Linear combinations of uncertain numbers

A first exercise

Imagine we have measured the two sides of an A4 paper, obtaining

$$
\begin{aligned}
a &= 29.73 \pm 0.03\,\text{cm} \\
b &= 21.45 \pm 0.04\,\text{cm}\,.
\end{aligned}
$$

1. Evaluate (expected values, standard uncertainty and correlation)
   - their difference ($d = a - b$);
   - their sum ($s = a + b$);

   assuming $\rho(a, b) = 0$ or $\rho(a, b) = +0.8$.
2. Evaluate the same quantities by Monte Carlo simulation.
3. Repeat points 1. and 2. changing $\sigma(a)$: $0.03,\ 0.04,\ 0.05\,\text{cm}$.

Previous exercise reviewed

Exercise

- Repeat the exercise of the measurements on the A4 paper solving it in a compact form:
  - write the covariance matrix (with the two values of ρ);
  - write the transformation matrix C;
  - apply the formula $V_Y = C\,V_X\,C^T$.

Linearization

exercise: Extending the A4 paper example

Imagine we have measured the two sides of an A4 paper, obtaining

$$a = 29.73 \pm 0.03\,\text{cm}$$
$$b = 21.45 \pm 0.04\,\text{cm}\,.$$

Evaluate (expected values, standard uncertainty and correlation)

- perimeter, $p = 2\,a + 2\,b$;
- Area, $A = a\,b$;
- diagonal, $d = \sqrt{a^2 + b^2}$

assuming both $\rho(a,b) = 0$ and $\rho(a,b) = +0.8$.

- Write directly C and then use the matrix formalism.

Linearization of monomial expressions of independent variables

Exercise

Imagine we want to measure g with a pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

from which it follows

$$g = (2\pi)^2\, l\, T^{-2}$$

Q.: How precisely we have to measure l and T if we require they contribute equally to r_g, that we want to keep $\leq 1\,\%$?

Try...

Previous exercise reviewd

Exercise

- Repeat the exercise of the measurements on the A4 paper solving it in a compact form:
  - write the covariance matrix (with the two values of ρ);
  - write the transformation matrix C;
  - apply the formula $V_Y = C\,V_X\,C^T$.

Problems on multidimensional conditioning

Imagine the angles of a triangle have been measured, resulting in

$$\begin{aligned}\alpha &= 24.2 \pm 0.5\,\text{degrees}\\ \beta &= 65.3 \pm 0.6\,\text{degrees}\\ \gamma &= 89.1 \pm 0.8\,\text{degrees}\,.\end{aligned}$$

$\Rightarrow$ Condition on $\alpha+\beta+\gamma = 180$ degrees.

Old problem:

$$\begin{aligned}\mu_1 &= 10.1 \pm 0.5\,\text{u}\\ \mu_2 &= 5.0 \pm 1.0\,\text{u}\\ \rho(\mu_1,\mu_2) &= -0.80\,.\end{aligned}$$

1. recondition on $\mu_1+\mu_2 = 14.00\,\text{u}$;
2. recondition on $\mu_1/\mu_2 = 2.20$.

Inferring the "Bernoulli's p"

Approximate solution using the 'Gaussian trick'

Exercise

- Given $f(p) \propto p^x\,(1-p)^{n-x}$,
- define $\varphi(p) = -\ln f(p)$
- and evaluate
  - $\frac{d\varphi}{dp}$
  - $\frac{d^2\varphi}{dp^2}$
- Then estimate
  - $\mathrm{E}(p) \approx p_m$ from minimum;
  - $\sigma^2(p)$ from second derivative at the minimum.