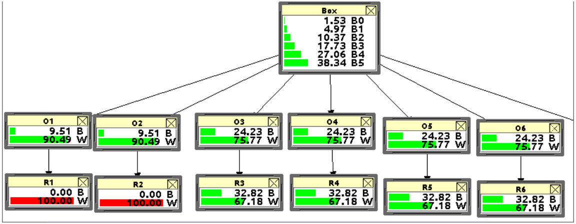
Effect of the testimony: R_1 followed by R_2



- Indeed we believe both at 90.5%!!

Six Boxes with reported evidence

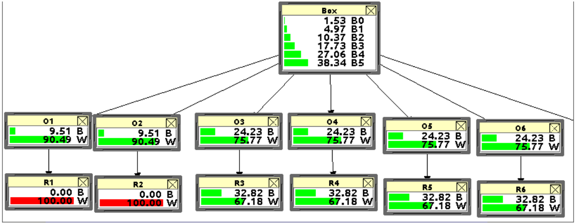
Effect of the testimony: R_1 followed by R_2



- Indeed we believe both at 90.5%!!
- Effect of mutual corroboration

Six Boxes with reported evidence

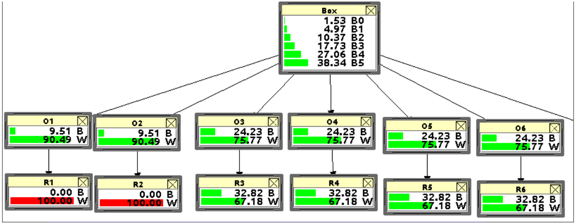
Effect of the testimony: R_1 followed by R_2



- Indeed we believe both at 90.5%!!
- Effect of mutual corroboration even if R_1 and R_2 are not reporting about the same extraction!

Six Boxes with reported evidence

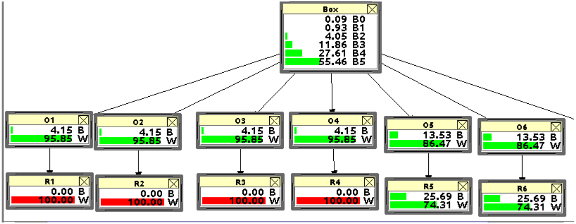
Effect of the testimony: R_1 followed by R_2



- Indeed we believe both at 90.5%!!
- Effect of mutual corroboration even if R_1 and R_2 are not reporting about the same extraction!
- But they are both indicating high probability of large number of white balls inside the same box.

Six Boxes with reported evidence

Effect of the testimony: R_1, R_2, R_3 and R_4 all reporting White



Corroboration effect continues.

Six Boxes with reported evidence

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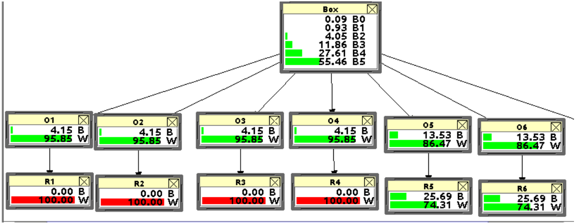
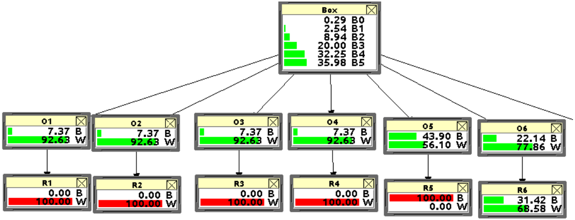


Corroboration effect continues. Then R_5 reports Black:

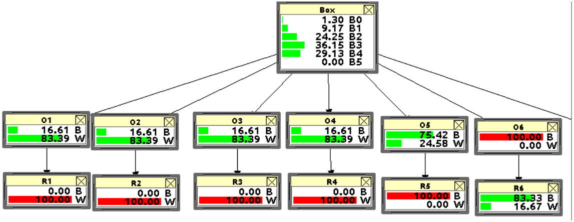


The poor R_5 is believed less than the others!

(And remember they are ‘talking’ about different outcomes.)

Six Boxes with reported evidence

Effect of the testimony: 4 reports followed by a certain evidence



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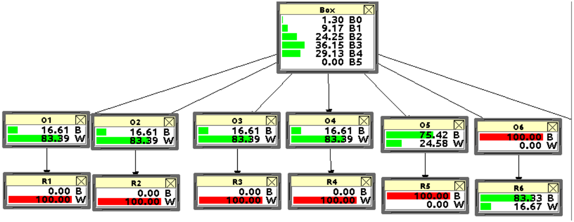

- Intuition fails (or at least it performs badly at quantitative levels).
- Formal guidance needed.

Conclusions

- Subjective probability recovers intuitive idea of probability.
- Nothing negative in the adjective 'subjective'. Just recognize, honestly, that probability depends on the status of knowledge, different from person to person.
- Most general concept of probability that can be applied to a large variety of cases.
- Bayesian networks are powerful conceptual/mathematical/ software tools to handle complex problems with variables related by ‘probabilistic’ links (not only ‘casual’ links).

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  "The celebrated Monsieur Leibnitz has observed it to be a defect in the common systems of logic, that they are very copious when they explain the operations of the understanding in the forming of demonstrations, but are too concise when they treat of probabilities, and those other measures of evidence on which life and action entirely depend, and which are our guides even in most of our philosophical speculations."

  (David Hume)

- The situation has not changed by much after three centuries!

More on the subject by the author

- *A defense of Columbo (and of the use of Bayesian inference in forensics): A multilevel introduction to probabilistic reasoning*, `http://arxiv.org/abs/1003.2086`
- *The Waves and the Sigmas (To Say Nothing of the 750 GeV Mirage)*, `http://arxiv.org/abs/1609.01668`
- *Bayesian reasoning in data analysis - A critical introduction*, World Scientific Publishing 2003 (soft cover 2013).
- *Così è... probabilmente. Il saggio, l'ingenuo e la signorina Bayes*, with Dino Esposito.
- *L'improbabile mondo del Mago di Odds*, with Gianluca Testa.

More on
`http://www.roma1.infn.it/~dagos/prob+stat.html` .