Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali di Frascati

CHANNELING:
From Crystal Undulators to Capillary Waveguides

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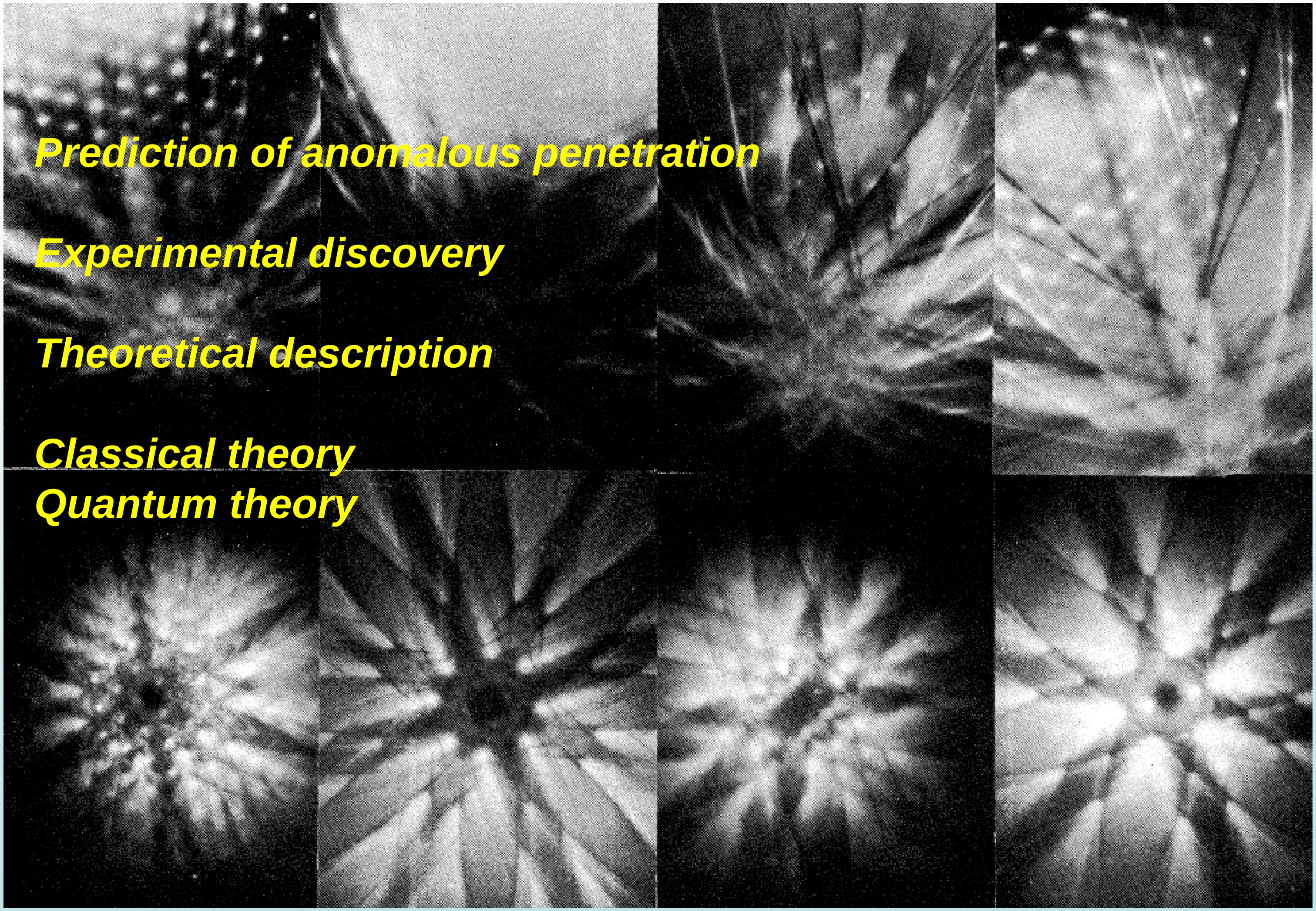
Channeling: Orientational effects of transmission

- 1962-63:
  - Robinson & Oen: *Prediction of anomalous penetration*
  - Piercy & Lutz: *Experimental discovery*
- 1965:
  - Lindhard: *Theoretical description*

Andersen
Kagan
Kononez
Firsov
Kumakhov
Beloshitsky
Bonderup
Gemmel
Appleton
Gibson
.....

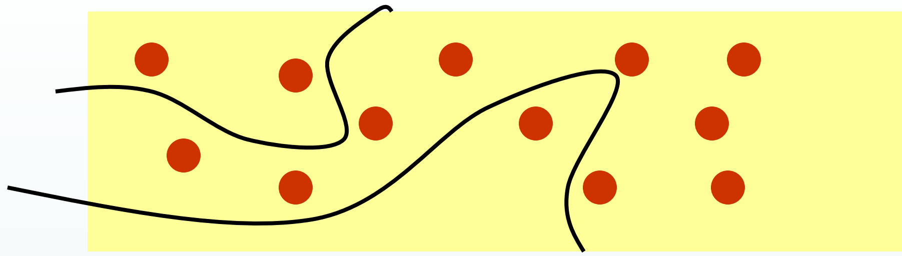
Classical theory
Quantum theory



1 MeV e^- @ Cu crystal

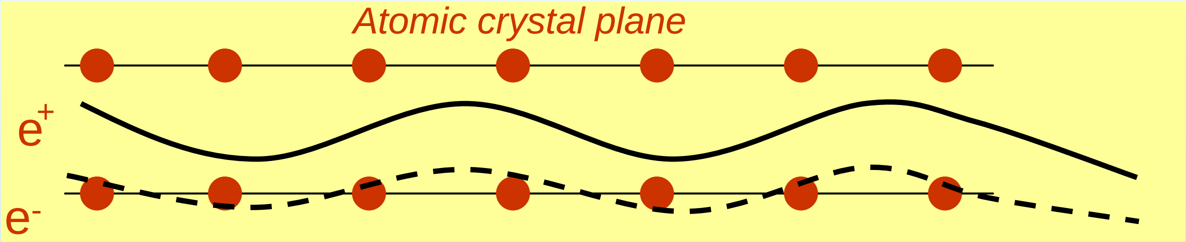
Channeling of Charged Particles

@ Amorphous:

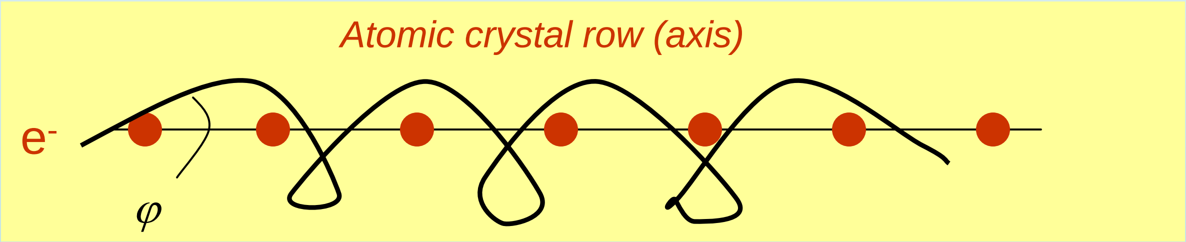


@ Channeling:

planar channeling



axial channeling



$\varphi << 1 \qquad (\varphi < \varphi_L \sim \sqrt{U/E})$ - *the Lindhard angle is the critical angle for the channeling*

Channeling: Continuum model

$$V(r) = \frac{Z_1 Z_2 e^2}{r} \varphi(r/a)$$

screening function of Thomas-Fermi type

$$a = .8853 a_0 \left(Z_1^{1/2} + Z_2^{1/2}\right)^{-2/3}$$

screening length

$$\varphi(r/a): \quad \sum_{i=1}^{3} \alpha_i \exp(-\beta_i r/a)$$

Molier's potential

$$1 - \left[1 + \frac{Ca}{r^2}\right]^{-1/2} \qquad C^2 \approx 3$$

Lindhard potential

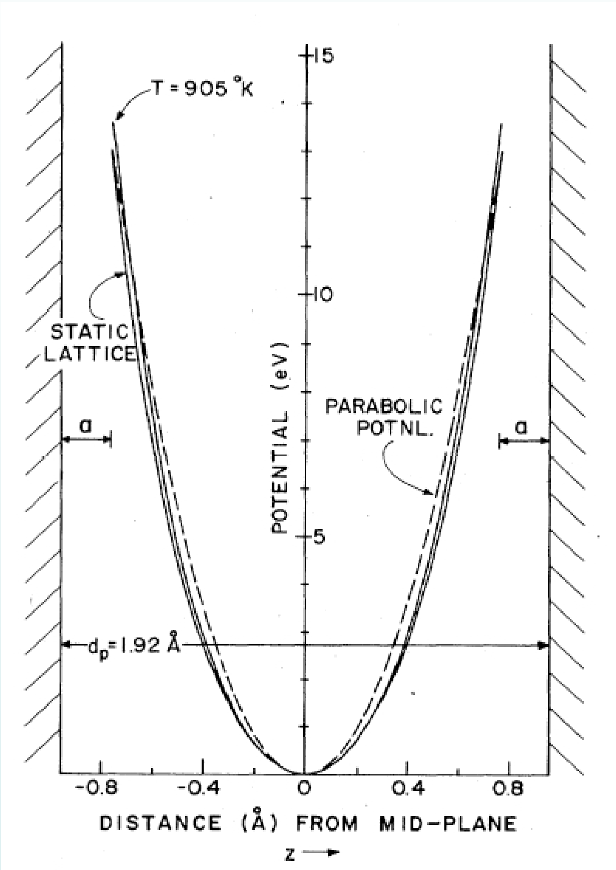
…… Firsov, Doyle-Turner, etc.

Lindhard: *Continuum model* *– continuum atomic plane/axis potential*

$$V_{RS}(\rho) = \frac{1}{d} \int_{-\infty}^{+\infty} V\left(\sqrt{\rho^2 + x^2}\right) dx$$

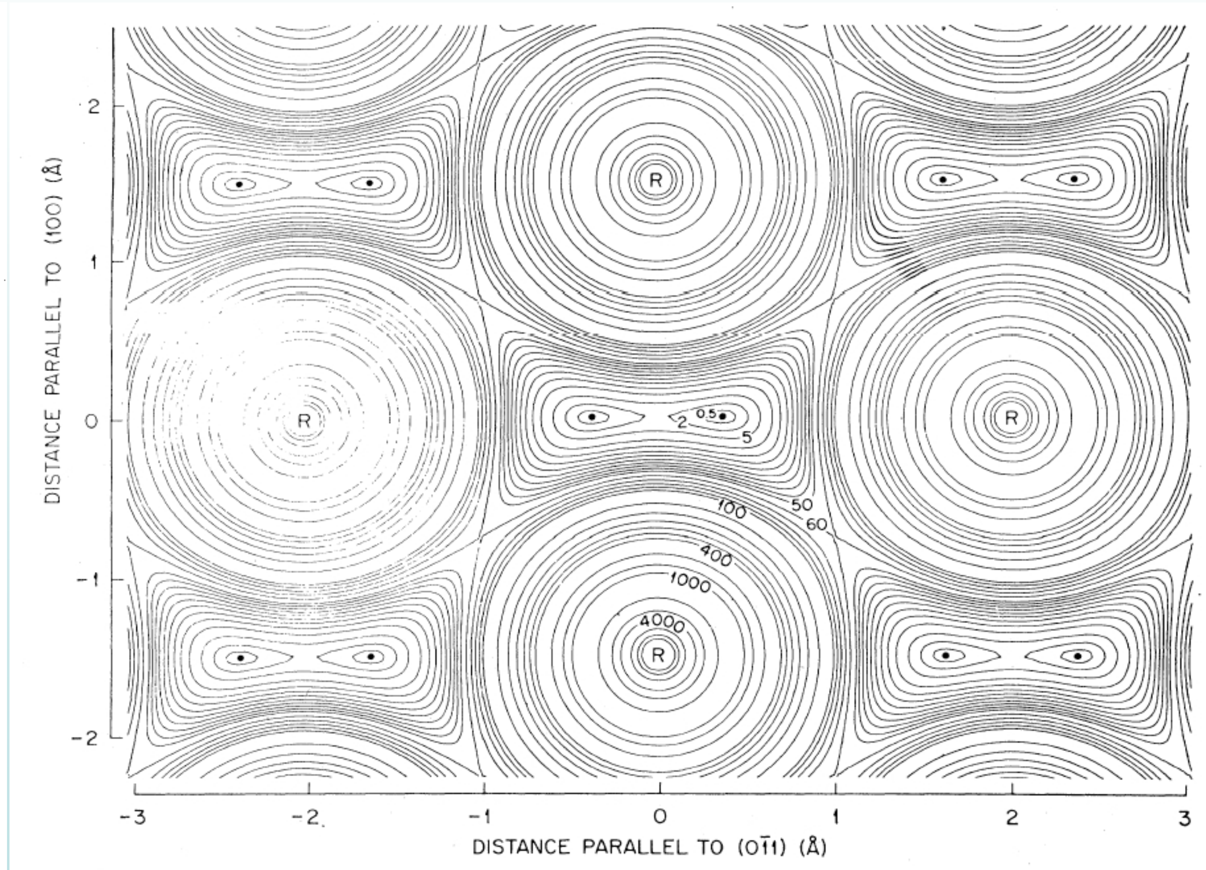
Channeling: Continuum axial and planar potentials

Planar potential



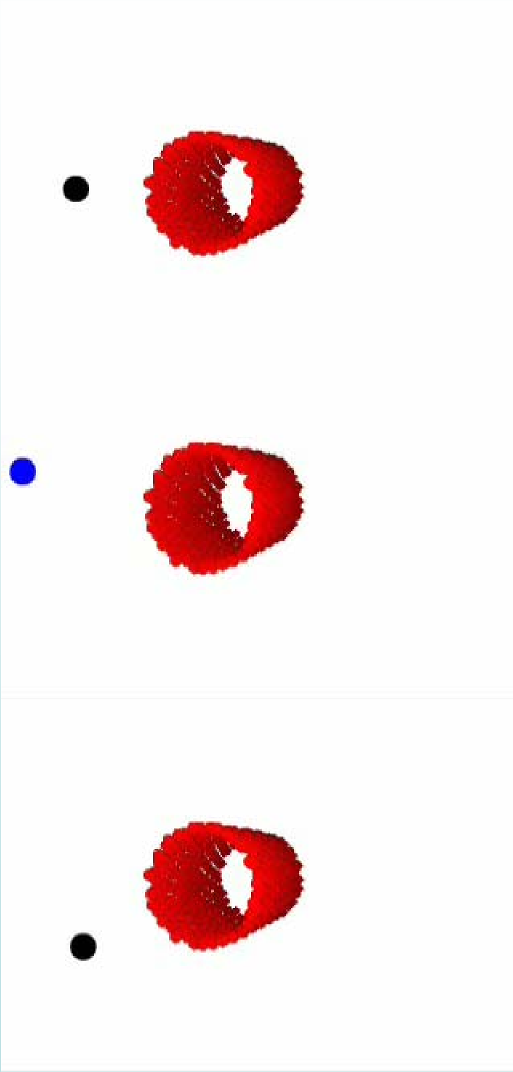
p → Si (100)

Axial potentials



I^+ → Ag

Variations in interaction: particle -> nanotube

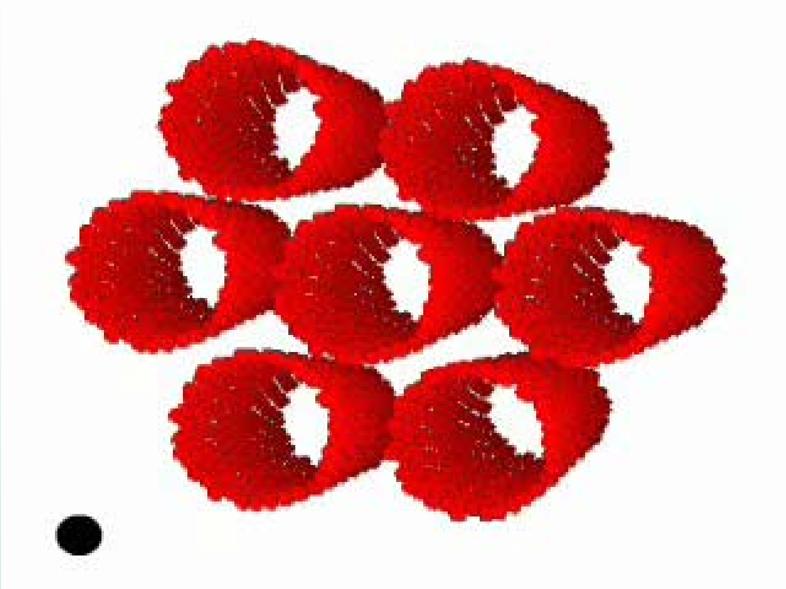


@ *scattering in single nanotube*

@ *tunneling in single nanotube - diffraction*

@ *channeling in single nanotube*

Variations in interaction: particle -> nanotube

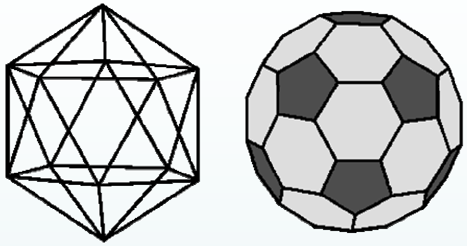


Particles channeling in space between various single nanotubes:

Averaged potential is formed by separate nanotubes

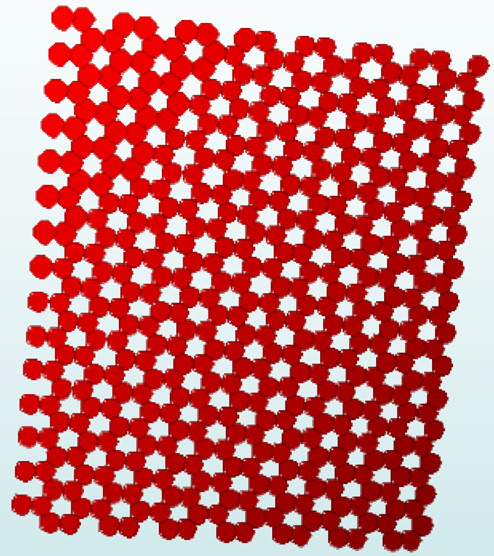
In reality we deal with various combinations of channeling types

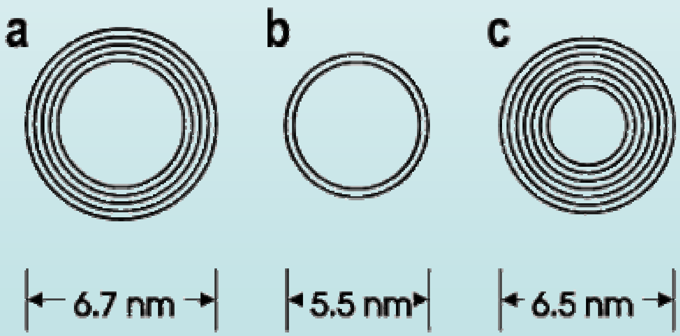
Nanotubes: continuum potential example



Base: *fullerene molecule C_{60}*
sphere of d ~ 0.7 nm

Nanosheet CC

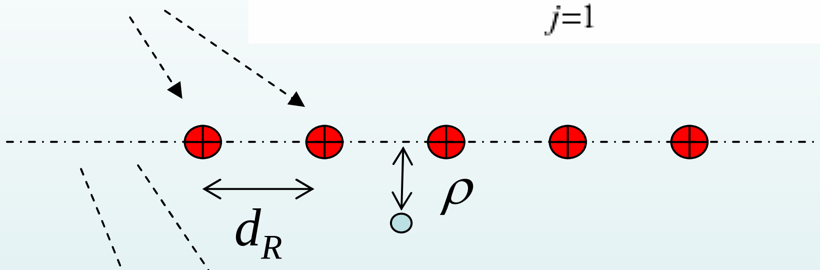




Roled graphite sheets:
nested nanotubes

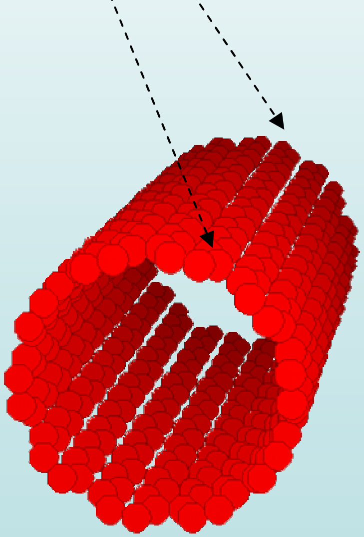
Potentials: Doyle-Turner approximation

$$f(\mathbf{k}) = 4\pi Ze \sum_{j=1}^{N} a_j \exp(-k^2/4b_j^2)$$ - *form-factor for the separate fullerene*



$$V_R(\rho) = (4Ze^2/d_R) \sum_{j=1}^{N} a_j b_j^2 \exp(-b_j^2 \rho^2)$$

$$U(\mathbf{r}) = \sum_i V_R(|\mathbf{r} - \mathbf{r}_i|)$$ *continuum potential as sum of row potentials*



$$U(r) = (16\pi d Ze^2/3\sqrt{3}\, l^2) \sum_{j=1}^{N} a_j b_j^2 \exp\{-b_j^2[r^2 + (d/2)^2]\} I_0(b_j^2 r d)$$

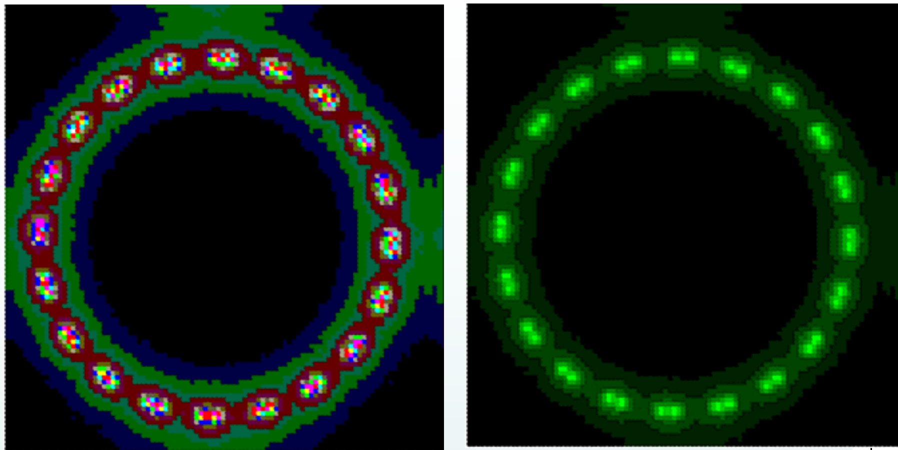
r – distance from the tube

$I_0(x)$ – mod. Bessel function

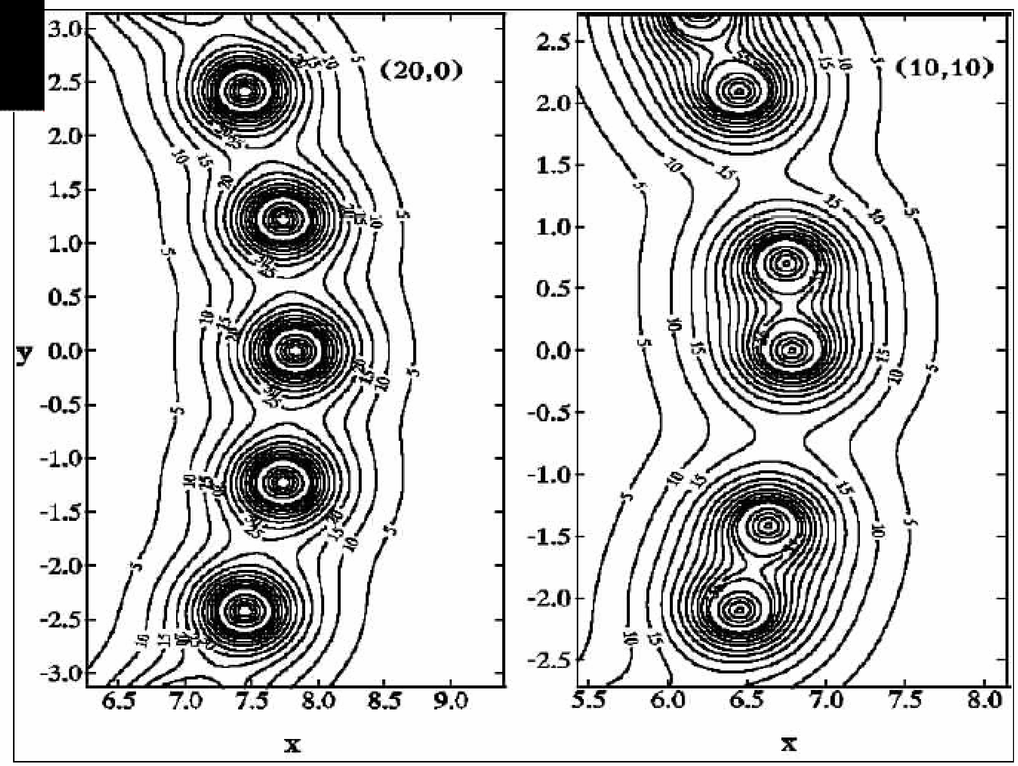
Phys. Lett. A250 (1998) 360

NIM B143 (1998) 584

Potentials: Doyle-Turner approximation



(Maisheev)



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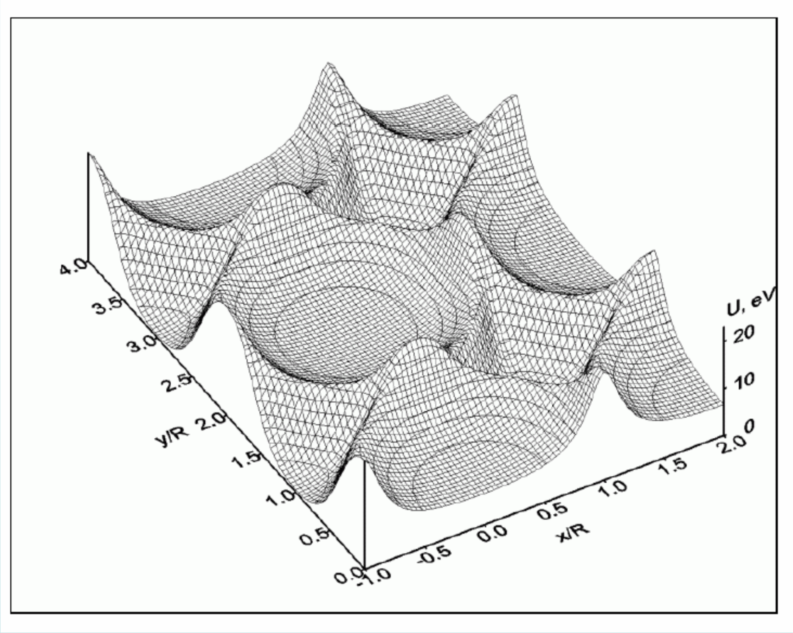
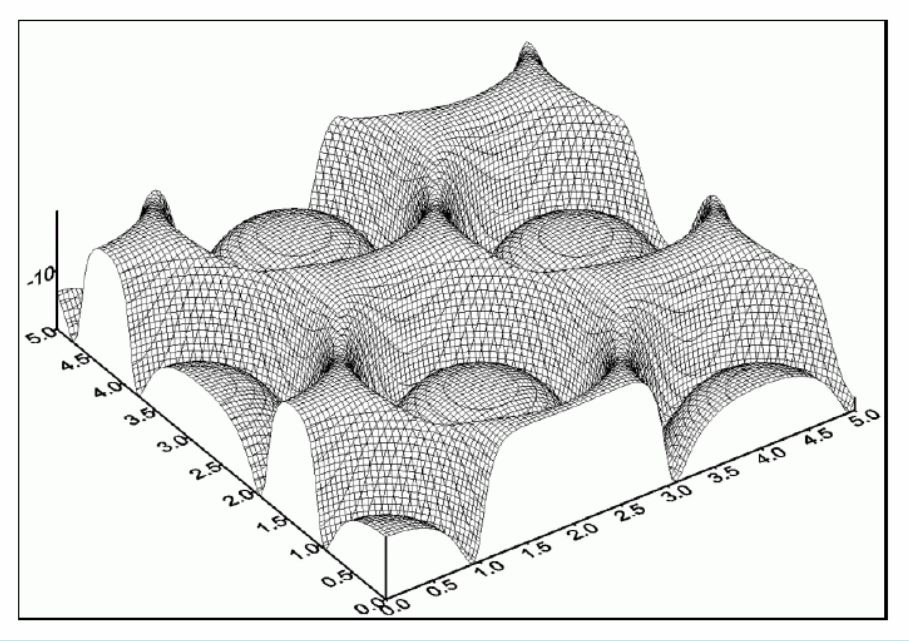
Potentials: Doyle-Turner approximation

Continuum potential in C60 fullerite: [100] and [110]

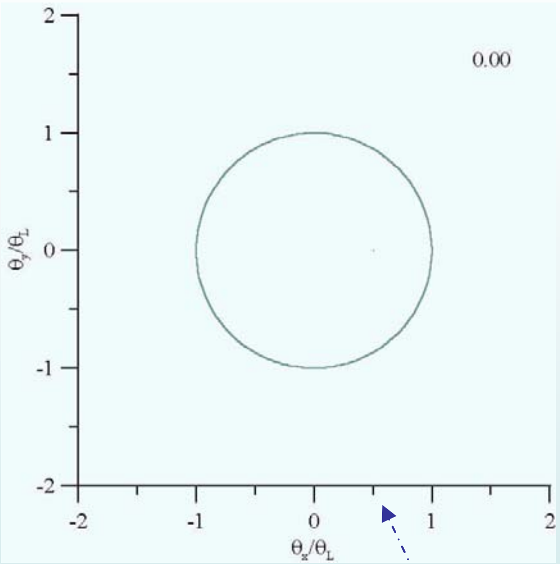


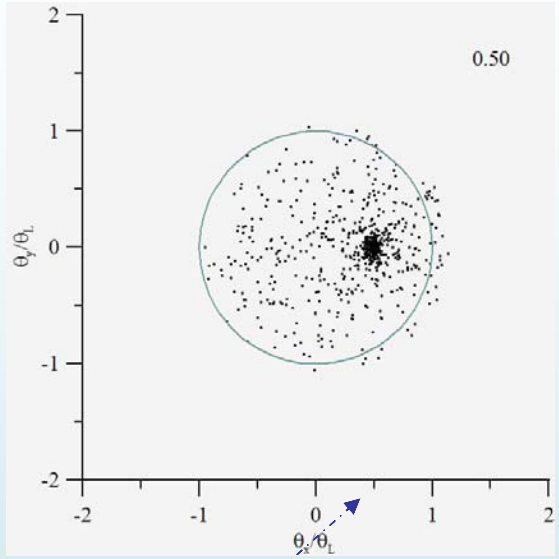


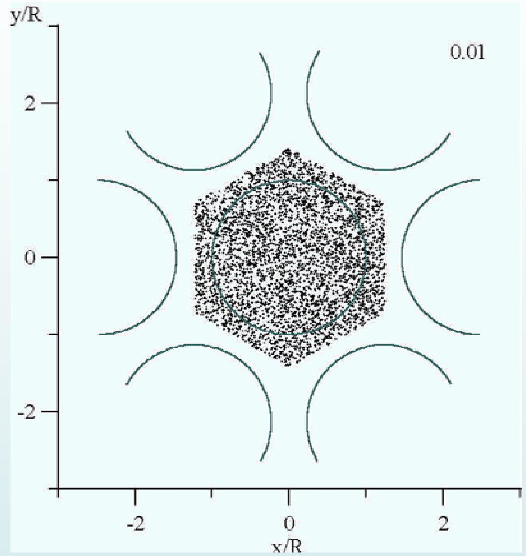
Simulations for particles channeling (straight)

Angular distributions

Spatial







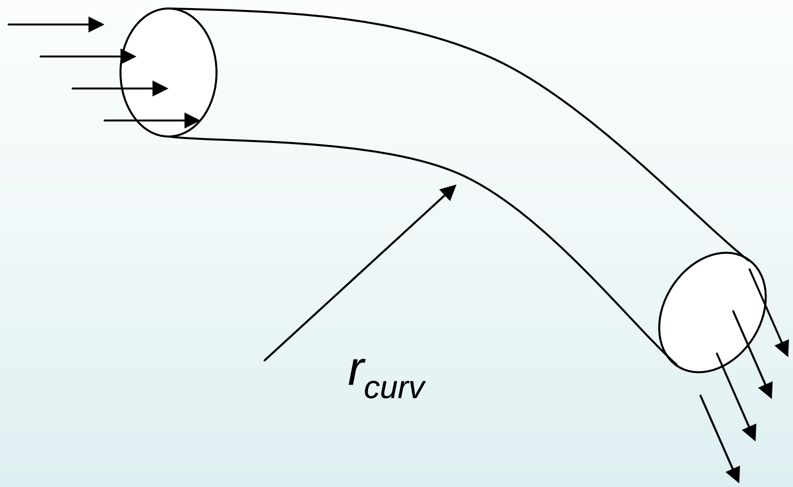
Coherent scattering:
$0\text{-}L_0$

Multiple reflections:
$L_0\text{-}20L_0\text{-}10^3L_0$

Flux redistribution:
Channeling & dechanneling

Angle of incidence – 0.5 critical angle of channeling

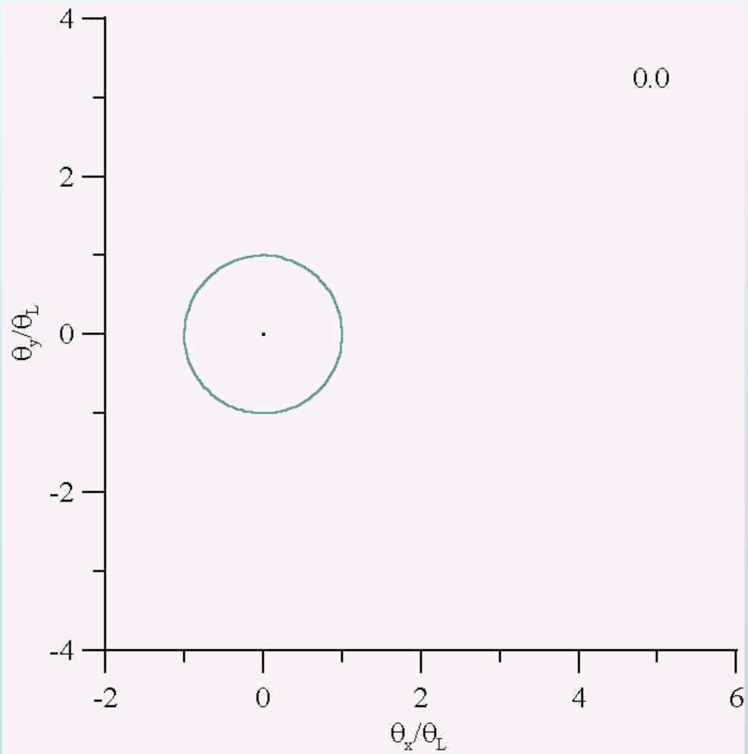
Simulations for particle channeling (bending)



Evolution of angular distribution

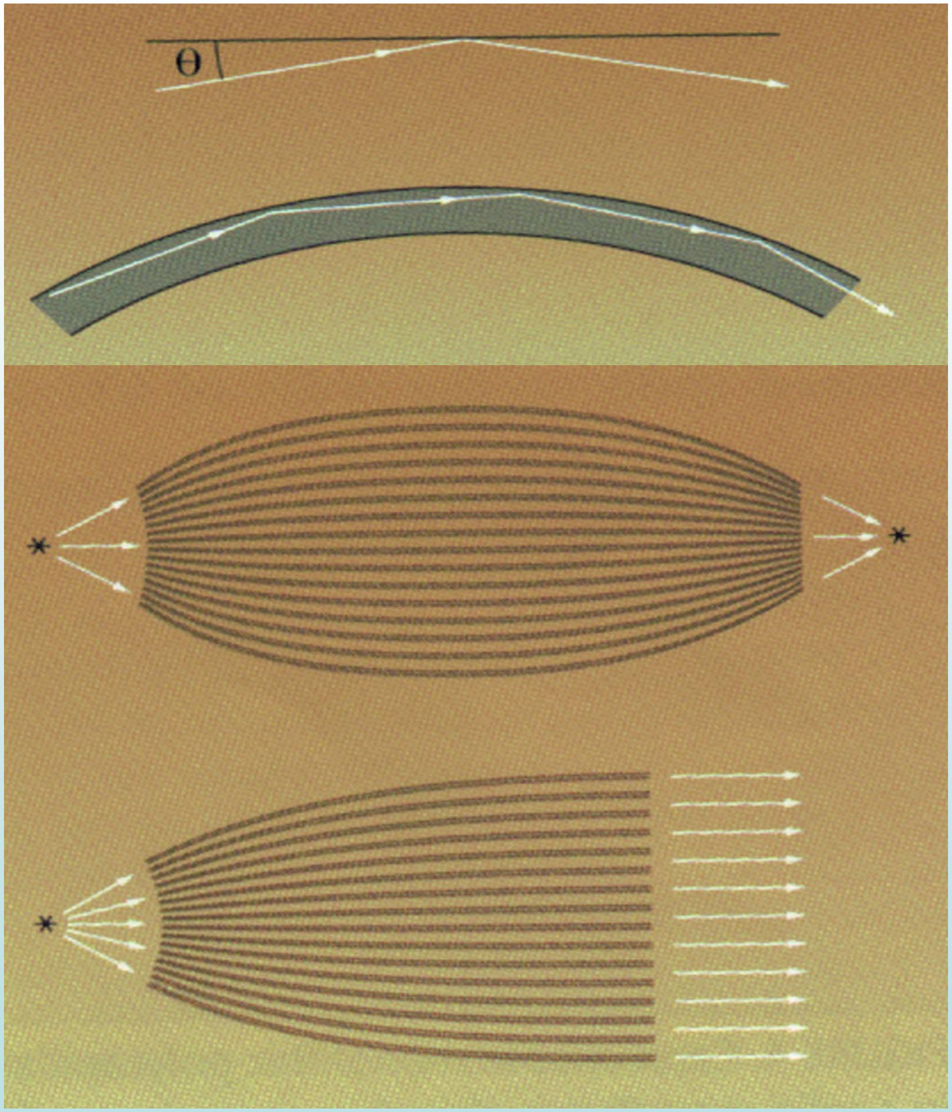
r_{curv} ~ 2 m :

Strong bending effect



X-ray and neutron capillary optics

@ Basic idea of **polycapillary optics** is very close to the phenomenon of charged **particle channeling**



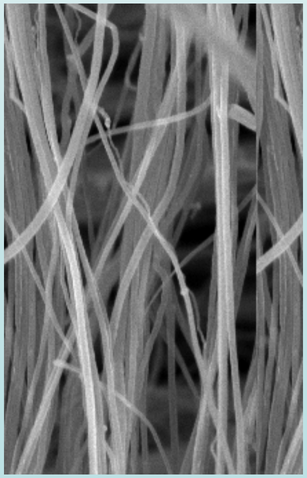
X-ray Channeling: samples of capillary optics

1st generation:
[m]

2d generation:
[cm]

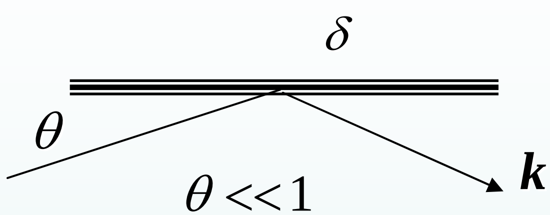
3d & 4th generations:
[mm]

5th generation:
[μm]



?n-capillaries?

http://www.unisantis.com
http://www.iroptic.com



$$\theta << 1$$

$$k_\perp \approx k\theta$$

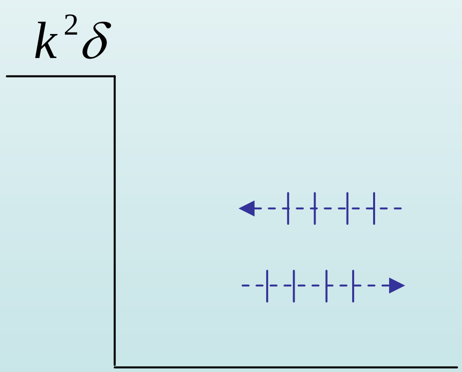
1st order: $\Delta\varepsilon(\vec{r}) = 0$ *- no roughness*

Wave equation:

$$\left(-\nabla^2 + \underbrace{k^2\delta(\vec{r}_\perp) - k_\perp^{\,2}}_{V_{eff}}\right)E(\vec{r}_\perp) = 0$$

$$k^2\left(\delta(\vec{r}_\perp) - \theta^2\right) =$$

$$= \begin{cases} -k^2\theta^2, & r_\perp < r_1 \\ \underbrace{k^2\left(\delta_0 - \theta^2\right)}, & r_\perp \geq r_1 \end{cases}$$

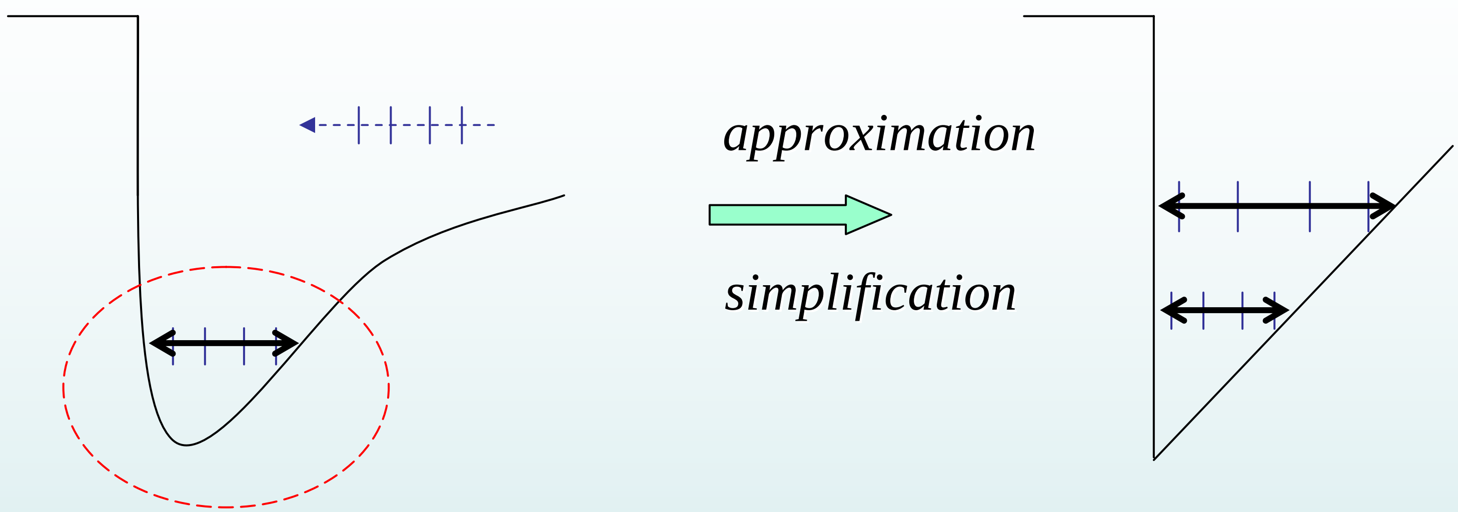


plane (flat) surface

Total external reflection

$$V_{eff} \equiv 0 \Rightarrow \theta_c \equiv \theta \approx \sqrt{\delta_0}$$

Quantum base (2) - curvature



additional term to

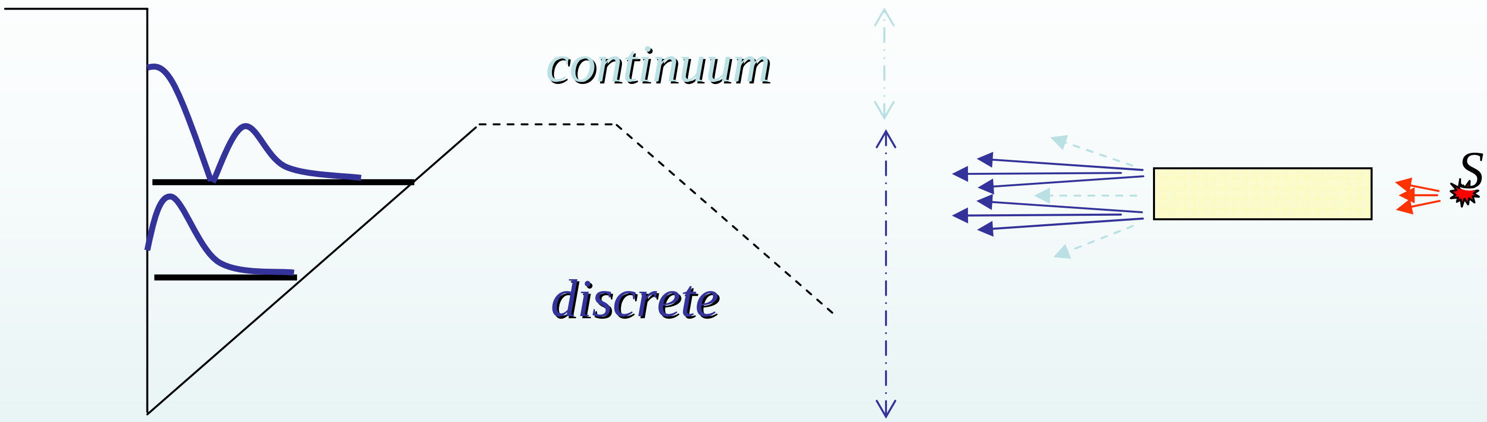
$$V_{eff} \longrightarrow 2\frac{-k^2 r_\perp}{r_{curv}}$$

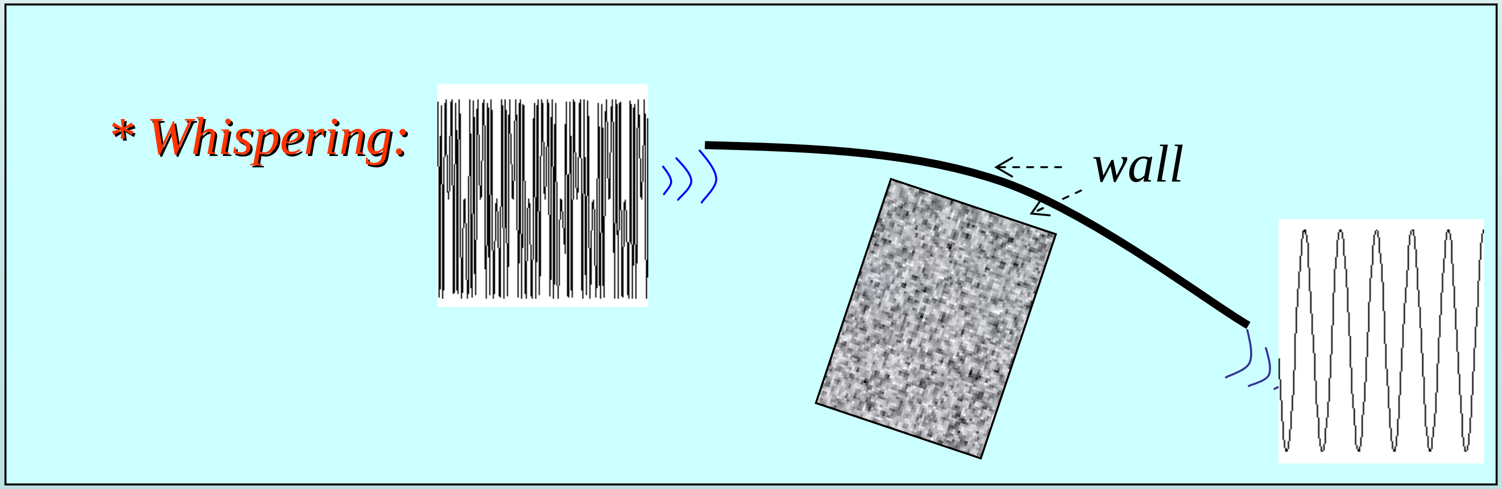
"potential energy": angular momentum of photon

$$k r_{curv} \varphi$$

$$V_{eff} = k^2 \left(\delta\left(\vec{r}_\perp\right) - \theta^2 - 2\frac{r_\perp}{r_{curv}} \right)$$

Surface channeling - "whispering X gallery"



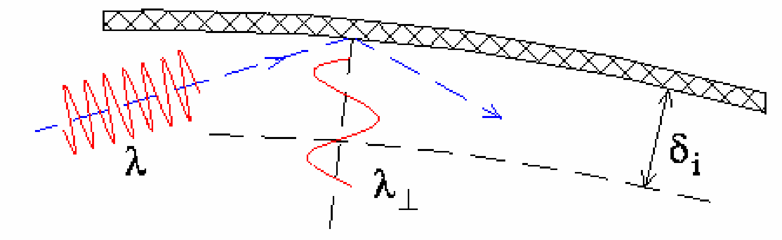


Modes of channeling along curved surfaces

$$\vec{\mathbf{k}} = (\mathbf{k}_\perp, \mathbf{k}_{||})$$

$$k_\perp \simeq k\theta \; (\theta < \theta_c)$$

$$\lambda_\perp = \lambda/\theta >> \lambda$$



Effective guide channel → $\delta_i(\theta) \simeq \lambda_\perp(\theta)$

$$(r_{curv})_i \theta^3 \sim \lambda$$

$$(r_{curv})_i \; = \; 1\,cm \div 1\,m$$

$$\theta \simeq 10^{-3} rad$$

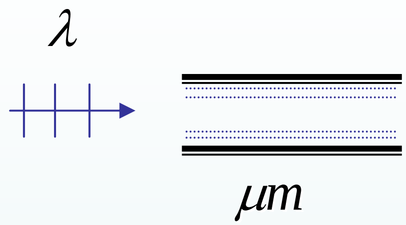
$$\lambda \simeq 0.1 \div 10\ \text{Å}$$

Upper limit of the curvature radius

$$(r_{curv})_m \sim 10\ cm$$

$$\lambda \sim 1\ \text{Å}$$

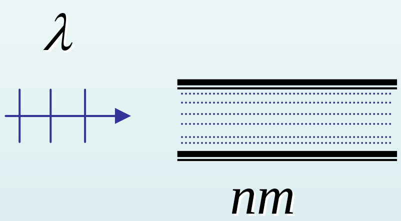
Down to bulk photon and neutron channeling



$\theta << 1 \quad (\theta_c \sim 10^{-3})$: *grazing incidence optics*

$\lambda \quad \rightarrow \quad \lambda_\perp >> \lambda$: *from nm to μm*

$d_0 \sim 1\mu m \div 10\mu m : \quad \lambda_\perp << d_0$: *surface channeling*

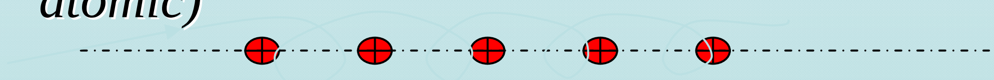


$\theta_d = \lambda / d_0 \sim \theta_c$: *diffraction angle approaches Fresnel angle*

$\lambda_\perp / d_0 \sim 1$: *bulk channeling*

* *Channeling: charged particles ⊕ crystals*

Example: e^- captured by the string potential (smeared atomic)



averaged potential

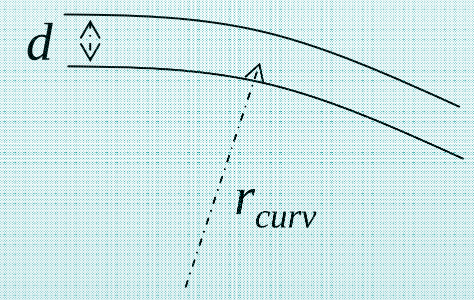
$\varphi << 1 \quad (\varphi < \varphi_L \sim \sqrt{U/E} \quad - \quad Lindhard \quad angle\ )$

Nanocapillary: Bending efficiency

$$n = \sqrt{1-\theta_c^2} \approx \sqrt{1-\frac{\omega_p^2}{\omega^2}}$$

$$\omega_p = \sqrt{\frac{4\pi N_e e^2}{m}} - \quad plasma \quad frequency$$

$$\omega - \quad photon \quad frequency$$



Effective bending:

$$\frac{r_{curv}\theta_c^2}{2d} \geq 1$$

μ-capillary: 10^0-30^0 through 10-20cm

n-capillary:the reduce of the dimensions by several orders with much higher efficiency

Nuovo Cimento ***B116*** *(2001) 361*

Potential for neutral particles: Moliere approximation

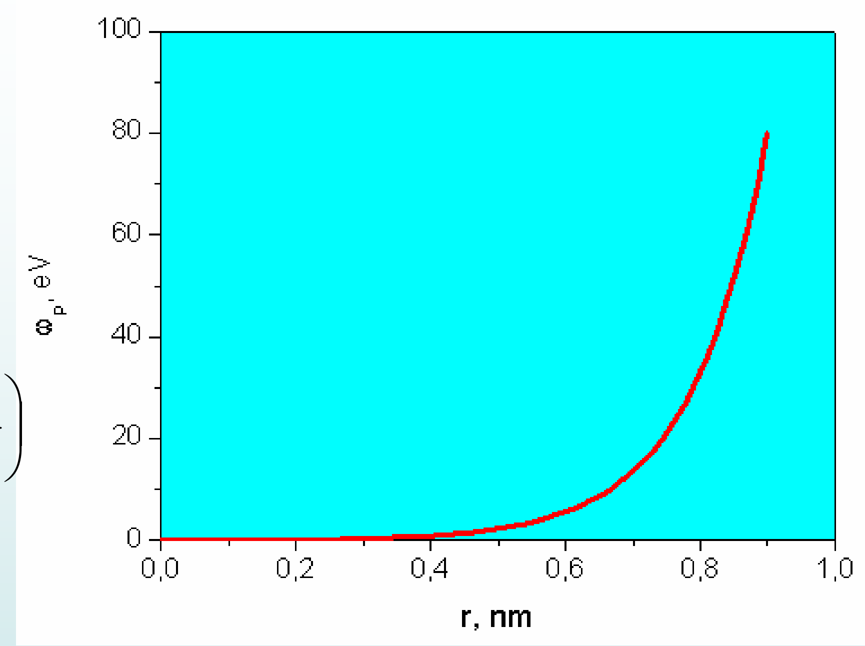
$$N_e(r) = \frac{Z}{4\pi a^2 r}\sum_{i=1}^{3}\alpha_i\beta_i^2 \exp\left(-\frac{\beta_i r}{a}\right)$$

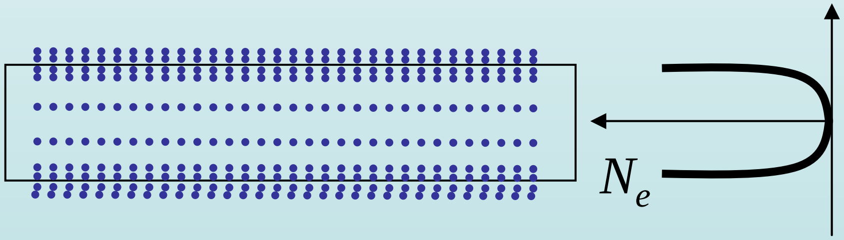
$C: \quad Z = 6$

$a \approx 0.05 Z^{-1/3} - \quad \textit{screening} \quad \textit{length}$

$$\overline{N}_e(r) \approx \frac{r_{curv} n_a Z}{\pi a^2}\sum_i \alpha_i \beta_i^2 \int_0^{\pi} d\theta\, K_0\left(\frac{\beta_i \rho}{a}\right)$$

$$\rho = \left(r^2 + r_{curv}^2 - 2 r r_{curv} \cos\theta\right)^{1/2}$$





"Continuous filtration"

Phys. Lett. ***A250*** *(1998) 360*
NIM ***B143*** *(1998) 584*

Channeling & Channeling Radiation

@ **Prediction of channeling radiation (ChR)**
Phys. Lett. 1976 (Kumakhov) –

@ **Experimental confirmation**: positron channeling in diamond crystal
USSR-USA collaboration, SLAC 1978
JETP Lett. 1979 (Miroshnichenko, Avakyan, Figut, et al.)

- **Classical theory** of scattering and radiation at channeling (Beloshitsky)
- **Quantum theory** of channeling and dechanneling & ChR (Andersen, Dabagov)

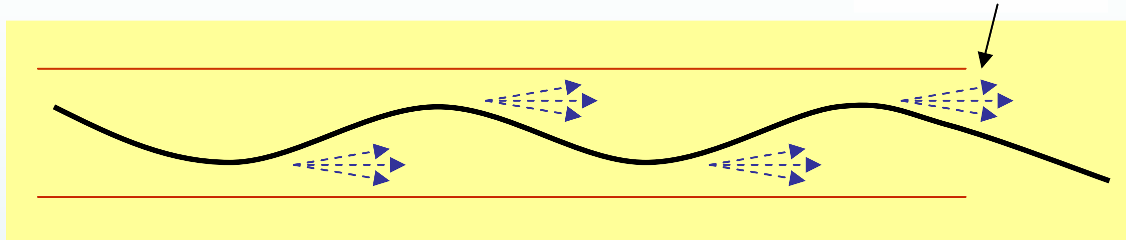
@ More than 1000 articles, a number of books
@ Starting from 1980 till 1990 each year/two - conference or school on channeling radiation
1991; 1993; 1996

Channeling Radiation…

@ Channeling Radiation:

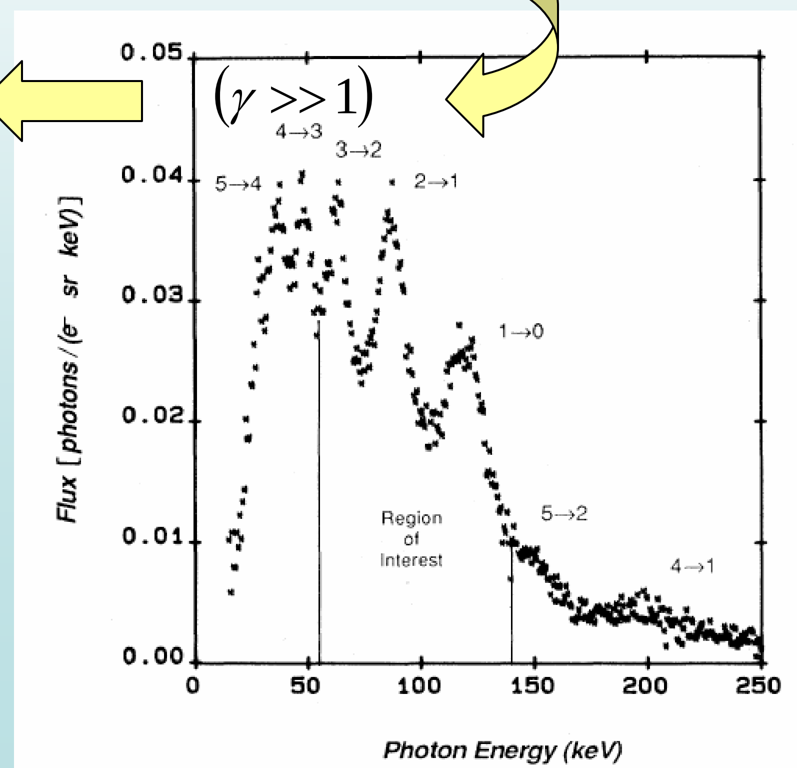
$$\Delta\vartheta \simeq \gamma^{-1}$$

$$\omega = \omega(\theta) = \frac{\omega_{fi}}{1-\beta_{\parallel}\cos\theta}$$

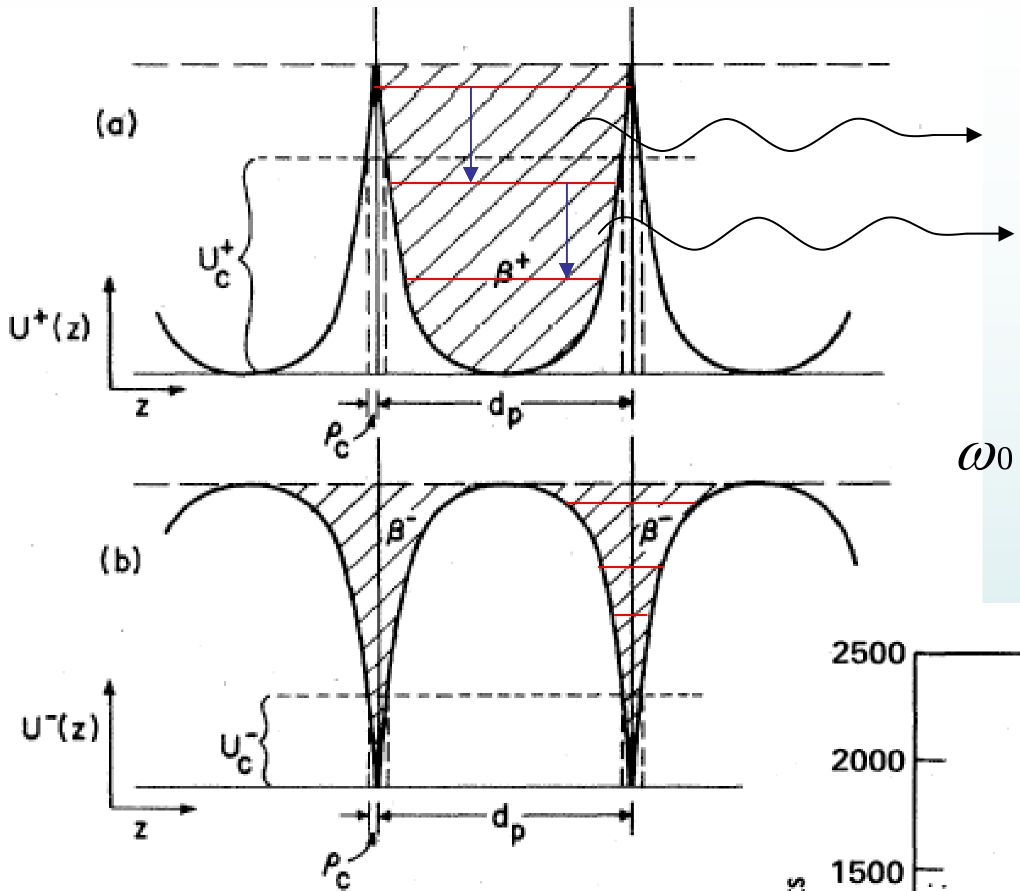


ω_{fi} - *optical frequency* → *Doppler effect* → $\omega_0\gamma^{3/2} \div \omega_0\gamma^2$

Powerful radiation source of X-rays and γ-rays:
- polarized
  - tunable
    - narrow forwarded

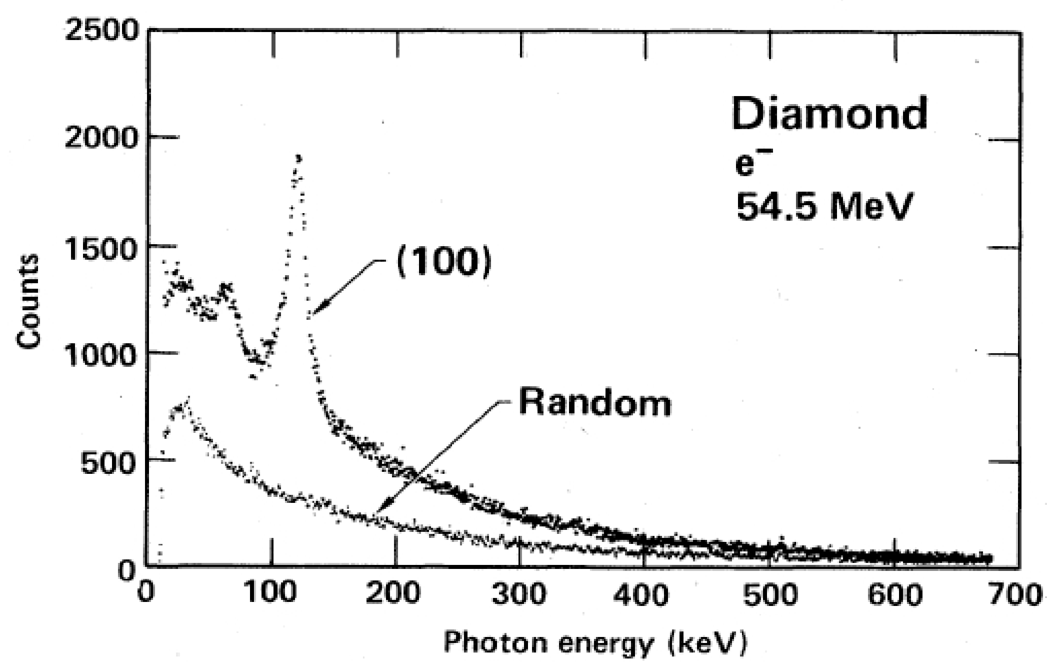


Channeling Radiation



$$\omega_0 \propto 10 \quad eV$$

$$\omega_0 \to 2\gamma^2 \max \quad forward \to 200 \quad keV$$



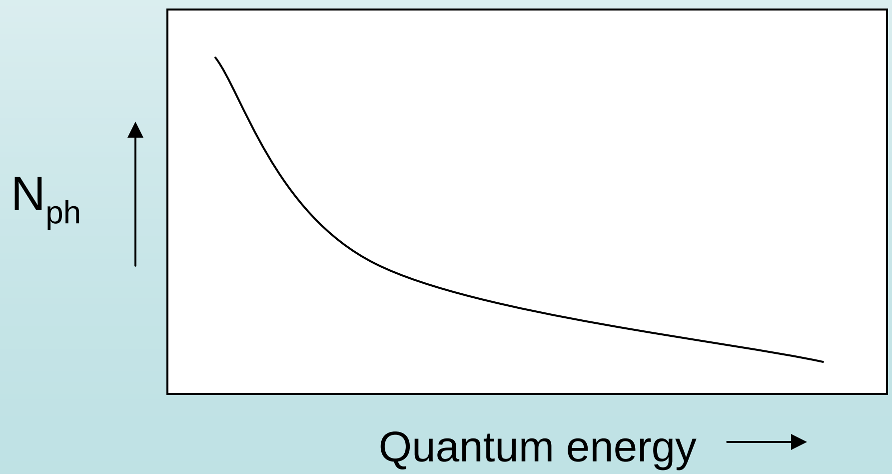
Bremsstrahlung & Coherent Bremsstrahlung vs Channeling Radiation

@ amorphous - electron:

- Radiation as sum of independent impacts with atoms
- Effective radius of interaction – a_{TF}
- Coherent radiation length $l_{coh} >> a_{TF}$
- Deviations in trajectory less than effective radiation angles:

$$\Delta\theta \propto a_{TF} / p \qquad \Delta\vartheta \simeq \gamma^{-1}$$

$$\left(\frac{d^2 I}{d\omega\Omega}\right)_{BR} \simeq (\pi L_R)^{-1}\gamma^2 \frac{1+\gamma^4\theta^4}{(1+\gamma^2\theta^2)^4} \longrightarrow \left(\frac{dI}{d\omega}\right)_{BR} \simeq \frac{4}{3} L_R^{-1}$$



Bremsstrahlung & Coherent Bremsstrahlung vs Channeling Radiation

@ interference of consequent radiation events:

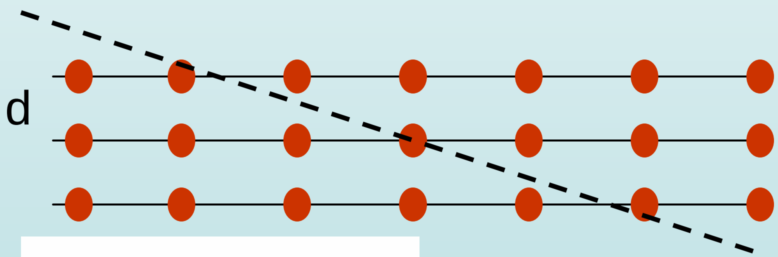
phase of radiation wave → $(\omega t - \mathbf{kr}(t))$

Radiation field as interference of radiated waves:

$$l_{coh} \approx \frac{v}{\omega - \mathbf{kr}} = \frac{\lambda\beta}{1 - \beta\cos\theta} \longrightarrow l_{coh} \propto \gamma^2\lambda$$

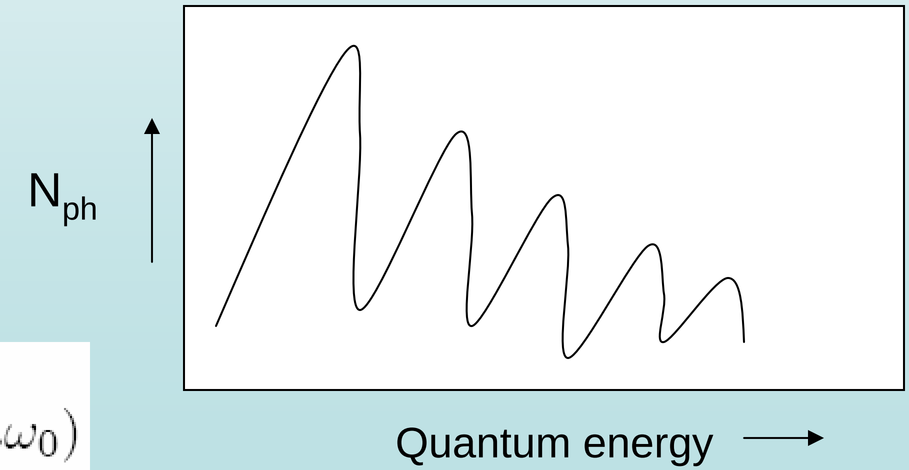
Coherent radiation length can be rather large even for short wavelength

@ crystal: $l_1 = n l_{coh}$ $l = d/\sin\alpha$ $l_1 = \frac{n\lambda\beta}{1 - \beta\cos\theta}$ $\omega_0 \equiv \beta/l_1$



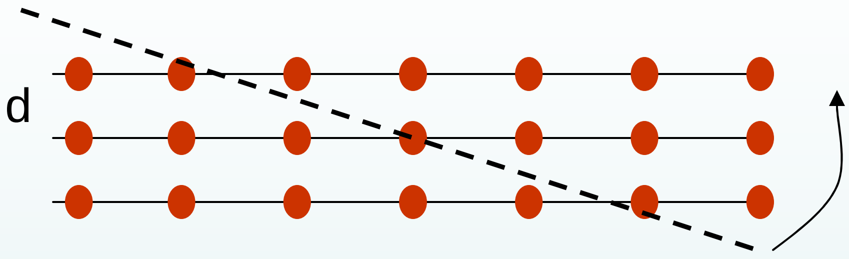
$$\omega = \frac{n\omega_0}{1 - \beta\cos\theta}$$

$$\left(\frac{d^2 I}{d\omega\Omega}\right)_{CBR} \propto \delta\left(\omega(1 - \beta\cos\theta) - n\omega_0\right)$$



Bremsstrahlung & Coherent Bremsstrahlung vs Channeling Radiation

@ crystal: $\alpha \to 0$



→ channeling

$$\omega = \frac{\omega_{fi}}{1 - \beta_{||} \cos\theta}$$

$$\left(\frac{dI}{d\omega}\right)_{CR} \propto \omega \left[1 - 2\left(\frac{\omega}{\omega_m}\right) + 2\left(\frac{\omega}{\omega_m}\right)^2\right], \quad \omega \le \omega_m \simeq 2\gamma^2 \omega_{fi}$$

$$\frac{ChR}{B} \propto \gamma^{1/2} Z^{-2/3}$$

at definite conditions channeling radiation can be significantly powerful than bremsstrahlung

B:	CB:	ChR:
	NZe	$N \leftrightarrow \quad l_{coh} \propto \gamma^2 / \omega \qquad N_{eff}$
$\propto NZ^2$	$\propto (NZ)^2$	$\propto (N_{eff} Z)^2$

Channeling Radiation vs Thomson Scattering

$$\omega_{lab}^{ChR} \approx \frac{2\gamma^2}{1+\theta^2\gamma^2}\omega_0^{ChR} \qquad \propto \gamma^{3/2}$$

- radiation frequency -

$$\omega_{lab}^{TS}\left\{\begin{array}{c}\vartheta=0\\ \vartheta=\pi/2\\ \vartheta=\pi\end{array}\right\} \simeq \left\{\begin{array}{c}1\\2\\4\end{array}\right\}\frac{\gamma^2}{1+\vartheta^2\gamma^2}\omega_0^{TS} \qquad \propto \gamma^2$$

$$\left(\frac{dN_{ph}}{dt}\right)_{ChR} \propto \gamma^{1/2}$$

- number of photons per unit of time -

$$\left(\frac{dN_{ph}}{dt}\right)_{TS} \propto Const$$

$$P \propto \gamma^2$$

- radiation power -

$$P \propto \gamma^2$$

@ comparison factor: $f \simeq \frac{\mathbf{A}_{Ch}^2}{\mathbf{A}_{TS}^2}\frac{L_{Ch}}{L_{TS}}$ → Laser beam size & mutual orientation

$$L_{Ch}(z) \simeq \int_0^z N_{ch}(z)\,dz$$

@ strength parameters – crystal & field:

	Si ⟨110⟩	C ⟨100⟩	W ⟨111⟩
$\mathbf{A}_{Ch}^2$, eV/Å^3	~ 520	~ 580	~ 10000

$\mathbf{A}_{TS}^2$ ~ 700 eV/Å^3 for the 10 TW laser with a beam diameter of 0.1 mm

Channeling Radiation vs Thomson Scattering

For X-ray frequencies: **100 MeV** electrons **channeled** in 105 μm Si (110) emit ~ 10^{-3} ph/e^{-}

⇕

Thomson scattering: **laser of 5 kW & d = 0.1 mm & L = 1 cm** can get ~ 10^{-8} ph/e^{-} at 1 μm wavelength

ChR – effective source of photons in very wide frequency range:

- in x-ray range – higher than B, CB, and TS
- however, TS provides a higher degree of monochromatization and TS is not undergone incoherent background, which always takes place at ChR

@ "**Channeling 2004**"
Workshop on Charged and Neutral Particles Channeling
(Frascati 2-6 November 2004)

- Radiation of relativistic charged particles in periodic structures
- Coherent scattering of electrons and positrons in crystals
- Channeling radiation of electrons and positrons in crystals
- Channeling of X-rays and neutrons in capillary systems (micro- and nano-channeling)
- Novel types of sources for electromagnetic radiation (FEL, powerful X-ray sources)
- Applications of channeling phenomena (novel radiation sources, X-ray waveguides, capillary/polycapillary optics)