Introducing

Bayesian Reasoning in Measurements

with a Toy Experiment

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"Probability is good sense reduced to a calculus" (S. Laplace)

"All models are wrong but some are useful" (G. Box)

Outline

- “Science and hypothesis” (Poincaré)
- Uncertainty, probability, decision.
- Causes $\longleftrightarrow$ Effects

  “The essential problem of the experimental method” (Poincaré).
- A toy model and its physics analogy: the six box game

  “Probability is either referred to real cases or it is nothing” (de Finetti).
- Probabilistic approach [but . . . What is probability?]
- Basic rules of probability and Bayes rule.
- Bayesian inference and its graphical representation:
  $\Rightarrow$ Bayesian networks
- From ball and boxes to real measurements
- Conclusions

What is measurement?

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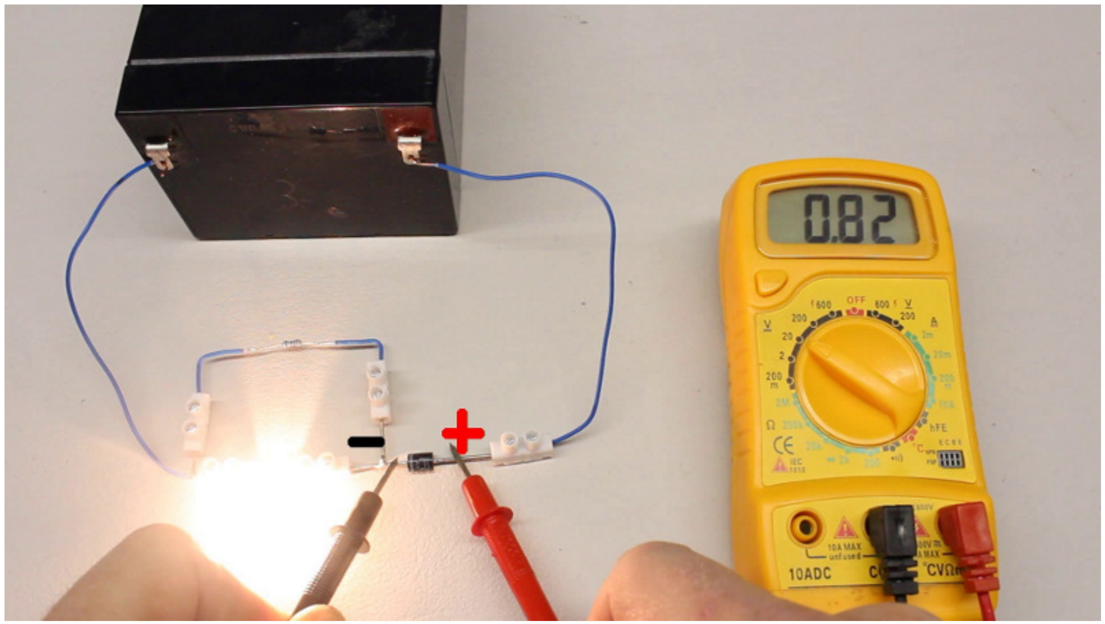

What is measurement?

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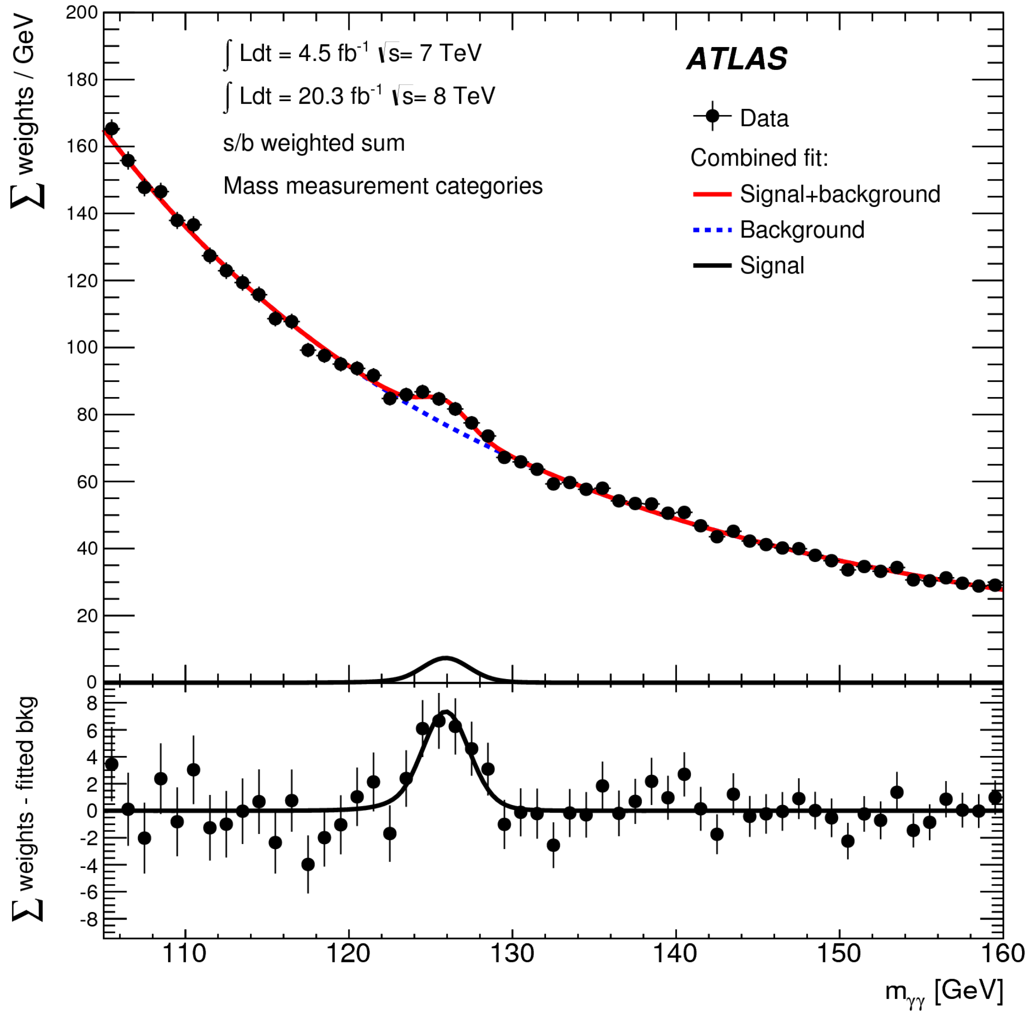
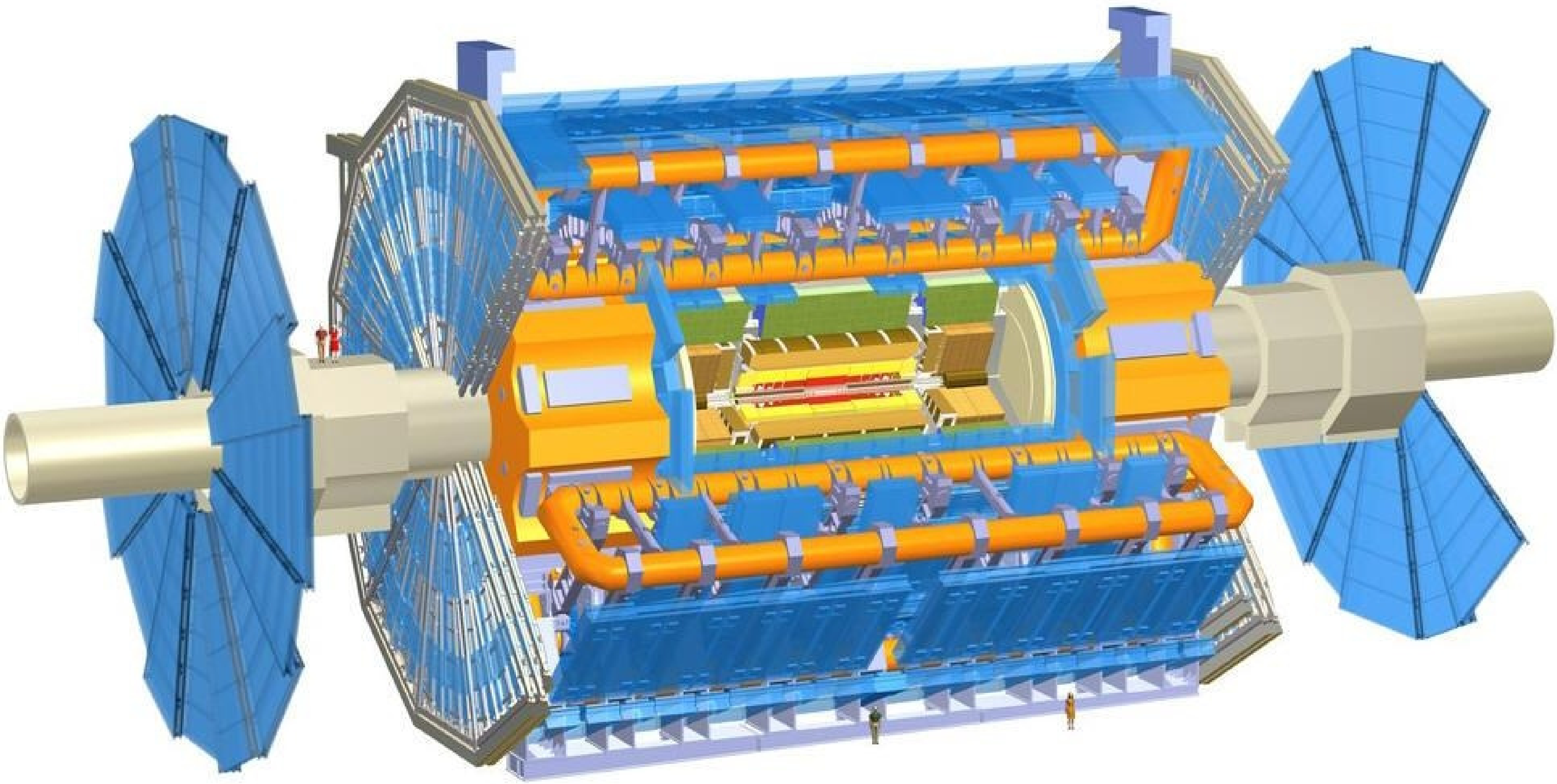
Higgs $\to \gamma\gamma$



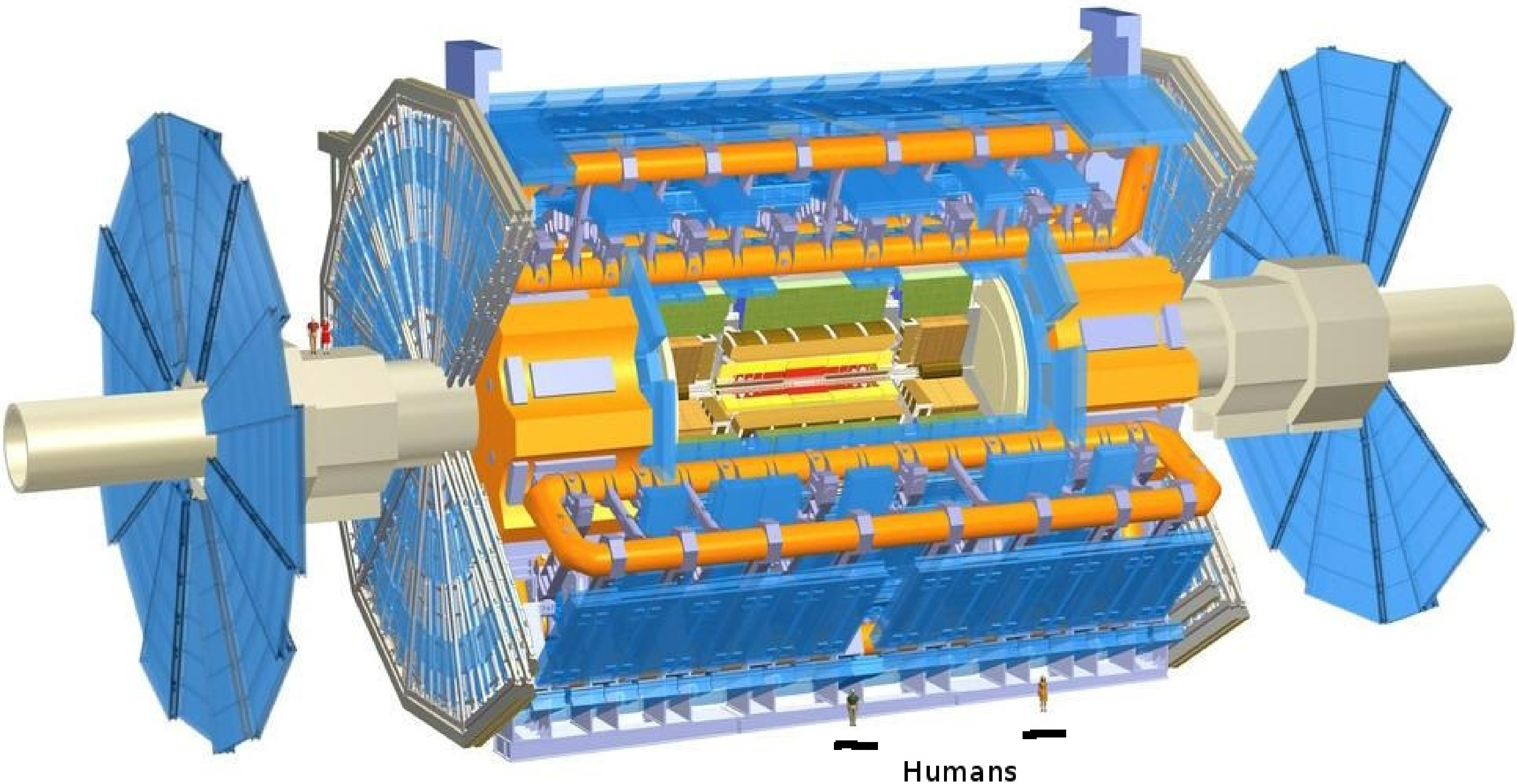
What is measurement?

ATLAS Experiment at LHC



What is measurement?

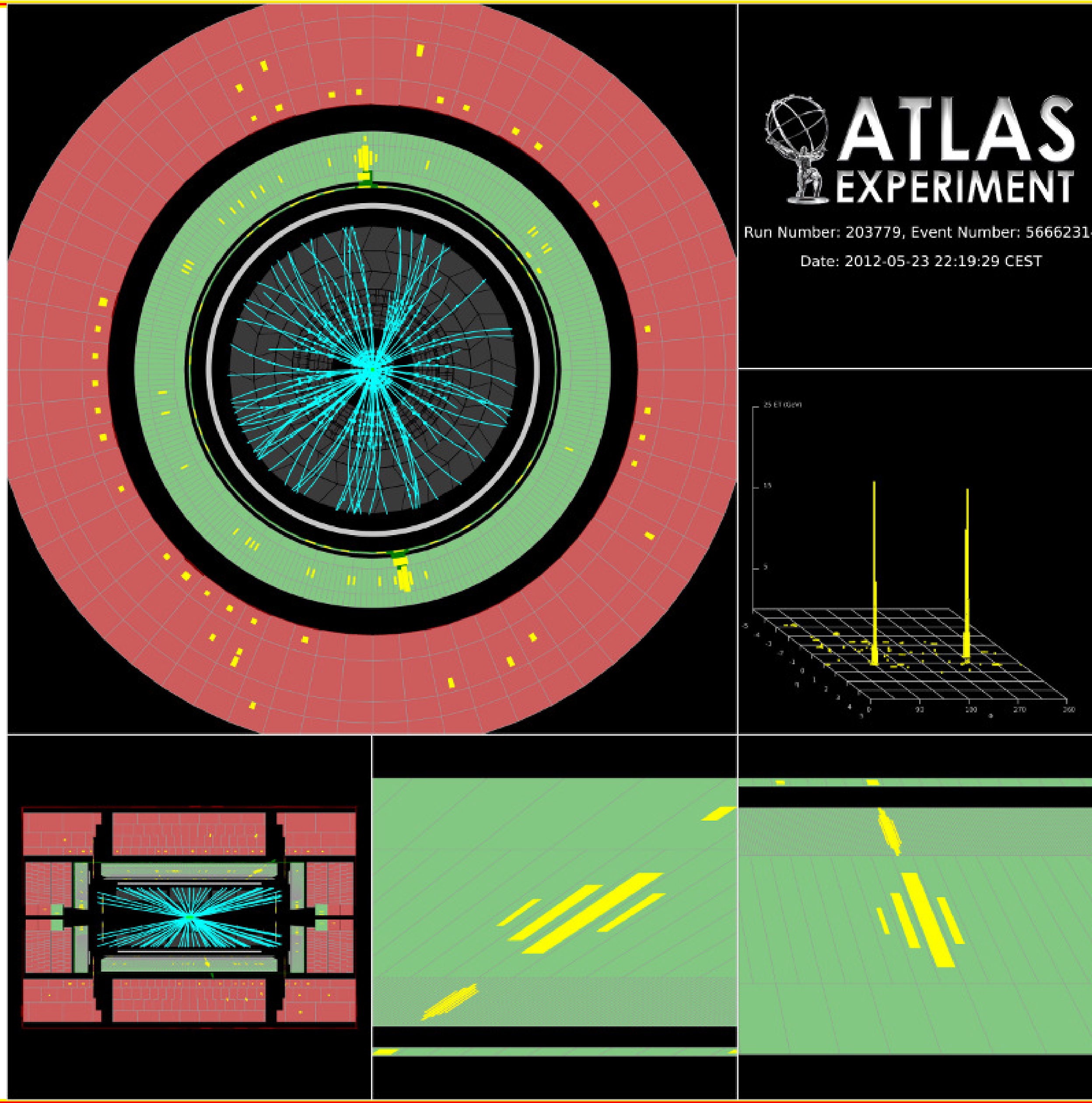
ATLAS Experiment at LHC [length: 46 m; $\varnothing$ 25 m]



≈ 3000 km cables

≈ 7000 tonnes ≈ 100 millions electronic channels

What is measurement?



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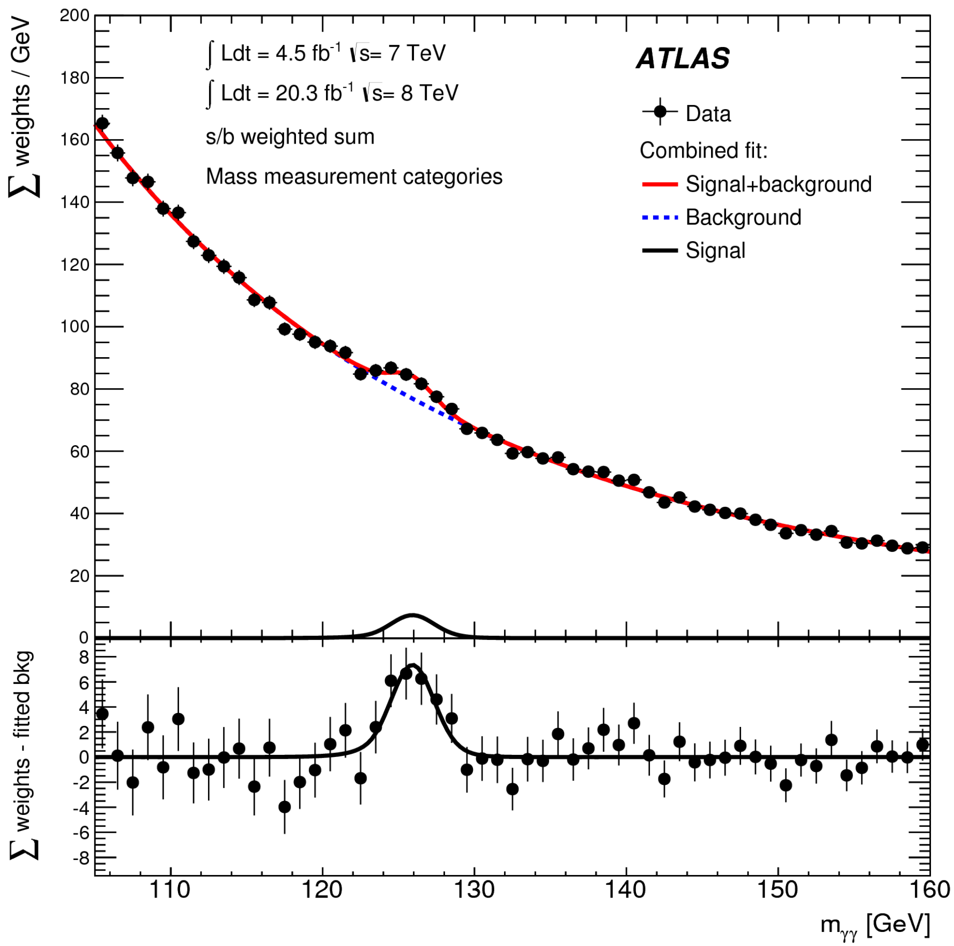

$\Rightarrow \left\{ \begin{array}{l} \text{Mass} \\ \text{Production rate} \end{array} \right.$

What is measurement?

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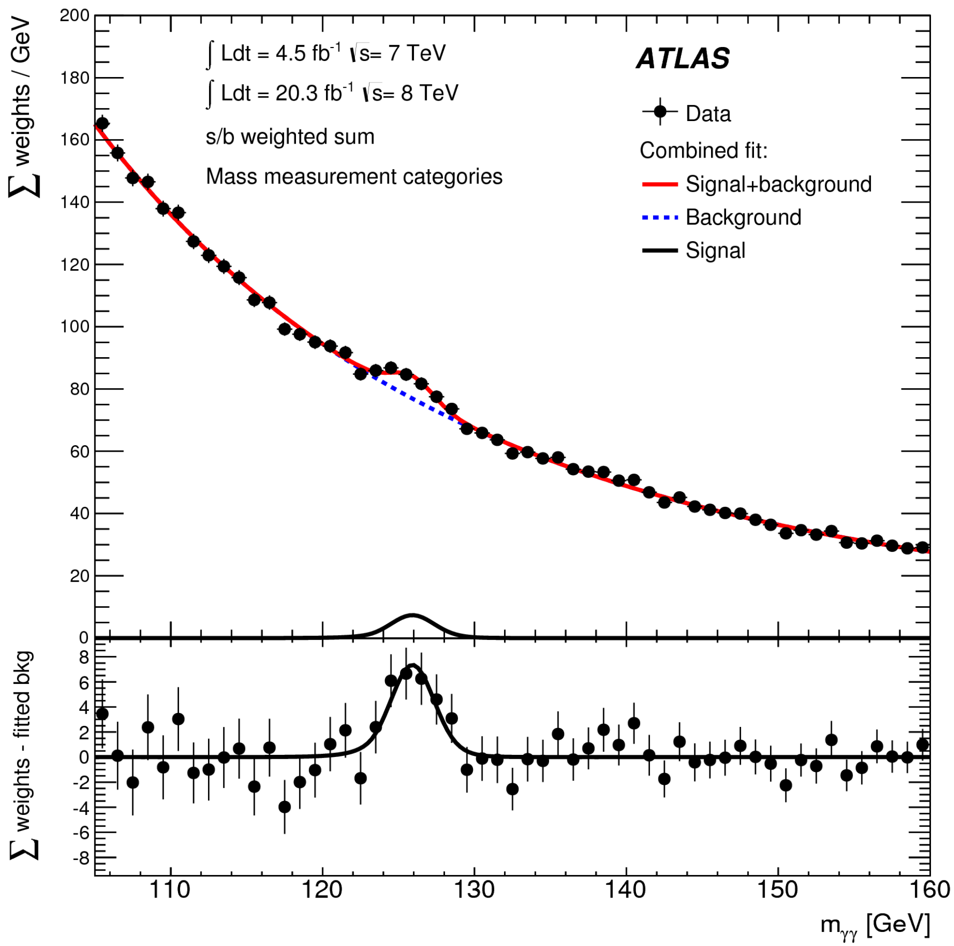


$\Rightarrow$ $\left\{\begin{array}{l}\text{Mass}\\ \text{Production rate}\end{array}\right.$

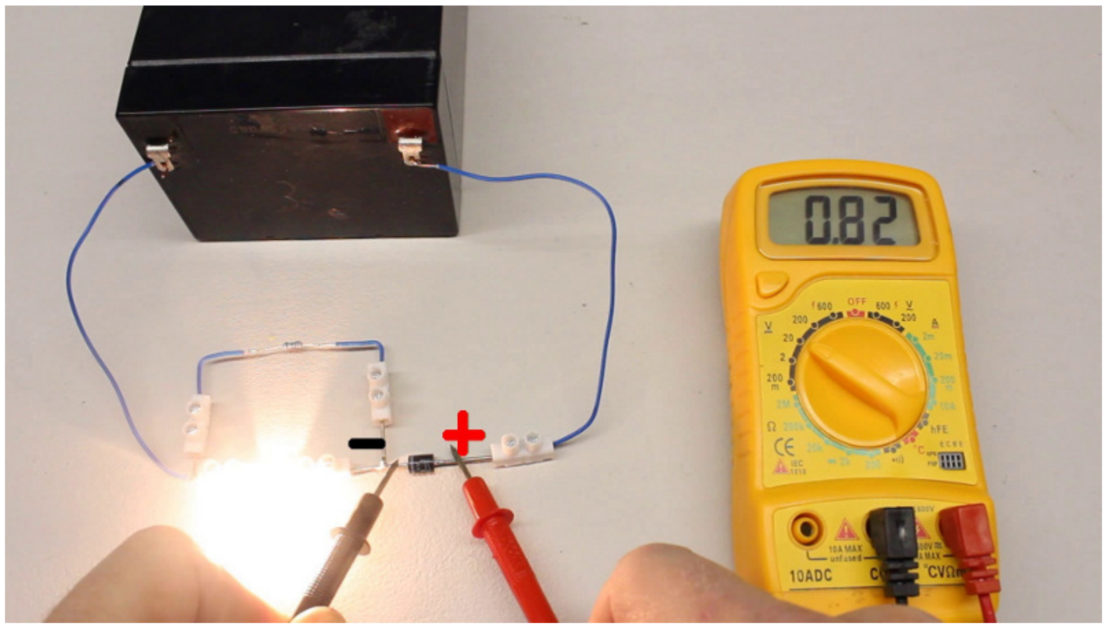
Quite indirect measurements of something we do not “see”!

Can we “see” physics quantities?

But, can we <u>see</u> our mass?

Can we “see” physics quantities?

. . . or a voltage?



Can we "see" physics quantities?

. . . or our blood pressure?

Can we "see" physics quantities?

Certainly not!

Can we “see” physics quantities?

Certainly not!

. . . although for some quantities we can have

a ‘vivid impression’ (in the David Hume’s sense)

Measuring a mass on a balance

Equilibrium:

$$\begin{aligned} {\color{red}m}g - k{\color{blue}\Delta x} &= 0 \\ {\color{blue}\Delta x} &\rightarrow {\color{blue}\theta} \rightarrow \text{{\color{blue}scale reading}} \end{aligned}$$

From the reading to the value of the mass:

$$\text{{\color{blue}scale reading}} \quad \xrightarrow[\textit{given } g,\, k,\, \text{“\textit{etc.}”}\ldots]{\hspace{5cm}} \quad {\color{red}m}$$

Measuring a mass on a balance

$$\text{scale reading} \xrightarrow[\textit{given } g,\, k,\, \text{“etc.”}\ldots]{} m$$

Dependence on ‘g’:

$$g \stackrel{?}{=} \frac{G M_{\oplus}}{R_{\oplus}^2}$$

Measuring a mass on a balance

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- Position is usually <u>not</u> at "$R_{\oplus}$" from the Earth center;
- Earth not spherical…
- …not even ellipsoidal…
- …and not even homogenous.
- Moreover we have to consider centrifugal effects
- …and even the effect from the Moon

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Certainly not to watch our weight ☺

But think about it!

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scale reading $\xrightarrow[\text{given } g,\ k,\ \text{“etc.”}\ldots]{} m$

Dependence on 'k':

- temperature
- non linearity
- ...

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$\mathbf{\Delta x} \rightarrow \theta \rightarrow$ **scale reading:**

- left to your imagination...

Measuring a mass on a balance

scale reading $\underset{\text{given } g,\ k,\ \text{“etc.”}\ldots}{\xrightarrow{\hspace{6em}}}$ m

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$+$ **randomic effects:**

- stopping position of damped oscillation;
- variability of all quantities of influence (in the ISO-GUM sense);
- reading of analog scale.

Measuring a mass on a balance

scale reading $\xrightarrow[\text{given } g,\, k,\, \text{"etc."}\ldots]{} m$

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$\Delta\mathbf{x} \to \theta \to$ **scale reading:**

- left to your imagination...

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$\Rightarrow \ m??$

Sources of uncertainties (from ISO GUM)

1 *incomplete definition of the measurand;*[†]

$\rightarrow$ g

$\rightarrow$where?

$\rightarrow$inertial effects subtracted?

2 *imperfect realization of the definition of the measurand;*

$\rightarrow$ scattering on neutron

$\rightarrow$how to realize a neutron target?

3 *non-representative sampling — the sample measured may not represent the measurand;*

4 *inadequate knowledge of the effects of environmental conditions on the measurement, or imperfect measurement of environmental conditions;*

5 *personal bias in reading analogue instruments;*

Sources of uncertainties (from ISO GUM)

6 *finite instrument resolution or discrimination threshold;*

7 *inexact values of measurement standards and reference materials;*

8 *inexact values of constants and other parameters obtained from external sources and used in the data-reduction algorithm;*

9 *approximations and assumptions incorporated in the measurement method and procedure;*

10 *variations in repeated observations of the measurand under apparently identical conditions.*

$\longrightarrow$ *"statistical errors"*

Note

- Sources not necessarily independent
- In particular, sources 1-9 may contribute to 10 (e.g. not-monitored electric fluctuations)

Pure empirical information?

A number, outside a contest, and denuted of all information the physicist or engineer has about its ‘production’ provides little (or zero) information: is not a measurement.

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mistrust the dogma of the dogma
Immaculate Observation!

Comparing hypotheses

We do measurements not only to ‘estimate’ the numeric value of a quantity.

Experimental observations are also used in order to

- “check hypotheses”
  (a generic expression that needs clarification. . .)

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Diagnostics, reliability, etc.

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Diagnostics, reliability, etc.

Diagnostics concerning health helps to clarify the issues $\Rightarrow$

AIDS test

An Italian citizen is selected at random
to undergo an AIDS test.
$\rightarrow$ Performance of clinical trial is not perfect, as customary:

$$
\begin{aligned}
P(\text{Pos}\,|\,\text{HIV}) &= 100\% \\
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\end{aligned}
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H_1='HIV' (Infected) $\qquad E_1$ = Positive

H_2='$\overline{\text{HIV}}$' (Healthy) $\qquad E_2$ = Negative

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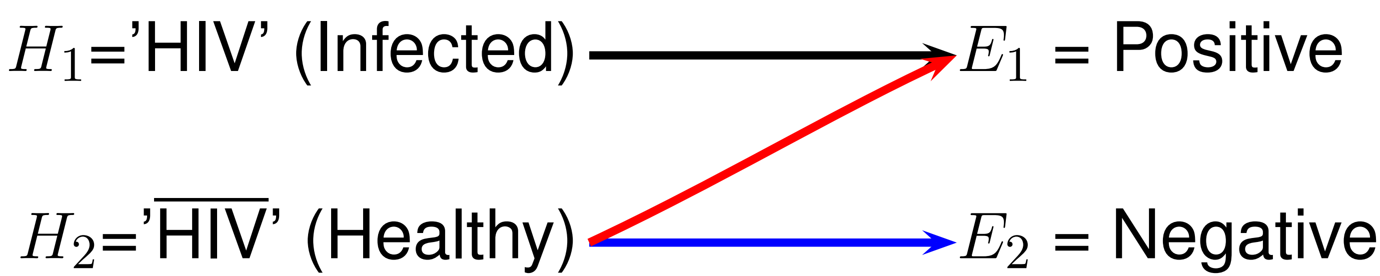

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Result: $\Rightarrow$ <u>Positive</u>

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Result: $\Rightarrow$ <u>Positive</u>

Infected or healthy?

AIDS test: how to interpret the result?

Being $P(\text{Pos} \,|\, \overline{\text{HIV}}) = 0.2\%$ and having observed 'Positive', can we say?

- "It is practically impossible that the person is healthy, since it was practically impossible that an healthy person would result positive"

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- ~~"The hypothesis H_1=Healthy is ruled out with 99.8% C.L."~~

NO

Instead, $P(\overline{\text{HIV}}\,|\,\text{Pos, random Italian}) \approx 45\%$

(We will learn in the sequel how to evaluate it correctly)

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NO

Instead, $P(\overline{\text{HIV}}\,|\,\text{Pos, random Italian}) \approx 45\%$

$\Rightarrow$ Serious mistake! (not just 99.8% instead of 98.3% or so)

AIDS test

???

Where is the problem?

AIDS test

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AIDS test

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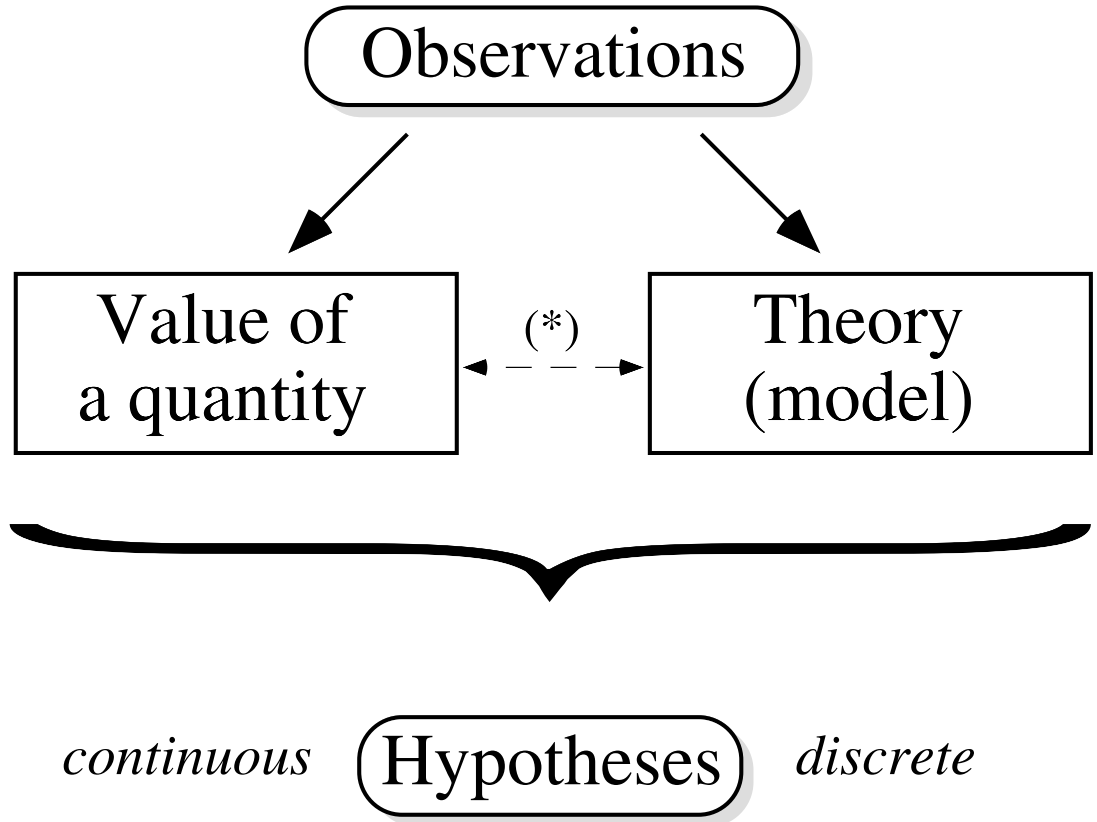
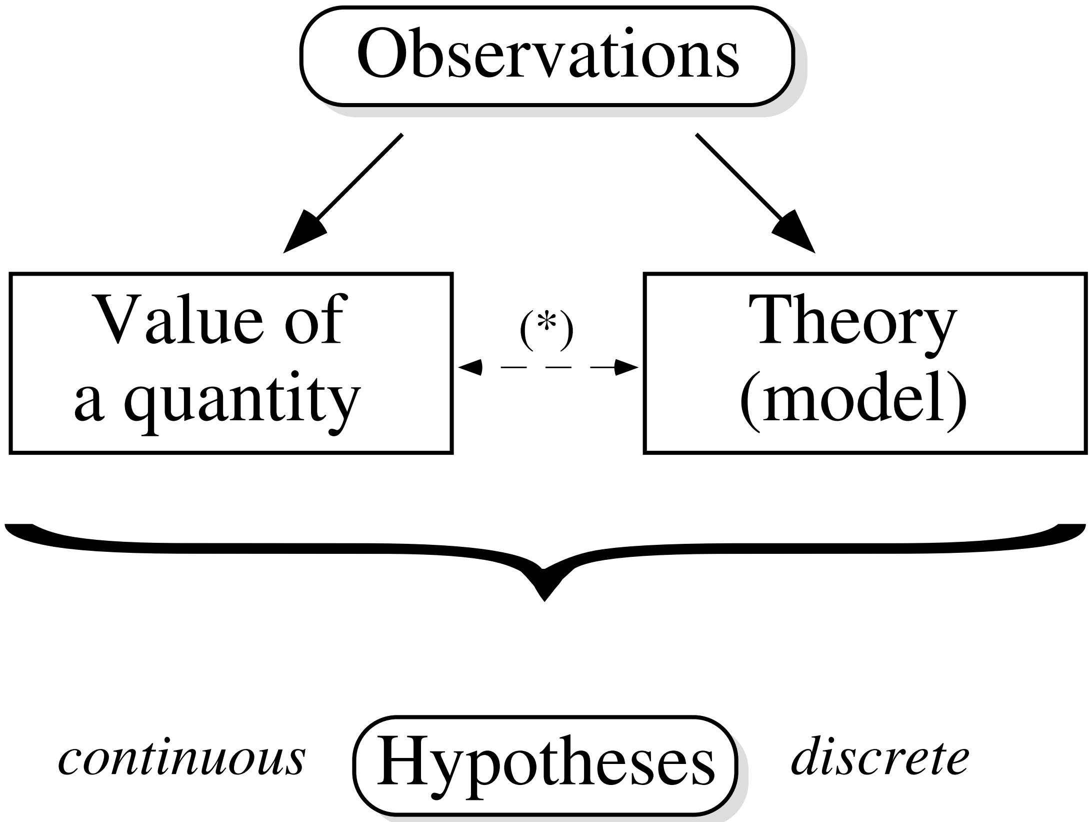
... and in these issues intuition can be fallacious!

$\Rightarrow$ A sound formal guidance can rescue us

Learning from data

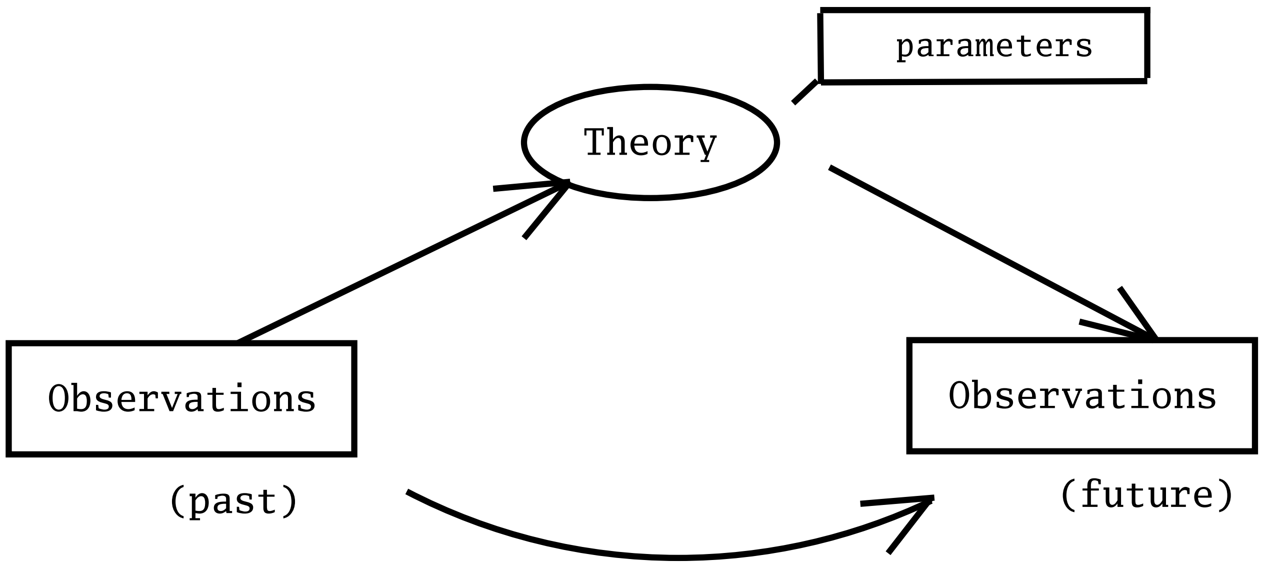


Learning from data



(*) A quantity might be meaningful only within a theory/model

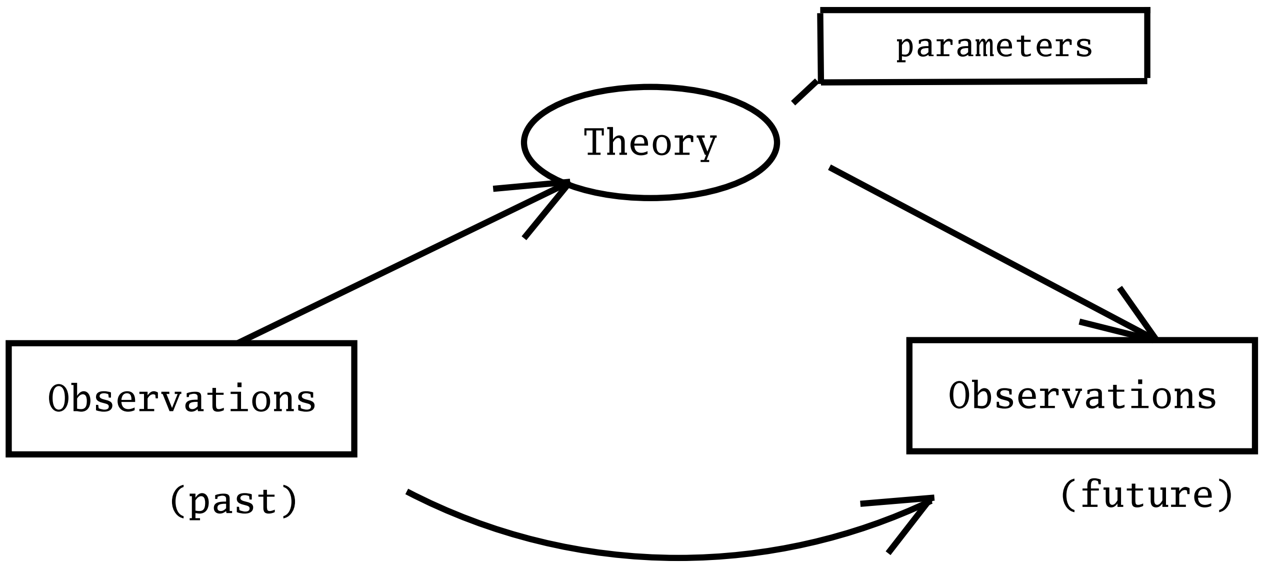
From past to future



Our task:

- Describe/understand the physical world
  $\Rightarrow$ inference of laws and their parameters
- Predict observations
  $\Rightarrow$ forecasting

From past to future



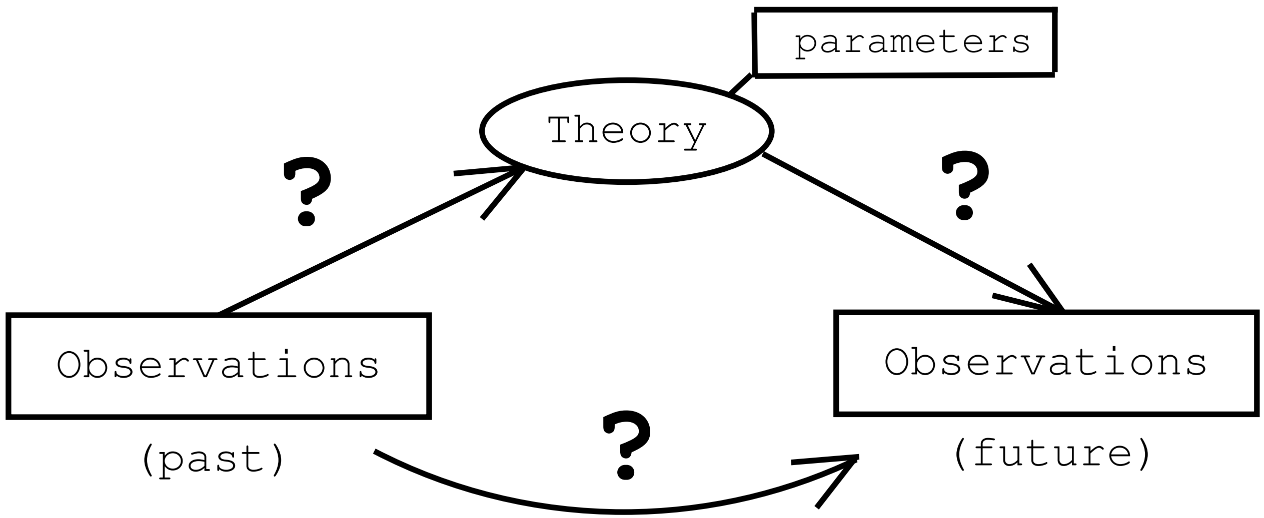
Process

- neither automatic
- nor purely contemplative

  $\rightarrow$ 'scientific method'

  $\rightarrow$ planned experiments ('actions') $\Rightarrow$ decision.

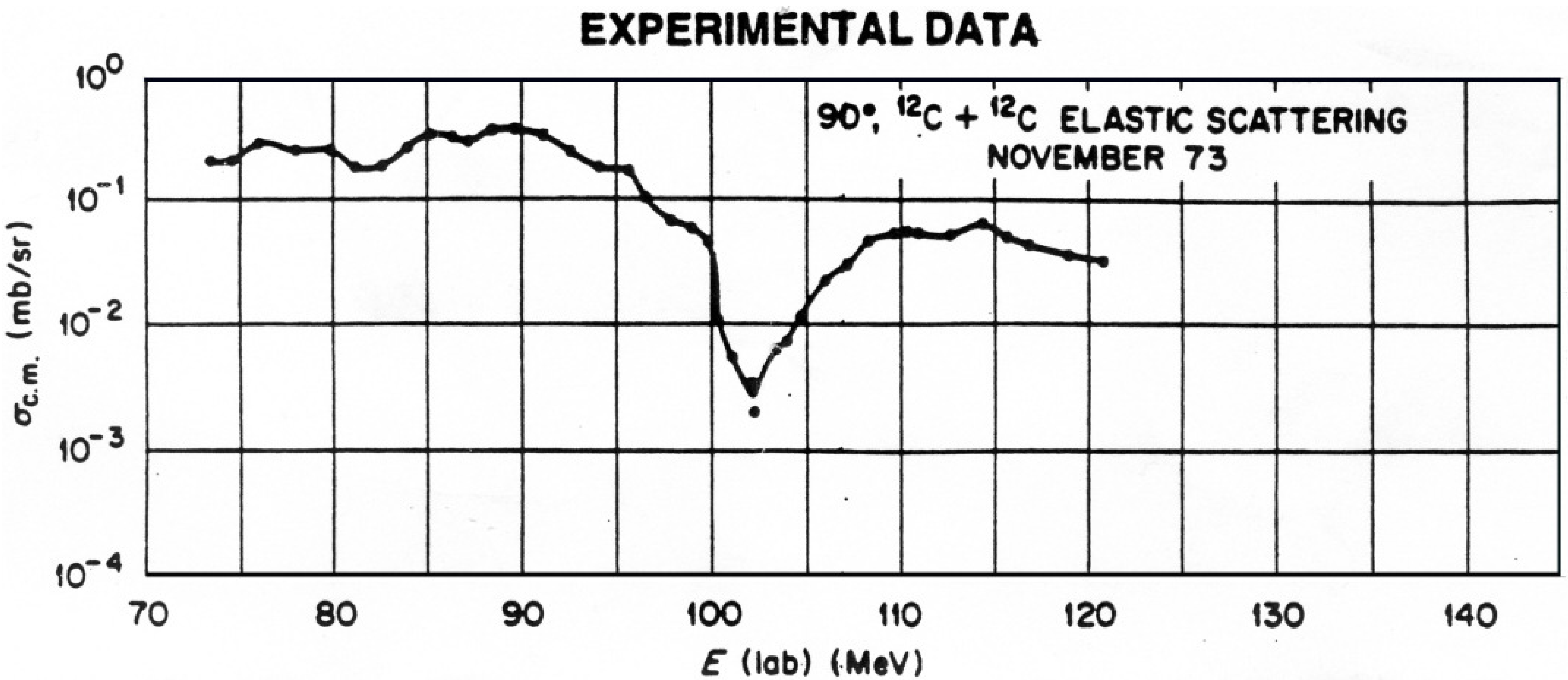
From past to future



⇒ Uncertainty:

1. Given the past observations, in general we are not sure about the theory parameters (and/or the theory itself)
2. Even if we were sure about theory and parameters, there could be internal (e.g. Q.M.) or external effects (initial/boundary conditions, ‘errors’, etc) that make the forecasting uncertain.

Inferential-predictive process



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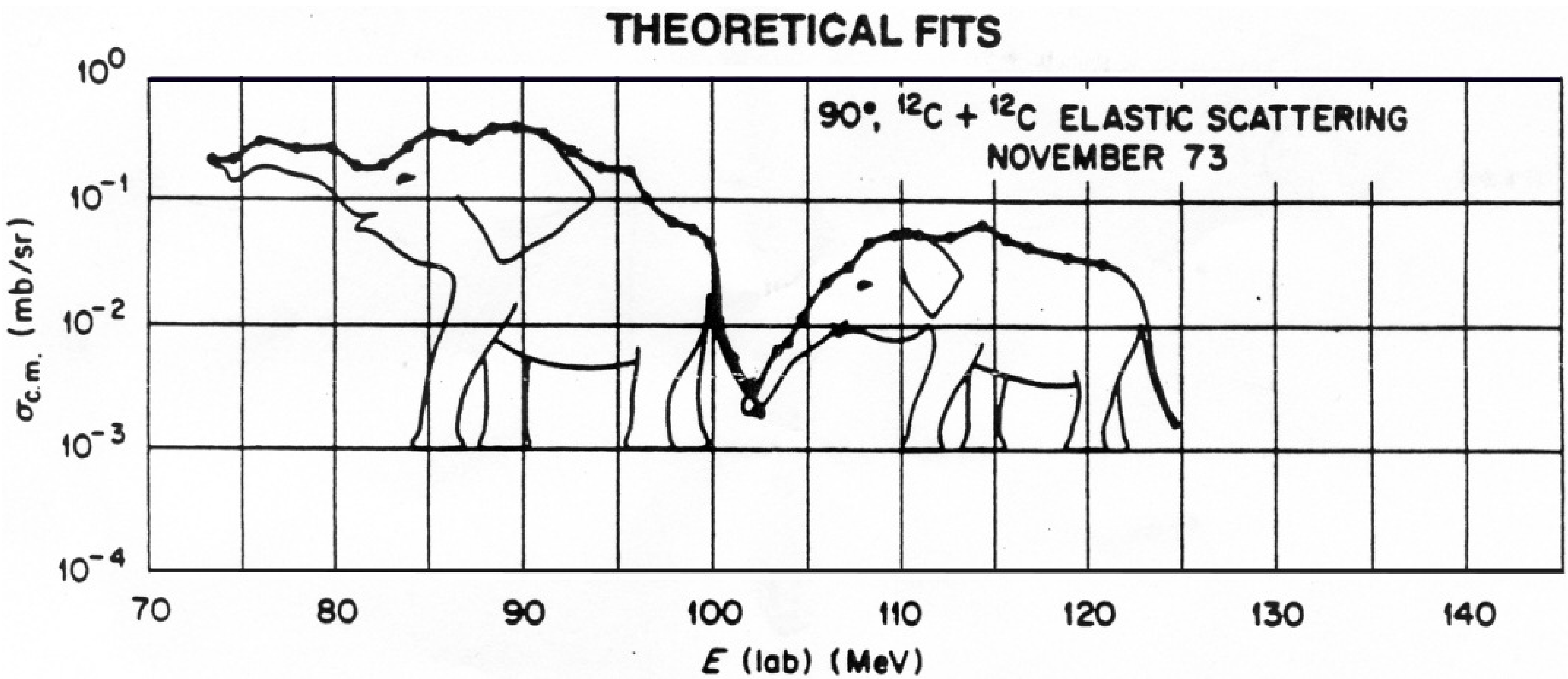

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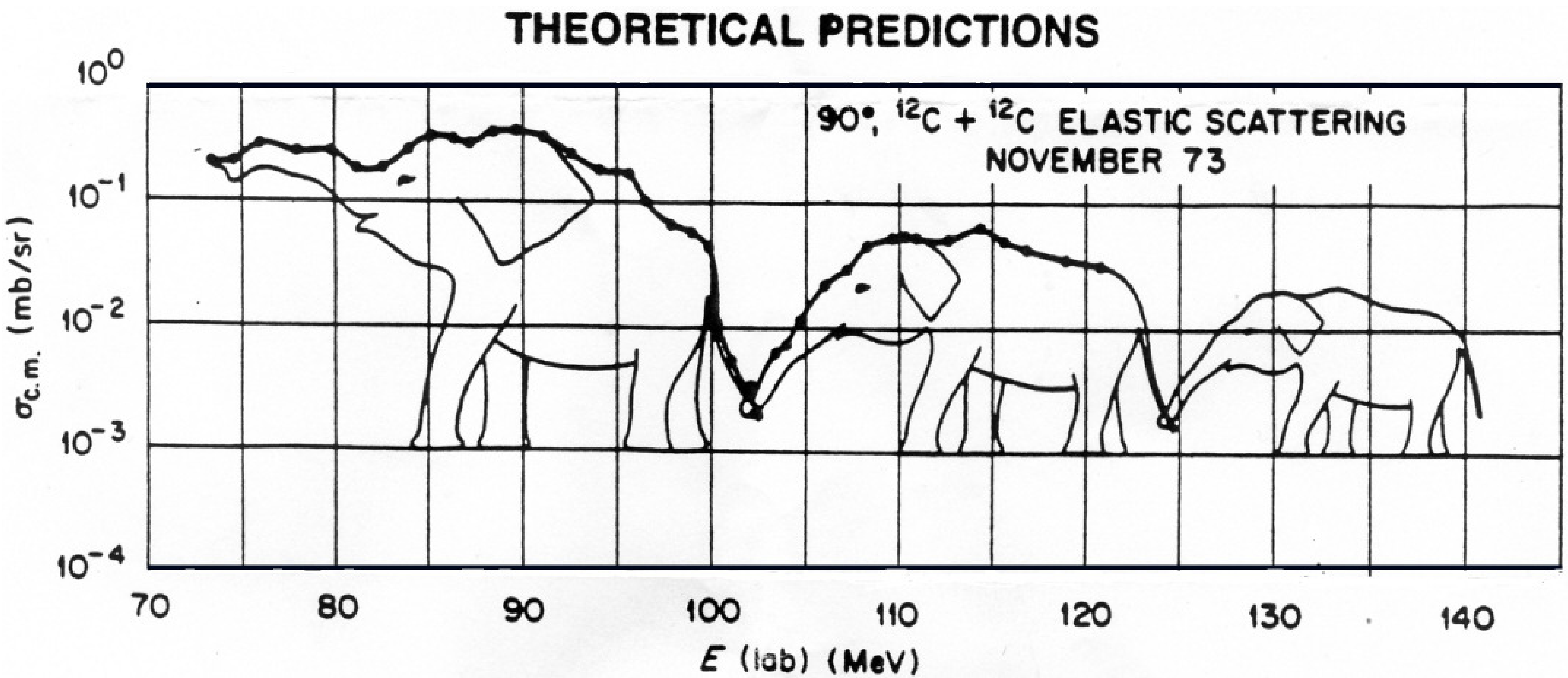
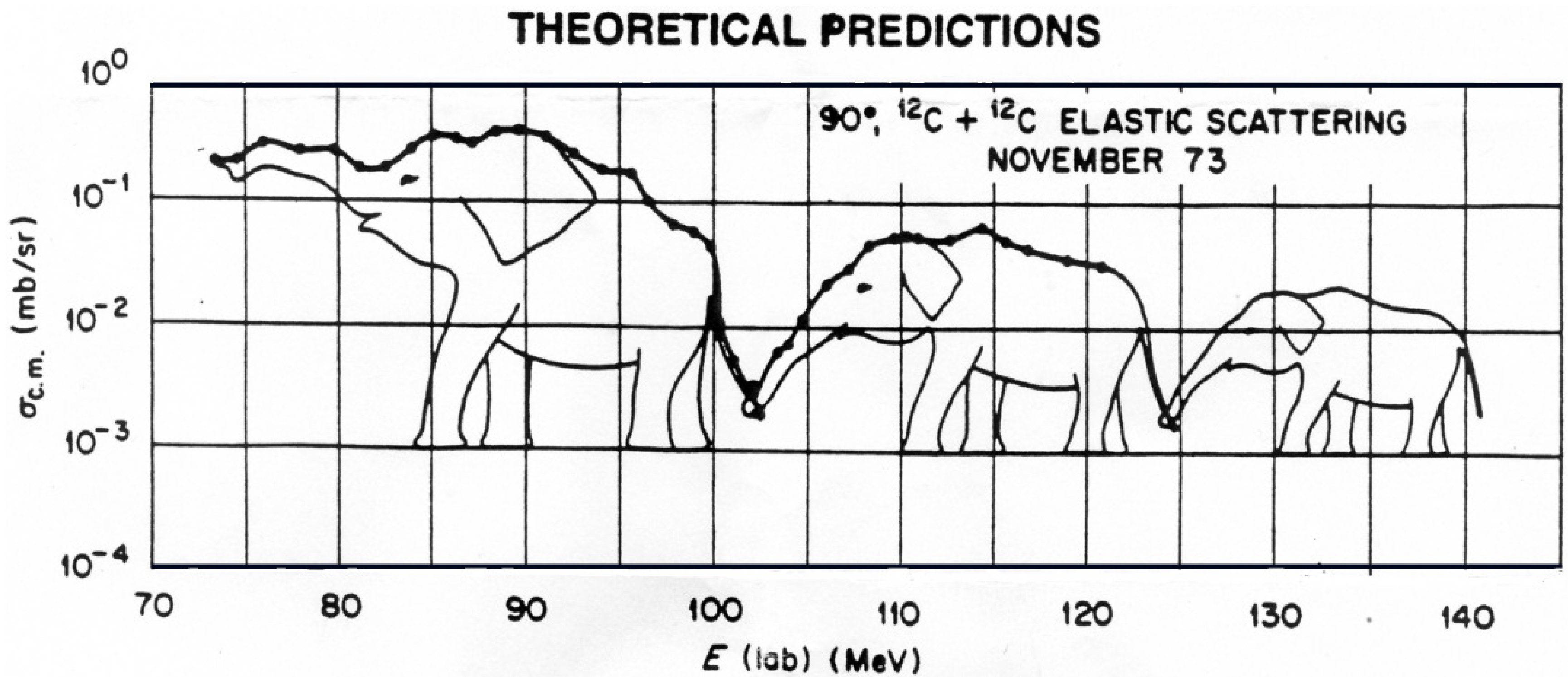


(S. Raman, *Science with a smile*)

Inferential-predictive process



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Even if the (*ad hoc*) model fits perfectly the data,
we do not believe the predictions
because we don't trust the model!

[Many 'good' models are *ad hoc* models!]

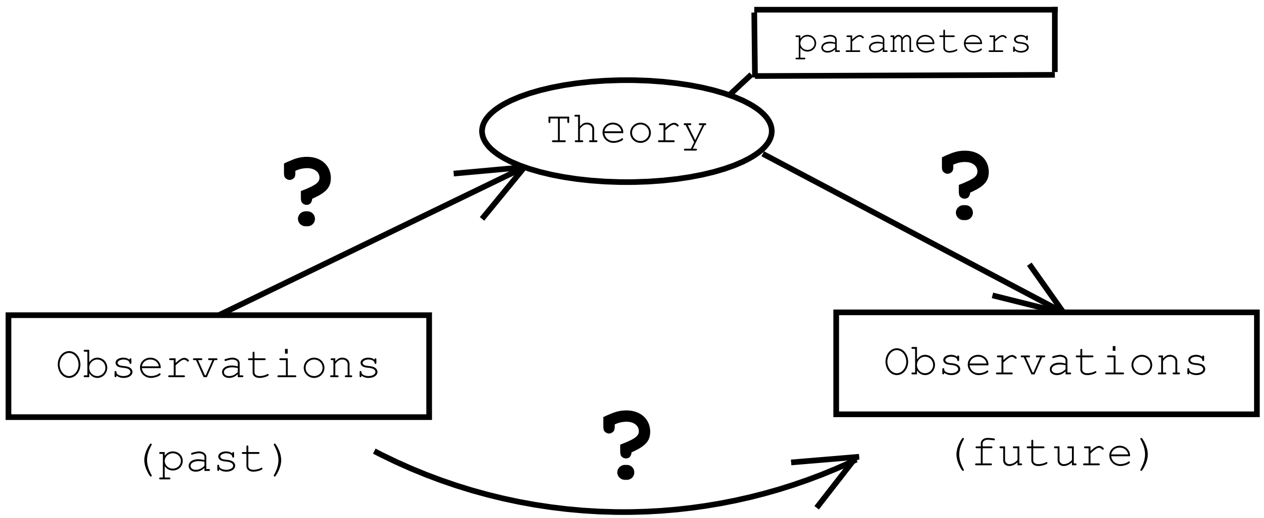
2011 IgNobel prize in Mathematics

- D. Martin of USA (who predicted the world would end in 1954)
- P. Robertson of USA (who predicted the world would end in 1982)
- E. Clare Prophet of the USA (who predicted the world would end in 1990)
- L.J. Rim of KOREA (who predicted the world would end in 1992)
- C. Mwerinde of UGANDA (who predicted the world would end in 1999)
- H. Camping of the USA (who predicted the world would end on September 6, 1994 and later predicted that the world will end on October 21, 2011)

2011 IgNobel prize in Mathematics

"For teaching the world to be careful when making mathematical assumptions and calculations"

Deep source of uncertainty



Uncertainty:

Theory	$-\,?\longrightarrow$	Future observations
Past observations	$-\,?\longrightarrow$	Theory
Theory	$-\,?\longrightarrow$	Future observations

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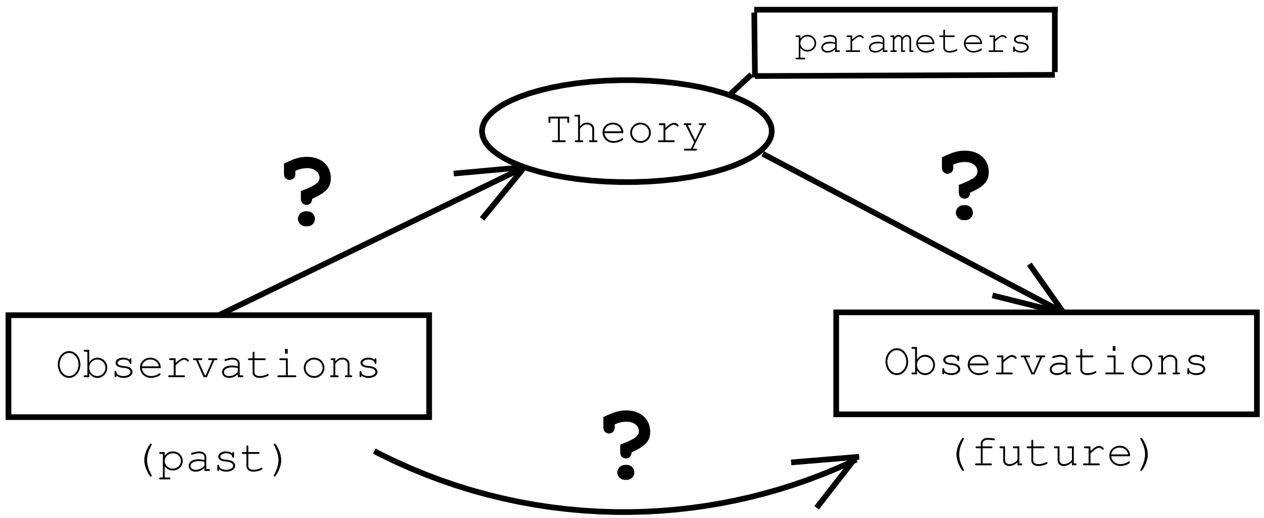


Uncertainty:

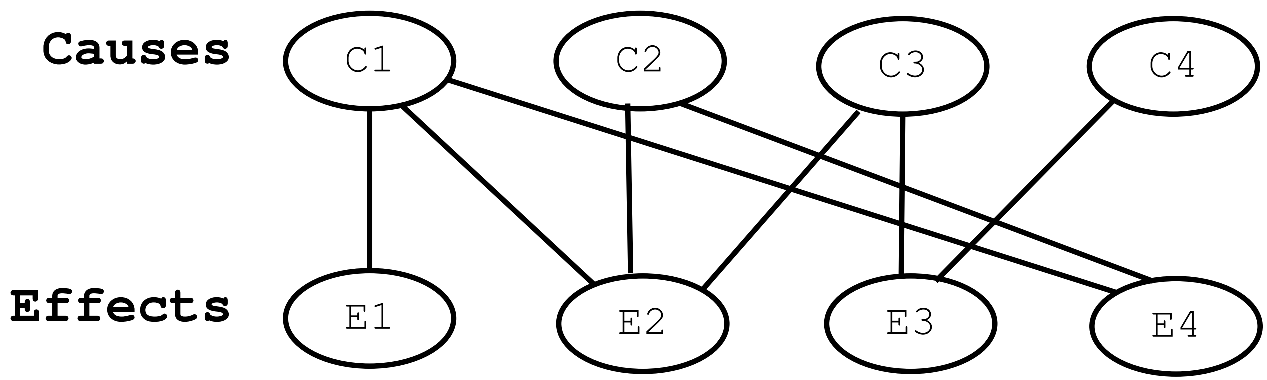
Theory	$\text{—}\,?\,\longrightarrow$	Future observations
Past observations	$\text{—}\,?\,\longrightarrow$	Theory
Theory	$\text{—}\,?\,\longrightarrow$	Future observations

$\Longrightarrow$ **Uncertainty about causal connections**

CAUSE $\Longleftrightarrow$ **EFFECT**

Causes $\rightarrow$ effects

The same *apparent* cause might produce several,different effects



Given an observed effect, we are not sure about the exact cause that has produced it.

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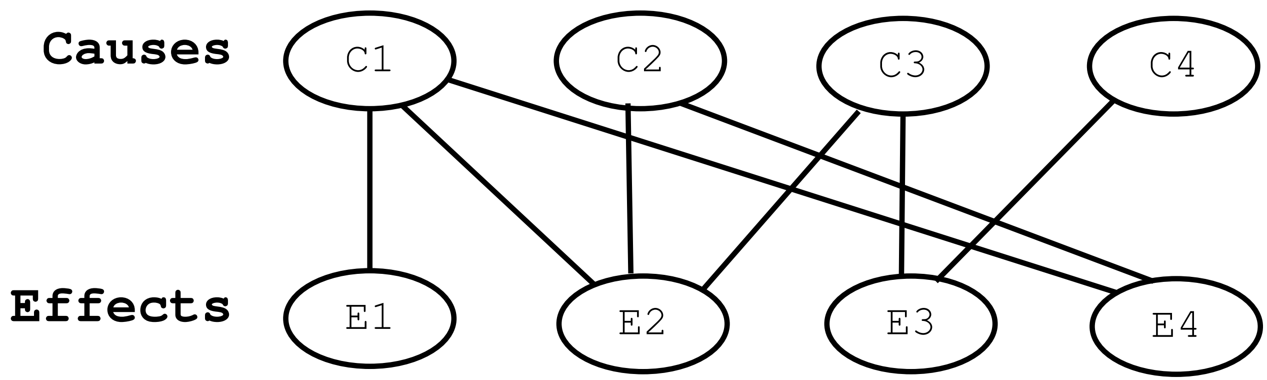

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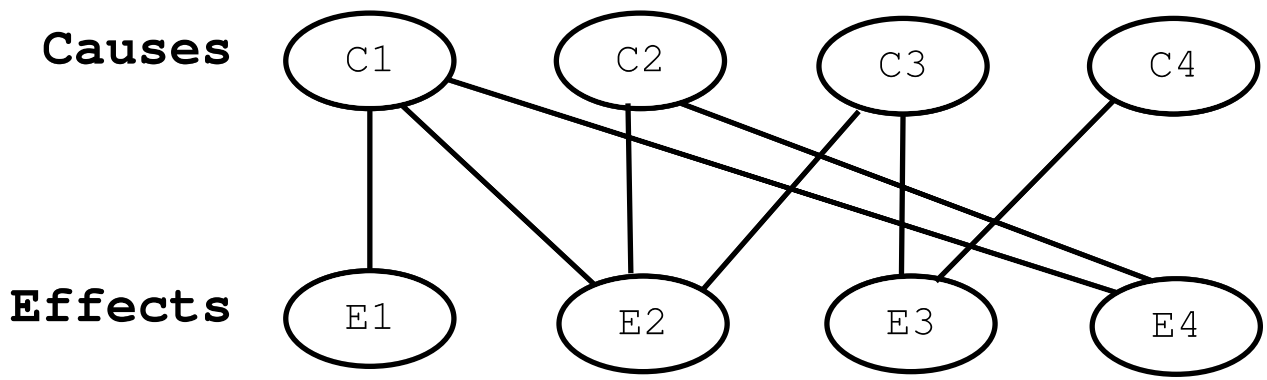

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$$\mathbf{E_2} \Rightarrow \{C_1, C_2, C_3\}?$$

The "essential problem" of the Sciences

"Now, these problems are classified as *probability of causes*, and are most interesting of all their scientific applications. I play at *écarté* with a gentleman whom I know to be perfectly honest. What is the chance that he turns up the king? It is 1/8. This is a problem of the probability of effects.

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I play with a gentleman whom I do not know. He has dealt ten times, and he has turned the king up six times. What is the chance that he is a sharper? This is a problem in the probability of causes. It may be said that it is the essential problem of the experimental method.”

(H. Poincaré – *Science and Hypothesis*)

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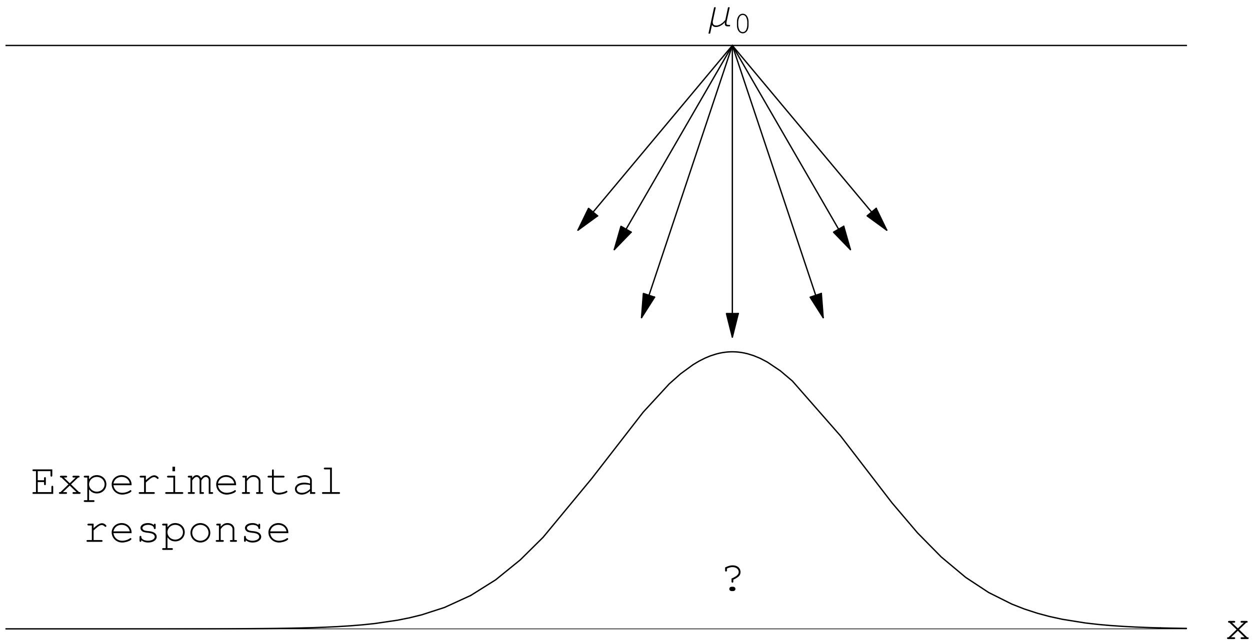
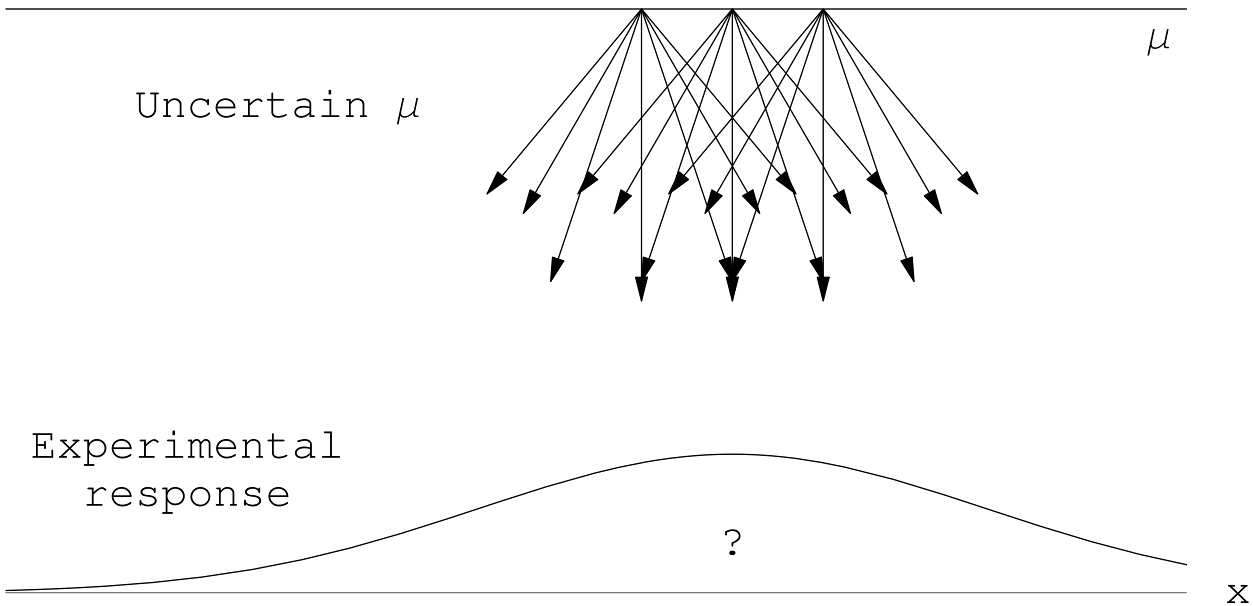
Why we (or most of us) have not been taught how to tackle this kind of problems?

From ‘true value’ to observations



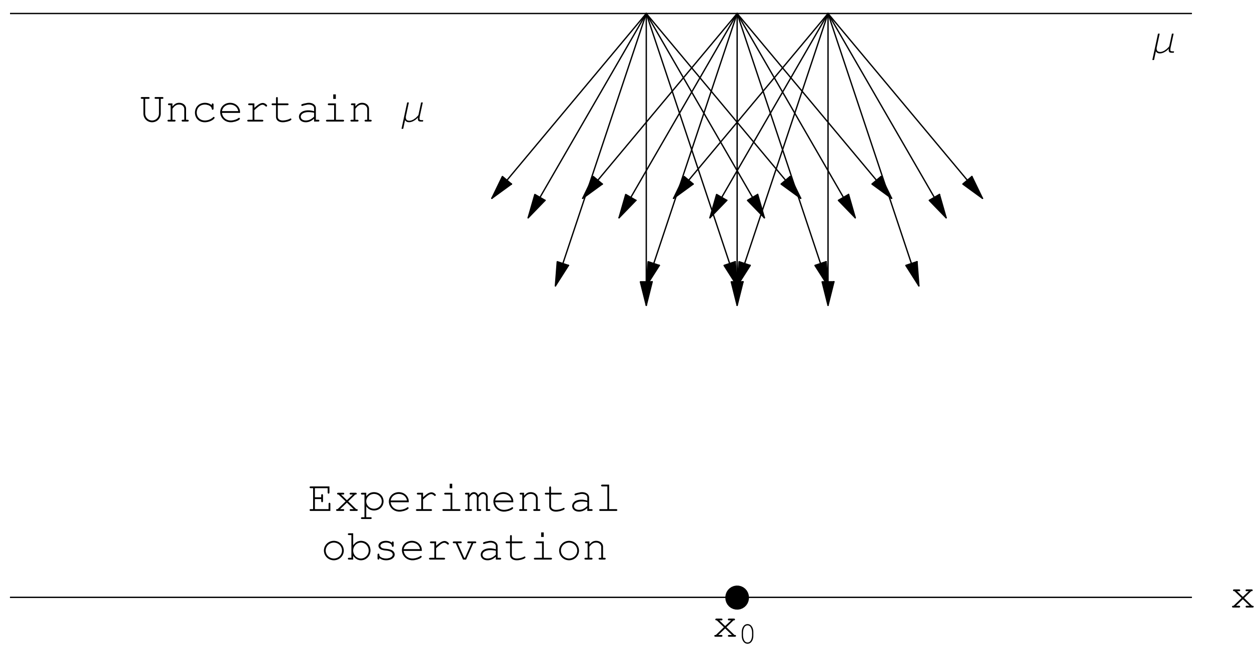
Given μ (exactly known) we are uncertain about x

From ‘true value’ to observations



Uncertainty about μ makes us more uncertain about x

. . . and back: Inferring a true value



The observed data is <u>certain</u>: → 'true value' uncertain.

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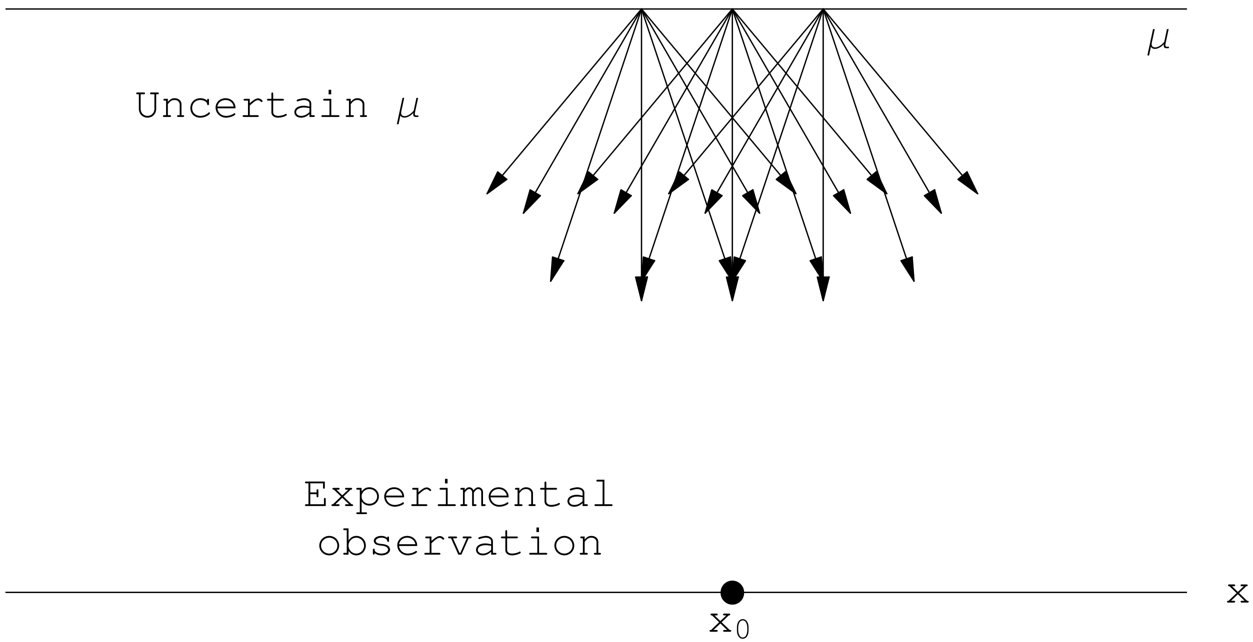

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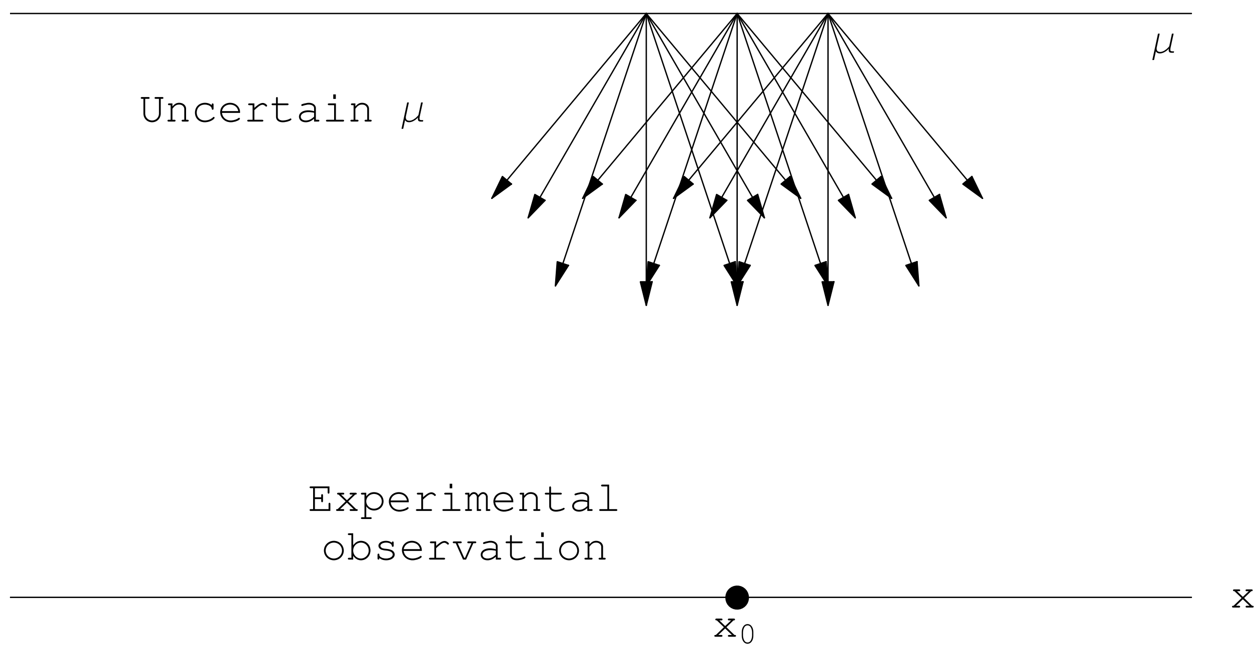
“data uncertainty” ?

. . . and back: Inferring a true value



The observed data is <u>certain</u>: $\rightarrow$ 'true value' uncertain.

"data uncertainty" ? Data corrupted?

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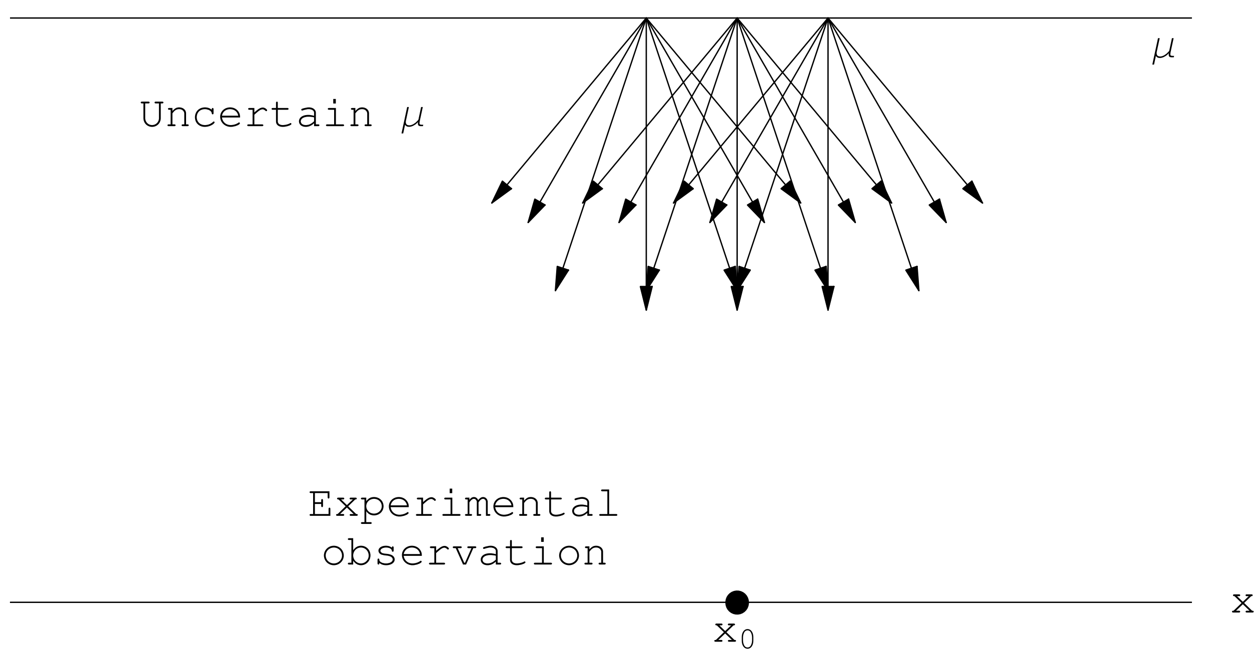
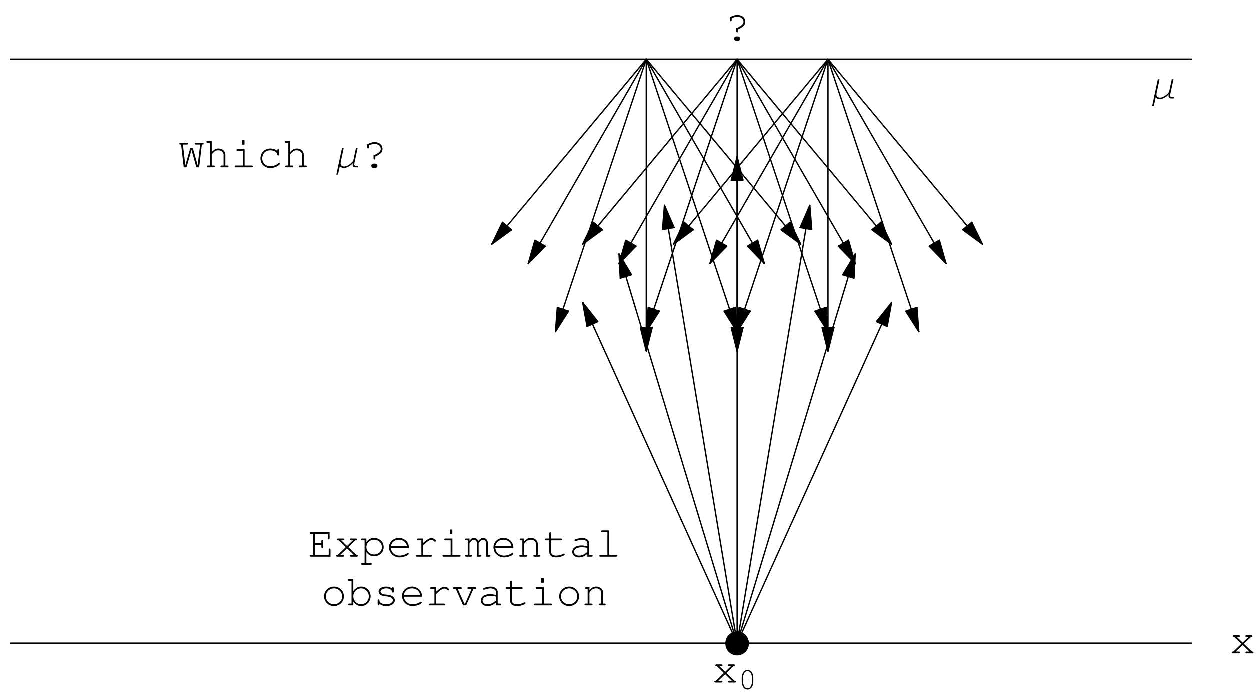


The observed data is <u>certain</u>: → 'true value' uncertain.

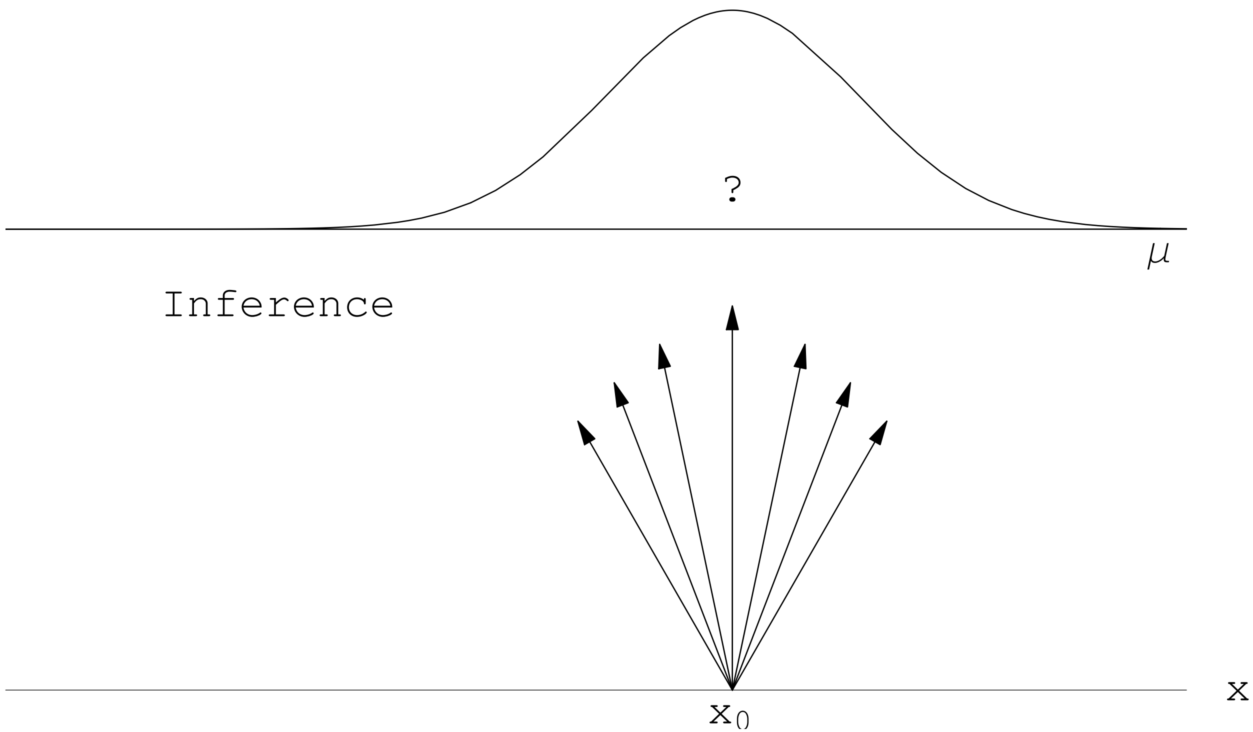
"data uncertainty" ? Data corrupted?
Even if the data were corrupted, the <u> data </u> were the corrupted data!! . . .

. . . and back: Inferring a true value



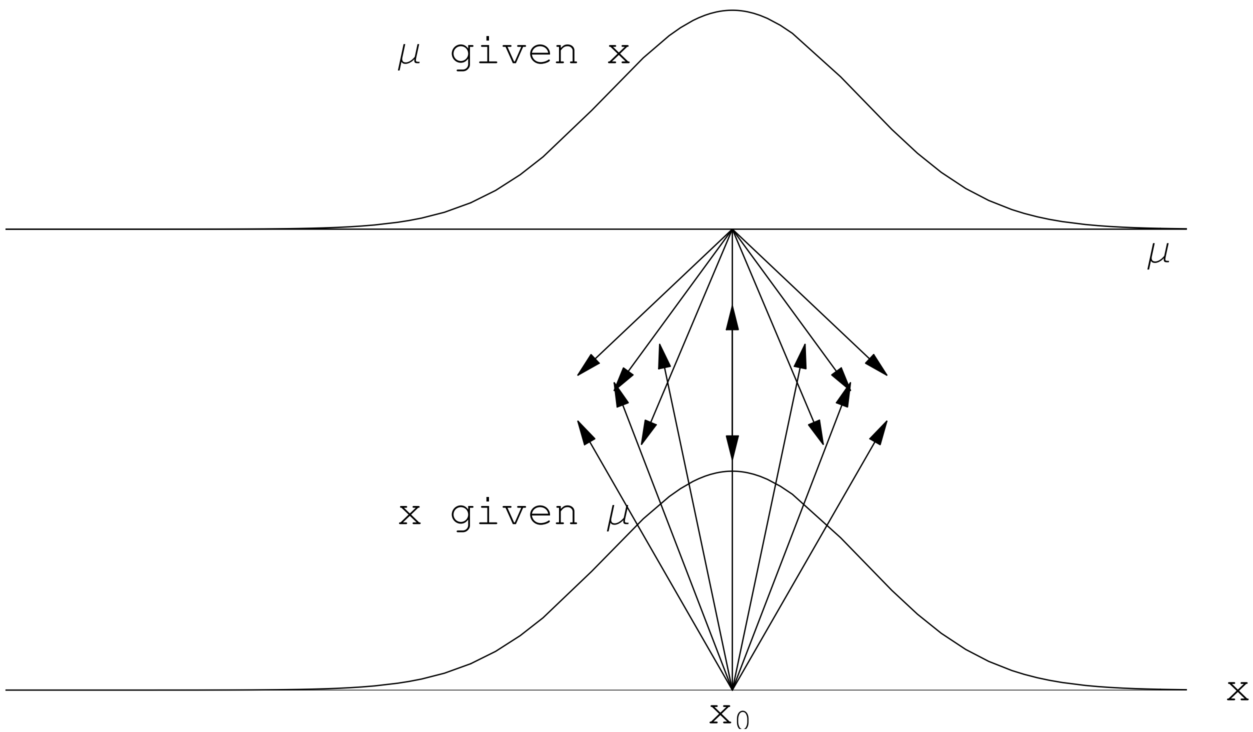
Where does the observed value of x comes from?

…and back: Inferring a true value



We are now uncertain about μ, given x.

…and back: Inferring a true value



Note the symmetry in reasoning.

A very simple experiment

Let's make an experiment

A very simple experiment

Let’s make an experiment

- Here
- Now

A very simple experiment

Let's make an experiment

- Here
- Now

For simplicity

- μ can assume only six possibilities:

$$\mathbf{0}, \mathbf{1}, \ldots, \mathbf{5}$$

- x is binary:

$$\mathbf{0}, \mathbf{1}$$

$$[\,(1, 2);\ \text{Black/White};\ \text{Yes/Not};\ \ldots\,]$$

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For simplicity

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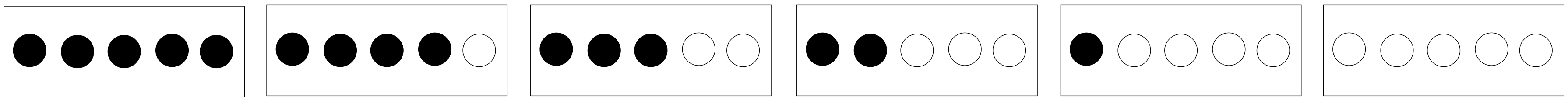
$\Rightarrow$ Later we shall make μ continous.

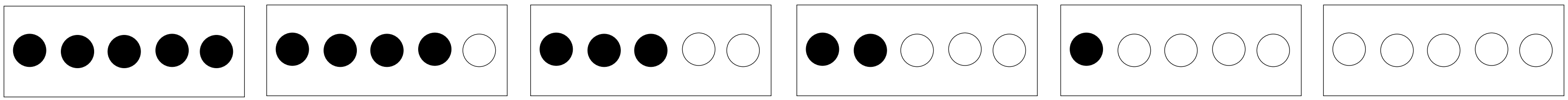
Which box? Which ball?



H_0 H_1 H_2 H_3 H_4 H_5

Let us take randomly one of the boxes.

Which box? Which ball?



H_0 H_1 H_2 H_3 H_4 H_5

Let us take randomly one of the boxes.

We are in a state of uncertainty concerning several *events*, the most important of which correspond to the following questions:

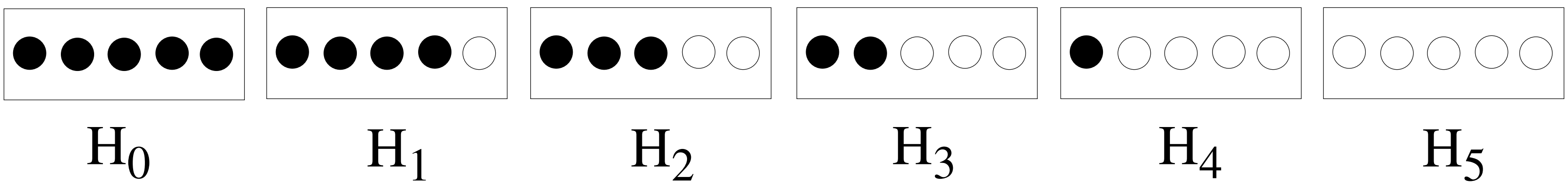
(a) Which box have we chosen, $H_0, H_1, \ldots, H_5$?

(b) If we extract randomly a ball from the chosen box, will we observe a white ($E_W \equiv E_1$) or black ($E_B \equiv E_2$) ball?

Our certainties:

$$\begin{aligned} \cup_{j=0}^{5} H_j &= \Omega \\ \cup_{i=1}^{2} E_i &= \Omega \,. \end{aligned}$$

Which box? Which ball?



Let us take randomly one of the boxes.

- What happens after we have extracted one ball and looked its color?
  - Intuitively feel *how to* roughly *change* our opinion about
    - the possible cause
    - a future observation

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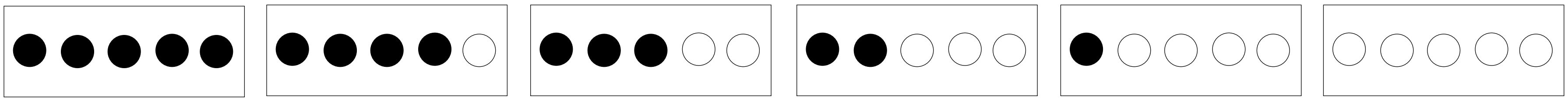

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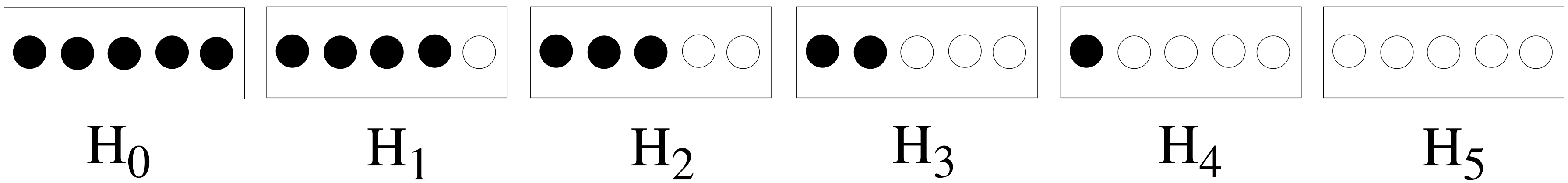

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    - the possible cause
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  - Can we do it *quantitatively*, in an ‘objective way’?
- And after a sequence of extractions?

The toy inferential experiment

The aim of the experiment will be to guess the content of the box without looking inside it, only extracting a ball, record its color and reintroducing in the box

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This toy experiment is conceptually very close to what we do in the pure and applied sciences

$\Rightarrow$ try to guess what we cannot see (the electron mass, a magnetic field, etc)

…from what we can see (somehow) with our senses.

The rule of the game is that we are not allowed to watch inside the box! (As we cannot open and electron and read its properties, unlike we read the MAC address of a PC interface.)

Where is probability?

We all agree that the experimental results change

- the probabilities of the box compositions;
- the probabilities of a future outcomes,

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Where is the probability?

Certainly <u>not</u> *in* the box!

Subjective nature of probability

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Probability depends on the status of information of the *subject* who evaluates it.

Probability is always conditional probability

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$$P(E) \longrightarrow P(E\,|\,I_s)$$

where I_s is the information available to *subject* s.

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"Given the state of our knowledge about everything that could possible have any bearing on the coming true. . . the numerical probability P of this event is to be a real number by the indication of which we try in some cases to setup a quantitative measure of the strength of our conjecture or anticipation, founded on the said knowledge, that the event comes true"

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$\Rightarrow$ How much we believe something

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$\rightarrow$ ‘Degree of belief’ $\leftarrow$

Beliefs and ‘coherent’ bets

Remarks:

- Subjective does not mean arbitrary!

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“His [Bouvard] calculations give him the mass of Saturn as 3,512th part of that of the sun. Applying my probabilistic formulae to these observations, I find that the odds are 11,000 to 1 that the error in this result is not a hundredth of its value.” (Laplace)

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$\rightarrow P(3477 \leq M_{Sun}/M_{Sat} \leq 3547 \,|\, I(\text{Laplace})) = 99.99\%$

Standard textbook definitions

$$p = \frac{\#\,\text{favorable cases}}{\#\,\text{possible equiprobable cases}}$$

$$p = \frac{\#\,\text{times the event has occurred}}{\#\,\text{independent trials under same conditions}}$$

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$$p = \frac{\#\,\text{favorable cases}}{\#\,\text{possible equally possible cases}}$$

$$p = \frac{\#\,\text{times the event has occurred}}{\#\,\text{independent trials under same conditions}}$$

Note!: *"lorsque rien ne porte à croire que l'un de ces cas doit arriver plutot que les autres" (Laplace)*

Replacing 'equi-probable' by 'equi-possible' is just cheating students (as I did in my first lecture on the subject. . .).

Standard textbook definitions

It is easy to check that ‘scientific’ definitions suffer of circularity, plus other problems

$$p = \frac{\#\text{ favorable cases}}{\#\text{ possible equiprobable cases}}$$

$$p = \lim_{n\to\infty} \frac{\#\text{ times the event has occurred}}{\#\text{ independent trials under same condition}}$$

Future $\Leftrightarrow$ Past (belief!)

$n\to\infty$: $\rightarrow$ *“usque tandem?”*
$\rightarrow$ *“in the long run we are all dead”*
$\rightarrow$ It limits the range of applications

‘Definitions’ $\rightarrow$ evaluation rules

Very useful evaluation rules

$$A) \quad p \;=\; \frac{\#\,\text{favorable cases}}{\#\,\text{possible equiprobable cases}}$$

$$B) \quad p \;=\; \frac{\#\,\text{times the event has occurred}}{\#\text{independent trials under same condition}}$$

If the implicit beliefs are well suited for each case of application.

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BUT they cannot define the concept of probability!

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In the probabilistic approach we are following

- Rule A is recovered immediately (under the assumption of equiprobability, when it applies).
- Rule B results from a theorem of Probability Theory (under well defined assumptions): $\Rightarrow$ Laplace’s rule of succession

Mathematics of beliefs

The good news:

> The basic laws of degrees of belief are the same we get from the inventory of favorable and possible cases, or from events occurred in the past.

It can be proved that

> the requirement of coherence leads to the famous 4 basic rules $\Longrightarrow$

[Details skipped...]

Basic rules of probability

1. $0 \leq P(A \,|\, I) \leq 1$
2. $P(\Omega \,|\, I) = 1$
3. $P(A \cup B \,|\, I) = P(A \,|\, I) + P(B \,|\, I) \quad [\text{if } P(A \cap B \,|\, I) = \emptyset]$

4. $P(A \cap B \,|\, I) = P(A \,|\, B, I) \cdot P(B \,|\, I) = P(B \,|\, A, I) \cdot P(A \,|\, I)$

Remember that probability is always conditional probability!

I is the background condition (related to information 'I'_s')

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Note: 4. does not define conditional probability.
(Probability is always conditional probability!)

Mathematics of beliefs

An even better news:

The fourth basic rule
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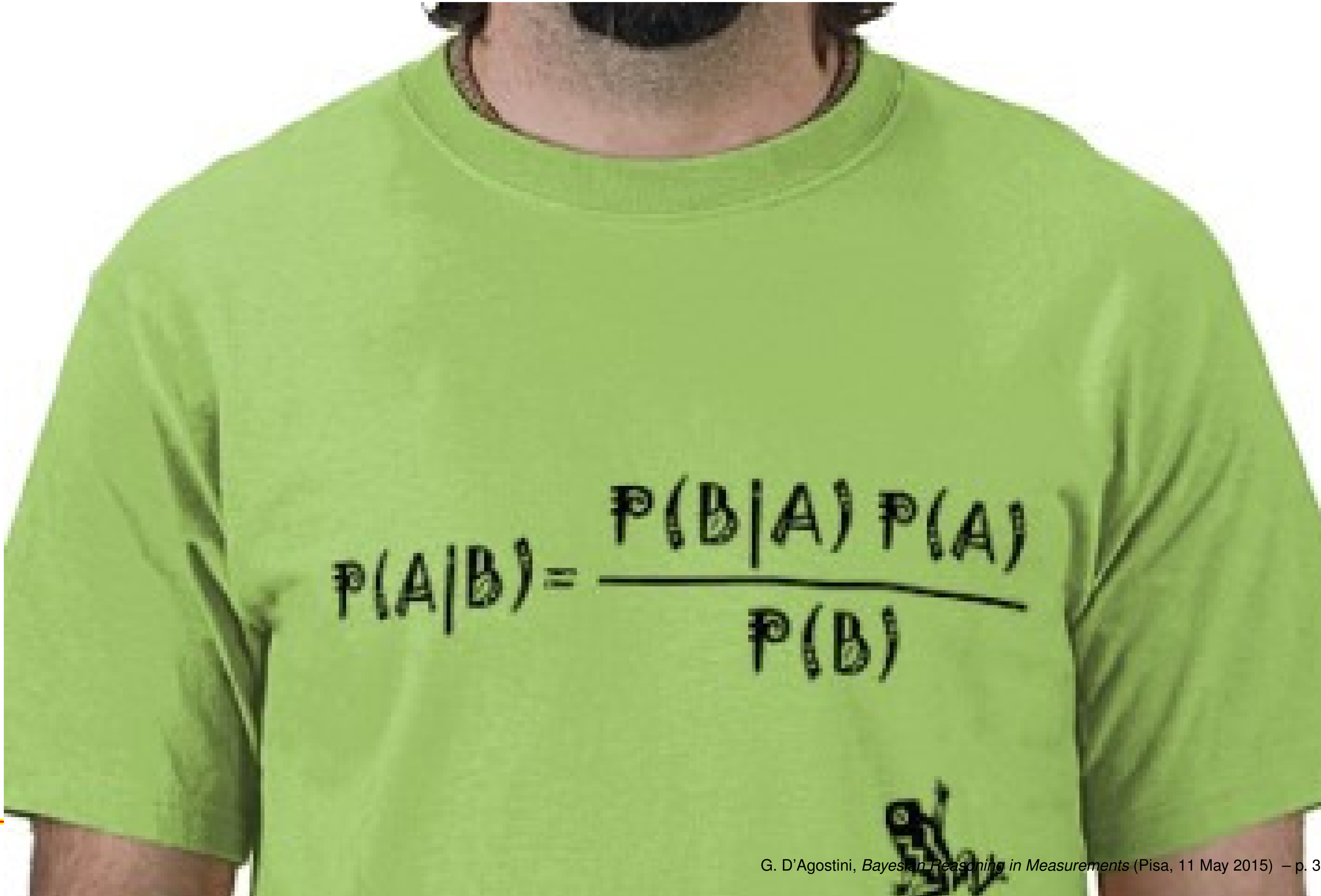
(Liberated by a curious ideology that forbits its use)

A simple, powerful formula



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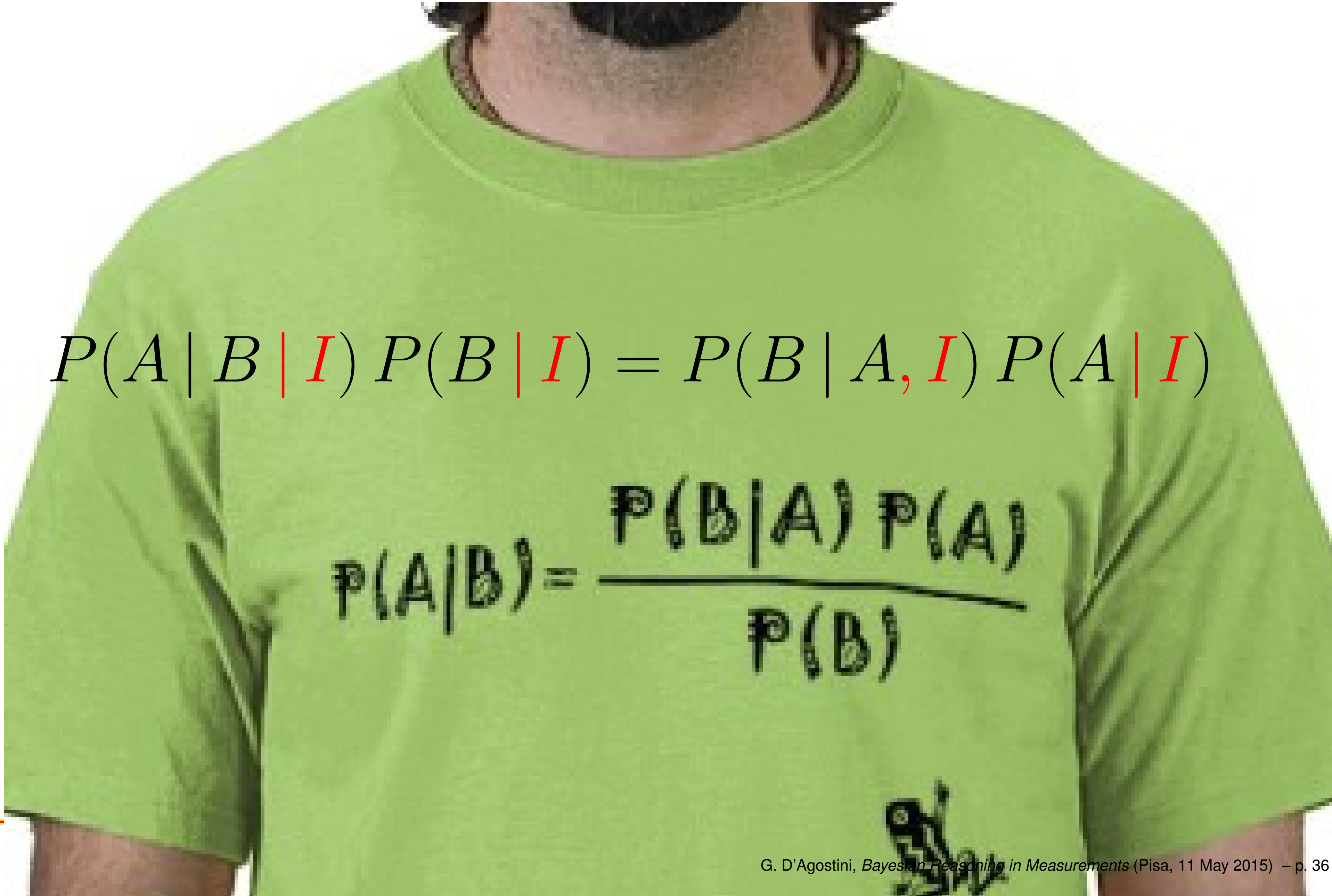
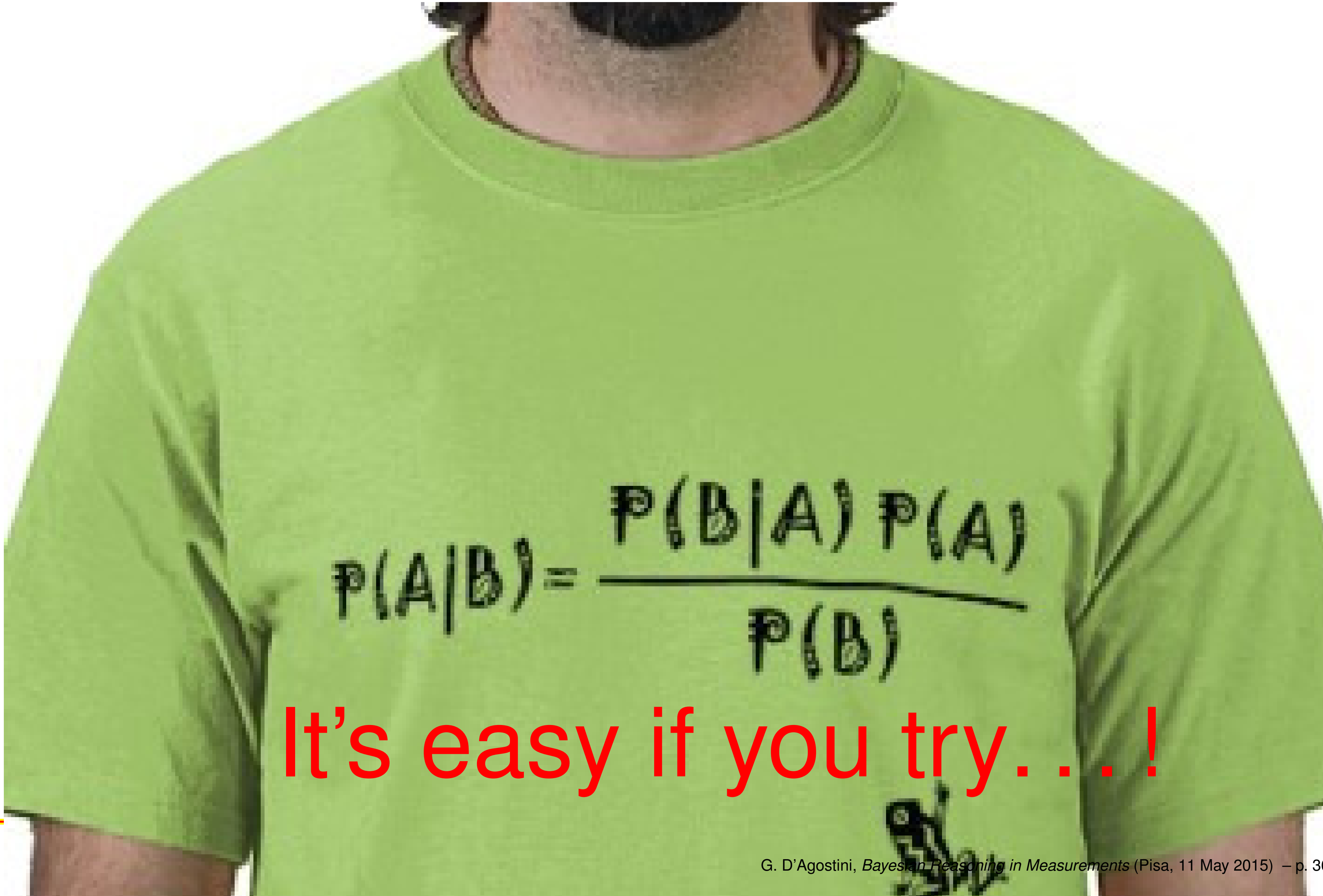


$$P(A\,|\,B\,|\,I)\,P(B\,|\,I) = P(B\,|\,A,I)\,P(A\,|\,I)$$

A simple, powerful formula

Take the courage to use it!

A simple, powerful formula



It's easy if you try. . . !

Laplace's "Bayes Theorem"

"The greater the probability of an observed event given any one of a number of causes to which that event may be attributed, the greater the likelihood of that cause {given that event}.

$$P(C_i\,|\,E) \propto P(E\,|\,C_i)$$

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$$P(C_i\,|\,E) = \frac{P(E\,|\,C_i)\,P(C_i)}{\sum_j P(E\,|\,C_j)\,P(C_j)}$$

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$$P(C_i \,|\, E) \;=\; \frac{P(E \,|\, C_i)\, P(C_i)}{\sum_j P(E \,|\, C_j)\, P(C_j)}$$

"This is the fundamental principle $^{(*)}$ of that branch of the analysis of chance that consists of reasoning *a posteriori* from events to causes"

(*) In his "Philosophical essay" Laplace calls 'principles' the 'fondamental rules'.

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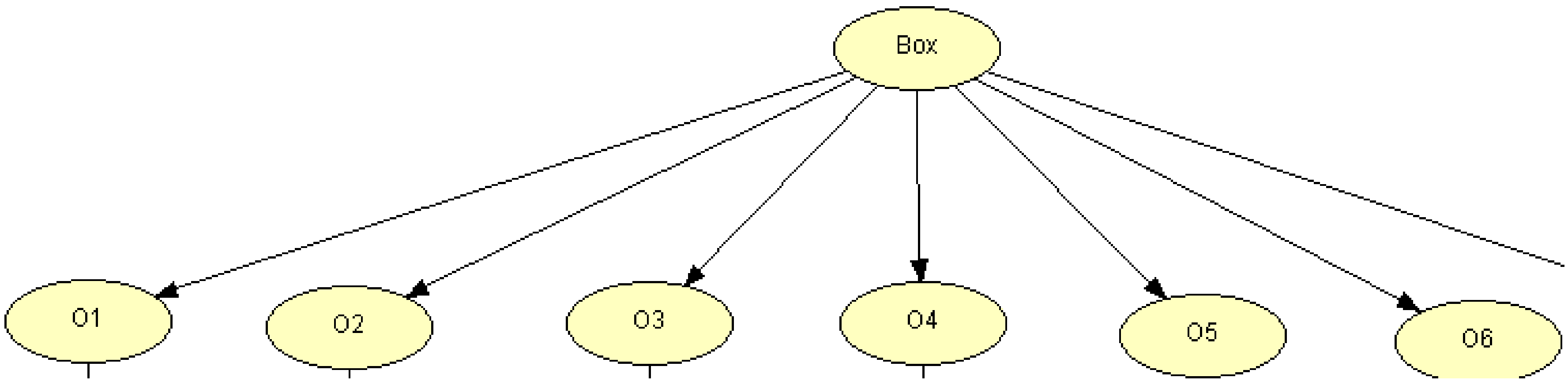
Most convenient way to remember Bayes theorem

Cause-effect representation

box content $\rightarrow$ observed color



$$P(B^{(1)}\,|\,H_j),\quad P(B^{(2)}\,|\,H_j),\ldots$$

$$P(W^{(1)}\,|\,H_j),\quad P(W^{(2)}\,|\,H_j),\ldots$$

Cause-effect representation

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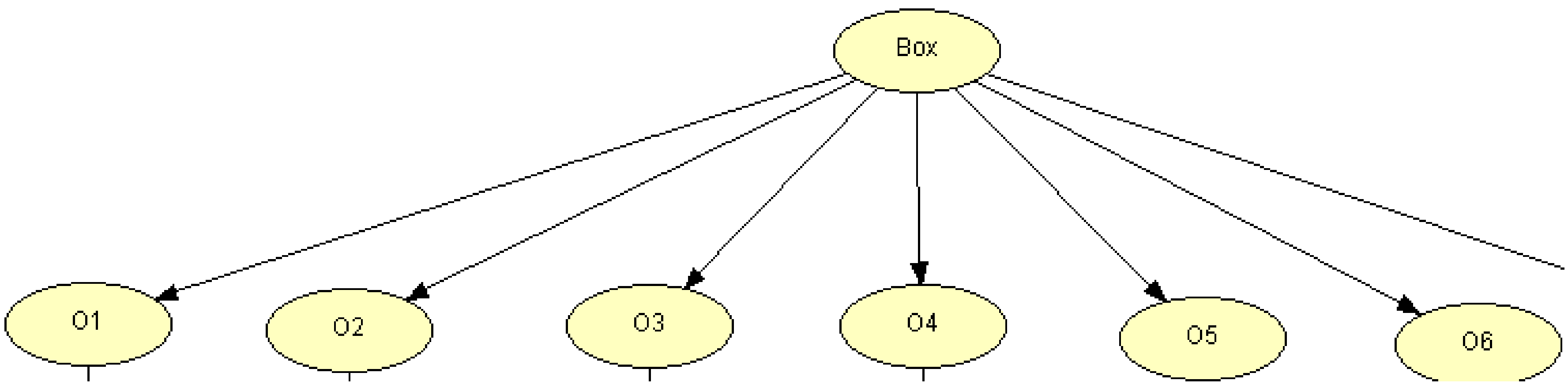
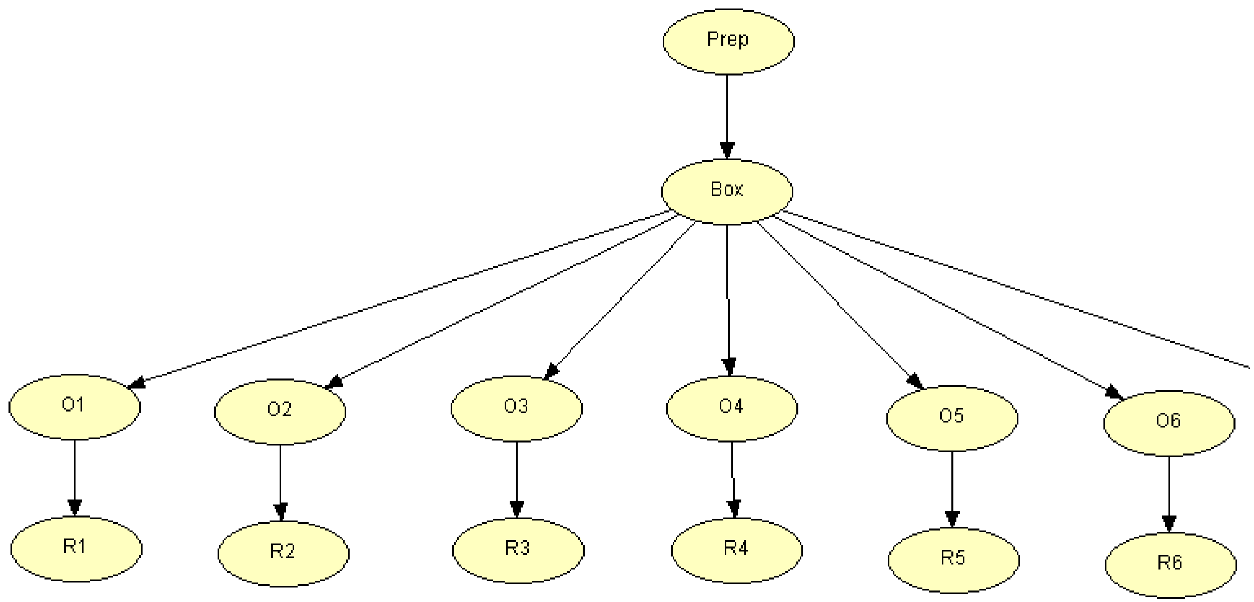


An effect might be the cause of another effect $\Longrightarrow$

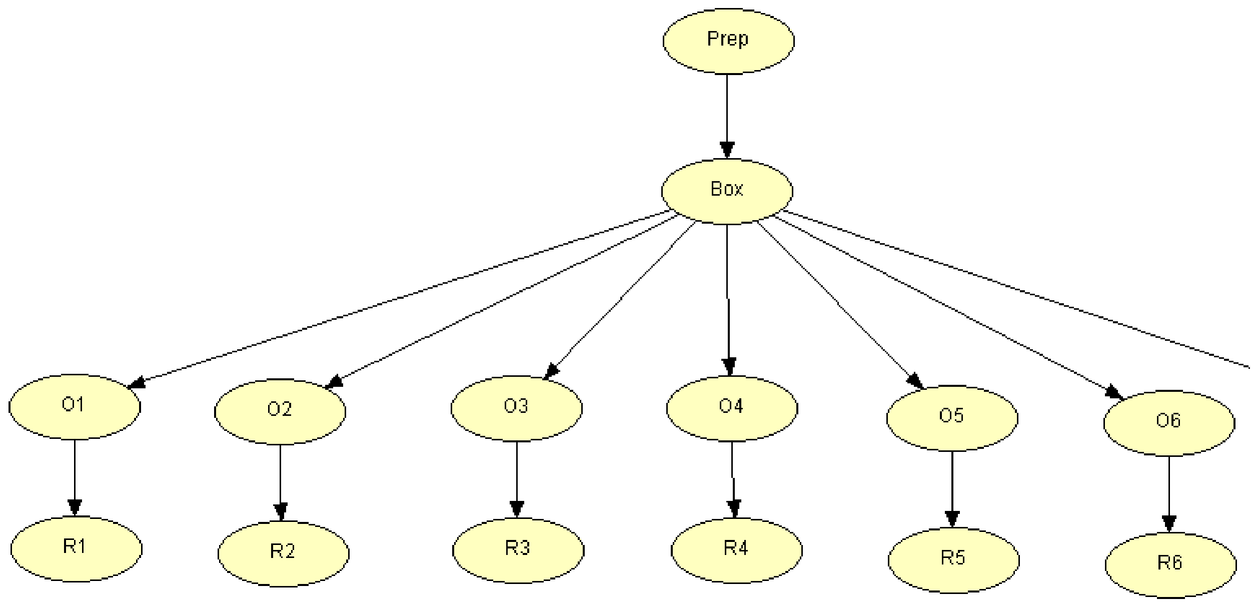
A network of causes and effects



A network of causes and effects

Preparation ‘node’ models prior knowledge about Box.

$\Rightarrow P(H_j \,|\, \text{Prep}_k)$



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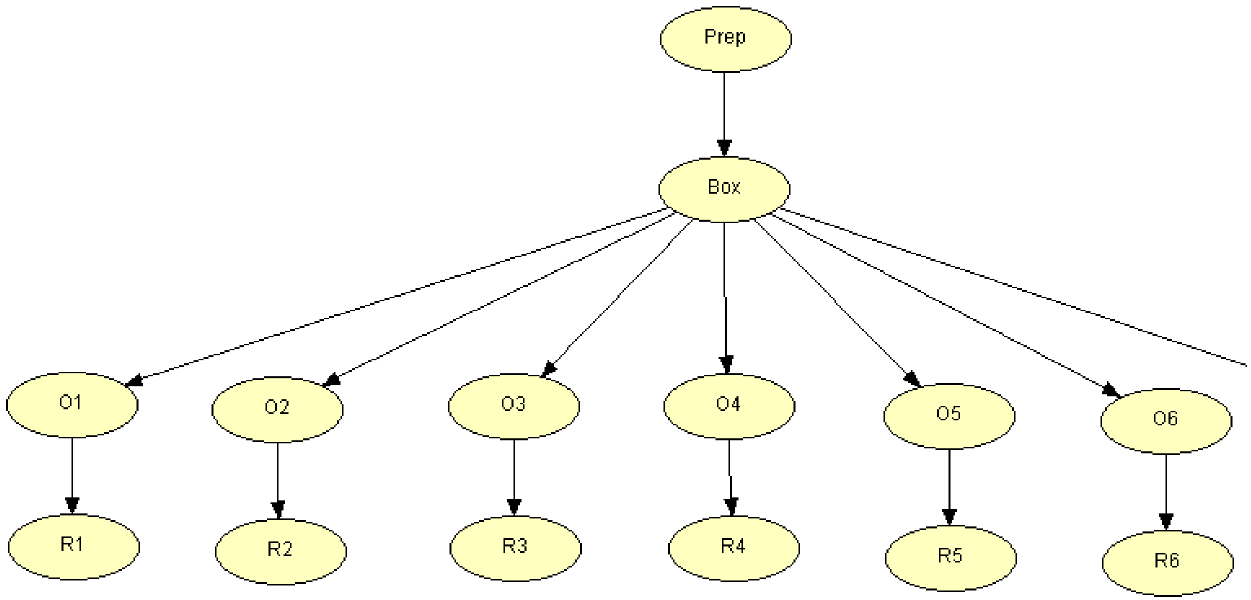

R_i model extra uncertainty in cascade.

$\Rightarrow P(W_R \,|\, W)$, $P(B_R \,|\, W)$, etc.

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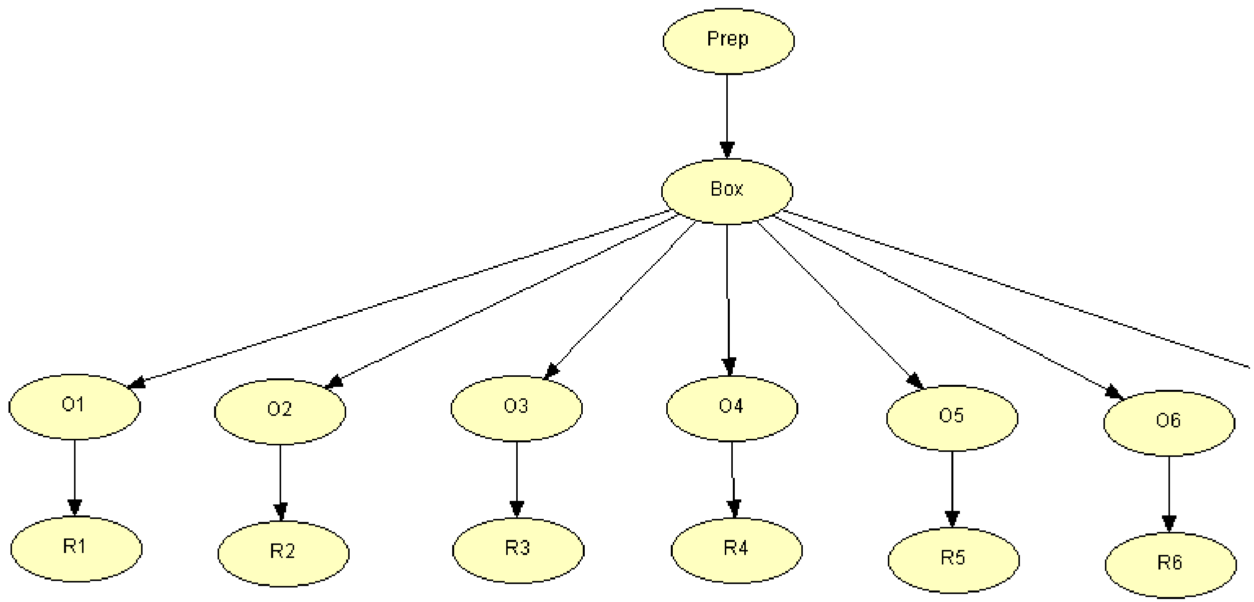

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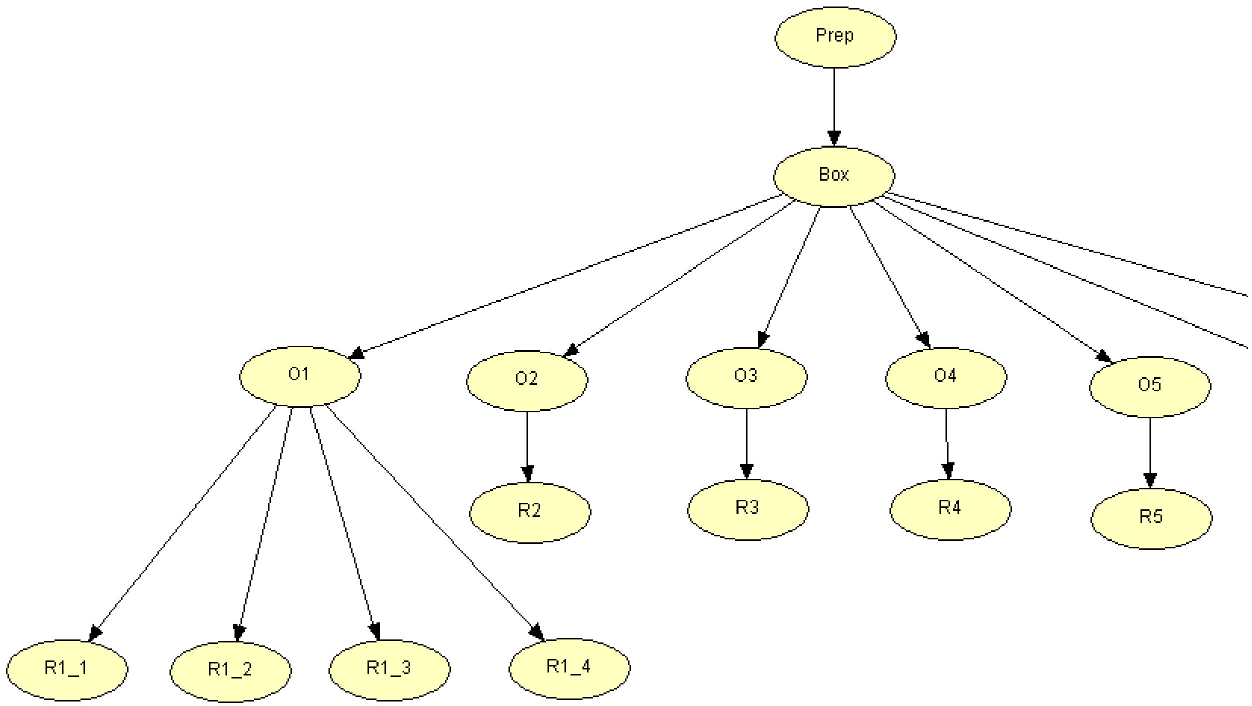
We shall also include multi-reporters and systematic effects

Multi-reporters

Multiple 'testimonies' of the same empirical fact.



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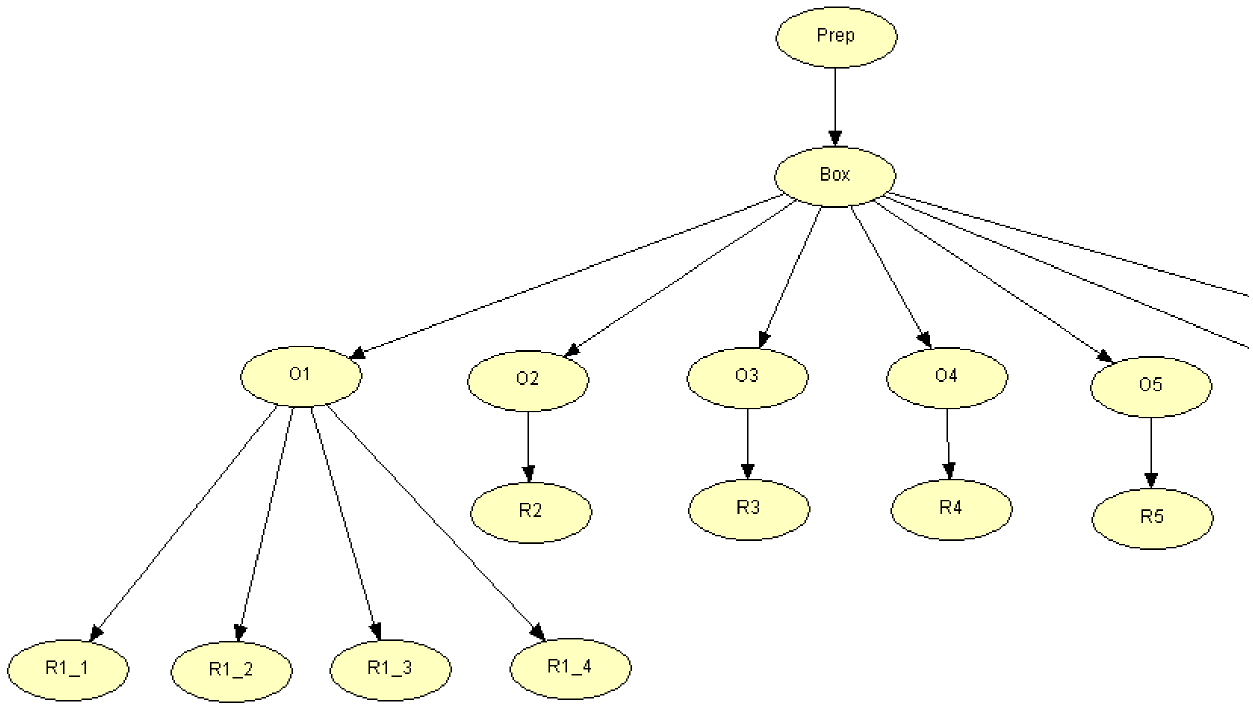
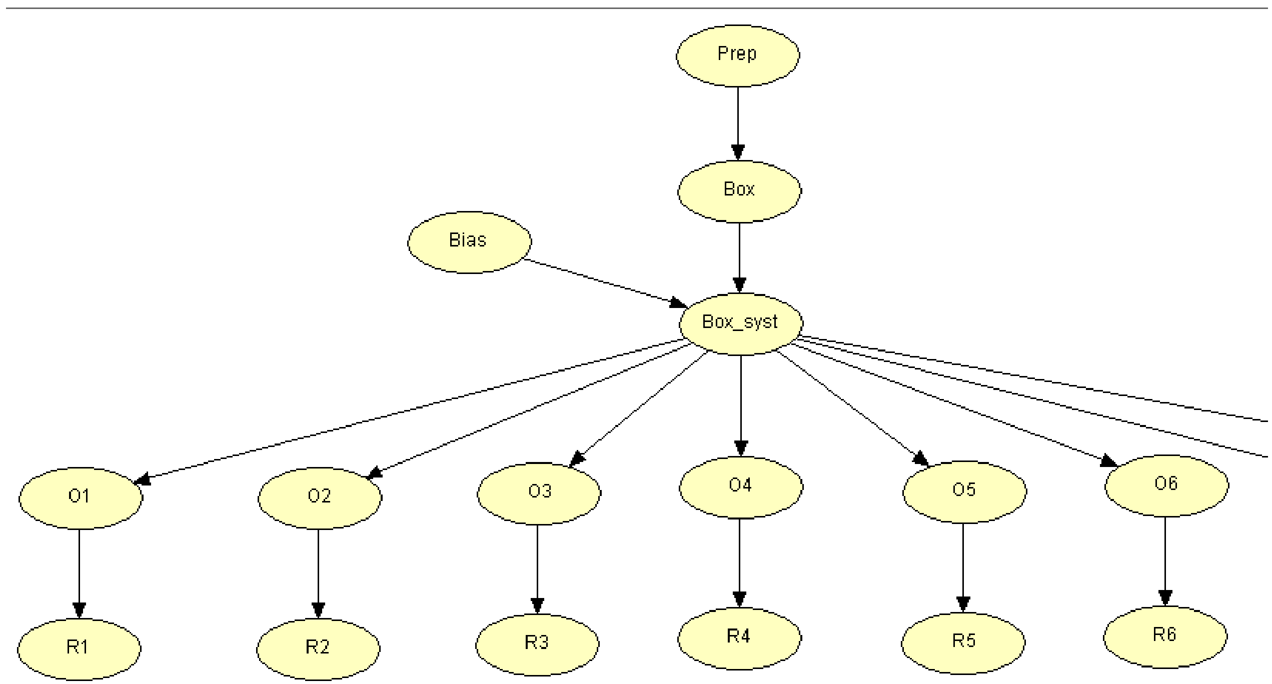


$\Rightarrow$ Our belief on O_1 being Black or White will depend on the consistencies of the 'testimonies'

Systematic effects

The box content could be biased. . .



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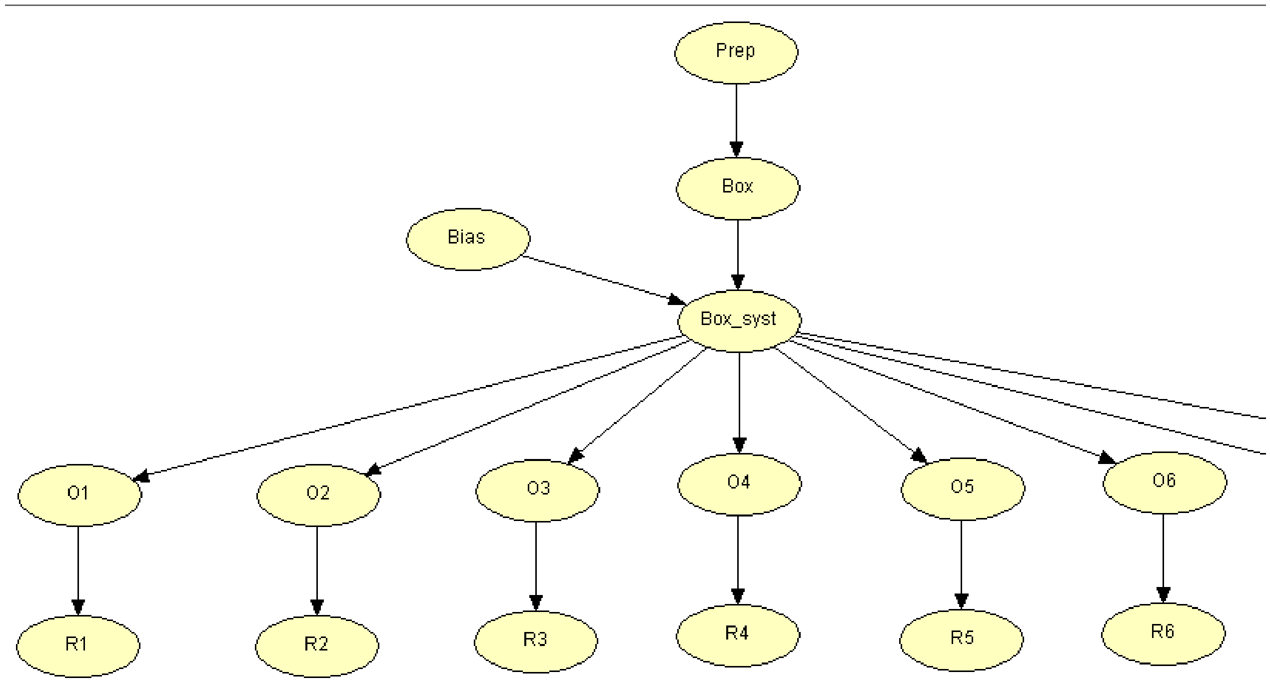
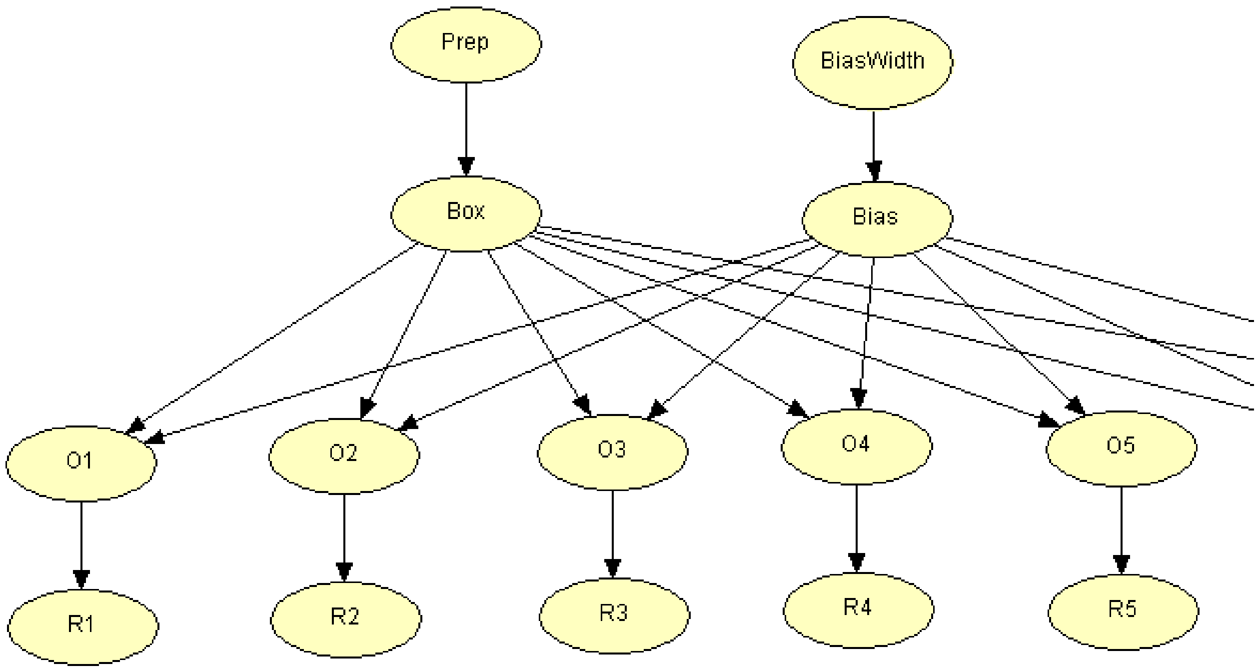


. . . if one or more balls of either color might be added to the original box content

Systematic effects

The box content could be biased. . .



[technical implementation of the bias – logically equivalent]

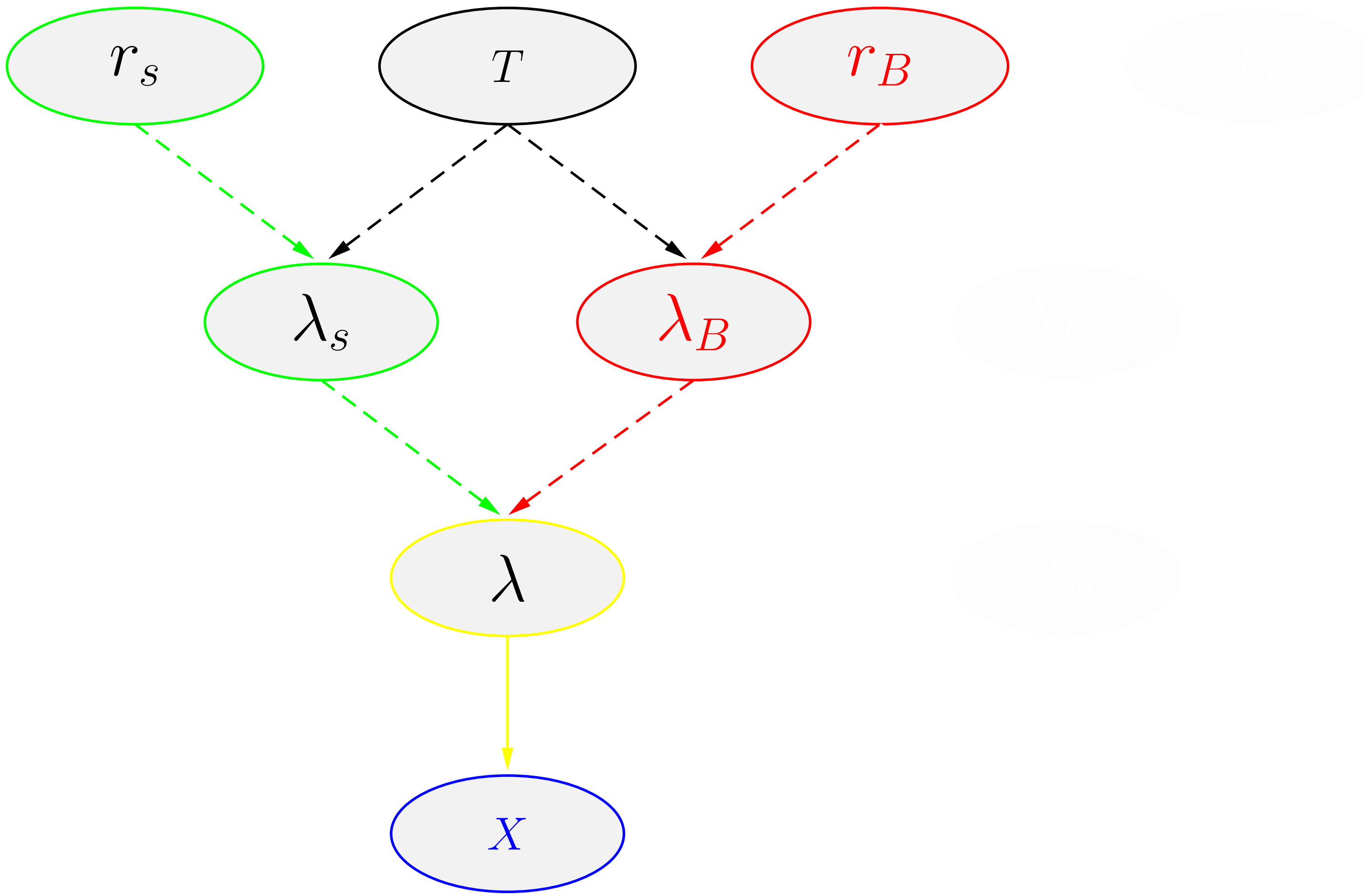
Graphical models

The importance of graphical models is that

⇒ Nowadays, thanks to progresses in mathematics and computing, drawing the problem as a 'belief network' is more than 1/2 step towards its solution!

Signal and background

Counting experiment ("Poisson process")



X affected by Signal and Bkgd

Signal and background

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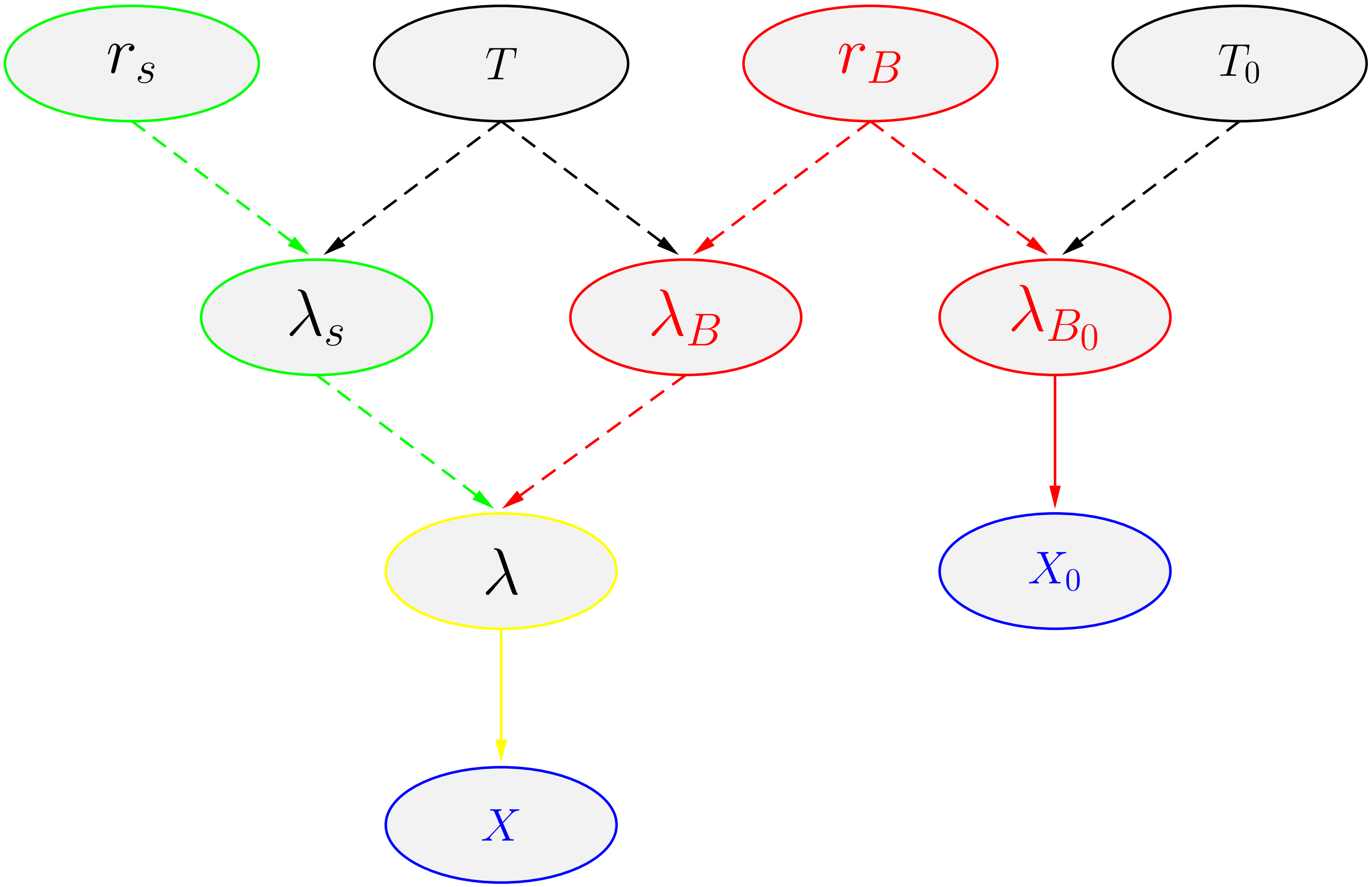
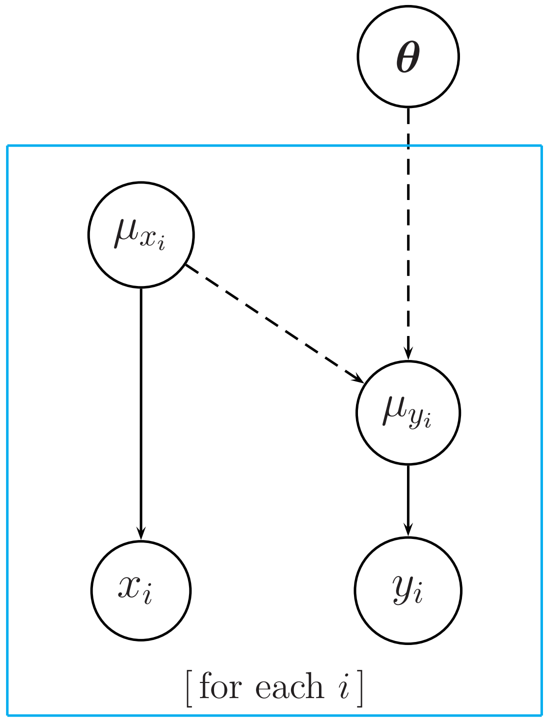


X affected by Signal and Bkgd $\quad$ X_0 only by Bkgd

A different way to view fit issues



Determistic link μ_x's to μ_y's

Probabilistic links $\mu_x \to x$, $\mu_y \to y$

(errors on both axes!)

$\Rightarrow$ aim of fit: $\{\boldsymbol{x}, \boldsymbol{y}\} \to \boldsymbol{\theta}$

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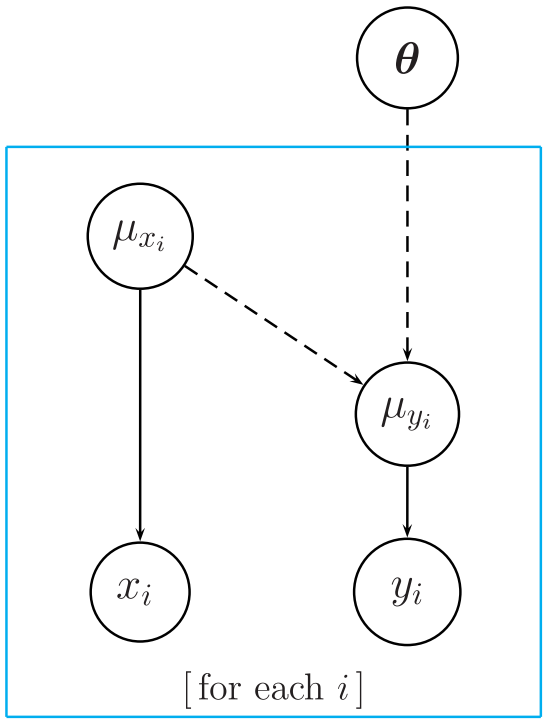

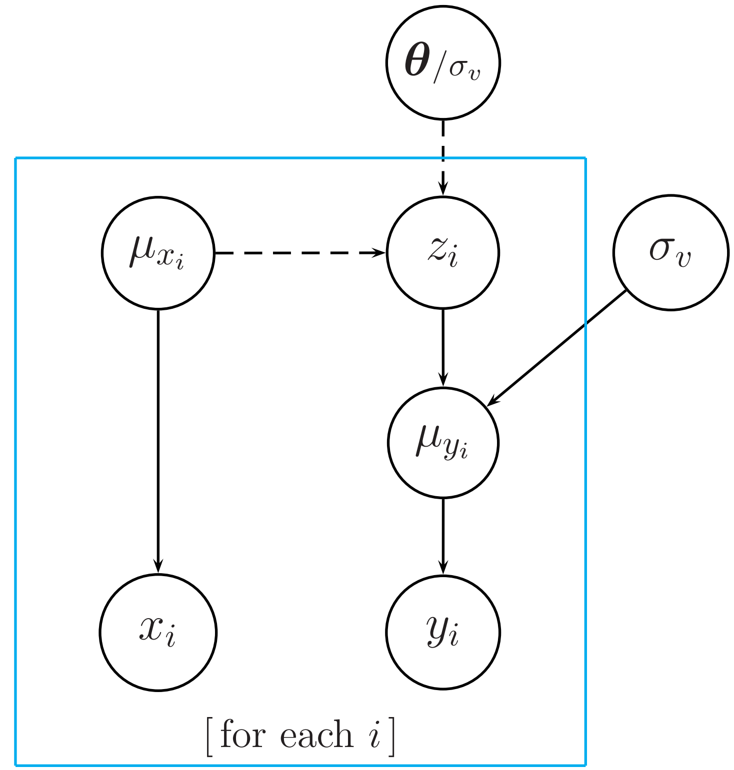

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Probabilistic links $\mu_x \rightarrow x$, $\mu_y \rightarrow y$
(errors on both axes!)
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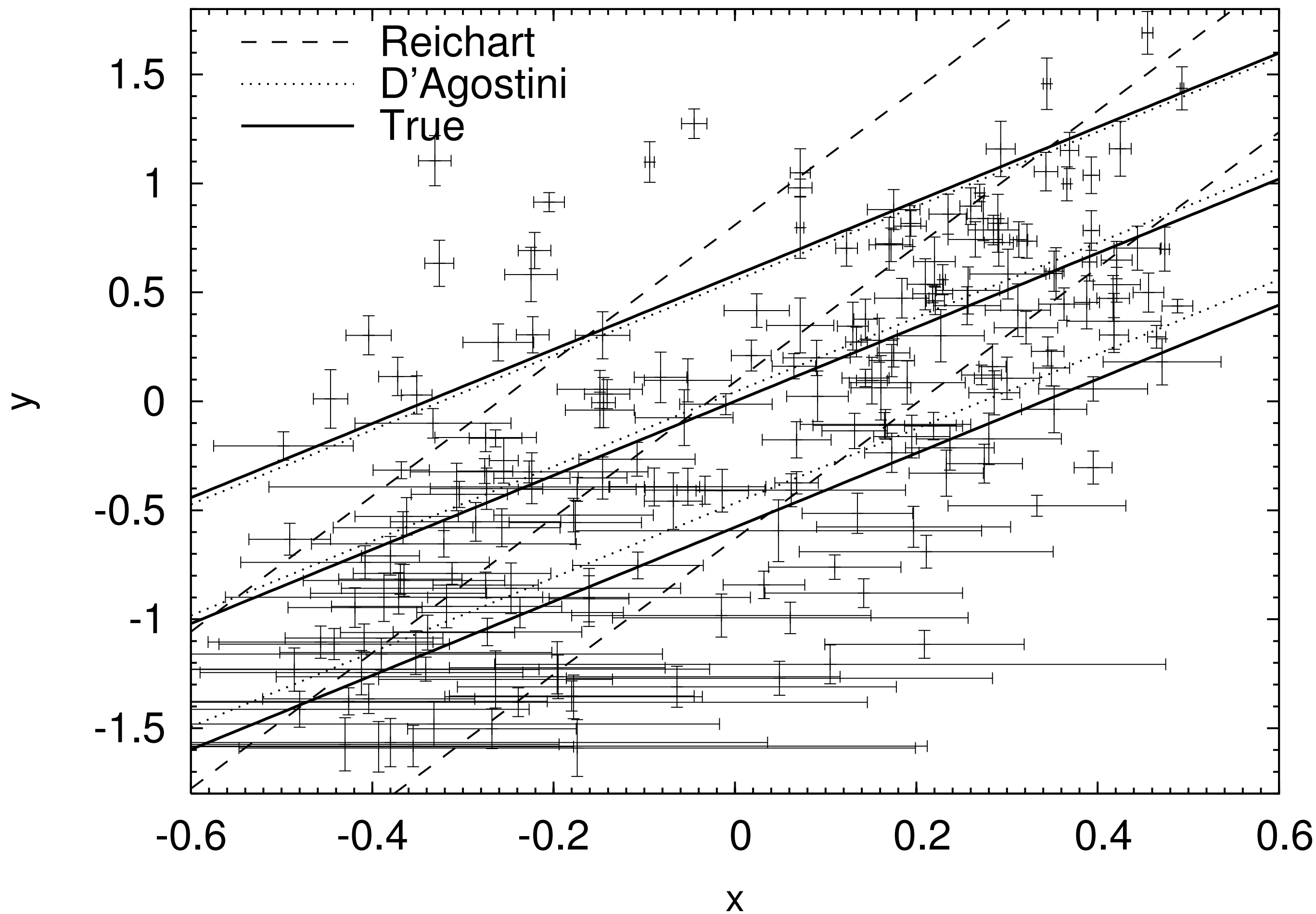
Extra spread
of the data points

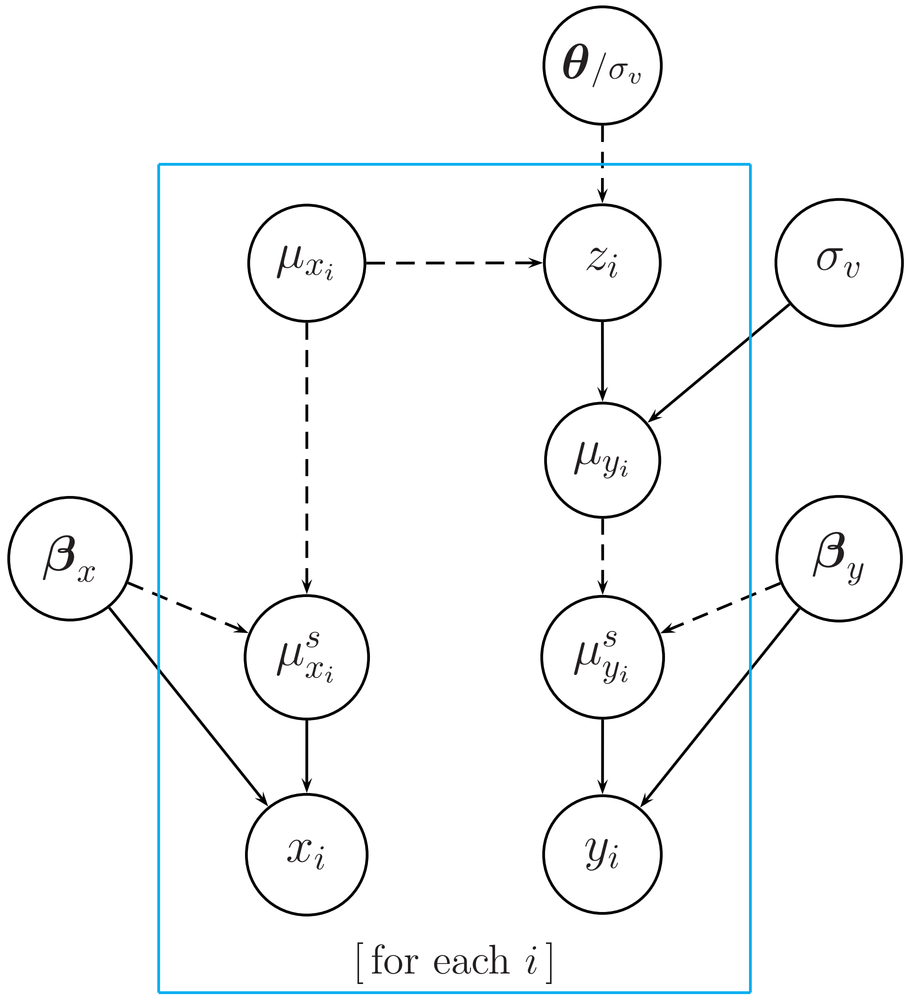
A different way to view fit issues

A physics case (from Gamma ray burts):



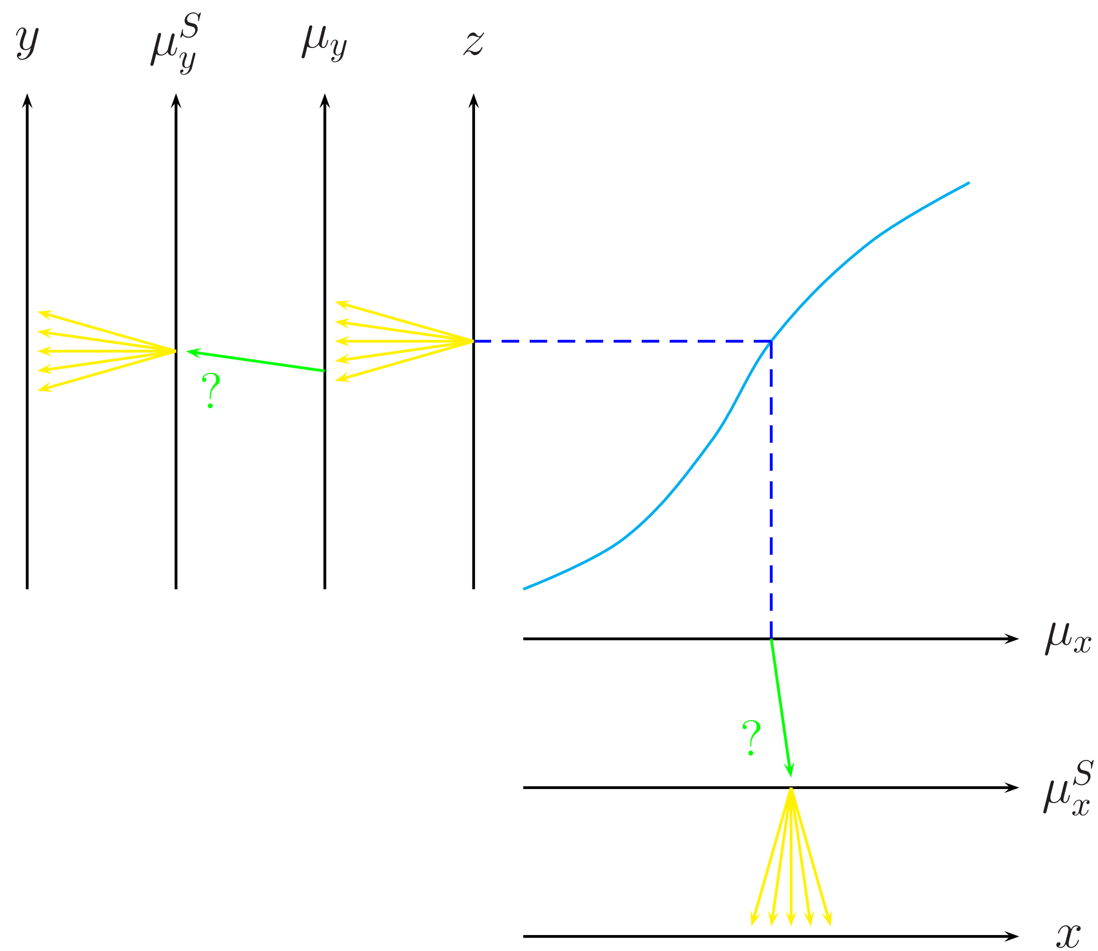
(Guidorzi et al., 2006)

A different way to view fit issues



Adding systematics

A different way to view fit issues



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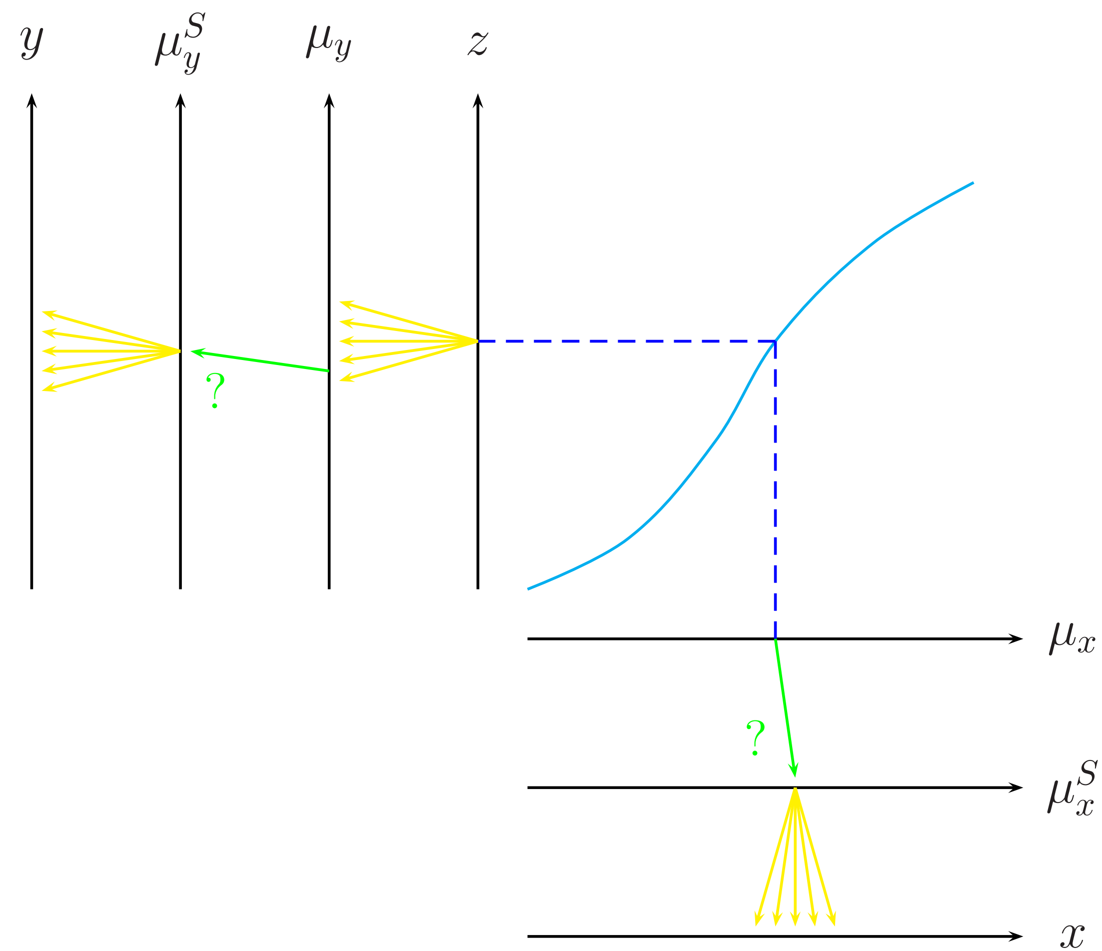
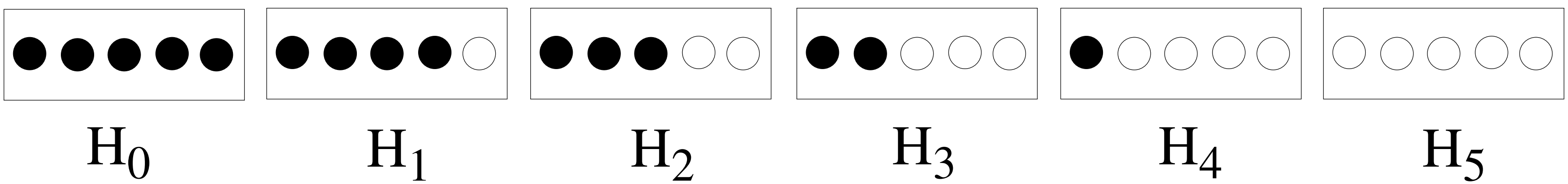


$\Rightarrow$ the mathematical function relating, generally speaking, "y to x" related the true values, not the observations!

Application to the six box problem



Remind:

- $E_1 =$ White
- $E_2 =$ Black

Collecting the pieces of information we need

Our tool:

$$\boxed{P(H_j \,|\, E_i, I) = \frac{P(E_i \,|\, H_j, I)}{P(E_i \,|\, I)}\, P(H_j \,|\, I)}$$

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Our prior belief about H_j

Collecting the pieces of information we need

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Probability of E_i under a well defined hypothesis H_j
It corresponds to the 'response of the apparatus in measurements.
$\rightarrow$ likelihood (traditional, rather confusing name!)

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$\rightarrow$ How much we are confident that E_i will occur.

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Probability of E_i taking account all possible H_j
$\rightarrow$ How much we are confident that E_i will occur.
We can rewrite it as
$P(E_i \,|\, I) = \sum_j P(E_i \,|\, H_j,\, I) \cdot P(H_j \,|\, I)$

We are ready

Now that we have set up our formalism, let's play a little

- analyse real data
- some simulations
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Let's play!

- Hugin Expert (Lite – demo version);
- R scripts

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Simply – and nothing more! – Probability Theory

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(Only some 'technical tricks' to factorize the problem when the number of 'states' becomes very large)

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  (Diffidate chi vi promette di far germogliar zecchini nel Campo dei Miracoli!)

My preferred conclusion

From the *ISO Guide on "the expression of uncertainty in measurement"*

"Although this Guide *provides a framework for assessing uncertainty, it cannot substitute for critical thinking, intellectual honesty, and professional skill. The evaluation of uncertainty is neither a routine task nor a purely mathematical one; it depends on detailed knowledge of the nature of the measurand and of the measurement. The quality and utility of the uncertainty quoted for the result of a measurement therefore ultimately depend on the understanding, critical analysis, and integrity of those who contribute to the assignment of its value."*

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- But it is now possible thank to progresses in applied mathematics and computation.
- It makes little sense to stick to old ‘ah hoc’ methods that had their *raison d’être* in the computational barrier.
- Mistrust all results that sound as ‘confidence’, ’probability’ etc about physics quantities, if they are obtained by methods that do not contemplate ’beliefs’.

References

- *Bayesian Reasoning in Data Analysis – a Critical Introduction*
- *Fits, and especially linear fits, with errors on both axes, extra variance of the data points and other complications*
- *Learning about probabilistic inference and forecasting by playing with multivariate normal distributions (with examples in R)*

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- ISO GUM
- Hugin 'Lite', `http://www.hugin.com/`

The End

FINE

Notes

The following slides should be reached by hyper-links, clicking on highlighted words marked by the symbol †

ISO dictionary

Measurand: *"particular quantity subject to measurement."*

Result of a measurement: *"value attributed to a measurand, obtained by measurement."*

Uncertainty: *"a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurement."*

Error: *"the result of a measurement minus a true value of the measurand."*

True value: *"a value compatible with the definition of a given particular quantity."*

Type A and Type B uncertainties →

Go back

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- *experience with or general knowledge of the behaviour and properties of relevant materials and instruments;*
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Go back

Solution of the AIDS test problem

$$
\begin{aligned}
P(\text{Pos}\,|\,\text{HIV}) &= 100\% \\
P(\text{Pos}\,|\,\overline{\text{HIV}}) &= 0.2\% \\
P(\text{Neg}\,|\,\overline{\text{HIV}}) &= 99.8\%
\end{aligned}
$$

We miss something: $P_\circ(\text{HIV})$ and $P_\circ(\overline{\text{HIV}})$: Yes! We need some input from our best knowledge of the problem. Let us take $P_\circ(\text{HIV}) = 1/600$ and $P_\circ(\overline{\text{HIV}}) \approx 1$ (the result is rather stable against *reasonable* variations of the inputs!)

$$
\begin{aligned}
\frac{P(\text{HIV}\,|\,\text{Pos})}{P(\overline{\text{HIV}}\,|\,\text{Pos})} &= \frac{P(\text{Pos}\,|\,\text{HIV})}{P(\text{Pos}\,|\,\overline{\text{HIV}})} \cdot \frac{P_\circ(\text{HIV})}{P_\circ(\overline{\text{HIV}})} \\
&= \frac{\approx 1}{0.002} \times \frac{0.1/60}{\approx 1} = 500 \times \frac{1}{600} = \frac{1}{1.2}
\end{aligned}
$$