Part IV

Concluding matter

Chapter 10

Conclusions

"You take your life in your own hands, and what happens? A terrible thing: no one to blame."
(Erica Jong)

10.1 About subjective probability and Bayesian inference

I hope to have been able to show that it is possible to build a powerful theory of measurement uncertainty starting from subjective probability and the rules of logics, from which the Bayes' theorem follows. Subjective probability is based on the natural concept of probability, as degree of belief, related to a status of uncertainty, whilst Bayes' theorem is the logical tool to update the probability in the light of new pieces of information.

The main advantages the Bayesian approach has over the others are (in addition to the non-negligible fact that it is able to treat problems on which the others fail):

- the recovery of the intuitive idea of probability as a valid concept for treating scientific problems;
- the simplicity and naturalness of the basic tool;
- the capability of combining prior knowledge and experimental information;
- the automatic updating property as soon as new information is available;
- the transparency of the method which allows the different assumptions on which the inference may depend to be checked and changed;
- the high degree of awareness that it gives to its user.

When employed on the problem of measurement errors, as a special application of conditional probabilities, it allows all possible sources of uncertainties to be treated in the most general way.

When the problems get complicated and the general method becomes too heavy to handle, it is often possible to use approximate methods based on the linearization to evaluate average and standard deviation of the distribution, while the central limit theorem makes the final distributions approximately Gaussian. Nevertheless, there are some cases in which the linearization may cause severe problems, as shown in Section 6.1. In such cases one needs to go back to the general method or to apply other kinds of approximations which are not just blind use of the covariance matrix.

Many conventional (frequentistic) methods can be easily recovered, like maximum likelihood or χ^2 fitting procedures, as approximation of Bayesian methods, when the (implicit) assumptions on which they are based are reasonable.

10.2 Conservative or realistic uncertainty evaluation?

Finally, I would like to conclude with some remarks about safe (or conservative) evaluation of the uncertainty. The normative rule of coherence requires that all probabilistic statements should be consistent with the beliefs. Therefore, if the uncertainty on a physical quantity is modeled with a Gaussian distribution, and one publishes a result as, for example, $\alpha_s = 0.119 \pm 0.03$, one should be no more nor less sure than 68 % that α_s is in that interval (and one should be 95% sure that the value is within ± 0.06, and so on). If one feels more sure than 68 % this should be explicitly stated, because the normal practice of HEP is to publish standard uncertainty in a normal probability model, as also recommended by the ISO Guide[3]. In this respect, the ISO recommendation can be summarized with the following quotation:

> *"This Guide presents a widely applicable method for evaluating and expressing uncertainty in measurement. It provides a realistic rather than a 'safe' value of uncertainty based on the concept that there is no inherent difference between an uncertainty component arising from a random effect and one arising from a correction for a systematic effect. The method stands, therefore, in contrast to certain older methods that have the following two ideas in common:*
>
> - *The first idea is that the uncertainty reported should be 'safe' or 'conservative' (. . .) In fact, because the evaluation of the uncertainty of a measurement result is problematic, it was often made deliberately large.*
> - *The second idea is that the influences that give rise to uncertainty were always recognizable as either 'random' or 'systematic' with the two being of different nature; (. . .) In fact, the method of combining uncertainty was often designed to satisfy the safety requirement."*
>
> *. . . When the value of a measurand is reported, the best estimate of its value and the best estimate of the uncertainty of that estimate must be given, for if the uncertainty is to err, it is not normally possible to decide in which direction it should err safe. An understatement of uncertainties might cause too much trust to be placed in the values reported, with sometimes embarrassing and even disastrous consequences. A deliberate overstatement of uncertainty could also have undesirable repercussions."*

The examples of the 'undesirable repercussions' given by the ISO Guide are of the metrological type. In my opinion there are other physical reasons which should be considered. Deliberately overstating uncertainty leads to a better (but artificial) agreement between results and 'known' values or results of other experiments. This prevents the identification of possible systematic effects which could have biased the result and which can only be identified by performing the measurement of the same physical quantity with a different instrument, method, etc. (the so-called 'reproducibility conditions'[3]). Behind systematic effects there is always some physics, which can somehow be 'trivial' (noise, miscalibration, row approximations, background, etc.), but also some new phenomenology. If the results of different experiments are far beyond their uncertainty the experimenters could compare their methods, find systematic errors and, finally, the combined result will be of a higher quality. In this respect, a quotation from Feynman is in order:

> *"Well, QED is very nice and impressive, but when everything is so neatly wrapped up in blue bows, with all experiments in exact agreement with each other and with the theory - that is when one is learning* **absolutely nothing**.*"*
>
> *"On the other hand, when experiments are in hopeless conflict - or when the observations do not make sense according to conventional ideas, or when none of the new models seems to work, in short when the situation is an unholy mess -* **that** *is when one is really making hidden progress and a breakthrough is just around the corner!"*
> (R. Feynman, 1973 Hawaii Summer Institute, cited by D. Perkins at the 1995 EPS Conference, Brussels).

10.3 Assessment of uncertainty is not a mathematical game

Finally, I would like to conclude with my favourite quotation concerning measurement uncertainty, taken from the ISO Guide [3]:

> *"Although this* Guide *provides a framework for assessing uncertainty, it cannot substitute for critical thinking, intellectual honesty, and professional skill. The evaluation of uncertainty is neither a routine task nor a purely mathematical one; it depends on detailed knowledge of the nature of the measurand and of the measurement. The quality and utility of the uncertainty quoted for the result of a measurement therefore ultimately depend on the understanding, critical analysis, and integrity of those who contribute to the assignment of its value."*

Acknowledgements

This report is based on the work of several years, during which I have had the opportunity to interact, directly or indirectly, with a large variety of persons, most of them physicists of all ages and of many nationalities, but also mathematicians, statisticians and metrologists. In particular, the interest shown by those who attended the lectures, and also the criticisms of those who had strong prejudices towards the approach I was presenting, has been highly stimulating. I take this opportunity to thank them all.

Special acknowledgements go to Romano Scozzafava of "La Sapienza" for many discussions about the fundamental aspects of probability theory. The many clarifications about DIN and ISO recommandations received by Klaus Weise of the PTB Braunschweig (Germany) have been particulary usefull. I would like to thank Fritz Fröhner, Frank Lad of Canterbury University (New Zealand), Gianni Penso of "La Sapienza", Stefan Schlenstedt of Zeuten (Germany), Pia Astone and Mirko Raso of INFN Roma1 for critical comments on the manuscript, as well on the on the old version of the primer. Finally I would like to thank Bruno Pellizzoni of INFN Roma1 for technical support with many of the drawings.

Bibliographic note

The state of the art of Bayesian theory is summarized in Refs. [19] and [86], where many references can be found. A comprehensive and eloquent presentation of the Bayesian approach in scientific reasoning, covering philosophical, mathematical and statistical aspects is given in Ref. [87], a short account of which can be found in a "Nature" article [8]. Very interesting and insightful philosophical and historical aspects of subjective probability are provided in the introduction of Ref. [80]. To get an idea of what present philosophers think about Bayesian theory see also Refs. [88] and [89] and references therein. For a short introduction on subjective probability, as well as its importance in the physics curriculum, see Ref. [21].

As a classical book on subjective probability, de Finetti's *"Theory of probability"* [11] is a must. I found Ref. [90] particularly stimulating and Ref. [32] very convincing (the latter represents, in my opinion, the only real introductory, calculus-based, textbook on subjective probability and Bayesian statistics available so far, with many examples and exercises). Unfortunately these two books are only available in Italian at the moment. For Italian readers, I also recommend Refs. [91] and [92].

I have consulted Refs. [30] and [31], which also contain many references. References [29], [93], [94], [95], [96] [97], [98], [99], [42] and [100] are well-known books among Bayesian. Some literature on Bayesian Networks can be found in Ref. [69], which also contains interesting URLs. Reference [101] is a recent Bayesian book close to the physicist's point of view. For developments on Bayesian theory and practical applications I recommend consulting the proceedings of "Valencia Meetings" [102] and "Maxent Workshops" [103]. An overview of maximum-entropy methods can also be found in Ref. [59]. This last reference and Ref. [36] show some applications of Bayesian reasoning in statistical mechanics. Other information on Bayesian literature methods can be found on web sites. As a starting point I would recommend Ref. [104], as well as other sites dedicated to Bayesian networks and artificial intelligence [69]. When integrals become complicated, the Markov Chain Monte Carlo (MCMC) technique becomes crucial: introductions and applications can be found, for example, in Refs. [95] and [105].

The applied part of these notes, as well as the critical part, is mostly original. References are given at the appropriate place in the text — only those actually used have been indicated. Reference [106] contains applications of some of the methods described here in analyses of HEP data. A concise critical overview of Bayesian reasoning versus frequentistic methods in HEP can be found in Ref. [107], whilst Ref. [22] is recommended to those who are still anxious about priors.

As far as measurement uncertainty is concerned, consultation of the ISO Guide [3] is advised. At present the BIPM recommendations are also followed by the American National Institute of Standards and Technology (NIST), whose guidelines [5] are also on the web.

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