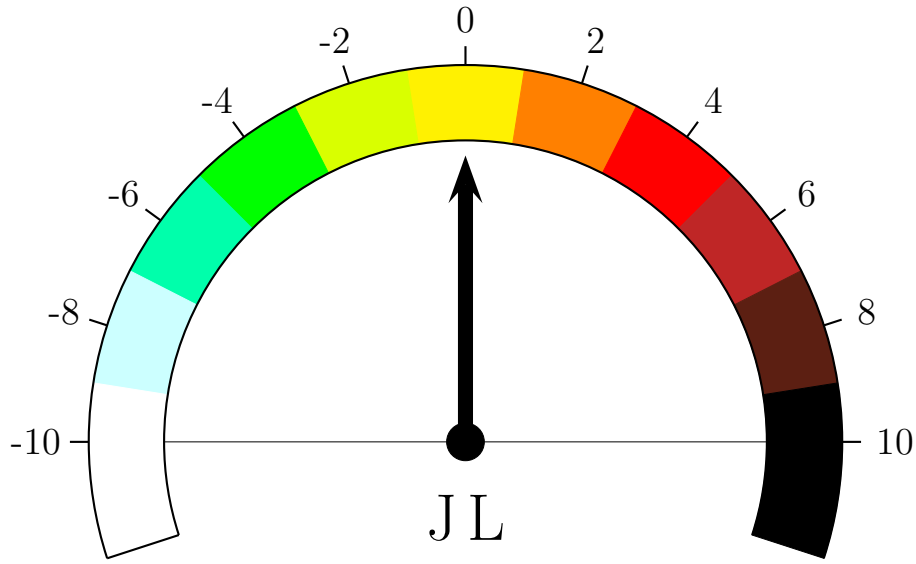
Judgment Leanings



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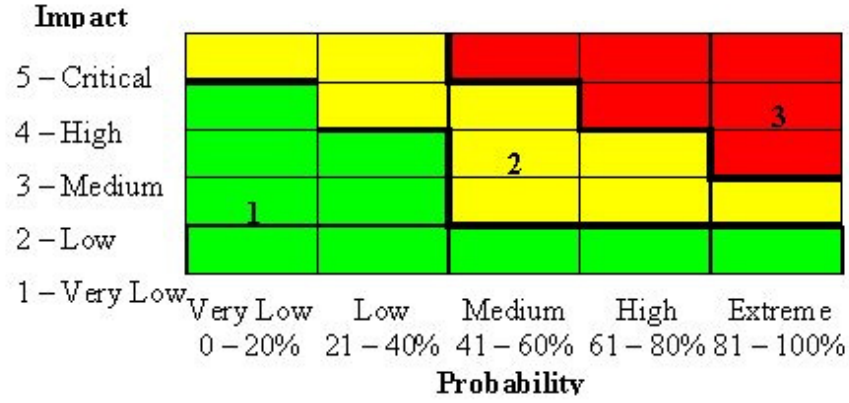
Communication experts $\leftrightarrow$ users

A typical problem in all fields in which there is a distinction of roles between

- experts who evaluates, somehow, *probabilities*;
- decision-makers who have to act on the basis of probability values as well of the related *impacts*.

Example from risk analysis



[And I have personally seen much worst in the risk protocol of a multi-billion enterprise specialized in quite critical products! (with just 4 probability slices)]

$\Rightarrow$ Calling 'extreme' the interval $[0.81, 1]$ is ridiculous!

$\surd$ OK, in Forensic Science such tables are not provided,
BUT what have judges in mind, when they say "above any reasonable doubt"?
[According to Kadane, not much more then 80-90%(!), priv. comm., UAI 2000)]

Linear versus log sensitivity

Well know that human perceptions are not linear with stimuli
(excellent evolutionary solution to adapt ourselves to a large variety of conditions):

- Our response to sound: $\rightarrow$ decibel

$$20 \log_{10} \frac{p}{20\,\mu\text{Pa}}$$

- Our response to light: $\rightarrow$ exposure value (EV)

$$\log_2 \frac{\text{f-number}^2}{t} \leftrightarrow \log_2 \frac{I}{2.5\,\text{lx}}$$

(full moon: ≈ 1 lx; direct sun: ≈ 100000 lx)

Extension to probability, in analogy to sound (Peirce, *The Probability of Induction*, 1878):

$\Rightarrow$ "*feeling of belief*", or "*intensity of beliefs*" as natural log of the odds ('chance' in *PoI*):

$$\log \frac{P(H)}{P(\overline{H})}$$

A “thermometer of beliefs”

Motivations of Peirce’s ”thermometer for the proper intensity of belief”, that

> *“should be as the logarithm of the chance, this latter being the expression of the state of facts which produces the belief”* (by ‘chance’ Peirce meant exactly ‘odds’)

- When the odds go to zero or to infinity, then the intensity of belief on either hypothesis goes to infinity; in particular when “an even chance is reached [the feeling of believing] should completely vanish and not incline either toward or away from the proposition.”
- Several independent pieces of evidence “ought to produce a belief equal to the sum of the intensities of belief which either would produce separately”
  “because we have seen that the chances of independent concurrent arguments are to be multiplied together to get the chance of their combination, and therefore the quantities which best express the intensities of belief should be such that they are to be *added* when the *chances* are multiplied. . . Now, the logarithm of the chance is the only quantity which fulfills this condition”
- Fechnel law:

  > *There is a general law of sensibility, called Fechner’s psychophysical law. It is that the intensity of any sensation is proportional to the logarithm of the external force which produces it*

Weights of evidence

Notes:

1. Peirce's 'chances' are introduced as if they were our odds, but are used if they were Bayes factors ("the chances of independent concurrent arguments are to be multiplied together to get the chance of their combination").
   Then he takes the *natural* logarithm of these 'chances', to which he also associates an idea of *weight of evidence* ("our belief ought to be proportional to the weight of evidence, in the sense, that two arguments which are entirely independent, neither weakening nor strengthening each other, ought, when they concur, to produce a belief equal to the sum of the intensities of belief which either would produce separately")
2. Decibel-like notation (Turing/Good), speaking by experience, tends to confuse, but I adopt Good's
   "Plausibility gained = weight of evidence"
3. Naming 'evidence' the log of the odds (à la Jaynes) is quite bad:
   - 'evidence' has already too many meanings (including – very bad choice! – the probability of the observed effect);
   - such a name should be anyway related to the piece of evidence and not to the final odds.

$\Rightarrow$ Weight of evidence: log of Bayes factor

Judgement Leanings

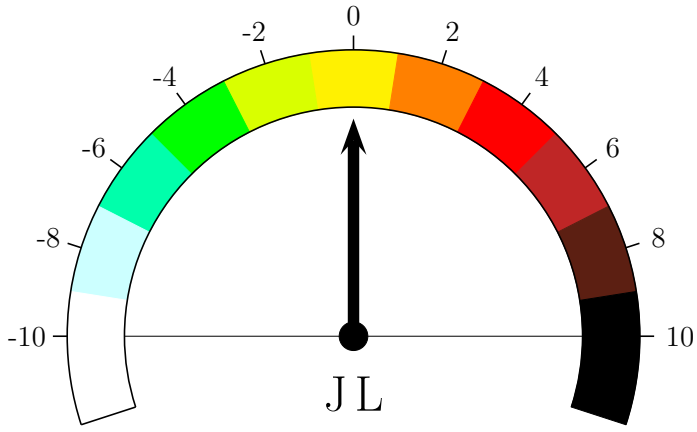
Aims:

1. essentially recover Peirce's intensity of beliefs for the log-odds, together with Peirce/Good's weight of evidence;
2. try a name not already 'overloaded';
3. use decimal log for easy conversion;
4. a graphical representation.

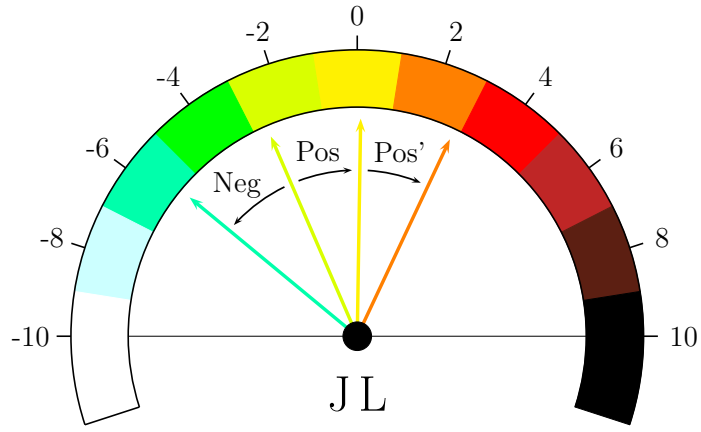
Noting that *Bayes factors can be interpreted as odds due only to an individual piece of evidence, if the two hypotheses were considered initially equally likely*, and indicating them by $\tilde{O}_{i,j}(E)$, while $O_{i,j}(I)$ and $O_{i,j}(E,I)$ are initial and final odds, give the observation E:

$$
\begin{aligned}
O_{i,j}(E,I) &= O_{i,j}(I) \times \tilde{O}_{i,j}(E) \\
\log_{10}[O_{i,j}(\boldsymbol{E},I)] &= \log_{10}[O_{i,j}(I)] + \sum_{k=1}^{n} \log_{10}[\tilde{O}_{i,j}(E_k)] \\
\mathsf{JL}_{i,j}(\boldsymbol{E},I) &= \mathsf{JL}_{i,j}(I) + \sum_{k=1}^{n} \Delta\mathsf{JL}_{i,j}(E_k) \\
&= \mathsf{JL}_{i,j}(I) + \Delta\mathsf{JL}_{i,j}(\boldsymbol{E})
\end{aligned}
$$

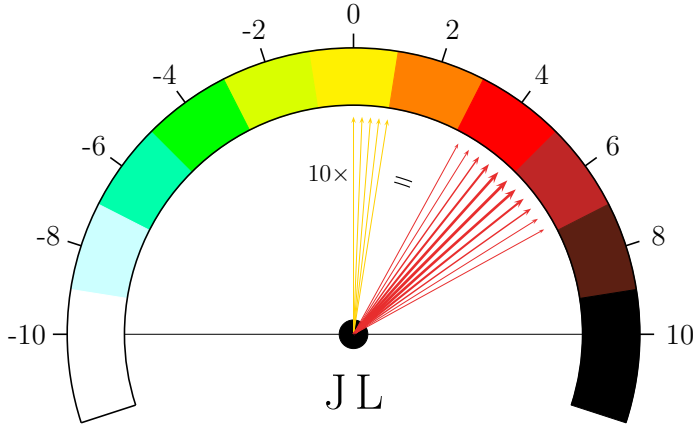
Examples



Neutral situation



An AIDS test



Imprecise ‘weights of evidence’

(“0.5 ± 0.5” $\Rightarrow$ “5.0 ± 1.8”)

More + references in

`http://arxiv.org/abs/1003.2086`