Particle Physics - Introduction
A.A. 2018 - 2019

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SAPIENZA
UNIVERSITÀ DI ROMA

AA 18-19

Contents

— Nostro figlio sta cambiando una lampadina... E' meraviglioso quello che insegnano all'università, al giorno d'oggi...

"Our son is changing a light bulb... What they teach at university nowadays is wonderful..."

slides / textbooks / original

- These slides have many sources (lectures in our + other Department(s), textbooks, seminars, …); many thanks to everybody, but all the mistakes are my own responsibility;
- download from http://www.roma1.infn.it/people/bagnaia/particle_physics.html
- comments and criticism to paolo.bagnaia@roma1.infn.it (**please !**)
- they are only meant to help you follow the lectures (and remember the items);
- i.e. NOT enough for the exam; students are also required to study on textbook(s) / original papers (see references);
- the original literature is always quoted; sometimes those papers offer a beautiful example of clarity; however, particularly in recent years, their technical level is difficult, probably more at PhD student level, than for an elementary presentation (i.e. **you**);
- however, students are strongly encouraged to attack the real stuff: these lectures are NOT meant for *amateurs* or interested public (which are welcome), but for future professionals !

Thanks !!!

Enjoy them !!!

PB

References

[BJ] **W. E. Burcham - M. Jobes – Nuclear and Particle Physics** – Wiley – 768 pag. [*clear, well-organized, old*];

[YN] **Yorikiyo Nagashima – Elementary Particle Physics** – Wiley VCH – 3 vol. [*clear, modern, complete, very expensive*];

[Bettini] **A.Bettini - Introduction to Elementary Particle Physics** [*another textbook*];

[MS] **B.R.Martin, G.Shaw – Particle Physics** [*ditto*];

[Perkins] **D.Perkins - Introduction to High Energy Physics**, 4th ed. [*ditto*];

[Povh] **Povh, Rith, Scholz, Zetsche - Particles and Nuclei** [*ditto, simpler*];

[Thoms] **M. Thomson – Modern Particle Physics** [*ditto*];

[CG] **R.Cahn, G.Goldhaber – The experimental foundation of particle physics** [*a collection of original papers + explanation, the main source for experiments*];

[FNSN1] **C.Dionisi, E.Longo - Fisica Nucleare e Subnucleare 1** – Dispense del corso [*in Italian, download it from our web – you are requested to know them*];

[MQR] **L.Maiani - O.Benhar – Meccanica Quantistica Relativistica** [*in Italian, the theory lectures of the previous semester*];

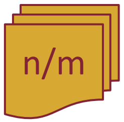
[IE] **L.Maiani – Interazioni elettrodeboli** [*ditto*];

[PDG] **The Review of Particle Physics –** latest: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) [*the bible; everything there, but more a reference, than a textbook, i.e. hard for newcomers*];

[original] **Original papers** are quoted in the slides [*try to read (some of) them* → help by [CG]].

quoted as [book, chapter] or [book, page]; *e.g.* [BJ, § 4] *: Burcham-Jobes, § 4*.

Symbols



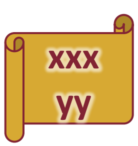
(in the <u>upper left</u> corner) this is page *n* of a total of *m* pages : read them all together;

(in the <u>upper right</u> corner) optional material;

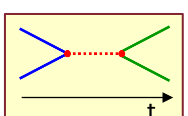
(in the <u>upper right</u> corner) tool, used also in other chapters;

summary;

animation (ppt/pptx only);



reference to a paper / textbook; [if textbook, you are requested to read it; if paper, try (at least some of) them];



in Feynman diagrams, time goes always left to right;

- "QM" : Quantum Mechanics;
- "SM" : Standard Model; here and there, the name and the history behind is explained;
- "bSM" : beyond Standard Model, i.e. the (until now unsuccessful) attempts to extend it, e.g. SUSY;
- ($\hbar$ = c = 1) whenever possible; i.e. mass, momentum and energy in MeV or GeV.
- m : scalar, E : component of a vector;
- $\mathbb{P}$: operator;
- $\vec{v}$: 3-vector, $\vec{v} = (x, y, z)$;
- p : 4-vector, $p = (E, p_x, p_y, p_z) = (E, \vec{p})$;
- if worth, the module is indicated $p = (E, p_x, p_y, p_z; m) = (E, \vec{p}; m)$;
- if irrelevant, the last component of a 3- or 4-vector is skipped : $p = (E, p_x, p_y) = (E, p_x, p_y; m)$.

room, time, …

Lecture time – aula Careri

- mon (lun) 12 - 14
- tue (mar) 11 - 13
- wed (mer) 11 - 13
- thu (gio) 12 - 14

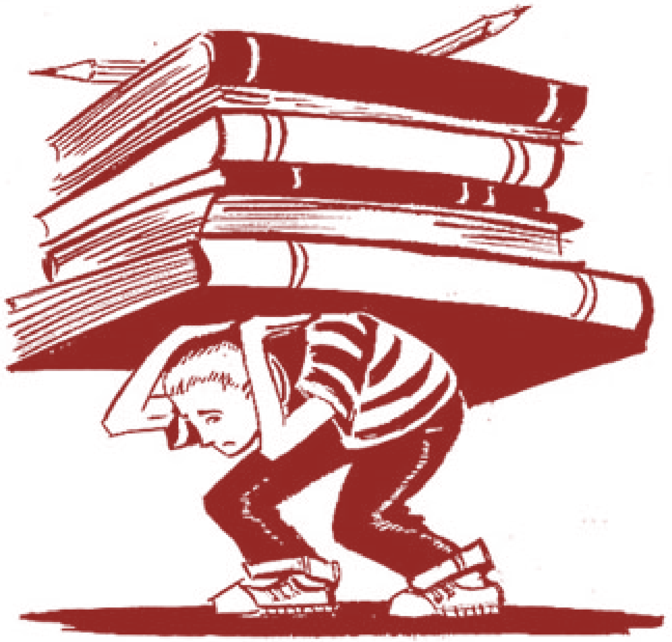
[*not ideal but acceptable*]

We have also this room on tue 14-16:

- not for independent lectures (too much);
- problems, exercises, …
- long questions from you, e.g. if you feel you need something you should know, but actually don't (relativistic kinematics ?)

exam

- questions [by me] and answers [*possibly by you*];
- 1st question known few days in advance by email [*I'll choose randomly, with a little bias*];
- if theoretician or experimentalist, you may [or may not] tell me [*I'll use it*];
- let me also know curriculum type (e.g. phenomenology, electronics, medical physics) [*I'll apply a stronger bias*];
- other rules after discussion and experiment [*I'm an experimentalist*].



Nota Bene

- Starting with **2017-2018** (one year ago), these lectures are delivered in English.
- No problem, we all know and love the Shakespeare idiom [*needless to say, we love Italian and Roman too*].
- As a minor consequence, the name of the course has changed – it was "*Fisica Nucleare e SubNucleare 2*".
- Apart from name and language, no major change [*I would love to improve, come and discuss your ideas with me*].
- Past years' students don't have to worry: students are officially bound (really) to the rules of the year of their registration (*anno di immatricolazione*). They only have to be careful with the registration(s), i.e. the *INFOSTUD* stuff.
- The exam (both this and past years' students) will be in Italian or English, at **your** choice.
- During the lecture, questions and comments in the language as you like. I will start answering by translating them into English.

> *and please never forget the RULE:*
> *"If you are going to sin, sin against God, not the bureaucracy. God will forgive you, but the bureaucracy won't."*
> *Hyman G. Rickover*

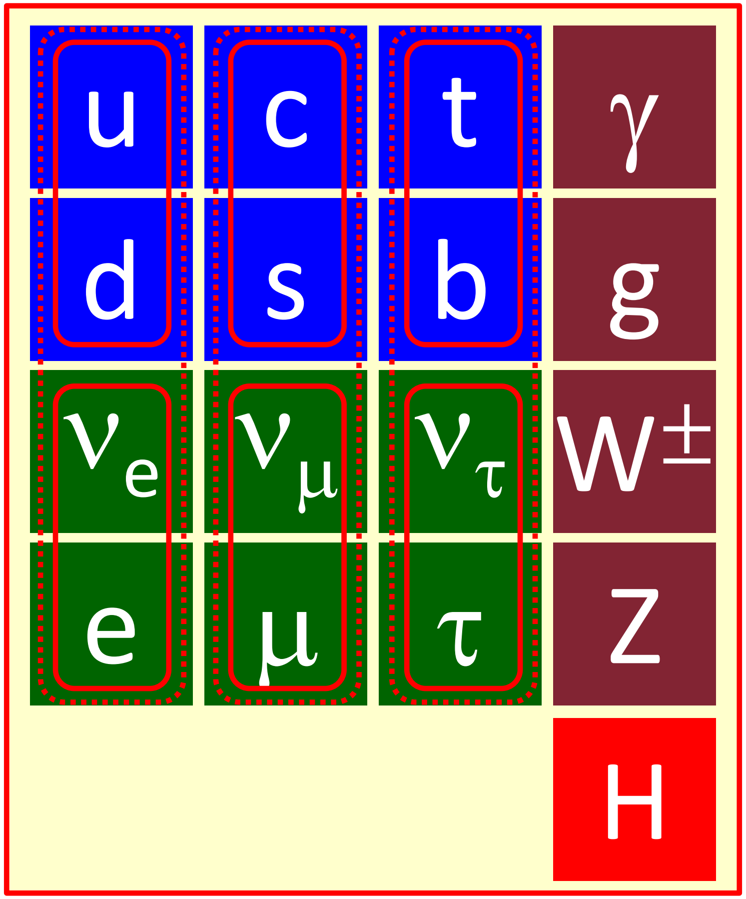
... and now ...

Let's start

Prologue

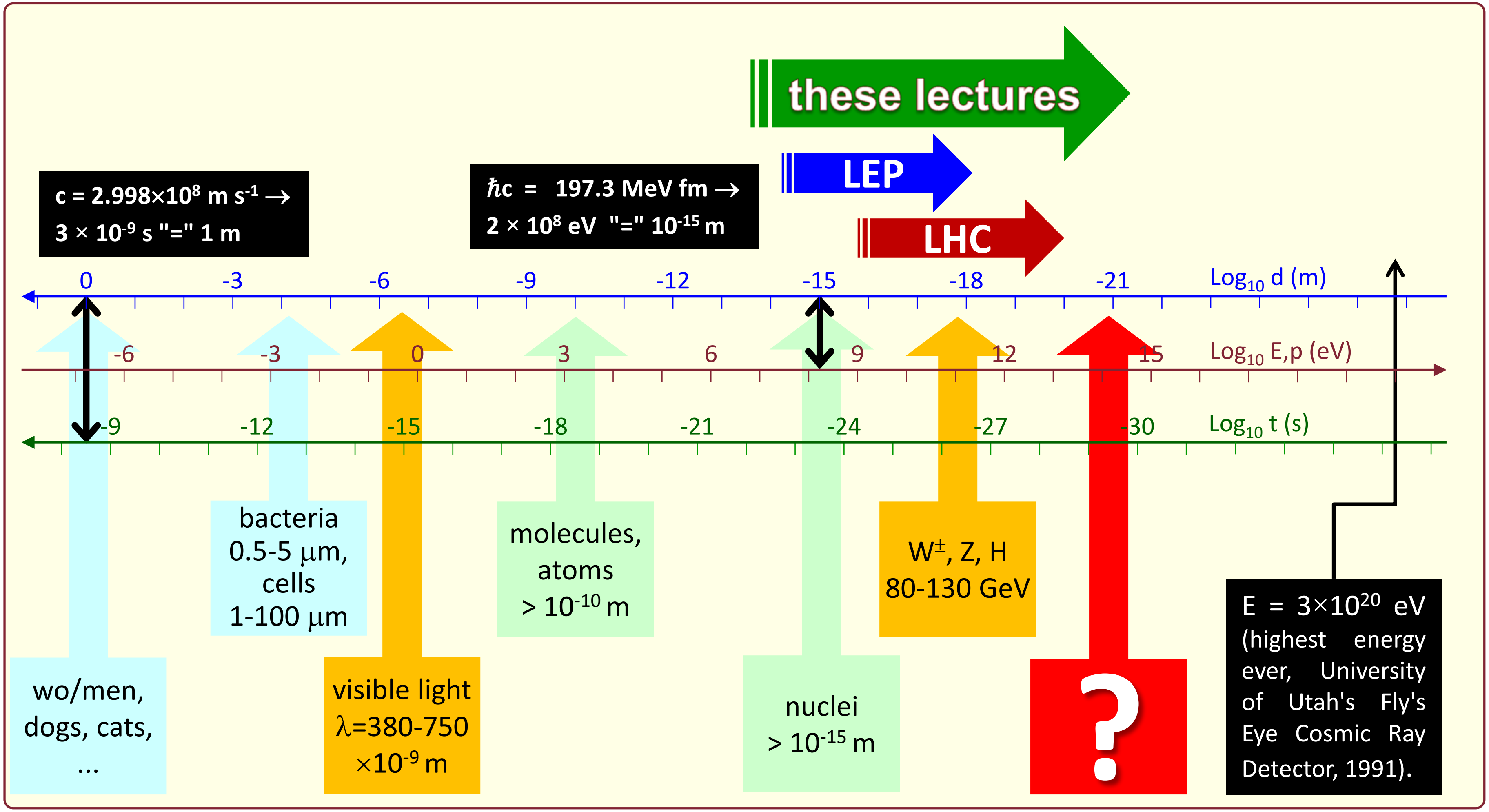
The present understanding of our world, in terms of its constituents and interactions, is much advanced:

- **fermions** (quarks/leptons) = matter:
  - "families" of doublets + antiparticles;
  - spin ½;
  - massive (large differences in mass);
  - charge ±⅔, ±⅓, 0, ±1;
- **bosons** = forces:
  - spin 1;
  - massless (γ, g) or massive ($W^{\pm}$, Z);
  - charged ($W^{\pm}$) or neutral (γ, g, Z);
  - some self-coupled;
- the mysterious **Higgs boson** carries the particle masses.



these lectures explain how

Prologue: twenty orders of magnitude

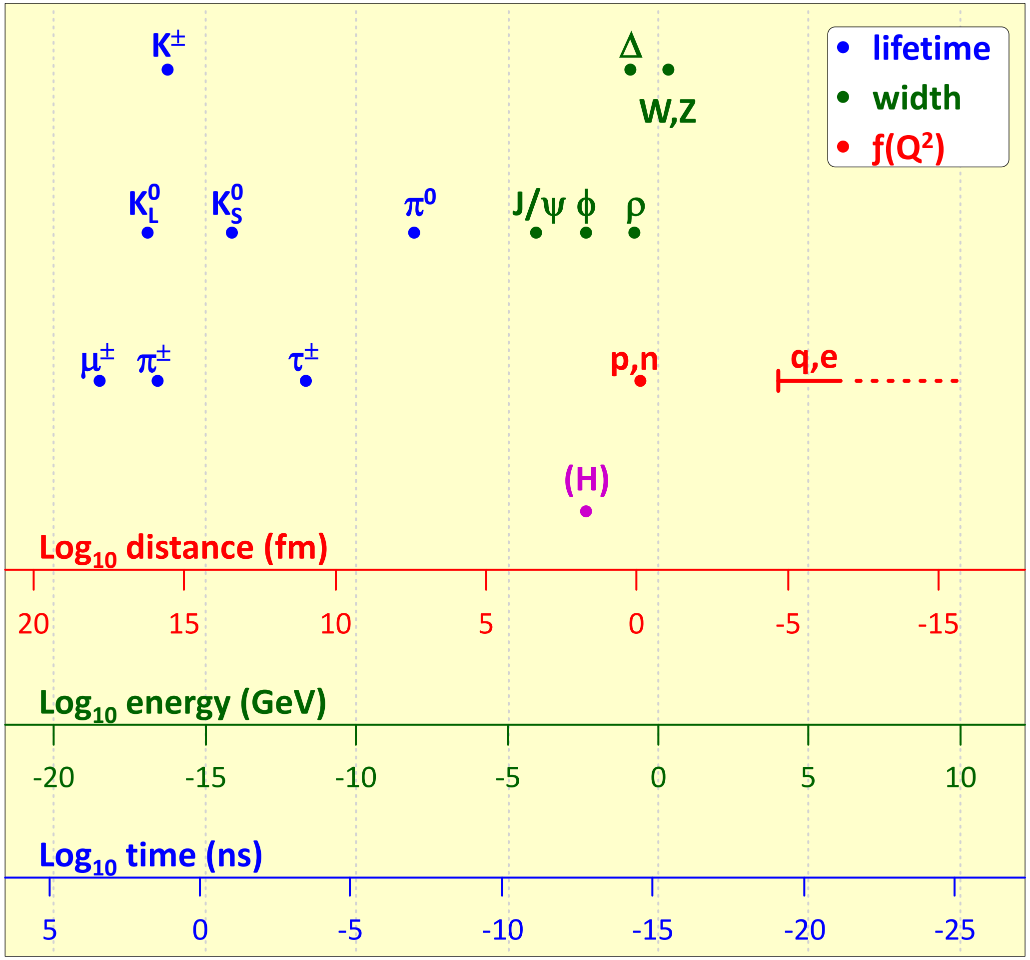


space / energy / time

Prologue: the realm of elementary particles

In these lectures, many phenomena. Consider the typical (rough) size/time/energy of the processes:

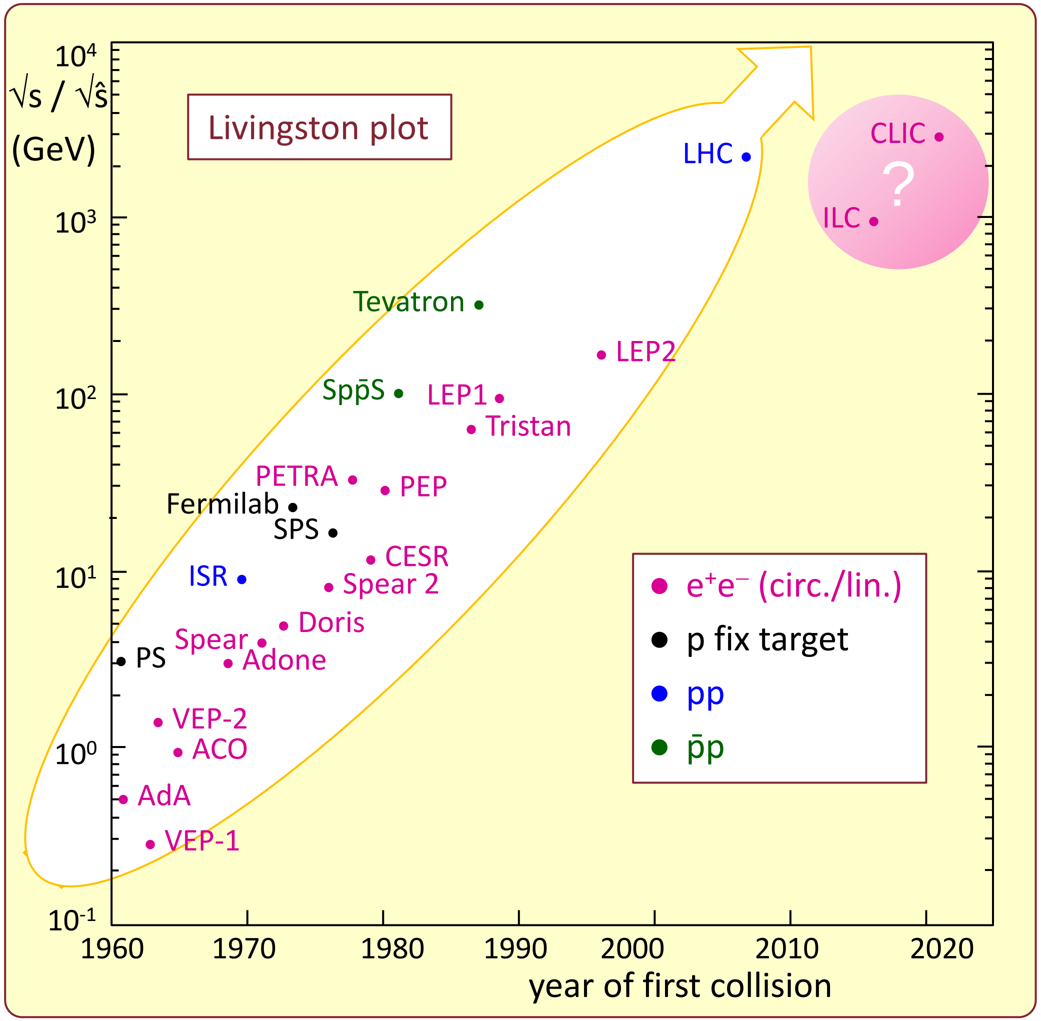
- **lifetimes** are measured in the rest system of the particles, i.e. in (nano-)s;
- the corresponding **distance** is the average space traveled by a particle with $\beta\gamma$=1 before decaying;
- the uncertainty principle relates a **width** to a lifetime: it is the fluctuation of the particle rest energy (= mass);
- $f(Q^2)$ deserves an explanation: sometimes the size of a particle is inferred "à la Rutherford", by a scattering experiment [*see chapter 2*] (only limits for q's and ℓ's: pointlike ?);
- the width of the Higgs boson (H) has not (yet ?) been measured and comes from theory.



*Do NOT panic: you are supposed to fully understand this plot only at the end of the lectures. Every single point in the figure will be **carefully** explained.*

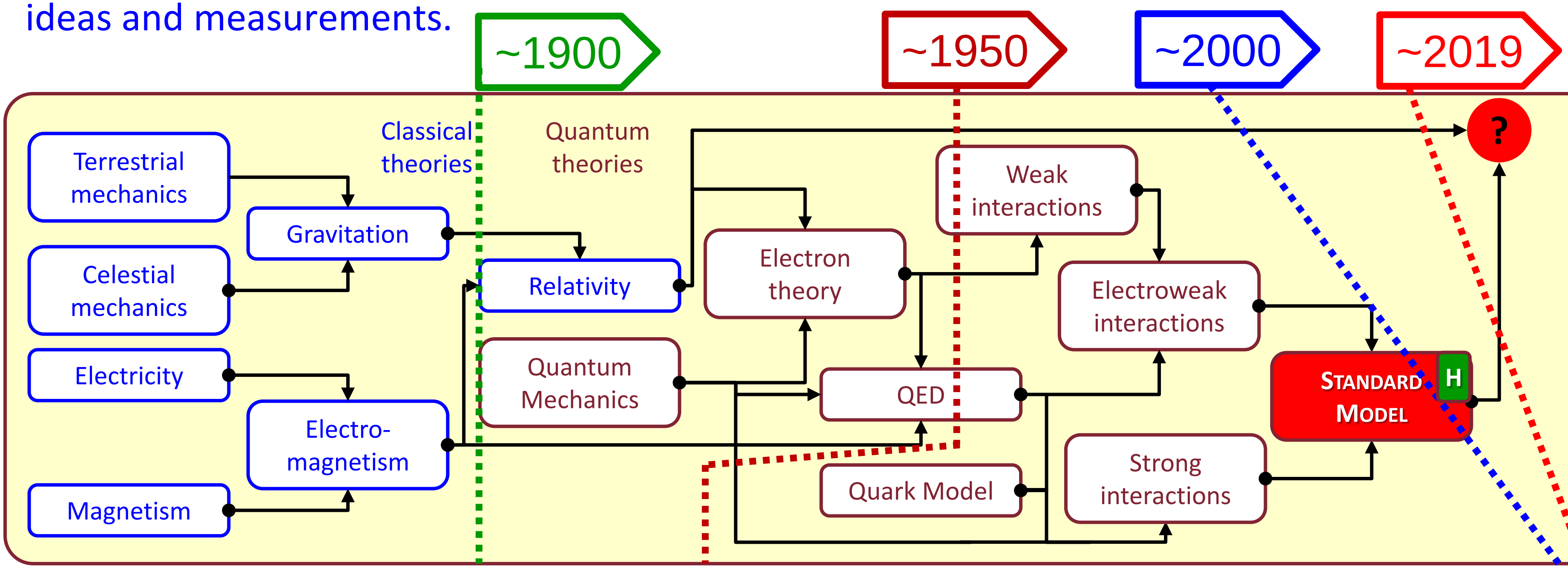
Prologue: the quest for higher energies

- Discovery range is limited by available data, i.e. by instruments and resources (an always improved microscope).
- The true variable is the resolving power [r.p.] of our microscope.
- From QM, r.p. $\propto \sqrt{Q^2}$ [i.e. $\propto \sqrt{s}$, the CM energy [*what ? why ? see § 2*].
- For non point-like objects, replace $\sqrt{s}$ with the CM energy at component level, called $\sqrt{\hat{s}}$ ($\sqrt{\hat{s}} < \sqrt{s}$).
- In the last half a century, the physicists have been able to gain a factor 10 in $\sqrt{s}$ (i.e. a factor ten in the quality of the microscope) <u>every 10 years</u> (see the "Livingston plot").
- Hope it will continue like that, but needs <u>IDEAS</u>, since not many $$$ (or €€€) will be available.



Prologue: the Standard Model

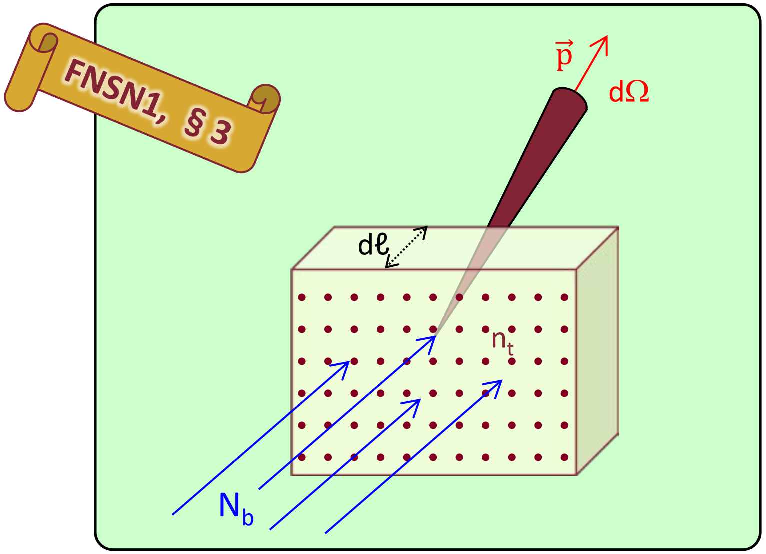
- The name **SM** (not a fancy name) designates the theory of the Electromagnetic, Weak and Strong interactions.
- The theory has grown in time, the name went together.
- The development of the SM is a complicated interplay between new ideas and measurements.
- Many theoreticians have contributed : since the G-S-W model is at the core of the SM, it is common to quote them as the main authors.
- The little scheme [BJ] of its time evolution may help (missing connections, approximations, …).



few subjects, well known, but … [skip next pages, if you can afford it]:

- the cross section σ;
- excited states (resonances);
- Gauss distribution.
- measurements:
  - spectrometers;
  - calorimeters;
  - particle id;

the cross section σ



A beam of N_b particles is sent against a thin layer of thickness dℓ, containing dN_t scattering centers in a volume $\mathcal{V}$ ("target", density $n_t = dN_t/d\mathcal{V}$).

The number of scattered particles dN_b is:

$$dN_b \propto N_b\, n_t\, d\ell \;\Rightarrow\; dN_b = N_b\, n_t\, \sigma_T\, d\ell$$

the number of particles left after a finite length ℓ is

$$N_b(\ell) = N_b(0) \exp(-\, n_t\, \sigma_T\, \ell).$$

The parameter σ_T is the total **cross section** between the particles of the beam and those of the target; it can be interpreted as *the probability of an interaction when a single projectile enters in a region of unit volume containing a single target*.

If many exclusive processes may happen (simplest case : elastic or inelastic), σ_T is the sum of many σ_j, one for each process:

$\sigma_T = \Sigma_j\, \sigma_j$ [e.g. $\sigma_T = \sigma_{elastic} + \sigma_{inelastic}$];

in this case σ_j is proportional to the *probability of process j*.

Common differential dσ/d... 's:

$$\frac{d\sigma}{d\Omega} = \frac{d^2\sigma}{d\cos\theta\, d\varphi} \xrightarrow{\text{no } \varphi \text{ dependence}} \frac{1}{2\pi}\frac{d\sigma}{d\cos\theta};$$

$$\frac{d\sigma}{d\vec{p}} = \frac{d^3\sigma}{dp_x dp_y dp_z} = \frac{d^3\sigma}{p_T dp_T dp_\ell d\varphi} \rightarrow \frac{1}{\pi}\frac{d^2\sigma}{dp_T^2 dp_\ell};$$

+ others.

the cross section σ: $\sigma_{inclusive}$

In a process ($a\ b \to c\ X$), assume:

- we are only interested in "c" and not in the rest of the final state ["X"];
- "c" can be a single particle (e.g. $W^\pm$, Z, Higgs) or a system (e.g. $\pi^+\pi^-$).

Define:

$\sigma_{inclusive}(ab \to cX) = \sum_k \sigma_{exclusive}(ab \to cX_k)$,
where the sum runs on all the **exclusive** processes which in the final state contain "c" + anything else [define also $d\sigma_{inclusive}/d\Omega$ wrt angles of "c", etc.].

The word *inclusive* may be explicit or implicit from the context. E.g., "*the cross-section for Higgs production at LHC*" is obviously $\sigma_{inclusive}(pp \to HX)$.

From the definition, if $\sigma_{inclusive} << \sigma_{total}$:

$\mathcal{P}_c$ = probability of "c" in the final state =
= $\sigma_{inclusive}(ab \to cX) / \sigma_{total}(ab)$.

Instead, if "c" is common:

$\langle n_c \rangle$ = <number of "c" in the final state> =
= $\sigma_{inclusive}(ab \to cX) / \sigma_{total}(ab)$.

e.g.

$\sigma_{Higgs}(LHC, 8\ TeV) = \sigma_{incl}(pp \to HX, \sqrt{s}=8\ TeV) = \approx 22.3\ pb$;

$\sigma_{total}(pp, \sqrt{s} = 8\ TeV) = 101.7 \pm 2.9\ mb$;

$\to\ \mathcal{P}_{Higgs}(LHC) \approx 2 \times 10^{-10}$;

[§ LHC]

$\sigma_{incl}(pp \to \pi^0 X, p_{LAB}=24\ GeV) = 53.5 \pm 3.1\ mb$;

$\sigma_{total}(pp, p_{LAB}=24\ GeV) = 38.9\ mb$;

$\to \langle n_{\pi^\circ}(pp, p_{LAB}=24\ GeV)\rangle \approx 1.37$

[V.Blobel et al. - Nucl. Phys., B69 (1974) 454].

Mutatis mutandis, define

- "inclusive width" $\Gamma(A \to BX)$;
- "inclusive BR" $BR(A \to BX)$.

the cross section σ: Fermi 2nd golden rule

- N_b, N_t : particles in beam(b) / target(t);
- $\mathcal{V}$: volume element;
- n_b, n_t : density of particles [= $dN_{b,t}/d\mathcal{V}$];
- v_b : velocity of incident particles;
- ϕ : flux of incident particles [= $n_b v_b$];
- p', E' : 4-mom. of scattered particles;
- $\rho(E')$: density of final states;
- $\mathcal{M}_{fi}$: matrix element between i→f state;
- dN/dt : number of events / time [= $\phi N_t \sigma$];
- W : rate of process [= $(dN/dt) / (N_b N_t)$].

FNSN1, 78

Fermi second golden rule

$$W = \frac{2\pi}{\hbar} |\mathcal{M}_{fi}|^2 \rho(E');$$

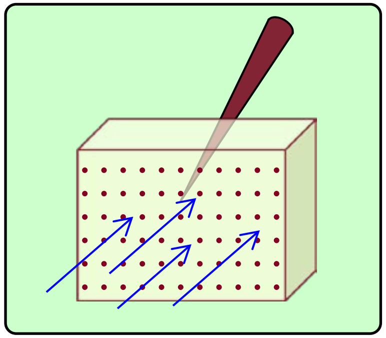
$$\rho(E') = \frac{dn(E')}{dE'} = \frac{\mathcal{V} 4\pi p'^2}{v'(2\pi\hbar)^3};$$

$$W = \frac{dN}{dt} \frac{1}{N_b N_t} = \frac{\phi N_t \sigma}{N_b N_t} = \frac{v_b \sigma}{\mathcal{V}}.$$

$$dn(p') = \frac{\mathcal{V} 4\pi p'^2}{(2\pi\hbar)^3} dp' = \frac{\mathcal{V} 4\pi p'^2}{(2\pi\hbar)^3} \frac{dE'}{v'}$$

$$\sigma = \frac{W\mathcal{V}}{v_b} = \frac{2\pi}{\hbar} |\mathcal{M}_{fi}|^2 \rho(E') \frac{\mathcal{V}}{v_b}$$

- the rule is THE essential connection (experiment ↔ theory);
- experiments measure event numbers → cross-sections;
- theories predict matrix elements → cross-sections;
- when we check a prediction, we are actually applying the rule;
- properly normalized, the rule is valid also for differential cases (i.e. $d\sigma/dk$, $d\mathcal{M}/dk$, dW/dk), where k is any kinematical variable, e.g. cosθ].



Excited states : decay pdf

Consider N (N large) unstable particles :

- independent decays;
- decay probability time-independent (e.g. no internal structure, like a timer);

Then :

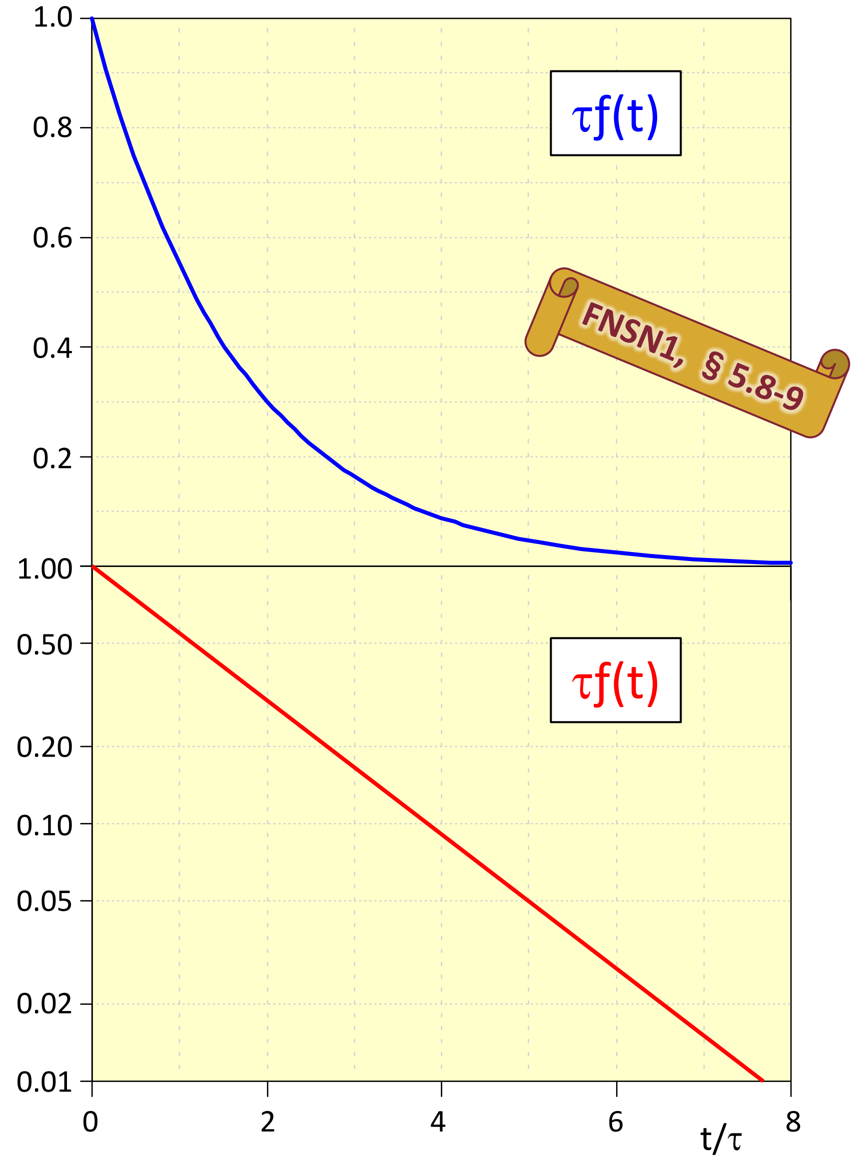
$$dN = -N\Gamma dt; \qquad \Gamma \equiv \frac{1}{\tau} = \text{const.} \qquad \Rightarrow$$

$$N(t) = N_0 e^{-\Gamma t} = N_0 e^{-t/\tau}.$$

The pdf of the decay for a single particle is

$$\int_0^\infty f(t)dt = 1 \quad \Rightarrow \quad \boxed{f(t) = \frac{1}{\tau} e^{-t/\tau}.}$$

- average decay time : $\left(\sum t_j\right)/n = \langle t \rangle = \tau;$
- likelihood estimate of τ, after n decays observed : $\tau^* = \langle t \rangle.$



Excited states : Breit-Wigner

If τ is small, the energy at rest (= mass) of a state is not unique (= δ_{Dirac}), but may vary as $\tilde{f}(E)$ around the nominal value E_0 = m :

Define $\psi(t<0) = 0$; $\psi(t=0) = \psi_0$;
width Γ [unstable] ;

$$\psi(t) = \psi_0 e^{(-im-\Gamma/2)t};$$

$$|\psi(t)|^2 = |\psi_0|^2 e^{-\Gamma t} = |\psi_0|^2 e^{-t/\tau};$$

$$\tilde{f}(E) = |\tilde{\psi}(E)|^2 = \frac{|\psi_0|^2}{2\pi} \frac{1}{(E-E_0)^2 + \Gamma^2/4}.$$

YN1, 2.1

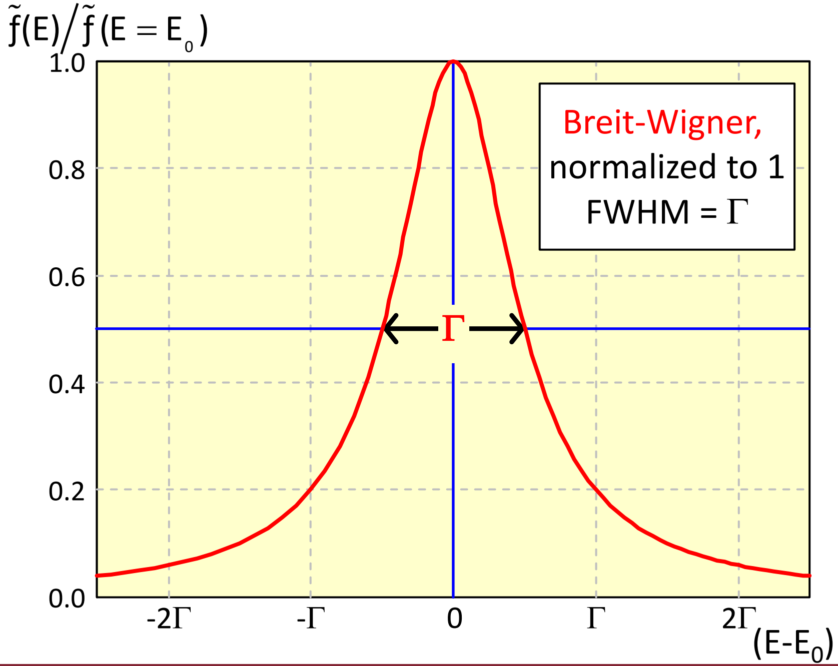
$$\tilde{\psi}(E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{iEt} \psi(t) dt =$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{iEt} \psi_0 e^{-i(E_0 - i\Gamma/2)t} dt =$$

$$= \frac{\psi_0}{\sqrt{2\pi}} \frac{-1}{i(E-E_0) - \Gamma/2} = \frac{\psi_0}{\sqrt{2\pi}} \frac{i(E-E_0) + \Gamma/2}{(E-E_0)^2 + \Gamma^2/4}.$$

The curve $(1 + x^2)^{-1}$ is called "Lorentzian" or "Cauchy" in math and "**Breit-Wigner**" in physics; it describes a RESONANCE and appears in many other phenomena:

- forced mechanical oscillations;
- electric circuits;
- accelerators;
- …



Excited states : BW properties

Cauchy (or Lorentz, or BW) distribution :

$$f(x) = BW(x\,|\,x_0,\gamma) = \frac{1}{\pi\gamma}\frac{\gamma^2}{(x-x_0)^2+\gamma^2};$$

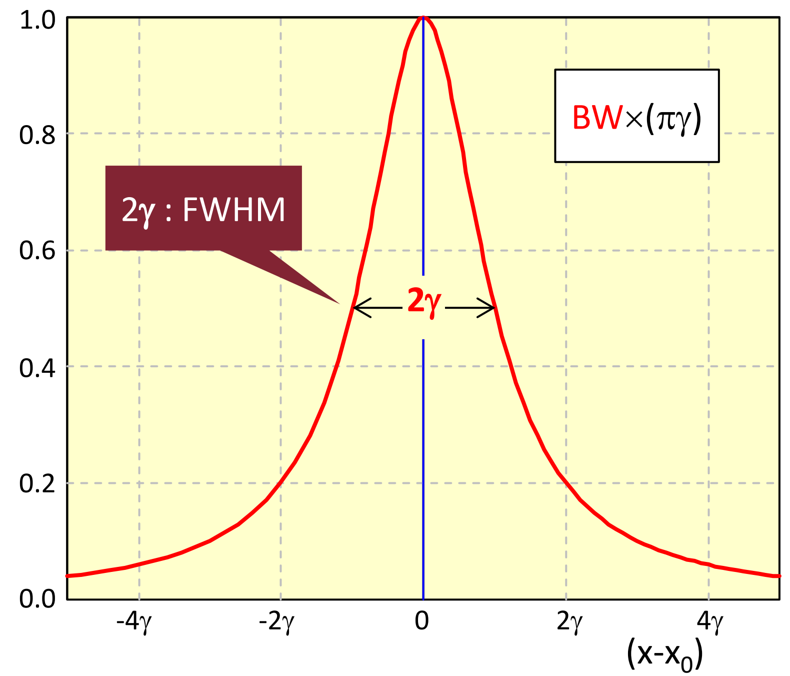
- median = mode = x_0;
- mean = math undefined [but use x_0];
- variance = really undefined [divergent]

This anomaly is due to

$$\langle x \rangle = \int_{-\infty}^{+\infty} x f(x) dx = \infty;$$

$$\langle x^2 \rangle = \int_{-\infty}^{+\infty} x^2 f(x) dx = \infty$$

The anomaly does NOT conflict with physics : the BW is an approximation valid only if $\gamma \ll x_0$ and in the proximity of x_0, e.g. in case of an excited state (mass m, width Γ), for ($\Gamma \ll m$) and ($|\sqrt{s}-m|$ < few Γ's).



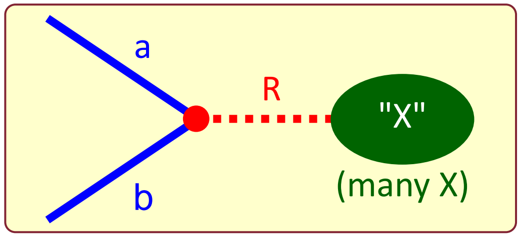
The "relativistic BW" is usually defined as

$$BW_{rel}(x\,|\,x_0,\gamma) = \frac{x_0^2\gamma^2}{(x^2-x_0^2)^2+x_0^2\gamma^2} \quad \left[\begin{matrix}\text{properly}\\ \text{normalized}\end{matrix}\right].$$

The formula comes from the requirement to be Lorentz invariant [see *Berends et al., CERN 89-08, vol 1*].

Resonance : σ_R

From first principles of QM ([FNSN1], [BJ 9.2.3], [YN1 13.3.3], [PDG])



$(E, \vec{p})$: CM 4-mom.

Γ_R : constant width

$\Gamma_{ab,\,X}$: couplings

M_R : E_0, mass

$$\sigma_{ab\to R\to X}(E_{CM}=\sqrt{s}) = \frac{\pi}{\left|\vec{p}_{a,b}\right|^2}\frac{(2J_R+1)}{(2S_a+1)(2S_b+1)}\frac{\Gamma_{ab}\Gamma_X}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4} \approx$$

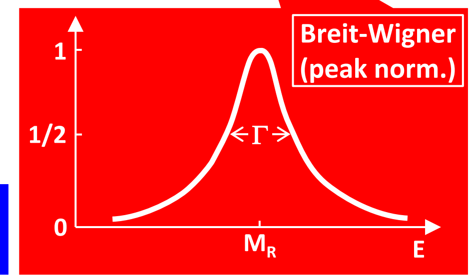
$$\approx \left[\frac{16\pi}{s}\right]\left[\frac{(2J_R+1)}{(2S_a+1)(2S_b+1)}\right]\left[\frac{\Gamma_{ab}}{\Gamma_R}\right]\left[\frac{\Gamma_X}{\Gamma_R}\right]\left[\frac{\Gamma_R^2/4}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4}\right]$$

scale factor (1/s)

statistical factor (particle spins)

= BR(R→ab)

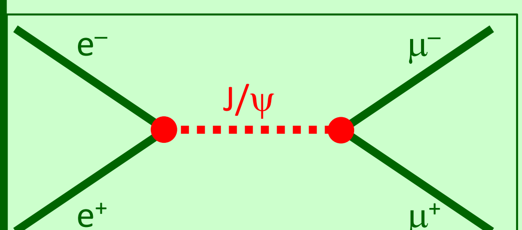
= BR(R→X)



e.g.
$e^+e^- \to J/\psi \to \mu^+\mu^-$

$\sigma_{peak} \propto 1/s$ ($\approx M_R^{-2}$), independent from coupling strength.

$$\sigma(e^+e^-\to J/\psi\to\mu^+\mu^-) = \left[\frac{16\pi}{s}\right]\left[\frac{3}{4}\right]\left[\frac{\Gamma_{ee}}{\Gamma_{tot}}\right]\left[\frac{\Gamma_{\mu\mu}}{\Gamma_{tot}}\right]\left[\frac{(\Gamma_{tot}/2)^2}{\left(\sqrt{s}-M\right)^2+(\Gamma_{tot}/2)^2}\right] =$$



$$= \frac{12\pi}{s} BR_{J/\psi\to e^+e^-} BR_{J/\psi\to\mu^+\mu^-}\left[\frac{(\Gamma_{tot}/2)^2}{\left(\sqrt{s}-M\right)^2+(\Gamma_{tot}/2)^2}\right].$$

Resonance : different functions

Many more parameterizations used in literature (semi-empirical or *theory inspired*), e.g.:

$$\sigma_0 = \left[\frac{16\pi}{(2p)^2}\right]\left[\frac{(2J_R+1)}{(2S_a+1)(2S_b+1)}\right]\left[\frac{\Gamma_{ab}}{\Gamma_R}\right]\left[\frac{\Gamma_{final}}{\Gamma_R}\right]\left[\frac{\Gamma_R^2/4}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4}\right]$$

original, non-relativistic

$$\sigma_1 = \left[\frac{16\pi}{s}\right]\left[\frac{(2J_R+1)}{(2S_a+1)(2S_b+1)}\right]\left[\frac{\Gamma_{ab}}{\Gamma_R}\right]\left[\frac{\Gamma_{final}}{\Gamma_R}\right]\left[\frac{\Gamma_R^2/4}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4}\right]$$

m_a, m_b << p

$$\sigma_2 = \left[\frac{16\pi}{M_R^2}\right]\left[\frac{(2J_R+1)}{(2S_a+1)(2S_b+1)}\right]\left[\frac{\Gamma_{ab}}{\Gamma_R}\right]\left[\frac{\Gamma_{final}}{\Gamma_R}\right]\left[\frac{\Gamma_R^2/4}{\left(\sqrt{s}-M_R\right)^2+\Gamma_R^2/4}\right]$$

if $M_R \gg \Gamma_R$, neglect s-dependence

$$\sigma_3 = \left[\frac{16\pi}{M_Z^2}\right]\left[\frac{3}{4}\right]\left[\frac{\Gamma_{ee}}{\Gamma_Z}\right]\left[\frac{\Gamma_{f\bar{f}}}{\Gamma_Z}\right]\left[\frac{M_Z^2\Gamma_Z^2}{\left(s-M_Z^2\right)^2+M_Z^2\Gamma_Z^2}\right]$$

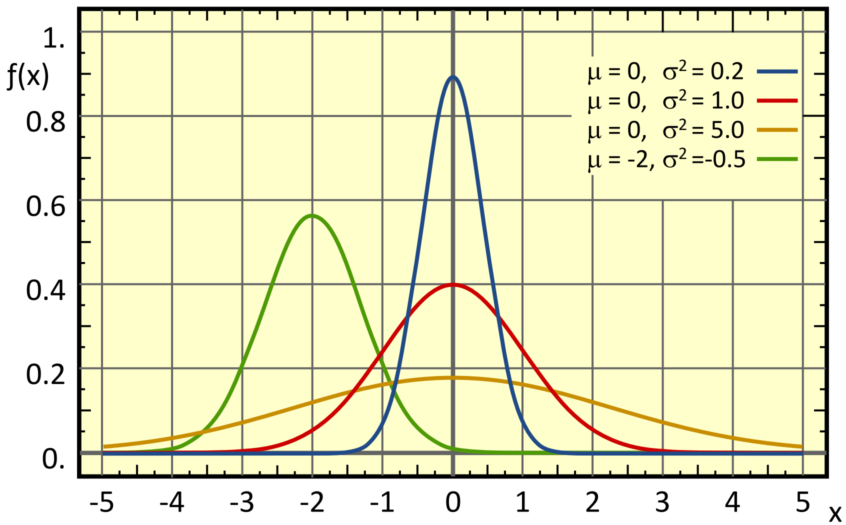
relativistic BW for $e^+e^- \to Z \to f\bar{f}$

$$\sigma_4 = \left[\frac{16\pi}{M_Z^2}\right]\left[\frac{3}{4}\right]\left[\frac{\Gamma_{ee}}{\Gamma_Z}\right]\left[\frac{\Gamma_{f\bar{f}}}{\Gamma_Z}\right]\left[\frac{s\Gamma_Z^2}{\left(s-M_Z^2\right)^2+s^2\Gamma_Z^2/M_Z^2}\right]$$

"s-dependent Γ_Z" (used at LEP for the Z lineshape)

Gauss distribution

$$f(x) = G(x\,|\,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



- mean = median = mode = μ;
- variance = σ^2;
- symmetric : $G(\mu+x) = G(\mu-x)$
- central limit theorem* : the limit of processes arising from multiple random fluctuations is a single G(x);
- similarly, in the large number limit, both the binomial and the Poisson distributions converge to a Gaussian;
- therefore $G(x|\ \mu=x_{meas},\ \sigma=error_{meas})$ is often used as the resolution function of a given experimental observation [*but as a good (?) first approx. only*].

* Consider n independent random variables $x = \{x_1, x_2, \ldots, x_n\}$, each with mean μ_i and variance σ^2_i; the variable

$$t = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{x_i - \mu_i}{\sigma_i}$$

can be shown to have a distribution that, in the large-n limit, converges to $G(t\,|\,\mu=0,\sigma=1)$.

Gauss distribution : hypothesis test

Given a measurement x with an expected value μ and an error σ, the value

$$F(x) = \int_x^{+\infty} G(t \mid \mu, \sigma) dt$$

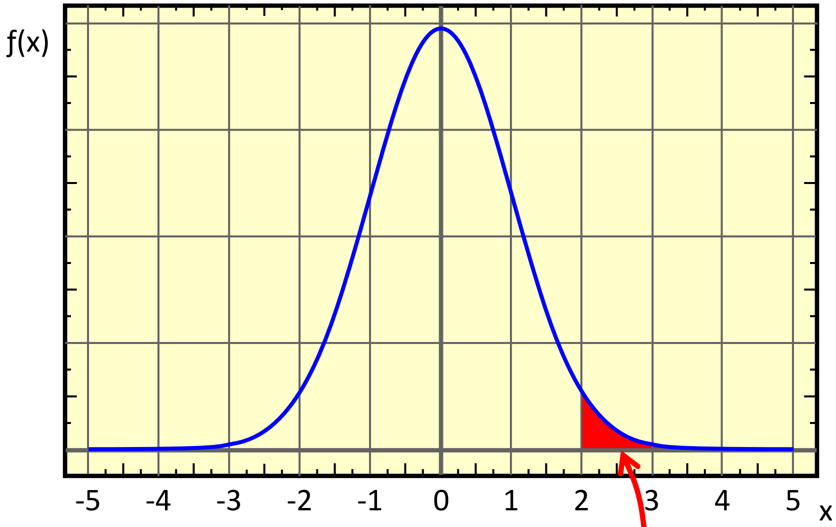
is often used as a "hypothesis test" of the expectation.

E.g. (see the ◣ plot): if the observation is at 2σ from the expectation, one speaks of a "2σ fluctuation" (not dramatic, it happens once every 44 trials – or 22 trials if both sides are considered).

The value of "5σ" * has assumed a special value in modern HEP [*see later*].

____

* if the expectation is not gaussian, one speaks of "5σ" when there is a fluctuation $\leq$ 2.87 E-7 in the tail of the probability, even in the <u>non-gauss case</u>.



x	G(x|0,1)	F(x)	$=1/n_{trial}$
0	3.989 E-01	5.000 E-01	2
1	2.420 E-01	1.587 E-01	6.3
2	5.399 E-02	**2.275 E-02**	44.0
3	4.432 E-03	1.350 E-03	741
4	1.338 E-04	3.167 E-05	31,500
5	**1.487 E-06**	**2.867 E-07**	**3.5 E+06**
6	6.076 E-09	9.866 E-10	1.0 E+09
7	9.135 E-12	1.280 E-12	7.8 E+11

Gauss distribution : the "Voigtian"

Assume :

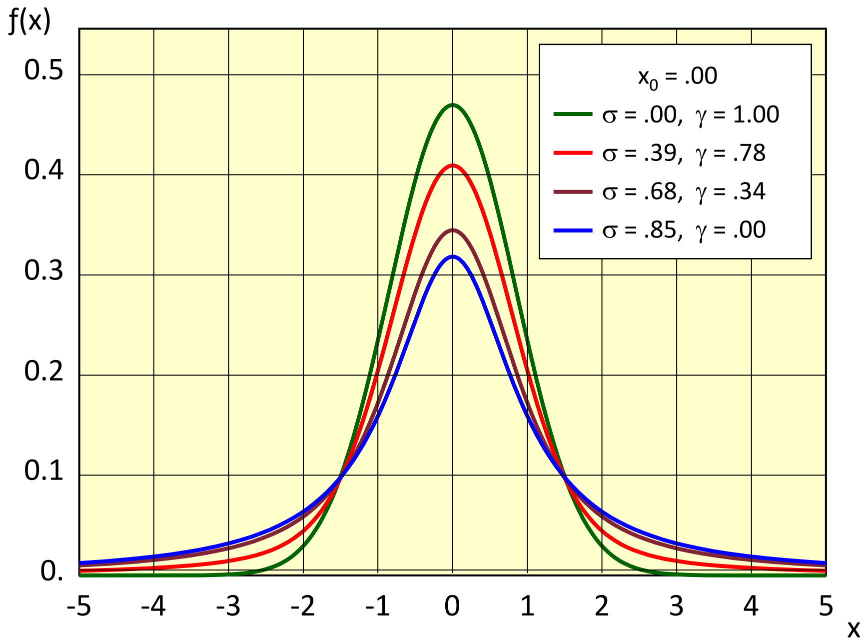
- a physical effect (e.g. a resonance) of intrinsic width described by a BW;
- a detector with a gaussian resolution;

→ the measured shape is a convolution "Voigtian" (after Woldemar Voigt).

- the V. is expressed by an integral and has no analytic form if $\gamma > 0$ AND $\sigma > 0$.
- however modern computers have all the stuff necessary for the numerical computations;
- mean = mathematically undefined [use x_0];
- variance = really undefined [divergent].

→ for real physicists : check carefully if resolution is gaussian, dynamics is BW, and γ and σ are uncorrelated .

$$f(x) = V(x|x_0,\gamma,\sigma) = $$
$$= \int_{-\infty}^{+\infty} dt G(t|0,\sigma) BW(x-t|x_0,\gamma) =$$
$$= \int_{-\infty}^{+\infty} dt \left[\frac{e^{\left(-\frac{t^2}{2\sigma^2}\right)}}{\sigma\sqrt{2\pi}} \right] \left[\frac{1}{\pi\gamma} \frac{\gamma^2}{(x-t-x_0)^2+\gamma^2} \right].$$



measurements

- Physics is an experimental science [*I would say "THE experimental science"*];
- therefore it is based on experimental verification;
- the "verification" is a sophisticated technique (see later & read Popper), but in essence it means that the theory has to be continuously confronted with experiments;
- … and when there are disagreements, the experiment wins(*);
- therefore, although this is NOT a course on experimental techniques, I find useful to remind a couple of formulæ about the main detectors of our science:
  - magnetic spectrometry;
  - calorimetry;
  - [do not forget Cherenkov's, scintillators, TRD's, …]
- although in real life the results do depend on experimental details and are obtained by complicated numerical evaluations, it is very instructive to study simple ideal cases.

(*) remember the Brecht poem "The Solution" :

(…) das Volk
Das Vertrauen der Regierung verscherzt habe
Und es nur durch verdoppelte Arbeit
zurückerobern könne. Wäre es da
Nicht doch einfacher, die Regierung
Löste das Volk auf und
Wählte ein anderes ?

[… the people had forfeited the confidence of the government and could win it back only by redoubled efforts. Would it not be easier in that case for the government to dissolve the people and elect another ?]

particle measurement: spectrometers

The Lorentz force bends a charged particle in a magnetic field ⇒ the particle momentum is computed from the measurement of a trajectory ℓ. Simple case:

- track ⊥ $\vec{B}$ (or ℓ = projected trajectory);
- $\vec{B}$ = constant (both mod. and dir.);
- ℓ ≪ R (i.e. α small, s ≪ R, arc ≈ chord);
- then (p in GeV, B in T, ℓ R s in m) :

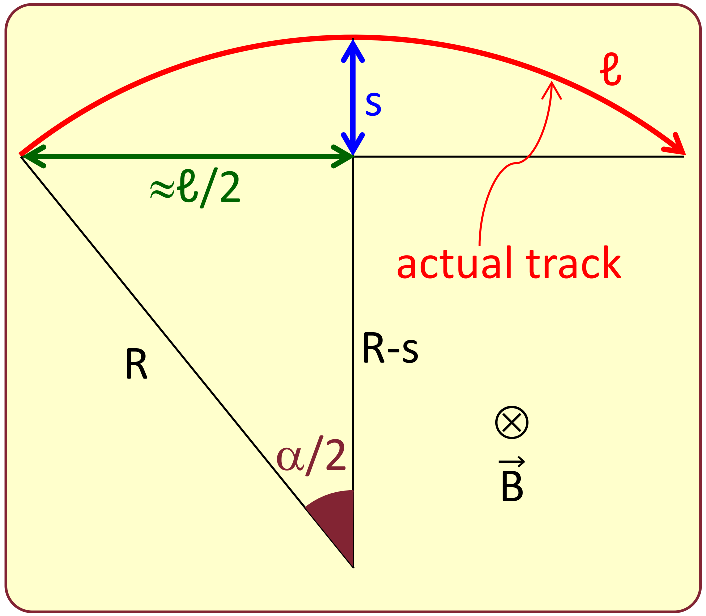
$$R^2 = (R-s)^2 + \ell^2/4 \rightarrow (R, \ell \gg s)$$

$$0 = \cancel{s^2} - 2Rs + \ell^2/4 \rightarrow$$

$$s = \frac{\ell^2}{8R} \simeq \frac{R\alpha^2}{8};$$

$$p = 0.3BR = 0.3B\frac{\ell^2}{8s};$$

$$\frac{\Delta p}{p} = \left|\frac{\partial p}{\partial s}\right|\frac{\Delta s}{p} = \frac{\cancel{p}}{s}\frac{\Delta s}{\cancel{p}} = \frac{\Delta s}{s} = \left(\frac{8\Delta s}{0.3B\ell^2}\right)p.$$



- e.g. B = 1 T, ℓ = 1.7 m, Δs = 200 μm →

  Δp/p =1.6 × 10^{-3} p (GeV);

- in general, from N points at equal distance along ℓ, each with error ε :

$$\frac{\Delta p}{p} \simeq \frac{\varepsilon p}{0.3B\ell^2}\sqrt{\frac{720}{N+4}}$$

(Gluckstern formula [PDG]).

particle measurement: spectrometers

[small difference] A track displaced by δ respect to a straight trajectory after ℓ; compute its momentum in the same case:

- track ⊥ $\vec{B}$ (or ℓ = projected trajectory);
- $\vec{B}$ = constant;
- ℓ ≪ R (i.e. β small, δ ≪ R, arc ≈ chord);
- then (p in GeV, B in T, ℓ R s in m) :

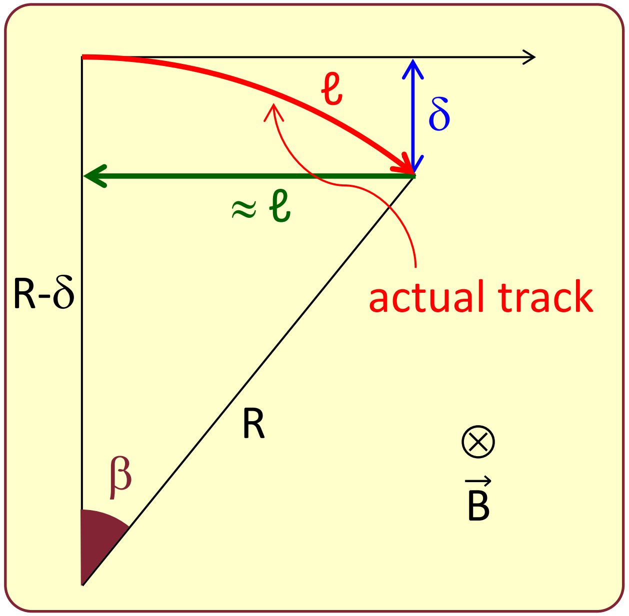
$$R^2 = (R-\delta)^2 + \ell^2 \rightarrow (R, \ell \gg \delta)$$

$$0 = \cancel{\delta^2} - 2R\delta + \ell^2 \rightarrow$$

$$\delta = \frac{\ell^2}{2R} = \frac{\ell\beta}{2};$$

$$p = 0.3BR = 0.3B\frac{\ell^2}{2\delta};$$

$$\frac{\Delta p}{p} = \left|\frac{\partial p}{\partial \delta}\right| \frac{\Delta\delta}{p} = \frac{\cancel{p}}{\delta}\frac{\Delta\delta}{\cancel{p}} = \frac{\Delta\delta}{\delta} = \left(\frac{2\Delta\delta}{0.3B\ell^2}\right)p.$$



- e.g. B = 1 T, ℓ = 1.8 m, Δδ = 200 μm →

  $\Delta p/p = 4 \times 10^{-4}$ p (GeV);

- Δp/p ∝ p → there exists a "maximum detectable momentum" (mdm), defined as the momentum with Δp/p = 1 (p_{mdm} = 2.5 TeV in the example);
- the mdm defines also the limit for charge identification.

particle measurement: spectrometers

- in presence of materials, the error depends also on the multiple scattering :

$$\Delta x = \frac{\ell}{\sqrt{3}} \frac{0.014}{\beta p(\text{GeV})} \sqrt{\frac{\ell}{X_0}} \left[1 + 0.038 \ell n \left(\frac{\ell}{X_0} \right) \right];$$

$$\left.\frac{\Delta p}{p}\right|_{m.s.}^{\ell} \propto p \Delta x \propto \text{constant};$$

e.g. ℓ = 1 m, air(X_0 = 300 m), p = 10 GeV :

($\rightarrow \beta$ = 1, ℓn term negligible)

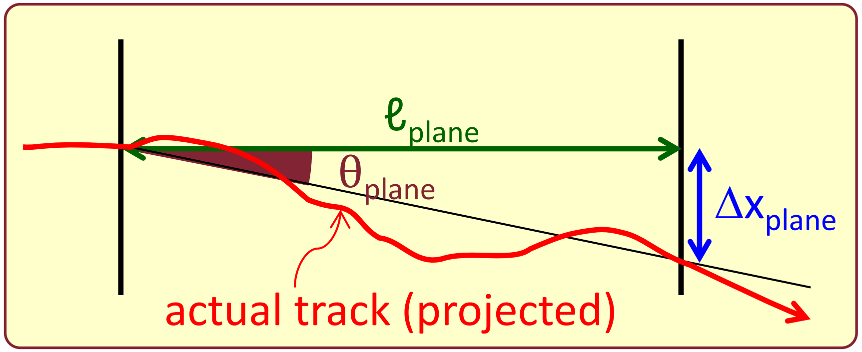
$$\Delta x \approx \frac{1}{\sqrt{3}} \frac{0.014}{10} \sqrt{\frac{1}{300}} = 47\ \mu\text{m};$$

(comparable with meas. error).

- the overall error is obtained by the sum in quadrature of all the contributions :

$$\left.\frac{\Delta p}{p}\right|_{tot} = \left(\left.\frac{\Delta p}{p}\right|_{meas} \right) \oplus \left(\left.\frac{\Delta p}{p}\right|_{m.s.} \right) = $$

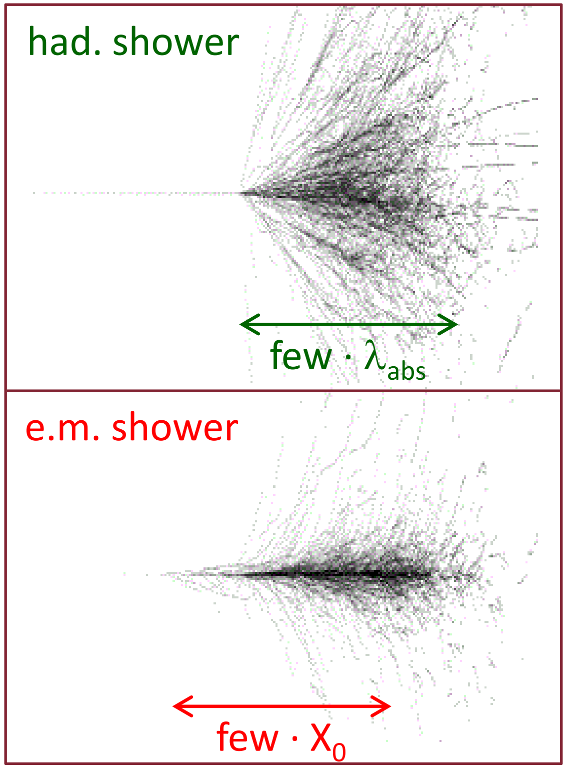
$$= \sqrt{\left(\left.\frac{\Delta p}{p}\right|_{meas} \right)^2 + \left(\left.\frac{\Delta p}{p}\right|_{m.s.} \right)^2}.$$



particle measurement: calorimeters

Based on the interactions of the particles in a dense material; the total length of the trajectories of the particles in the shower (= the signal) is proportional the primary energy :

E = calib × track_length = calib' × signal.



Errors depend on

- stochastic effects on shower development ;
- different response to different particles ($e^\pm \leftrightarrow \mu^\pm \leftrightarrow$ hadrons);
- shower physics [e.g. different amount of ($\gamma+e^\pm$) $\leftrightarrow$ (hadrons) in had showers];
- systematics of the detectors ("calibration" errors).

Formulas :

$$\lambda_{abs}(g/cm^2) \approx 35(g/cm^2)A^{1/3};$$

for solid heavy materials : $\lambda_{abs} = O(100\text{ cm})$;

discrimination (+ shape)

$$X_0(g/cm^2) \approx \frac{716(g/cm^2)A}{Z(Z+1)\ell n[287/\sqrt{Z}]};$$

for solid heavy materials : $X_0 = \text{few} \times 1\text{ cm}$.

particle measurement: calorimeters

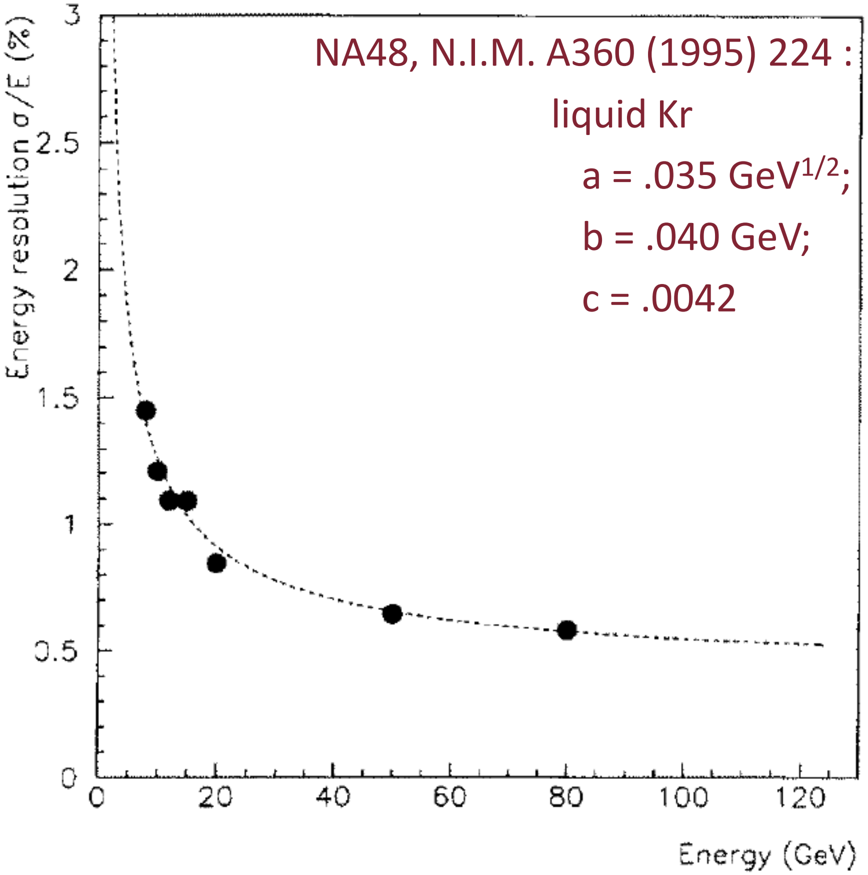
Energy errors, especially in e.m. calorimetry, are parametrized as :

$$\left.\frac{\Delta E}{E}\right|_{tot} = \left(\left.\frac{a}{\sqrt{E}}\right|_{stochastics}\right) \oplus \left(\left.\frac{b}{E}\right|_{noise}\right) \oplus \left(c|_{constant}\right).$$

- the stochastic term comes from the statistical fluctuations in the shower development;
- the noise term from the readout noise and pedestal fluctuations;
- the constant term from the non-uniformity and calibration error.

Other sources of error :

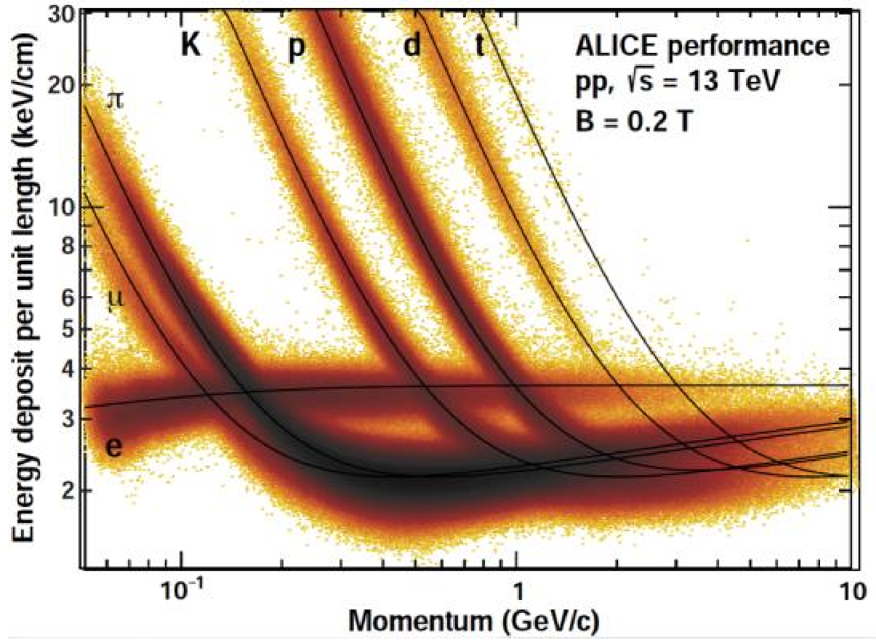
- shower leakage (longitudinal, lateral);
- upstream material;
- non-hermeticity;
- cluster algorithm (+ software approx.);
- e/π ratio [for hadr. non-compensating calos];
- non-linearity;
- nuclear effects;
- …



particle measurement: partid

The particle identification (*partid*) is a fundamental component of modern experiments; many algorithms are embedded in the event reconstruction [*no details*]:

- the gas detectors of the spectrometers detect the amount of ionization, which, for a given momentum, is a function of the particle mass (see fig.);
- the calorimeters select $e^{\pm}$ and γ from hadrons, thanks to the differences between e.m. and hadron showers;
- the $\mu^{\pm}$ are identified by their penetration through thick layers of material;
- the Cherenkov and TRD detectors measure the particle velocity (β and γ respectively), which allows for the determination of the mass;
- powerful kinematical algorithms put all the information together and combine it with known constraints (e.g. known decay modes);
- …



particle measurement: mass errors

Problem – For a given particle, assume independent measures of momentum ($p \pm \Delta p$) and velocity ($c\beta \pm c\Delta\beta$) [*e.g.* $|\vec{p}|$ *from magnetic bending and* β *from time-of-flight*]. Compute its mass ($m \pm \Delta m$).

$$m = \sqrt{E^2 - p^2} = \frac{p}{\beta\gamma} = p\frac{\sqrt{1-\beta^2}}{\beta};$$

$$\frac{\partial m}{\partial p} = \frac{1}{\beta\gamma};$$

$$\frac{\partial m}{\partial \beta} = p\left[-\frac{\sqrt{1-\beta^2}}{\beta^2} + \frac{1}{\beta}\frac{1}{2\sqrt{1-\beta^2}}(-2\beta)\right] =$$

$$= -p\left[\frac{\sqrt{1-\beta^2}}{\beta^2} + \frac{1}{\sqrt{1-\beta^2}}\right] =$$

$$= -p\left[\frac{1-\beta^2+\beta^2}{\beta^2\sqrt{1-\beta^2}}\right] = \frac{-p\gamma}{\beta^2}.$$

$$(\Delta m)^2 = \left(\frac{\partial m}{\partial p}\right)^2 (\Delta p)^2 + \left(\frac{\partial m}{\partial \beta}\right)^2 (\Delta\beta)^2 =$$

$$= \left(\frac{\Delta p}{\beta\gamma}\right)^2 + \left(\frac{p\gamma\Delta\beta}{\beta^2}\right)^2;$$

$$\left(\frac{\Delta m}{m}\right)^2 = \left(\frac{\Delta p}{\beta\gamma}\frac{\beta\gamma}{p}\right)^2 + \left(\frac{p\gamma\Delta\beta}{\beta^2}\frac{\beta\gamma}{p}\right)^2 = \ldots$$

$$m = \frac{p}{\beta\gamma} = p\frac{\sqrt{1-\beta^2}}{\beta};$$

$$\left(\frac{\Delta m}{m}\right)^2 = \left(\frac{\Delta p}{p}\right)^2 + \gamma^4\left(\frac{\Delta\beta}{\beta}\right)^2.$$

End - Introduction