Particle Physics - Chapter 5
K^0 mesons - CKM matrix

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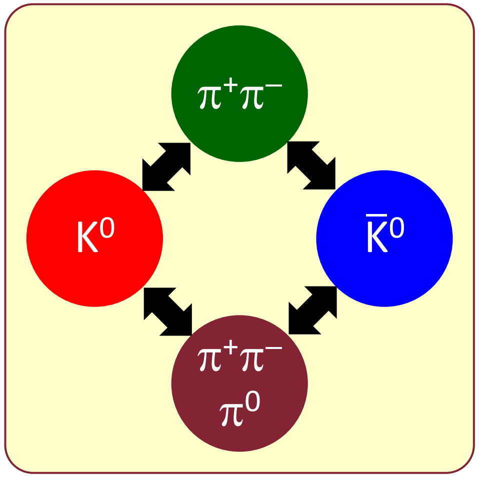
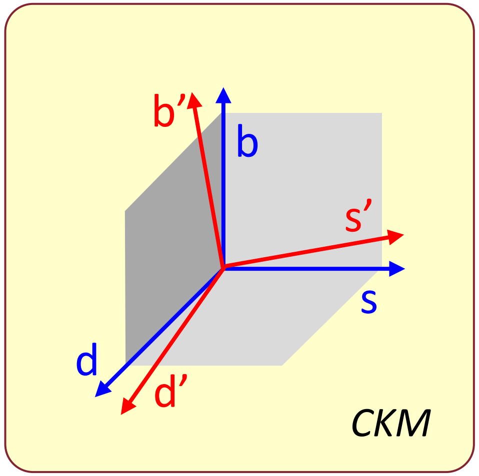
5 – K^0 mesons – CKM matrix

this section belongs to another chapter: It is here because of the similarity between ν and K^0 oscillations.





introduction

- The neutral mesons K^0 and $\bar{K}^0$ are special quark systems, in which unusual and surprising phenomena are generated.
- The mathematical interpretation of these phenomena is based almost exclusively on the application of the fundamental principles of q.m., in particular the principle of quantum superposition.
- The experimental observation of the effects of oscillation and regeneration is a further elegant confirmation of the validity of these principles.
- The successes of the experimental physics of the '50s and '60s have been based both on the confirmation of accurate theoretical predictions (like oscillations) and to new and unexpected phenomena (like $\mathbb{CP}$ violation).
- They have been possible thanks to new techniques (e.g. regeneration), and to new experimental methods (e.g. the new accelerators, bubble / spark chambers) and by data analysis via computer.
- The study of these particles is possible only by analyzing the symmetry of Nature; K^0 physics emerges from the analysis of CPT symmetries, strangeness and isospin.
- In successive years, the K^0 meson system has been replicated by the B^0 mesons, with further fundamental studies.
- The interpretation in the SM of the flavor and $\mathbb{CP}$ violations requires the weak interactions theory and the CKM matrix.
- *… but we hope that experiments show also physics* ***bSM*** *!!!*

introduction : quantum states

- Quarks and antiquarks of the u and d type can form two different neutral mesons : $(u\bar{u})$ $(d\bar{d})$, or linear combinations like π^0 or η [*see § quark model*].
- The same mechanism holds when heavier families, like (cs) (tb), are considered. Each heavy flavor has a quantum number which identifies it and its $\bar{q}$.
- These states make sense in a quantum basis of distinct conserved flavors, as in strong interactions.
- In different quantum bases (e.g. the one where $\mathbb{CP}$ is conserved, but not $\mathbb{C}$ and $\mathbb{P}$ separately), different states appear, which are linear superposition of the above.
- These states may offer a more natural description of the phenomena.

	K^0	$\bar{K}^0$	D^0	$\bar{D}^0$	B_d^0	$\bar{B}_d^0$	B_s^0	$\bar{B}_s^0$
$q\bar{q}$	$d\bar{s}$	$s\bar{d}$	$c\bar{u}$	$u\bar{c}$	$d\bar{b}$	$b\bar{d}$	$s\bar{b}$	$b\bar{s}$
S	+1	−1	0	0	0	0	−1	+1
C	0	0	+1	−1	0	0	0	0
B	0	0	0	0	+1	−1	+1	−1

quantum numbers of $q\bar{q}$ neutral mesons.

Warning: K^0 and K^+ are in the same doublet and contain $\bar{s}$; B^0/B^+ contain $\bar{b}$, while D^0 and D^+ contain c (not $\bar{c}$).

Questions (simple):
- other neutral mesons with heavy quarks ? [yes, D_s^0 and $\bar{D}_s^0$ $\rightarrow$ *write their q.n.*;
- why states like $t\bar{u}$, $t\bar{c}$, ..., are not listed ?

production of K^0 mesons: the problem

- The K^0-mesons are produced by <u>strong interactions</u> with a fixed strangeness S :

  $|K^0\rangle = |d\bar{s}\rangle$, S = +1; $|\bar{K}^0\rangle = |s\bar{d}\rangle$, S = −1.

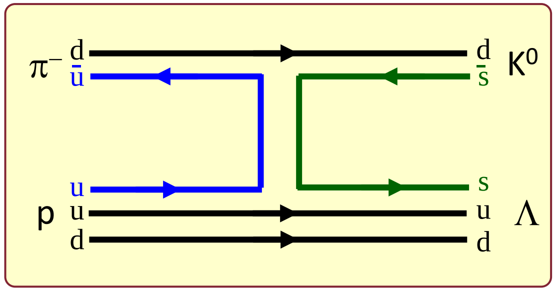
- Problem : get a <u>pure sample</u> of K^0's.
- A K^0 sample is created, e.g. ($\pi^- p \to \Lambda K^0$), with a "threshold energy" [*next slide*] :

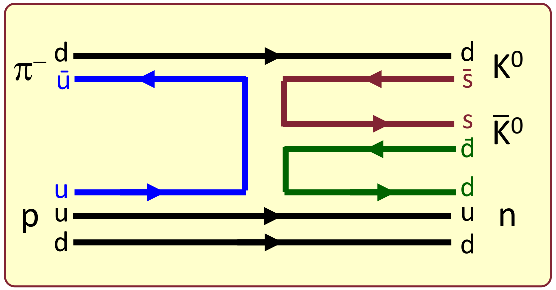
$$E_{\pi^-}^{min} = \frac{(m_\Lambda + m_K)^2 - (m_\pi^2 + m_N^2)}{2m_N} = 0.91 \text{ GeV},$$

to be compared with ($\pi^- p \to K^0 \bar{K}^0 n$):

$$E_{\pi^-}^{min} = \frac{(2m_K + m_N)^2 - (m_\pi^2 + m_N^2)}{2m_N} = 1.50 \text{ GeV},$$

- Since these processes are the simplest for K^0 / $\bar{K}^0$ respectively, with <u>$0.91 < E_\pi < 1.50$ GeV</u> only K^0's are produced [*the observation of the products of the interaction confirms the conservation of S*]
- However, even when selecting pure K^0's, some unexpected $\bar{K}^0$ mesons show up among the final state particles;





- this effect demonstrates that production and "life" (i.e. decay) of K^0 / $\bar{K}^0$ mesons follow different rules.
- [*the <u>weak interactions</u> do NOT conserve S, therefore they do NOT distinguish* K^0 *from* $\bar{K}^0$ → *once produced, their S is "forgotten" and they behave as the same particle, a superposition of different states*]

production of K^0 mesons : kinematics

general case

Study the reaction a b → c d (e.g. $\pi^- p \to \Lambda K^0$).

If $(m_c + m_d) > (m_a + m_b)$, it requires some kinetic energy to happen.

Study the process in the LAB system, i.e. the system where b (the proton) is at rest:

- the projectile a hits the target b, producing c and d :
- define E_a^{min} = the minimum energy of a IN THE LAB, such that the process happens
- in this case, c and d are at rest in the CM frame.

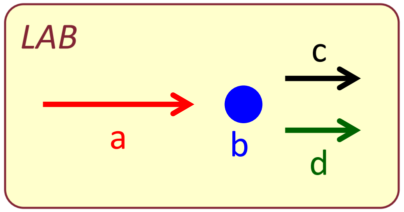
$$\text{LAB system} \begin{cases} a & (E_a, & p_a, & 0) \\ b & (m_b, & 0, & 0) \end{cases}$$

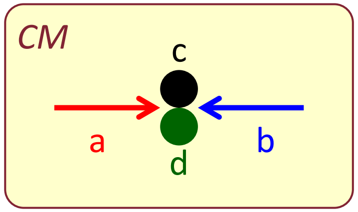
$$\text{CM system} \begin{cases} c & (m_c, & 0, & 0) \\ d & (m_d, & 0, & 0) \end{cases}$$

$$s\big|_{LAB}^{ini} = m_a^2 + m_b^2 + 2E_a^{min} m_b = s\big|_{CM}^{fin} = (m_c + m_d)^2;$$

$$E_a^{min} = \frac{(m_c + m_d)^2 - (m_a^2 + m_b^2)}{2m_b}.$$

- what, if $E_a^{min} < m_a$??? (an easy question);
- the result does NOT depend on the dynamics, but only on general kinematical constraints : it will be used in similar cases.





production of K^0 mesons : comments

To be specific, these strong interactions are allowed, because they conserve S :

a. $K^+ n \;\to K^0 p$;

b. $K^- p \;\to \bar{K}^0 n$;

c. $K^0 p \;\to K^+ n$;

d. $\bar{K}^0 p \;\to \pi^0 \Sigma^+$;

- instead, the following s.i. are forbidden :

e. $K^+ n \;\to \bar{K}^0 p$;

f. $K^- p \;\to K^0 n$;

g. $\bar{K}^0 p \;\to K^+ n$;

h. $K^0 p \;\to \pi^0 \Sigma^+$.

- Reactions (e-h) are only forbidden by S conservation;
- for a particle-antiparticle pair, because of the $\mathbb{CPT}$ symmetry, all the intrinsic properties are exactly correlated (equal or opposite mass, spin, charge, baryon-lepton number, decay channels, BR's).

- However, sometimes, the K^0 particle, generated via reaction (a), re-interacts as a $\bar{K}^0$ via reaction (d), or (b) $\to$ (c) :

i. $K^+ n \to$ "X^0" p, "X^0" p $\to \pi^0 \Sigma^+$;

ii. $K^- p \to$ "Y^0" n, "Y^0" p $\to K^+ n$;

[$X^0/Y^0 = K^0$ or $X^0/Y^0 = \bar{K}^0$?]

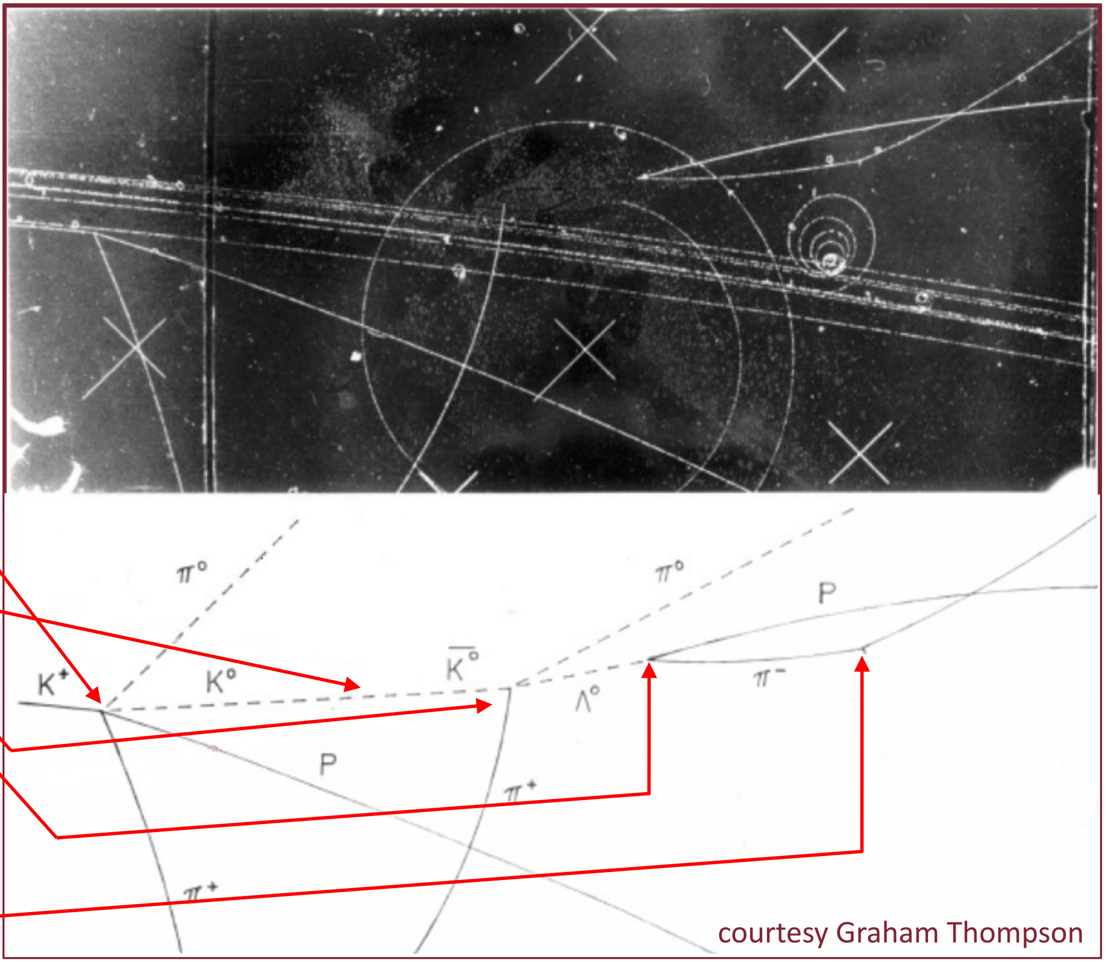
- it seems that there are transitions "in flight" (i.e. oscillations) $K^0 \leftrightarrow \bar{K}^0$.
- Can this effect show up also in their decay ?

NB Transitions ($n \leftrightarrow \bar{n}$) *are forbidden because of baryon number,* ($e^+ \leftrightarrow e^-$) *because of electric charge and lepton number. All these "charges" are conserved by all known interactions. Instead the oscillations* ($K^0 \leftrightarrow \bar{K}^0$) *are only forbidden by S conservation.*

the $K^0 \leftrightarrow \bar{K}^0$ puzzle : an event

A nice oscillation $K^0 \to \bar{K}^0$:

1. beam of K^+;
5. main vertex $K^+ p \to K^0 p \pi^+ \pi^0$;
6. $\boxed{K^0 \to \bar{K}^0 \text{ (???)}}$;
4. $\bar{K}^0 p \to \Lambda\pi^+\pi^0$;
3. $\Lambda \to p\pi^-$ (decay);
2. $\pi^- p \to X$;



courtesy Graham Thompson

[end/right $\to$ start/left] K^0 and $\bar{K}^0$ unambiguously identified, no other explanation.

???

the $K^0 \leftrightarrow \bar{K}^0$ puzzle : solution

In addition, the decay of K^0 and $\bar{K}^0$ was not understood and created a puzzle.

- Both K^0 and $\bar{K}^0$ can decay into $(\pi^+\pi^-)$ and $(\pi^+\pi^-\pi^0)$ [2π *and* 3π *states have different G-parity, but G is NOT conserved in w.i.*].
- The explanation was provided by Gell-Mann and Pais [Phys. Rev. 97, 1387 (1955)], before the discovery that w.i. violate parity:
  - K^0 and $\bar{K}^0$ are eigenstates of the strong interactions;
  - each is the antiparticle of the other, the $\mathbb{C}$ operator transforms $(K^0 \leftrightarrow \bar{K}^0)$;
  - they have opposite strangeness S;
  - if S were not there, they would mix (like in π^0 and η);
  - w.i. do not conserve S;
  - … and see **a mixture of K^0 and $\bar{K}^0$**.

Consequences:

- the mixture is interpreted as two new states, quantum superpositions of $K^0/\bar{K}^0$;
- if w.i. conserve $\mathbb{CP}$, the two new states must be $\mathbb{CP}$ eigenstates$^{(*)}$;
- since the new states are NOT a particle-antiparticle pair, they may have different properties (masses, lifetimes, decays);
- if the mass difference allows for that, the states oscillate between themselves;
- the only known decay was ("K^0"$\to \pi^+\pi^-$); a possible transition, generated via w.i., is then $[K^0 \leftrightarrow (\pi^+\pi^-) \leftrightarrow \bar{K}^0]$;
- another "K^0" must exist, "K^0" $\to \pi\pi\pi$.

$^{(*)}$ Today we know that the w.i. violate also $\mathbb{CP}$, but this violation is small, so provisionally we do not take it into account.

the $K^0 \leftrightarrow \bar{K}^0$ puzzle: predictions

(more formally ...)

TWO "K^0" STATES:

- different values of CP → CP = ± 1;
- one with CP=+1 and decay →($\pi\pi$), another with CP=–1 and decay →($\pi\pi\pi$);
- other decays are allowed for both states, but they have to conserve $\mathbb{CP}$ (e.g. no → $\pi\pi$ for the state CP=–1);
- the state ($\pi\pi\pi$) is near the kinematical threshold ($m_K \approx 3m_\pi$ + 70 MeV) → the lifetime of the ($\pi\pi\pi$) state is <u>much longer</u> than the lifetime of the ($\pi\pi$) one.
- the obvious proposal was to call "short" the CP=+1 state and "long" the CP=–1;
- so, two new particles have born:
  - they have been discovered;
  - their lifetimes and properties have been measured and found in agreement with the predictions :

1) K^0_S : CP = +1, $\tau = 0.90 \times 10^{-10}$ s, decay → $\pi^+\pi^-$, → $\pi^0\pi^0$;

2) K^0_L : CP = –1, $\tau = 0.51 \times 10^{-7}$ s, decay → $\pi^+\pi^-\pi^0$, $\pi^0\pi^0\pi^0$.

J.W. Cronin and M.S. Greenwood, Physics Today (July 1982) :

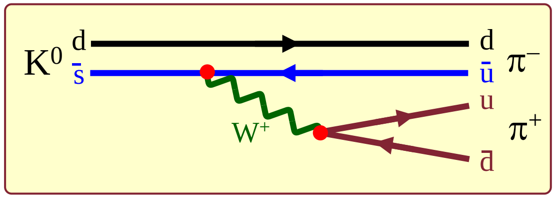
"So these gentlemen, Gell-Mann and Pais, predicted that in addition to the short-lived K mesons, there should be long-lived K mesons. They did it beautifully, elegantly and simply.

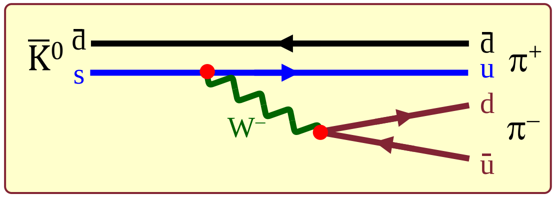
I think theirs is a paper one should read sometime just for its pure beauty of reasoning. It was published in Physical Review in 1955. A very lovely thing ! You get shivers up and down your spine, especially when you find you understand it. At the time many of the most distinguished theoreticians thought this prediction was really baloney."

the $K^0 \leftrightarrow \bar{K}^0$ puzzle : oscillations

In q.m. or quark model language:

- Both the K^0 and $\bar{K}^0$ decay via w.i. in the same final states; the $\pi^+\pi^-$ diagram is shown in the figure, while the others ($\pi^0\pi^0$; $\pi^+\pi^-\pi^0$; $\pi\ell\nu$) are similar :

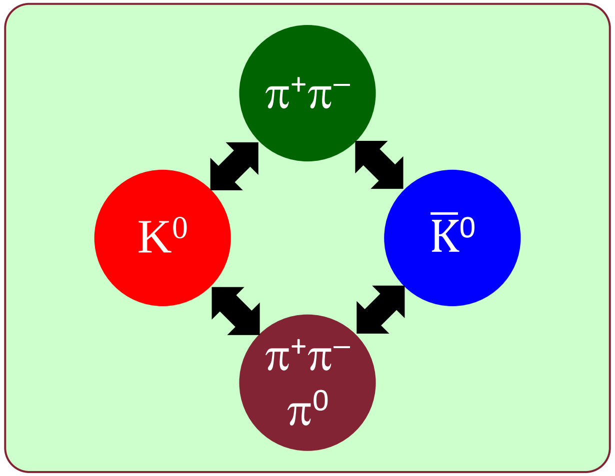


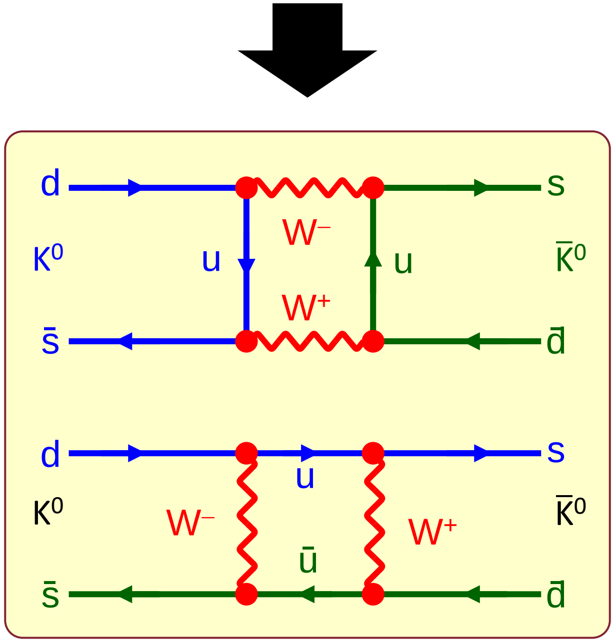


- The oscillations can be understood as a continuous transformation between the K^0 and $\bar{K}^0$ themselves, via the second order box-diagrams, or as a mixture, with time-dependent coefficients $\alpha(t)$, $\beta(t)$:

$$|K(t)\rangle = \alpha(t)\ |K^0\rangle + \beta(t)\ |\bar{K}^0\rangle\ ;$$

$\alpha(t)^2 + \beta(t)^2 = 1$ [$\times$ *a decreasing function of t, to account for their decay*]

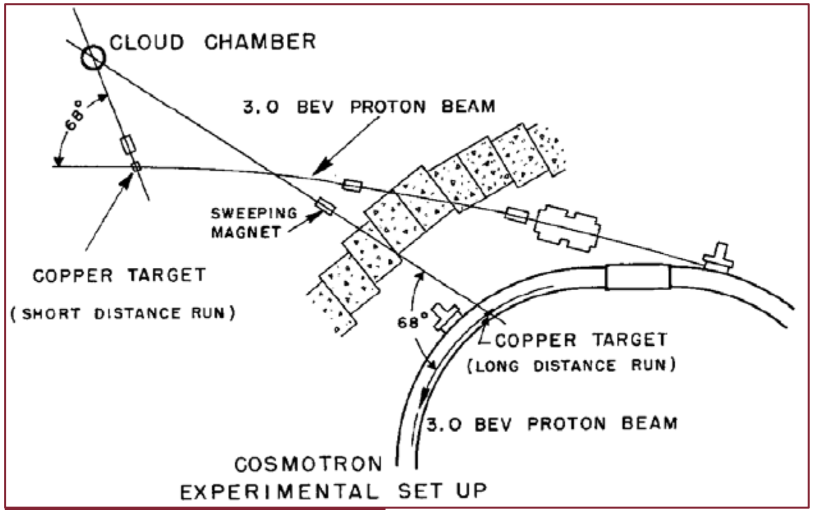




the $K^0 \leftrightarrow \bar{K}^0$ puzzle : K^0_L

- The K^0_L was first observed in 1956 by Lande and coll. with a cloud chamber.
- Brookhaven Cosmotron (3 GeV protons).
- Path between the beam and the cloud chamber (6 meters) is ~100 K^0_S / Λ lifetimes.
- This path is therefore sufficient for the decay of all strange particles known at the time.
- A few months later the same authors confirmed the result. They also observed in the cloud chamber interactions of these particles with the nuclei of He, producing final states with total S $\neq$ 0, like ($\bar{K}^0\ {}^4He \to \Sigma^- p p n \pi^+$).
- These states cannot be generated by a K^0, because of the value of S.

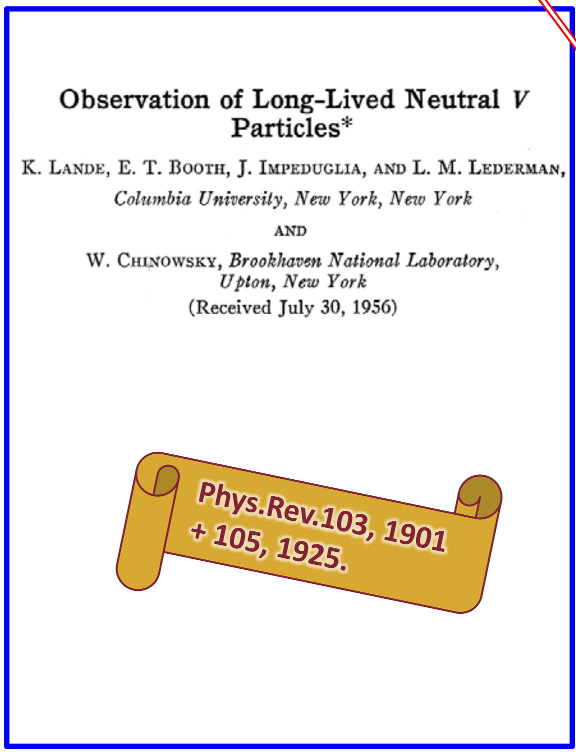
- However, no $\bar{K}^0$ should be present, because the primary proton energy was chosen to be below the energy threshold for $\bar{K}^0$ production, which is higher than for K^0 [*same argument as before*] .
- For some reason, $\bar{K}^0$ mesons have "appeared" $\to$ oscillation.



modern : "BEV" = GeV

the $K^0 \leftrightarrow \bar{K}^0$ puzzle : K^0_L results

- *The K^0_L was first observed in 1956 by Lande and coll. with a cloud chamber.*
- They found 26 events with a "V-zero", incompatible to be $(\pi^+\pi^-)$ because of their Q^2 (one shown on the right).
- [*today we interpret these events as decays* $(\pi^\pm e^\mp \nu_e)$, $(\pi^\pm \mu^\mp \nu_\mu)$, $(\pi^\pm \pi^\mp \pi^0)$].
- Events consistent with 3 body decays of neutral mesons of mass ~ 500 MeV.
- First estimate of the lifetime : 10^{-9} s < τ < 10^{-6} s, now $\tau = 0.53 \times 10^{-7}$ s.
- Another beautiful and "impossible" event (no $\bar{K}^0$ in the beam, see previous pages).

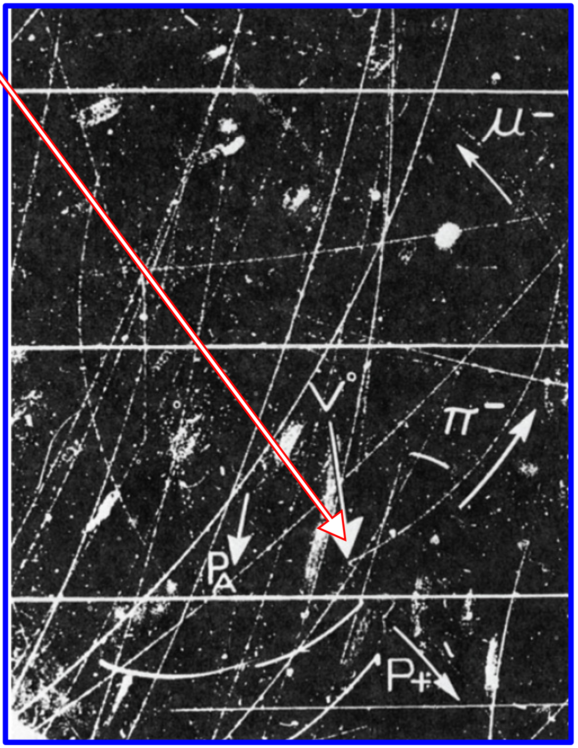

Observation of Long-Lived Neutral *V* Particles*

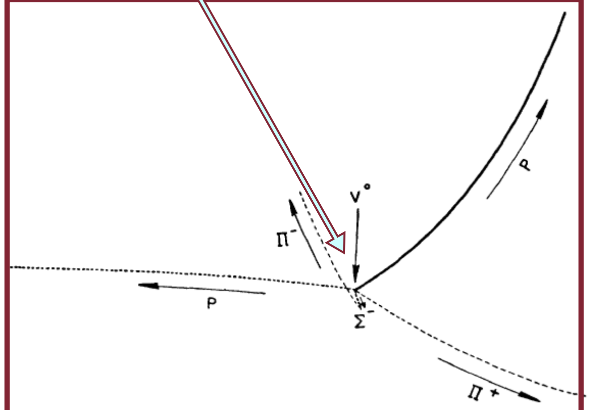
K. Lande, E. T. Booth, J. Impeduglia, and L. M. Lederman, *Columbia University, New York, New York*

and

W. Chinowsky, *Brookhaven National Laboratory, Upton, New York*

(Received July 30, 1956)





$\bar{K}^0\ {}^4He \rightarrow \Sigma^- ppn\pi^+$;
i.e. $\bar{K}^0\ n[ppn] \rightarrow \Sigma^-\pi^+[ppn]$
$\Sigma^- \rightarrow n\pi^-$;

[modern : $V^0=K^0$; $\Pi^\pm=\pi^\pm$]

K^0 decays in $\mathbb{CP}$ eigenstates : caveat

- In the following slides we assume that the K^0 decay conserve $\mathbb{CP}$, i.e. that both K^0_S and K^0_L are $\mathbb{CP}$ eigenstates with eigenvalues = ± 1.
- Although this is not true (see later), the violation is small and therefore the results obtained with this approximation are in fair agreement with (almost) all observations.
- To remember that, the next pages are marked by a little sign "$\mathbb{CP}$" in the upper right corner.

warning : the sign of ζ in
$\mathbb{C}\,|K^0\rangle = \zeta\,|\bar{K}^0\rangle$; $\mathbb{C}\,|\bar{K}^0\rangle = \zeta\,|K^0\rangle$;
is non-physical; in literature both $\zeta = \pm 1$;
here we (try to) stick to $\zeta = -1$.

K^0 decays in $\mathbb{CP}$ eigenstates : K^0_S and K^0_L

- The states $|K^0\rangle$ and $|\bar{K}^0\rangle$ are strong interactions (s.i.) eigenstates:

$\mathbb{C}\,|K^0\rangle = -\,|\bar{K}^0\rangle$; $\quad \mathbb{C}\,|\bar{K}^0\rangle = -\,|K^0\rangle$;

$\mathbb{P}\,|K^0\rangle = -\,|K^0\rangle$; $\quad \mathbb{P}\,|\bar{K}^0\rangle = -\,|\bar{K}^0\rangle$;

$\mathbb{CP}\,|K^0\rangle = +\,|\bar{K}^0\rangle$; $\quad \mathbb{CP}\,|\bar{K}^0\rangle = +\,|K^0\rangle$;

- these equations show that the s.i. states K^0 / $\bar{K}^0$ are NOT $\mathbb{CP}$ eigenstates;

- $|K^0_1\rangle$ and $|K^0_2\rangle$ are linear combinations of $|K^0\rangle$ and $|\bar{K}^0\rangle$, which are $\mathbb{CP}$ eigenstates :

$$|K^0_1\rangle = 1/\sqrt{2}\,[\,|K^0\rangle + |\bar{K}^0\rangle\,];$$
$$|K^0_2\rangle = 1/\sqrt{2}\,[\,|K^0\rangle - |\bar{K}^0\rangle\,];$$
$$|K^0\rangle = 1/\sqrt{2}\,[\,|K^0_1\rangle + |K^0_2\rangle\,];$$
$$|\bar{K}^0\rangle = 1/\sqrt{2}\,[\,|K^0_1\rangle - |K^0_2\rangle\,].$$

$\mathbb{CP}\,|K^0_1\rangle = +\,|K^0_1\rangle$; $\quad \mathbb{CP}\,|K^0_2\rangle = -\,|K^0_2\rangle$.

- The $(\pi\pi)$ and $(\pi\pi\pi)$ give (next slide) :

$\mathbb{CP}\,|2\pi\rangle = +\,|2\pi\rangle$;

$\mathbb{CP}\,|3\pi\rangle = -\,|3\pi\rangle$;

$K^0_1 \to 2\pi$

$K^0_2 \to 3\pi$

- Therefore :

$K^0_S \equiv K^0_1$; $\quad K^0_L \equiv K^0_2$.

if $\mathbb{CP}$ not conserved, **NOT** true !!!

> - K^0 and $\bar{K}^0$ are eigenstates of the strong interactions;
> - therefore, the creation process generates one of them [NOT the other];
> - but, as soon as they are created, they behave as a linear combination of K^0_S and K^0_L;
> - therefore they "live" (i.e. decay) as them;
> - then $K^0_S \to 2\pi$ (lot of phase space, small τ);
> - and $K^0_L \to 3\pi$ (small phase space, long τ);
> - if $K^0_{S,L}$ interact via strong interactions, they come back to the s.i. eigenstates, as K^0 or $\bar{K}^0$ with a given probability each.

K^0 decays in $\mathbb{CP}$ eigenstates : eigenvalues

Compute the eigenvalues of $\mathbb{CP}$.

For 2π systems :

- Since $J^{PC}(\pi^0) = 0^{-+}$:

  $\mathbb{P}\ |\pi^0\pi^0\rangle = (-)^2 (-)^L\ |\pi^0\pi^0\rangle = +|\pi^0\pi^0\rangle$;

  $\mathbb{C}\ |\pi^0\pi^0\rangle = (+)^2\ |\pi^0\pi^0\rangle = +|\pi^0\pi^0\rangle$;

  $\mathbb{CP}\ |\pi^0\pi^0\rangle = +\ |\pi^0\pi^0\rangle$;

- if $L = S_1 = S_2 = 0$:

  $\mathbb{PC}\ |\pi^+\pi^-\rangle = \mathbb{P}\ |\pi^-\pi^+\rangle = +|\pi^+\pi^-\rangle$;

- i.e. $CP(2\pi) = +1$, both for the $(\pi^0\pi^0)$ and $(\pi^+\pi^-)$ systems.

For 3π systems :

- $P(\pi^0\,\pi^0\,\pi^0) = (-)^3\,(-)^{L1}\,(-)^{L2} = -1$;

  $C(\pi^0\,\pi^0\,\pi^0) = (+)^3 = +1$;

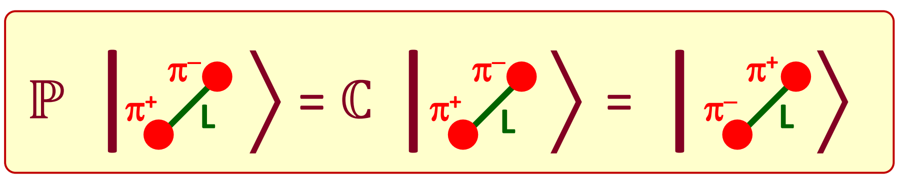
  $CP(\pi^0\,\pi^0\,\pi^0) = -1$;

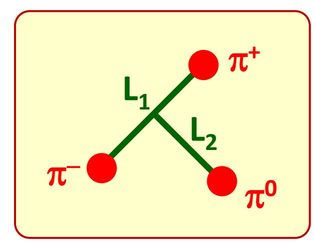
- $P(\pi^+\,\pi^-\,\pi^0) = (-)^3\,(-)^{L1}\,(-)^{L2} = -1$;

  $C(\pi^+\,\pi^-\,\pi^0) = (+)\,(-)^{L1} = +1$;

  $CP(\pi^+\,\pi^-\,\pi^0) = -1$;

- i.e. $CP(3\pi) = -1$, both for the $(\pi^0\pi^0\pi^0)$ and $(\pi^+\pi^-\pi^0)$ systems.





K^0 decays in $\mathbb{CP}$ eigenstates : Γ and τ

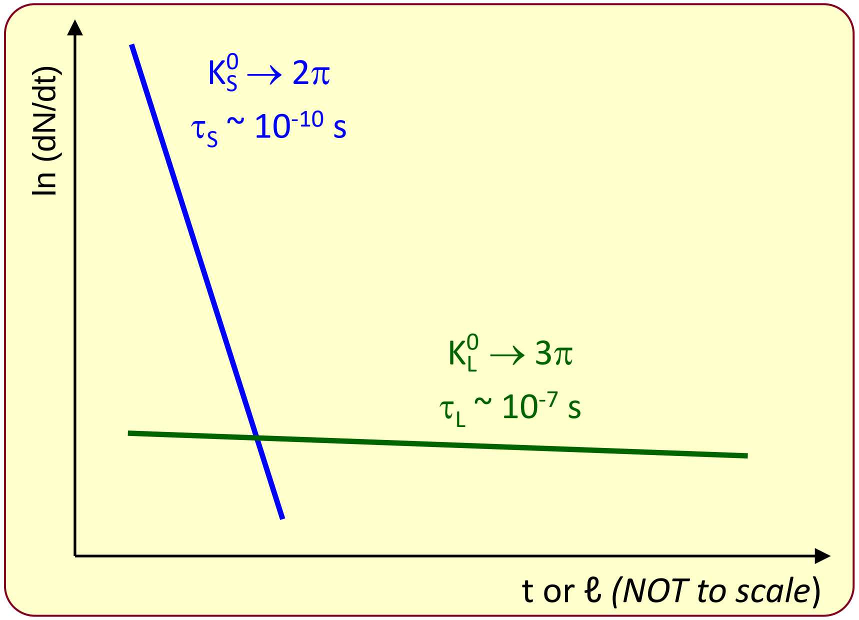
Conclusion : after strange particle production, expect two neutral particles of (not exactly, but almost) equal mass [*actually 498 MeV*] :

- the shorter (K^0_S) with
  - CP = +1;
  - decay into 2π;
  - "short" lifetime;
  - [$\tau_S = 0.90 \times 10^{-10}$ s = 7.4 μeV^{-1}, $\ell_S = c\tau_S$ = 2.68 cm];
- the longer (K^0_L) with
  - CP = –1;
  - decay into 3π;
  - "long" lifetime [$580 \times \tau_S$];
  - [$\tau_L = 0.51 \times 10^{-7}$ s = 0.013 μeV^{-1}, ℓ_L = 15.5 m]

- therefore :
  - $\Delta\Gamma_K \equiv \Gamma_L - \Gamma_S \approx -\Gamma_S$ = –7.4 μeV = = –11.2 ns^{-1}.

1 μeV = 1.52 ns^{-1};

1 ns^{-1} = 0.66 μeV.



K^0 oscillations

- While the K^0 and $\bar{K}^0$ masses are equal because of ℂℙ𝕋, no symmetry equalizes the masses and lifetimes of K^0_S and K^0_L;
- the measurement gives [*see later*] :

  $\Delta m_K = m(K^0_L) - m(K^0_S) = 3.51 \pm 0.018\ \mu eV$

  $= 5.303 \pm 0.009\ ns^{-1}$;
- $\Delta m_K \approx - \frac{1}{2}\,\Delta\Gamma_K$ [*no explanation, but deep phenomenological consequences*];
- the mass difference means that the two states [K^0_L *and* K^0_S] evolve with <u>different time constants</u>;
- following the evolution on the basis (K^0, $\bar{K}^0$), a "desynchronization" is observed between the K^0_S and K^0_L components, interpreted as oscillations ($K^0 \leftrightarrow \bar{K}^0$);
- a little algebra shows that, instead of a pure evolution of a particle of width Γ, which would give rise to an intensity N(t) $\propto$ exp $(-\Gamma t)$ = exp $(-t/\tau)$, we have a different phenomenon :

$$\psi_S(t) = \psi_S^0 \exp\left[-\left(\Gamma_S/2 + i m_S\right)t\right];$$

$$\psi_L(t) = \psi_L^0 \exp\left[-\left(\Gamma_L/2 + i m_L\right)t\right];$$

- take a pure K^0 beam at t=0 : then, in case of no decay (Γ = 0, τ = ∞), the probability $\mathcal{P}$ to find a K^0 or a $\bar{K}^0$, function of t, is:

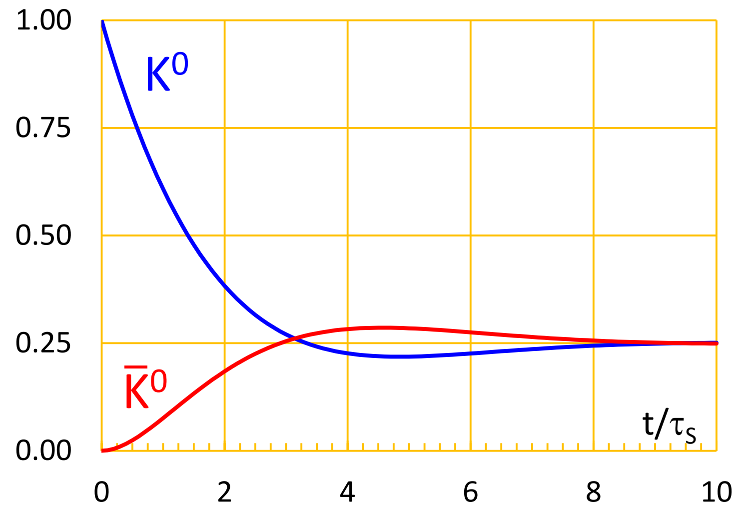
$$\mathcal{P}_{K^0}(t) = \frac{1}{4}\left|e^{(-i m_S t)} + e^{(-i m_L t)}\right|^2 = \cos^2\left(\frac{\Delta m_K}{2}t\right);$$

$$\mathcal{P}_{\bar{K}^0}(t) = \frac{1}{4}\left|e^{(-i m_S t)} - e^{(-i m_L t)}\right|^2 = \sin^2\left(\frac{\Delta m_K}{2}t\right).$$

- In addition, the oscillations are damped by the occurrence of the decays (τ_L=1/Γ_L >> τ_S=1/Γ_S); Γ_S dominates, because of the shorter lifetime [*next slide*].

K^0 oscillations: formulæ

- The amount of K^0 and $\bar{K}^0$ can be computed as a function of (proper) time, by simple considerations of quantum mechanics.
- E.g. starting with pure K^0 (fig.), there is an "oscillation" between the two states, according to τ_S, τ_L, Δm ($=|m_S-m_L|$).
- The figure is made with $\tau_S \ll \tau_L$ and $\Delta m = 1/(2\tau_S)$ (not exact, but realistic and simple).
- For the computations, see next page.



$$R(K^0)(t) = \left|\langle K^0 | \psi(t)\rangle\right|^2 = \frac{1}{4}\left[\exp\left(-\frac{t}{\tau_S}\right) + \exp\left(-\frac{t}{\tau_L}\right) + 2\exp\left(-\frac{\tau_L+\tau_S}{2\tau_L\tau_S}t\right)\cos(\Delta m_K t)\right];$$

$$R(\bar{K}^0)(t) = \left|\langle \bar{K}^0 | \psi(t)\rangle\right|^2 = \frac{1}{4}\left[\exp\left(-\frac{t}{\tau_S}\right) + \exp\left(-\frac{t}{\tau_L}\right) - 2\exp\left(-\frac{\tau_L+\tau_S}{2\tau_L\tau_S}t\right)\cos(\Delta m_K t)\right].$$

K^0 oscillations: math

CP

Some (simple and tedious) algebra. Start with f K^0 and (1–f) $\bar{K}^0$. Then put f=1:

$$|\psi(t=0)\rangle = f|K^0\rangle + (1-f)|\bar{K}^0\rangle = \frac{f}{\sqrt{2}}\left(|K^0_S\rangle + |K^0_L\rangle\right) + \frac{1-f}{\sqrt{2}}\left(|K^0_L\rangle - |K^0_S\rangle\right) = \frac{2f-1}{\sqrt{2}}|K^0_S\rangle + \frac{1}{\sqrt{2}}|K^0_L\rangle;$$

$$|\psi(t)\rangle = \frac{2f-1}{\sqrt{2}} e^{-\left(\frac{\Gamma_S}{2}+im_S\right)t}|K^0_S\rangle + \frac{1}{\sqrt{2}} e^{-\left(\frac{\Gamma_L}{2}+im_L\right)t}|K^0_L\rangle \xrightarrow{\boxed{f=1}} \frac{1}{\sqrt{2}} e^{-\left(\frac{\Gamma_S}{2}+im_S\right)t}|K^0_S\rangle + \frac{1}{\sqrt{2}} e^{-\left(\frac{\Gamma_L}{2}+im_L\right)t}|K^0_L\rangle;$$

$$R(K^0)(t) = \left|\langle K^0|\psi(t)\rangle\right|^2 = \frac{1}{4}\left|\left[\langle K^0_S| + \langle K^0_L|\right]\left[e^{-\left(\frac{1}{2\tau_S}+im_S\right)t}|K^0_S\rangle + e^{-\left(\frac{1}{2\tau_L}+im_L\right)t}|K^0_L\rangle\right]\right|^2 =$$

$$\boxed{\left|x\cdot\exp(iy)\right|^2 = (x\cdot\cos y)^2 + (x\cdot\sin y)^2}$$

$$\boxed{\langle K^0_i|K^0_j\rangle = \delta_{ij}}$$

$$= \frac{1}{4}\left[e^{\frac{-t}{2\tau_S}}\cos(m_S t) + e^{\frac{-t}{2\tau_L}}\cos(m_L t)\right]^2 + \frac{1}{4}\left[e^{\frac{-t}{2\tau_S}}\sin(m_S t) + e^{\frac{-t}{2\tau_L}}\sin(m_L t)\right]^2 =$$

$$= \boxed{\frac{1}{4}\left[e^{\frac{-t}{\tau_S}} + e^{\frac{-t}{\tau_L}} + 2e^{-\frac{(\tau_L+\tau_S)t}{2\tau_L\tau_S}}\cos(\Delta m_K t)\right]} \boxed{\xrightarrow[\tau_S\to\infty,\,\tau_L\to\infty]{} \cos^2\left(\frac{\Delta m_K t}{2}\right)}.$$

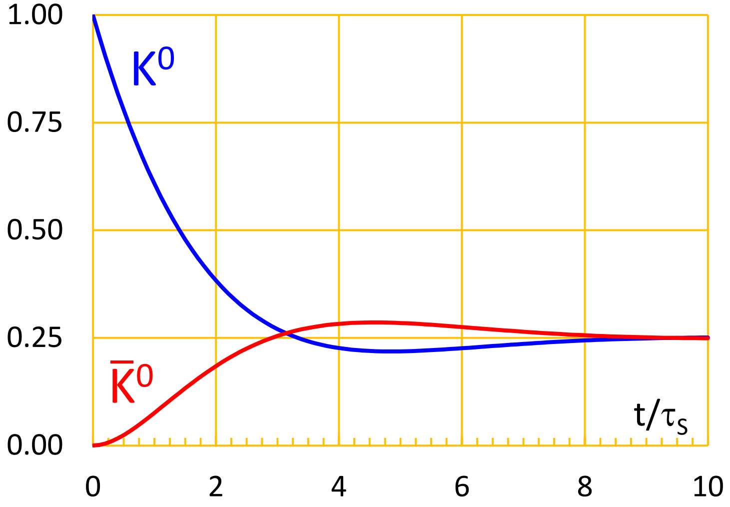
$$\boxed{\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta)}$$

$$\boxed{1+\cos\alpha = 2\cos^2(\alpha/2)}$$

Damped oscillation (previous slide). If both τ_L and τ_S >> $1/\Delta m_K$ (not true) → simple oscillation.

The computations for $R(\bar{K}^0)(t)$ and for f≠1 are left to the (patient) reader.

K^0 oscillations: semileptonic decays



- To test this prediction, the experimental problem [Bettini] is to distinguish $K^0 \leftrightarrow \bar{K}^0$ when they decay. It is not possible from the 2π or 3π states, because these channels have definite CP, not definite strangeness.

- To select definite strangeness states, select semileptonic decays of K^0_L. These decays obey the "$\Delta S = \Delta Q$ rule": *the difference between the strangeness of the hadrons in the final and initial states is equal to the difference of their electric charges*. The rule is a consequence of the quark contents of the states [$K^0 = \bar{s}d$] :

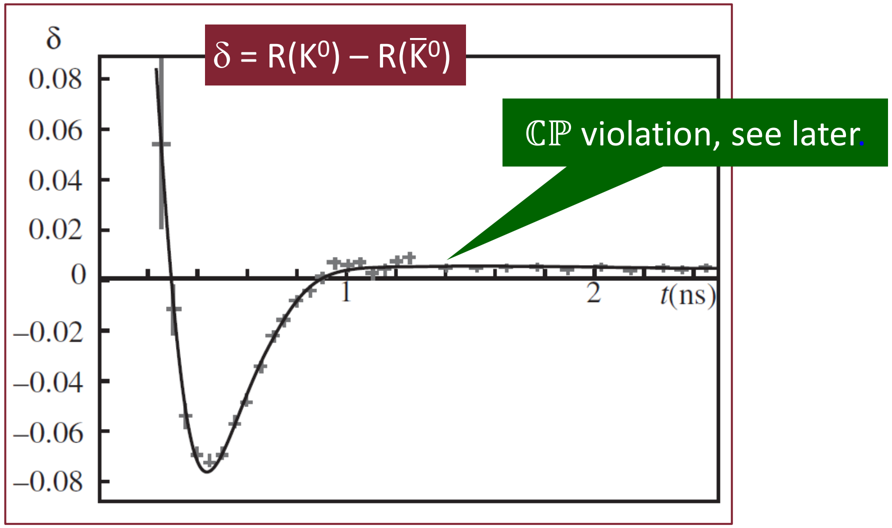
$$\bar{s} \to \bar{u}\ell^+\nu_\ell \Rightarrow K^0 \to \pi^-\ell^+\nu_\ell;\; K^0 \nrightarrow \pi^+\ell^-\bar{\nu}_\ell;$$

$$s \to u\ell^-\bar{\nu}_\ell \Rightarrow \bar{K}^0 \to \pi^+\ell^-\bar{\nu}_\ell;\; \bar{K}^0 \nrightarrow \pi^-\ell^+\nu_\ell.$$

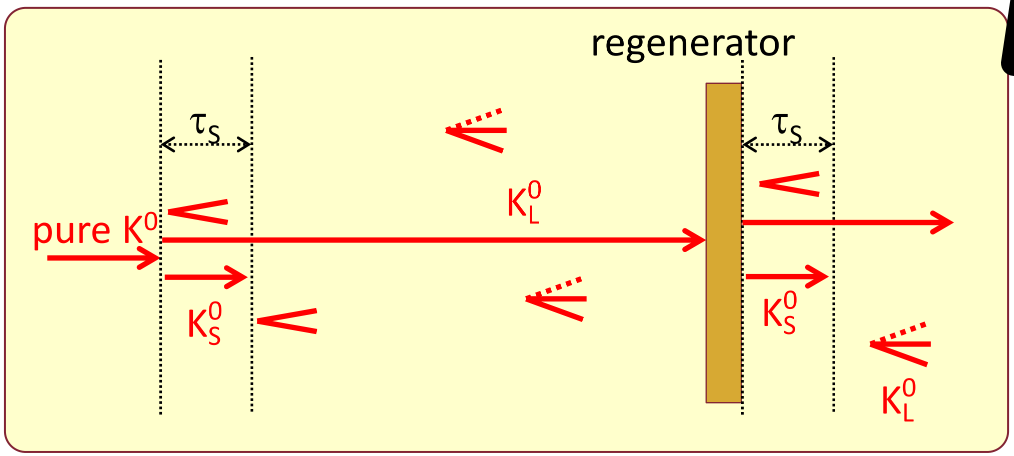
- The sign of the charged lepton flags the strangeness of the $K^0/\bar{K}^0$. The semileptonic decays are called $K^0{}_{e3}$ and $K^0{}_{\mu3}$ depending on the lepton. Their branching ratios are large:

  $BR(K^0{}_{e3})$ = 41%, $BR(K^0{}_{\mu3})$ = 27%.

- The experimental measure regards the charge asymmetry δ, i.e. the difference between +ve and –ve leptons, which is directly related to the oscillations. The results agree very well with the expectations, but the tail.



K^0 regeneration



[the correct computation takes into account the L-factor $\beta\gamma$]

The regeneration (Pais and Piccioni, 1956) consisted in a clever use of an absorber (the "regenerator"), positioned at a distance determined by τ_S and τ_L, to demonstrate the superposition of K^0 and $\bar{K}^0$.

[explanation on the next slide]

Abraham Pais Oreste Piccioni

K^0 regeneration : the idea

- Start with a pure K^0 beam in vacuum (equal amounts of K_S^0 and K_L^0).
- After $t \approx 10\ \tau_S$ the K_S^0 intensity down by factor $e^{(-t/\tau S)} = e^{-10} \approx 45 \times 10^{-6}$ (none left).
- [*For* K^0 *with 1 GeV momentum this corresponds* to ~*0.5 m.*]
- The K_L^0 intensity is down by $e^{(-t/\tau L)} \approx 0.98$, i.e. all left.
- After 0.5 m, 100% K_L^0 (50% K^0 + 50% $\bar{K}^0$).
- If we put another target at [say] $t = 20\ \tau_S$ [1 m downstream], we will get K^0 interactions as well as $\bar{K}^0$.
- K^0 and $\bar{K}^0$ interact (strongly) differently in the target :

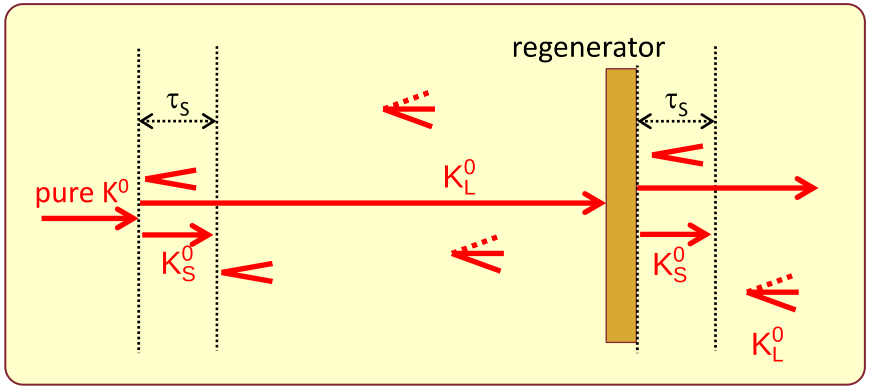
  $K^0\, p \to K^0\, p,\ K^+\, n;$

  $K^0\, n \to K^0\, n;$

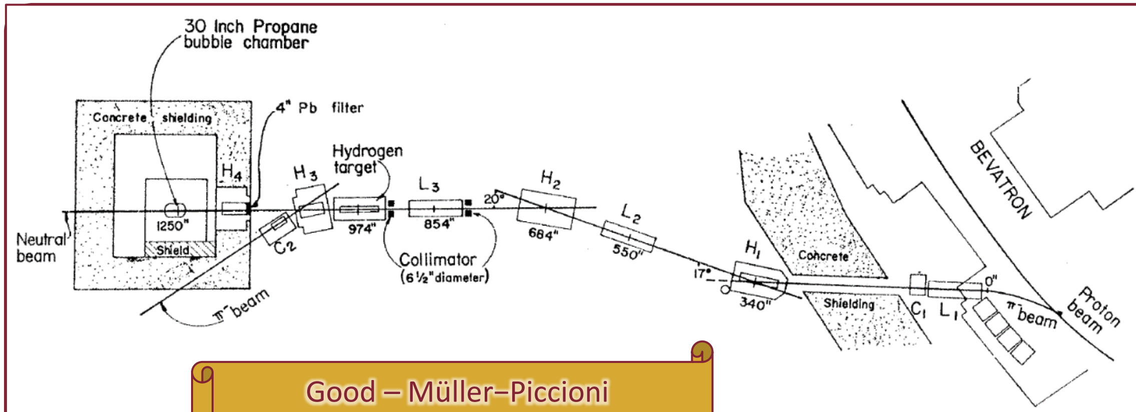
  $\bar{K}^0\, p \to \bar{K}^0\, p,\ \Lambda\, \pi^+; \to \Sigma^0\, \pi^+,\ \Sigma^+\, \pi^0;$

  $\bar{K}^0\, n \to \bar{K}^0\, n,\ \Lambda\, \pi^0; \to \Sigma^+\, \pi^-,\ \Sigma^0\, \pi^0,\ \Sigma^-\, \pi^+;$

- The s quark from the $\bar{K}^0$ can swap with one of the quarks in the proton or neutron, but the $\bar{s}$ from the K^0 cannot [e.g. $\bar{K}^0\, p \to \Lambda\, X$, but $\cancel{K^0\, p \to \Lambda\, X}$] .
- Hence there are more $\bar{K}^0$ processes, so the $\bar{K}^0$ are more strongly absorbed.
- Then, no longer 50% K^0 +50% $\bar{K}^0$ (as in K_L^0), but an amount of K_S^0 has "born".
- So will have some K_S^0 decays again.



K^0 regeneration : experiment

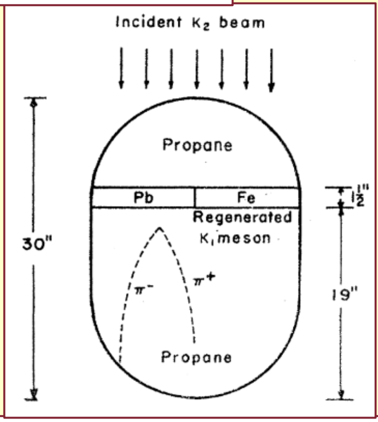


Good – Müller–Piccioni
Phys. Rev., 124, 1223 (1961).

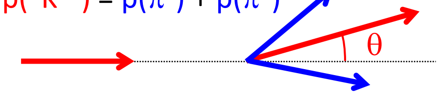
The experiment used a beam of 1.1 GeV π^- from the "Bevatron", the 6.2 GeV ("BeV", old American) proton synchrotron at LNL, Berkeley.

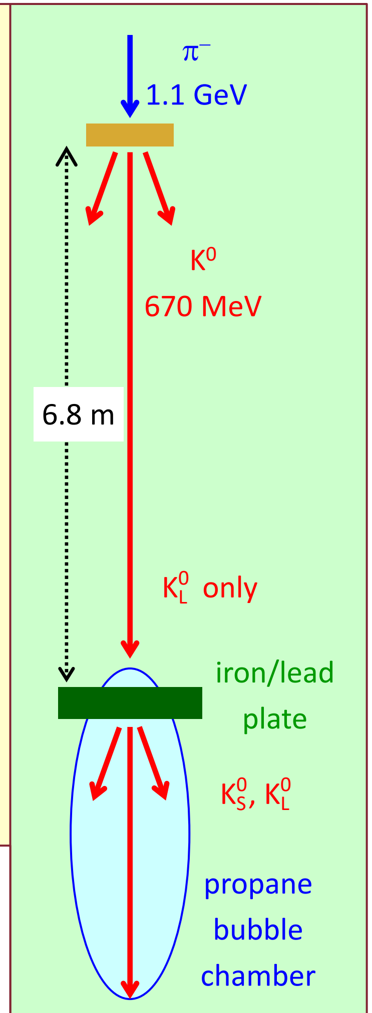
The propane bubble chamber was able to measure the $\pi^\pm$ momenta by their curvature in magnetic field.

Therefore the angle θ (shown in the fig) is measured.



$\vec{p}(\text{"}K^0\text{"}) = \vec{p}(\pi^+) + \vec{p}(\pi^-)$





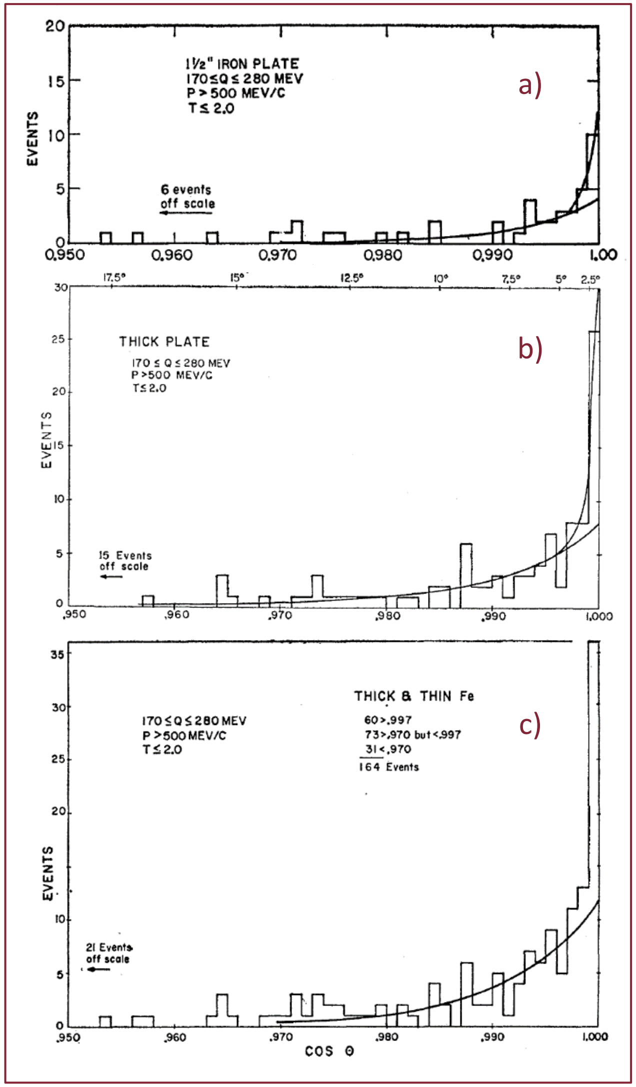
K^0 regeneration : results

A study of the phenomenon by M. Good (1957) considered three types of regeneration, with different distributions of the angle θ between the incoming and the regenerated particle :

1. Regeneration for transmission ("forward") : $\theta = 0$. No momentum transfer to the nucleus : coherent.
2. Regeneration for diffraction : elastic scattering, θ distribution as in diffraction.
3. Inelastic regeneration : interaction with individual nucleons, θ distribution as in scattering.

- The relative amount of the three depends on the small mass difference $\Delta m_K = m(K^0_L) - m(K^0_S)$;
- 200 observed 2π decays;
- they were able to confirm oscillations and regeneration;
- … and to measure the mass difference (units $\hbar/\tau_s$) :
  $\Delta m_K = 0.84^{+0.89}_{-0.22}$;

[*very clever result, despite present best value is 2 σ smaller*]



$\mathbb{CP}$ violation

Redefine the K^0 mesons system :

- K^0 and $\bar{K}^0$ as the particle produced in strong interactions (i.e. s.i. eigenstates) :
  - $|K^0\rangle = |d\bar{s}\rangle$, $S = +1$; $|\bar{K}^0\rangle = |s\bar{d}\rangle$, $S = -1$;
  - $\mathbb{C}\ |K^0\rangle = -|\bar{K}^0\rangle$; $\mathbb{C}\ |\bar{K}^0\rangle = -|K^0\rangle$;
- K_1^0 and K_2^0 as the $\mathbb{CP}$ eigenstates :
  - $|K_1^0\rangle = 1/\sqrt{2}\ [\ |K^0\rangle + |\bar{K}^0\rangle\ ]$;
  - $|K_2^0\rangle = 1/\sqrt{2}\ [\ |K^0\rangle - |\bar{K}^0\rangle\ ]$;
  - $\mathbb{CP}\ |K_1^0\rangle = +\ |K_1^0\rangle$;
  - $\mathbb{CP}\ |K_2^0\rangle = -\ |K_2^0\rangle$;
- K_S^0 and K_L^0 as the states with lifetimes τ_S, τ_L [*NOT necessarily $\mathbb{CP}$ eigenstates*] :
  - $\tau_S = 0.90 \times 10^{-10}$ s; $\tau_L = 0.51 \times 10^{-7}$ s;
- the $(\pi^+\pi^-)$, $(\pi^0\pi^0)$, $(\pi^+\pi^-\pi^0)$ systems are $\mathbb{CP}$ eigenstates:
  - $\mathbb{CP}\ |2\pi\rangle = +\ |2\pi\rangle$; $\mathbb{CP}\ |3\pi\rangle = -|3\pi\rangle$;

- Clearly, if $K_1^0 = K_S^0$, $K_2^0 = K_L^0$, then $\mathbb{CP}$ is conserved in the K^0 decays; i.e. $\mathbb{CP}$ conservation implies

$$K_S^0 \to 2\pi,\ K_L^0 \to 3\pi;$$

???

- On the contrary, decays

$$K_L^0 \to 2\pi,\ K_S^0 \to 3\pi$$

with small, but non-0 BR, would be an experimental evidence of **the NON-CONSERVATION of $\mathbb{CP}$**.

In the following slides we do NOT assume $\mathbb{CP}$ conservation in K^0 decays.

The little "$\mathbb{CP}$" in the upper right corner has disappeared.

$\mathbb{CP}$ violation: test of the theory

A textbook "experimentum crucis".

Consider three possible interactions:

a. $\mathbb{C}$ and $\mathbb{P}$ conserved ["strong i."] :
- $\mathbb{C}$, $\mathbb{P}$ conserved separately,
- strangeness conserved;
- eigenstates K^0, $\bar{K}^0$;

b. $\mathbb{CP}$ conserved :
- $\mathbb{C}$, $\mathbb{P}$ not conserved separately, but $\mathbb{CP}$ conserved;
- strangeness NOT conserved;
- eigenstates $K_1^0 \to 2\pi$, $K_2^0 \to 3\pi$ [because 2π and 3π states are $\mathbb{CP}$ eigenstates];

c. $\mathbb{CP}$ non conserved ["weak i."] :
- K_S^0, K_L^0 decay with lifetimes τ_S, τ_L;
- strangeness NOT conserved;
- eigenstates K_S^0, K_L^0 [K_S^0 and K_L^0 NOT $\mathbb{CP}$ eigenstates].

Strong interactions follow [a].

If weak interactions conserve $\mathbb{CP}$, then they follow [b]:

$$\boxed{|K_1^0\rangle = |K_S^0\rangle\ ,\ |K_2^0\rangle = |K_L^0\rangle, \quad K_S^0 \to 2\pi\ ,\ K_L^0 \to 3\pi.}$$

Instead, if $\mathbb{CP}$ is violated in w.i., then [b] is only a first approx. of [c].

The discriminant is the existence (at least with a small BR) of the decays: $K_S^0 \to 3\pi$, $K_L^0 \to 2\pi$.

Conclusion :

since a small amount of ($K_S^0 \to 3\pi$) is not observable, due to the background ($K_L^0 \to 3\pi$), the key observation is $\boxed{(K_L^0 \to 2\pi)}$.

$\mathbb{CP}$ violation: summary

a) Flavor eigenstates :

$|K^0\rangle = d\bar{s}$; $S = +1$; $\mathbb{CP}\ |K^0\rangle = +|\bar{K}^0\rangle$;

$|\bar{K}^0\rangle = s\bar{d}$; $S = -1$; $\mathbb{CP}\ |\bar{K}^0\rangle = +|K^0\rangle$.

(strong interactions)

b) CP eigenstates :

$|K^0_1\rangle = 1/\sqrt{2}\,[\,|K^0\rangle + |\bar{K}^0\rangle\,]$; CP = +1;

$|K^0_2\rangle = 1/\sqrt{2}\,[\,|K^0\rangle - |\bar{K}^0\rangle\,]$; CP = –1;

$|K^0\rangle = 1/\sqrt{2}\,[\,|K^0_1\rangle + |K^0_2\rangle\,]$;

$|\bar{K}^0\rangle = 1/\sqrt{2}\,[\,|K^0_1\rangle - |K^0_2\rangle\,]$.

(K^0 oscillations+decay, regeneration)

c) Mass eigenstates in vacuum :

$|K^0_S\rangle = (|K^0_1\rangle + \varepsilon|K^0_2\rangle) / \sqrt{1+|\varepsilon|^2}$;

$|K^0_L\rangle = (\varepsilon|K^0_1\rangle + |K^0_2\rangle) / \sqrt{1+|\varepsilon|^2}$

($\mathbb{CP}$ violation in vacuum)

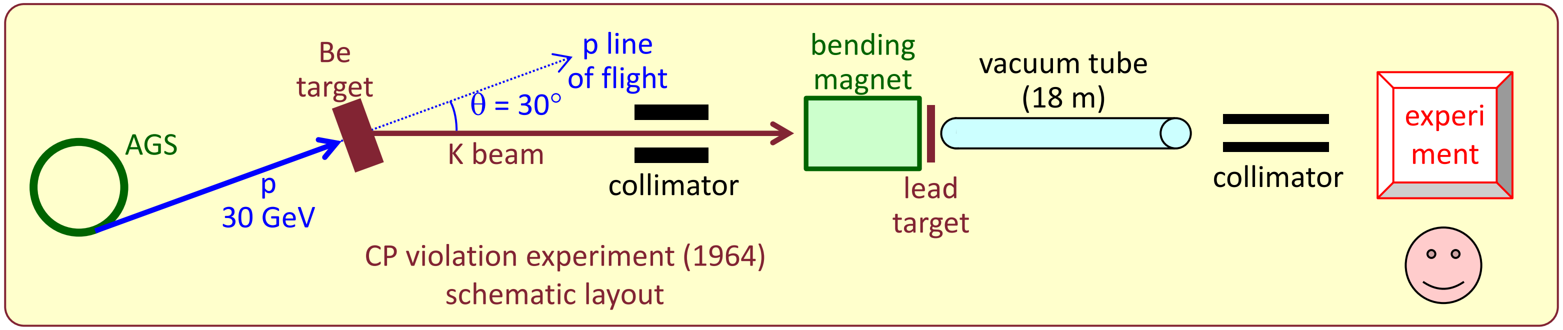
d) Mass eigenstates in matter :

$|K^0_{S,M}\rangle = (|K^0_1\rangle + \varepsilon^M|K^0_2\rangle)/\sqrt{1+|\varepsilon^M|^2}$;

$|K^0_{L,M}\rangle = (\varepsilon^M|K^0_1\rangle + |K^0_2\rangle)/\sqrt{1+|\varepsilon^M|^2}$.

($\mathbb{CP}$ violation in matter)

$\mathbb{CP}$ violation: experimental layout



In 1964 an experiment was built to search for $\mathbb{CP}$ violation at the Brookhaven AGS (Alternating Gradient Synchrotron).

The schematic layout is shown in the fig.:

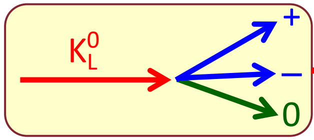
- the primary proton beam (30 GeV) hits a beryllium target;
- secondaries at θ = 30° are selected;
- if charged, collimated and bent away;
- if neutral, collimated and let decay;
- the resultant K^0_L (long lifetime) hit a second lead target, regenerate and are let decay again in a long decay tube;

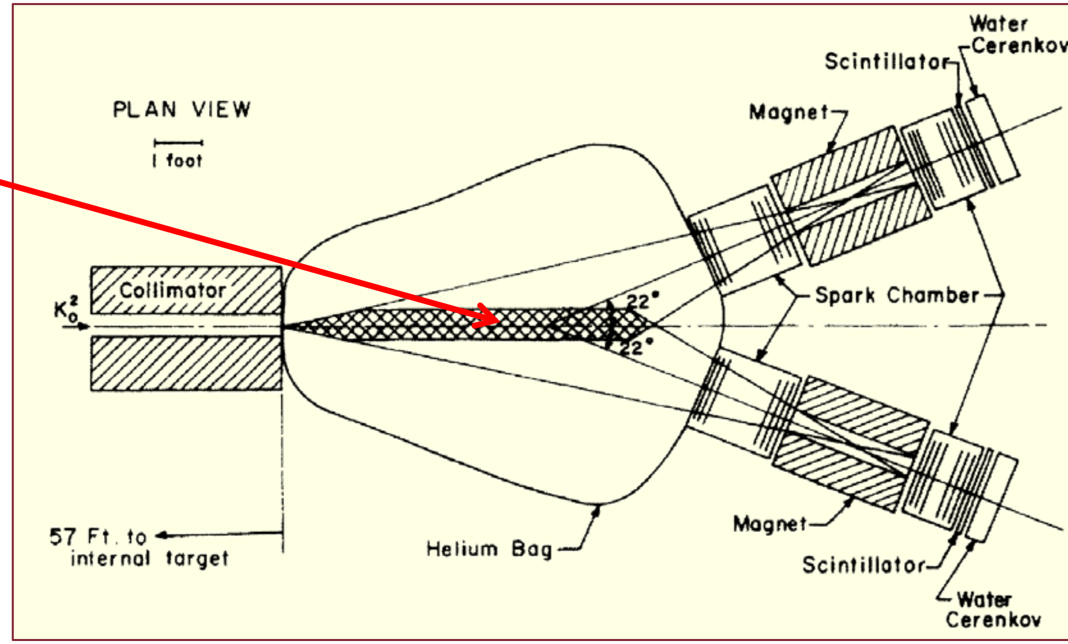
- no K^0_S left → if $\mathbb{CP}$ is conserved, only long lifetime K^0_L [= K^0_2] should remain and decay → 3π;
- if (2π) observed → $\boxed{\mathbb{CP} \text{ is violated}}$!!!

- *16 years after, in Stockolm*

James Cronin Val Fitch

CP violation: the experiment





Helium bag for K^0_L decays + two-arm-spectrometer.

Each of the two arms :

- spark chambers ($\rightarrow$ position);
- magnetic field ($\rightarrow$ momentum measurement);
- scintillators ($\rightarrow$ trigger + tof);
- water Cerenkov ($\rightarrow$ particle id);

main background : n ($\rightarrow$ tof rejects).

Other selection criteria :

- two opposite charged particles, one for each arm;
- measure $\vec{p}_+$ and $\vec{p}_-$ (direction and module);
- assume $m_+ = m_- = m_\pi \rightarrow m_{+-} \approx m_K \rightarrow$ test;
- angle θ between $\vec{p}_{sum}$ (= $\vec{p}_+ + \vec{p}_-$) and $\overrightarrow{dir}_{collimator} \approx 0 \rightarrow$ test.

The three-body decays (e.g. $K^0_L \rightarrow \pi^+\pi^-\pi^0$) do NOT satisfy those conditions :

- ($\vec{p}_+ + \vec{p}_- = \vec{p}_K - \vec{p}_0$) not collinear with $\overrightarrow{dir}_{collimator}$;
- $m_{+-} \leq (m_K - m_\pi) < m_K$.

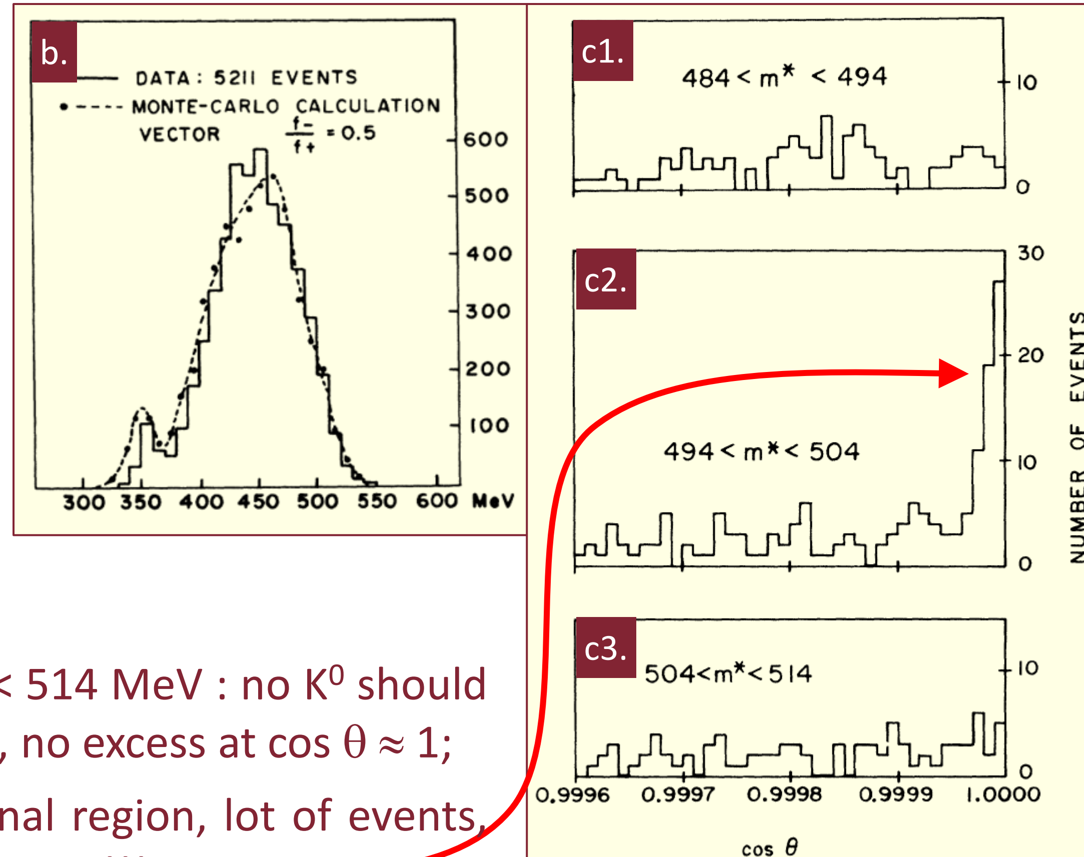
ℂℙ violation: results

a. (not in figs.) just for calibration, a tungsten plate was put in front of the spectrometer for K^0 regeneration: $\pi^\pm$ identification and mass reconstruction [OK !];

b. distribution of m* [=mass($\pi^+\pi^-$)] for real events and MC simulation [OK!];

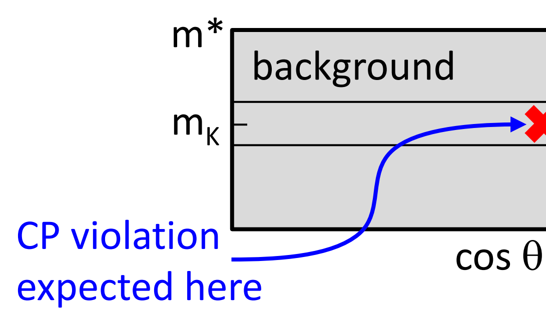
c. distribution of cos θ for 3 mass bins, with improved resolution :

- 484 < m* < 494 and 504 < m* < 514 MeV : no K^0 should be there : therefore few events, no excess at cos $\theta \approx 1$;
- 494 < m* < 504 MeV : the signal region, lot of events, clear peak at cos $\theta \approx 1$: THE SIGNAL !!!



d. final result (similar result for the neutral decay $\to\pi^0\pi^0$) :

$$R = BR(K^0_L \to \pi^+\pi^-) \,/\, BR\,(K^0_L \to \text{charged}) = (2.0 \pm 0.4) \times 10^{-3}$$



⇒ ℂℙ is violated !!!

CP violation: $K^0_L \to \pi^+\pi^-$, $\pi^+\pi^-\pi^0$, $\pi^\pm e^\mp \nu/\nu$

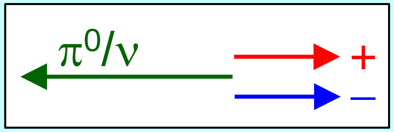
Q.: study the mass m*

[a typical kin. problem with ambiguities + mass hypoteses]

- work in the K^0_L ref. system;
- define m^* = mass(+ve, –ve);
- approx. : $m_\nu \approx 0$, $m_e^2 << m_\pi^2$;
- look at the box

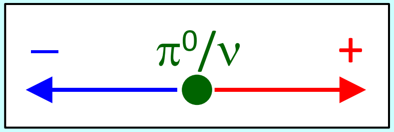
min(m^*) happens when + and – are at rest wrt each other:

$m^*|_{min} = m_+ + m_-$.



max(m^*) happens when the neutral is at rest:

$m^*|_{max} = m_K - m_0$.



a) $K^0_L \to \pi^+\pi^-$

$m^* = m_K$ [easy, no problem];

b) $K^0_L \to \pi^+\pi^-\pi^0$

$m^*|_{min} = 2\ m_\pi \approx 270$ MeV;

$m^*|_{max} = m_K - m_\pi \approx 360$ MeV;

c) $K^0_L \to \pi^\pm e^\mp \nu$

$m^*|_{min} = m_\pi + m_e \approx m_\pi$;

$m^*|_{max} = m_K - m_\nu \approx m_K$;

[*apparently easy, but ...*]

d) $K^0_L \to \pi^\pm e^\mp \nu/\bar{\nu}$, "$e^\mp$" interpreted as $\pi^\mp$:

"m^*"$_{min}$ = m_π + "m_e" = $2m_\pi \approx 270$ MeV;

for "m^*"$_{max}$ compute $|\vec{p}_{\pi/e}|$ and $E_{\pi/e}$ when $|\vec{p}_\nu| \approx 0$:

$$p_\pi = p_e = \frac{m_K^2 - m_\pi^2}{2m_K} \text{ [see e.g. § 4];}$$

$$E_\pi = "E_e" = \sqrt{m_\pi^2 + p_p^2} = \sqrt{m_\pi^2 + \frac{m_K^4 + m_\pi^4 - 2m_K^2 m_\pi^2}{4m_K^2}} =$$

$$= \sqrt{\frac{m_K^4 + m_\pi^4 + 2m_K^2 m_\pi^2}{4m_K^2}} = \frac{m_K^2 + m_\pi^2}{2m_K};$$

$$"m^*"_{max} = E_\pi + "E_e" = 2E_\pi \approx m_K\left(1 + m_\pi^2/m_K^2\right).$$

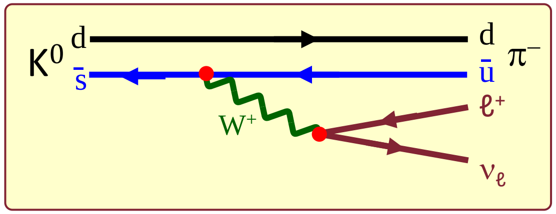
$m^*_{max} \approx 534$ MeV $> m_K$!!!

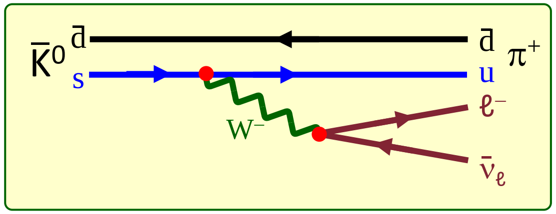
$\mathbb{CP}$ violation: semileptonic decays

- The $(K^0_L \to \pi^+\pi^-)$ is NOT the only channel, which shows $\mathbb{CP}$ violation;
- another important process is the semileptonic decay $(K^0_L \to \pi^\pm \ell^\mp \nu_\ell)$;
- it is an important channel, since :

  $BR(K^0_L \to \pi^\pm e^\mp \nu_e) \approx 40.6\ \%$;

  $BR(K^0_L \to \pi^\pm \mu^\mp \nu_\mu) \approx 27.0\ \%$;
- if $\mathbb{CP}$ were conserved, the rate with the +ve and the –ve charge would be the same, since they are connected by a $\mathbb{CP}$ transformation;





- instead, they are different; it is customary to express the difference as :

$$\delta_L = \frac{\Gamma\left(K^0_L \to \ell^+ \nu_\ell \pi^-\right) - \Gamma\left(K^0_L \to \ell^- \bar{\nu}_\ell \pi^+\right)}{\Gamma\left(K^0_L \to \ell^+ \nu_\ell \pi^-\right) + \Gamma\left(K^0_L \to \ell^- \bar{\nu}_\ell \pi^+\right)};$$

it is measured $\delta_L = (3.32 \pm 0.06) \times 10^{-3}$.

- NOT "just another boring number".
- First evidence for difference matter-antimatter :

  "*the real matter contains the electron with smaller BR in the* $K^0_L \to \pi^\pm e^\mp \nu_e$ *decay*".
- In fact, some mechanism MUST have generated the asymmetry matter-antimatter of the Universe [*if primordial universe was symmetric*].
- However $\delta \sim 10^{-3}$ is too small to account for the large asymmetry of our world.
- In addition, if the K^0_L decay is the only source, at the big bang time who provided all these K^0_L's ?

ℂℙ violation: the Sandro's view

From [Bettini] :

[… *A*]t late times, when only K_L's survive, they decay through $K_L \rightarrow \pi^- \ell^+ \nu_\ell$ a little more frequently than through the ℂℙ conjugate channel $K_L \rightarrow \pi^+ \ell^- \bar{\nu}_\ell$. […] This shows, again and independently, that matter and antimatter are somewhat different.

Let us suppose that we wish to tell an extraterrestrial being what we mean by matter and by antimatter. We do not know whether his/her world is made of the former or the latter.

We can tell him/her : "prepare a neutral K meson beam and go far enough from the production point to be sure to have been left only with the long-lifetime component." At this point s/he is left with K_L mesons, independently of the matter or antimatter constitution of her/his world.

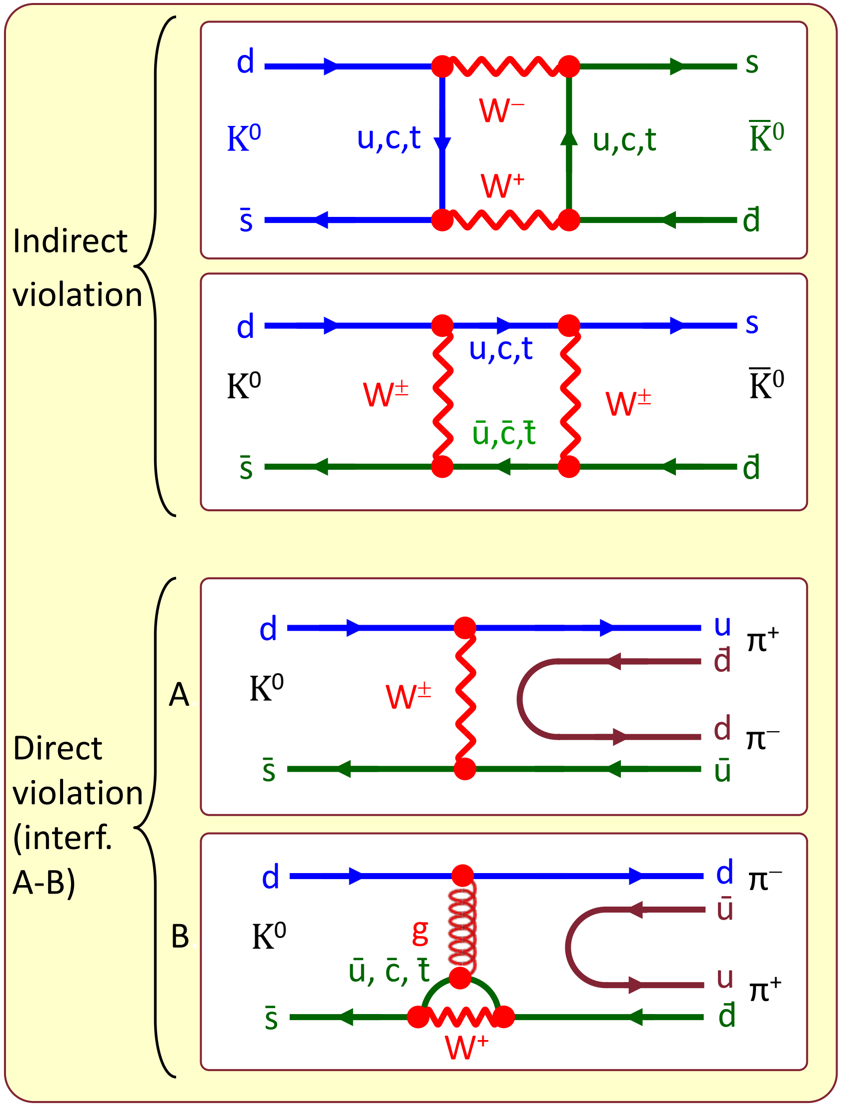
We continue: "count the decays with a lepton of one or the other charge and call positive the charge of the sample that is about three per thousand larger. Humans call matter the one that has positive nuclei."

If, after a while, our correspondent answers that his nuclei have the opposite charge, and comes to meet you, be careful, apologize, but do not shake his/her hand.

Sandro's version of the famous Feynman joke [see § 4].

Direct/indirect $\mathbb{CP}$ violation

- The previous examples/experiments show $\mathbb{CP}$ violations in the decay of neutral flavored mesons (K^0, in the following B^0).
- In fact, three different types of $\mathbb{CP}$ violation have been identified and measured:
  a. in the mixing of neutral mesons ($M \leftrightarrow \bar{M}$) (**indirect violation**);
  b. difference in the decay of a particle: $\Gamma(M \to X) \neq \Gamma(\bar{M} \to \bar{X})$ (**direct violation**);
  c. **interference** between direct and indirect violation : $\Gamma(M \to X) \neq \Gamma(M \to \bar{M} \to X)$.
- in the K^0 system (a) is important, while in the B^0 system b/c dominate; the relative importance of the effect is determined by the values of the V_{CKM} matrix [*see later*];
- (a) and (b) are usually parametrized by the coefficients ε and ε'.



[the indirect violation has been discussed before, e.g. for the 1964 experiment; the couplings qqW are regulated by the V_{CKM} matrix, see later]

Direct/indirect $\mathbb{CP}$ violation: ε and ε'

- The complex parameter ε is associated with the indirect $\mathbb{CP}$ violation;
- this parameter decouples the states with definite lifetimes from the $\mathbb{CP}$ eigenstates :

$$\left|K_S^0\right\rangle = \frac{\left|K_1^0\right\rangle + \varepsilon\left|K_2^0\right\rangle}{\sqrt{1+|\varepsilon|^2}} = \frac{(1+\varepsilon)\left|K^0\right\rangle + (1-\varepsilon)\left|\overline{K}^0\right\rangle}{\sqrt{2\left(1+|\varepsilon|^2\right)}};$$

$$\left|K_L^0\right\rangle = \frac{\left|K_2^0\right\rangle + \varepsilon\left|K_1^0\right\rangle}{\sqrt{1+|\varepsilon|^2}} = \frac{(1+\varepsilon)\left|K^0\right\rangle - (1-\varepsilon)\left|\overline{K}^0\right\rangle}{\sqrt{2\left(1+|\varepsilon|^2\right)}};$$

- no $\mathbb{CP}$ violation → ε = 0 →
→ ($|K_S^0\rangle = |K_1^0\rangle$, $|K_L^0\rangle = |K_2^0\rangle$);
- other commonly used parameters are :

$$\eta_{00} \equiv |\eta_{00}|\exp(i\phi_{00}) \equiv \frac{\left\langle \pi^0\pi^0 \middle| \mathbb{H} \middle| K_L^0 \right\rangle}{\left\langle \pi^0\pi^0 \middle| \mathbb{H} \middle| K_S^0 \right\rangle};$$

$$\eta_{+-} \equiv |\eta_{+-}|\exp(i\phi_{+-}) \equiv \frac{\left\langle \pi^+\pi^- \middle| \mathbb{H} \middle| K_L^0 \right\rangle}{\left\langle \pi^+\pi^- \middle| \mathbb{H} \middle| K_S^0 \right\rangle};$$

- the direct violation is parametrized by a complex parameter ε' :

$$\eta_{+-} = \varepsilon + \varepsilon'; \quad \eta_{00} = \varepsilon - 2\varepsilon';$$

- no direct $\mathbb{CP}$ violation → ε' = 0 and $|\eta_{00}| \approx |\eta_{+-}| \approx \varepsilon$;
- ε' is an important parameter for our understanding of Nature;
- as of today, the best measurement, assuming $\mathbb{CPT}$ invariance, are :

$$\begin{aligned}
|\eta_{+-}| &= (2.232 \pm 0.011) \times 10^{-3};\\
|\eta_{00}| &= (2.221 \pm 0.011) \times 10^{-3};\\
|\phi_{+-}| &= (43.51 \pm 0.05)^\circ;\\
|\phi_{00}| &= (43.7 \pm 0.8)^\circ;\\
|\varepsilon| &= (2.228 \pm 0.011) \times 10^{-3};\\
\Re e(\varepsilon'/\varepsilon) &= (1.65 \pm 0.26) \times 10^{-3};
\end{aligned}$$

which are obtained in a long series of dedicated experiments on $\mathbb{CP}$ violation.

Direct/indirect $\mathbb{CP}$ violation: summary$_1$

D. Kirkby (UC Irvine), Y. Nir (Weizmann Inst.) [PDG 2012] :

- The $\mathbb{CP}$ transformation combines charge conjugation $\mathbb{C}$ with parity $\mathbb{P}$.
- Under $\mathbb{C}$, particles and antiparticles are interchanged, by conjugating all internal quantum numbers, e.g., $Q \to -Q$ for electromagnetic charge.
- Under $\mathbb{P}$, the handedness of space is reversed, $\vec{x} \to -\vec{x}$. [… A] left-handed electron e^-_L is transformed under $\mathbb{CP}$ into a right-handed positron e^+_R.
- If $\mathbb{CP}$ were an exact symmetry, the laws of Nature would be the same for matter and for antimatter. We observe that most phenomena are $\mathbb{C}$- and $\mathbb{P}$-symmetric, and therefore, also $\mathbb{CP}$-symmetric.
- In particular, these symmetries are respected by the gravitational, electromagnetic, and strong interactions.
- The weak interactions, on the other hand, violate $\mathbb{C}$ and $\mathbb{P}$ in the strongest possible way. For example, the charged W bosons couple to left-handed electrons, e^-_L, and to their $\mathbb{CP}$-conjugate right-handed positrons, e^+_R, but to neither their $\mathbb{C}$-conjugate left-handed positrons, e^+_L, nor their $\mathbb{P}$-conjugate right-handed electrons, e^-_R.

(*… continue …*)

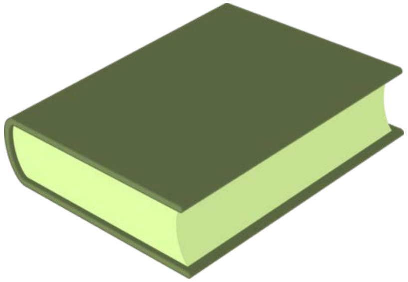
Direct/indirect $\mathbb{CP}$ violation: summary$_2$

D. Kirkby (UC Irvine), Y. Nir (Weizmann Inst.) [PDG 2012] :

(... continued ...)

- While weak interactions violate $\mathbb{C}$ and $\mathbb{P}$ separately, $\mathbb{CP}$ is still preserved in most weak interaction processes.
- The $\mathbb{CP}$ symmetry is, however, violated in certain rare processes, as discovered in neutral K decays in 1964 [...], and observed in recent years in B decays. A K_L meson decays more often to $\pi^- e^+ \nu_e$ than to $\pi^+ e^- \bar{\nu}_e$, thus allowing electrons and positrons to be unambiguously distinguished, but the decay-rate asymmetry is only at the 0.003 level.
- The $\mathbb{CP}$-violating effects observed in B decays are larger: the $\mathbb{CP}$ asymmetry in $B^0/\bar{B}^0$ meson decays to $\mathbb{CP}$ eigenstates like $J/\psi K_S$ is about 0.7 [...].
- These effects are related to $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing, but $\mathbb{CP}$ violation arising solely from decay amplitudes has also been observed, first in $K \to \pi\pi$ decays [...], and more recently in various neutral [...] and charged B [...] decays.
- Evidence for $\mathbb{CP}$ violation in the decay amplitude at a level higher than 3σ (but still lower than 5σ) has also been achieved in neutral D [...] and B_s [...] decays.
- $\mathbb{CP}$ violation has not yet been observed in the lepton sector.

LHCb observed $\mathbb{CP}$ violation in D decays in 2019 at 5.3σ.



CKM matrix

Reinterpret the ℂℙ violation using the CKM matrix [§ 4]:

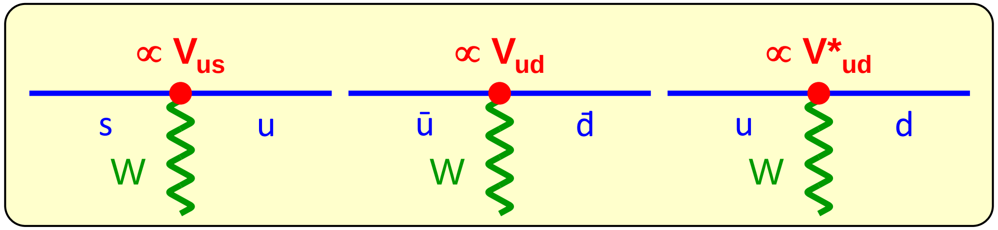
NB

V_{CKM} is a fundamental ingredient of the SM; the actual values V_{ij} are observable (→ measurable, see later), but not predictable inside the SM (like fermion masses, number of families, …)

- the weak charged current for quarks
  [d,s,b are down-quark spinors and ū,c̄,t̄ are the adjoint spinors for up-quarks]

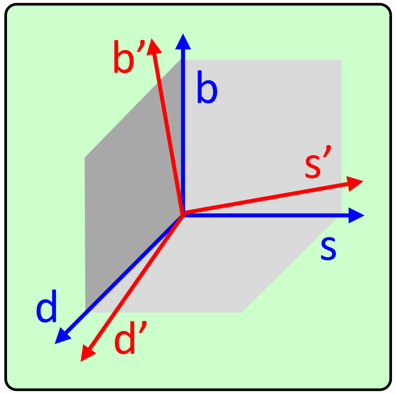
$$j^{\mu}_{qq} = -i\frac{g}{\sqrt{2}}(\bar{u},\ \bar{c},\ \bar{t})\gamma^{\mu}\frac{1}{2}\left(1-\gamma^{5}\right)\boxed{V_{CKM}}\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- therefore, e.g. [notice the "*"; the definition is "V_{ij} when bds is a spinor and ūc̄t̄ the adjoint spinor" and "V^*_{ij} when uct is a spinor and b̄d̄s̄ the adjoint spinor".]



- the V_{CKM} matrix represents the rotation, i.e. the amount of mixing among rotated quarks.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



CKM matrix: α_{ij}, δ

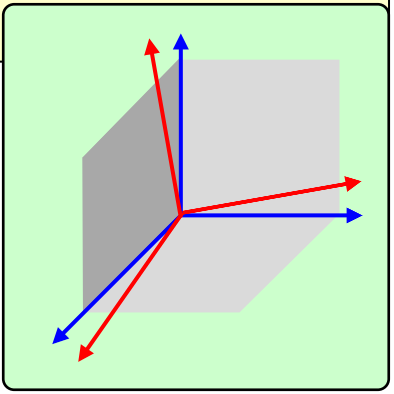
- in a N-family scheme with N=3, V_{CKM} requires n_{rot}=3 real rotations α_{ij} and n_{ph}=1 imaginary phase δ (see box);
- the rotations α_{ij} are "Euler angles" in the quark space ("Cabibbo angles in 3-dim");
- $\delta \neq 0$ $\to$ some V_{ij} complex
  $\to$ $\mathbb{CP}$ violation [next slides];
- many representations, give the most common [PDG] ($c_{ij} \equiv \cos\alpha_{ij}$, $s_{ij} \equiv \sin\alpha_{ij}$):

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{+i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$

the K-M approach [IE, §9]:

$$\begin{pmatrix} n_{rot} = N(N-1)/2 \\ n_{ph} = (N-1)(N-2)/2 \\ \mathbb{CP} \text{ violation} \end{pmatrix} \to (n_{ph} \geq 1) \to (N \geq 3).$$



CKM matrix: phenomenology

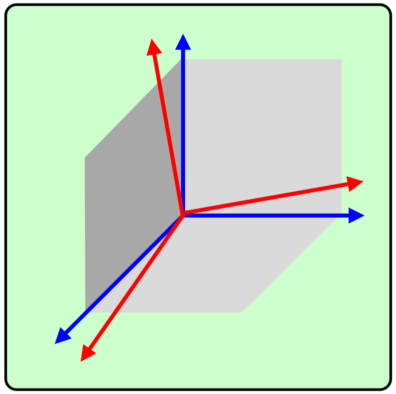
The representation is chosen to highlight the agreement with experimental data:

- α_{ij} small $\to$ cos α_{ij} >> sin α_{ij}
  $\to V_{CKM} = \mathbb{1}$ + "small rotations"
  $\to$ q'-dynamics = q-dynamics
  + small effects;
- α_{13} small $\to \alpha_{12} \cong \theta_c$;

- Cabibbo theory works well, when considering N=2 (udsc only);
- s_{12} and s_{13} small $\to$ matrix almost real
  $\to$ $\mathbb{CP}$ violation small.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$

$$\begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \cong \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \end{pmatrix}.$$



CKM matrix: Wolfenstein parameters

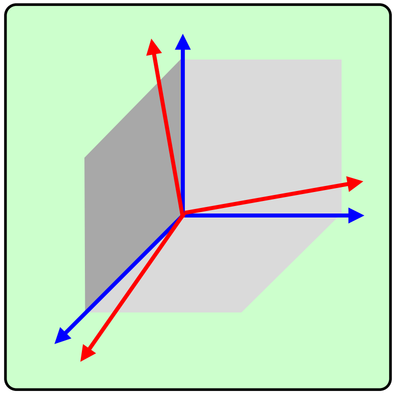
The violations associated with V_{CKM} are usually studied with the Wolfenstein parameterization V^W_{CKM}, which singles out the "small" terms and their physical meaning:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cong V^W_{CKM} + \mathcal{O}\left(\lambda^4\right);$$

$$V^W_{CKM} \equiv \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & A\lambda^2 & 1 \end{pmatrix}.$$

As the "Euler" parameterization, V^W_{CKM} has 4 independent real parameters (**λ A ρ η**):

- $\lambda \cong s_{12}$ ($\rightarrow \sin\theta_c$, mixing $1^{st}/2^{nd}$);
- $A\lambda^2 \cong s_{23}$ ($\rightarrow$ mixing $2^{nd}/3^{rd}$);
- $A\lambda^3(\rho + i\eta) \cong s_{13}e^{i\delta}$ ($\rightarrow \delta \cong \tan^{-1} \eta/\rho$);
- i.e. $\eta=0 \rightarrow \delta=0 \rightarrow V_{CKM}$ real
  $\rightarrow$ no $\mathbb{CP}$ violation.



CKM matrix: $\mathbb{CP}$ violation in K^0

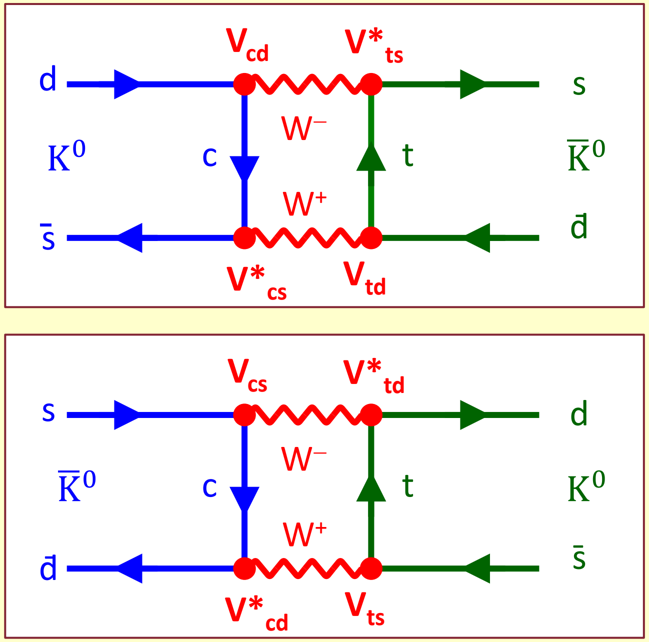
The indirect $\mathbb{CP}$ violation in the K^0 system can be explained with the CKM formalism [Thoms, 393]:

- for each of the $K^0 \leftrightarrow \bar{K}^0$ diagrams
  - *look the t-channel exchange: 9 couples of diagrams (uu, uc, ut, cu, cc, ct, …);*
  - *here discuss only (ct) case, others similar*;
- $\mathcal{M}(K^0 \to \bar{K}^0) \propto V_{cd}\, V^*_{ts}\, V^*_{cs}\, V_{td}$;
- $\mathcal{M}(\bar{K}^0 \to K^0) \propto V^*_{cd}\, V_{ts}\, V_{cs}\, V^*_{td}$;
- V_{ij} real $\to \mathcal{M}(K^0 \to \bar{K}^0) = \mathcal{M}(\bar{K}^0 \to K^0)$
  $\to$ no ~~$\mathbb{CP}$ violation~~;
- V_{ij} complex $\to \mathcal{M}(K^0 \to \bar{K}^0) \neq \mathcal{M}(\bar{K}^0 \to K^0)$
  $\to \mathbb{CP}$ violation.
- in this case $\mathcal{M}(K^0 \to \bar{K}^0) \neq \mathcal{M}(\bar{K}^0 \to K^0)$:

  $$\mathcal{M}(K^0 \to \bar{K}^0) - \mathcal{M}(\bar{K}^0 \to K^0) \propto i\Im(V_{td}) = i\eta A\lambda^3;$$

  [$\Delta\mathcal{M}$ imaginary, small, $\propto \eta$]

  $\to$ CP violation $\propto \eta A^2\lambda^6$ [Jarlskog invariant]

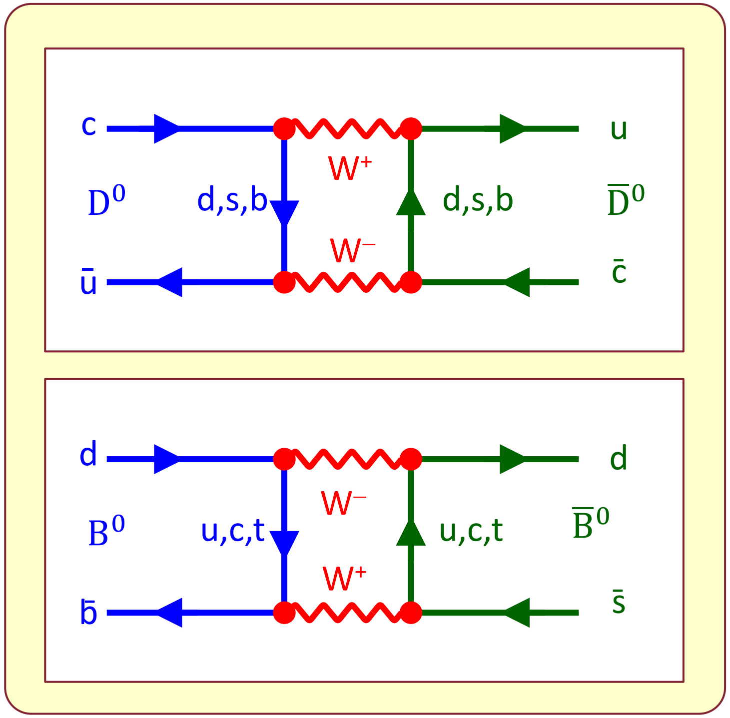


It can be shown [Thoms 403] that the ε parameter of the $\mathbb{CP}$ violation can be written as:

$$|\varepsilon| \propto \eta\,(1 - \rho + \text{const.})$$

CKM matrix: $\mathbb{CP}$ violation in D^0 / B^0

- The $\mathbb{CP}$ violation is expected to occur in the SM also in the D^0–$\bar{D}^0$ and B^0–$\bar{B}^0$ systems through the same dynamical mechanism [*see box*].
- However the importance of the phenomenon depends on the value of the CKM matrix elements, i.e. by the quark mixing.
- In the D^0–$\bar{D}^0$ case:
  - main contribution from b quark exchange;
  - but product $V_{cb}V_{ub}$ very small;
  - therefore predicted D^0–$\bar{D}$ mixing minute;
  - only been observed in 2019 by LHCb.
- Instead B^0–$\bar{B}^0$ mixing:
  - dominated by t quark exchange;
  - expected substantial level of mixing;
  - [*see next slides for some results*].



it could be a golden opportunity: since the SM prediction is small (and computable), a bSM effect would not be obscured.

CKM matrix: measure $|V_{ij}|$

How to measure (the real part of) V_{ij} ?

- from decays ([YN2, §6], [PDG]):
  - $|V_{ud}|$: $p \to ne\bar{\nu}$ and other β decays;
  - $|V_{cs}|$: c-mesons C(abibbo)-allowed;
  - $|V_{us}|$: s-mesons (e.g. $K^\pm$);
  - $|V_{cd}|$: c-mesons C-suppressed,
    : dileptons in ν scattering;
  - $|V_{ub}|$: b-mesons $\to$ non_c-mesons;
  - $|V_{cb}|$: b-mesons $\to$ c-mesons;
  - $|V_{td}|, |V_{ts}|$: $(B^0 \leftrightarrow \bar{B}^0)$ oscillations;
  - $|V_{tb}|$: $t \to W^\pm b$ [*not accurate*];
- conceptually simple, the problem is to disentangle the clean weak decay from the dirty hadron corrections;
- semi-leptonic decays cleaner;
- a technically difficult job (hundreds of papers, theses, conferences…);

- nice final result [PDG 2016]:
  - V_{CKM} quasi-diagonal, as expected;
  - well consistent with SM (unitary, 3 families).

$$|V_{CKM}| \equiv \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} =$$

$$= \begin{pmatrix} .97417 & .2248 & .0409 \\ .220 & .995 & .0405 \\ .0082 & .0400 & 1.009 \end{pmatrix} \pm$$

$$\pm \begin{pmatrix} .00021 & .0006 & .0039 \\ .005 & .016 & .0015 \\ .0006 & .0027 & .0031 \end{pmatrix}.$$

CKM matrix: $|V_{ij}|$ and SM

How to interpret V_{CKM} ?

$$|V_{CKM}| \equiv \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} =$$

$$= \begin{pmatrix} .97417 & .2248 & .0409 \\ .220 & .995 & .0405 \\ .0082 & .0400 & 1.009 \end{pmatrix} \pm$$

$$\pm \begin{pmatrix} .00021 & .0006 & .0039 \\ .005 & .016 & .0015 \\ .0006 & .0027 & .0031 \end{pmatrix}.$$

- tests of SM from $V^\dagger V = \mathbb{1}$:

  $\sum_i V_{ij} V_{ik}^* = \delta_{jk}; \quad \sum_j V_{ij} V_{kj}^* = \delta_{ik}.$

  (e.g. $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$;
- if (a) test(s) fail(s)
  - more generations (missing pieces) ?
  - general breakdown of the model ?
- if all tests succeed
  - general fit imposing unitarity;
  - improved accuracy;
  - stricter tests;
  - more accuracy;
  - and so on, forever [see §LEP].

Unitarity triangle

- from one of the unitarity relations:

$$\sum_i V_{i1}V_{i3}^* = V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = \delta_{13} = 0;$$

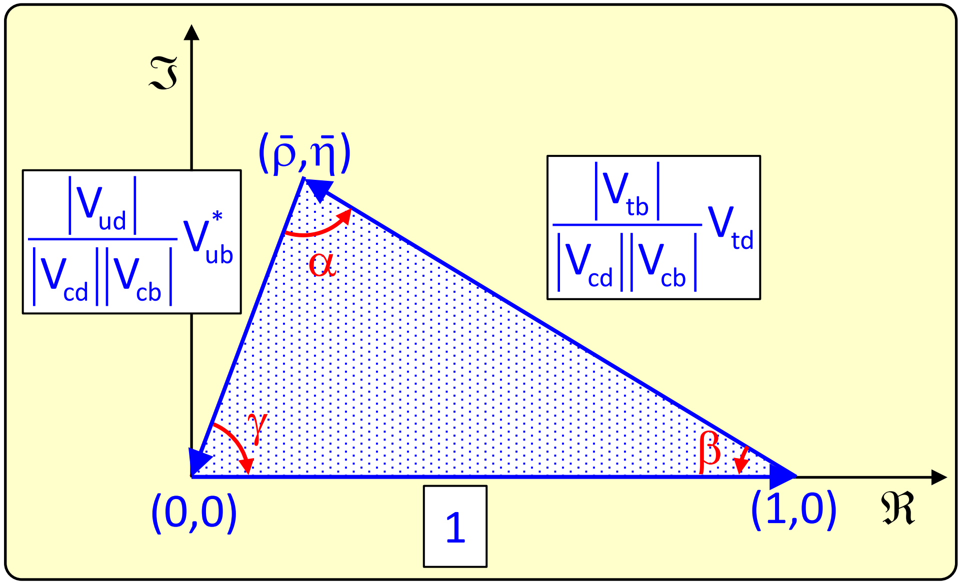
- add some simple math:

V_{ud}, V_{cb}, V_{tb} real > 0;

V_{cd} real < 0 (see V^W_{cd});

$$\rightarrow |V_{ud}|V_{ub}^* - |V_{cd}||V_{cb}| + V_{td}|V_{tb}| = 0;$$

$$\rightarrow \boxed{1 - \frac{|V_{tb}|}{|V_{cd}||V_{cb}|}V_{td} - \frac{|V_{ud}|}{|V_{cd}||V_{cb}|}V_{ub}^* = 0;}$$



- put the relation in complex plane $\Re\Im$;
- interpreted it as a triangle (unitarity triangle, u.t.);
- define angles (α, β, γ) (see fig.);
- relate $V_{ij} \rightarrow$ Wolfenstein param. ρ^W, η^W;
- the vertex is at ($\bar\rho \cong \rho^W$, $\bar\eta \cong \eta^W$)

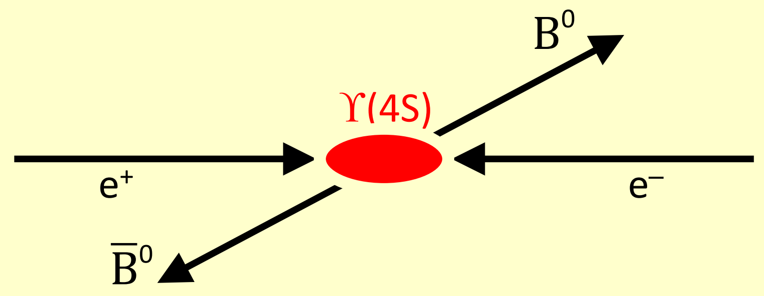
The exact relation is [*check it !*] :

$$\bar\rho + i\bar\eta = (\rho + i\eta)\left(1 - \frac{\lambda^2}{2}\right) + \mathcal{O}(\lambda^4).$$

Note:

- u.t. defined by using V_{ij} only;
- nice adimensional parameters (ratios);
- experiments measure triangle "geometry" (sides, angles);
- lot of relations (e.g. $\alpha+\beta+\gamma=180°$):
  - consistency tests of SM,
  - global fits to parameters assuming SM.

Unitarity triangle: meas β at BaBar, Belle

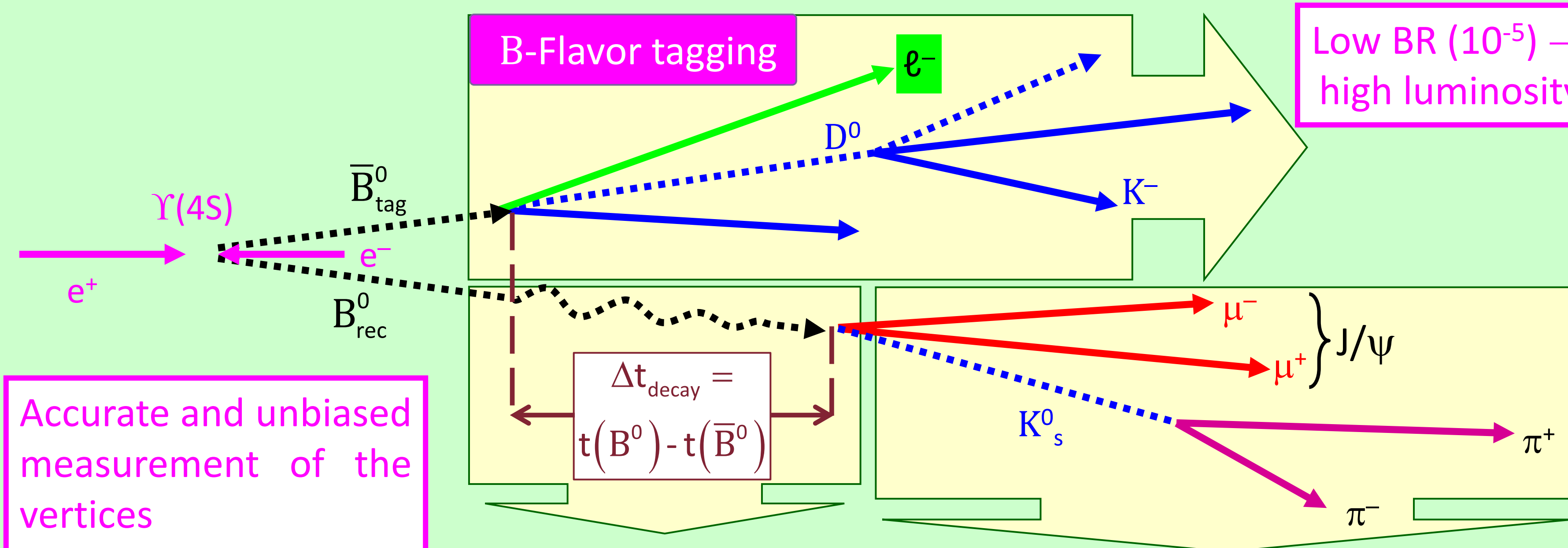


A typical event used for $\mathbb{CP}$ violation in asymmetric e^+e^- at $\sqrt{s} = m(\Upsilon_{4S}) \approx 10.579$ GeV :

$$e^+e^- \to \Upsilon(4S) \to \overline{B}^0 B^0;$$

$$\overline{B}^0 \to \ell^- D^0 X^+; \quad D^0 \to K^- X^+;$$

$$B^0 \to J/\psi K^0_S; \quad J/\psi \to \mu^+\mu^-; \quad K^0_S \to \pi^+\pi^-.$$



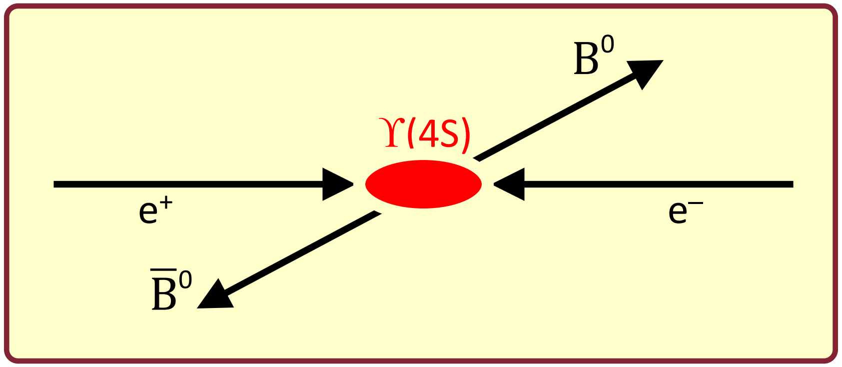
Low BR (10^{-5}) $\to$ high luminosity

Accurate and unbiased measurement of the vertices

Vertex reconstruction

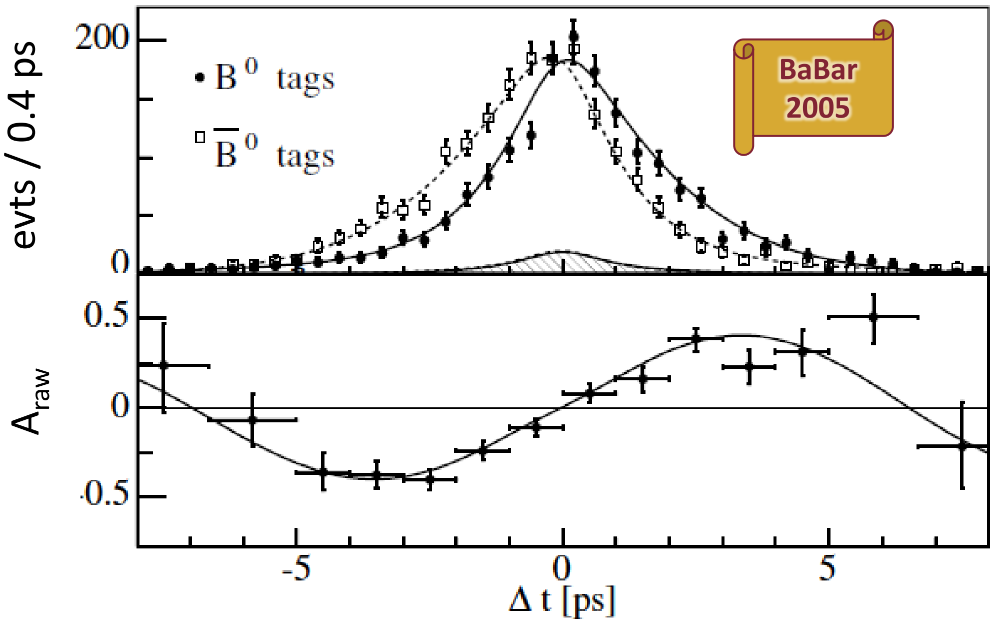
Exclusive B meson reconstruction

Unitarity triangle: results for β at BaBar



$$A_{raw} = \frac{n\left[\overline{B}(\Delta t) \to J/\psi K_s^0\right] - n\left[B^0(\Delta t) \to J/\psi K_s^0\right]}{n\left[\overline{B}(\Delta t) \to J/\psi K_s^0\right] + n\left[B^0(\Delta t) \to J/\psi K_s^0\right]} \propto$$

$$\propto \sin(2\beta)\sin(\Delta m \Delta t).$$



$\sin 2\beta = 0.722 \pm 0.040$

± 0.023

[*now improved*]

NB: $\sin\beta > 0$

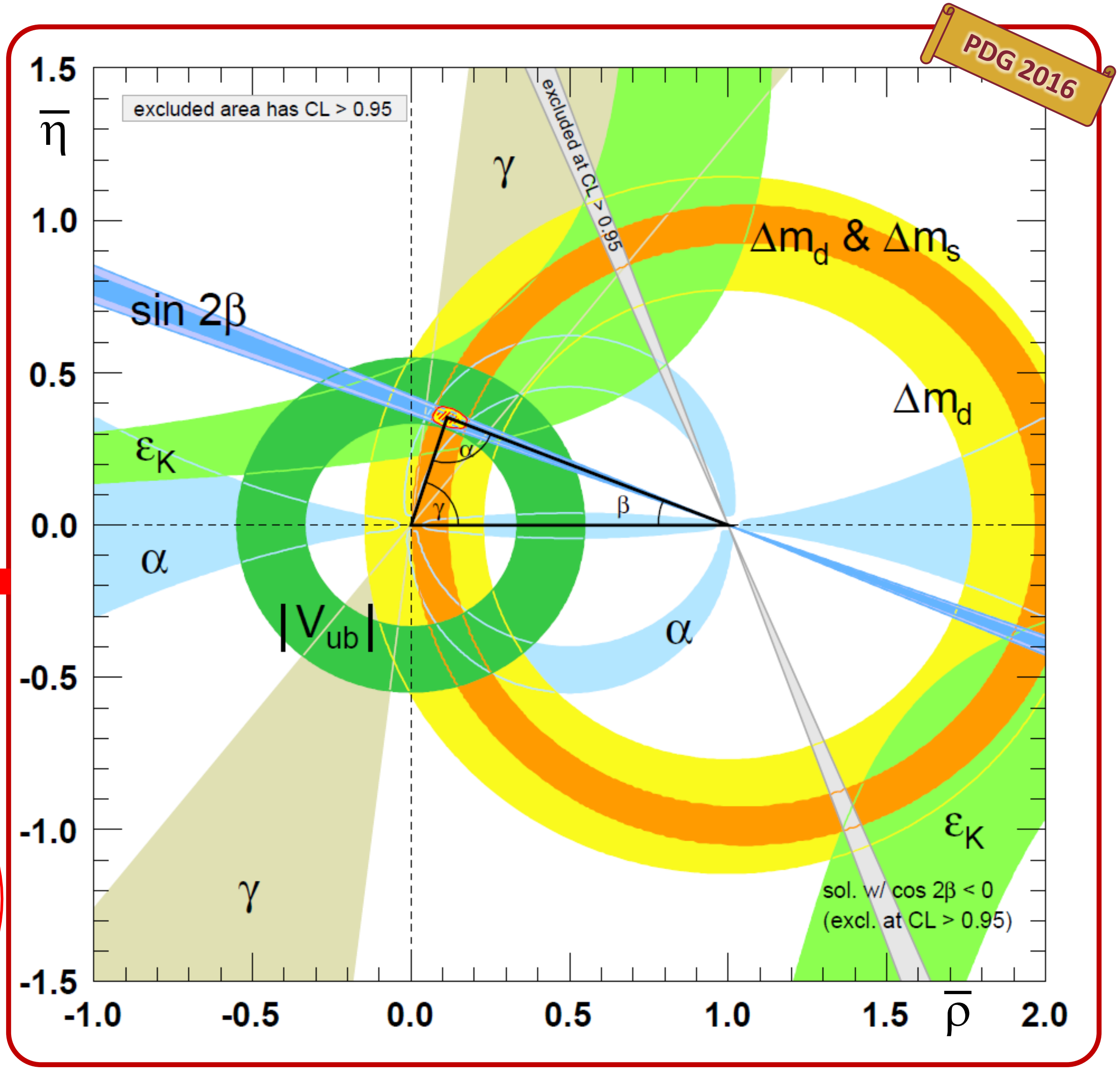
$\to \eta > 0$

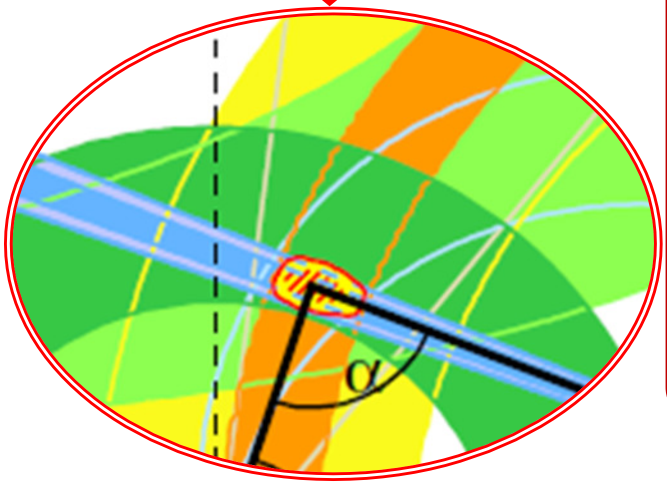
$\to \mathbb{CP}$ violated !!!

Unitarity triangle: measure ρ,η

As of today [PDG 2016]:

- converging measurements (mainly asymmetric e^+e^- factories BaBar, Belle);
- no deviation from 3_f-SM, e.g. $[\alpha+\beta+\gamma]_{fit} = (183\pm8)°$;
- try harder, one of the most promising frontiers !!!



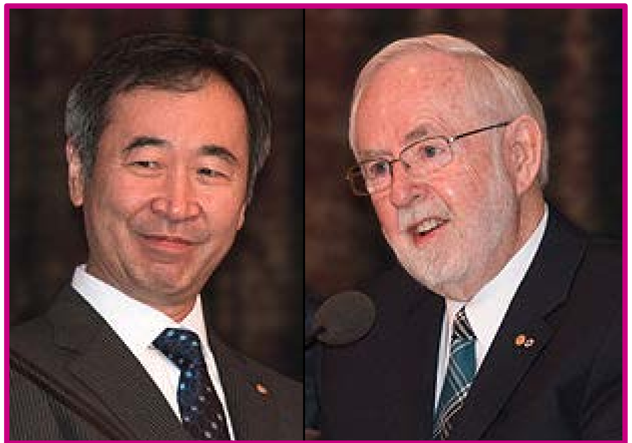


ν oscillations

Quarks of same charge and different flavor mix together → composite hadrons "oscillate" (e.g. $K^0 \leftrightarrow \bar{K}^0$).

The CKM matrix parameterizes the process in the context of the SM.

And the lepton sector ? Do the ν's oscillate ?



T. Kajita – A. McDonald

The answer to the previous question is **YES**.

The results are **important** (Nobel Prize 2015):

- $m_\nu > 0$ (at least for two of them);
- there is mixing in the lepton sector;
- and possibly $\mathbb{CP}$ violation (not easy to see);
- the first discovery bSM (even though, if ν's are Dirac fermions, they can be easily incorporated in the SM).

In the following the ν's will be considered as massive neutral Dirac fermions (sort of neutral electrons), sometimes called "Weyl ν's":

- *this hypothesis is simple, but not the favorite of most physicists;*
- *(as of today) it is NOT falsified by the exp.;*
- *other comments on § Standard Model.*

The ν's are very complicated objects! many (most ?) of the important discoveries in particle physics of the last 80 years came from them !!!

ν oscillations: toy model

Assume mixing in the ν sector and look for possible observables.

notice: $\nu_{1,2}$ = mass eigenstates (= $K^0_{S,L}$) with $m_{1,2}$, $\nu_{e,\mu}$ = lepton eigenstates (= K^0, $\bar{K}^0$) with $n_{e,\mu}$.

Simple toy model, inspired to Cabibbo angle:

- 2 families ($\nu_1, \nu_2 \rightarrow \nu_e, \nu_\mu$);

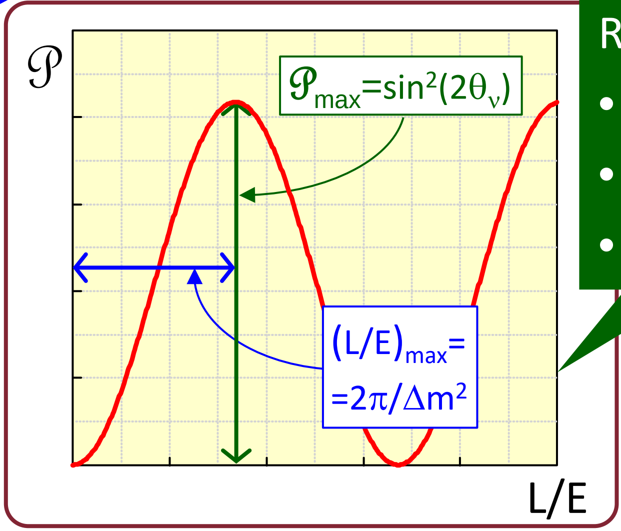
$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_\nu & \sin\theta_\nu \\ -\sin\theta_\nu & \cos\theta_\nu \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix};$$

- free parameters: masses, mixing angle θ_ν;
- same formalism as in the ($K^0_1 \leftrightarrow K^0_2$) case;
- time evolution of a pure $\nu_{e,\mu}$ at t=0:

$$|\nu_e(t)\rangle = \cos\theta_\nu e^{-iE_1t} |\nu_1\rangle + \sin\theta_\nu e^{-iE_2t} |\nu_2\rangle$$
$$|\nu_\mu(t)\rangle = -\sin\theta_\nu e^{-iE_1t} |\nu_1\rangle + \cos\theta_\nu e^{-iE_2t} |\nu_2\rangle$$

- the oscillation probability $\mathcal{P}$ is [*next slide*]:

$$\boxed{\mathcal{P}_L(\nu_e \rightarrow \nu_\mu) = \sin^2[2\theta_\nu]\sin^2\left[\frac{\Delta m^2 L}{4E}\right]; \quad \frac{\Delta m^2 L}{4E} \approx \frac{1.27 \times (m_2^2 - m_1^2)[eV^2] \times L[km]}{E[GeV]}.}$$



Required:
- large θ_ν;
- $m_\nu > 0$;
- large L/E;

→ since θ_ν and $m_{1,2}$ are not up to us, the relevant exper. parameter is **L/E**; with present technologies, the observation is:

- difficult (= impossible) with accelerators;
- needs astrophysical exp.

[actual experiments are NOT discussed here: they belong to the astroparticle course]

ν oscillations: the math

$$\left|\langle \nu_e(t) | \nu_e(0)\rangle\right|^2 = \left|\left(\cos\theta_\nu e^{-iE_1t}\langle\nu_1| + \sin\theta_\nu e^{-iE_2t}\langle\nu_2|\right)\left(\cos\theta_\nu|\nu_1\rangle + \sin\theta_\nu|\nu_2\rangle\right)\right|^2 =$$

$$= \left|\cos^2\theta_\nu e^{-iE_1t} + \sin^2\theta_\nu e^{-iE_2t}\right|^2 =$$

$$= \left|\cos^2\theta_\nu\cos(E_1t) - \imath\cos^2\theta_\nu\sin(E_1t) + \sin^2\theta_\nu\cos(E_2t) - \imath\sin^2\theta_\nu\sin(E_1t)\right|^2 =$$

$$= \cos^4\theta_\nu\cos^2(E_1t) + \sin^4\theta_\nu\cos^2(E_2t) + 2\sin^2\theta_\nu\cos^2\theta_\nu\cos(E_1t)\cos(E_2t) +$$

$$+ \cos^4\theta_\nu\sin^2(E_1t) + \sin^4\theta_\nu\sin^2(E_2t) + 2\sin^2\theta_\nu\cos^2\theta_\nu\sin(E_1t)\sin(E_2t) =$$

$$= \cos^4\theta_\nu + \sin^4\theta_\nu + 2\sin^2\theta_\nu\cos^2\theta_\nu\cos\left[(E_2 - E_1)t\right]\boxed{+1-(\cos^2\theta_\nu+\sin^2\theta_\nu)^2} =$$

$$= 1 - 2\sin^2\theta_\nu\cos^2\theta_\nu\left\{1 - \cos\left[(E_2-E_1)t\right]\right\} = 1 - 4\sin^2\theta_\nu\cos^2\theta_\nu\sin^2\left[(E_2-E_1)t/2\right] =$$

$$= 1 - \sin^2(2\theta_\nu)\sin^2\left(\frac{\Delta m^2 L}{4E}\right).$$

$$\mathcal{P}_L(\nu_e \to \nu_\mu) = 1 - \left|\langle \nu_e(t) | \nu_e(0)\rangle\right|^2 =$$

$$= \sin^2(2\theta_\nu)\sin^2\left(\frac{\Delta m^2 L}{4E}\right).$$

$\mathcal{P}_L$ is the oscillation probability after a distance L.

$$(E_2 - E_1)t = \left(\sqrt{p^2+m_2^2} - \sqrt{p^2+m_1^2}\right)t \approx$$

$$\approx p\left[\left(1+\frac{m_2^2}{2p^2}\right) - \left(1+\frac{m_1^2}{2p^2}\right)\right]\frac{L}{c} \approx$$

$$\approx \frac{m_2^2 - m_1^2}{2p}L \approx \frac{\Delta m^2 L}{2E}.$$

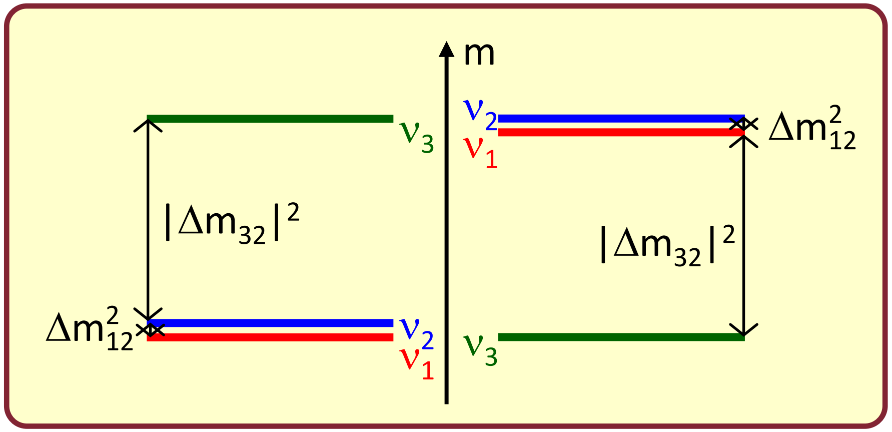
ν oscillations: results

Current ν oscillation experiments measure:

$$\Delta m^2_{12} = m^2_2 - m^2_1 \approx 7.37 \times 10^{-5}\ \text{eV}^2;$$

$$|\Delta m_{32}|^2 = |m^2_3 - m^2_2| \approx 2.56 \times 10^{-3}\ \text{eV}^2;$$

compatible with the two "hierarchies" shown in the box (ambiguity still not solved).



Q. why ν's from the sky and not from an accelerator ? compute the value of L/E for the oscillation maxima using these values.

In the SM there are three families $\rightarrow$ the ν mixing matrix is 3×3, with the same math properties of the CKM one (three angles + a CP-violating phase).

It is called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix:

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = V_{PKMS} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix};$$

the present best measurements are [PDG]:

$$|V_{PKMS}| = \begin{pmatrix} 0.826 & 0.544 & 0.151 \\ 0.427 & 0.642 & 0.635 \\ 0.368 & 0.540 & 0.757 \end{pmatrix}.$$

The CP-violating phase (δ_ν) is $\approx 3\pi/2$.

$\mathbb{CPT}$ theorem

If (Quantum field theory) and (Special relativity) and ($\mathbb{H}$ invariant under Lorentz transformation),

then

the physical states are $\mathbb{CPT}$ invariant, i.e. invariant under the consecutive application of the operators $\mathbb{C}$harge-conjugation, $\mathbb{P}$arity and $\mathbb{T}$ime-reversal.

Nota bene :

- The states may be invariant for the application of any of the three, like in strong interaction processes.
- In this case, *a fortiori*, they will be invariant under the three together.
- But even processes which violate one (left-handed neutrinos, K^0 oscillations) or even two (K^0 semileptonic decays), must be invariant under the combined application of the three together.

Consequences of the $\mathbb{CPT}$ theorem :

- mass, charge and lifetime of a particle and its antiparticle are exactly equal :
  - $|m(K^0) - m(\bar{K}^0)|$ / aver. $< 6 \times 10^{-19}$;
  - $|m(e^+) - m(e^-)|$ / aver. $< 8 \times 10^{-9}$;
  - $|q(p) - q(\bar{p})|$ / $q(e^-) < 2 \times 10^{-9}$;
  - $[\tau(\mu^+) - \tau(\mu^-)]$ / aver. $= (2 \pm 8) \times 10^{-5}$;
- any violation in an individual or pair of symmetries must be compensated by an asymmetry in the other operation(s), so to save exact symmetry under $\mathbb{CPT}$.
- (e.g.) The weak interactions violate $\mathbb{C}$ and $\mathbb{P}$ separately but in general they are invariant under the combined operation of $\mathbb{C}$ and $\mathbb{P}$ (and $\mathbb{T}$ alone).
- (e.g.) The weak decays of the K^0 mesons violate $\mathbb{CP}$, but this is accompanied by a corresponding violation of $\mathbb{T}$, so that [$\mathbb{CP}$ $\mathbb{T}$] is respected.

References

1. [BJ, 11.13]], [YN1, 16];
2. the CPT theorem is discussed in [MQR, 12];
3. the $\mathbb{CP}$ violation and the FCNC are discussed in [IE, 12-13]

Gian Lorenzo Bernini – Apollo and Daphne – 1622-25 – Galleria Borghese

End of chapter 5