Particle Physics - Chapter 8
Colliders : $\bar{p}p$ – e^+e^- – pp

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8 – Colliders : p̄p – LEP – pp

i. Accelerators

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   5) Rapidity and pseudo-rapidity

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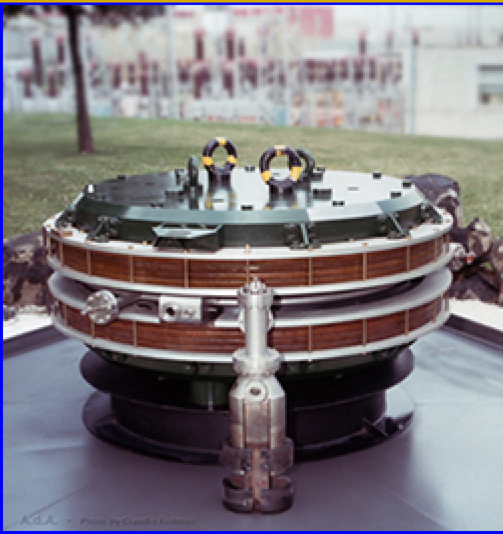
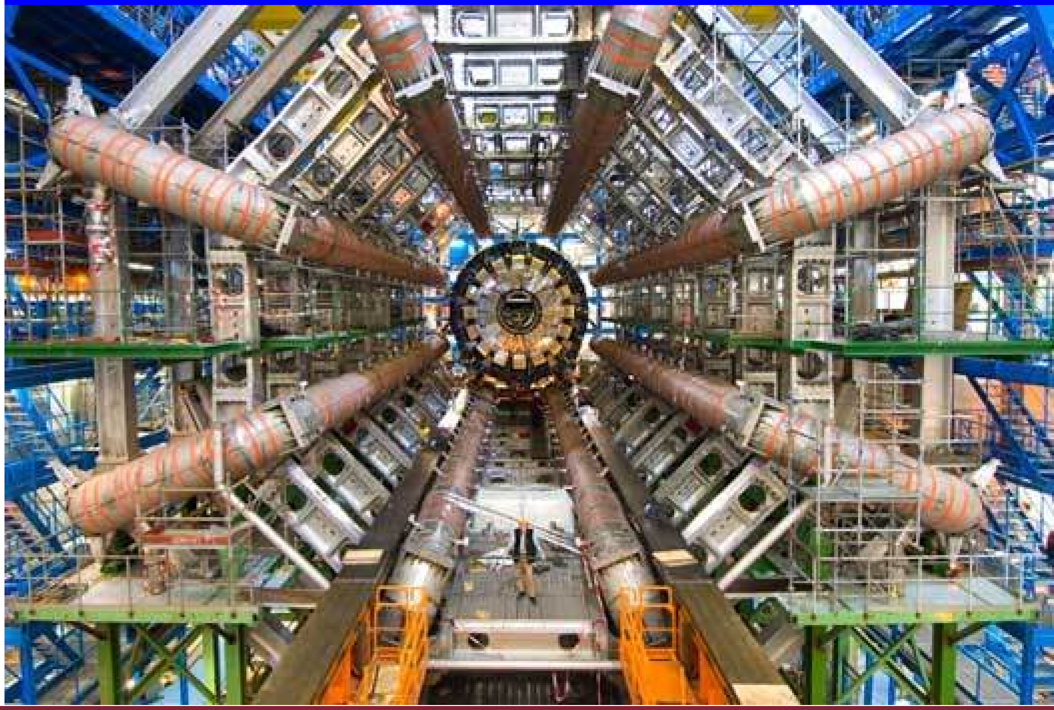
iii. comparisons

   9) $e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$.

An introduction to collider physics: specific items + a discussion of $\ell^+\ell^-$ vs had colliders.

The full machine ADA (e^+e^-, R=65 cm) and a single detector like ATLAS (pp, R=12 m) at LHC (R = 4.2 km).





i. Accelerators

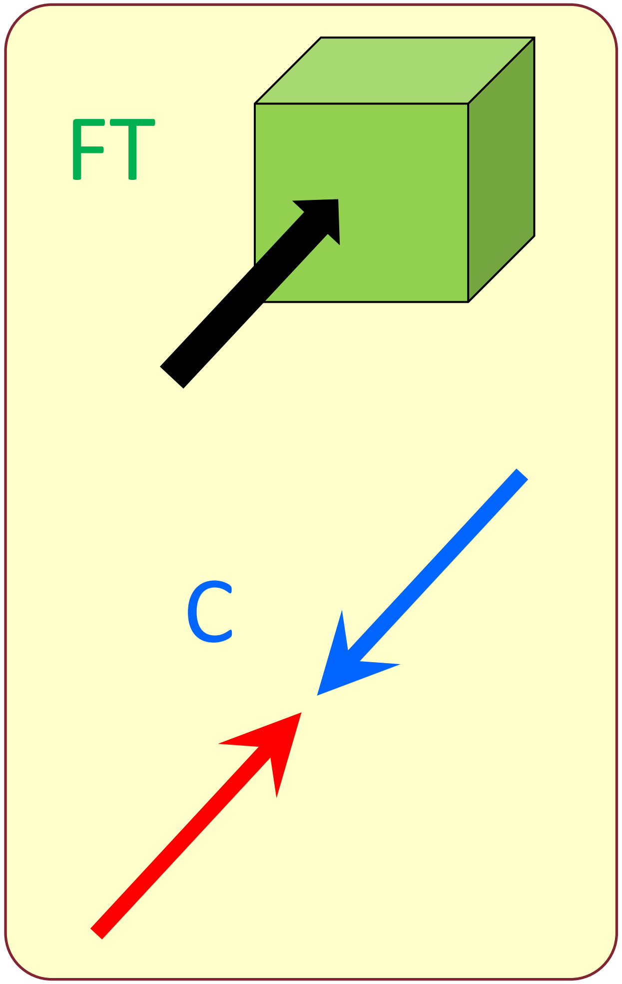
In modern particle physics, a good machine is the essential part of the game. E.g. the $W^\pm$, Z, H discoveries are mainly due to two new extraordinary colliders (the CERN $\bar{p}p$ and LHC respectively).

Colliders : introduction

- Hadronic collisions (Sp$\bar{p}$S + LHC at CERN, TeVatron at Fermilab) share common dynamical and kinematical features, different from e^+e^- (Spear, LEP, …).
- Hadrons are composite, as explained by the QCD-quark-parton model :
  - coherent pp ($\bar{p}$p) scattering at low p_T;
  - qq/$\bar{q}\bar{q}$/q$\bar{q}$/qg/$\bar{q}$g/gg scattering at high p_T, dominated by t-channel gg.
- Instead in e^+e^- Colliders only point-like interactions, dominated by s-channel.
- The historical order Sp$\bar{p}$S – LEP – LHC is unnatural (hadrons, leptons, hadrons), but we will follow it, at the price of some repetitions and logical leaps.
- In the Sp$\bar{p}$S and LHC chapters, the order will be the traditional one, increasing p_T and decreasing cross-section :
  1. [*total cross-section*],
  2. low-p_T interactions,
  3. high-p_T hadronic processes,
  4. high-p_T electro-weak;
  5. [*searches for new physics, if any*].
- For LEP, the order will be the history, i.e. the increasing beam energy :
  1. Z-pole electroweak physics,
  2. W^+W^- pair creation,
  3. [a digression on the method of searches and the analysis of negative results, the "limits"],
  4. Higgs searches;
  5. [*searches for new physics, if any*].
- In this first chapter, there are some definitions and discussions, useful for all the following parts, especially for hadron colliders.

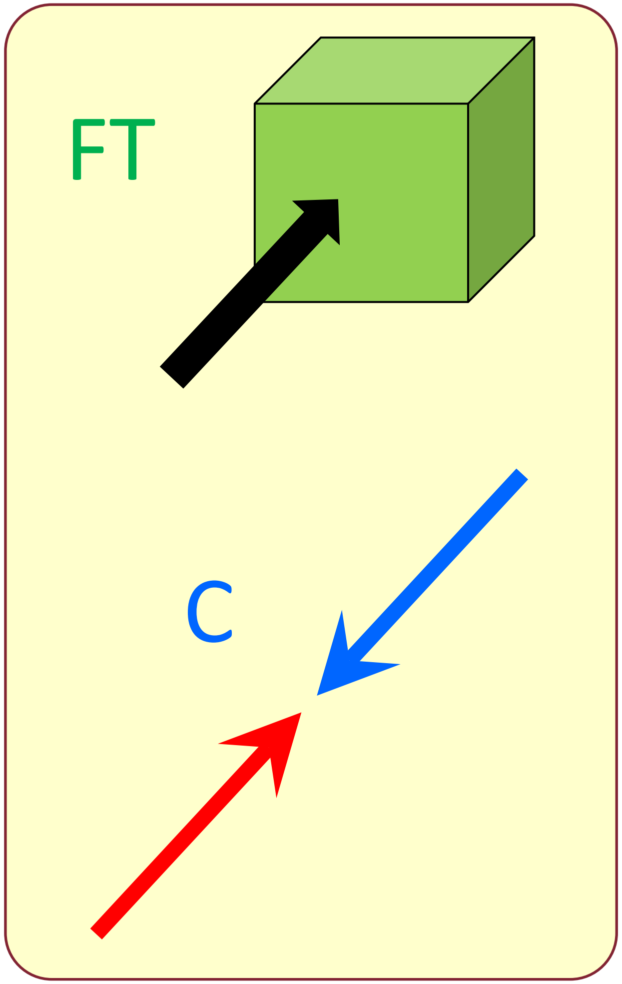
Colliders : vs fixed target

- Dynamics is invariant under a Lorentz boost; the processes depend on the relative motion of particles only : fixed target experiments (**FT**) and colliders (**C**) are dynamically equivalent;
- however, the explored kinematical region (and the experiments) are very different;
- a general (simplified) discussion of the relative merits of FT vs C in the next slides;
- for general purpose experiments, the quest for higher energy gives C a definitive advantage over FT [*imho, but widely shared*];
- the [*obvious*] reason is the CM energy $\sqrt{s}$:
  - **FT** : $s \approx 2\, m_N\, E_{beam} \to \sqrt{s} \propto \sqrt{E_{beam}}$;
  - **C** : $s = (2E_{beam})^2 \to \sqrt{s} \propto E_{beam}$;
- future alternatives : e^+e^- linear C, $\mu^+\mu^-$ circular C.



Colliders : types

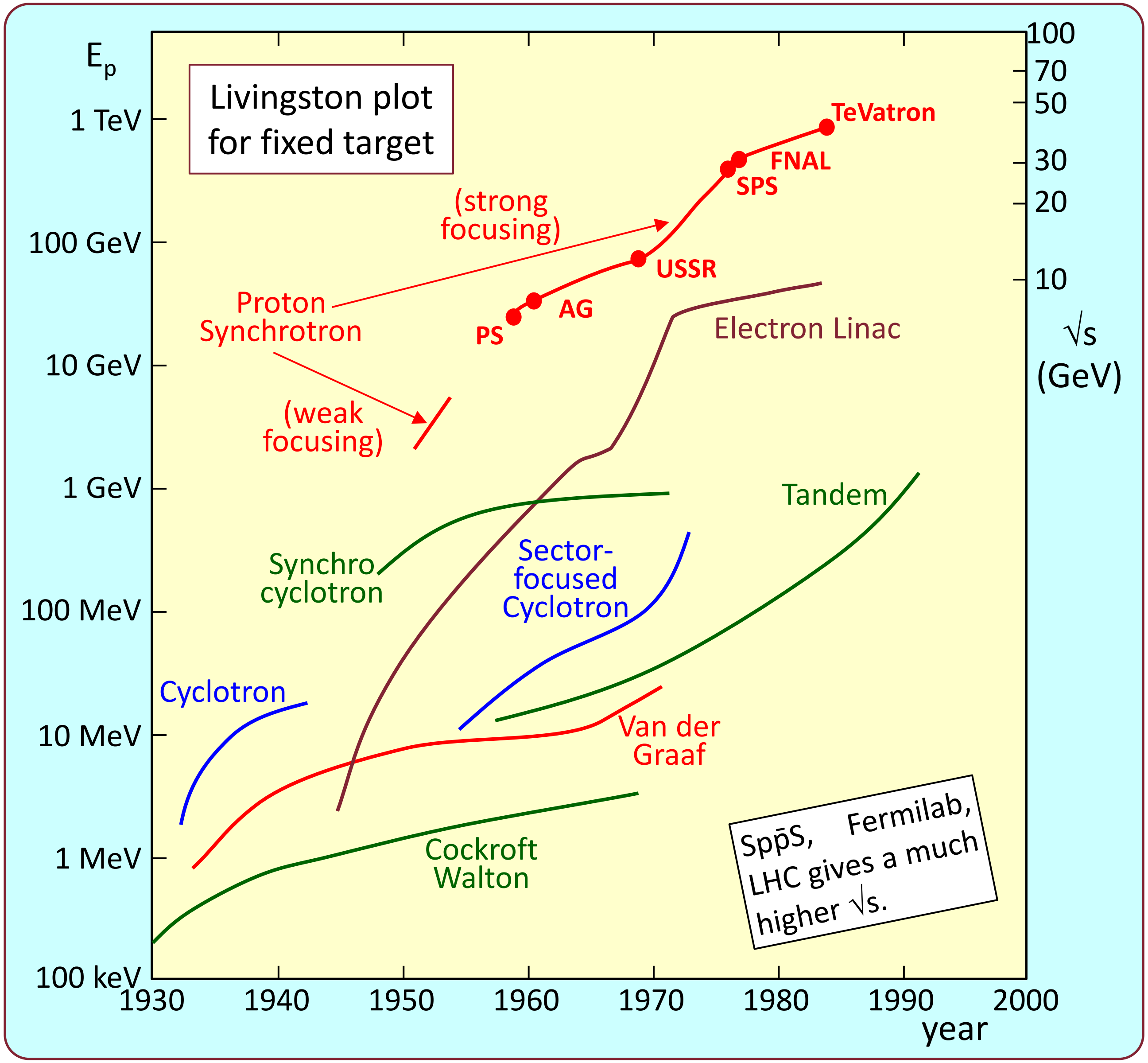
- FT's offers a plethora of initial states (nucleons, mesons, charged and neutral leptons, …), while C's have been realized with only few initial states:
  - e^+e^- AdA, ADONE, SPEAR, DESY, LEP, DAΦNE, …;
  - $\bar{p}p$ CERN and Fermilab Colliders;
  - pp ISR, LHC;
  - $e^\pm p$ Hera;
  - (+ heavy ions and specialized machines);
- projects for $\mu^+\mu^-$ Colliders; $\mu^\pm$ are dynamically equal to $e^\pm$, but produce (much) smaller brem; so they can be accelerated to higher energy;
- colliders e^+e^- have been realized since 50 years; they have discovered new leptons (τ), new hadrons (J/ψ, charm), new dynamics …
- The successes of pp ($\bar{p}p$) are $W^\pm$, Z, top, H.
- The swan songs of FT have been J/ψ and b quark (+ ν physics, which is a special case).



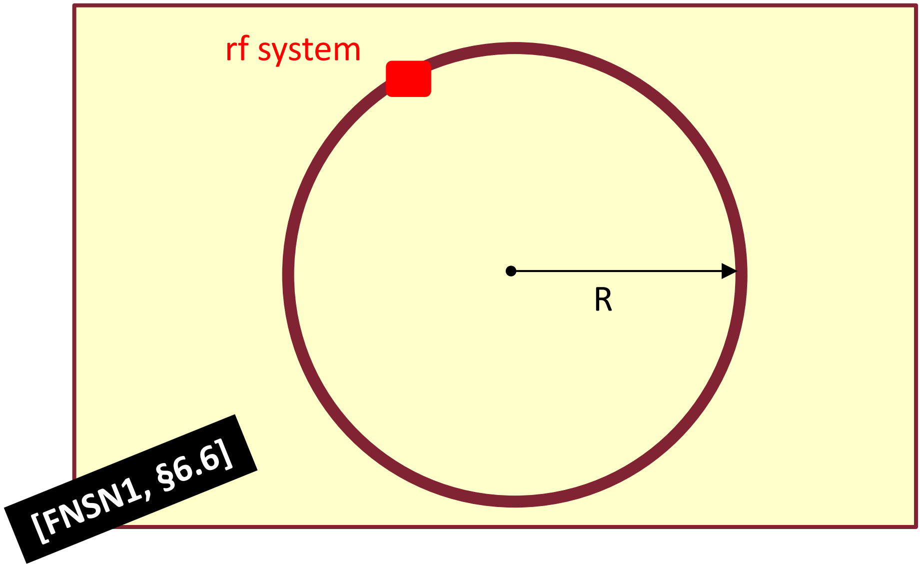
Colliders: Livingston plot

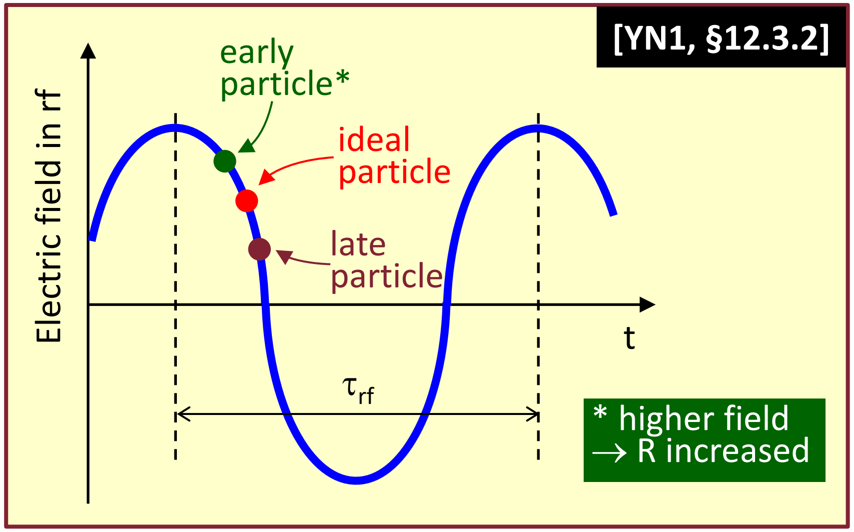
In addition, FT has plenty of applications out of the "energy frontier".

[our department, together with INFN and the SBAI department, hosts a PhD programme in accelerator physics ("dottorato in Fisica degli acceleratori")]



Synchrotron





Build a machine with a circular tube of small size and large radius, instrumented with dipoles and radiofrequencies of small-aperture and big power (+ auxiliaries) :

- from Lorentz force:

  p (GeV) = mβγ = 0.3BR (T,m);

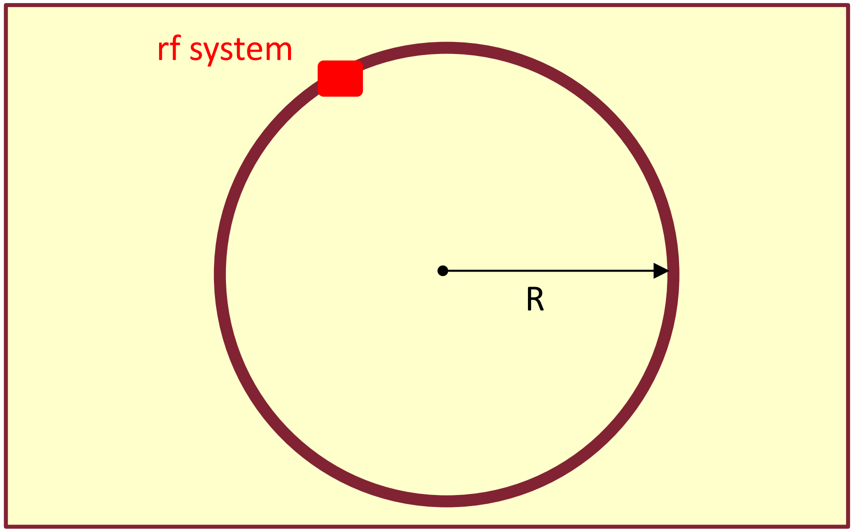
→ the mag. field $|\vec{B}|$ must be continuosly **synchro**nized to keep the beam on the same R, by varying the current ι in the magnet coils ($|\vec{B}| = \mu_0 n\iota$).

- the revolution period must be an integer multiple n_R of the radio-frequency period τ_{rf} [Povh, § A.1] :

$$t_R = \frac{2\pi R}{p/E} = n_R \tau_{rf} = \frac{2\pi n_R}{\omega_{rf}} \rightarrow \omega_{rf} = \frac{n_R p}{RE};$$

→ ω_{rf} must be continuosly re-adjusted (i.e. **synchro**nized) to follow the beam velocity (β=p/E), in order to always get the beam in the correct phase;

Synchrotron: parameters



Present limitations for parameters :

- mag. field B < 1.4 T (warm, iron core) or B < 10 T (superconductivity, but requires cryo magnets);
- R limited by civil engineering (costs, availability) to few (max tens) Km;
- radiofrequency limited by energy costs;
- brem problem for electrons [§ LEP].

Results:

- beam(s) bunched : $n_{bunch} < n_{bucket}$ (= n_R);
- $\sqrt{s}_{collider}$ (TeV) ≈ 2p ≈ 0.6 B(T) R(Km);
- $\sqrt{s}_{fixed}$ (GeV) ≈ $\sqrt{2M_pE} \approx \sqrt{0.6BR}$ (T,m).

Problems:

- beam manipulation is complicated (next);
- interaction rate [see *Luminosity* in the following] is smaller wrt continuous accelerators;
- however, in practice this is the only known method to achieve high energy/high intensity;

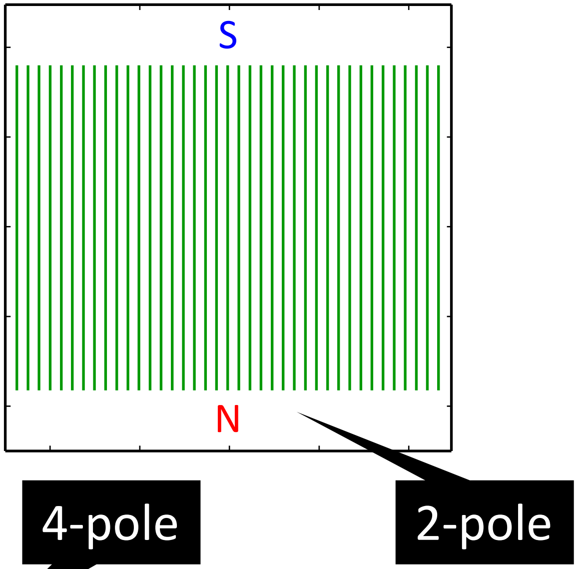
→ all modern accelerators are based on the principle of synchrotrons.

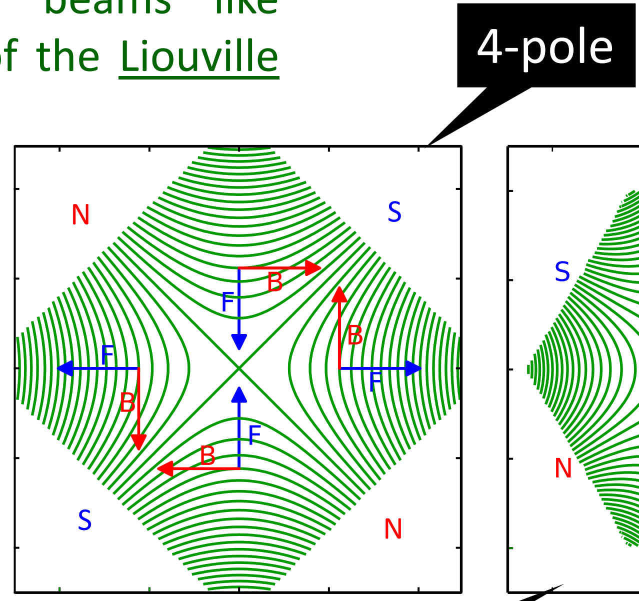
Synchrotron: magnets

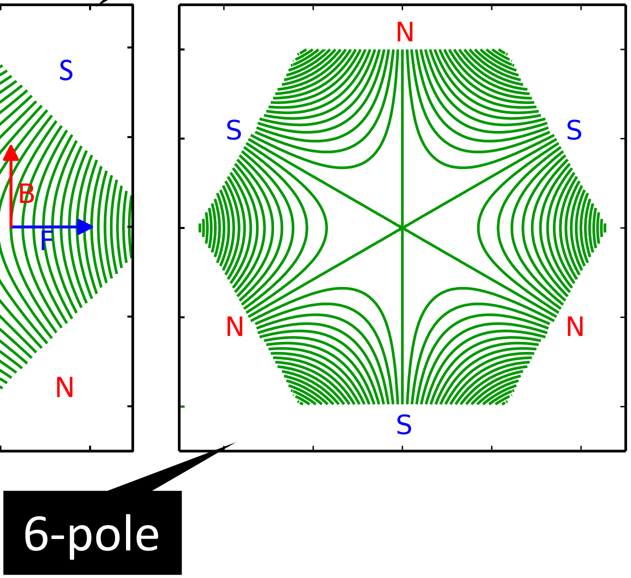
The conventional approach to particle beam manipulations is to treat them as light rays (**beam optics**). The "lenses" are magnets :

- **dipoles** for beam bending; the dipoles are the main elements; if all the particles behave as their average ("ideal trajectory") no other elements were necessary;
- higher multipoles, like **quadrupoles**, **sextupoles**, for (de)focalization; they (de-)focus the beams like (di/con)vergent lenses (but be aware of the Liouville theorem !!!);
- the overall control is in the hands of very smart physicists/engineers, fast and big computers, under the goddess Fortuna.

Liouville's theorem: if the particles obey the canonical equations of motion, then every element of a volume phase space is constant with respect to time.
[in this case: every gain in space density has to be compensated by a loss in momentum density]



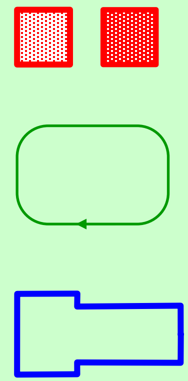




Synchrotron: magnet coils

The magnets are built with two different techniques :

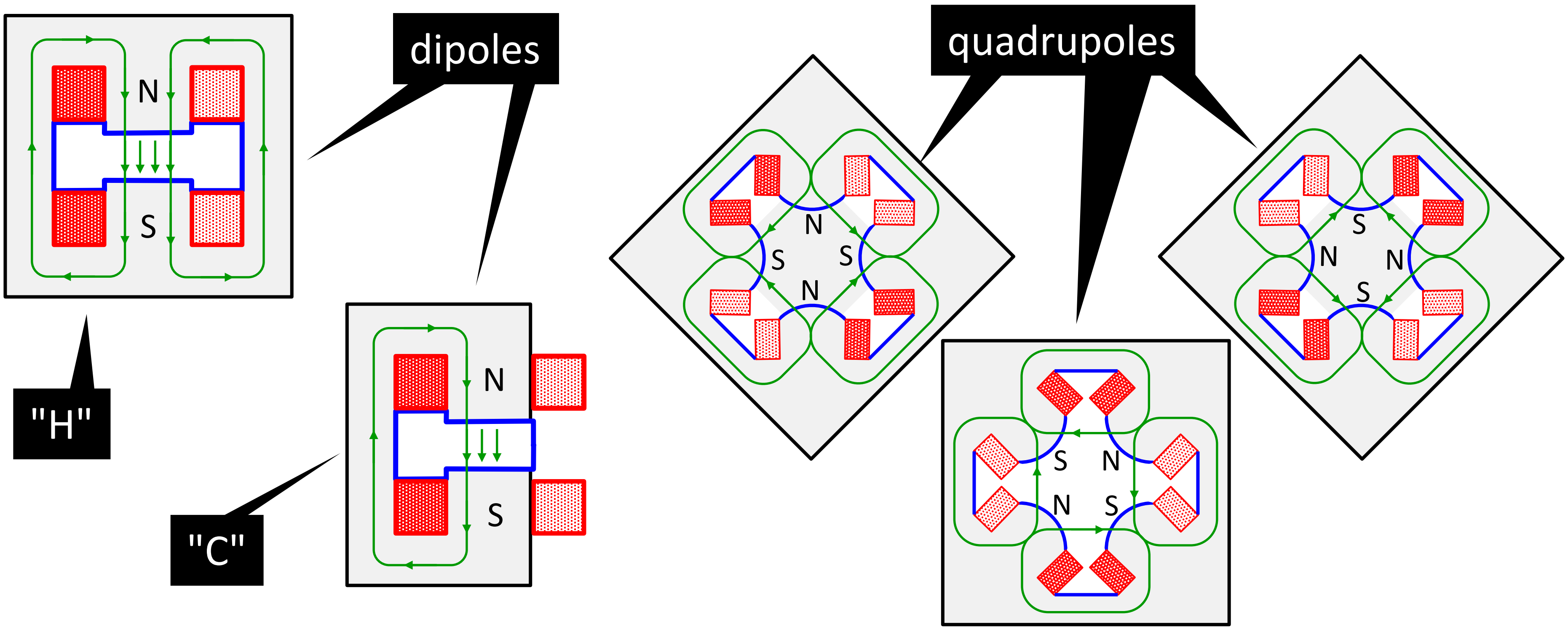
- warm : coils with high continuous currents + iron yoke;
- cold : superconducting coils at cryo temperature and (almost) no iron.



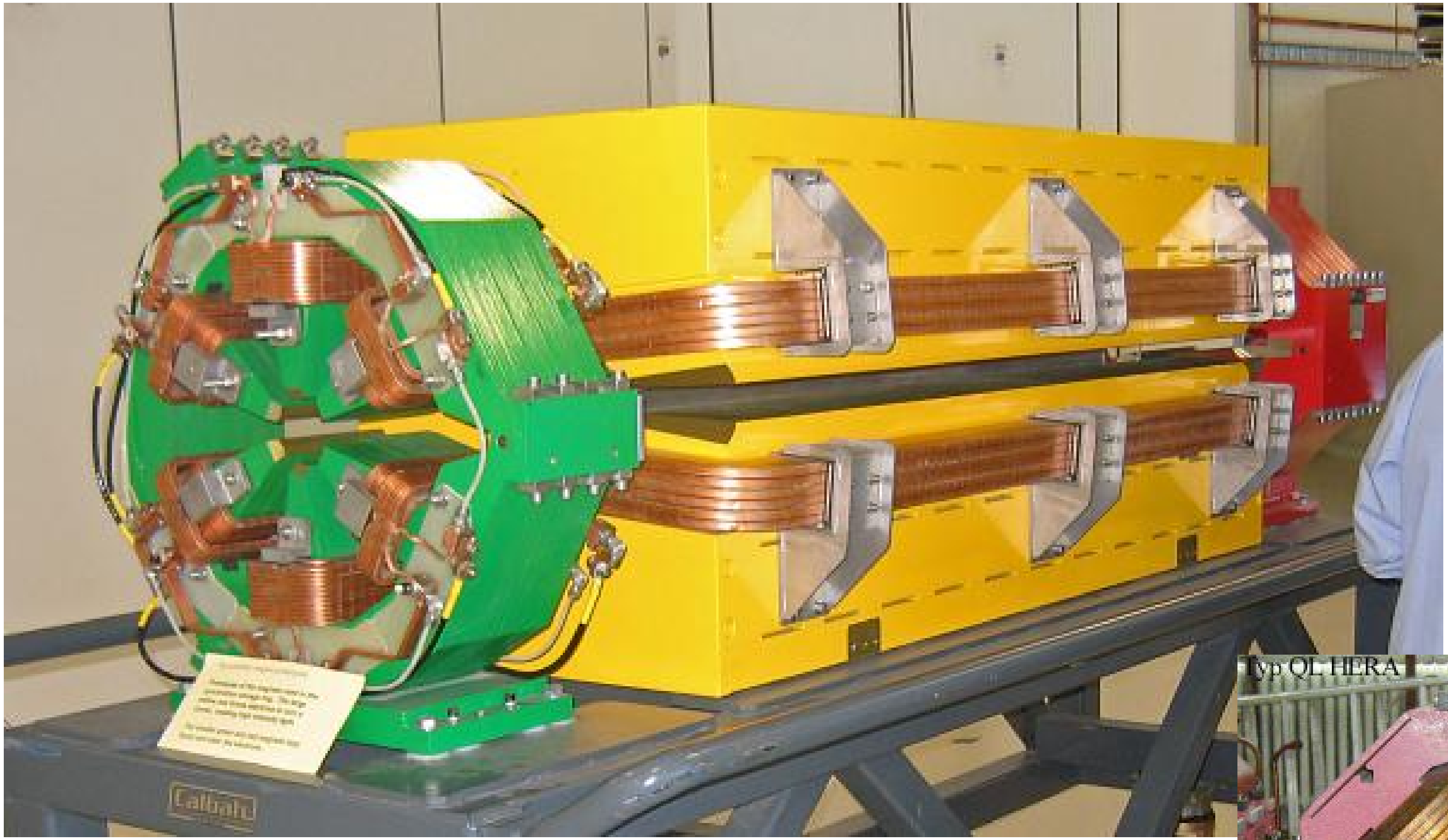
Coils (currents);

B-field lines;

Open for beam circulation.



Synchrotron: examples of magnets



a sextupole in front of a "C" dipole

a quadrupole of the HERA accelerator

Synchrotron: the brem effect

[FNSN1, §6.6]

$$W_{Larmor} = \frac{1}{6\pi\varepsilon_0}\frac{e^2 a^2 \gamma^4}{c^3};$$

$$a(\text{circle}, \beta \approx 1) = \frac{v^2}{R} \approx \frac{c^2}{R};$$

$$W_{Larmor} = \frac{1}{6\pi\varepsilon_0}\frac{e^2 c \gamma^4}{R^2};$$

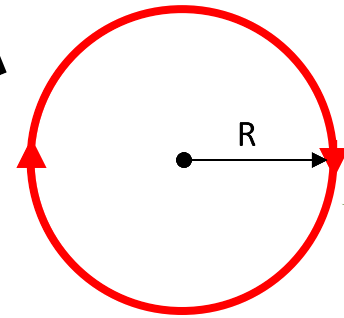
$$T_{1orbit} = \frac{2\pi R}{v} \approx \frac{2\pi R}{c};$$

$$\Delta E_{1orbit} = W_{Larmor} T_{1orbit} = \frac{1}{3\varepsilon_0}\frac{e^2\gamma^4}{R} \propto \frac{E^4}{M^4 R};$$

$(m_p/m_e)^4 \approx 1.1\times10^{13}$

$$\Delta E^{electron}_{1orbit} = 8.85\times10^{-5}\left(\frac{E_e}{1\text{GeV}}\right)^4 \Big/ \left(\frac{R}{1\text{Km}}\right) \text{MeV};$$

$$\Delta E^{proton}_{1orbit} = 7.8\times10^{-3}\left(\frac{E_p}{1\text{TeV}}\right)^4 \Big/ \left(\frac{R}{1\text{Km}}\right) \text{KeV}.$$



in practice $R_{tunnel} > R_{magnet}$.

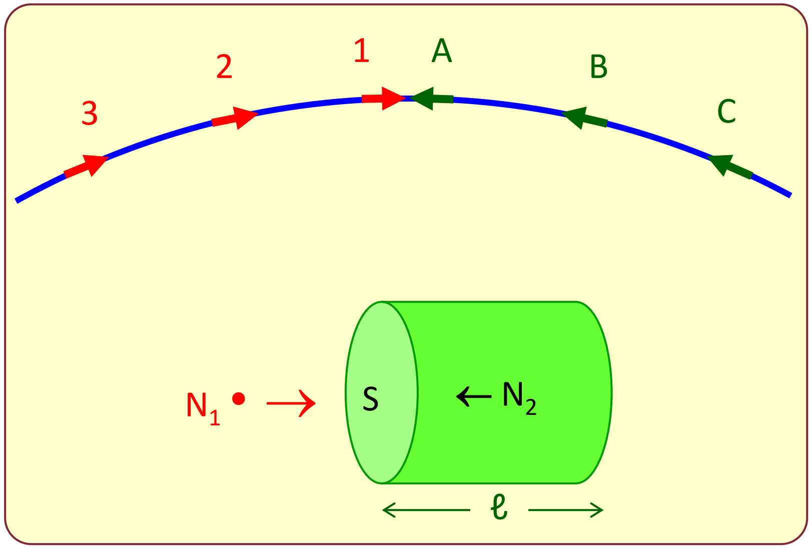
$\Delta E \propto M^{-4}$		$\sqrt{s}$ (GeV)	ΔE
LEP 1	e^+e^-	90	**121 MeV**
LEP 2	e^+e^-	200	**2,500 MeV**
LHC	pp	14,000	**6.9 KeV**

in circular e^+e^- colliders $R_{best} \propto s$ (severe limitation, see § LEP).

therefore, in future try :

- **$\mu^+\mu^-$ colliders;**
- **linear e^+e^- colliders.**

Luminosity: toy model



The fundamental figure to quantify collider performances is the **Luminosity** $\mathcal{L}$. Define it with a toy model:

- N_1 particles/bunch turning "clockwise";
- N_2 … "anti-clockwise";
- cylindrical bunches $S \times \ell$, ρ = const. [*this is the toy assumption*];
- for each of N_1, while traveling inside the cylinder N_2 for a small step x, the probability of interaction is:

$$\mathcal{P}_1(x) = 1 - e^{-\rho\sigma_T x} \cong \rho\,\sigma_T\, x = N_2\sigma_T x/(S\,\ell);$$

- the average number of interactions / crossing is :

$$\langle n_I\rangle = N_1\,\mathcal{P}_1(\ell) = N_1 N_2 \sigma_T / S;$$

[$\langle n_I\rangle$ independent from ℓ]

- the crossings rate is

$n_c = k \times f$

[k = bunch number, f = revolution frequency]

therefore, the interaction rate is :

$$R \equiv \mathcal{L}\,\sigma_T = \langle n_I\rangle \times n_c = N_1 N_2\, k\, f\, \sigma_T / S,$$

where $\mathcal{L}$, the "**luminosity**", contains the parameters of the machine, while σ_T reflects the particle dynamics:

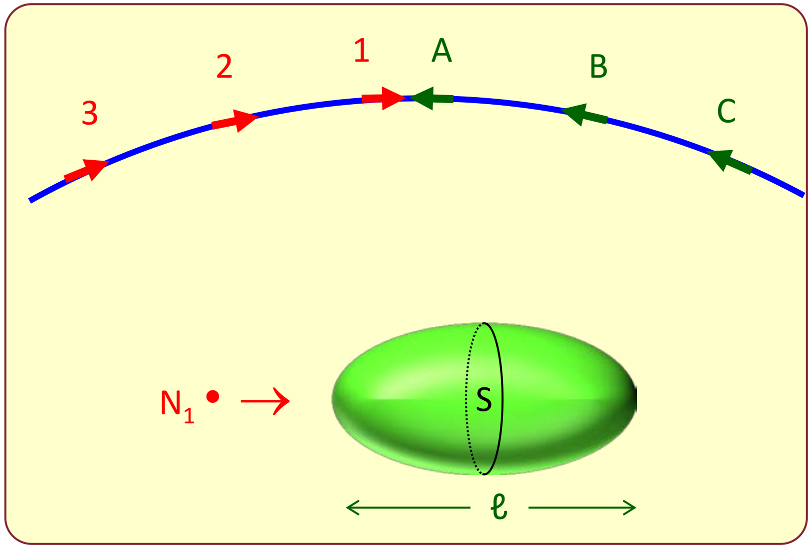
$$\boxed{\mathcal{L}^{toy} = \frac{N_1 N_2 k f}{S}.}$$

Luminosity: comments

The toy model is too naïve, however some of the conclusions are correct.

The luminosity is defined as $\mathscr{L} \equiv R/\sigma_T$, the ratio between the interaction rate and the total cross section(*). $\mathscr{L}$ is:

- **NOT** dependent (for head-on collisions) on the **bunch length ℓ**;
- proportional to the **inverse of the bunch section** (use an effective bunch section $S = 4\pi\sigma_x\sigma_y$);
- proportional to the **number of particles / bunch** of both beams (N_1N_2);
- proportional to the **number of bunch crossings / second** (kf);
- [not in formula] dependent on **centroids displacement** and **beam lifetime**.



$$\mathscr{L} = \frac{N_1 N_2 k f}{4\pi\sigma_x\sigma_y}.$$

NB the total number of interactions seems to grow $\propto k^2$; however, in a given interaction point, it grows $\propto$ **k**. *Is it clear ?* from this consideration, many clever machine developments, e.g. the *pretzel scheme*.

____

(*) for a process x : $R_x/R_T = \sigma_x/\sigma_T \rightarrow R_x = \mathscr{L}\,\sigma_x$.

Luminosity: collisions at angle α

- In case of an angle α between the beams (LHC), the formula becomes

$$\mathcal{L} = \frac{k f N_1 N_2}{4\pi\sigma_x\sigma_y} f(\alpha) \equiv \mathcal{L}_0 f(\alpha);$$

$$f(\alpha = 0) = 1; \quad f(\alpha \neq 0) < 1.$$

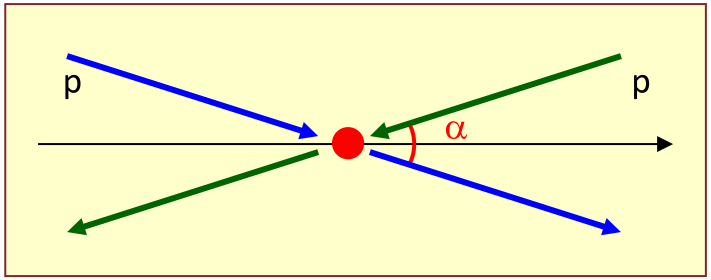
- It turns out(*) :

$$f(\alpha) = 1 \Big/ \sqrt{1 + \left(\alpha\sigma_\ell / 2\sigma_T\right)^2}$$

where σ_ℓ(σ_T) is the longitudinal (transv.) effective dimension of a bunch.

- Notice the dependence on σ_ℓ/σ_T; short bunches have other pros (better definition of the interaction point) and cons (e.g. in case of many overlapping events in the same bunch-crossing).

- At LHC, $\alpha \approx 300$ μrad $\rightarrow f(\alpha) = 0.83$.



- *Problem : the effect of α on $\sqrt{s}$ and p_T :*

  in LAB sys (≠ CM !!!) :

  [2E, 0, -2p sin(α/2),0] ≈ [2E, 0, - Eα, 0];

  $\rightarrow \sqrt{s} = 2E\sqrt{1-\alpha^2/4} \approx 2E\,(1 - \alpha^2/8);$

  $\rightarrow \Delta\sqrt{s} \approx - E\,\alpha^2/4$ *(negligible at LHC);*

  $\rightarrow |p_T| \approx E\alpha \approx 2$ *GeV at LHC (also negligible).*

  → *CONCLUSION : at LHC, in practice,* **LAB. sys. = CM sys., $\sqrt{s}$ = 2E, only $\mathcal{L}$ affected by α.**

(*) e.g. CERN CAS 2003, YR 2006-002, page 361.

Luminosity: $\langle n_{int} \rangle$

Problem. How many interactions / bunch-crossing [b.c.] ? [n_{int}, also "μ", *a bad choice for an overused symbol*].

Solution [τ_{bc} = time between b.c.] :

$$\langle \mu \rangle = \frac{N_1 N_2 \sigma_T}{4\pi\sigma_x\sigma_y} = \frac{\mathscr{L}\sigma_T}{kf} \approx \mathscr{L}\tau_{bc}\sigma_T = \mathscr{L}_{bc}\sigma_T \;^{(*)};$$

The effects of μ depend on its value:

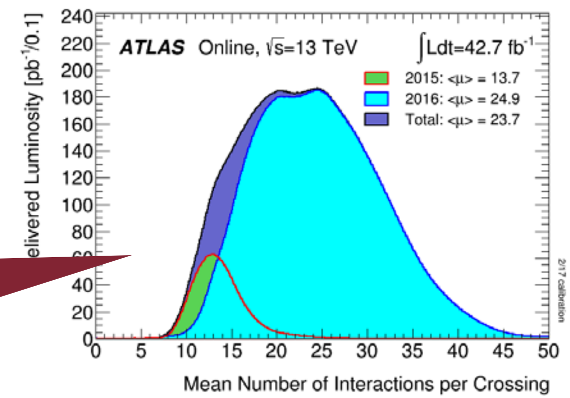
- <u>**$\langle\mu\rangle \ll 1$**</u> (Sp$\bar{\text{p}}$S, LEP): the probability of an interaction in a given b.c.; then "μ^2" is the probability of two events in the same b.c. (a known and not-very-important bckgd for Sp$\bar{\text{p}}$S and LEP);
- <u>**$\langle\mu\rangle > 1$**</u> (LHC): the <u>average</u> number of overlapped events in a b.c.; the actual number is Poisson-distributed, with average $\langle\mu\rangle$.

$^{(*)}$ some buckets are empty $\rightarrow$ larger $\mathscr{L}_{bc}$ and μ.

Comments:

- for hadronic colliders, it is better to consider $\mu_{inelastic}$ [$\sigma_T \rightarrow \sigma_{inel}$], which decreases μ by ~20%, because elastic collisions do not produce secondaries in the detectors;
- some old machines (e.g. CERN ISR) had "debunched" beams, i.e. particle uniformly spread over the whole ring; in this case the very definition of $\langle n_{int} \rangle$ is meaningless; however, for LHC this setup is simply impossible [*why ? try to answer*].

μ @ LHC, see § 12b



Luminosity : ε, β, β*

The dynamics of a real beam :

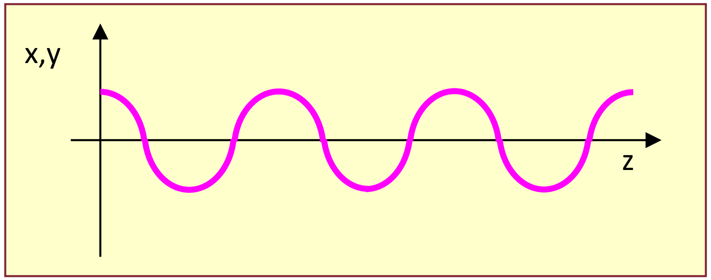
- real particles oscillate around the ideal trajectory (betatron oscillations);

- Reference system and definitions :
  - z : line of flight of the ideal particle;
  - x,y : deflections from ideal orbit;
  - $x' \equiv p_x / p_z$; $y' \equiv p_y / p_z$;
  - $\sigma_x \equiv$ rms beam size in x (also σ_y, $\sigma_{x'}$, $\sigma_{y'}$);
  - $\varepsilon_x = \pi \cdot \sigma_x \cdot \sigma_{x'}$ = "transverse emittance";
  - $\beta_x = \sigma_x / \sigma_{x'}$ = "amplitude function";
  - $\varepsilon_y = \pi \cdot \sigma_y \cdot \sigma_{y'}$; $\beta_y = \sigma_y / \sigma_{y'}$.

- Therefore (for the *, see on this page):

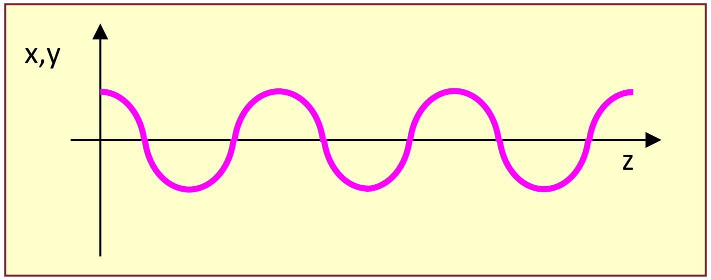
$$\mathcal{L} = \frac{k f N_1 N_2}{4\pi\sigma_x\sigma_y} f(\alpha) = \frac{k f N_1 N_2}{4\sqrt{\varepsilon_x\beta_x^*\varepsilon_y\beta_y^*}} f(\alpha);$$

- From Liouville's theorem :
  - $V(\text{6-dim}) = \sigma_x \cdot \sigma_y \cdot \sigma_z \cdot \sigma_{px} \cdot \sigma_{py} \cdot \sigma_{pz} =$ $=$ constant;
  - $\varepsilon_{x,y}$ = const. (modulo stochastic effects, which increase it with time);
  - $\beta_{x,y}$ can be modified by accelerator devices (e.g. quadrupoles) : it MUST be SMALL in the interaction regions ("low-beta", β*), and large far from them ("high-beta", β) [next slide].



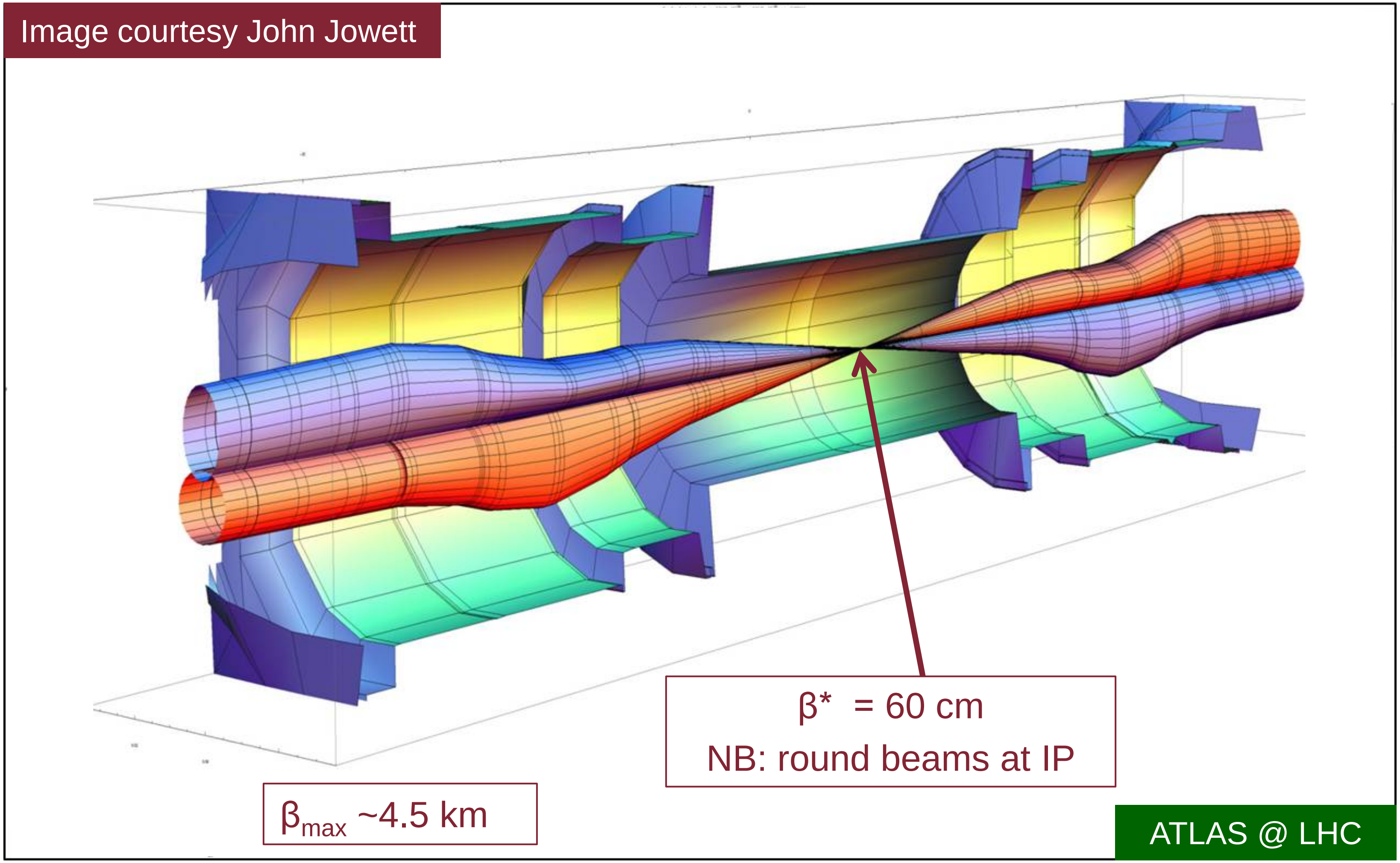
Luminosity: values of ε, β, β*

- At the **CERN Sp$\bar{p}$S** :
  - $\varepsilon_p \approx 9 \times 10^{-9}\ \pi$ rad m; $\varepsilon_{\bar{p}} \approx 5 \times 10^{-9}\ \pi$ rad m;
  - $\beta^*_H \approx 0.60$ m; $\beta^*_V \approx 0.15$ m.
- At **LEP** (remember the electron brem) :
  - $\varepsilon_H \approx (20 \div 45) \times 10^{-9}\ \pi$ rad m;
  - $\varepsilon_V \approx (0.25 \div 1.0) \times 10^{-9}\ \pi$ rad m;
  - $\beta^*_H \approx 1.50$ m; $\beta^*_V \approx 0.05$ m.
- At **LHC** ($\geq$ 2012) :
  - $\varepsilon_x \approx \varepsilon_y \approx 0.5 \times 10^{-9}\ \pi$ rad m;
  - $\beta^*_x \approx \beta^*_y \approx 0.55$ m;
  - [*see next page, from a beautiful CERN Academic training by Mike Lamont*].



Luminosity : β squeeze

Image courtesy John Jowett



ATLAS @ LHC

Luminosity: better toy model

A mechanical analogy [Ed Wilson, 28] :

- a little ball on a falling guide [*see*];
- two forces :
  1. gravity toward z (= "acceleration");
  2. a force orthogonal to z, which depends on the local shape of the guide (e.g. elastic $\propto |x|$);
- choose two parameters ε, β:

$$x = \sqrt{}$$

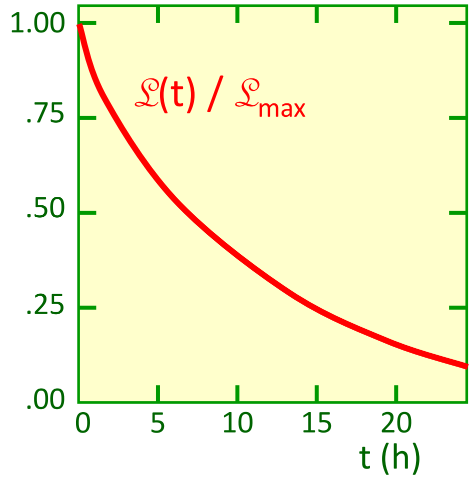
Luminosity: Liouville's theorem

- Because of the Liouville's theorem, for an "ideal fluid of balls", the [iper-] volume of the ellips[oid] keeps constant during the motion :

$$V = \pi\sqrt{}$$

Luminosity: evolution with time

- Many effects deteriorate the luminosity during a long data-taking. [following figures from LHC, but the effects are similar for all colliders].
- Parameterize as $d\mathscr{L} = -\mathscr{L}\, dt/\tau_i$; at LHC :
  - collisions $\tau_{coll} \cong 29$ h;
  - increase of emittance $\tau_{IBS} \cong 80$ h;
  - residual gas $\tau_{gas} \cong 100$ h;
  - (many other minor effects ...)
- Global effect on luminosity :



$$\mathscr{L}(t)=\mathscr{L}_{max}e^{(-t/\tau)}\ ; \qquad \frac{1}{\tau}=\sum\frac{1}{\tau_j}\approx 1\,/\,(15\text{ h}).$$

Integrated luminosity after a time T :

$$\mathscr{L}_{INT}(T)=\int_0^T \mathscr{L}(t)dt \approx \mathscr{L}_{MAX}\tau\left[1-e^{-(T/\tau)}\right];$$

$$N(T)=\int_0^T \mathscr{L}(t)\cdot\sigma_{TOT}dt=\mathscr{L}_{INT}(T)\cdot\sigma_{TOT}.$$

- After few hours, new injection and acceleration [*see* § *LHC*].
- I.e. $\mathscr{L}_{max,effective} \approx ½\ \mathscr{L}_{max}$.
- The decision to dump the beam and restart the cycle (inject – accelerate – squeeze – data-taking) is crucial :
  - At the Sp$\bar{p}$S was dramatic (high level officials), due the scarcity of $\bar{p}$.
  - Even at LHC (plenty of protons everywhere) is a major concern.

Luminosity: $\mathscr{L}$ vs $\sqrt{s}$

Summary :

$$R = \frac{dN}{dt} \equiv \mathscr{L}\sigma_{TOT}; \quad R_x = \mathscr{L}\sigma_x;$$

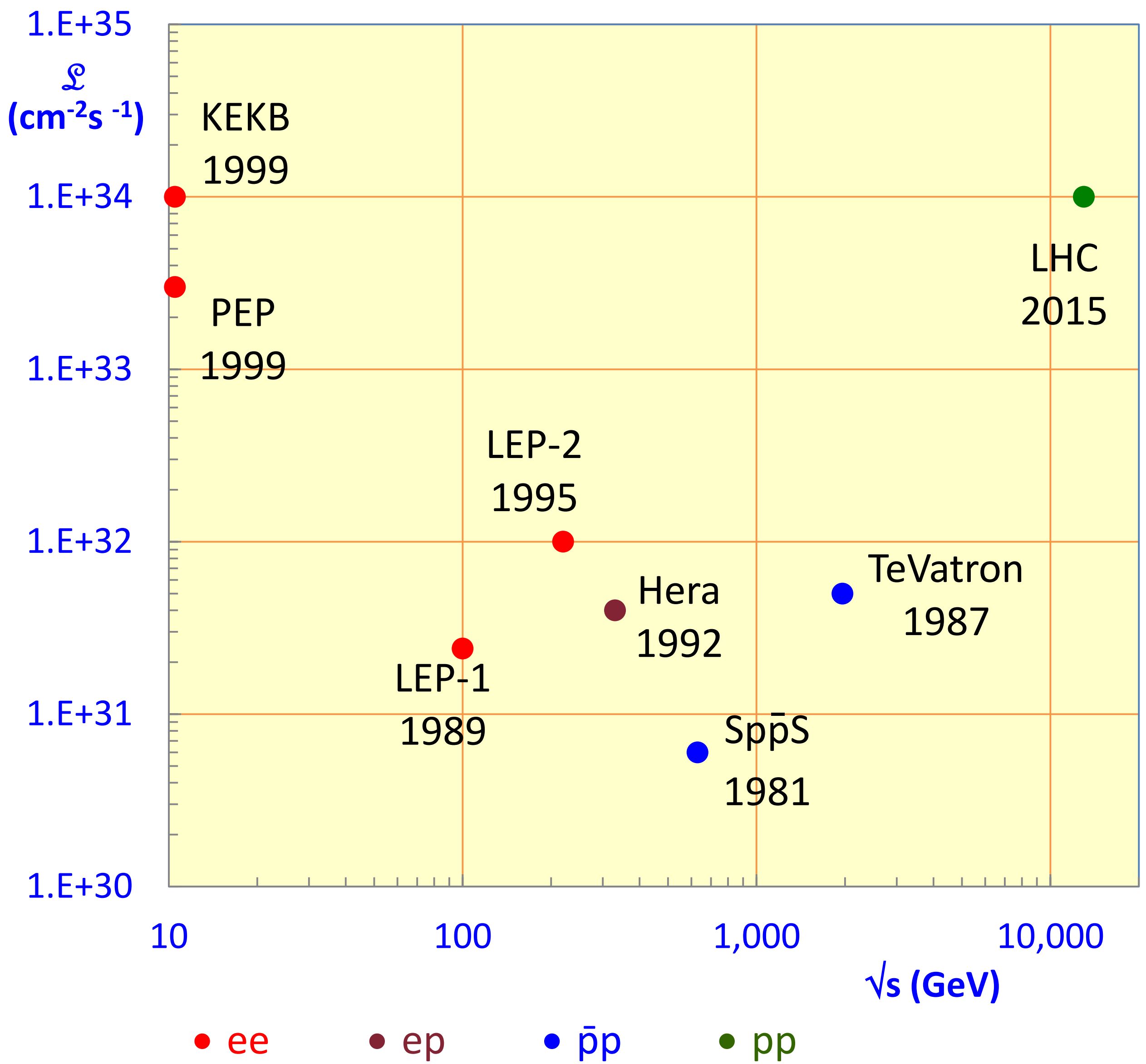
$$\mathscr{L} = \mathscr{L}(\alpha = 0) = \frac{N_1 N_2 k f}{4\pi\sigma_x\sigma_y} = \frac{k f N_1 N_2}{4\sqrt{\varepsilon_x\beta_x^*\varepsilon_y\beta_y^*}};$$

$$\mathscr{L}(t,\alpha) = \frac{\mathscr{L}(t,\alpha=0)}{\sqrt{1+\left[\alpha\sigma_\ell/(2\sigma_T)\right]^2}};$$

$$\langle n_{int}\rangle = \mathscr{L}\tau_{bc}\sigma_{TOT} \quad [\text{or } \sigma_{inel}];$$

$$\mathscr{L}_{INT} = \int \mathscr{L}(t)dt;$$

$$N_{TOT} = \mathscr{L}_{INT}\sigma_{TOT}; \quad N_x = \mathscr{L}_{INT}\sigma_x.$$



ii. Physics

Five parts:

a. Scattering: collisions in non-relativistic q.m., mainly the optical theorem and its consequences [*a memo*].

b. (Pseudo-)rapidity: kinematical variables used both at low- and high-Q^2 [*the math looks crazy, but it is very useful*].

c. Log s physics: a synonym of "low-Q^2 physics", i.e. when hadrons behave as coherent non-point-like particles [*an old subject, difficult, no clean results, but unavoidable, because it is the main source of events in hadronic physics*].

d. The quark parton model: the QCD theory and its approx., applied to the data [*the real subject of the discussion*].

e. High-p_T processes: the kinematical analysis of high-Q^2 events [*Mandelstam variables, x, $\sqrt{s}$ & c., both at parton and hadron level*].

NB. The sequence is dictated by understanding; (a-c-d-e-b) would have been more logical, but also more difficult.

scattering

- The electromagnetic processes, treated in § 2, are a special privileged case :
  - the potential is derived from a well-known and tested theory;
  - the model is based on symmetries;
  - the dimensionless coupling constant $\alpha_{em} << 1$.
- The treatment of **nuclear interactions** is much more complex :
  - there is no classical analogue;
  - the analytic form of the interaction is [was] unknown;
  - the coupling is much larger than in electromagnetism : the perturbative approach does not give results at small Q^2 (= large distances).
- Much experimental information comes from nuclear reactions and **scattering** processes. This study is therefore crucial.

- Examine the simplest case :
  - two particles;
  - spinless;
  - non-relativistic approximation;
  - potential only dependent from relative position.

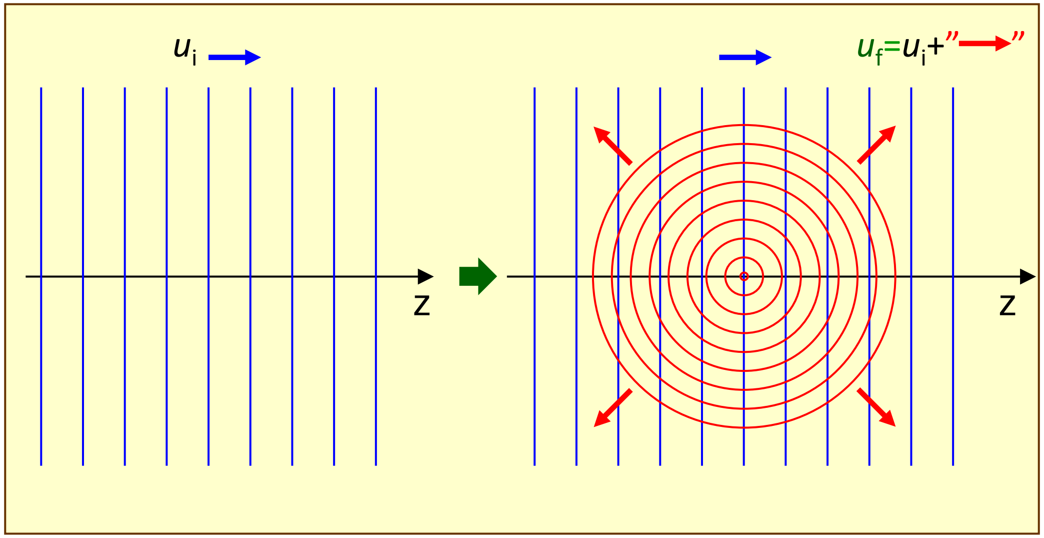
scattering: partial waves

References (many, but e.g.) :
- Sakurai, Modern q.m., 397;
- Weinberg, Lectures on q.m., 211;
- Burcham – Jobes, 286;
- Messiah, vol 2, 866;
- Perkins (ed. 1971), 265.

- Two particles, mass m_1 and m_2, both spin 0, collide with a potential V(x,y,z).
- The particles are observed far from the collision region, i.e. where $V \approx 0$.
- Define :

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}; \quad \vec{r} = \vec{r}_1 - \vec{r}_2;$$

$$M = m_1 + m_2; \qquad \mu = \frac{m_1 m_2}{m_1+m_2}.$$



- If $V(\vec{r})$ depends only on $\vec{r}$, i.e. on the relative positions of $m_{1,2}$, the Schrödinger equation splits in two parts :
  - a function $\psi_{CM}(\vec{R})$, for the free motion of the CM, which behaves as a free particle, with mass M and energy E_R;
  - a function $\psi(\vec{r})$, for the motion of a particle with reduced mass μ and energy E_r, subject to $V(\vec{r})$.

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2M}\nabla_R^2 - \frac{\hbar^2}{2\mu}\nabla_r^2 + V(\vec{r})\right]\Psi;$$

$$-\frac{\hbar^2}{2M}\nabla_R^2\psi_{CM}(\vec{R}) = E_R\psi_{CM}(\vec{R});$$

$$\left[-\frac{\hbar^2}{2\mu}\nabla_r^2 + V(\vec{r})\right]\psi(\vec{r}) = E_R\psi(\vec{r}).$$

scattering: partial waves

- The initial state is described by a plain wave along z :

$$\psi_i = e^{ikz} = \sum_\ell i^\ell \sqrt{4\pi(2\ell+1)}\; j_\ell(kr) Y_\ell^0(\theta) \xrightarrow{kr\to\infty}$$

$$\approx \frac{1}{kr}\sum_\ell i^\ell (2\ell+1) P_\ell(\cos\theta) \times \left(\frac{e^{i(kr-\ell\pi/2)} - e^{-i(kr-\ell\pi/2)}}{2i} \right).$$

exp(ikz) = mixture of different ℓ → expand [no φ → m=0 only].

$j_\ell(kr)$ = spherical Bessel functions, $P_\ell(\cos\theta)$ Legendre polynomials.

- … and the final state by the superposition of a plane and a spherical wave, modulated by f(θ,φ) :

$$\psi_f = e^{ikz} + \frac{e^{ikr}}{r} f(\theta,\varphi); \quad \psi_f^{scatt} = \psi_f - \psi_i = \frac{e^{ikr}}{r} f(\theta,\varphi);$$

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = \frac{\text{outgoing flux}@(\theta,\varphi)}{\text{ingoing flux}} = |f(\theta,\varphi)|^2;$$

definition of σ and flux.

$$\psi_f = \frac{1}{kr}\sum_\ell i^\ell \eta_\ell e^{2i\delta_\ell} (2\ell+1) P_\ell(\cos\theta) \times$$

$$\times \left(\frac{e^{i(kr-\ell\pi/2)} - e^{-i(kr-\ell\pi/2)}}{2i} \right);$$

complex factor, for each ℓ :
- $\Im(\ldots_\ell)$ = change in phase;
- $\Re(\ldots_\ell)$ = … in amplitude.

$$f(\theta,\varphi) = f(\theta) =$$

spin-0 particles + central potential = no dependence on φ.

$$= \frac{1}{2ik}\sum_\ell (\eta_\ell e^{2i\delta_\ell} - 1)(2\ell+1) P_\ell(\cos\theta);$$

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = |f(\theta)|^2 =$$

this σ refers to elastic scattering

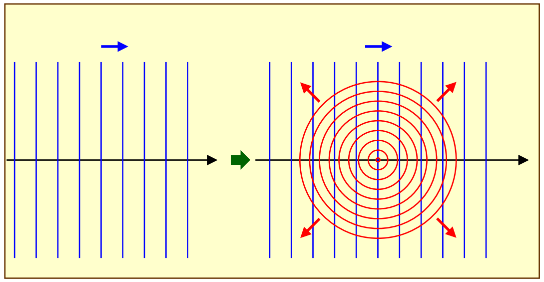
$$= \frac{1}{k^2}\left| \sum_\ell \left(\frac{\eta_\ell e^{2i\delta_\ell} - 1}{2i}\right)(2\ell+1) P_\ell(\cos\theta) \right|^2;$$

$$\sigma_{el} = \int \frac{d\sigma(\theta,\varphi)}{d\Omega} d\Omega = \int |f(\theta)|^2 d\Omega =$$

$$= \frac{\pi}{k^2}\sum_\ell (2\ell+1) \left| \eta_\ell e^{2i\delta_\ell} - 1 \right|^2.$$

$$\int d\Omega P_\ell(\cos\theta) P_{\ell'}(\cos\theta) = 4\pi\delta_{\ell\ell'}/(2\ell+1)$$

[δ = Kronecker symbol].



scattering: the optical theorem

- the *phase shifts* δ_ℓ pass through a resonance when $\delta_\ell = \pi/2$:
  - $\eta_\ell \exp(2i\delta_\ell)$; $0 \le \eta_\ell \le 1$;
  - only elastic scattering $\rightarrow \eta_\ell = 1 \rightarrow$

$$\sigma_{el}^{only} = \frac{4\pi}{k^2}\sum_\ell (2\ell+1)\sin^2\delta_\ell .$$

- Finally, calculating the flux associated with ψ_f, the value of σ_{tot} is :

$$\sigma_{tot} = \sigma_{el} + \sigma_{inel} = \frac{2\pi}{k^2}\sum_\ell (2\ell+1)\left[1-\eta_\ell\cos(2\delta_\ell)\right];$$

$$\Im\left[f_{el}(\theta=0)\right] = \Im\left[\frac{1}{2ik}\sum_\ell (2\ell+1)(\eta_\ell e^{2i\delta_\ell}-1)P_\ell(\cos\theta=1)\right] =$$

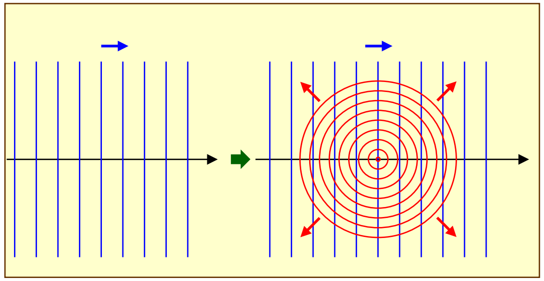
$$= \frac{-1}{2k}\sum_\ell (2\ell+1)\left[\eta_\ell\cos(2\delta_\ell)-1\right];$$

$$\sigma_{inel} = \int r^2 d\Omega\left(|\psi_i|^2 - |\psi_f^{scatt}|^2\right) = \frac{\pi}{k^2}\sum_\ell (2\ell+1)\left(1-\eta_\ell^2\right).$$

$$\boxed{\sigma_{tot} = \frac{4\pi}{k}\Im\left[f_{el}(\theta=0)\right].}$$

"**optical theorem**"
[Sellmeier, Rayleigh 1871; Bohr, Peierls, Placzek 1939; Bethe, de Hoffman 1955]

- [*warning : the theorem looks very smart; however, it is only a relation, based on wave mechanics, between two unknown quantities.*]
- The dynamics, carried by the potential $V(\vec{r})$, rests in **f(θ)** [**the scattering amplitude**], or, alternatively, in the **inelasticity parameters** $\boldsymbol{\eta_\ell}$ and in the **phase shifts** $\boldsymbol{\delta_\ell}$.



scattering: σ_{tot}

In hadron colliders, the standard method to measure the total cross section, e.g. at LHC **σ_{tot}(pp)**, uses the optical theorem:

a. $$\boxed{\sigma_{tot} = \frac{4\pi}{k}\Im[f_{el}(\theta=0)];}$$

$$\boxed{\theta=0\rightarrow\cos\theta=1\rightarrow t=0}$$

$$\Im[f_{el}(\theta=0)] \equiv \Im\left[f_{el}^{t=0}\right] = \frac{k\sigma_{tot}}{4\pi} \approx \frac{\sigma_{tot}\sqrt{s}}{8\pi}.$$

b. Define the elastic cross section in terms of $f_{el}(\theta)$ and t(Mandelstam):

$$\frac{d\sigma_{el}}{d\Omega} \equiv \frac{d^2\sigma_{el}}{d\varphi d\cos\theta} = \left|f_{el}(\theta)\right|^2;$$

$$\boxed{t=-\frac{s}{2}(1-\cos\theta)} \rightarrow \cos\theta = 1+\frac{2t}{s};$$

$$\frac{d\sigma_{el}}{dt} = \int d\varphi\left(\frac{d^2\sigma_{el}}{d\varphi d\cos\theta}\right)\left|\frac{\partial\cos\theta}{\partial t}\right| =$$

$$= 2\pi\left|f_{el}(\theta)\right|^2\frac{2}{s} = \frac{4\pi}{s}\left|f_{el}(s,t)\right|^2.$$

c. Define $\rho = \Re[f_{el}^{t=0}] / \Im[f_{el}^{t=0}]$ and put it in the equations :

$$\left|f_{el}^{t=0}\right|^2 = \left|\Re\left[f_{el}^{t=0}\right]\right|^2 + \left|\Im\left[f_{el}^{t=0}\right]\right|^2 =$$

$$= \left|\Im\left[f_{el}^{t=0}\right]\right|^2\left(1+\rho^2\right) = \frac{\sigma_{tot}^2 s}{64\pi^2}\left(1+\rho^2\right).$$

d. From the definition of the luminosity $\mathscr{L}$, for each process x, the rate is

$\sigma_x = R_x / \mathscr{L} \rightarrow \sigma_{el} = R_{el} / \mathscr{L}$; $\sigma_{tot} = R_{tot}/\mathscr{L}$;

$\rightarrow (\sigma_{tot})^2 = R_{tot}\sigma_{tot} / \mathscr{L}$.

e. Equating (b) = (c), and using (d) :

$$\left|f_{el}^{t=0}\right|^2 = \frac{\cancel{s}}{4\pi}\left.\frac{d\sigma_{el}}{dt}\right|_{t=0} = \frac{R_{tot}\sigma_{tot}\cancel{s}}{64\pi^2\mathscr{L}}\left(1+\rho^2\right).$$

f. The final equation is :

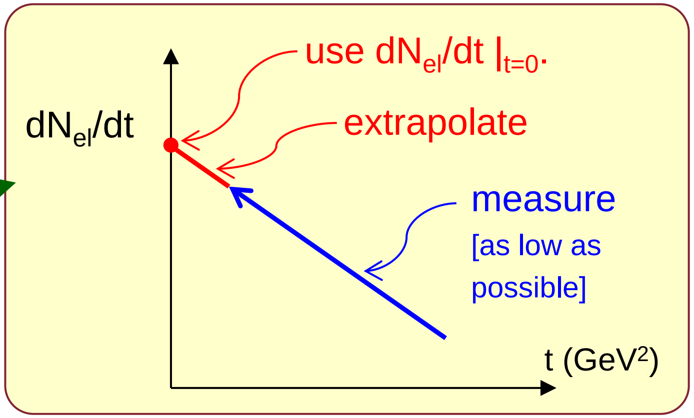
$$\boxed{\sigma_{tot} = \frac{16\pi(\hbar c)^2}{1+\rho^2}\frac{1}{R_{tot}}\left.\frac{dR_{el}}{dt}\right|_{t=0}.}$$

scattering: measure σ_{tot}

$$\sigma_{tot} = \frac{4\pi}{k}\Im\left[f_{el}(\theta=0)\right] = \frac{16\pi(\hbar c)^2}{1+\rho^2}\frac{1}{R_{tot}}\left.\frac{dR_{el}}{dt}\right|_{t=0}.$$

Since everything (but ρ) is directly measurable, σ_{tot} can be measured:

- R_{el} and R_{tot} :
  - ➢ absolute rates in arbitrary units (only the ratio counts, i.e. use N_{el} and N_{tot}, integrated over the same time interval $\rightarrow$ smaller stat. errors);
  - ➢ systematics due to dead time, faults in data-taking, … cancels in the ratio;
- the term "$dR_{el}/dt\,|_{t=0}$" :
  - ➢ produce a plot R_{el} (or N_{el}) vs $t_{Mandelstam}$;
  - ➢ N(t=0) is non-measurable $\rightarrow$ go as low as possible in t and extrapolate $\rightarrow$ t=0;
  - ➢ units do NOT count, but extrapolation errors do;



  - ➢ the histogram requires t $\rightarrow$ must know $\vec{p}_{init}$ $\rightarrow$ high-β is preferable, even if $\mathcal{L}$ (and N) are smaller;
- the ratio ρ [*a personal pessimistic view*] :
  - ➢ can be computed [*maybe "guessed"*] from first principles;
  - ➢ turns out small ($\approx$ 0.14 @ LHC) $\rightarrow$

$$\Delta\sigma/\sigma \approx 2\rho\Delta\rho \leq 1\%;$$

  - ➢ so ρ [*is not well-understood, but it*] <u>does not harm</u> the result.

scattering: $\mathbb{S}$ matrix

The $\mathbb{S}$ matrix ($\mathbb{S}$ for "scattering") was introduced indipendently by J.Wheeler in 1937 and W.Heisenberg in 1940.

The following definitions and properties are discussed in [MQR § 11] in the Interaction Picture ("IP", $|\rangle_I$) :

- $\lim_{t\to\pm\infty} \mathbb{H}_I(t) = 0$;
- $\lim_{t\to\pm\infty} |\psi(t)\rangle_I \equiv |\psi(t=\pm\infty)\rangle_I = \text{const.}$;
- $|\psi(t)\rangle_I = \mathbb{U}_I(t,t_0)|\psi(t_0)\rangle_I$;
- $|\, i\,\rangle \equiv |\psi(t=-\infty)\rangle_I$;
- $|\, f\,\rangle \equiv |\psi(t=+\infty)\,\rangle_I \equiv \mathbb{S}|\, i\,\rangle$;
- $\mathbb{S} \equiv \lim_{t_2\to+\infty, t_1=-\infty} \mathbb{U}_I(t_2,t_1)$;
- $\mathbb{S}\,\mathbb{S}^\dagger = \mathbb{S}^\dagger\,\mathbb{S} = \mathbb{1}$.

The following properties follow :

- $\mathcal{S}_{fi} \equiv \langle\, f\,|\,\mathbb{S}\,|\, i\,\rangle$;
- $\Sigma_f |\mathcal{S}_{fi}|^2 = 1$ [conservation of probability];
- $\mathbb{S} \equiv \mathbb{1} + 2i\mathbb{T}$;
- $\mathbb{T} = (\mathbb{S} - \mathbb{1}) / (2i)$;
- $\langle f|\mathbb{S}|i\rangle = \delta_{fi} + i(2\pi)^4\delta^4(p_f\text{-}p_i)\langle f|\mathbb{T}|i\rangle$;
- $d\sigma = \frac{1}{v}\frac{d\vec{p}_f}{(2\pi)^3}|\mathfrak{M}_{fi}|^2\, 2\pi\delta(E_f - E_i)$.

It is interesting to note that, starting from there, the optical theorem follows (almost) immediately :

- $\sigma_T = -2\,\Re[\mathfrak{M}_{ii}] / v_i = 4\pi\,\Im[f(0,\varphi)] / p_I$.

The analytical properties of the $\mathbb{S}$ matrix have been extensively studied in the '50s and '60s. After that, the success of the field theory and the SM have terminated the approach, even if some addicts are still around.

(pseudo-)rapidity

- The *rapidity* ϕ was introduced by Minkowski (NOT in particle physics):

  $\phi = \tanh^{-1}(v/c)$,

  many properties : i.e. it reduces to v/c for low speed, it is additive (unlike v), ….

- In particle physics a similar variable (y) defined by Feynman for a particle $m \neq 0$, relative to an axis z (usually the beam) :

$$\boxed{y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z};}$$

- define also :
  - $m_T^2 = m^2 + p_x^2 + p_y^2$ (transverse mass);
  - $\eta = -\ln[\tan(\theta/2)]$ (pseudo-rapidity);
  - $x = 2\, p_z / \sqrt{s}$ ("Feynman x");

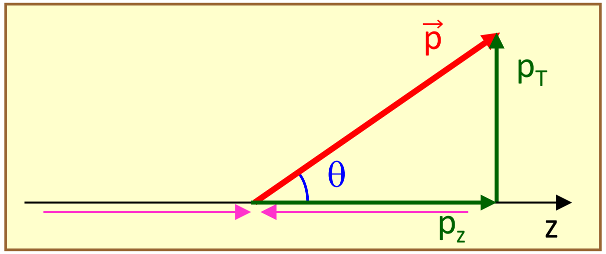
Use $p = [E, p_x, p_y, p_z; m]$; other variables will be defined. [*Unfortunately, with only 26 letters available, there is a lot of repetition, e.g. the rapidity y has nothing to do with the inelasticity y.*]

It follows (next slides) :

- $p_z \to -p_z \Rightarrow \theta \to (180°-\theta) \Rightarrow y \to -y$;
- $E = m_T \cosh(y)$; $p_z = m_T \sinh(y)$;
- $y = \ln[(E+p_z) / m_T] = \tanh^{-1}(p_z/E)$;
- $dy = dp_z / E$;
- if $(p \gg m) \to y \approx \eta$.
- given a Lorentz transformation $\mathbb{L}$ along z, with velocity β_z :

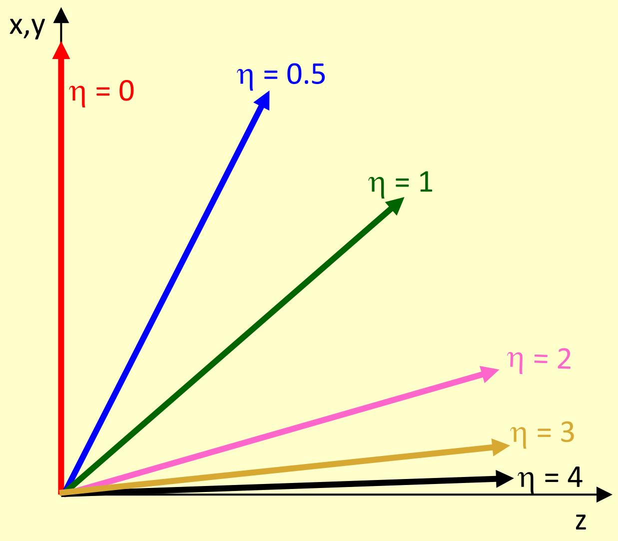
  $y' = \mathbb{L}(y) = y - \tanh^{-1}\beta_z$; $\Delta y' = \Delta y$;

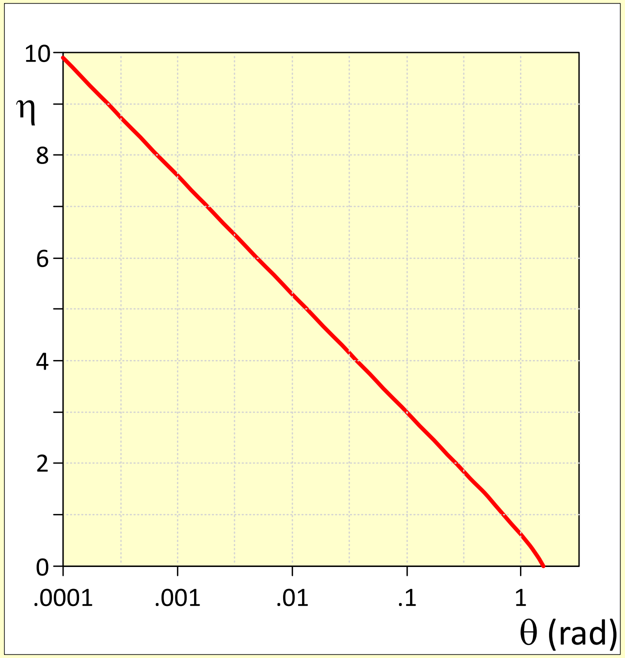
i.e. y is the variable, whose differential dy is invariant for $\mathbb{L}$-transformations along z.



(pseudo-)rapidity: plot

- The pseudorapidity η is important.
- Sometimes physicists assume to be in the extreme relativistic case, and call it "rapidity".
- Roughly, it represents the zenith θ, with a scale much expanded towards the beam axis.
- But its properties are many, and …





$\theta = 2\tan^{-1}(e^{-\eta})$

η	θ (°)
0.0	90.000
0.5	62.476
1.0	40.395
1.5	25.157
2.0	15.415
2.5	9.385
3.0	5.700
3.5	3.459
4.0	2.099
4.5	1.273
5.0	0.772
5.5	0.468

For small θ (large η) : $\eta\ [\approx y] \quad = -\ln\,[\tan\,(\theta/2)] \rightarrow$

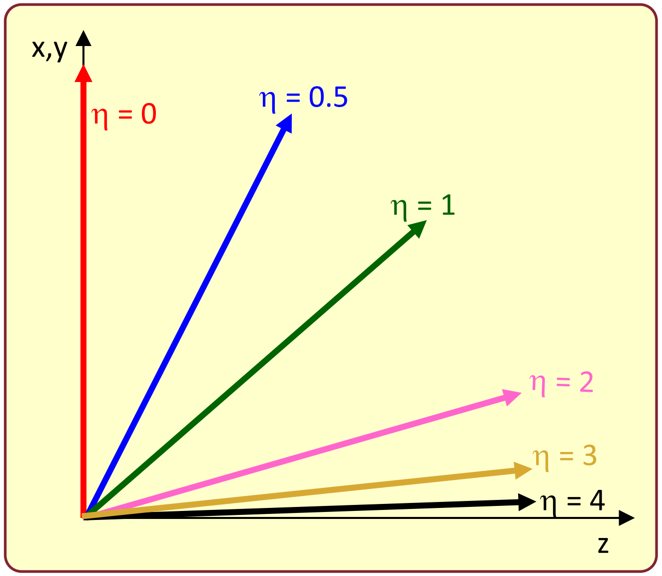
$$\approx \ln(2) - \ln[\theta(\text{rad})] = \ln(360/\pi) - \ln[\theta(\text{deg})] = 4.741 - \ln[\theta(\text{deg})].$$

(pseudo-)rapidity: properties (1)

Simple computations:

$$a)\ y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) \xrightarrow{p>>m} \frac{1}{2}\ln\left(\frac{p+p_z}{p-p_z}\right) =$$

$$= \frac{1}{2}\ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right) = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right] = \eta;$$



$$b)\ y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) = \frac{1}{2}\ln\left[\frac{(E+p_z)^2}{E^2-p_z^2}\right] = \ln\left(\frac{E+p_z}{m_T}\right) =$$

$$= \frac{1}{2}\ln\left[\frac{E^2-p_z^2}{(E-p_z)^2}\right] = \ln\left(\frac{m_T}{E-p_z}\right);$$

$$c)\ E+p_z = m_T e^{y}; \qquad E-p_z = m_T e^{-y};$$

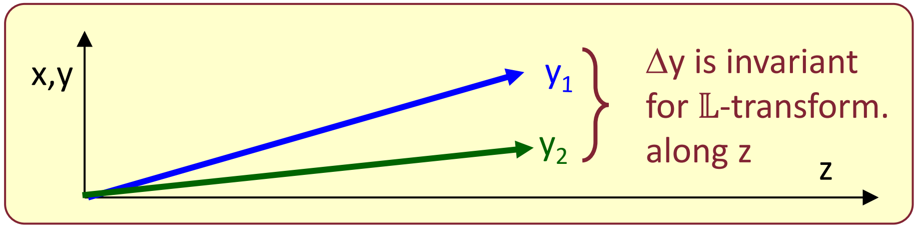
$$E = m_T\frac{e^{y}+e^{-y}}{2} = m_T\cosh(y);$$

$$p_z = m_T\frac{e^{y}-e^{-y}}{2} = m_T\sinh(y);$$

$$\rightarrow\ y = \tanh^{-1}\left(\frac{p_z}{E}\right).$$

(pseudo-)rapidity: properties (2)

… And some others, quite long :



a) $\mathbb{L}$ transform : $p'_z = \gamma(p_z - \beta E);$
$E' = \gamma(E - \beta p_z);$

b) $$y' = \quad (y) = \frac{1}{2}\ln\left(\frac{E'+p'_z}{E'-p'_z}\right) =$$

$$= \frac{1}{2}\ln\left(\frac{\gamma E - \beta\gamma p_z + \gamma p_z - \beta\gamma E}{\gamma E - \beta\gamma p_z - \gamma p_z + \beta\gamma E}\right) =$$

$$= \frac{1}{2}\ln\left[\frac{E(1-\beta) + p_z(1-\beta)}{E(1+\beta) - p_z(1+\beta)}\right] =$$

$$= \frac{1}{2}\ln\left[\frac{(1-\beta)(E+p_z)}{(1+\beta)(E-p_z)}\right] = \frac{1}{2}\ln\left[\frac{(1-\beta)}{(1+\beta)}\right] + \frac{1}{2}\ln\left[\frac{(E+p_z)}{(E-p_z)}\right] =$$

$$= y + \tanh^{-1}(\beta).$$

c) $\Delta y = y_2 - y_1 = \Delta y' = y'_2 - y'_1;$

i.e. y is the variable, whose differential (even the finite Δy) is invariant for $\mathbb{L}$-transf. along z.

(pseudo-)rapidity: properties (3)

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

- Start from well-known math : $E = \sqrt{p_z^2 + p_T^2 + m^2}$;

$$dE = \frac{\partial E}{\partial p_z} dp_z = \frac{p_z dp_z}{E} \rightarrow \frac{dp_z}{E} = \frac{dE}{p_z}.$$

- Then :

$$dy = \frac{\partial y}{\partial p_z} dp_z + \frac{\partial y}{\partial E} dE = \frac{1}{2}\left(\frac{E-p_z}{E+p_z}\right)\left[\left(\frac{1}{E-p_z} + \frac{E+p_z}{(E-p_z)^2}\right)dp_z + \left(\frac{1}{E-p_z} - \frac{E+p_z}{(E-p_z)^2}\right)dE\right] =$$

$$= \frac{1}{2}\left(\frac{E-p_z}{E+p_z}\right)\left[\left(\frac{E-p_z+E+p_z}{(E-p_z)^2}\right)dp_z + \left(\frac{E-p_z-E-p_z}{(E-p_z)^2}\right)\frac{p_z dp_z}{E}\right] =$$

$$= \frac{1}{2}\left(\frac{dp_z}{E^2-p_z^2}\right)\left[2E - \frac{2p_z}{E}p_z\right] = \frac{1}{2}\left(\frac{dp_z}{E^2-p_z^2}\right)2\left(\frac{E^2-p_z^2}{E}\right) = \frac{dp_z}{E} = \frac{dE}{p_z}.$$

- i.e. the differential $dy = dp_z / E = dE / p_z$ at constant p_T.

- Definition of the **invariant cross section** ["invariant" under $\mathbb{L}$-transform. along z] :

$$\frac{E d^3\sigma}{dp_x dp_y dp_z} = \frac{d^3\sigma}{p_T dp_T d\varphi dy}\left[= \frac{1}{\pi}\frac{d^2\sigma}{dp_T^2 dy}\right] = \frac{E' d^3\sigma}{dp'_x dp'_y dp'_z}.$$

(pseudo-)rapidity: properties (4)

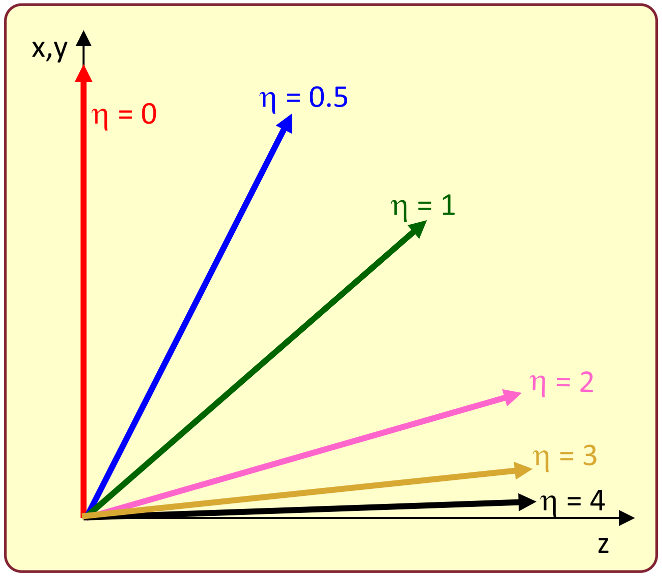
- [curiosity : an alternative way to show that y is invariant for $\mathbb{L}$-transf. along z :

$$\begin{cases} p'_z = \gamma(p_z - \beta E); \\ E' = \gamma(E - \beta p_z); \end{cases}$$

$$dp'_z = \frac{\partial p'_z}{\partial p_z} dp_z + \frac{\partial p'_z}{\partial E} dE = \gamma dp_z - \beta\gamma dE = \gamma dp_z - \beta\gamma \frac{p_z dp_z}{E} =$$

$$= \gamma dp_z \left(1 - \frac{\beta p_z}{E}\right) = \frac{\gamma dp_z}{E}(E - \beta p_z);$$

$$\text{i.e. } \frac{dp'_z}{E'} = dy' = \frac{dp_z}{E} = dy].$$

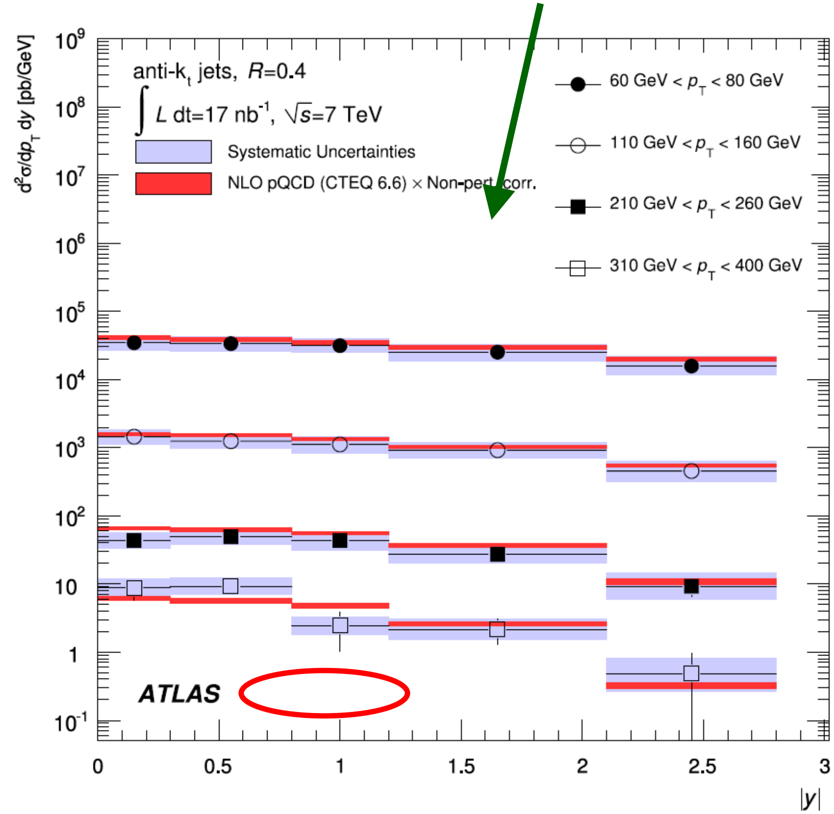


(pseudo-)rapidity: why

Why are hadronic interactions often analyzed in terms of (pseudo-)rapidity ?

Angular variables depend on each other : jacobian transformations relate all distributions; however, y looks "natural" (and produces simpler plots).

- The "Feynman argument" :
  - at high-p_T the real interaction happens at parton level;
  - the values of the parton momenta vary for each event, but they are (in 1st approx) along z;
  - therefore y is the correct variable in the lab., e.g. for jets and IVB analysis.
- The "Rutherford argument" :
  - in the parton CM, the scattering is dominated by t-channel processes;
  - the dominant processes are NOT flat in y, but $\propto t^{-2}$;
  - σ is a mixture of processes, with many t-dependences, indistinguishable on an event-by-event basis;
  - the rapidity, which expands the scale at $\theta \approx 0°$ is welcome : dσ/dy is ~ flat.



(pseudo-)rapidity: how

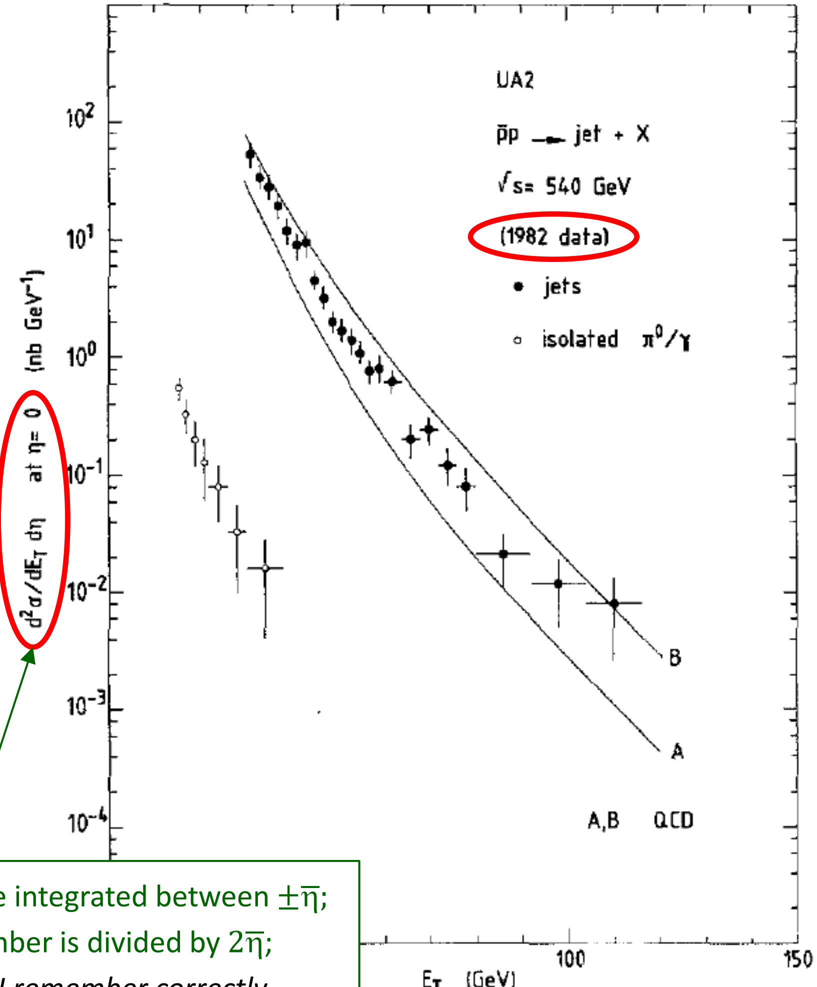
Why are soft hadronic interactions often analyzed in terms of (pseudo-)rapidity ?

The phenomenology of low-p_T :
- [*maybe* reasons based on low-p_T physics, related to the invariant cross-section];
- the inclusive y distributions are ~ flat;
- so, y is very handy for fast background computations.

Why is η used often, instead of y ?
- y has important physical properties;
- y is difficult to measure, since is a small difference of two large quantities (E, p_z);
- η depends on an angle, exper. friendly;
- worst : in the literature sometimes η is given the properties of y [*but it is ALMOST correct*].

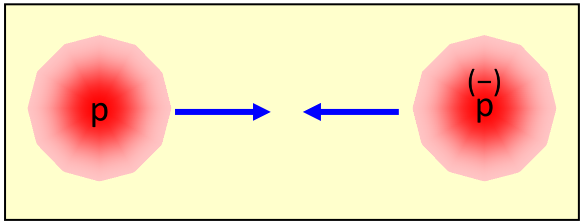
Instead, e^+e^- interactions, where partons ($=e^\pm$) interact in the LAB at x=1, are usually analyzed in terms of cos θ.

How to do it ? "typical example" : a hard interaction studied in terms of $d^2\sigma/dp_Td\eta|_{\eta=0}$.



it means : jets are integrated between $\pm\bar{\eta}$;
the resultant number is divided by $2\bar{\eta}$;
we used $\bar{\eta}$ = 1, *if I remember correctly*.

Log s physics



- An intuitive toy-model, with surprisingly good results :

  $\sigma_{tot}(pp \text{ } or \text{ } \bar{p}p) \approx \pi R^2 \approx \pi\ (\hbar c/m_\pi)^2 =$
  $= \pi(197 \text{ MeV}\cdot\text{fm} / 140 \text{ MeV})^2 = 62 \text{ mb}.$

- A limit ("Froissart bound") on the increase of cross-section for any pairs of particles, when $\sqrt{s}$ increases :

  for any two particles (ab) [e.g. pp, p̄p] :

  $$\lim_{s\to\infty} \sigma_{ab} \leq \text{const} \times \left(\ln^2 s\right),$$

  i.e."*at sufficiently high energies, the* *total cross-section* *for scattering on a given target* [*e.g* . $\sigma(\bar{p}p)$, $\sigma(pp)$, $\sigma(\pi^\pm p)$, $\sigma(\pi^\pm n)$] *cannot grow faster than* $\ln^2 s$".

- A theorem, based on quantum field theory (NOT on dynamical assumptions, i.e. valid for any type of interaction), knows as the "Pomeranchuk theorem" :

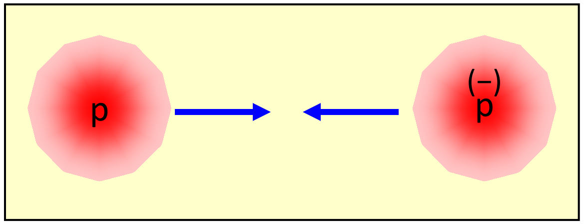
  $$\lim_{s\to\infty} \left(\frac{\sigma_{ab}}{\sigma_{\bar{a}b}}\right) = 1, \text{ for any two particles (a,b).}$$

  i.e. "*at sufficiently high energies, the* *total cross-section* *on a given target is the same for particle and antiparticle*" [*e.g.* $\sigma(\bar{p}p) \approx \sigma(pp)$, $\sigma(\pi^+ n) \approx \sigma(\pi^- n)$].

- The (unexpected) experimental behavior that indeed *hadron cross-sections grow with* $\sqrt{s}$, [$\propto \ln(s)$ or maybe $\propto \ln^2(s)$], and that the "Pomeranchuk regime" is reached at accelerator energies.

↓

Log s physics: comments



- … gave rise (50 years ago) to much excitement and phenomenological models of low p_T hadronic interactions ("Regge poles", "Pomeron", "cylindrical phase space", ...).

- Then, no real breakthrough for many years …

there are books with an extensive treatment of the subject; instead we summarize everything here.

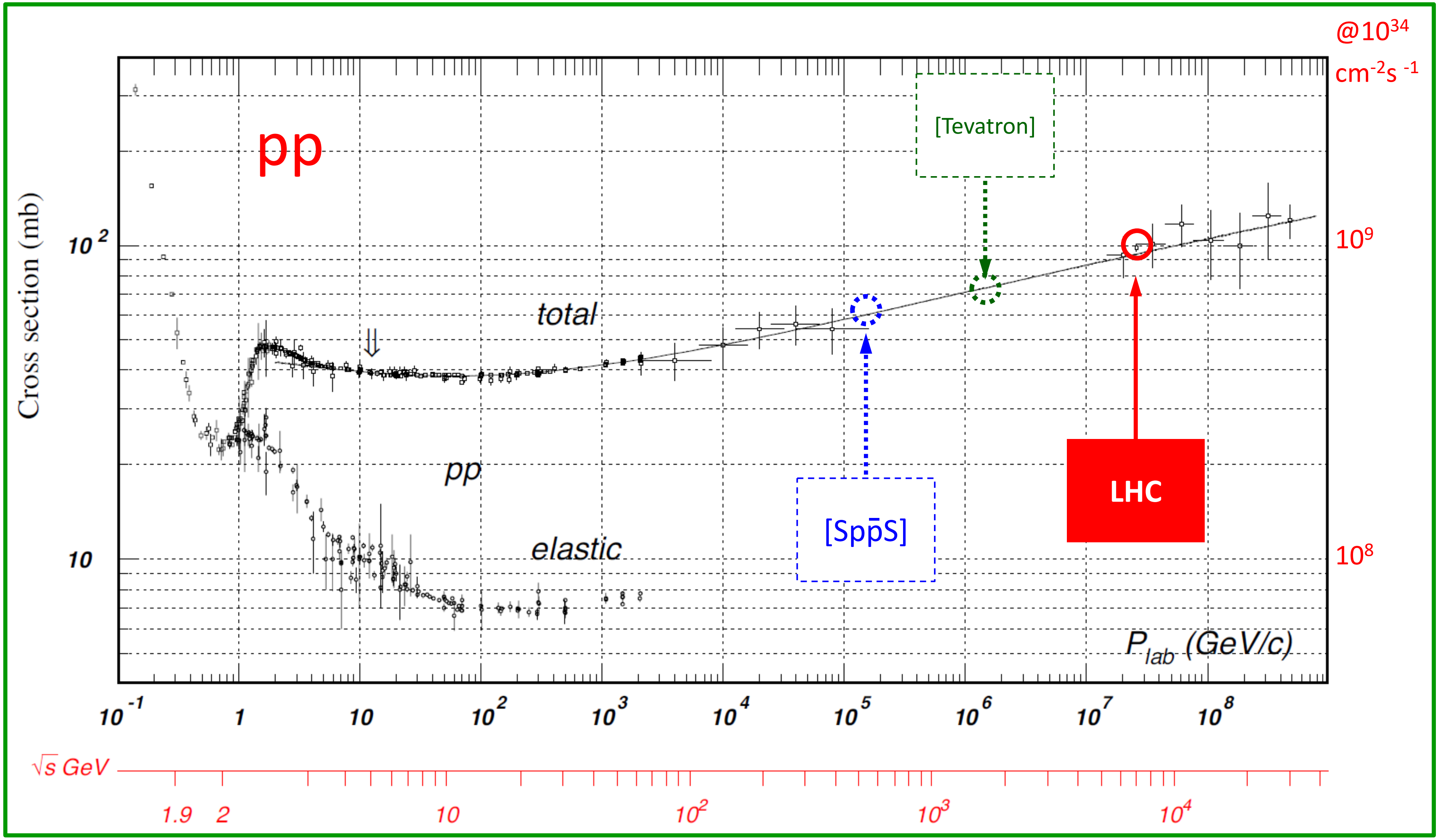
Comments (*very personal*) :

- physics born many years ago ('50s + CERN ISR), before the advent of QCD;
- poor conceptual foundations, but many phenomenological successes;
- many mysteries remain (perhaps no mystery, only complex many-body interactions, e.g. chemistry);
- today the main motivation of the study is to predict, parameterize and filter out the background.

In the following, we will assume this attitude.

The funny name "Log s physics" comes from the fact that, in low-p_T processes, the evolution with s of many quantities is logarithmic; the reasons are not really understood (Froissart ?).

Log s physics: $\sigma_{tot}(pp)$

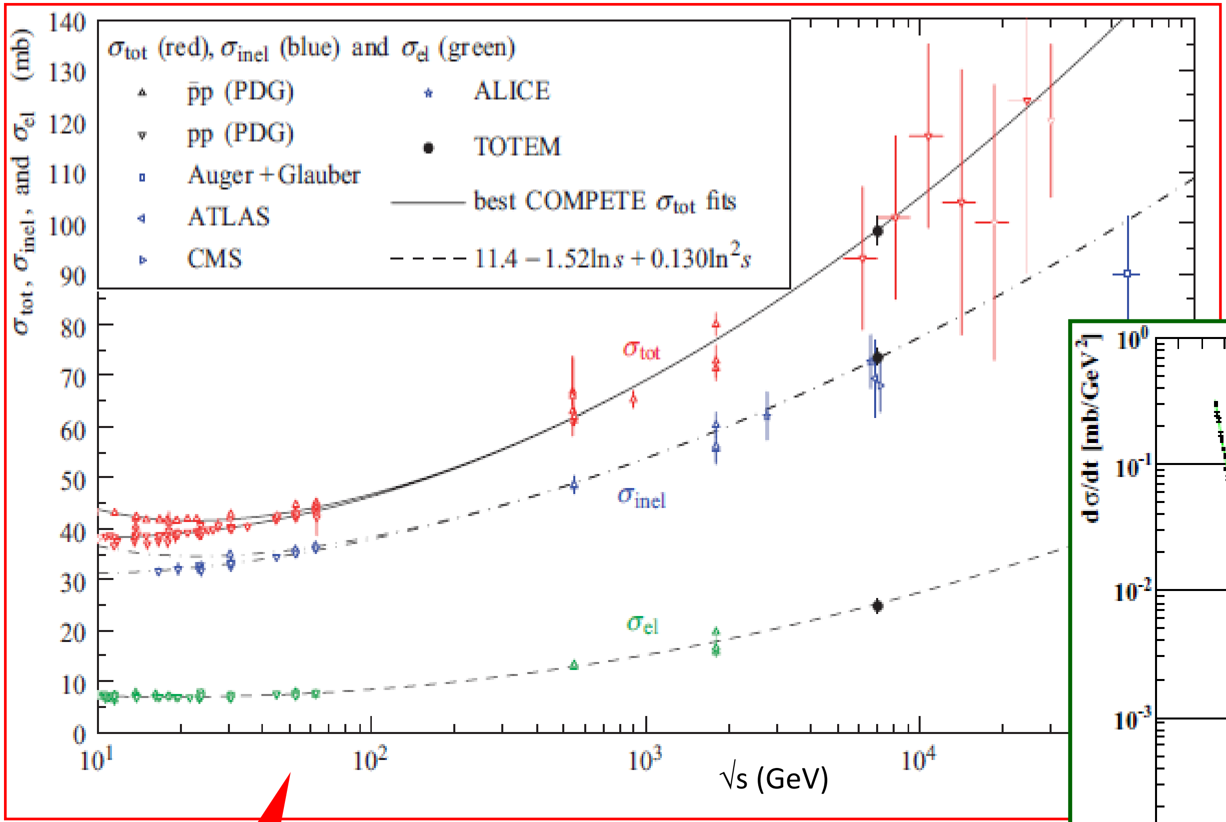


1 b = 10^{-28} m^2 = 10^{-24} cm^2

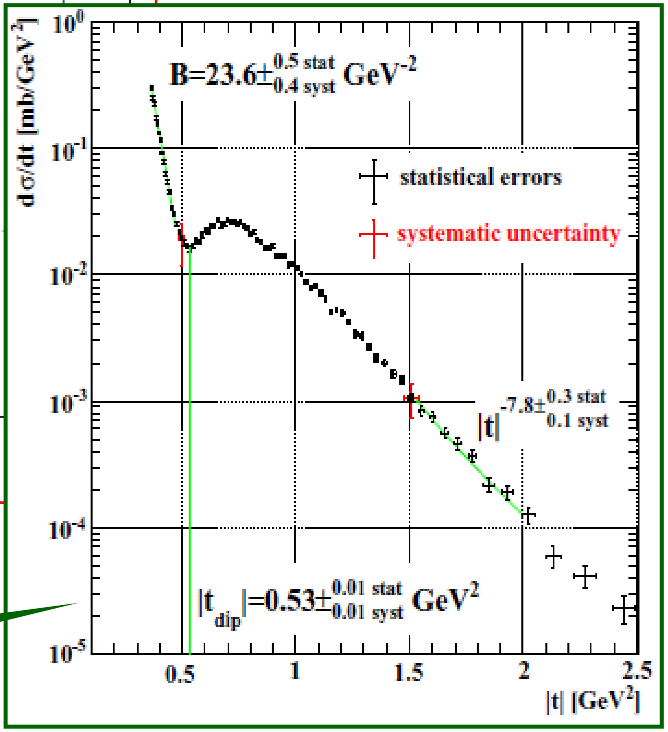
1 mb = 10^{-31} m^2 = 10^{-27} cm^2

The data of $\sigma(\bar{p}p)$, i.e. Sp$\bar{p}$S and Tevatron, are dashed, to show the similarity of the cross sections.

Log s physics: σ_{tot}, $d\sigma_{el}/dt$ @ LHC



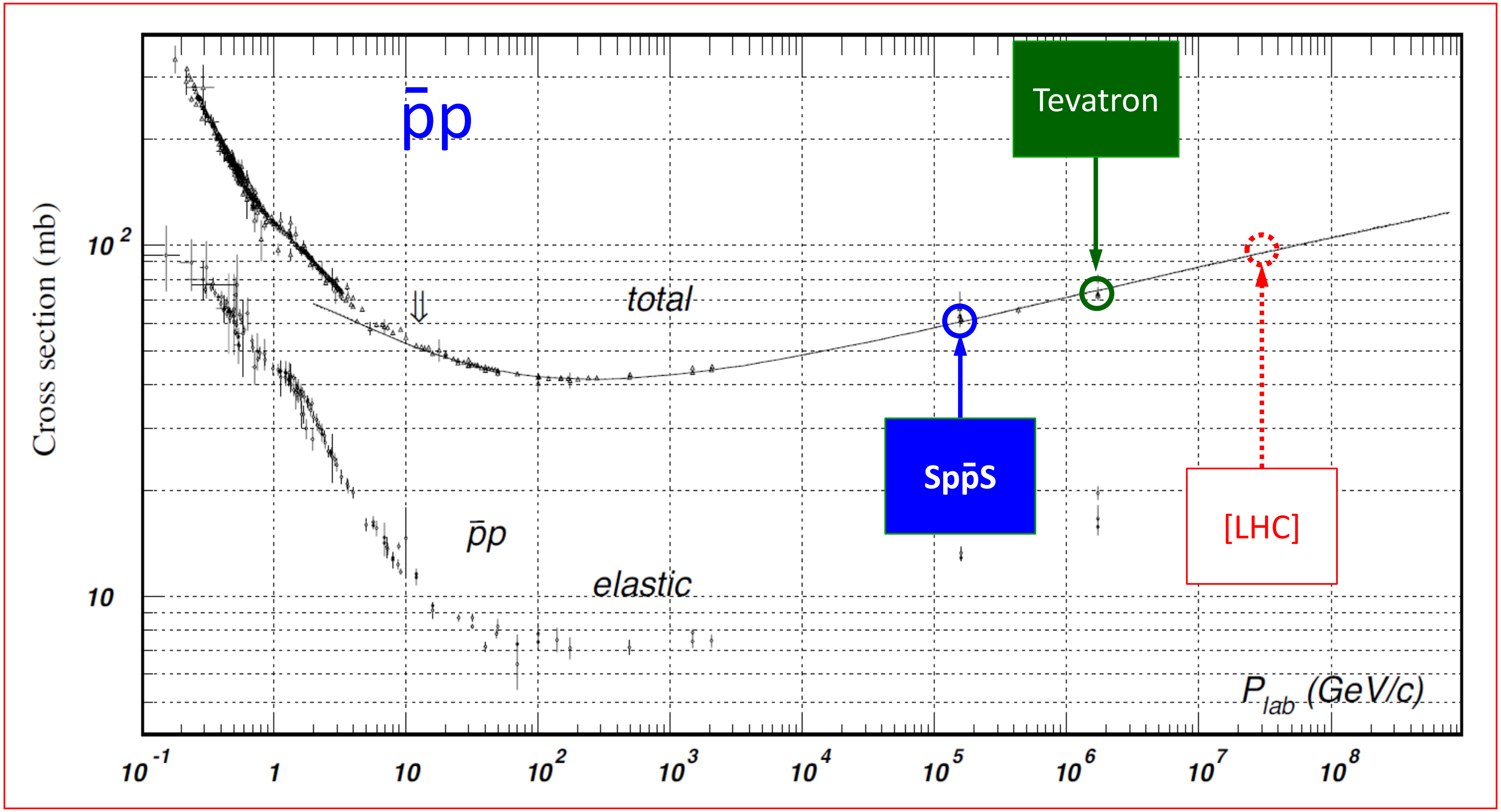
TOTEM (LHC)
EPL, 96 (2011) 21002



σ_T, σ_{anel}, σ_{el}

$d\sigma_{el}/dt$

Log s physics: $\sigma_{tot}(\bar{p}p)$



The data of σ(pp), i.e. LHC, do NOT belong to this plot; they are plotted dashed, to show the similarity of the cross sections ("Pomeranchuk theorem").

Log s physics: "rapidity plateau"

A heuristic computation :

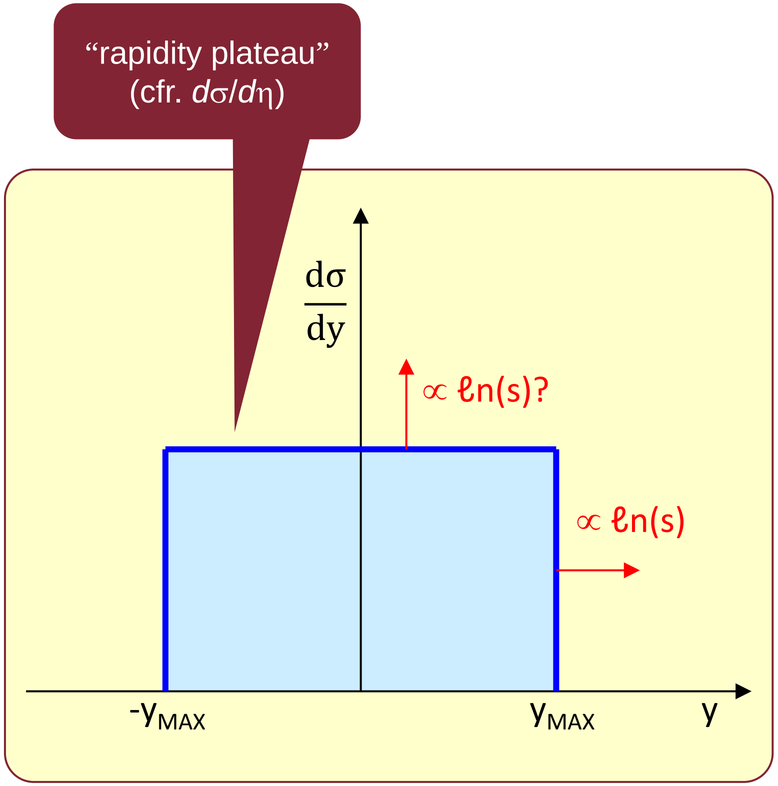
- Compute the limits on y :

$$y = \ln\left(\frac{E+p_z}{m_T}\right) \leq \ln\left(\frac{\sqrt{s}}{m_T}\right) \leq \frac{1}{2}\ln\left(\frac{s}{m^2}\right) \equiv y_{MAX};$$

- i.e. y_{max} increases $\propto \ln(s)$;
- if there is a "rapidity plateau", the total cross section is represented by the area of the rectangle :

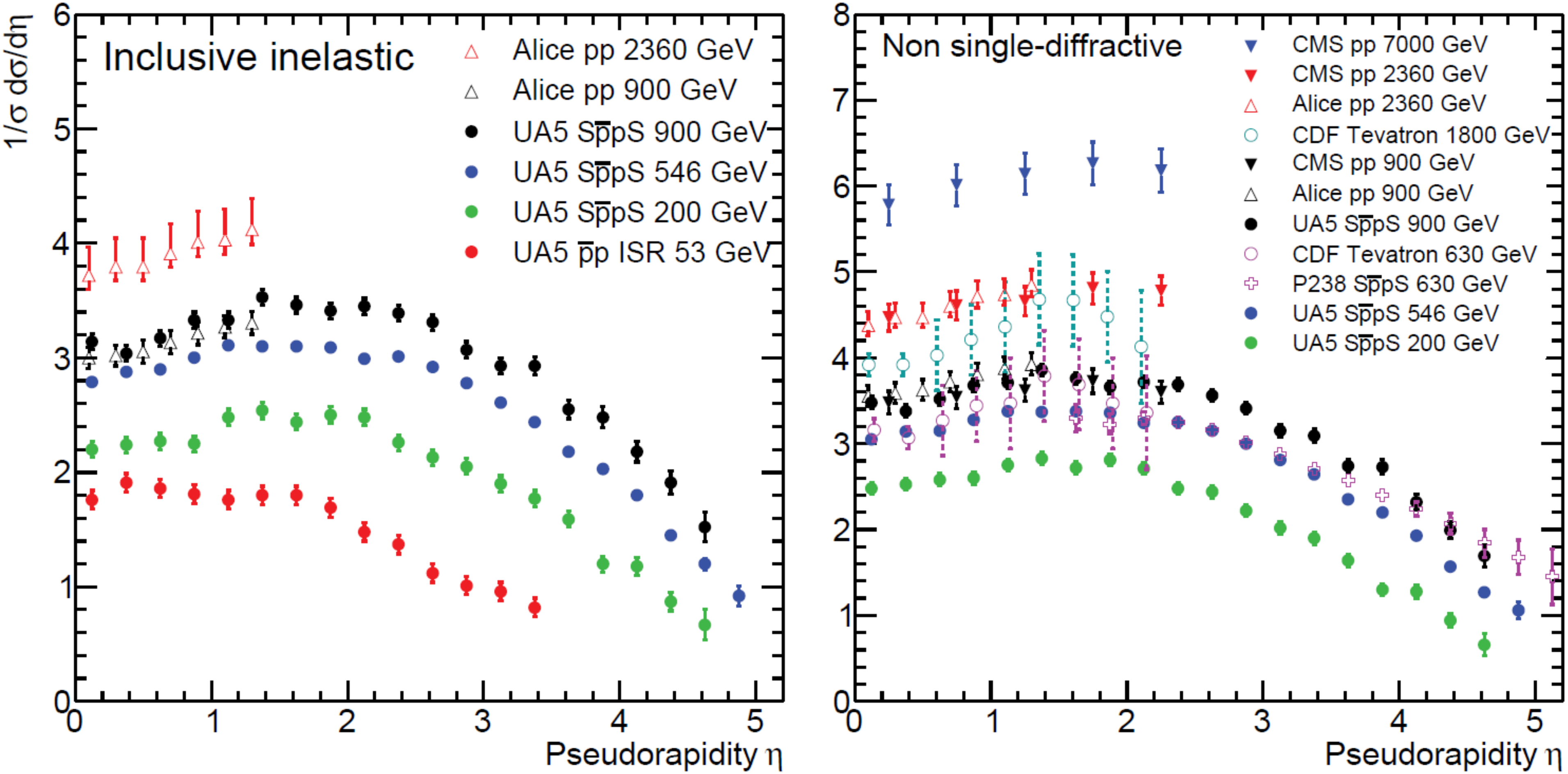
$$\sigma_{tot} = \int_{-y_{MAX}}^{-y_{MAX}} \left(\frac{d\sigma}{dy}\right) dy \approx \text{const} \times \left(\frac{d\sigma}{dy}\right) \times \ln(s);$$

- if the plateau grows $\propto$ ln s, then $\sigma_{tot} \propto \ln^2 s$, and "saturates" the Froissart bound;
- actually, this seems to be the case : both width and height of the rectangle grow $\propto$ ln s.



The real question is : why $d\sigma/dy \propto \ln s$?

Log s physics: $d\sigma/d\eta|_{particles}$

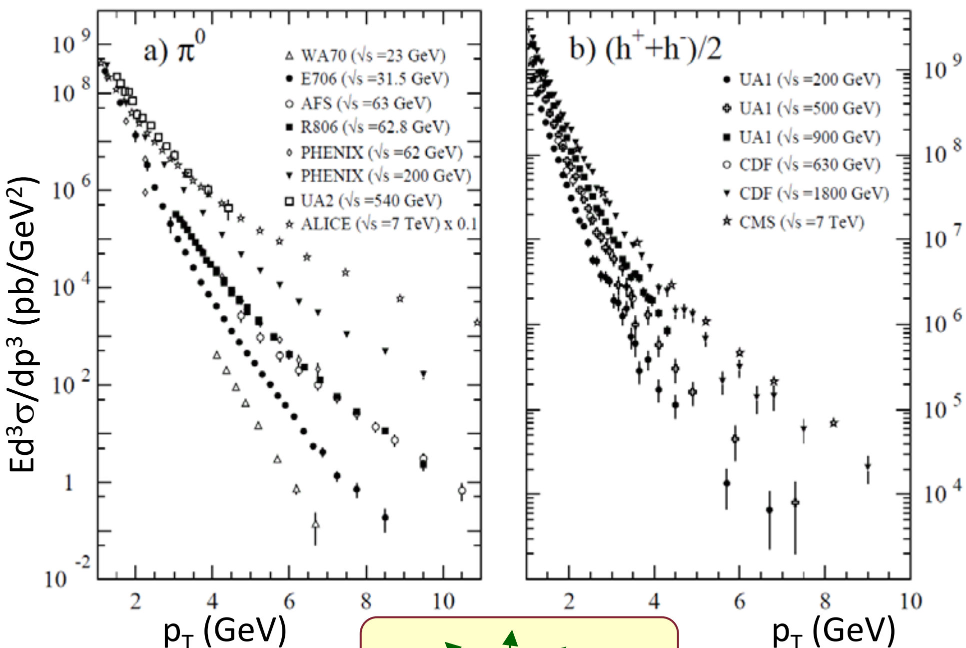


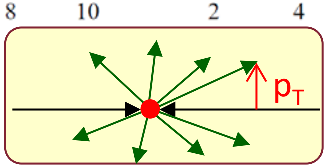
The η distributions of charged particles exhibit typical "rapidity plateaus", which increases ∝ log s.

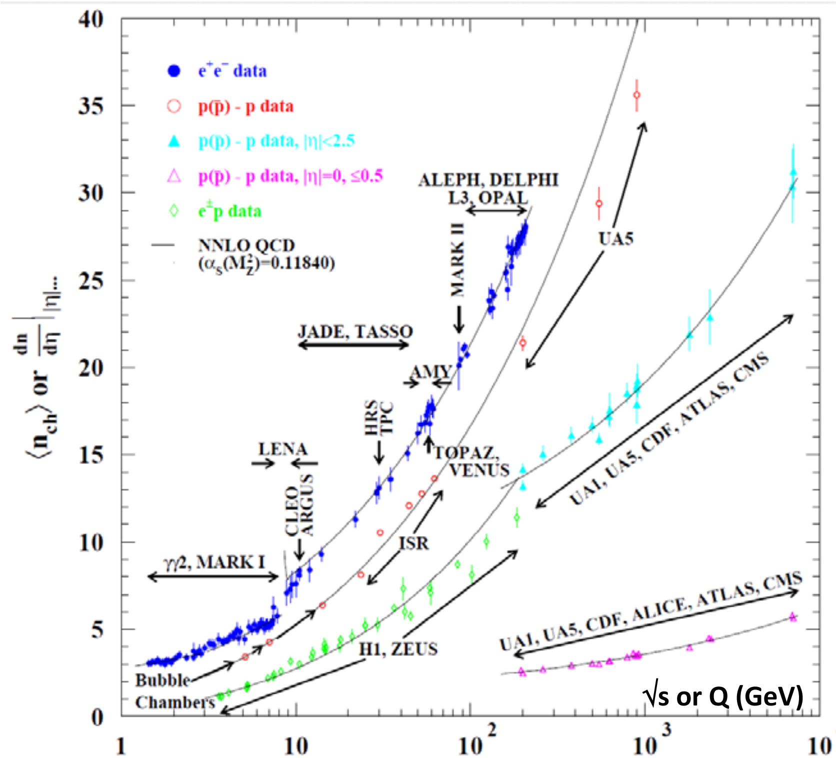
Log s physics: inclusive data

The number and p_T distribution of the charged particles of the final state exhibits interesting properties :

- they seem to follow a general law;
- the law is independent from the primary state (e^+e^-, pp, $\bar{p}p$, $e^\pm p$);
- it scales (approx) $\propto \ln s$ or $\propto \ln^2 s$.







Suggestion of a general "factorization property" of single particle production at low-p_T ["Feynman scaling"] :

$$\frac{Ed^3\sigma}{p_T dp_T dy} = f(s, p_T, y) \approx f_s(s) f_{p_T}(p_T) f_y(y);$$

In turn, the single f_i exhibits interesting properties (like the log-dependence of f_s).

The quark parton model

Hadronic collisions at high p_T (= short distance) are studied in terms of the "**quark-parton model**" (*) :

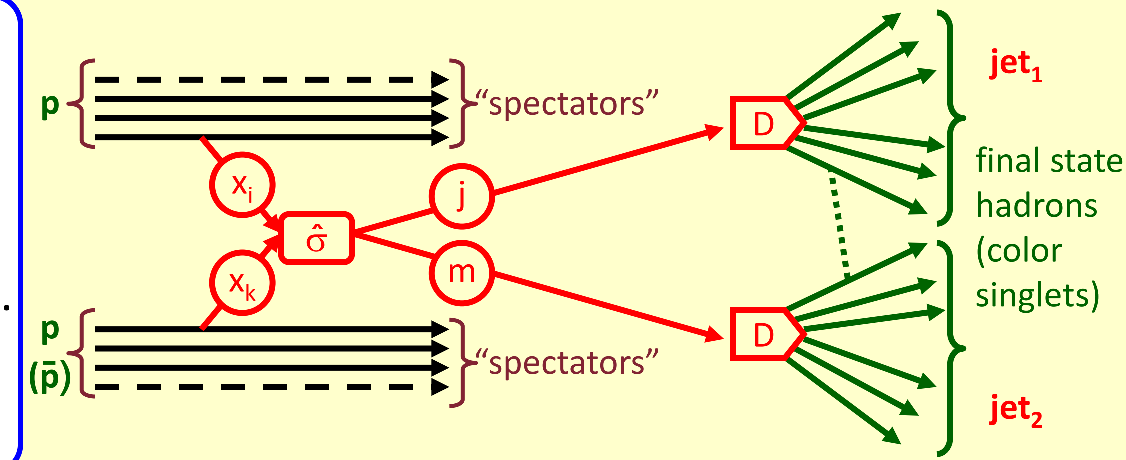
- the process take place in phases, that "**factorize**" (= take place one after the other, without mutual interference);
- the hadrons of the initial state are an **incoherent mixture** of elementary partons (= quarks and gluons of QCD);
- the partons behave as **point-like** particles **quasi- free** (like the electrons in e^+e^-);
- because of the sea contribution, the "**number**" of partons in a hadron is **not defined**; only their total momentum (= the hadron momentum) is measurable.

(… continue …)

(*) hadronic collisions at low p_T (= great distance, Q^2<[few-GeV]^2) correspond to interactions between non-point-like hadrons; they do NOT belong to this picture.

$$\sigma(\bar{p}p\,[pp] \to \mathrm{jet}_1\mathrm{jet}_2) =$$

$$= \sum_{i,k} \int \left[\begin{array}{l} dx_i dx_k F_i^p(x_i, Q^2) F_k^{\bar{p}}(x_k, Q^2) \\ \hat{\sigma}\left(ik \to jm\ @\sqrt{\hat{s}} = \sqrt{s x_i x_k}\right) \end{array} \right].$$



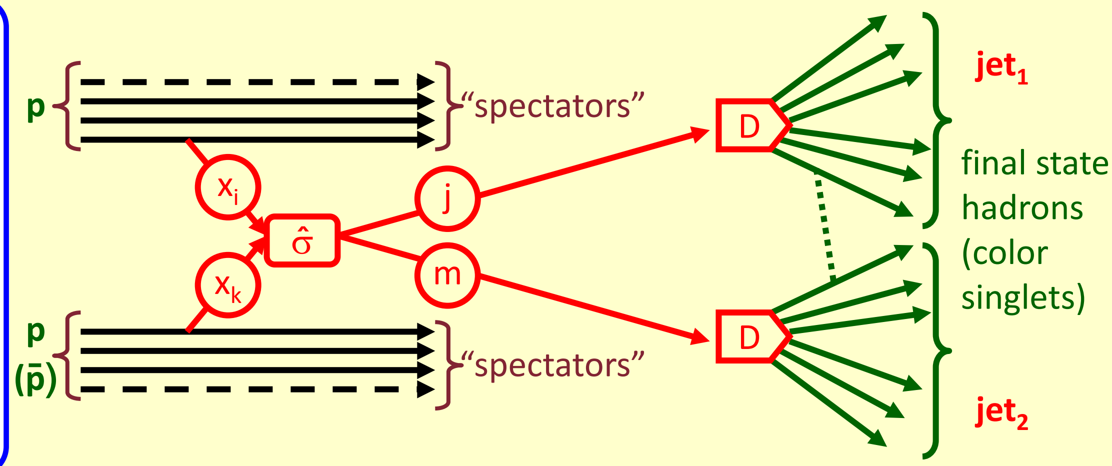
The quark parton model: initial state

- in first approximation, partons have **only longitudinal momentum** (the "Fermi motion" of partons in the hadron is small);
- each parton shares **a fraction x** of the momentum of its parent :
  $\vec{p}_{parton} = (0, 0, \pm x\, p_{hadron})$;
- the distribution function of x [$\mathbf{F_i^h(x,Q^2)}$, for the parton i in the hadron h] are called **pdf [= parton distribution functions**, and depend both on x and Q^2 [§ 2 and 7];
- the evolution in (x, Q^2) of the pdf is regulated in non-perturbative QCD by the equation **GLAP** (Gribov – Lipatov – Altarelli – Parisi).

(… continue …)

$$\sigma(\bar{p}p\,[pp] \rightarrow jet_1 jet_2) =$$

$$= \sum_{i,k} \int \left[\begin{array}{c} dx_i dx_k F_i^p\left(x_i, Q^2\right) F_k^{\bar{p}}\left(x_k, Q^2\right) \\ \hat{\sigma}\left(ik \rightarrow jm @ \sqrt{\hat{s}} = \sqrt{s x_i x_k}\right) \end{array} \right].$$



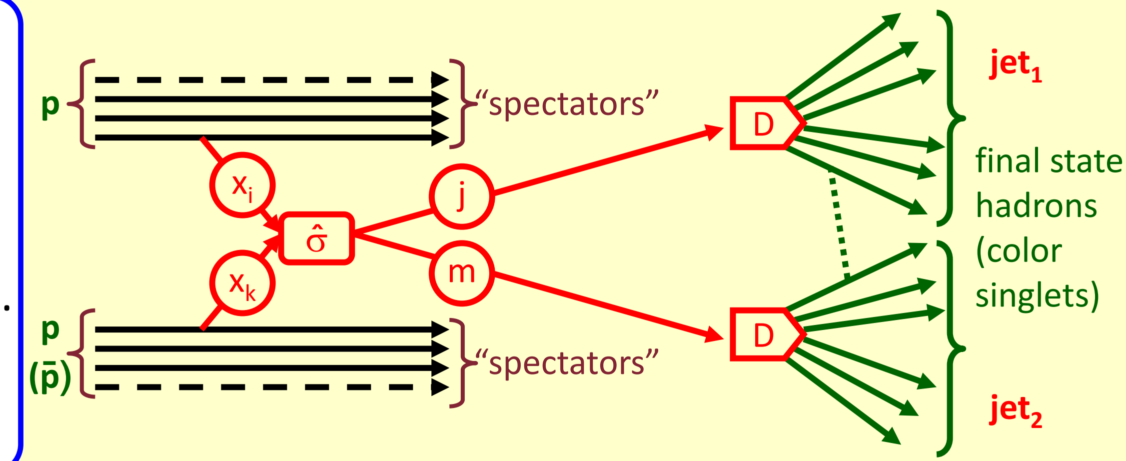
The quark parton model: collision

- collisions at high-p_T between elementary partons are two-body scatterings ("ab → cd"), to be studied in **perturbative QCD**;
- parton energy in their CM : $\hat{s} = sx_1x_2$;
- most of the partons of the hadrons do NOT participate in the collision ("**spectator partons**"); they continue in a direction (quasi-)parallel to the hadrons of the initial state;
- after the collision, the partons of the final state "hadronize" ("**fragment**"), i.e. give rise to the hadrons of the final state;
- those particles emerge as collimated sprays ("**jets**") of particles with high p_T;
- the 4-vector sum of the momenta of the hadrons of a **jet** is identified with the 4-vector momentum of the **parton**.

(…continue…)

$$\sigma(\bar{p}p\,[pp] \to jet_1 jet_2) =$$

$$= \sum_{i,k} \int \left[\begin{array}{c} dx_i dx_k F_i^p(x_i, Q^2) F_k^{\bar{p}}(x_k, Q^2) \\ \hat{\sigma}\left(ik \to jm @ \sqrt{\hat{s}} = \sqrt{s x_i x_k}\right) \end{array} \right].$$

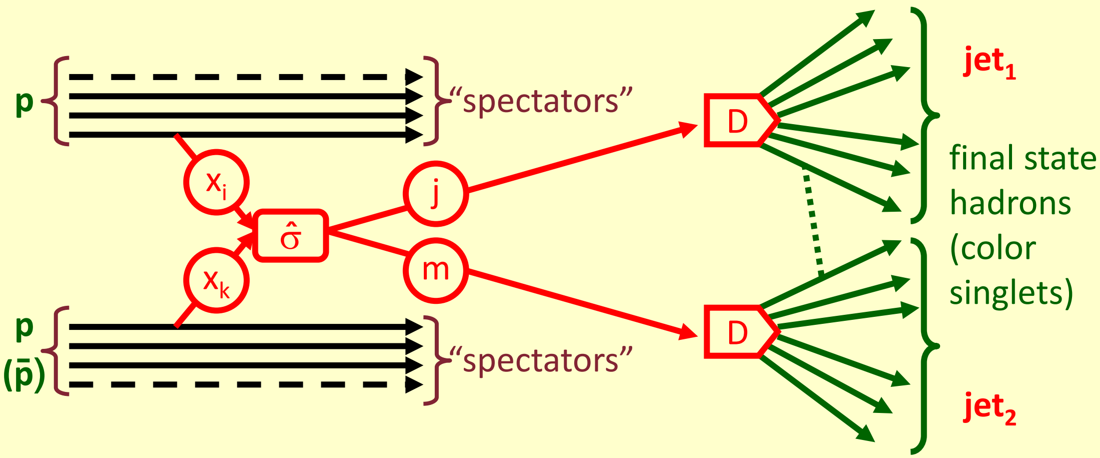


The quark parton model: fragmentation

- The distributions of the final state hadrons are called "**fragmentation functions**";
- they are functions $[D_p^h(z,Q^2)]$ of the variable z (= p_{hadron} / p_{parton}), which defines the distribution of hadron "h" in a jet from parton "p";
- they do NOT depend, to a good approximation, neither on the initial state, nor on the elementary collision, but only on the final state parton and the value of Q^2;
- however, unlike the partons of the elementary collision, the hadrons are color singlets; therefore in the process of fragmentation particles of different jets must interact.

$$\sigma(\bar{p}p\,[pp] \rightarrow jet_1 jet_2) =$$

$$= \sum_{i,k} \int \left[\begin{array}{l} dx_i dx_k F_i^p\left(x_i, Q^2\right) F_k^{\bar{p}}\left(x_k, Q^2\right) \\ \hat{\sigma}\left(ik \rightarrow jm @ \sqrt{\hat{s}} = \sqrt{s x_i x_k}\right) \end{array} \right].$$



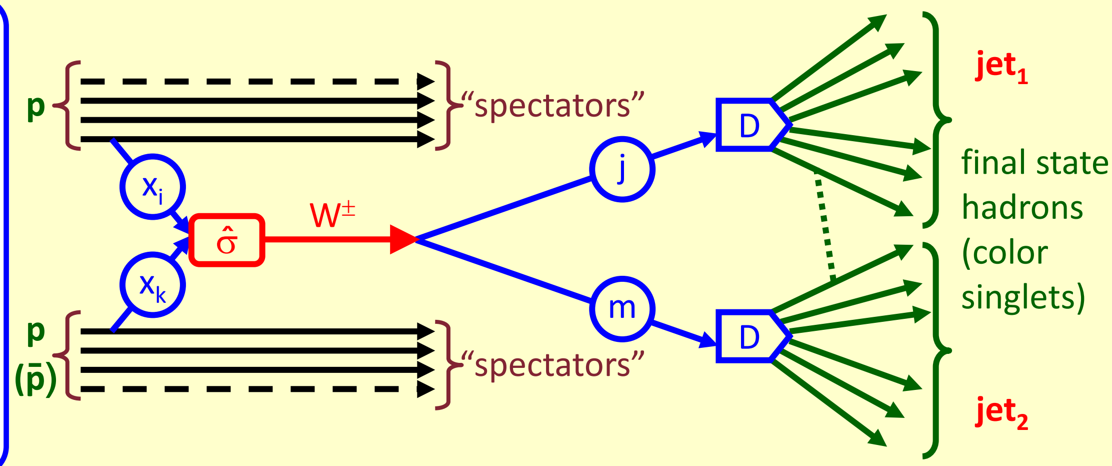
The quark parton model: electroweak

- In (few but) interesting cases, non-QCD processes happens [e.g. $\bar{u}d \to W^-$, followed by W decay into quarks];
- these processes are rare (e.g. $10^{-5} \div 10^{-6}$ of pQCD at LHC), but very valuable; they are at the origin of both the Sp$\bar{p}$S and LHC construction;
- the analysis proceeds in the same way: the two-body QCD parton scattering is replaced by the appropriate electroweak (or SUSY, or whatever) theory;

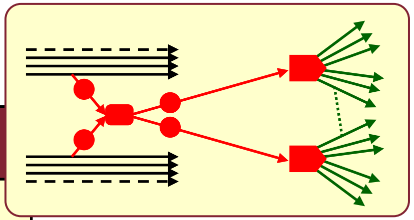
[the figure represents a Drell-Yan process (see § Sp$\bar{p}$S*), with the creation of a $W^\pm$ and its successive decay into a $q\bar{q}$ pair, which fragments into two jets; other processes are treated in the same way.]*

$$\sigma(\bar{p}p\,[pp] \to W^\pm) =$$

$$= \sum_{i,k} \int \left[dx_i dx_k F_i^p(x_i, Q^2) F_k^{\bar{p}}(x_k, Q^2) \; \hat{\sigma}\left(ik \to W^\pm @ \sqrt{s x_i x_k} = m_w\right) \right]$$



The quark parton model: score



process	prediction ?	theory $\leftrightarrow$ exp.	why
$\sigma_{tot}(\bar{p}p \to \bar{p}p)$	no	the optical theorem is a relation, NOT a prediction.	low-p_T
$\sigma_{tot}(pp \to pp)$	no		
$\sigma_{incl}(pp/\bar{p}p \to \pi^+ X)$	no	$\ell n\ s$ model ?	
$\sigma_{incl}(pp/\bar{p}p \to \text{jet } X)$	yes	fair	pQCD
$\sigma_{incl}(pp/\bar{p}p \to Z\ X)$	yes	good	electro-weak
$\sigma_{incl}(pp/\bar{p}p \to W\ X)$	yes	good	
$\sigma_{incl}(pp/\bar{p}p \to H\ X)$	yes	very good	
$\sigma_{incl}(pp/\bar{p}p \to \text{SUSY})$	if …	???	???

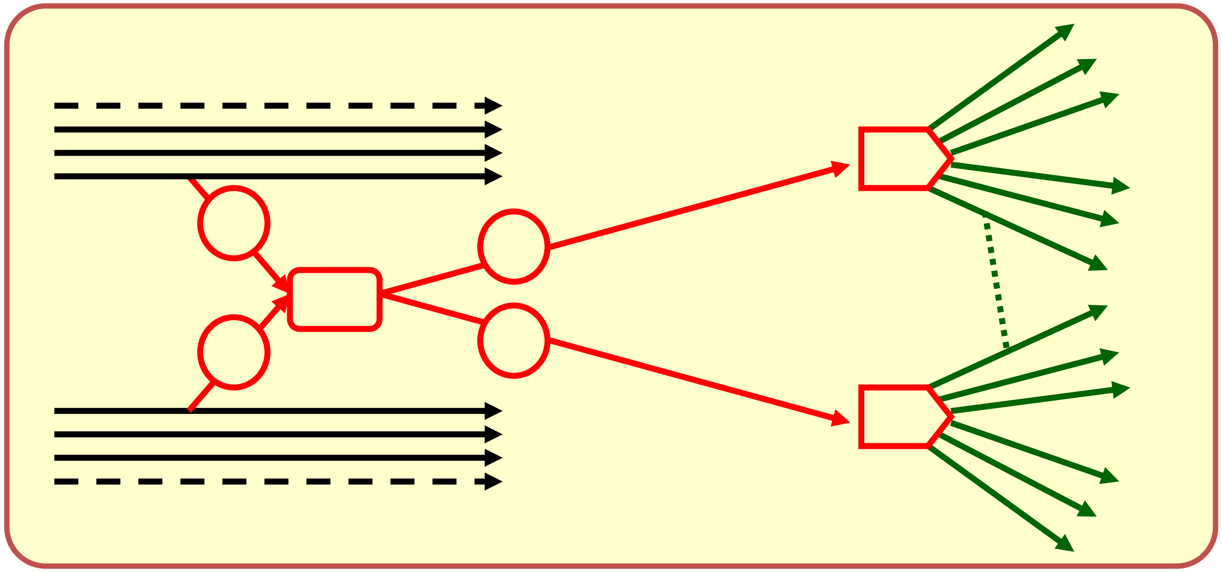
cfr. similar e.w. processes:

$\sigma_{tot}(e^+e^- \to \mu^+\mu^-)$	yes	perfect	pure e.w.
$\sigma_{tot}(e^+e^- \to Z \to f\bar{f})$	yes	perfect	pure e.w.
$\sigma_{tot}(e^+e^- \to HZ \to f\bar{f}f\bar{f})$	yes	[it will be perfect, I know]	pure e.w.

The quark parton model: method

- The scheme works for all known interactions of quarks and gluons, both e.w. and strong, if the correct definition of the elementary process ($\hat{\sigma}$) is applied.
- The present method is to reproduce the process, via Montecarlo generation of events, later analyzed as real data.
- When, according to q.m., a distribution function (e.g. $\hat{\sigma}$, pdf) appears, the random function of the computer is used.
- Many events are generated, so the a-posteriori analysis is able to predict/reproduce the statistical result.
- A single event is built in successive steps, according to the "factorization approximation":

(continue ...)



The quark parton model: procedure

a. a **parton of a given type** is generated out of the first hadron; its x is also generated, according to its pdf;

b. ditto for the **second** init. state parton;

c. the elementary parton process is computed, using the appropriate **cross section at parton level**(1);

d. (as a part of this step) **the angular distribution** of the final state partons is generated, according to the dynamics of the elementary process;

e. each parton of the final state is fragmented, with its **fragmentation functions** (or a fragmentation model(2));

f. the hadrons from **spectator** partons are added (few methods exist);

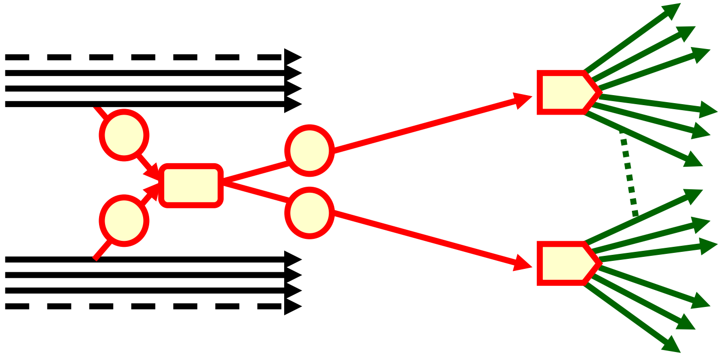
g. all the hadrons of the final state are recorded for successive analysis.

---

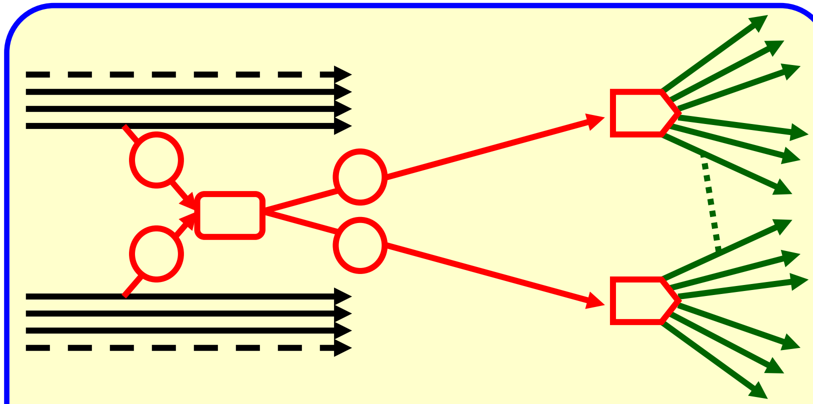
(1) In case of electroweak decays ($W^\pm$, Z, H), with production of leptons, the treatment of the final state has to be appropriate (in fact, it is easier, since the fragmentation step is absent or simpler).

(2) "Fragmentation models" like Lund (Pythia), Herwig, are a mixture of theory (perturbative and non-pertubative QCD), parameterization of measurements (fragmentation functions) and computing skill for easy management. They are very well-done and successful, but are NOT based on a complete reproduction of the theory.

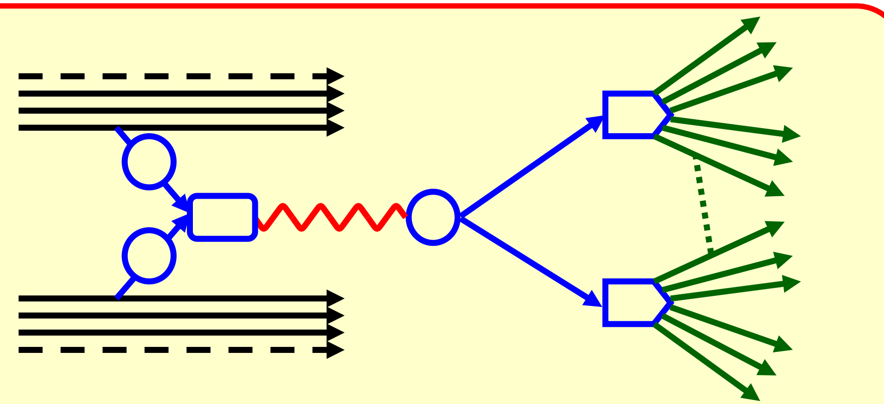
NB. The procedure just described contains some loopholes, e.g. pdf's (a-b) depend on Q^2, which is generated later (c-d); there are appropriate tricks, not described here.



The quark parton model: examples



$$\sigma(\bar{p}p\,[pp] \to jet_1 jet_2) = \sum_{i,k}\int \left[\begin{array}{l} dx_i dx_k F_i^p\left(x_i,Q^2\right) F_k^{\bar{p}}\left(x_k,Q^2\right) \\ \hat{\sigma}\left(ik \to jm @ \sqrt{\hat{s}} = \sqrt{s x_i x_k}\right) \end{array} \right].$$



$$\sigma(\bar{p}p\,[pp] \to W \to q_1 \bar{q}_2) = \sum_{i,k}\int \left[\begin{array}{l} dx_i dx_k F_i^p\left(x_i,Q^2\right) F_k^{\bar{p}}\left(x_k,Q^2\right) \times \\ \hat{\sigma}\left(ik \to W @ \sqrt{\hat{s}} = \sqrt{s x_i x_k}\right) \times BR(W \to q_1 \bar{q}_2) \end{array} \right].$$

Two test-case processes for the q-p model :

a) two-jet production;

b) W (or Z) production and decay into jets.

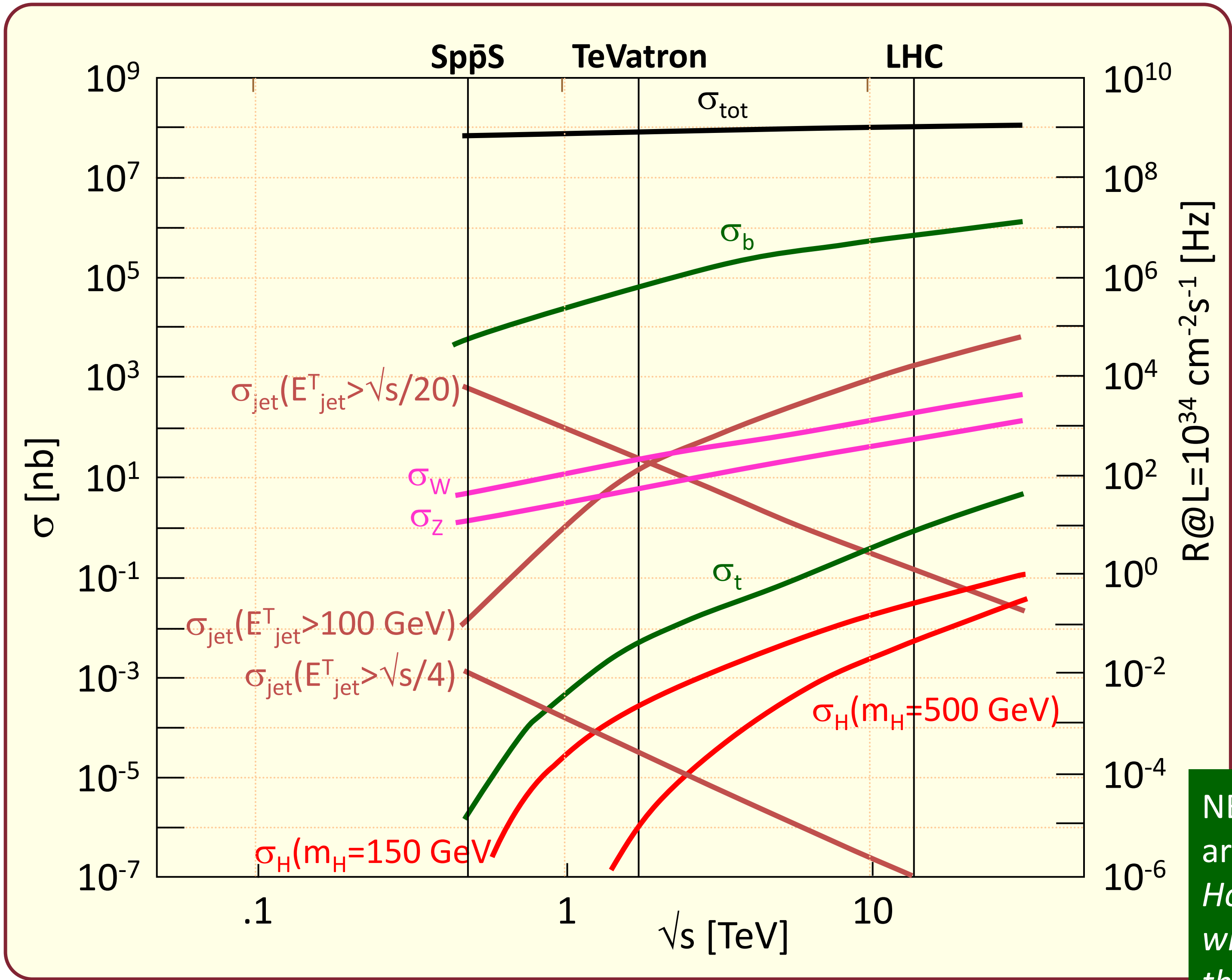
Notice the correspondence between the scheme and the corresponding formula.

The sums run over all the partons which may generate the final state, and the integrals between the kinematical limits.

The pdf's "weight" the processes, giving each parton and each x the correct share.

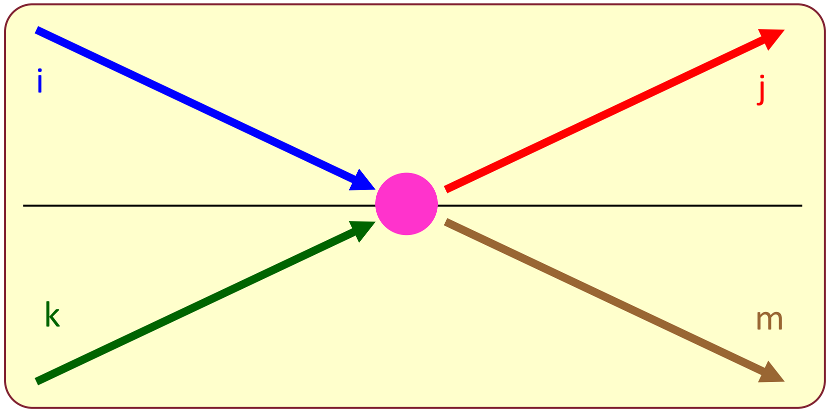
NB. a) in principle the parton type is observable $\to$ sum the σ's, NOT the amplitudes;
b) σ_W is strongly peaked for real W's $\to$ x_i, x_k are NOT kinematically independent]

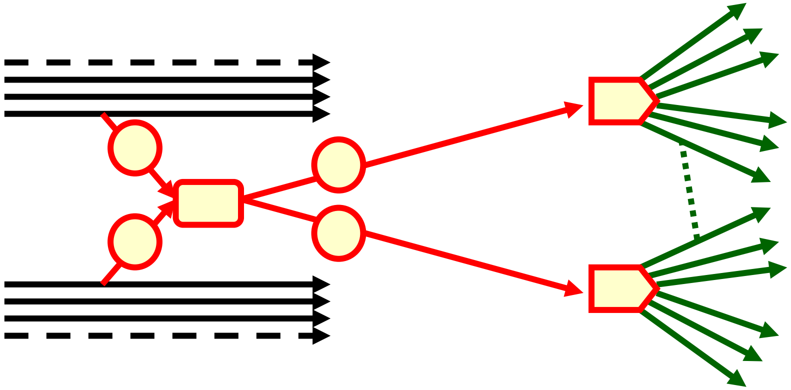
The quark parton model: Sp$\bar{p}$S → LHC



NB. Sp$\bar{p}$S and Tevatron are $\bar{p}$p, LHC is pp. *However, no difference within the accuracy of this plot.*

High-p_T: kinematics





➢ initial state in pp [p̄p] CM :

p_{hadron_1}= [½√s, ½√s, ~0, ~0]; p_{parton_i} = [½x_i√s, ½x_i√s, ~0, ~0];

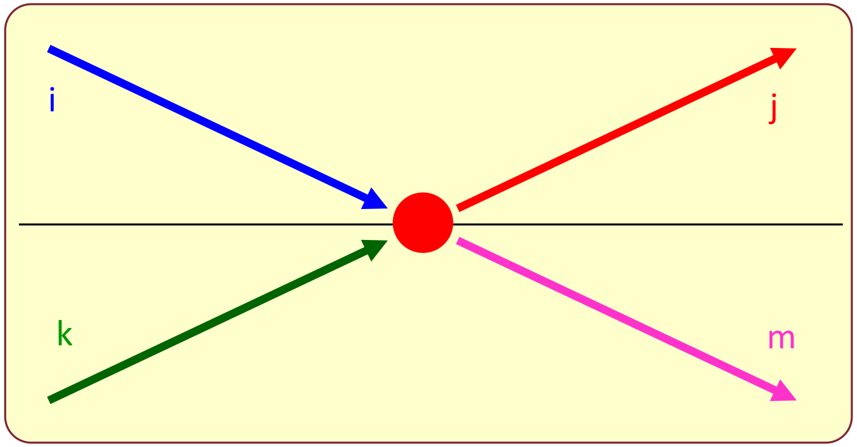
p_{hadron_2}= [½√s,-½√s, ~0, ~0]; p_{parton_k} = [½x_k√s, -½x_k√s, ~0, ~0];

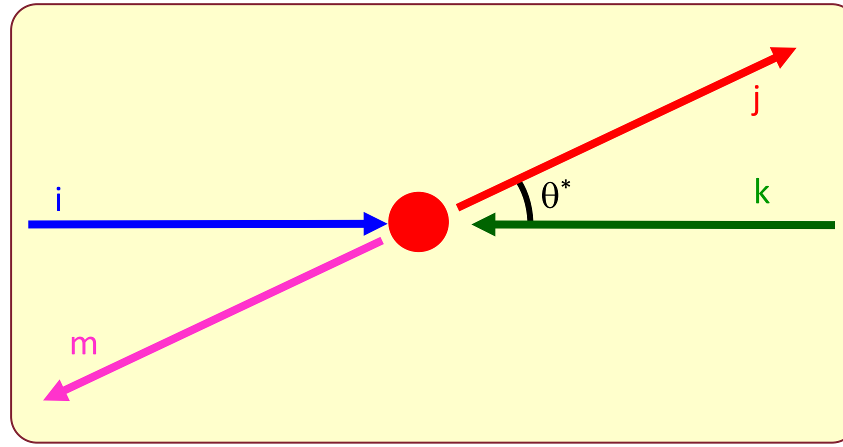
@LHC not head-on collisions + Fermi motion of partons

➢ sum : ik in CM_{12}: [½√s(x_i + x_k), ½√s(x_i - x_k), ~0, ~0];

ik in CM_{ik} : [$\sqrt{\hat{s}}$, 0, 0, 0] → $\hat{s} = \frac{1}{4}s[(x_i + x_k)^2 - (x_i - x_k)^2] = s\, x_i\, x_k$.

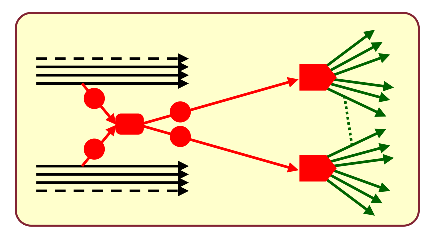
High-p_T: parton variables



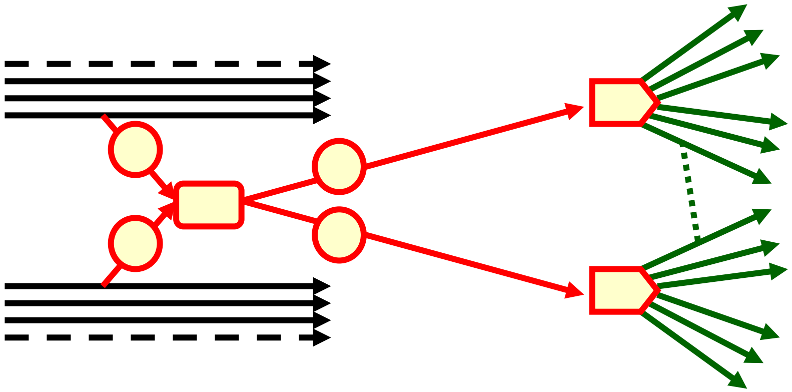


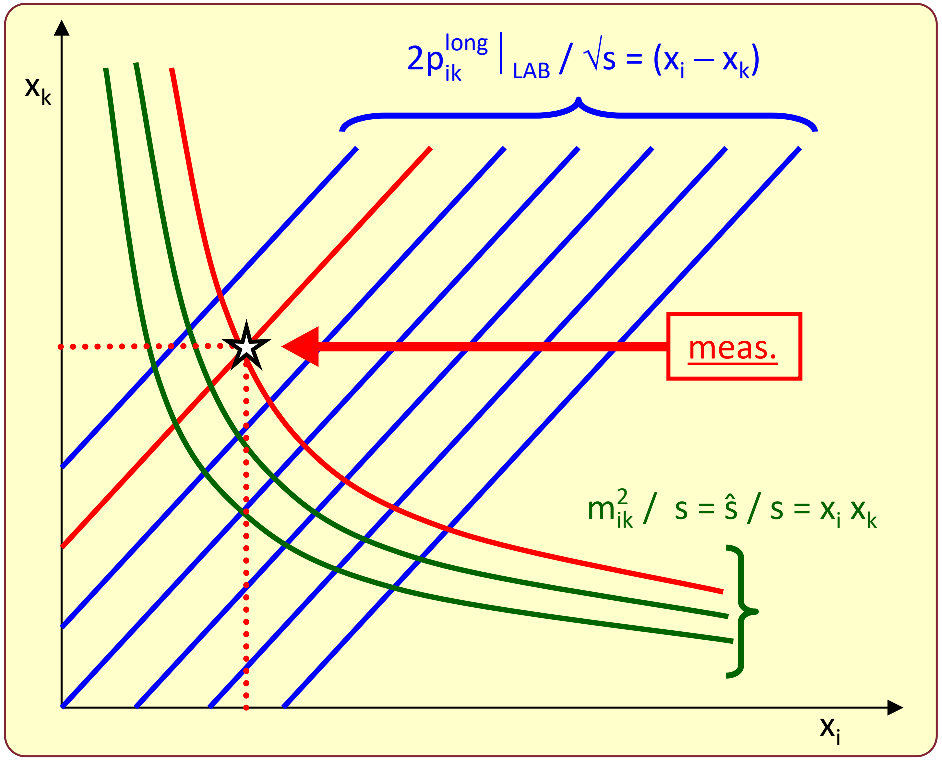
- $p_i = [\tfrac{1}{2}\sqrt{\hat{s}},\quad \tfrac{1}{2}\sqrt{\hat{s}},\quad 0,\quad 0];$
- $p_k = [\tfrac{1}{2}\sqrt{\hat{s}},\quad -\tfrac{1}{2}\sqrt{\hat{s}},\quad 0,\quad 0];$
- $p_j = [\tfrac{1}{2}\sqrt{\hat{s}},\quad \tfrac{1}{2}\sqrt{\hat{s}}\cos\theta^*,\quad \tfrac{1}{2}\sqrt{\hat{s}}\sin\theta^*,\quad 0];$
- $p_m = [\tfrac{1}{2}\sqrt{\hat{s}},\quad -\tfrac{1}{2}\sqrt{\hat{s}}\cos\theta^*,\quad -\tfrac{1}{2}\sqrt{\hat{s}}\sin\theta^*,\quad 0];$
- $\hat{s} = (p_i + p_k)^2 = (p_j + p_m)^2 = s\, x_i\, x_k;$
- $\hat{t} = (p_i - p_j)^2 = (p_m - p_k)^2 = -\tfrac{1}{2}\hat{s}\,(1 - \cos\theta^*);$
- $\hat{u} = (p_i - p_m)^2 = (p_k - p_j)^2 = -\tfrac{1}{2}\hat{s}\,(1 + \cos\theta^*);$
- $\hat{s} + \hat{t} + \hat{u} = 0$ ($\rightarrow$ in parton CM, two independent variables).

Comments:
- see § 3 for similar discussion for not-composite particles;
- zero mass approx for all partons [for $m \neq 0$, § 3 and PDG § 43.5].



High-p_T: solve the kinematics





- The overall transverse momentum MUST be balanced. A p_T imbalance is attributed to non interacting particles (ν's) or, most likely, to measurement errors.
- By measuring the 4-momenta of the final state (e.g. two jets), it is possible to compute $\hat{s}$ and p_{long}. From there, x_i and x_k and the full kinematics at parton level.

Compute $(\vec{p}_i+\vec{p}_k)$:

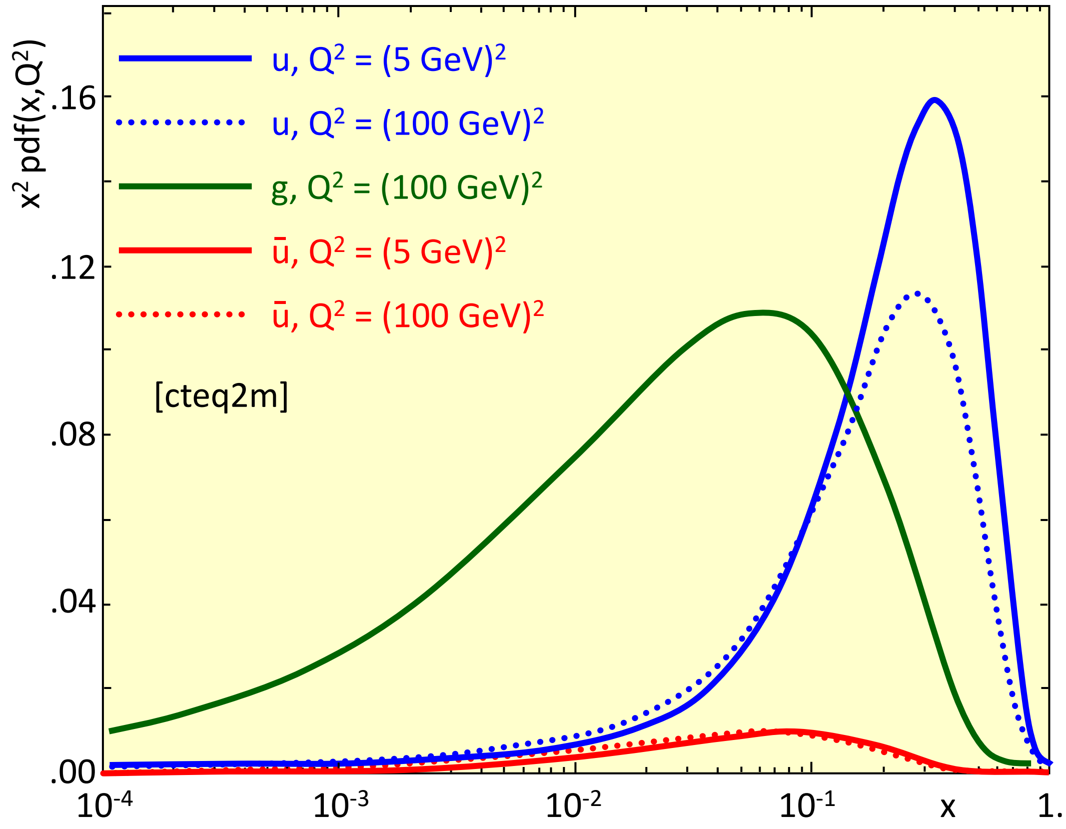
- LAB : [$\frac{1}{2}\sqrt{s}(x_i+x_k)$, $\frac{1}{2}\sqrt{s}(x_i-x_k)$, ~0, ~0];
- CM_{ik} : [$\sqrt{\hat{s}}$, 0, 0, 0];

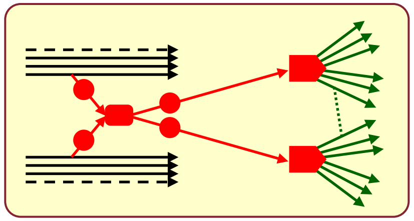
$\rightarrow \hat{s} = \frac{1}{4}s[(x_i+x_k)^2 - (x_i-x_k)^2] = s\, x_i\, x_k$.

High-p_T: structure functions (pdf)

- in the quark parton model, hadrons are "wide-band beams" of elementary partons;

- in first approximation, structure functions do NOT depend on Q^2 : $\partial F_i(x, Q^2) / \partial Q^2 = 0$;
- but scaling violations do exist.





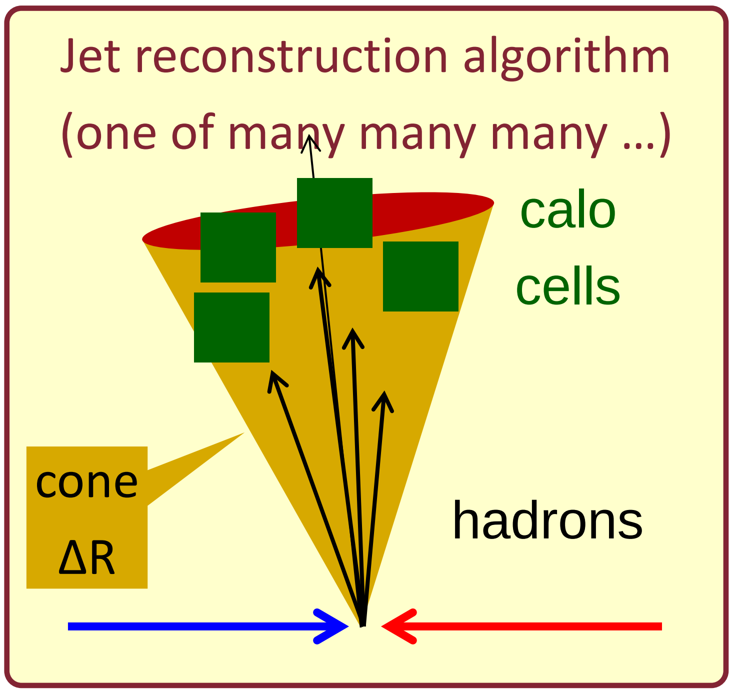
www.zebu.uoregon.edu/~parton/partongraph.html

when Q^2 increases :

- partons "get closer";
- q_{sea} and g increase at small x;
- $q_{valence}$ decreases at all x;
- at x fixed and large, rates @LHC smaller than @Sp$\bar{p}$S.

High-p_T: partons → jets

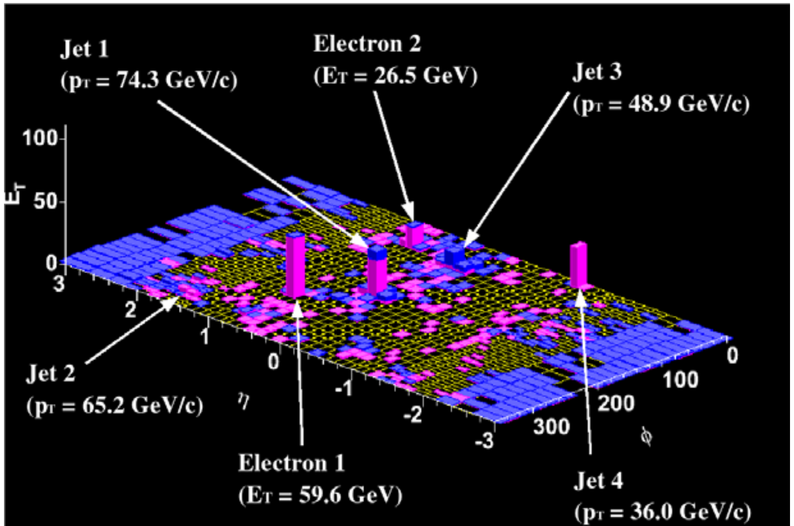
- reconstruct the jets via an algorithm :
  - simple clustering of nearby calo cells;
  - cone algo. (see fig) with fixed ΔR (very popular $\Delta R^2 = \Delta\phi^2 + \Delta\eta^2 = 1$);
  - "Durham"
  - anti-Kt
  - …



- more refined cooking (split, sum, …)
- reconstruct 4-momentum :

$$\vec{p}_{jet} = \sum \vec{p}_{hadrons}; \quad E_{jet} = \sum E_{hadrons};$$

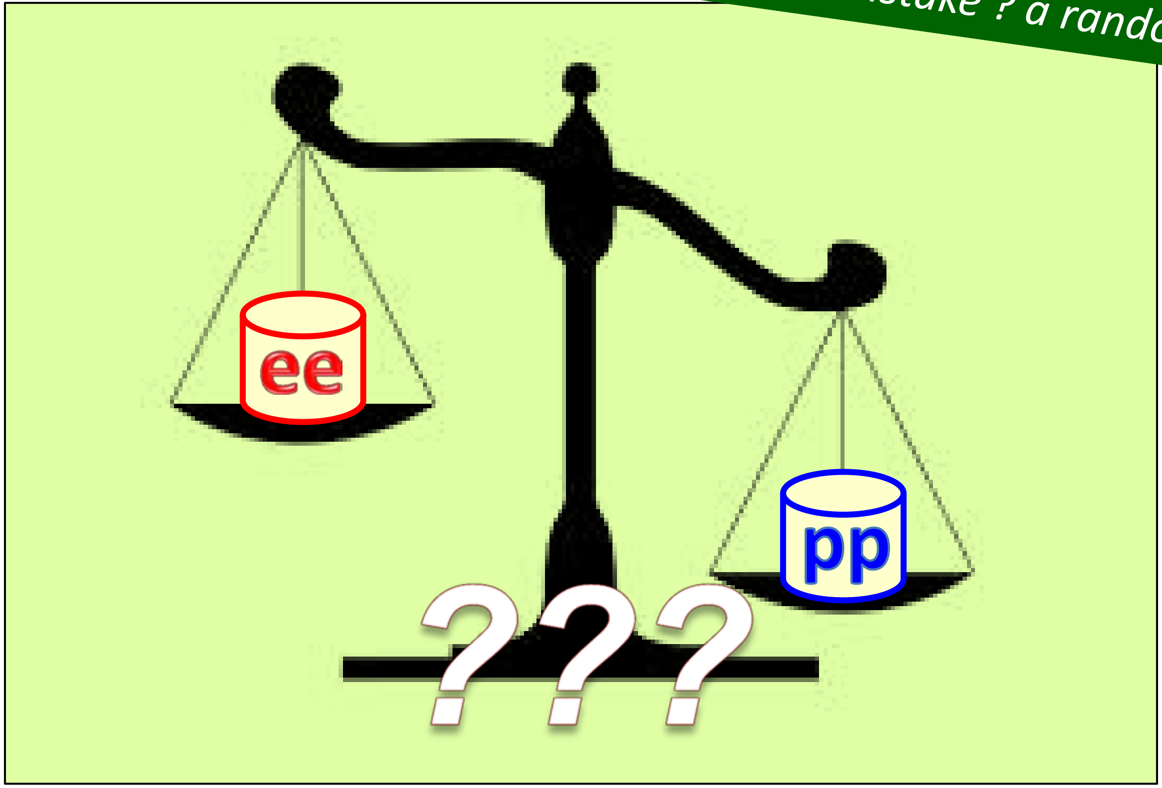
- [*notice that the above definition gives jets a mass ≠ 0, generally much larger than the tiny parton mass → more cooking …*]
- identify (jet → parton) and play with its 4-momentum;
- check the manipulations with known cases ($W^\pm$, Z → jets) and montecarlo.



CDF – Z→ e^+e^- + jets

iii. Comparisons

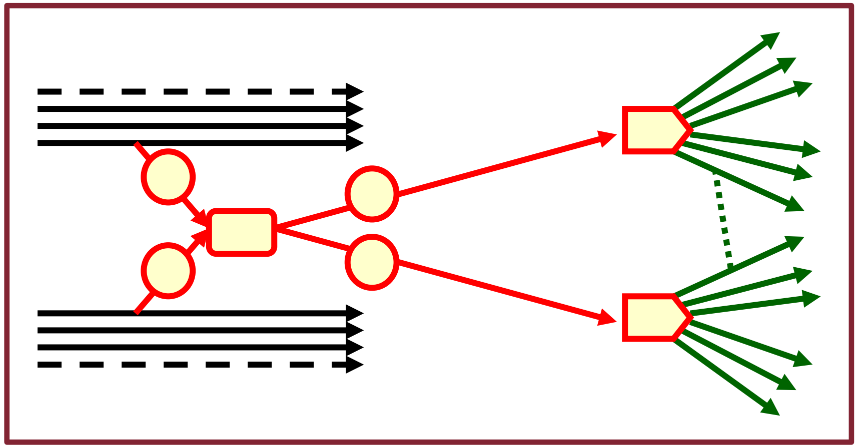
why sometimes e^+e^-, sometimes $\bar{p}p$, sometimes pp ?
and possibly $\mu^+\mu^-$? a mistake ? a random choice ?



$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$

Few general arguments : the REAL answer is in the complete set of lectures.

- ☺ a hadron is a bundle of many different partons (valence+sea quarks, sea anti-quarks, gluons);
- ☺ many initial states are simultaneously available in pp/p̄p, i.e. hadron machines are much richer in physics;
- ☺ in pp/p̄p, no need to scan in $\sqrt{s}$: at high Q^2, the pdf's provide a large range of $\sqrt{\hat{s}}$ simultaneously (see the J/ψ story);
- ☺ it is therefore possible to define a "differential luminosity" $d\mathcal{L}_i/d\sqrt{\hat{s}}$ for partons of type "i" (quarks, gluons) as a function of $\sqrt{\hat{s}}$ for the same $\sqrt{s}$;
- ☹ $d\mathcal{L}_i/d\sqrt{\hat{s}}$, integrated in small intervals of $\sqrt{\hat{s}}$, is small; it also decreases for $\sqrt{\hat{s}} \to \sqrt{s}$ (i.e. $x_1x_2 \to 1$), because of the pdf's;
- ☹ because of all that, the experiments and analysis are **much more difficult** in hadron machines.



$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$: soft vs hard collisions

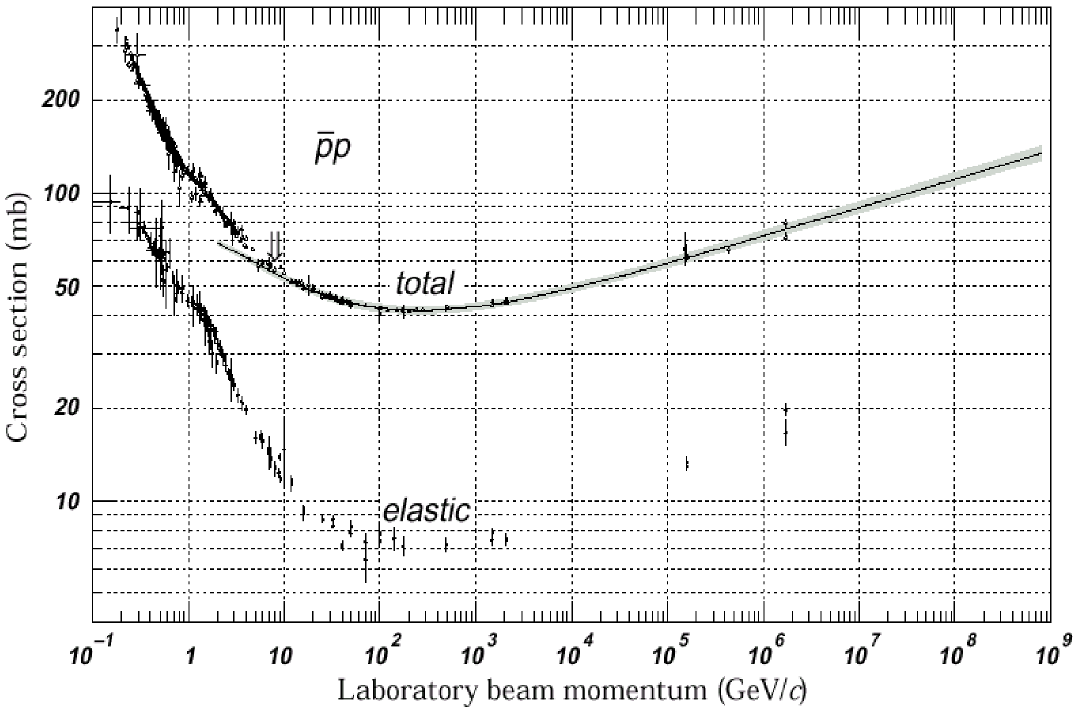
ex. : σ(LEP II, $e^+e^- \to$ hadr., $\sqrt{s}$ = 200 GeV) ≈ **100 pb**;

σ(LHC, pp $\to$ total, $\sqrt{s}$ = 14 TeV) ≈ **100 mb**;

σ(LHC, pp $\to$ jet X, E_T^{jet} > 250 GeV) ≈ **100 nb**.

[actual thresholds quite arbitrary, retain the order of magnitude]

~ 1 ÷ 10^9 ÷ 10^3 (!!!)

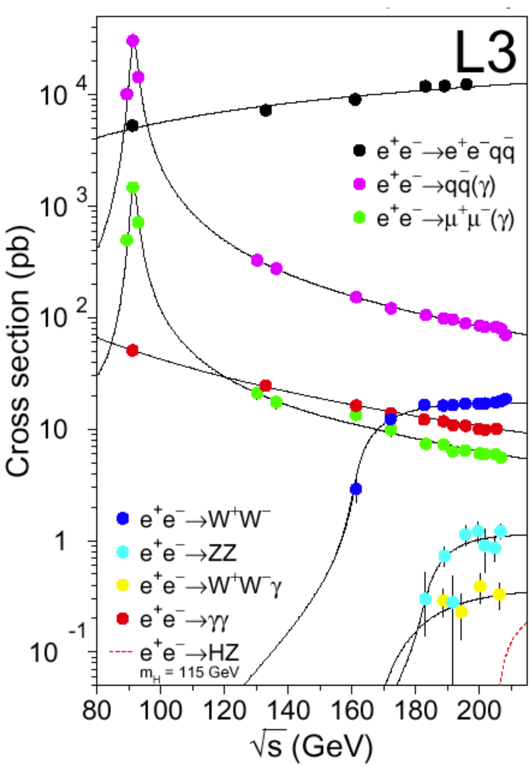


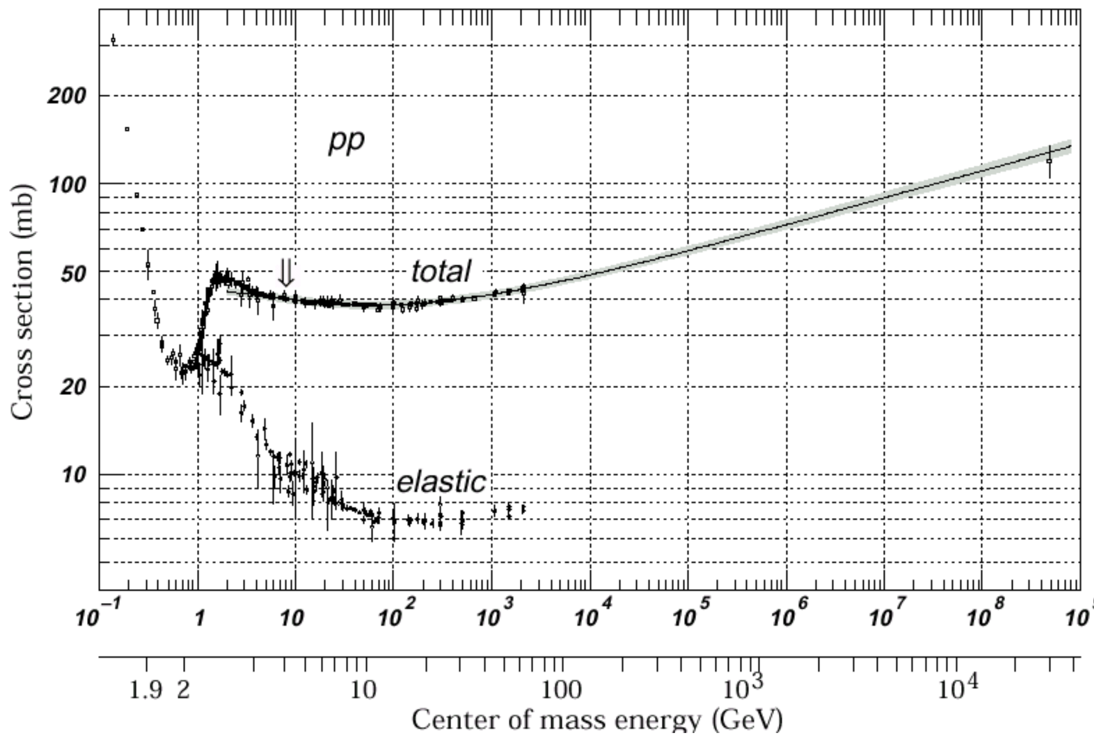
- nucleons, when coherent, are "one billion times" larger than electrons;
- however, when individual partons have to play, they are only "1,000 times" (the actual number depends on Q^2) larger;
- the factor 1,000 is due to the strength of the coupling ($\alpha_s \leftrightarrow \alpha_{em}$).

$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$: small vs large σ

- in ee, "small" σ_{tot} (~pb, $\propto$ 1/s away from the Z pole), dominated by high-Q^2 processes mainly in the s-channel;
- therefore few events (rate ~1 Hz), all very interesting $\rightarrow$ **event trigger**;

- in pp/$\bar{p}$p, much higher σ_{tot} (~100 mb over many orders of magnitude), dominated by low-Q^2 processes (t-channel);
- therefore very high rate (~10^9 Hz), rare interesting events $\rightarrow$ **high-p_T triggers**.

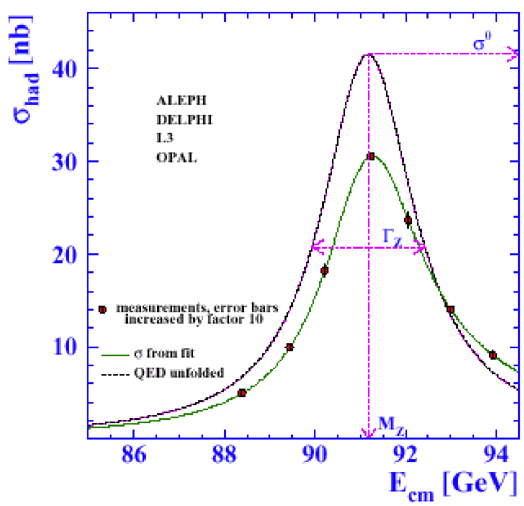




$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$: data analysis

In detector and analysis many differences between e^+e^- and $pp/\bar{p}p$:

- in ee "partonic" energy $\sqrt{s}$ is fixed by the machine, and known precisely;
- in $pp/\bar{p}p$ partonic energy $\sqrt{\hat{s}}$ changes for each event by a large factor;
- for a given $\sqrt{s}$, the average $\sqrt{\hat{s}}$ in a $pp/\bar{p}p$ collision is much lower;
- in ee, kinematical fits in 4D, constraints known to 10^{-5};
- in $pp/\bar{p}p$, fits in 2D, (because of spectators), constraints to %;
- but $\sqrt{s}$ in ee machines is severely limited by brem.



LEP I, $e^+e^- \to Z$

m_Z from $\sqrt{s}$ (LEP)

width = Γ_Z = 2.5 GeV

$\Delta m \approx$ few MeV

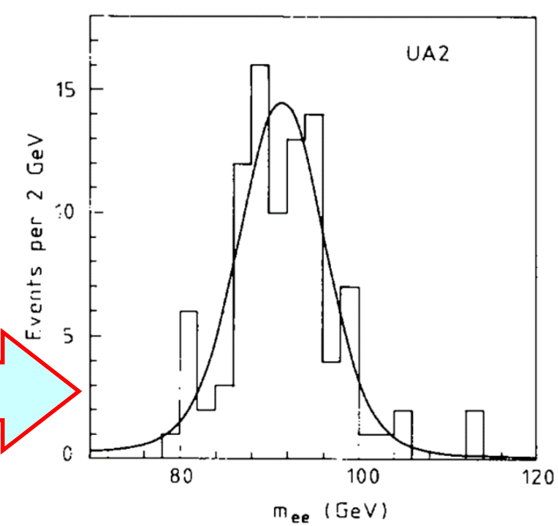
UA2, $\bar{p}p \to Z$

m_Z from $m(e^+e^-)$

width = $\Gamma_Z \oplus \sigma_Z \approx$ 4.3 GeV,

i.e. Γ_Z and σ_Z comparable,

$\Delta m \approx$ few × 100 MeV



$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$: a personal conclusion

In a given moment, with similar technology (and resources, *don't forget*) :

A pp/p̄p machine :

- needs a smaller ring (because of brem);
- more difficult to build (both the magnets and the detectors);
- (much) higher $\sqrt{s}$ and (fairly) higher $\sqrt{\hat{s}}$;
- analysis difficult, higher systematics;
- larger variety of both initial and final states (not only vacuum q.n.);

Therefore [*imho, but largely shared*]:

- (ee) and (pp/p̄p) are complementary, NOT competitive;
- (pp/p̄p) an exploratory machine, for first generation experiments;
- (ee) a "second generation" machine, for systematics and consolidation (and surprises in the precision meas);

This has been the CERN strategy in the last half a century :

1. (pp/p̄p) (re-using an old machine);
2. civil engineering for a new ring (the long and expensive step);
3. (ee) in the new ring;
4. [back to step (1), restart the cycle].

It happens that, e.g., the value of $\sqrt{s}$ in step (3) is similar to $\hat{s}_{eff}$ in step (4/1) [e.g. both the Sp$\bar{p}$S and LEP had $W^\pm$ and Z as their main purpose.

The "luminosity frontier" (Babar, Daφne, …) is a different approach : a dedicated machine, especially optimized wrt intensity and systematics, for (a) very important (single) measurement(s).

$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$: matter vs antimatter

Last question : $pp \leftrightarrow \bar{p}p$?

- **pp** has major problems :
  - it needs two independent magnet rings;
  - at the same $\sqrt{s}$, the effective $\sqrt{\hat{s}}$ is smaller for $q\bar{q}$ channels (valence-sea instead of valence-valence);
- however, **$\bar{p}p$** has a larger problem:
  - antiprotons do NOT exist in nature (at least in our proximity);
  - therefore $\bar{p}$'s have to be "built", starting from pp collisions;
  - they are scarce, and have an incredible "price" (in the $Sp\bar{p}S$, one good $\bar{p}$ / 3×10^5 pp collisions);
  - they have to be cooled and stored (AA, stochastic cooling, van der Meer);
  - the resultant luminosity is small (in 1983, the golden year, $\mathcal{L}(Sp\bar{p}S) < 10^{30}\ cm^{-2}s^{-1}$);

- Therefore, in spite of all the successes of the $\bar{p}p$ machines, both at CERN and Fermilab, the quest for higher energies and (consequently) higher luminosities makes the pp option really superior for present and future colliders.
- The $\bar{p}p$ option will probably be reserved for dedicated single-task machines at sub-TeV energy.

$e^+e^- \leftrightarrow pp \leftrightarrow \bar{p}p$: e^+e^- linear or circular ?

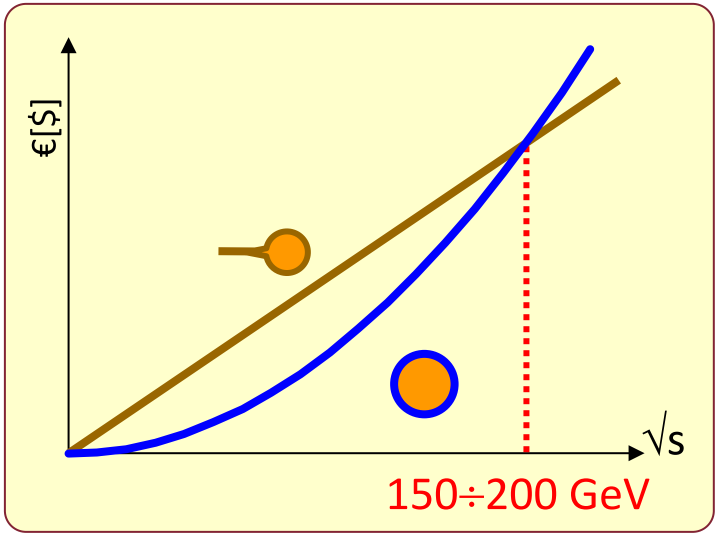
- Smart idea (SLAC '80s): build/use a powerful e^+e^- linear collider, add two arcs and produce the equivalent of a circular electron collider [*see § LEP*].
- In this way, essentially NO BREM (e^+/e^- only once in a curved path).

Pros/cons : *[thanks to Gary Feldman]*

- Circular colliders (like ADA, ADONE, SPEAR, LEP, …) :
  - cost $\propto$ radius,
  - energy to exploit $\propto E^4 / R$ *(brem)*,
  - $\$ = \alpha R + \beta E^4 / R$;
    $d\$ / dR = 0 \rightarrow \alpha = \beta E^4 / R^2 \rightarrow$
    $R_{best} = \sqrt{\beta/\alpha}\, E^2$; $\quad \$_{min} = \sqrt{\alpha\beta}\, E^2$;
  - best choice: $R \propto E^2$; $\$ \propto E^2$.
- Linear colliders (SLC, next CERN ?) :
  - both machine and energy $\propto$ length;
  - $R \propto E$; $\$ \propto E$.

- Coefficients α,β depend on technology and market; at present the crossing is at $E_{beam} \approx 150 \div 200$ GeV;
- ~~possibly LEP is the highest energy e^+e^- circular collider ever built~~ [*never say never …* read the CERN strategy plan];

- p, $\bar{p}$, $\mu^\pm$, etc., are different (see § LHC).



References

1. e.g. [BJ, 14];
2. for the results, see next 3 chapters;
3. accelerator physics : [BJ, 2], [Povh, appendix];
4. better accelerator physics : Ed. Wilson, An introduction to particle accelerators.

God the Geometer, Frontispiece of Bible Moralisee Codex Vindobonensis 2554 (French, ca. 1250) [Österreichische Nationalbibliothek]

End of chapter 8