Particle Physics - Chapter 6
The Standard Model

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6 – The Standard Model

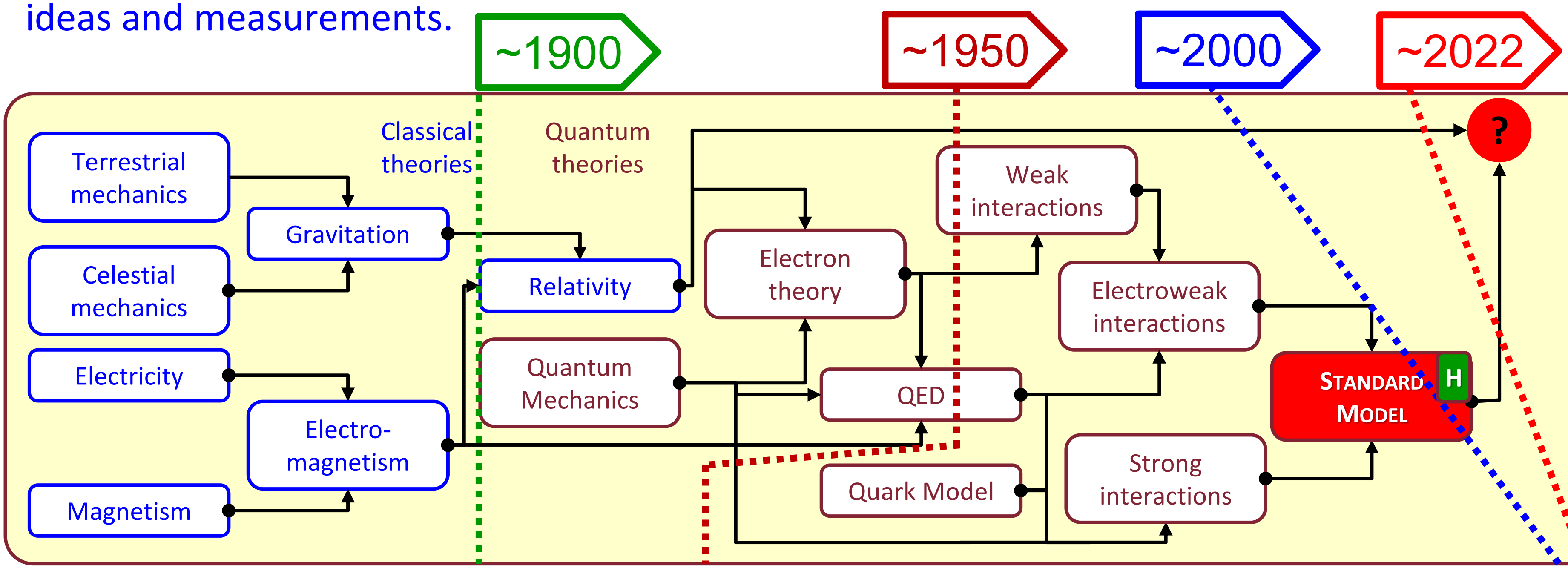
The style of this chapter depends on our CdL:

- the electroweak part is in another course
  → here just for reference;
- the QCD part is different: still amateurish, but I'm the first to tell you;
- the Higgs part is in next year lectures.

u	c	t	γ
d	s	b	g
ν_e	ν_μ	ν_τ	Z
e	μ	τ	$W^\pm$
			H

The Standard Model

- The name **SM** (not really nice) designates the theory of the Electromagnetic, Weak and Strong interactions.
- The theory has grown in time, the name went together.
- The development of the SM is a complicated interplay between new ideas and measurements.
- Many theoreticians have contributed : since the G-S-W model is at the core of the SM, it is common to quote them as the main authors.
- The little scheme [BJ] of its time evolution may help (missing connections, approximations, …).



the electroweak theory

- Glashow (1961), Salam (1964), Weinberg (1967) provided the main ingredients for the unification of weak and electromagnetic interactions.
- The fundamental interactions are described by field theories, invariant under local gauge transformations.
- Technically, by a Lagrangian $\mathscr{L}$, invariant under the appropriate symmetries.
- The symmetries correspond, via the Noether theorem, to the conservation laws of the Theory.
- The conservation laws are local [*i.e. in a given space-time point*]: electric charge is the usual example of such a quantity.
- In the Standard Model, the electro-magnetic and weak interactions (both CC and NC) are related to the symmetry group SU(2) $\otimes$ U(1).
- The parameters of the theory controls all the phenomena: "few" independent masses and couplings for the full theory.
- The dynamics is fully regulated : ($e^{\pm}$, $\mu^{\pm}$, ν) DIS, e^+e^- processes (LEP), IVB and Higgs production and decay (Sp$\bar{p}$S, LHC) are fully described by the e.w. Theory.
- Among the successful predictions neutral currents, $W^{\pm}$, Z, Higgs.
- [in the '60s/'70s no strong interactions theory, but now QCD occupies the role.]
- [as of today, no quantum gravity theory.]

Sheldon Lee Glashow	Abdus Salam (عبد السلام)	Steven Weinberg

the e.w. theory: properties

- Any theory (including the e.w.) has to be free from logical and mathematical inconsistencies.
- In mathematical terms, it MUST be renormalizable, i.e. it must exist a mathematically correct procedure, that eliminates the infinities that arise in calculations of physical observables, such as cross sections and decay rates.
- As a consequence, the e.w. $\mathscr{L}$ must not contain explicitly mass terms; i.e., at the $\mathscr{L}$ level, both the Gauge bosons (the "fields") and Fermions (the "matter") must be massless.
- The proof of the renormalizability of the theory was provided by 't Hooft and Veltman (in 1971, Nobel Prize in 1999).
- The masses are then generated in the theory, without destroying the renormalizability, with the mechanism of spontaneous symmetry breaking, usually called *Higgs mechanism*, proposed by Englert & Brout (1964), Higgs (1964) and Gularnik, Hagen & Kibble (1964).
- The mechanism predicts the existence of (at least) one scalar, the Higgs boson H.
- The values of the fermion masses are left as free parameters; however, once they are fixed, all the couplings of the H boson to the other bosons and fermions are predicted by the theory.


G.t'Hooft – M.Veltman

the e.w. theory: I_W, Y

The properties of the **gauge bosons** **$W^\pm$, Z** and **γ** come out from the theory.

- The fundamental representation of SU(2) ⊗ U(1) is given by three [SU(2)] and one [U(1)] Gauge fields.
- The quantity called "weak isospin" **I_W** [*here called simply "isospin"*(*)] belongs to the SU(2) sector.
- For U(1), there is the "weak hypercharge" **Y_W** [*here "hypercharge"*].
- All the members of the same isospin multiplet have the same hypercharge.
- Similarly to the flavor case, the hypercharge is defined as twice the difference between the electric charge and the third component of the isospin :

  $Y_W \equiv 2\,(Q - I_{Wz})$.
- The triplet of fields corresponding to SU(2) is called W = (W_1, W_2, W_3). The fields W have I_W = 1 and Y_W = 0. They interact with the weak isospin of the particles.
- The field corresponding to U(1) is called **B**. Its isospin, electric charge and hypercharge are zero. It interacts with the weak hypercharge of the particles.
- These four fields (W_i, B) are NOT the physical fields which mediate the interactions.
- The CC weak interactions are mediated by $W^\pm$, which are linear combinations of W_1 and W_2.
- The photon and the Z, mediators of the electromagnetic and NC weak interactions, are linear combinations of W_3 and B.

(*) *Notice that the weak isospin and hypercharge do NOT have any dynamical relation with those defined in precedence for the hadrons, although their mathematical properties are the same.*

the e.w. theory: fermions

The value of I_W and Y_W of the particles depends on the fact that the $W^\pm$, the mediators of the CC, are coupled only to states with negative chirality.

The leptons. In each family there are two left-handed leptons in a I_W = ½ doublet :

$$I_W = ½,\ I_{Wz} = +½ \quad : \nu_{eL},\ \nu_{\mu L},\ \nu_{\tau L};$$

$$I_W = ½,\ I_{Wz} = -½ \quad : e^-_L,\ \mu^-_L,\ \tau^-_L.$$

- the ν's have a (small but non-zero) mass and mix together (mixing matrix 3×3);
- unlike the charged currents, the neutral currents also interact with the charged right-handed fermions, but NOT with right-handed neutrinos;
- the right-handed charged lepton of each family is an isospin singlet (I_W = 0) :

  $$I_W = 0,\ I_{Wz} = 0 : e^-_R,\ \mu^-_R,\ \tau^-_R.$$
- right-handed ν's DO NOT EXIST [*more precisely, if existing, they have* ($I_W = Y_W$ = 0) *and do NOT interact with anything except possibly through gravity*].

The quarks. Their structure is similar, apart from a different mixing (the CKM matrix) and the color :

- The $W^\pm$ is universally coupled with the CKM-rotated states d', s' and b'.
- [three isospin doublets, one for each family] × [three colors] = nine doublets :

  $$I_W = ½,\ I_{Wz} = +½ \quad : u_L,\ c_L,\ t_L;$$

  $$I_W = ½,\ I_{Wz} = -½ \quad : d'_L,\ s'_L,\ b'_L;$$
- the singlets (18 in total) are :

  $$I_W = 0 \ : d_R,\ u_R,\ s_R,\ c_R,\ b_R,\ t_R;$$
- for NC, the quark mixing is irrelevant; therefore we can study the interactions of the "non-rotated" states.

the e.w. theory: a remark on ν's

Some alternative hypotheses on **ν's** :

a) Dirac particles, charge=0, spin=½, mass=0, helicity = –1, partners of charged leptons:
   → ν's do not mix;

b) as (a), but $m_{\nu's} > 0$, although very small:
   → define the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) mixing matrix;
   → helicity NOT intrinsic, depends on L-system [*but see comment*];

   b$_1$) PMNS diagonal → ν's do not mix;

   b$_2$) PMNS NOT diagonal + complex
   → ν's do mix + $\mathbb{CP}$ violation;

c) any hypothesis bSM (e.g. Majorana ν's).

in [MQR] and [IE], (a) and (b) are called "Weyl ν's", while "Majorana ν's " are in (c).

Results:

(a) believed to be correct for most of the XX century; **falsified** in 1998 by discovery of ν-oscillations [ν-o.];

(b$_1$) [*ugly*] **falsified** by ν-o.;

(b$_2$) **current working hypothesis** [*because minimal extension of the SM*]; however it looks unlikely to most [*???*] physicists;

(c) **much appreciated**; however, as of today, no data supports it (many new ongoing experiments: ***good luck !!!***).

Comment:

- ν oscillations (→ mixing) appear only in astro-physical or long-baseline experiments;
- in all other experiments, data (**until today**) consistent with (a);
- no contradiction: ($m_{\nu's} << E_{\nu's}$) → (ν's ultra-relativistic) → (a) good approx. of (b).

Conclusion (as of today):

- (at least two) ν's have mass > 0;
- ν's can and do oscillate (PMNS ≠ $\mathbb{1}$);
- for most exp., approx. $m_{\nu's}$=0, no-oscillation, fixed helicity (–1 for ν, +1 for $\bar{\nu}$);
- ***hope for new exp., or more precise data***.

the e.w. theory: antiparticles

- **The antiparticles.** For each particle, there exists an antiparticle, with opposite quantum numbers.
- In the lepton sector, for CC there are the following three doublets of antileptons :

  $I_W = ½,\ I_{Wz} = +½\ \ : e^+_R,\ \ \mu^+_R,\ \ \tau^+_R;$

  $I_W = ½,\ I_{Wz} = -½\ \ : \bar{\nu}_{eR},\ \ \bar{\nu}_{\mu R},\ \ \bar{\nu}_{\tau R};$

- Plus the following singlets :

  $I_W = 0,\ I_{Wz} = 0 : e^+_L,\ \mu^+_L,\ \tau^+_L.$

- For the $\bar{\nu}$'s, the same rules apply as for ν's.
- In the antiquark sector, three doublets of isospin and six singlets for each family (9 plus 18 in total) :

  $I_W = ½,\ I_{Wz} = +½\ \ : \bar{d}'_R, \bar{s}'_R, \bar{b}'_R;$

  $I_W = ½,\ I_{Wz} = -½\ \ : \bar{u}_R,\ \bar{c}_R,\ \bar{t}_R;$

  $I_W = 0\ \ : \bar{d}_L,\ \bar{u}_L,\ \bar{s}_L,\ \bar{c}_L,\ \bar{b}_L,\ \bar{t}_L.$

P.A.M. Dirac, 1933 Nobel Lecture:

"If we accept the view of complete symmetry between positive and negative electric charge so far as concerns the fundamental laws of Nature, we must regard it rather as an accident that the Earth (and presumably the whole solar system), contains a preponderance of negative electrons and positive protons.

It is quite possible that for some of the stars it is the other way about, these stars being built up mainly of positrons and negative protons. In fact, there may be half the stars of each kind. The two kinds of stars would both show exactly the same spectra, and there would be no way of distinguishing them by present astronomical methods."

Great !!! But presently we know much more (more precisely, **we ignore** much more).

Find where we **improved** in the last 90 years.

the e.w. theory: summary of q.n.

	Spin	I_W	I_{Wz}	Y_W	Q (*)
$\nu_{\ell L}$	½	½	+½	−1	0
ℓ^-_L	½	½	−½	−1	−1
ℓ^-_R	½	0	0	−2	−1
u_L	½	½	+½	+⅓	⅔
d'_L	½	½	−½	+⅓	−⅓
u_R	½	0	0	+4/3	⅔
d_R	½	0	0	−⅔	−⅓
W^+	1	1	+1	0	+1
W^-	1	1	−1	0	−1
Z	1	1,0	0	0	0
γ	1	1,0	0	0	0
H	0	0	0	0	0

(*) $Q = I_{wz} + ½\, Y_W$.

- Weak isospin (I_W) and weak hypercharge (Y_W) are conserved in all known interactions.
- I_W and Y_W have nothing to do with those of hadrons.
- I_W is the source of the weak charged fields $W^\pm$.
- Y_W and I_{Wz} are the sources of the weak neutral field Z and of the e.m. field γ.
- The L(eft) components of the spinors have $I_W \neq 0$; they emit and absorb $W^\pm$.
- The R(ight) components have $I_W = 0$; they do not emit or absorb $W^\pm$.
- Both components have $Y_W \neq 0$; they emit and absorb Z.
- the ν_R have $I_W = 0$ and $Y_W = 0$; they do not exist or are not observable (in the m=0 limit).

the e.w. theory: the IVBs W/Z

The field W_μ = ($W_{\mu 1}$, $W_{\mu 2}$, $W_{\mu 3}$) is a 4-vector in the space-time(*), and a vector in the space of the weak isospin I_W of SU(2) (index $_{123}$), because it has I_W = 1 :

- The fields of the physical charged bosons:
  $W^\pm = (W_1 \mp iW_2) / \sqrt{2}$;
- For each doublet of fermions there is a 4-vector, which is at the same time a 3-vector in the I_W space, which represents the weak current :
  $j_\mu \equiv (j_{\mu 1}, j_{\mu 2}, j_{\mu 3})$;
- The field W_μ is coupled to j_μ as ($g\, W_\mu\, j_\mu$) through the dimensionless coupling constant g.
- The charged currents are linear combinations of two current components
  $j^\pm = (j_1 \pm j_2)$.
- E.g., consider the doublet (ν_{eL}, e^-_L); the corresponding charged currents are
  $j^+_{e\mu} = \bar{\nu}_{eL} \gamma_\mu e^-_L$; $\quad j^-_{e\mu} = \bar{e}^-_L \gamma_\mu \nu_{eL}$.

The field B_μ is a 4-vector in space-time and a scalar in isospin (I_W = 0). It interacts with the neutral current of the leptons j_μ (4-vector - isoscalar) through the coupling constant g'.

- The current generated by the hypercharge is twice the difference between the electric current j^{EM}_μ and the neutral component of the NC :
  $Y_W = 2\,(Q - I_{Wz}) \rightarrow j^Y_\mu = 2 j^{EM}_\mu - 2 j_{3\mu}$.
- The first term is the electromagnetic current, which for charged fermions is
  $j^{EM}_{f\mu} = \bar{f} \gamma_\mu f$.
- The chirality is not specified because the electro-magnetic interactions do not depend on it.

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(*) *warning : here the μ index refers to the space-time dimensions, NOT to the $\mu^\pm$ lepton.*

the e.w. theory: mixing angle θ_w

Call **A** and **Z** respectively the physical fields that mediate the electromagnetic and neutral currents.

- They are two mutually orthogonal linear overlap of W_3 and B, which can be determined by requiring that the photon does not couple to the neutral particles, contrary to the Z.
- The transformation is given as a function of two couplings g and g', i.e. as a rotation of an angle θ_w, the mixing angle of the weak interactions [a.k.a. the *Weinberg angle*] :

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix};$$

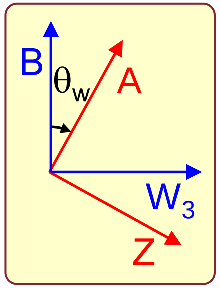
$$\tan\theta_w = g'/g.$$

- The mixing angle θ_w is an observable, and is measured to be $\theta_w \approx 29°$ (*).
- The interaction Lagrangian, being symmetric under the gauge group, is an isoscalar :

$$\mathscr{L} = g\,(j_{\mu 1}W_{\mu 1} + j_{\mu 2}W_{\mu 2} + j_{\mu 3}W_{\mu 3}) + + \tfrac{1}{2}g' j_\mu^Y B_\mu,$$

which can also be written as:

$$\mathscr{L} = g/\sqrt{2}\,(j_\mu^- W_\mu^+ + j_\mu^+ W_\mu^-) + + j_\mu^3\,(gW_{\mu 3} + g'B_\mu) + + g'\, j_\mu^{EM} B_\mu.$$



(*) usually experiments measure $\sin^2\theta_w$ (see next chapter).

the e.w. theory: interactions

- Then, introducing the neutral physical fields:

$$\mathscr{L} = \frac{g}{\sqrt{2}}\left(j_\mu^- W_\mu^+ + j_\mu^+ W_\mu^-\right) + \qquad \textbf{CC}$$

$$+\frac{g}{\cos\theta_w}\left(j_{\mu 3} - \sin\theta_w j_\mu^{EM}\right) Z_\mu + \qquad \textbf{NC}$$

$$+ g \sin\theta_w j_\mu^{EM} A_\mu . \qquad \textbf{EM}$$

- The equation contains three terms :

  CC : the charged current interactions;

  NC : the neutral current interactions;

  EM : the electromagnetic interactions.

- The constant which multiplies the last term has to be proportional to the electrical charge, to ensure that the photon is NOT coupled to neutral particles ($\hbar$ = c = 1) :

$$g \sin\theta_W = q_e = \sqrt{4\pi\alpha}.$$

- All the interactions, mediated by the four vector bosons, are expressed in terms of two constants, the electric charge q and the weak angle θ_w.

- However, the model does not predict the values of the two fundamental constants, which must be determined experimentally.

- The numerical relations between the fundamental constants, obtained from low-energy value of α (≈ 1/137) and the best measurement of θ_w) ($\sin^2\theta_w$ ≈ 0.232), are :

$$\frac{1}{\alpha} = \frac{4\pi}{g^2} + \frac{4\pi}{g'^2}; \quad \frac{4\pi}{g^2} = 31.8; \quad \frac{4\pi}{g'^2} = 105.2.$$

- The second term in the equation gives the coupling between the Z and the fermions.

the e.w. theory: summary of formulæ

- The Z coupling is "universal“: it only depends on the electric charge and weak isospin:

$$g_Z \equiv \frac{g}{\cos\theta_W}\left[I_{W_Z} - Q\sin^2\theta_W\right] =$$
$$= \frac{\sqrt{4\pi\alpha}}{\sin\theta_W \cos\theta_W}\left[I_{W_Z} - Q\sin^2\theta_W\right].$$

warning: these formulæ (and all those of this chapter) are simple and elegant, but only valid in 1st order; *the devil lies in the details* (= higher order corrections).

Useful formulæ:

$$\boxed{m_W^2 = \frac{\sqrt{2}g^2}{8G_F} = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2\theta_W} = \left(\frac{37.3}{\sin\theta_W}\ \text{GeV}\right)^2}$$

$$\boxed{G_F = \frac{\sqrt{2}g^2}{8m_W^2}}$$

$$\boxed{\tan\theta_W = \frac{g'}{g}}$$

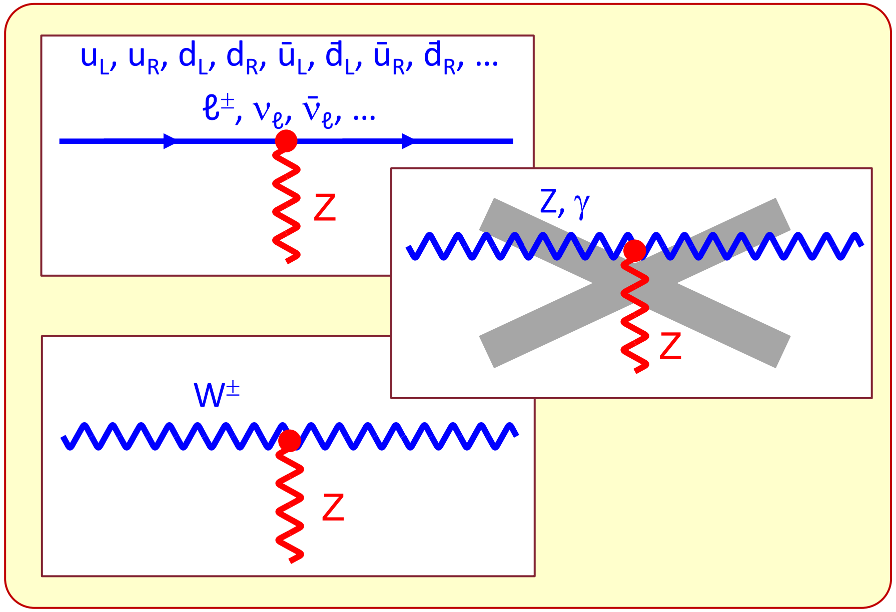
$$\boxed{g\sin\theta_W = q_e = \sqrt{4\pi\alpha}}$$

$$\boxed{\frac{1}{\alpha} = \frac{4\pi}{g^2} + \frac{4\pi}{g'^2}}$$

$$\boxed{m_W = m_Z\cos\theta_W}$$

the e.w. theory: NC

- **The Neutral Currents (NC)** have important differences compared to CC.
- NO FCNC, i.e. fermions are only coupled with themselves (e.g. $e^- \leftrightarrow e^-$, $u_{red} \leftrightarrow u_{red}$, NOT $u_{red} \leftrightarrow u_{blue}$, NOT $u \leftrightarrow c$, etc).
- They do not have the simple coupling [$\gamma_\mu(1-\gamma_5)$, i.e. "V – A"], but are a mixture of both left and right couplings.
- The currents of the 1st family (the other families are similar) are :



$$J(\nu) = \frac{1}{2} g_L^{\nu_e} \bar{\nu}_e \gamma_\mu (1-\gamma_5) \nu_e = g_L^{\nu_e} \bar{\nu}_{eL} \gamma_\mu \nu_{eL};$$

$$J(e) = \frac{1}{2} g_L^{e} \bar{e} \gamma_\mu (1-\gamma_5) e + \frac{1}{2} g_R^{e} \bar{e} \gamma_\mu (1+\gamma_5) e = g_L^{e} \bar{e}_L \gamma_\mu e_L + g_R^{e} \bar{e}_R \gamma_\mu e_R;$$

$$J(u) = \frac{1}{2} g_L^{u} \bar{u} \gamma_\mu (1-\gamma_5) u + \frac{1}{2} g_R^{u} \bar{u} \gamma_\mu (1+\gamma_5) u = g_L^{u} \bar{u}_L \gamma_\mu u_L + g_R^{u} \bar{u}_R \gamma_\mu u_R;$$

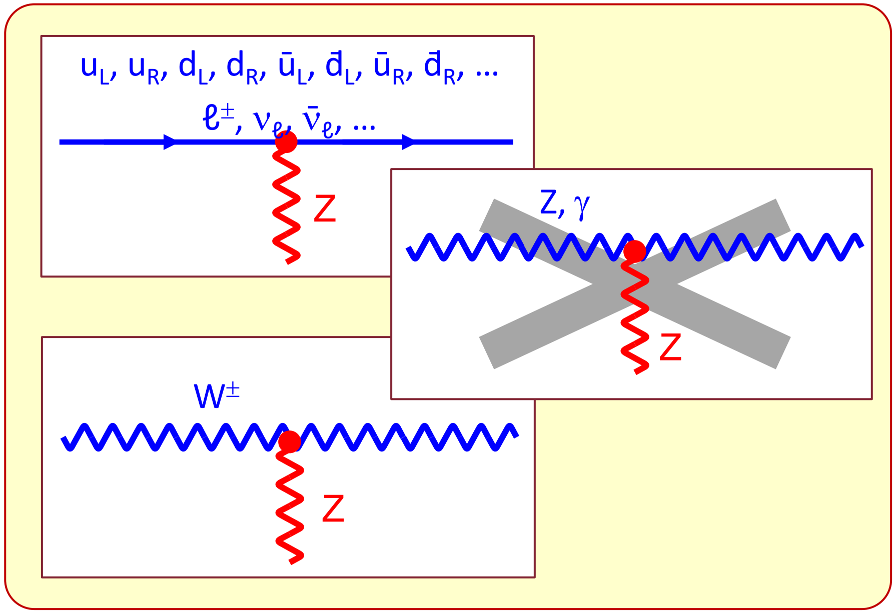
$$J(d) = \frac{1}{2} g_L^{d} \bar{d} \gamma_\mu (1-\gamma_5) d + \frac{1}{2} g_R^{d} \bar{d} \gamma_\mu (1+\gamma_5) d = g_L^{d} \bar{d}_L \gamma_\mu d_L + g_R^{d} \bar{d}_R \gamma_\mu d_R.$$

i.e. 7 parameters $g_L^\nu + g_{L,R}^{e,u,d}$ [see § 7]:

the e.w. theory: NC couplings

- In the SM the 7 couplings are equal for the 3 families and functions of only two parameters (α_{em} and θ_w).
- The Z couples with quarks/leptons:
  - ➢ charged fermions, both L and R;
  - ➢ ν's and $\bar{\nu}$'s, even if they have no charge, because they have $I_{Wz} \neq 0$;
  - ➢ $W^\pm$.
- The Z does NOT couple (in lowest order) to particles with both Q = 0 and I_{Wz} = 0, i.e. the γ, the Z itself (and the gluons).
- In NC processes, the unification of the weak and electromagnetic interactions is particularly evident.
- The following tests have been performed [*those with "➢" will be discussed in these lectures or in Collider Physics*]:

- parity violation in atoms (scale = eV);
- ➢ DIS ν_μ on electron (scale = MeV);
- scattering of polarized electrons on D_2 (GeV);
- ➢ asymmetries in $e^+e^- \to \mu^+\mu^-$ (from 10 GeV to 200 GeV);
- ➢ DIS ν_μ and $\bar{\nu}_\mu$ on nuclei (several GeV);
- ➢ the measurement of Z parameters themselves.



electroweak results: theory ↔ exp

- All the couplings can be expressed in terms of the values of g and g' (usually α and θ_w).
- The experiments measure observables (cross sections, decay rates, …) and compare calculated ↔ measured quantities.
- The calculation is based on a "perturbative series" : lowest order (= "tree level"), higher orders (= "radiative corrections").
- The table shows an incomplete set of e.w. results since '70s: **hundreds** of measurements, **no inconsistency** found, **no disagreement**.
- In our lectures only a small part of the measurements will be examined, in the context of their experimental environment.
- The overall picture is impressive.

recently, observed a possible disagreement between the μ magnetic moment and its SM prediction: be cautious, wait'n see.

Score

✓ CC processes at low energy : well described by Fermi theory.

✓ NC processes : direct test of unification.

✓ Gauge boson ($W^{\pm}$, Z) existence.

✓ Gauge boson ($W^{\pm}$, Z, γ) coupling.

✓ **Fermion mass generation (Higgs boson existence).**

? **Higgs boson couplings** $^{(*)}$.

✓ Quark mixing and CP violation $^{(*)}$.

? **Neutrino masses.**

? **Neutrino mixing.**

$^{(*)}$ *Looks OK, with some possibility of surprises.*

QCD

- The **color** quantum number was introduced [*see* §1] to avoid Pauli principle violation for **the Δ^{++}**;
- color is necessary to explain the value of **R** [$=\sigma_{hadrons}/\sigma_{\mu+\mu-}$] in e^+e^- interactions, which shows an excess of a factor 3 [*see* §3];
- in a similar way, it is necessary to explain the decay rate **$\pi^0 \to \gamma\gamma$** [*next slide*];
- all these observations have convinced the physicists of the existence of "static" color;
- is color also important for **dynamics** ?
- the theory must also explain **confinement** (= *no free quarks*) and **asymptotic freedom** (= *qpm, i.e. quarks almost free at high* Q^2);
- the modern **QCD** is a gauge theory, based on the symmetry $SU(3)_{color}$, mathematically equivalent to $SU(3)_{flavor}$, but based on a completely different dynamics;
- the carriers of the force are 8 colored massless vector (= spin 1) bosons, called **gluons** [*"glue" as an example of a strong force with short range*];
- compared to QED, the differences are in the behavior of the fields, i.e. **gluons $\leftrightarrow \gamma$** :

	"matter" = fermions			"fields" = bosons				
	name	spin	"charge"	name	spin	"charge"	mass	self-coupling
QED	ch. leptons +quarks	½	yes	γ	1	no	no	no
QCD	quarks	½	yes	**8 gluons**	1	**yes**	no	**yes**

QCD : π^0 decay

[the π^0 decay is an e.m. process, NOT a strong one; we discuss it here because it critically depends on the number of colors, i.e. on a QCD parameter.]

- An independent test of the color charge of the quarks comes from a completely different measurement, the **π^0 decay**;
- compute the decay amplitude, by introducing an (a-priori unknown) arbitrary normalization factor "N_c" :

$$\langle \gamma\gamma | \mathbb{H}_{em} | \pi^0 \rangle = \left\langle \gamma\gamma \left| \mathbb{H}_{em} \right| \frac{(u\bar{u} - d\bar{d})}{\sqrt{2}} \right\rangle$$

$$\propto f_\pi \frac{N_c}{\sqrt{2}} \left(\frac{4}{9} e^2 - \frac{1}{9} e^2 \right);$$

where f_π is the decay constant of the π^0, which is related to the wave-function overlap of the quark and antiquark;

- the full computation gives(*) :

$$\Gamma(\pi^0 \to \gamma\gamma)\Big|_{\text{theo.}} = \left(\frac{q_u^2 - q_d^2}{e^2} \right)^2 \frac{N_c^2 \alpha^2 m_\pi^3}{32\pi^3 f_\pi^2} =$$

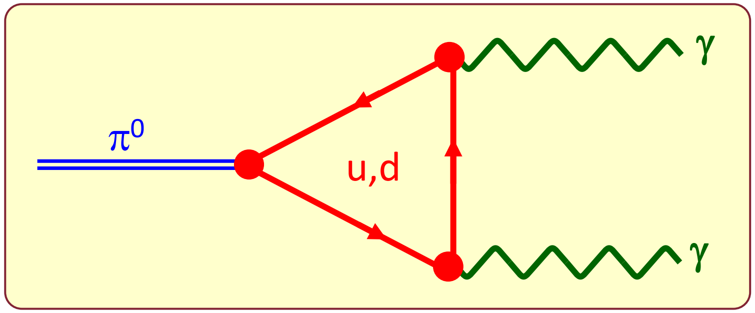
$$= 7.64 \left(\frac{N_c}{3} \right)^2 \text{ eV};$$

$$\Gamma(\pi^0 \to \gamma\gamma)\Big|_{\text{exp.}} = (7.84 \pm 0.6) \text{ eV};$$

$$\to N_c = 2.96 \pm 0.11,$$

not compatible with N_c=1, but with the QCD prediction N_c = 3.

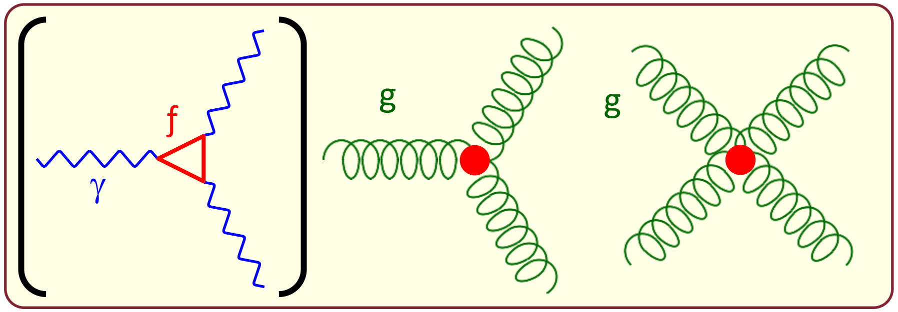
(*) warning : "$32\pi^3 f_\pi^2$" depends on the definition of f_π; in the literature also "16" or "64".



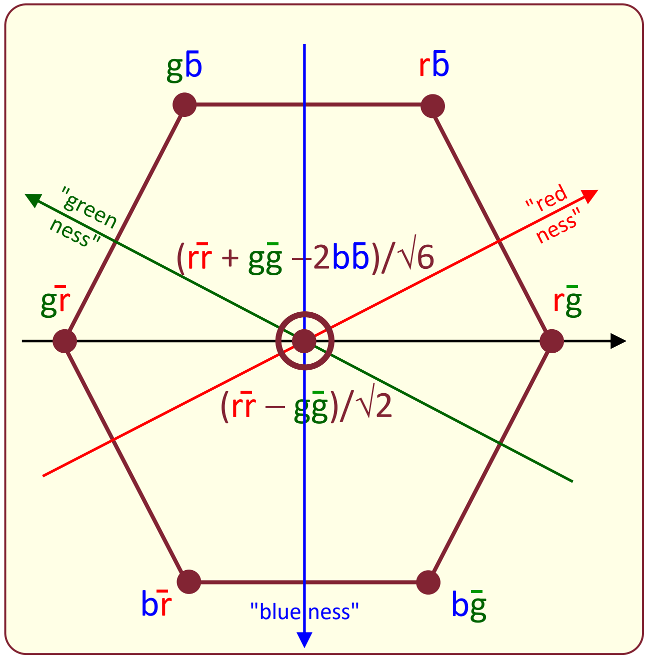
QCD : color

The **color** is a charge, equivalent to the electric charge :

- it is exactly conserved in strong processes;
- it obeys the "video-display" rgb rules (e.g. red + green = yellow, so yellow = $\overline{\text{blue}}$);
- a gluon carries two colors (which is equivalent to an anti-color, see above);
- gluons are colored, therefore **self-coupled**; the vertex with three bosons is allowed in QCD, while in QED it happens only on higher orders (with a triangle of fermions); also 4-gluons verteces are allowed;



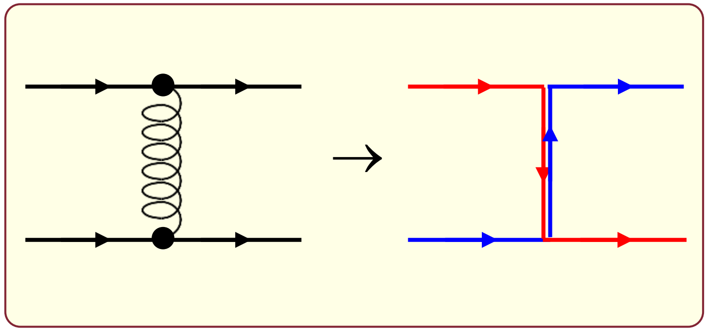
- the number of gluons (8) comes from the number of generators of SU(3) (Gell-Mann matrices [*see* § *1*]);
- similarly, it comes from the independent combinations of two rgb ($r\bar{r}$, $g\bar{g}$, $b\bar{b}$, $r\bar{b}$, $r\bar{g}$, $g\bar{b}$, $g\bar{r}$, $b\bar{r}$, $b\bar{g}$), after removing the singlet combination [$(r\bar{r} + g\bar{g} + b\bar{b}) / \sqrt{3}$];
- the gluon octet is similar to the $q\bar{q}$ one :

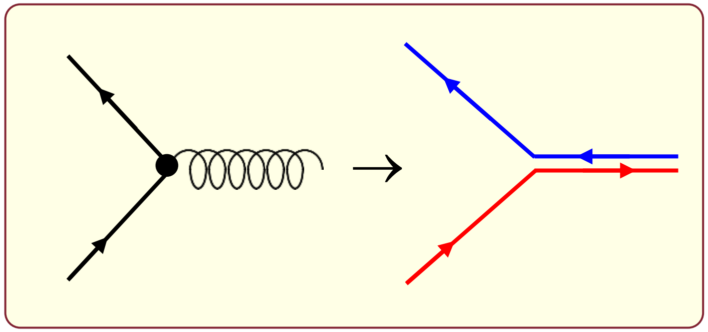


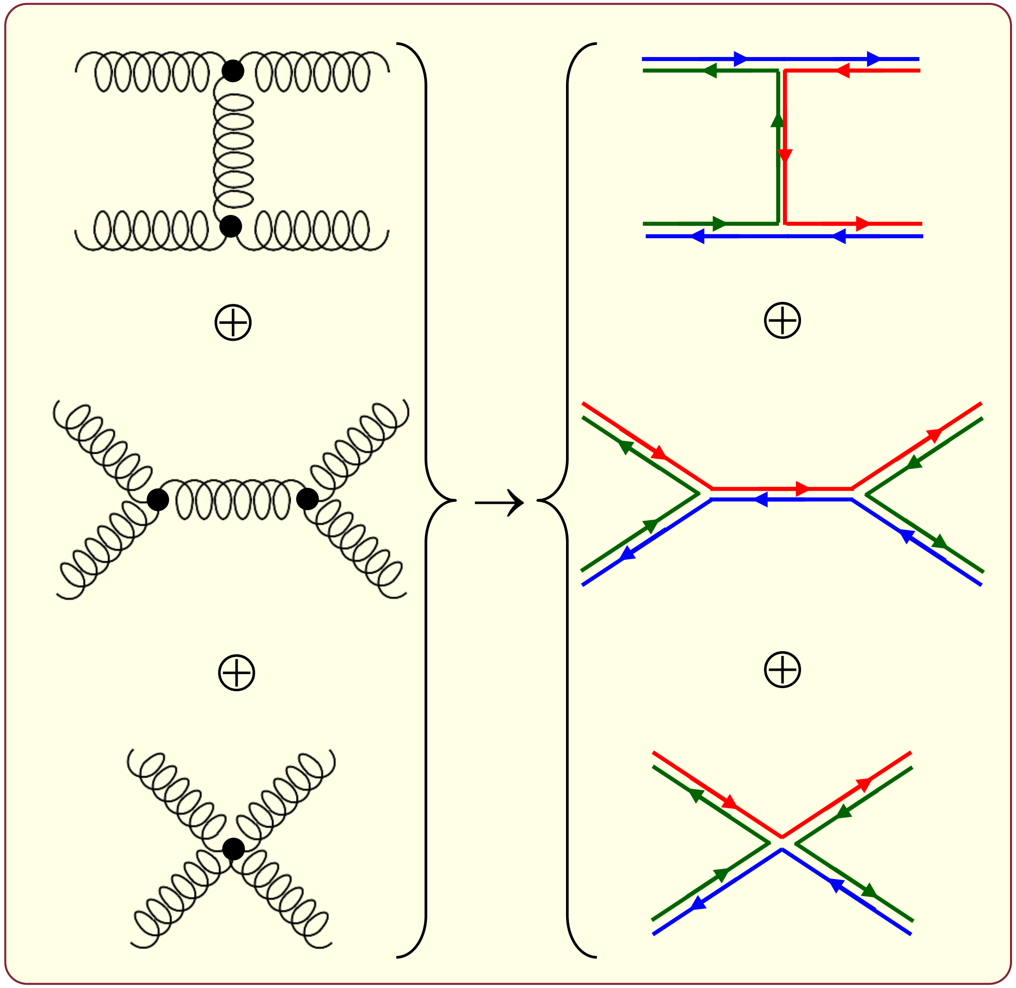
QCD : gluon color

Examples of quark-gluon diagrams with emphasis on color conservation :

- in this page, color is "QCD-color";
- only one shown of the many color permutations;

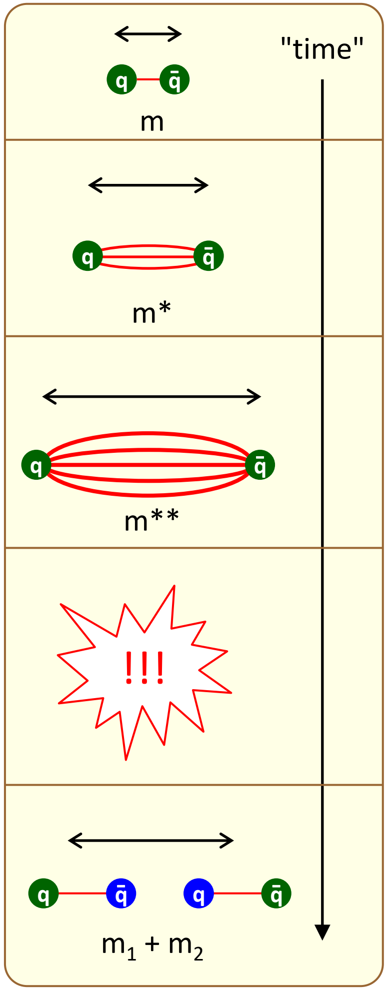






QCD : confinement

- The color does NOT manifest directly in an observable property of the particles (something like a "red" particle has never been observed).
- The standard explanation of this fact requires that only "white" ("*color singlets*") states be physically existent.
- The consequence is that quarks and gluons themselves cannot be observed as free states (***confinement***); they exist only inside "molecules" (= hadrons)
- The mathematical formalism of QCD gives an account for that.
- Some naïve classical models, with similarity to springs and magnetism (the "broken magnet") are often quoted.
- An important consequence is that partons (quarks and gluons), created in e^+e^- or hadronic scattering, must undergo a complicated mechanism which finally produces only color singlets (hadrons) in a spray of particles (***jets***).



QCD : asymptotic freedom

- The study of the DIS [§ 2] shows that at high Q^2, the projectile "sees" smaller objects inside the nucleon.
- At small distances the force between quarks and gluons is apparently smaller and smaller : the quarks behave as free objects; the scattering onto free quarks is the origin of the Bjorken scaling [§ 2].
- This effect is called **asymptotic freedom** and is the core of QCD [Gross, Politzer, Wilczek – Nobel Prize 2004].
- With increasing distance among the quarks (i.e. lower Q^2), the intensity of the strong force increases, keeping the quarks "confined" in the nucleon.
- At some distance the available energy becomes sufficient to create a new quark-antiquark pair, eventually leading to the production of new hadron(s), but PREVENTING the emission of quarks as free particles.
- Summary : among quarks there exists a "color" field. The gluons that mediate this force act as additional sources of the color field ("*gluons are non-abelian*"). The gluon-gluon interaction "pulls" the lines of force of the color in a narrow tube, a sort of a "string", similar to a spring, whose tension ("= *potential energy*") increases with length.

David Gross | David Politzer | Frank Wilczek

Strong interactions at low Q^2: hadrons

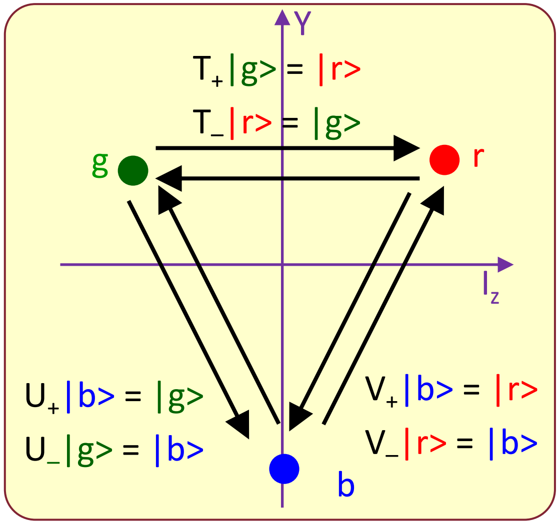
remember §1, "color"

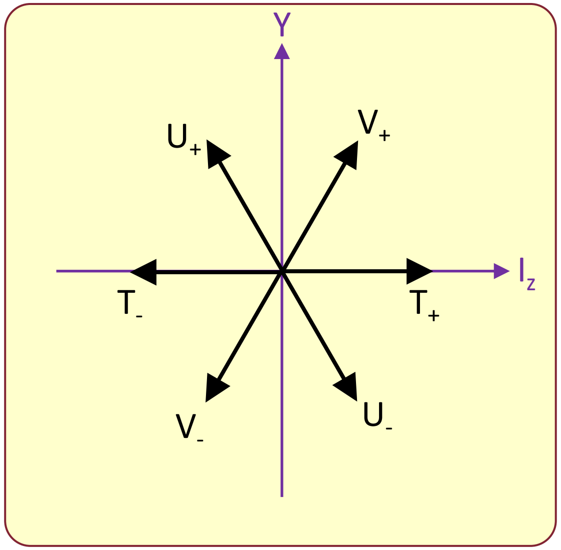
here "rgb" are quarks with the appropriate flavor.

The particles of the theory are built from the SU(3) rules.

Technically, introduce the ladder operators $T_\pm$, $U_\pm$, $V_\pm$ [§1 + **the QCD dynamics**]:

- mesons are **color singlets** [a $q\bar{q}$ pair with symmetric wave function: $(r\bar{r} + g\bar{g} + b\bar{b}) / \sqrt{3}$];
- baryons are also color singlets [qqq with antisymmetric w.f.: $(rgb + gbr + brg - grb - bgr - rbg) / \sqrt{6}$];
- [mesons are their own anti-particles]
- [anti-baryons are $\bar{q}\bar{q}\bar{q}$ states with the same rules]
- *puzzling : there are also other possible color singlets : $q\bar{q}q\bar{q}$, $qqqq\bar{q}$* [*next slide*], *or glue-glue bound states …*
- *no (QCD-based) rule forbids their existence; in the past there have been (well founded ?) claims of discovery (tetraquarks, pentaquarks, glueballs, …).*

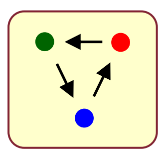




S.i. at low Q^2: $SU(3)_C$

C	$\mathcal{B}$	I_{Cz}	Y_C	C	$\mathcal{B}$	I_{Cz}	Y_C
r	+⅓	+½	+⅓	$\bar{r}$	–⅓	–½	–⅓
g	+⅓	–½	+⅓	$\bar{g}$	–⅓	+½	–⅓
b	+⅓	0	–⅔	$\bar{b}$	–⅓	0	+⅔

- mathematically identical to $SU(3)_{flavor}$ (see);
- define a "color isospin" (I_C, I_{Cz}) and a "color hypercharge" (Y_C);
- rgb just names, no connection with ordinary "colors";
- for baryons, ψ_{color} :
  - $\mathcal{B}$ = 1 $\Rightarrow$ 3-quarks (or 3 antiquarks);
  - normalized;
  - overall color = 0;
  - anti-symmetric (Pauli principle);
- therefore, only one solution :



$$\psi_{color}^{baryons} = \frac{1}{\sqrt{6}}\begin{pmatrix} r_1g_2b_3 + g_1b_2r_3 + b_1r_2g_3 + \\ -r_1b_2g_3 - b_1g_2r_3 - g_1r_2b_3 \end{pmatrix};$$

The most general quark combination is :

$\psi_{color} = r^\alpha g^\beta b^\gamma \bar{r}^{\bar{\alpha}} \bar{g}^{\bar{\beta}} \bar{b}^{\bar{\gamma}}$; [$\alpha,\beta$,... integer]

define (whithout loss of generality) :

$m \equiv \alpha+\beta+\gamma \geq n \equiv \bar{\alpha}+\bar{\beta}+\bar{\gamma}$;

$I_{Cz} = (\alpha-\bar{\alpha})/2 - (\beta-\bar{\beta})/2 = 0$;

$Y_C = (\alpha-\bar{\alpha})/3 + (\beta-\bar{\beta})/3 - 2(\gamma-\bar{\gamma})/3 = 0$;

$\Rightarrow (\alpha-\bar{\alpha}) = (\beta-\bar{\beta}) = (\gamma-\bar{\gamma}) \equiv p$;

$\Rightarrow \alpha+\beta+\gamma-\bar{\alpha}-\bar{\beta}-\bar{\gamma} = m-n = 3p; \quad p \geq 0$;

$\Rightarrow \psi_{color} \equiv q^m\bar{q}^n = q^{3p+n}\bar{q}^n = (qqq)^p(q\bar{q})^n$.

The simplest cases are :

- p = 1, n = 0 $\rightarrow$ baryons qqq (+ anti–);
- p = 0, n = 1 $\rightarrow$ mesons $q\bar{q}$;
- many other possibilities NOT forbidden, e.g. (p=n=1; p=0,n=2) $\rightarrow$ ($qqq\,q\bar{q}$; $q\bar{q}q\bar{q}$);
- searches (and claims...).

S.i. at low Q^2: multiquarks ? glueballs ?

Modern remark on tetra- and penta-quarks

M.Karliner et al, Annu. Rev. Nucl. Part. Sci. 68:17-44 (2018) [*emphasis mine*]:

Why do we see certain types of elementary particles and not others ? This question was posed more than 50 years ago in the context of the quark model. Gell-Mann and Zweig proposed that the known mesons were $q\bar{q}$ and baryons qqq, with the quarks known at the time, u (up), d (down), and s (strange), having charges of ⅔, −⅓, and −⅓, respectively. Mesons and baryons would then have integral charges.

Mesons such as $qq\bar{q}\bar{q}$ and baryons such as $qqqq\bar{q}$ would also have integral charges. Why weren't they seen? *They have now been seen*, but only with additional heavy quarks and under conditions that tell us a lot about the strong interactions and how they manifest themselves.

(...)

A look back at the experimental developments in hadron spectroscopy in the new millennium shows that heavy quarks have done it again! After converting us into firm believers in the quark model in the 1970s, heavy quark systems have taught us a new lesson: *Not all hadronic states are minimal quark combinations*. In addition to $q\bar{q}$ mesons, four-quark $qq\bar{q}\bar{q}$ configurations become important, especially near and above the $q\bar{q}$ + $q\bar{q}$ meson thresholds. Similarly, not all baryons are qqq states; $qqqQ\bar{Q}$ configurations also play a role.

Theoretical disputes continue as to whether the observed multiquark configurations are tightly bound tetra- and penta-quarks or loosely bound meson-meson and baryon-meson *molecules*. ***In our opinion***, the case for the latter is stronger. It is also beyond dispute that baryon-baryon *molecules* exist and have been known for a long time as nuclei.

(...)

[*they are comparing e.g. a deuterium molecule (D_2, i.e. $^2H_2$) with ^4He, both bound states (ppnn)*]

S.i. at low Q^2: non-relativistic potential

A semi-classical approach for the QCD potential from experimental data :

- for small distances ($r \to 0$), Coulomb shape, with a stronger coupling α_s (instead of α_{em}) :

$$V(\text{small } r) = -4\alpha_s / (3\, r)$$

- at high distances ($r \to \infty$), a linearly increasing function, responsible for confinement :

$$V(\text{large } r) = k\, r$$

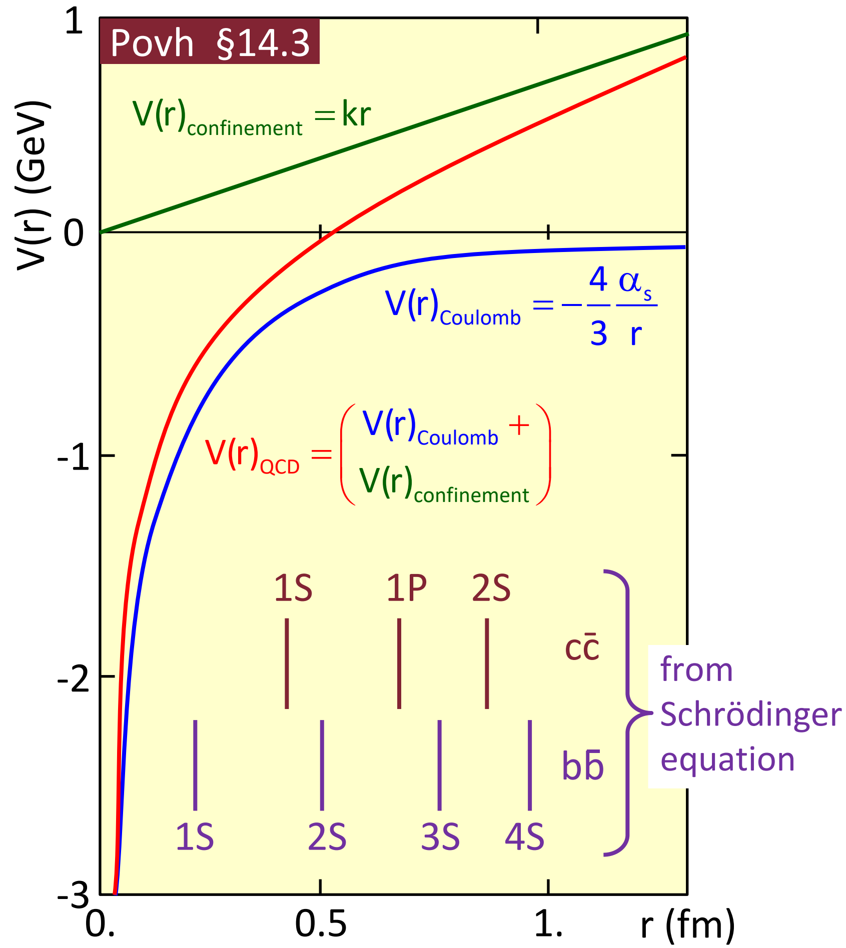
- all together (see fig) :

$$V(r) = -4\alpha_s / (3r) + k\, r$$

- parameters α_s and k adjusted to fit data : $\alpha_s \approx 0.15 \div 0.25$, $k \approx 1$ GeV fm^{-1};
- then (numerically) solve the Schrödinger equation and derive (e.g.) the properties of bound states;

see also [MS, 6.4.3] for $V(r) = A \ln(r/B)$.

- approximation supposed to work better in non-relativistic case, V << m;
- (fair) agreement with reality, especially in the heavy quark sector.



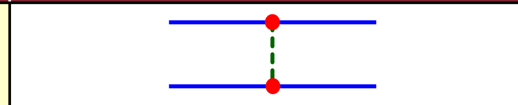
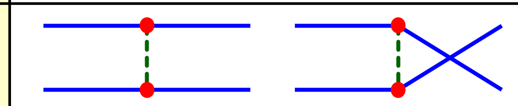
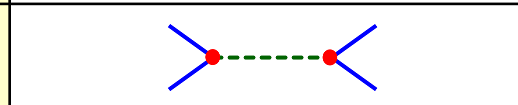
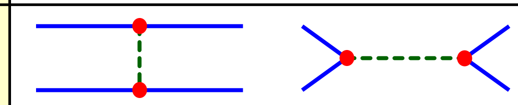
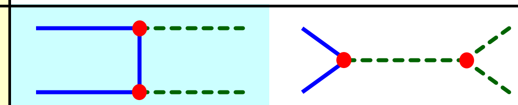
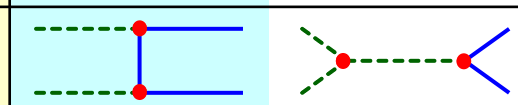
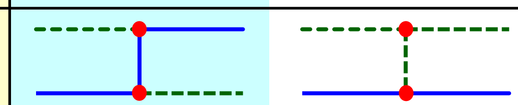
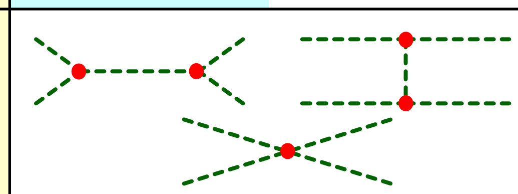
Strong interactions at high Q^2

2-body process: **{q$\bar{q}$g} {q$\bar{q}$g} → {q$\bar{q}$g} {q$\bar{q}$g}**, e.g. (q$\bar{q}$ → q$\bar{q}$) or (qg → qg).

- picture only valid <u>at high Q^2</u>: at low Q^2 hadrons scatter coherently (see § 2), → rest of discussion assumes high Q^2;
- <u>8 cases</u>, according to {q$\bar{q}$g} [next slide];
- impossible to distinguish on an event-by-event basis (compare QED $e^+e^+ \leftrightarrow e^+e^-$ vs QCD qq ↔ q$\bar{q}$, always present because of the sea) [a bonus, but a difficult one];
- therefore all processes mixed together, difficult (≈ impossible) to disentangle on an event-by-event basis: only statistical mixtures measurable;
- weights of stat. mixture are **couplings** at parton level (get from theory) * **PDF** (parametrize/evolve) [a difficult game];
- in hadronic initial states (h.i.s.) the energy at parton level (**ŝ**) is different from energy at hadron level (s); same for **t̂** and **û** ↔ t and u [next slide uses stu, but means ŝt̂û];
- in h.i.s. s from beam energy, but ŝ difficult to measure and different for each event;
- jets, not single partons in final state: in general (q ↔ $\bar{q}$ ↔ g) not distinguishable, single quarks (e.g. b ↔ u d) difficult;
- t̂−channels much more abundant than ŝ-channels; gluon channels more abundant than quark- : a disgrace for the search of $W^\pm$, Z, H, which mainly come from q-$\bar{q}$ processes in the ŝ channel.

<u>Conclusion</u>: a rich and difficult game, which requires a lot of events, strong computing power, intelligent analysis [*i.e.* ***<u>YOU</u>***].

S.i. at high Q^2: $2 \to 2$ processes

process	$\left[\frac{d\sigma}{d\Omega} = \alpha_s^2 f(s,t,u)/(9s)\right]$ $f(s,t,u)$	$f(\theta=90°)$ $[t=u=-s/2]$	diagram(s)	QED equivalent
$qq' \to qq'$ $\bar{q}q' \to \bar{q}q'$	$(s^2+u^2)/t^2$	5		$e^-\mu^- \to e^-\mu^-$ $e^+\mu^- \to e^+\mu^-$
$qq \to qq$ $\bar{q}\bar{q} \to \bar{q}\bar{q}$	$(s^2+t^2)/u^2+(s^2+u^2)/t^2-2s^2/(3ut)$	$7+\frac{1}{3}= 7.3$		$e^-e^- \to e^-e^-$ $e^+e^+ \to e^+e^+$
$q\bar{q} \to q'\bar{q}'$	$(t^2+u^2)/s^2$	$\frac{1}{2}= 0.5$		$e^+e^- \to \mu^+\mu^-$
$q\bar{q} \to q\bar{q}$	$(t^2+u^2)/s^2+(s^2+u^2)/t^2-2u^2/(3st)$	$5+\frac{5}{6}= 5.8$		$e^+e^- \to e^+e^-$
$q\bar{q} \to gg$	$8/3(t^2+u^2)[1/(tu) -9/(4s^2)]$	$2+\frac{1}{3}= 2.3$		$e^+e^- \to \gamma\gamma$
$gg \to q\bar{q}$	$3/8(t^2+u^2)\ [1/(tu) -9/(4s^2)]$	$\frac{21}{64}= 0.3$		$\gamma\gamma \to e^+e^-$
$gq \to gq$ $g\bar{q} \to g\bar{q}$	$(s^2+u^2)[9/(4t^2)-1/(su)]$	$13+\frac{3}{4}=$ 13.8		$\gamma e^- \to \gamma e^-$ $\gamma e^+ \to \gamma e^+$
$gg \to gg$	$81/8[3-ut/s^2-su/t^2-st/u^2]$	$68+\frac{11}{32}=$ 68.3		$[\gamma\gamma \to \gamma\gamma]$

The **lowest order processes of the strong interactions in QCD:**

- s,t,u,θ at parton level ($\hat{s}$, $\hat{t}$, $\hat{u}$);
- $q' \neq q$.

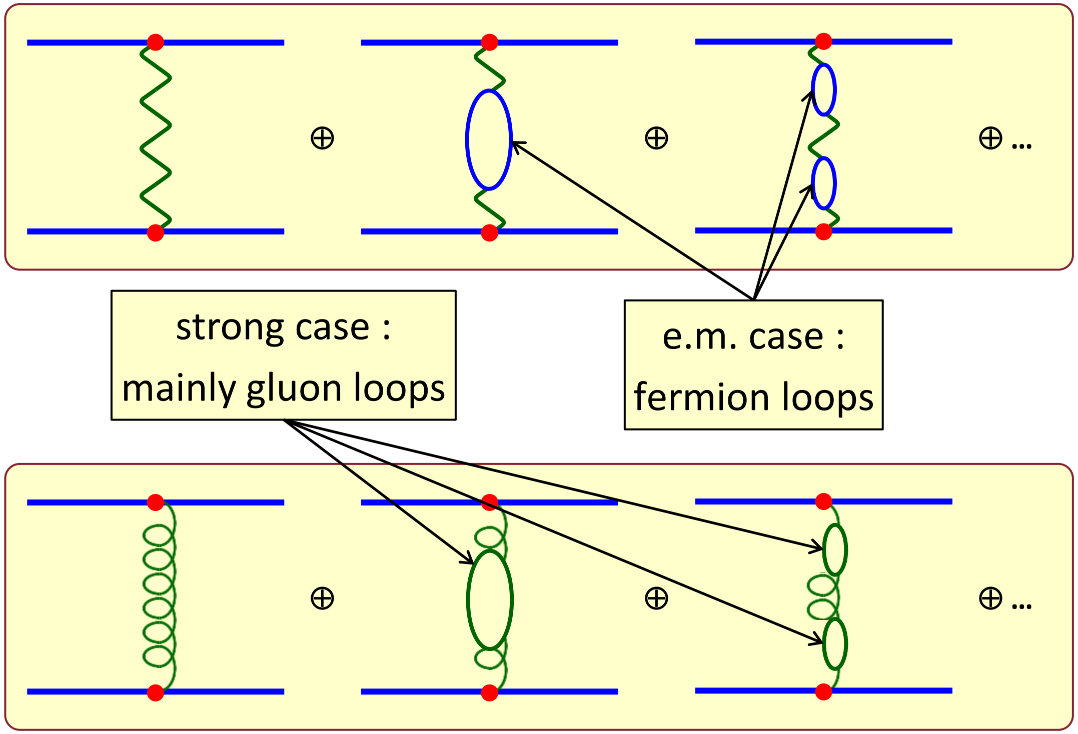
——— q or $\bar{q}$;
- - - - γ_{QED} or g_{QCD}.

S.i. at high Q^2: $\alpha_s = \alpha_s(Q^2)$

More effective approach for scattering processes : *reabsorb higher orders* into an effective α_s :

$\rightarrow$ loops increases for higher Q^2 ;

$\rightarrow$ evolution of the coupling α_s from its low-Q^2 value, with standard Feynman techniques;



Important difference $\alpha_s \leftrightarrow \alpha_{em}$:

- higher order loops in α_{em} only due to fermions $\rightarrow$ *increase* of α_{em} as a function of Q^2;
- instead, since the gluons are self-coupled, loops in α_s mainly due to bosons $\rightarrow$ *decrease* of α_s with Q^2;
- the formulæ show the "*running*" of α_{em} and α_s with Q^2 :
- (confinement and asymptotic freedom automatically produced).

$$\alpha_{em}\left(Q^2\right) \approx \alpha_{em}\left(Q_0^2\right) \Big/ \left[1 - \frac{\alpha_{em}\left(Q_0^2\right)}{3\pi} \ell n\left(\frac{Q^2}{Q_0^2}\right)\right];$$

$$\alpha_s\left(Q^2\right) \approx \alpha_s\left(Q_0^2\right) \Big/ \left[1 + \alpha_s\left(Q_0^2\right) \frac{11N_c - 2N_f}{12\pi} \ell n\left(\frac{Q^2}{Q_0^2}\right)\right].$$

S.i. at high Q^2: $\alpha_s \leftrightarrow \alpha_{e.m.}$

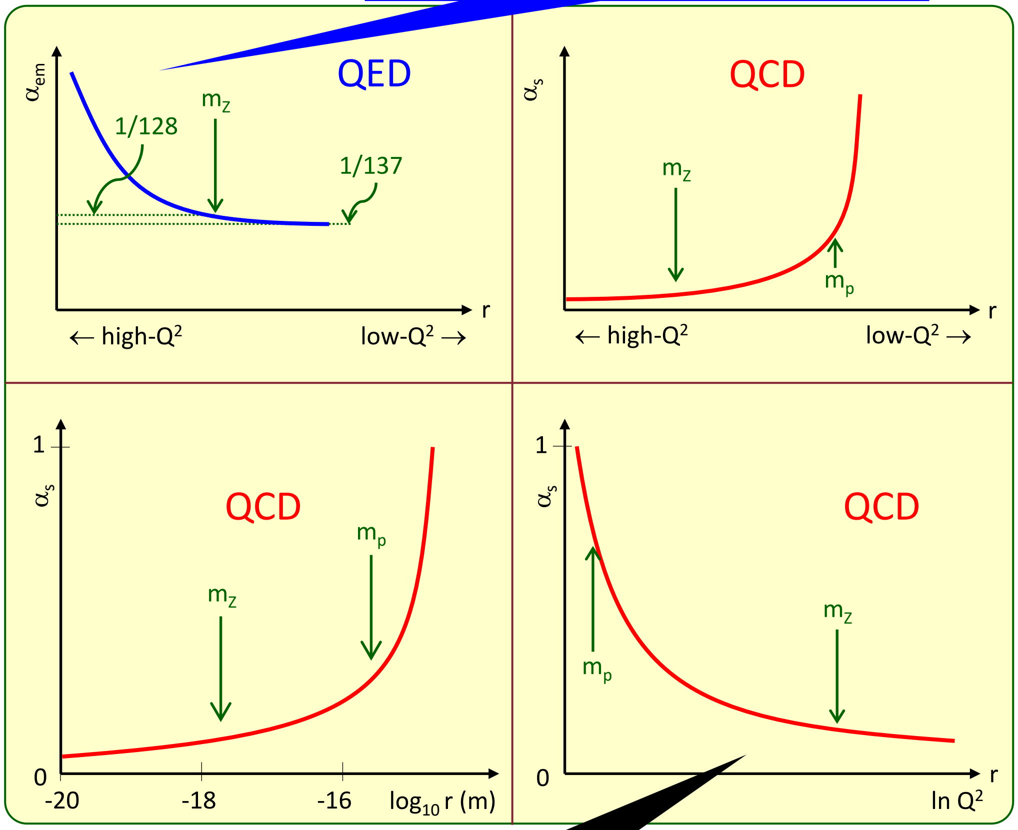
$$\alpha_s(Q^2) = \alpha_s(Q_0^2) \Big/ \left[1 + \alpha_s(Q_0^2)\frac{11N_c - 2N_f}{12\pi}\ell n\left(\frac{Q^2}{Q_0^2}\right)\right]; \quad N_c = 3; \quad N_f = 3 \to 4 \to 5 \to 6$$

$$\alpha_{em}(Q^2) = \alpha_{em}(Q_0^2) \Big/ \left[1 - \frac{\alpha_{em}(Q_0^2)}{3\pi}\ell n\left(\frac{Q^2}{Q_0^2}\right)\right]$$

The effective value of α_s decreases as a function of Q^2, thus explaining the nucleons (strongly bound partons) and the DIS (quasi-free partons).

Comments on the coefficients :

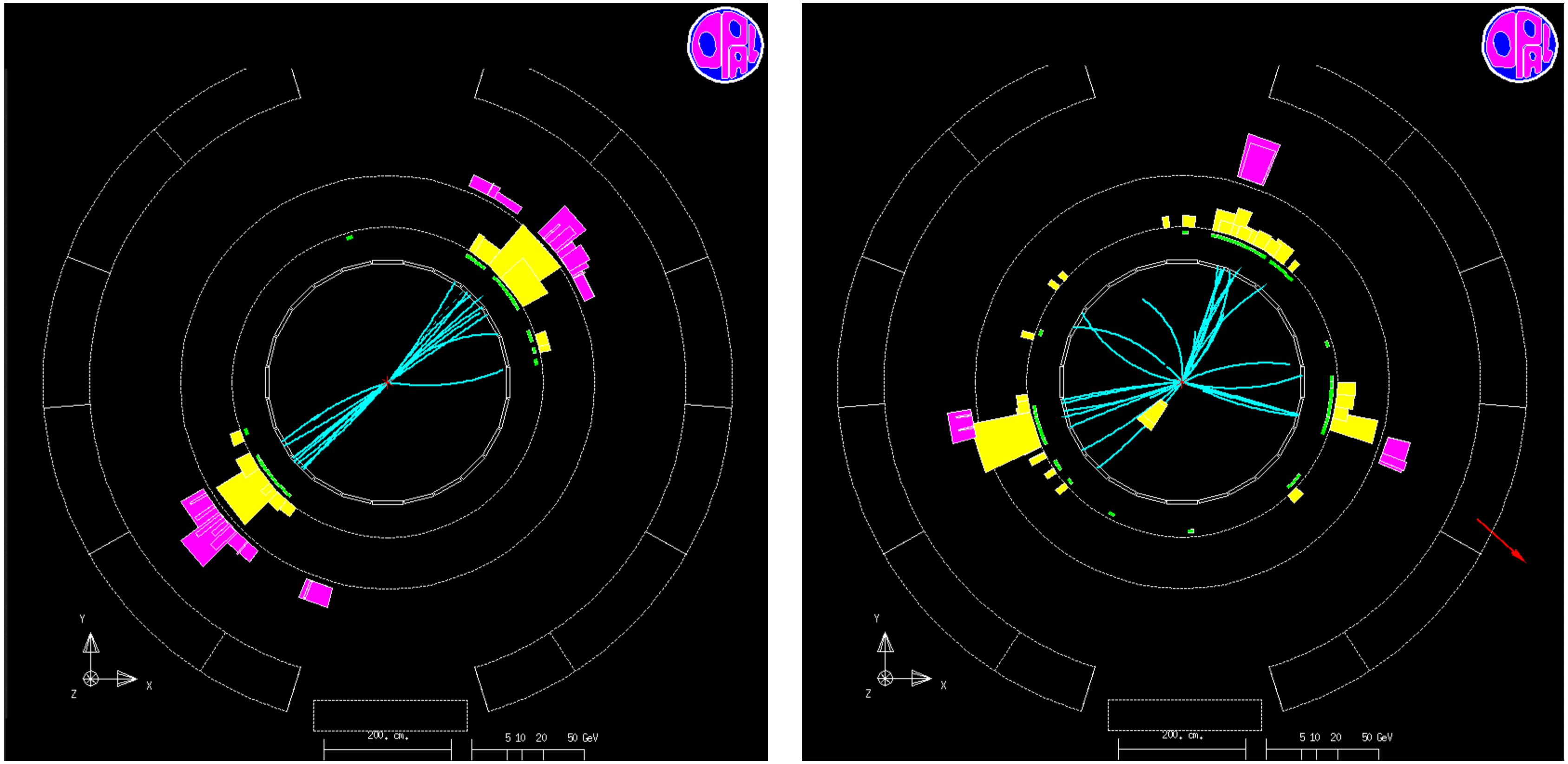
- [QCD effect $\propto \alpha_s$] >> [QED $\propto \alpha_{em}$];
- "N_c" = number of colors = 3;
- "N_f" = number of flavors = 6 (?);
- but N_f = $N_f(Q^2)$, i.e. the EFFECTIVE number of flavors at a given Q^2;
- to be simple (but not entirely correct), at a given value of Q^2, only flavors with $(2m_f)^2 < Q^2$ enter in the computation of α_s.

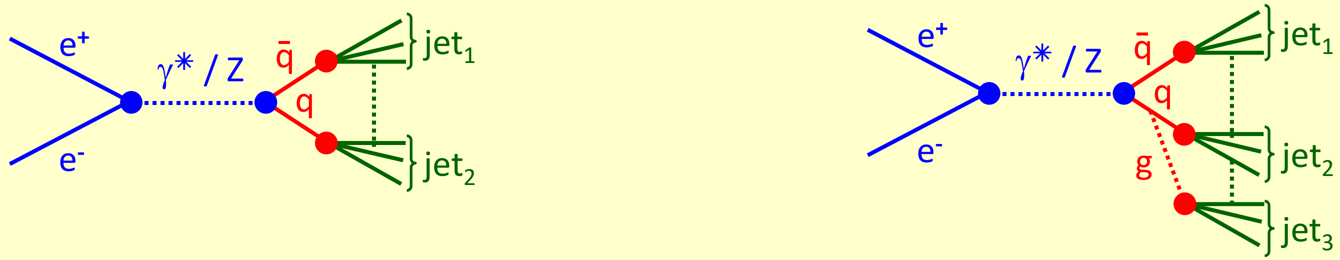


for the exp. data, see later

Partons, Jets, Hadrons

A two-jet and a three-jet event in OPAL at LEP:



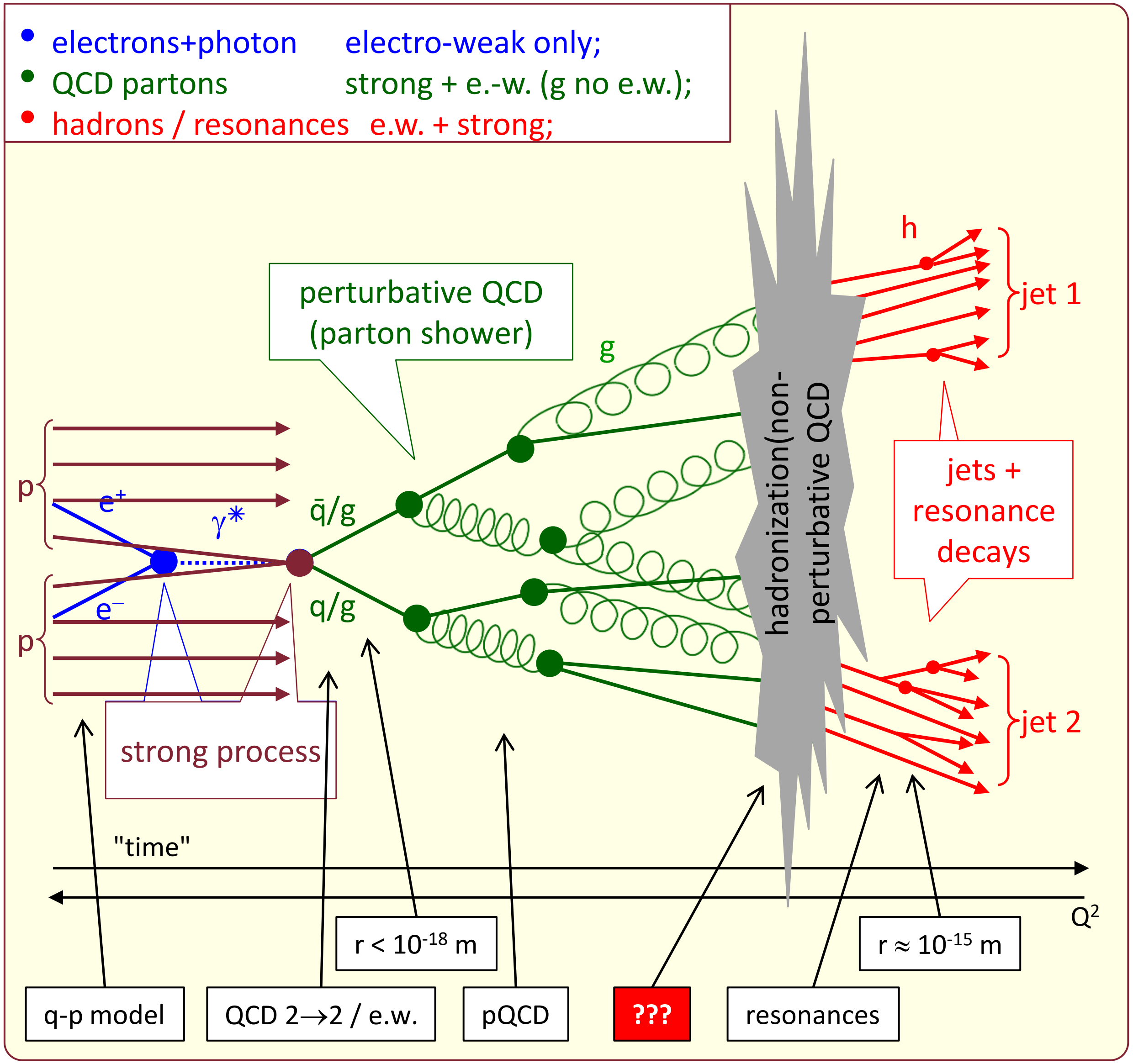


Partons, Jets, Hadrons: fragmentation

Three phases after elementary process :

- **parton shower** : perturbative cascade of (q q̄ g); notice the gluon self-coupling (non-abelian);
- **hadronization** : low-Q^2 parton processes, <u>no well-funded calculation</u>;
- **hadrons** : decays of resonances and emergence of jets.

NB Lot of work in parameterizations, fitting, algorithms, speculations …

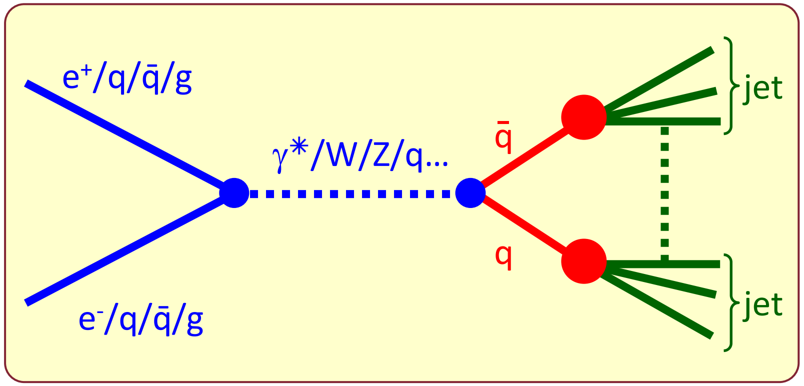


Partons, Jets, Hadrons: partons vs jets

The jets can be identified with the partons of the final state;

- problems :
  - to preserve the color, the two jets in the final state must "talk" each other (e.g. by exchange of gluons);
  - so it is impossible, *strictly speaking*, to assign in a given event a hadron (and hence a jet) to a "father" parton;
- however, as soon as Q^2 > (few GeV)^2, the majority of the events presents two (rarely three) well identified jets, with essentially no ambiguity;
- from the experimental point of view, the situation is relatively simple:
  - *(in practice all)* the events $e^+e^- \to$ hadrons for $\sqrt{s} \geq$ (few GeV) [SPEAR 1975] have two collimated jets of particles, opposite in θ and φ.

- the direction and momentum of the partons can be reconstructed from the vector sum of the 4-momenta of the hadrons (many subtleties, but the essence is simple);
- it is also possible to measure the "fragmentation function" of the partons : $f(z)$, $z = E_{hadron} / E_{parton}$;
- more discussions in *Coll.Phys.*.



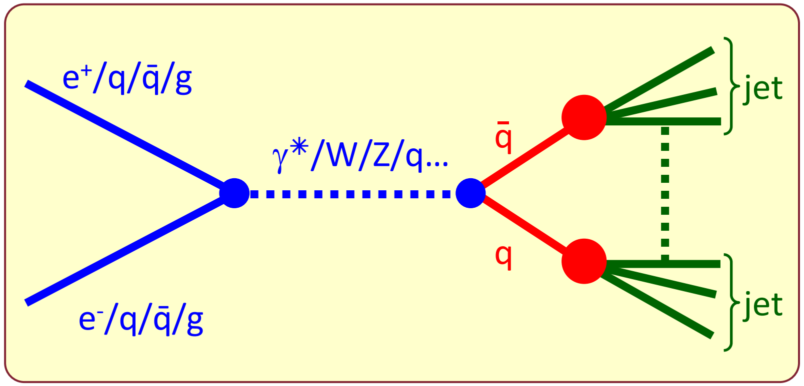
Partons, Jets, Hadrons: comments

The "correct" approach

- generate MC with zillions of events
  - in e^+e^- according to e.w. theory;
  - in $\bar{p}p$/pp to PDF + QCD cross-sections;
- for each event
  - generate QCD parton shower
  - … and QCD hadronization [perturbative or non-perturbative, according to α_s];
- get zillions of real events;
- compare MC and real data;
- success ☺☺☺ (or failure ☹☹☹).

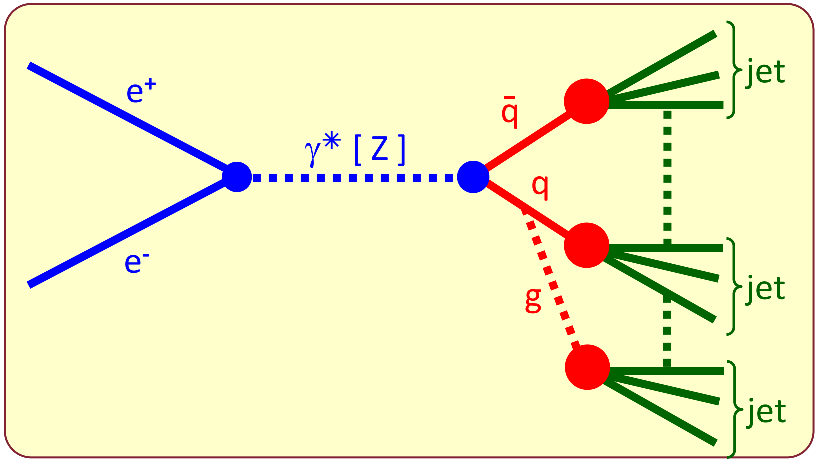
The actual approach

- generate MC with (e.w.) / (PDF + QCD)
  - unable to perform computations with non-perturbative QCD;
  - fudge the MC with phenomenological parameterizations "QCD inspired" ☹☹;
- get real data;
- compare MC and real data
  - improve the parameterizations;
  - claim agreement QCD ↔ reality;
- derive results, e.g. value of α_s, study of the fragmentation mechanism.

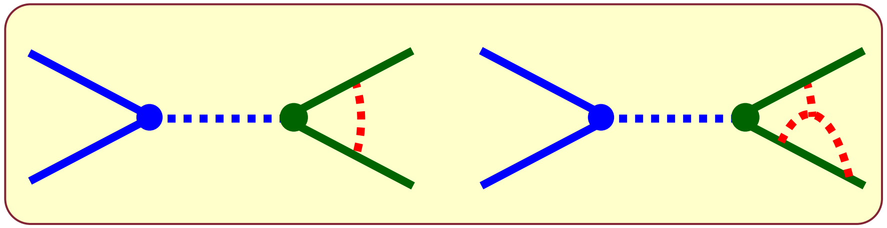


- Not clean, some logical loopholes;
- but [*imho*] acceptable;
- *anyway, that's the rule of the land;*
- more discussions in *Coll.Phys.*.

Partons, Jets, Hadrons: three-jet events



- Sometimes, a parton emits a gluon of bremsstrahlung, at an angle and with an energy such as to produce a third jet, well separated from the other two;
- usual litany : "*the fraction of three-jet events* $\propto \alpha_s$"; however :
  - ➢ jets are "ill-defined" quantities : the number and 4-mom. of jets in an event depends on the analysis (the so-called jet-finding algorithm, *JFA*);
  - ➢ the real meaning is that one has to compute (e.g. via montecarlo) the yield of multi-jet events with a given JFA and a given value of α_s; then the comparison with the data, analysed with the same JFA, is a "meas." of α_s (*e.g. too few three-jets in MC wrt data* $\rightarrow \alpha_s^{MC} < \alpha_s^{true}$);
- similarly, 4-jet, 5-jet, ...;
- with the previous caveats :
  - ➢ σ(2-jet) $\propto \alpha_{em}^2$; σ(3-jet) $\propto \alpha_{em}^2 \times \alpha_s$;
  - ➢ σ(3-jet) / σ(2-jet) $\propto \alpha_s$;
  - ➢ α_s can be measured by the ratio 3-jet/2-jet [also many other ways];
- high value of α_s [> 0.10] $\rightarrow$ importance of higher orders of the strong interactions, particularly true for multi-jets final states.



QCD results : quark spin

- If quark-spin = lepton-spin = ½, in $e^+e^- \to$ jets[(1)], $d\sigma/d\Omega \propto d\sigma/d\Omega|_{\mu\mu} \propto (1+\cos^2\theta)$;
- however, the heavy quarks have a non-negligible mass; their θ dependence is :

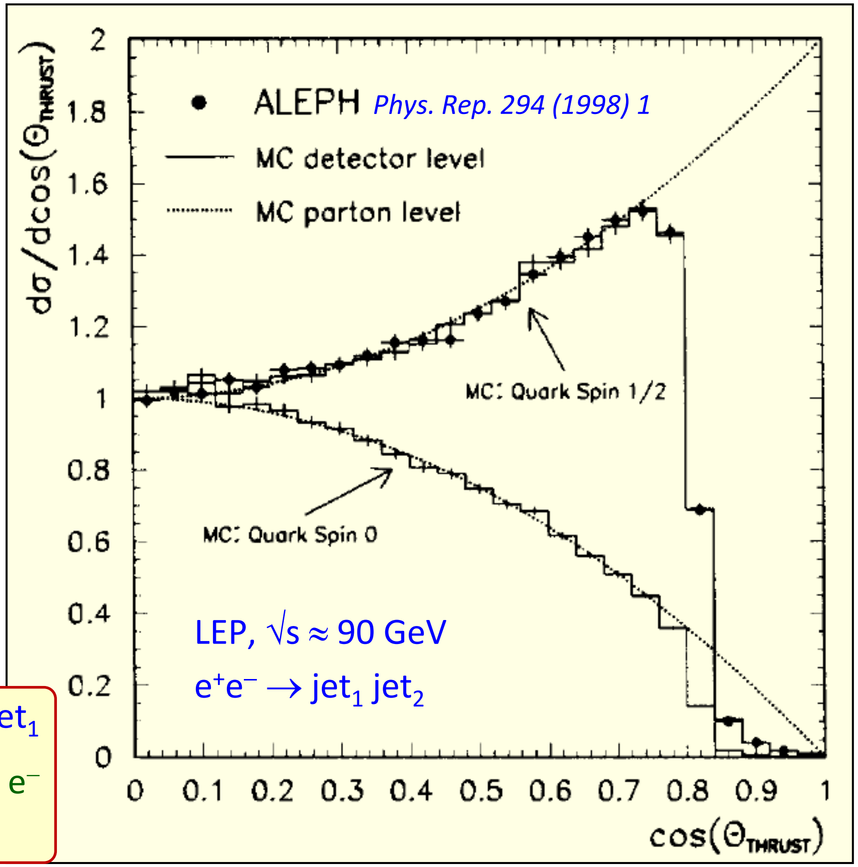
$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2 e_q^2 N_c \beta_q^*}{2s}\left(2 - \beta_q^{*2} + \beta_q^{*2}\cos^2\theta\right);$$

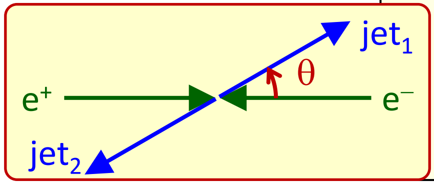
$$\beta_q^* = \sqrt{1 - 4m_q^2/s}.$$

- in reality, cannot distinguish jets from q and $\bar{q}$ → exp. ambiguity ($\theta \leftrightarrow 180° - \theta$), ($\cos\theta \leftrightarrow -\cos\theta$) → plot $|\cos\theta|$;
- the value of θ, i.e. the quark direction, is given by the jet axis [see previous pages], usually identified with the "thrust" axis[(2)];
- after all that, the comparison is possible, and definitive (e.g. ALEPH, 1998).

(1) True for e.m. and not for NC; but at the Z pole, the $q\bar{q}$ asymmetry is small.

(2) In the CM, the thrust axis is the direction which minimizes the sum of the transverse momenta of the final state particles respect to it.





QCD results : the gluon

Naïvely, the existence (both σ and $d\sigma/d\Omega$) of three-jet events (apart from pedantic caveats on the JFA) is a convincing test of the existence of the gluon.

Other "proofs" include :

- the integral of the structure function $F_2(x)$, which demonstrates that ~50% of the nucleon momentum is NOT carried by charged partons;
- the overall agreement between QCD and measurements, e.g. for hadron colliders;

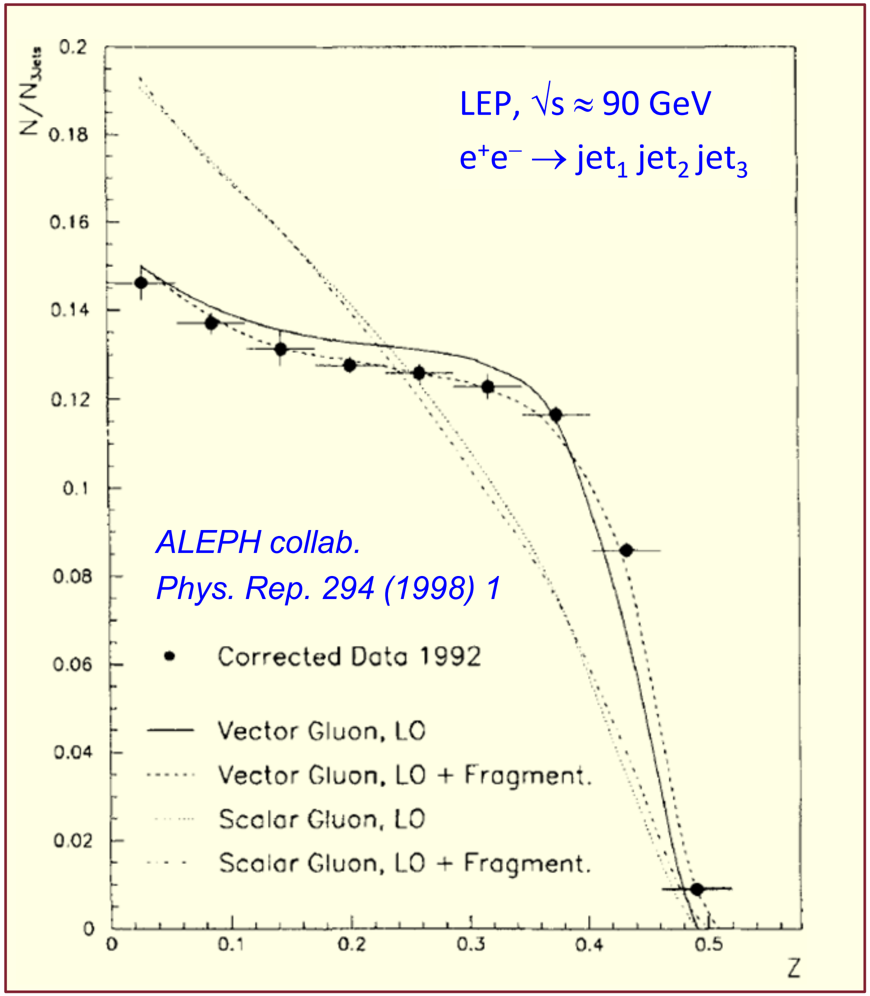
The spin of the gluon is measured :

- in e^+e^-, the third jet in three-jet events comes from gluon brem (theory : 75%);
- after ordering the jets according to energy, the variable

$$Z = 2\,(E_2 - E_3)\,/\,\sqrt{3s}$$

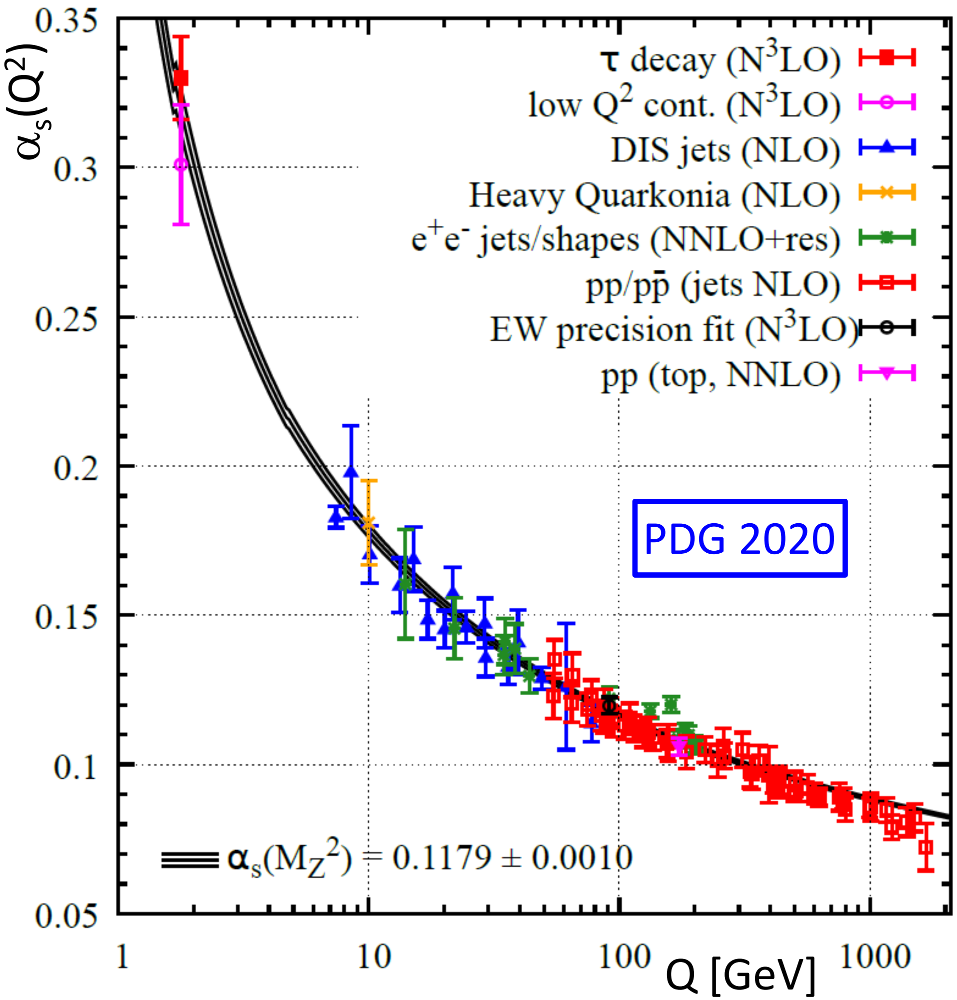
is sensitive to the gluon spin value.

- OK !!! (e.g. ALEPH, notice the quality of the result, insensitive to fragmentation) ["vector"/"scalar" : spin 1/0].



QCD results : the running of α_s

- Actually the running of $\alpha_s = \alpha_s(Q^2)$ has been shown, by measuring the strength of the coupling at different Q^2.
- The data of the figure show a variation of 1000 in $\sqrt{Q^2}$, which ranges from τ decay to jets at LHC energies.
- The measurements are compared with predictions, normalized to the value with smallest error, i.e. at $Q^2 = m^2(Z)$
- [only the "running" can be computed in QCD, not the value].
- The funny acronyms (N^3LO, NNLO) refer to the computations : they are performed at a given order of Feynman diagrams : NLO = "next to leading order", NNLO = "next to next …"…



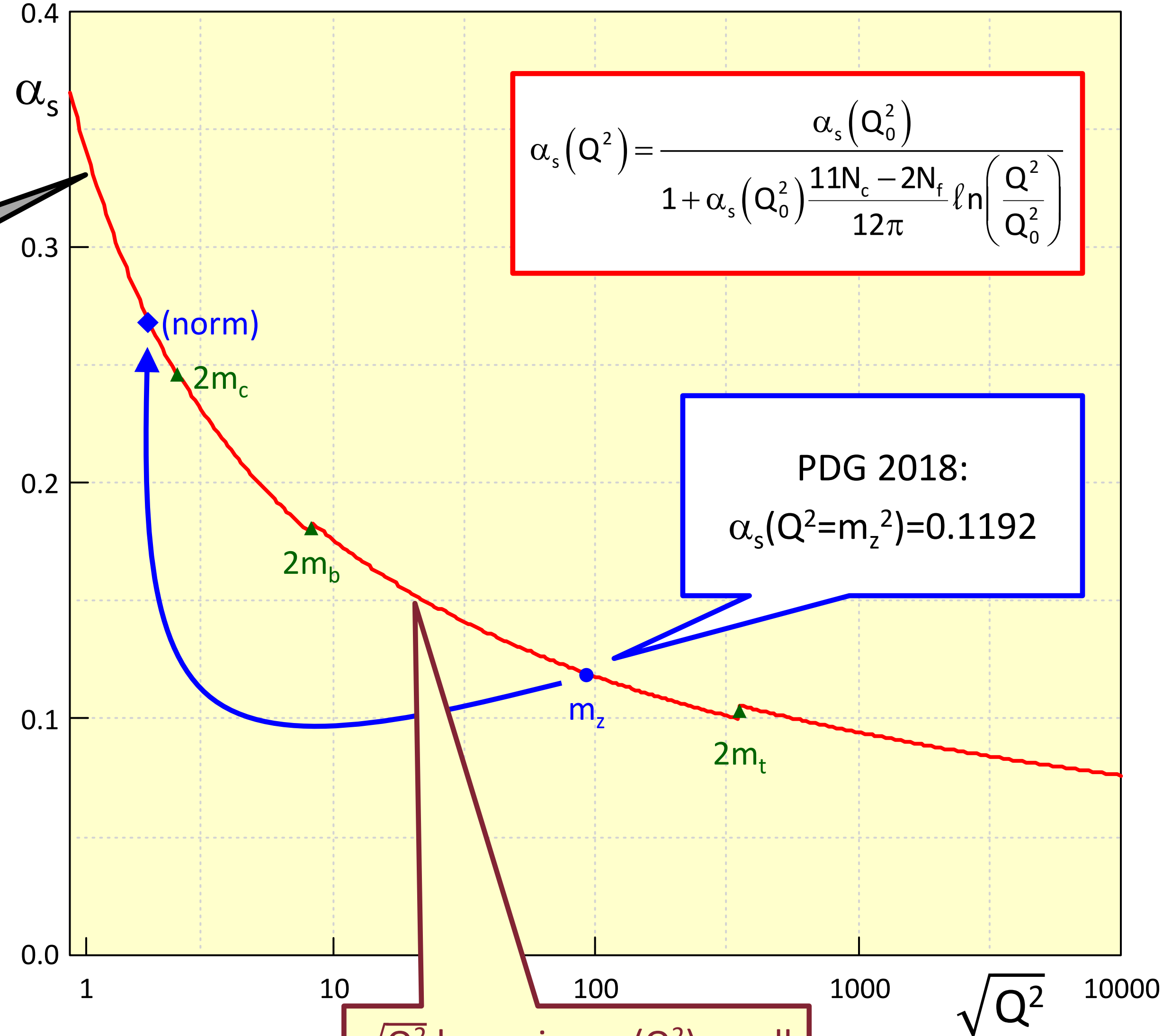
$$\alpha_s(Q^2) = \alpha_s(Q_0^2) \Big/ \left[1 + \alpha_s(Q_0^2)\frac{11N_c - 2N_f}{12\pi}\ell n\left(\frac{Q^2}{Q_0^2}\right)\right]$$

QCD results : $\alpha_s = \alpha_s(Q^2)$

do NOT use *for real computations,*
only useful as an exercise !!!

$\sqrt{Q^2}$ small, i.e. $\alpha_s(Q^2)$ large
$\rightarrow$ perturbative QCD fails

- A "home-made" plot, just for understanding;
- notice the steps at $\sqrt{Q^2} = 2m_q$, clearly an artefact of the approx.



$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + \alpha_s(Q_0^2)\frac{11N_c - 2N_f}{12\pi}\ell n\left(\frac{Q^2}{Q_0^2}\right)}$$

PDG 2018:
$\alpha_s(Q^2{=}m_z^2) = 0.1192$

$\sqrt{Q^2}$ large, i.e. $\alpha_s(Q^2)$ small
$\rightarrow$ perturbative QCD is OK

QCD results: parameter Λ_{QCD}

The running of α_s can be expressed with a parameter Λ_{QCD} (e.g. [Bettini, §6]):

- it makes the equations (apparently) simpler;
- actually $\Lambda_{QCD} = \Lambda_{QCD}(Q^2)$, because of the effective $N_f = N_f(Q^2)$;
- its measured value ranges from 340 MeV for $N_f = 3$ to 210 MeV for $N_f = 5$;
- for $Q^2 \gg (\Lambda_{QCD})^2$, $\alpha_s \propto 1/[\ell n Q^2 - \text{const}]$; therefore Λ_{QCD} can be related to the "border" between perturbative and non-perturbative regimes [*notice the log-dependence*];
- [*theoreticians do not like it anymore* $\rightarrow$ Λ_{QCD} *is disappearing from the literature, but a future revival is possible*]

In the perturbative region $\sqrt{Q^2} \gg \Lambda_{QCD}$

$$\boxed{\begin{aligned}&\alpha_s\left(Q^2\right) \equiv \alpha_s, \quad \alpha_s\left(Q_0^2\right) \equiv \alpha_s^0,\\ &\alpha_s = \alpha_s^0 \Bigg/ \left[1 + \alpha_s^0 \frac{11N_c - 2N_f}{12\pi} \ell n\left(\frac{Q^2}{Q_0^2}\right)\right]\end{aligned}}$$

$$\boxed{\ell n \Lambda_{QCD}^2 \equiv \ell n Q_0^2 - \frac{12\pi}{11N_c - 2N_f} \frac{1}{\alpha_s^0}} \rightarrow$$

$$\boxed{\ell n Q_0^2 = \ell n \Lambda_{QCD}^2 + \frac{12\pi}{11N_c - 2N_f} \frac{1}{\alpha_s^0}};$$

$$\frac{1}{\alpha_s} = \frac{1}{\alpha_s^0} + \frac{11N_c - 2N_f}{12\pi}\left[\ell n Q^2 - \boxed{\ell n Q_0^2}\right] =$$

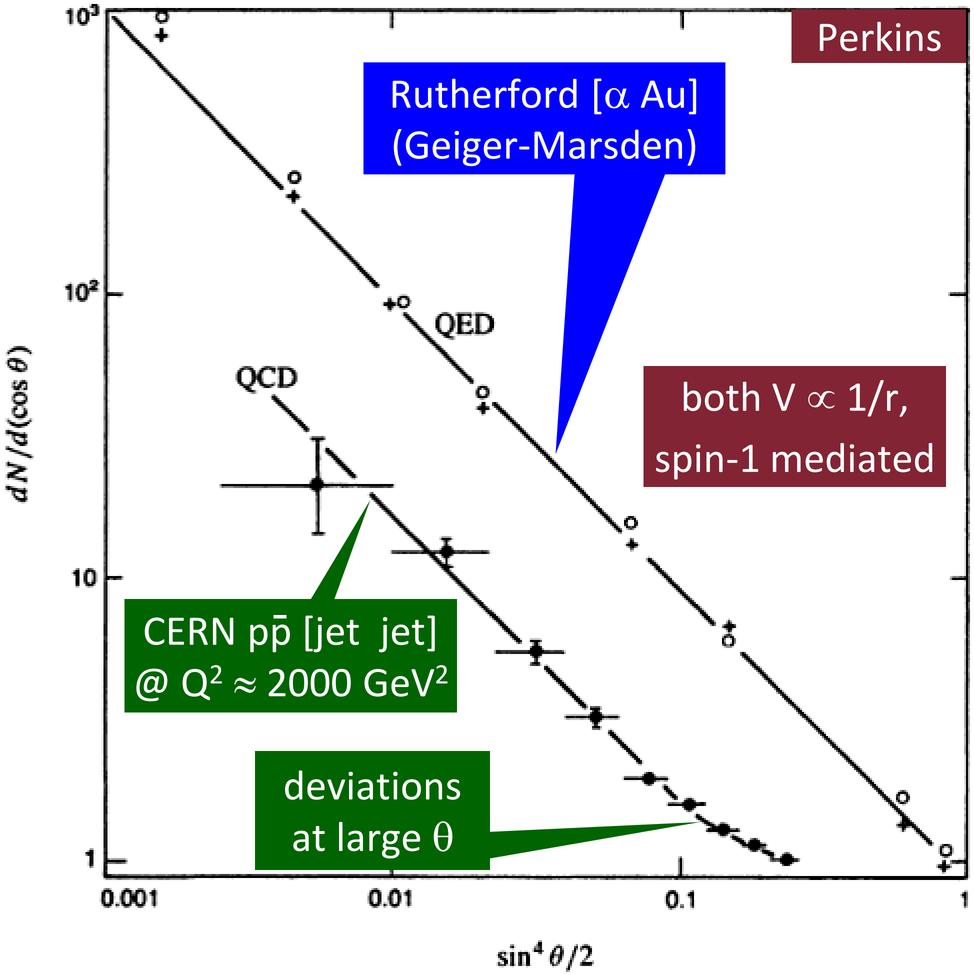
$$= \frac{1}{\alpha_s^0} + \frac{11N_c - 2N_f}{12\pi}\left[\ell n Q^2 - \ell n \Lambda_{QCD}^2\right] - \frac{1}{\alpha_s^0};$$

$$\rightarrow \boxed{\alpha_s\left(Q^2\right) = \frac{12\pi}{11N_c - 2N_f} \frac{1}{\ell n\left(Q^2/\Lambda_{QCD}^2\right)}}.$$

QCD results : angular distribution

Finally, an angular comparison is shown between QED and QCD :

- upper : Rutherford scattering (QED) in the famous Geiger-Marsden plot; the dependence $\propto 1 / \sin^4(\theta/2)$ is evident;
- lower (arbitrary normalization): the same angular plot for $\bar{p}p$ QCD jets at $Q^2 \approx 2000$ GeV^2;
- [notice that, one century ago, it was not customary to show errors on the plots; *maybe in good ole time, they did not make errors*].



"Grand unification bSM" ?

Two curiosities on α_s.

- Asymptotic freedom requires

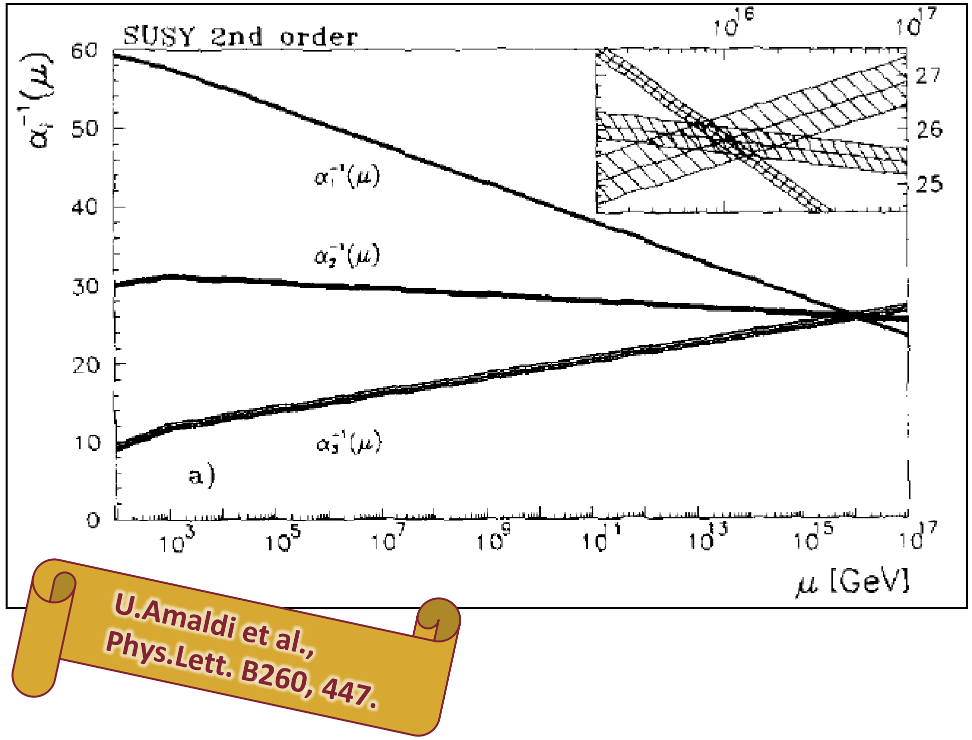
  $11\ N_c - 2\ N_f > 0 \rightarrow$

  $\boxed{N_f < 11\ N_c / 2 = 16.5;}$

  i.e. an upper limit on the number of flavors; after the LEP measurement of Γ_Z, the argument has lost importance, even though there is a technical possibility (new heavy flavors with heavy ν's);

- Since $\alpha_s = \alpha_s(Q^2)$ is decreasing with Q^2, while α_{em} is increasing, do they "meet" each other ? and, at this value of Q^2, what happens to gravity ?

- It turns out that in a bSM model (SUSY) at $\sqrt{Q^2} \approx 10^{15}$ GeV (μ in the fig.), the three couplings $[SU(3)_c \otimes SU(2)_L \otimes U(1)]$ have the same value, therefore suggesting a possible "grand unification".



- Notice that in the SM the three constants all run, but badly miss each other (!!!).

References

1. e.g. [BJ, 13], [YN2, 1], [YN2, 7];
2. the e-w theory is fully discussed in [IE];
3. QCD experimental tests in [BJ, 14].

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End of chapter 6