A Comparative Study of Nucleon Structure in Light-Front Quark Models in AdS/QCD

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Abstract

We present the nucleon electromagnetic form factors, using the light-front wave functions of a quark-diquark model for nucleon predicted by the soft-wall model of AdS/QCD. The results are compared with the soft-wall AdS/QCD model. Then we show a comparative study of the nucleon charge and anomalous magnetization densities in the transverse plane. Flavor decompositions of the form factors and transverse densities are also presented.

LF quark-diquark model

- In the quark-diquark model, nucleon is considered to be a bound state of a single quark and a scalar diquark state.
- For a spin $\frac{1}{2}$ composite system the Dirac and Pauli form factors $F_1(q^2)$ and $F_2(q^2)$ are indentified to the helicity-conserving and helicity-flip matrix elements of the vector current J^+

$$\langle P+q,\uparrow|\frac{J^{+}(0)}{2P^+}|P,\uparrow\rangle = F_1(q^2),$$
$$\langle P+q,\uparrow|\frac{J^{+}(0)}{2P^+}|P,\downarrow\rangle = -(q^1-iq^2)\frac{F_2(q^2)}{2M_n},$$

- Using the two particle Fock states for $J^z=+\frac{1}{2}$ and $J^z=-\frac{1}{2}$, the Dirac and Pauli form factors for the quarks can be written in the light-front representation as

$$F_1^q(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\Big[\psi_{+q}^{+*}(x,\mathbf{k}'_\perp)\psi_{+q}^{+}(x,\mathbf{k}_\perp) + \psi_{-q}^{+*}(x,\mathbf{k}'_\perp)\psi_{-q}^{+}(x,\mathbf{k}_\perp)\Big],$$

$$F_2^q(Q^2) = -\frac{2M_n}{q^1-iq^2}\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\Big[\psi_{+q}^{+*}(x,\mathbf{k}'_\perp)\psi_{+q}^{-}(x,\mathbf{k}_\perp) + \psi_{-q}^{+*}(x,\mathbf{k}'_\perp)\psi_{-q}^{-}(x,\mathbf{k}_\perp)\Big],$$

where $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$. For the frame $q=(0,0,\mathbf{q}_\perp)$, $Q^2=\mathbf{q}_\perp^2$.

- $\psi_{\lambda q}^{\Lambda}$ are the LFWF with nucleon helicity Λ and quark q with helicity λ. Wavefunctions are constructed from LFWF predicted by AdS/QCD [1].

AdS/QCD Model I

- This model is originally proposed by Brodsky and Téramond [2].
- Action for Dirac fields in AdS_5 in this model

$$S = \int d^4x dz\sqrt{g}\Big(\frac{i}{2}\bar{\Psi}e_A^M\Gamma^A D_M\Psi - \frac{i}{2}(D_M\bar{\Psi})e_A^M\Gamma^A\Psi - (\mu+U(z))\bar{\Psi}\Psi\Big)$$

where $e_A^M=(z/R)\delta_A^M$, $\sqrt{g}=(R/z)^5$, and $\Gamma_A=\{\gamma_\mu,-i\gamma_5\}$.

- The action leads to the AdS solutions $\psi_+(z)$ and $\psi_-(z)$ (correspond to different orbital angular momentum $L^z=0$ and $L^z=+1$),

$$\psi_+(z) = \frac{\sqrt{2}\kappa^2}{R^2}z^{7/2}e^{-\kappa^2z^2/2}, \qquad \psi_-(z) = \frac{\kappa^3}{R^2}z^{9/2}e^{-\kappa^2z^2/2}.$$

- The Dirac (spin non-flip) form factors

$$F_1^p(Q^2) = R^4\int\frac{dz}{z^4}V(Q^2,z)\psi_+^2(z),$$
$$F_1^n(Q^2) = -\frac{1}{3}R^4\int\frac{dz}{z^4}V(Q^2,z)(\psi_+^2(z)-\psi_-^2(z)).$$

- The Pauli (spin flip) form factor is modeled as

$$F_2^{p/n}(Q^2) = \kappa_{p/n}R^4\int\frac{dz}{z^3}\psi_+(z)V(Q^2,z)\psi_-(z).$$

- We use the value of $\kappa=0.4$ GeV and $V(Q^2,z)$ is the bulk to boundary propagator, related to electormagnetic field.

AdS/QCD Model II

- This model has been proposed by Abidin and Carlson [3].
- They add the following extra gauge invariant term to the action

$$\eta\int d^4x dz\sqrt{g}\,\frac{i}{2}\bar{\Psi}e_A^M e_B^N\,[\Gamma^A,\Gamma^B]\,F_{MN}\Psi.$$

- This term produces the Pauli (spin flip) form factors F_2.
- Also provides a contribution to the Dirac (spin non-flip) form factor F_1.

$$F_1^p(Q^2) = C_1(Q^2)+\eta_pC_2(Q^2), \qquad F_1^n(Q^2)=\eta_nC_2(Q^2),$$
$$F_2^p(Q^2) = \eta_pC_3(Q^2), \qquad F_2^n(Q^2)=\eta_nC_3(Q^2).$$

- $\eta_pC_2(Q^2)$ is an additional contribution to the Dirac form factor.
- In this model, the value of $\kappa=0.350$ GeV and the other parameters $\eta_p=0.224$ and $\eta_n=-0.239$.

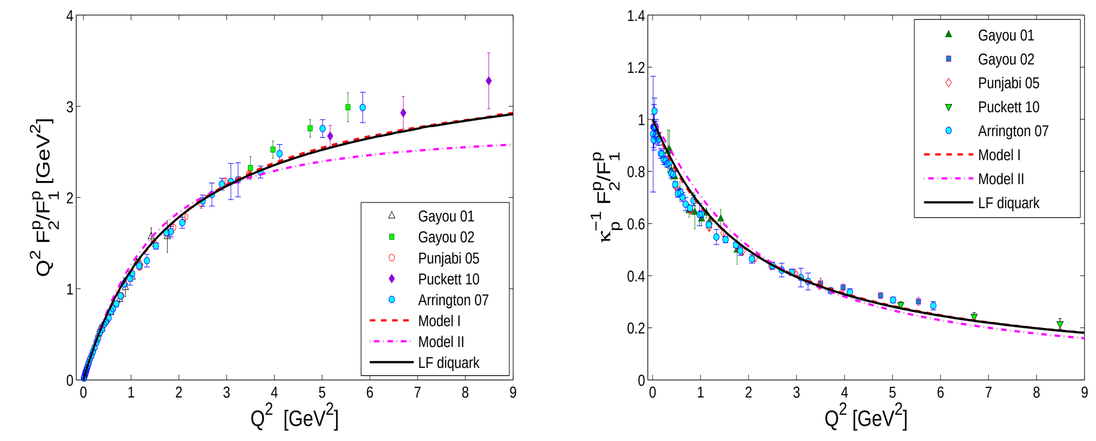


FIGURE 1: Ratio of Pauli and Dirac form factors for proton [4].

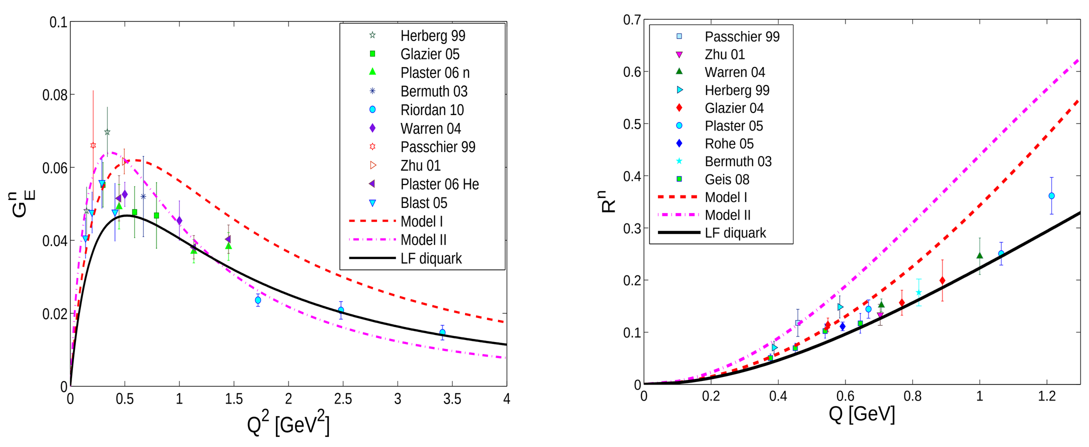


FIGURE 2: Neutron Sach form factor $G_E^n(Q^2) = F_1^n(Q^2) - \frac{Q^2}{4M^2}F_2^n(Q^2)$ and the ratio $R^n=\frac{\mu_n G_E^n}{G_M^n}$ [4].

Flavor decompositions

- Under the charge and isospin symmetry, the flavor decompositions of the nucleon FFs [Cates et. al. PRL 106, 252003 (2011)]

$$F_i^p = e_uF_i^u + e_dF_i^d \qquad F_i^n = e_uF_i^d + e_dF_i^u$$

- Normalizations : $F_1^u(0)=2$, $F_1^d(0)=1$ and $F_2^u(0)=\kappa_u$, $F_2^d(0)=\kappa_d$.
- Anomalous magnetic moments : $\kappa_u=2\kappa_p+\kappa_n=1.673$ and $\kappa_d=\kappa_p+2\kappa_n=-2.033$.

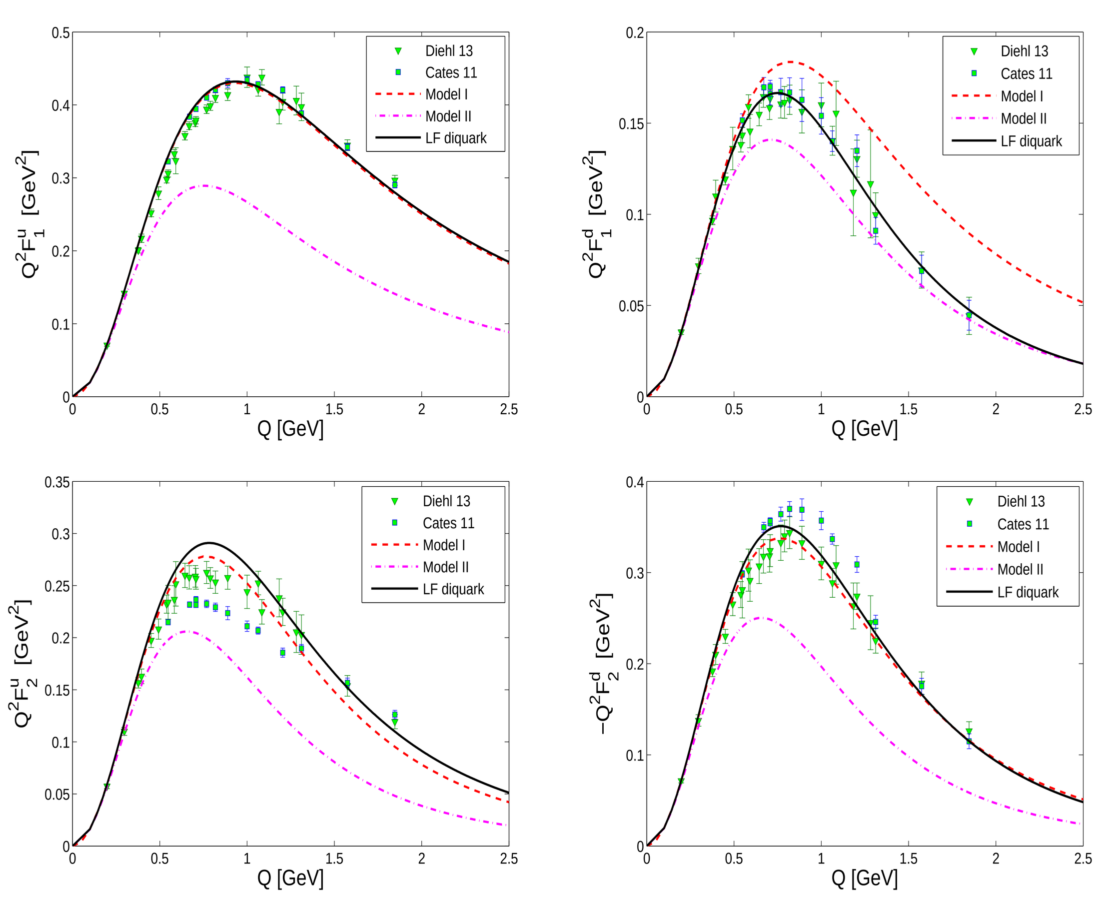


FIGURE 3: Plots of flavor dependent form factors for u and d quarks [4].

Transverse densities

- The transverse charge density inside the nucleons :

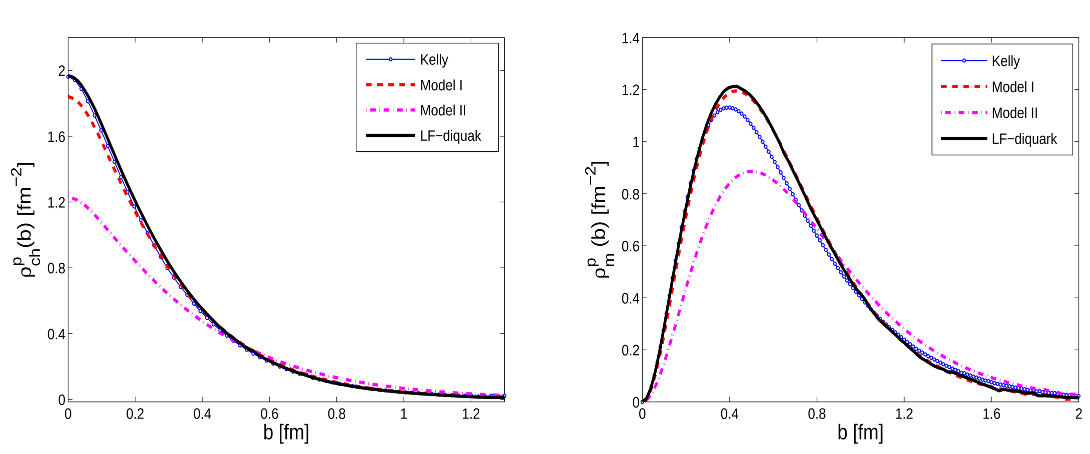
$$\rho_{ch}(b) = \int\frac{d^2q_\perp}{(2\pi)^2}F_1(q^2)e^{iq_\perp.b_\perp} = \int_0^\infty\frac{dQ}{2\pi}QJ_0(Qb)F_1(Q^2).$$

- The anomalous magnetization density :

$$\rho_m(b) = -b\frac{\partial\widetilde{\rho}_M(b)}{\partial b} = b\int_0^\infty\frac{dQ}{2\pi}Q^2J_1(Qb)F_2(Q^2).$$

- For transversely polarized nucleon with transverse spin : $\vec{S}_\perp=\cos\phi_s\hat{x}+\sin\phi_s\hat{y}$., the charge density is given by

$$\rho_T(b) = \rho_{ch}(b) - \sin(\phi_b-\phi_s)\frac{1}{2Mb}\rho_m(b).$$



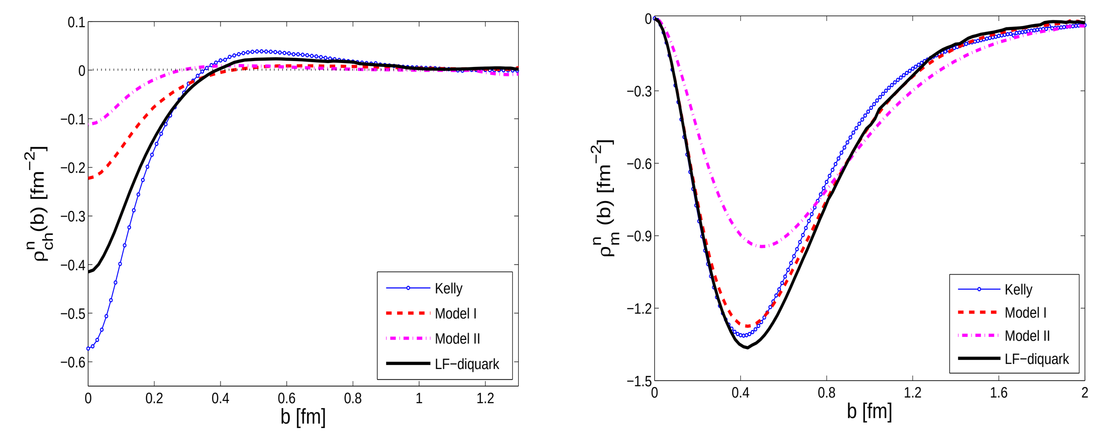


FIGURE 4: Plots of charge and anomalous magnetization densities for proton and neutron in transverse plane [4].

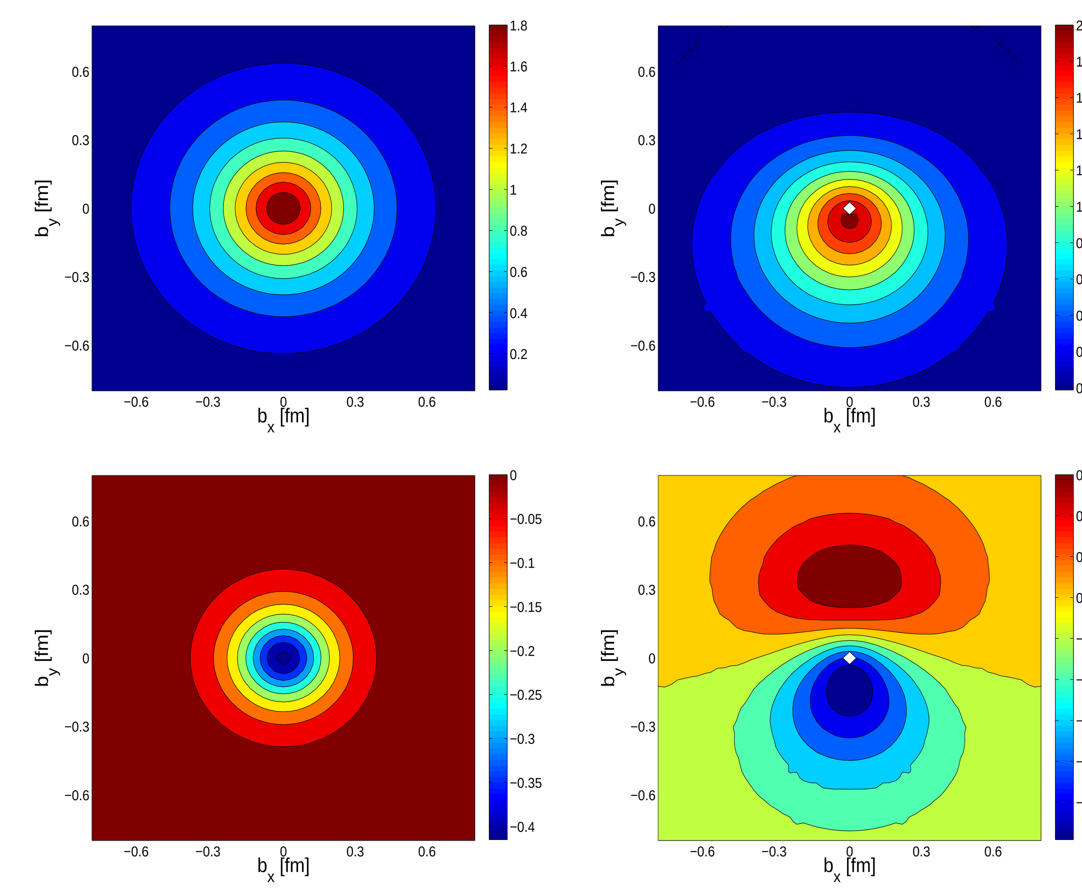


FIGURE 5: Unpolarized and transversely polarized charge densities for proton(upper) and neutron(lower) in LF diquark model [4].

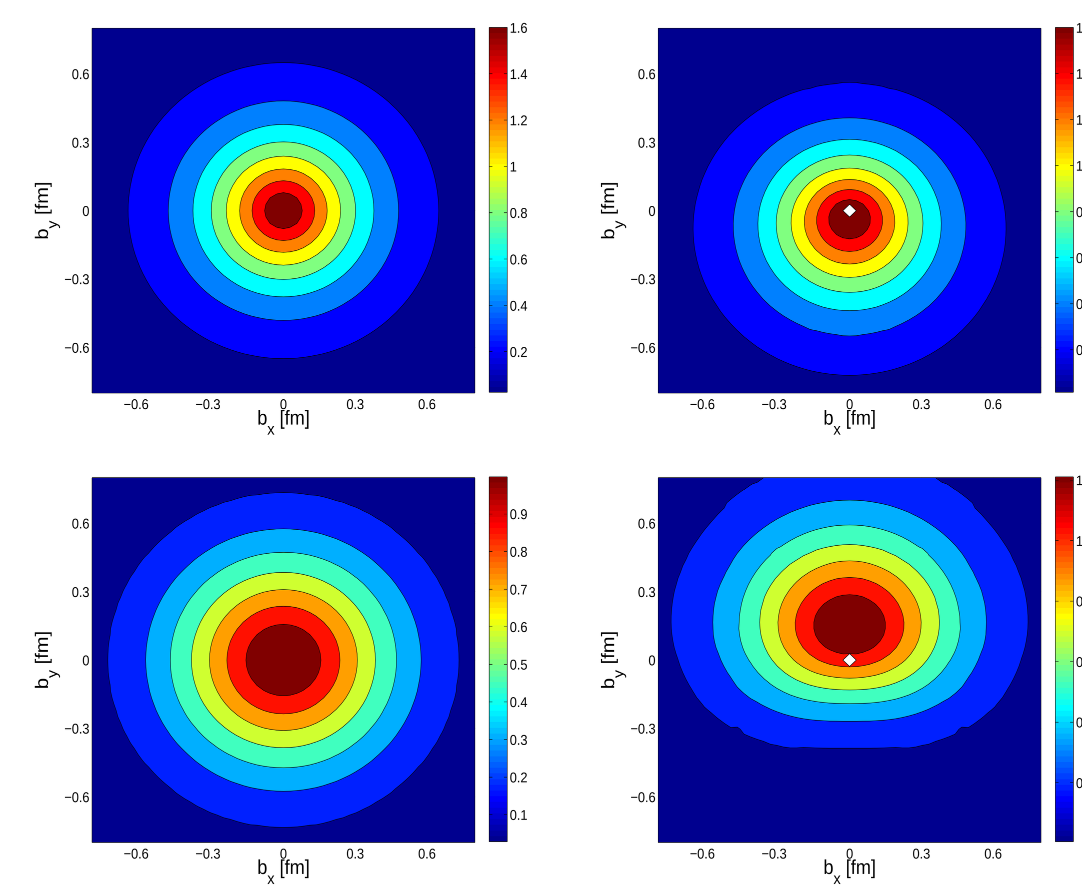


FIGURE 6: u(upper) and d(lower) quarks charge densities in a unpolarized and transversely polarized nucleon in LF diquark model [4].

Electromagnetic radii

$$\langle r_E^2\rangle^N = -6\frac{dG_E^N(Q^2)}{dQ^2}\Big|_{Q^2=0},$$
$$\langle r_M^2\rangle^N = -\frac{6}{G_M^N(0)}\frac{dG_M^N(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

The Sachs form factors are defined as

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4M_N^2}F_2^N(Q^2),$$
$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2).$$

Quantity	Model I	Model II	LF diquark	Measured data
r_E^p (fm)	0.810	0.980	0.786	0.877 ± 0.005
r_M^p (fm)	0.782	0.921	0.772	0.777 ± 0.016
$\langle r_E^2\rangle^n$ (fm^2)	−0.088	−0.123	−0.085	-0.1161 ± 0.0022
r_M^n (fm)	0.796	0.937	0.7596	$0.862^{+0.009}_{-0.008}$

References

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