A detail in Gauss' derivation

If $\begin{cases} \sum_i \psi(z_i) = 0 \\ \sum_i z_i = 0 \end{cases}$ then, $\forall\, \alpha, \beta \neq 0$, $\Rightarrow \begin{cases} \alpha \sum_i \psi(z_i) = 0 \\ \beta \sum_i z_i = 0 \end{cases}$

Then (making the sum would be the same, or even better, because later it would imply $k/2 = h^2$),

$$\alpha \sum_i \psi(z_i) - \beta \sum_i z_i \;=\; 0$$

$$\sum_i \left[\alpha\, \psi(z_i) - \beta\, z_i\right] \;=\; 0\,.$$

Since this relation must hold independently of n and the values of z_i, then each term in the sum must vanishy, i.e.

$$\alpha\, \psi(z_i) - \beta\, z_i \;=\; 0\,,$$

or, $\forall\, z$,

$$\psi(z) \;=\; \frac{\beta}{\alpha}\, z = k\, z$$