22/12/2015

# The ATLAS Muon System

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# Summary

- Overall design
- Track reconstruction
- Performance measurements
- Trigger
- Outlook

# “specifications”

“Physics Requirements” from the Technical Design Report (TDR) :

- Identify and reconstruct muon tracks, measure their momenta, and provide matching information for association with inner-detector data [...].
- Trigger on single- or multi-muon event topologies [...].
- Unambiguously associate the muon with its parent bunch crossing.

The scale of the performance requirements is set by a number of benchmark reactions:

1) H->ZZ*->μμ*ll* SM (120<$m_H$< 170 GeV) [...]

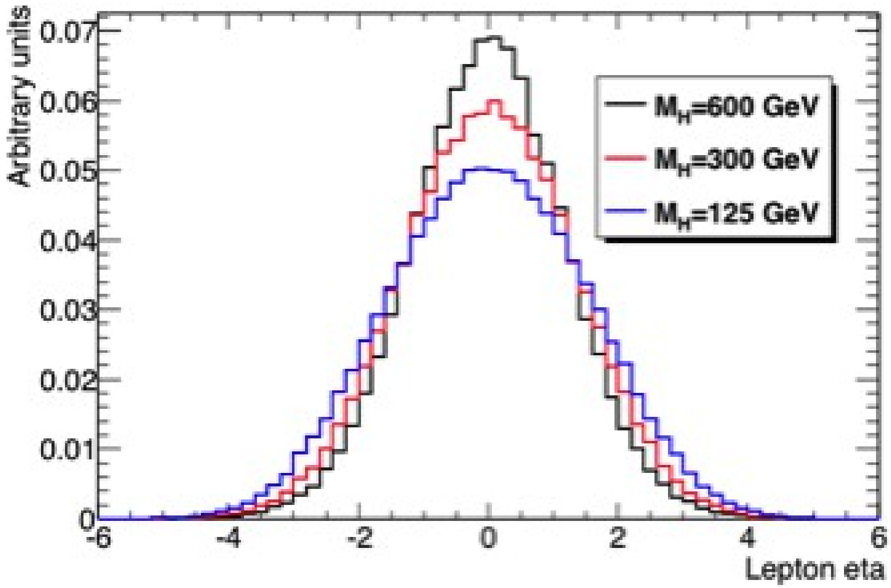
2) H->ZZ*->μμ*ll*, *A*->μμ MSSM (180<$m_H$<2$m_{top}$) [...]

3)New vector bosons Z’->μμ, W’->μν (1<m<5 TeV);

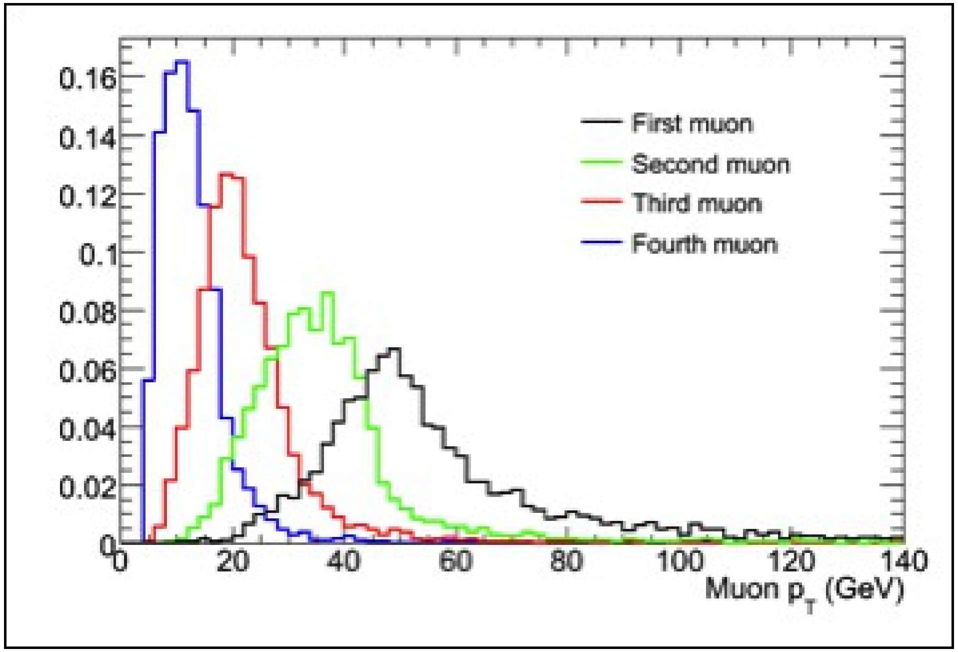
4) B-physics

# Example: muons frm H->ZZ*->4μ

Pseudo-rapidity η :




Transverse momentum $p_T$



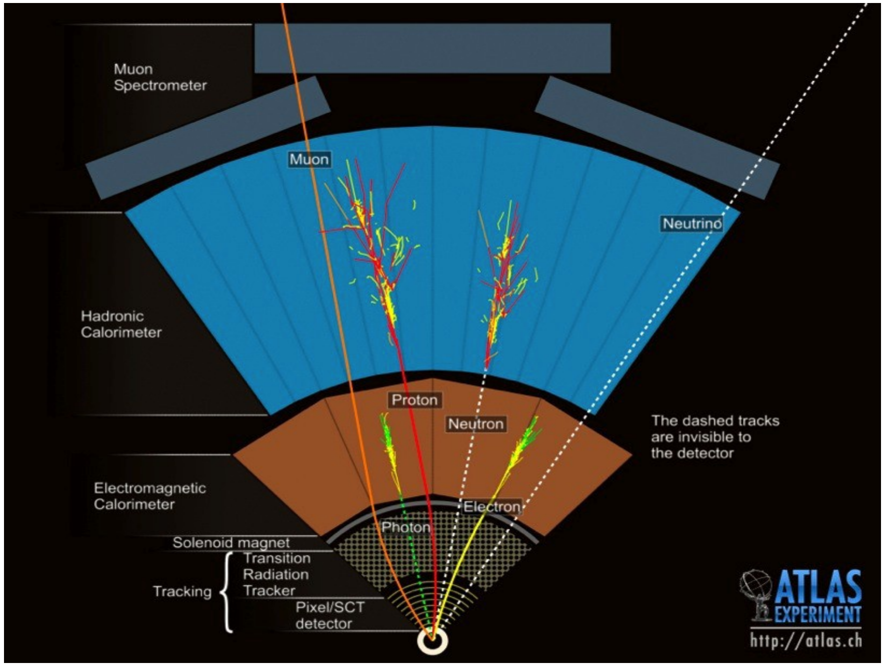

In practice for Higgs analysis we would like a detector with
- coverage |η|<~3
- Reco from $p_T$ > ~5 GeV
- Trigger: 1 mu with $p_T$ > ~25 GeV or 2 mu with $p_T$ > ~10 GeV
- Best possible momentum resolution

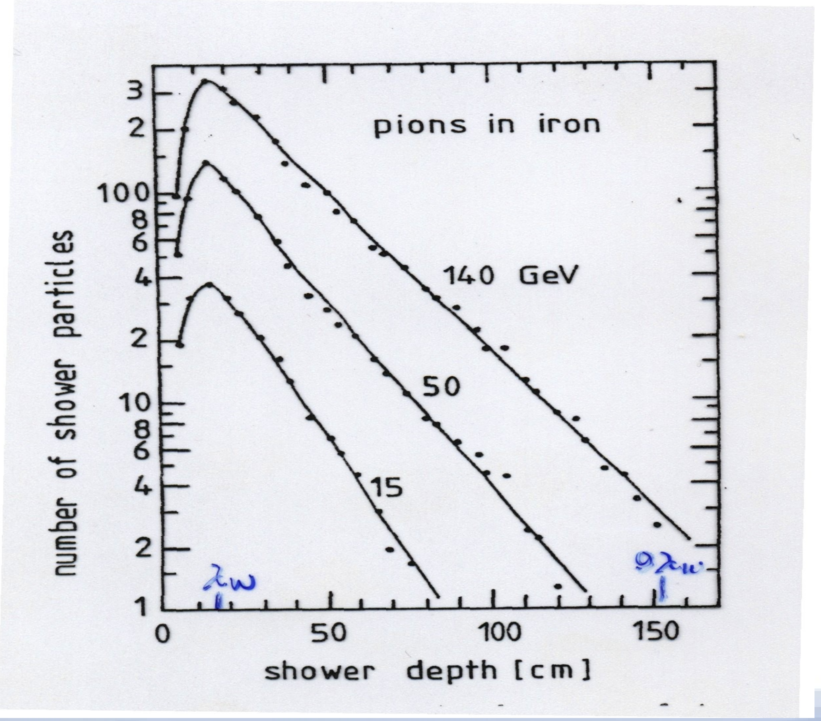
The actual choices are driven by cost-performance optimization

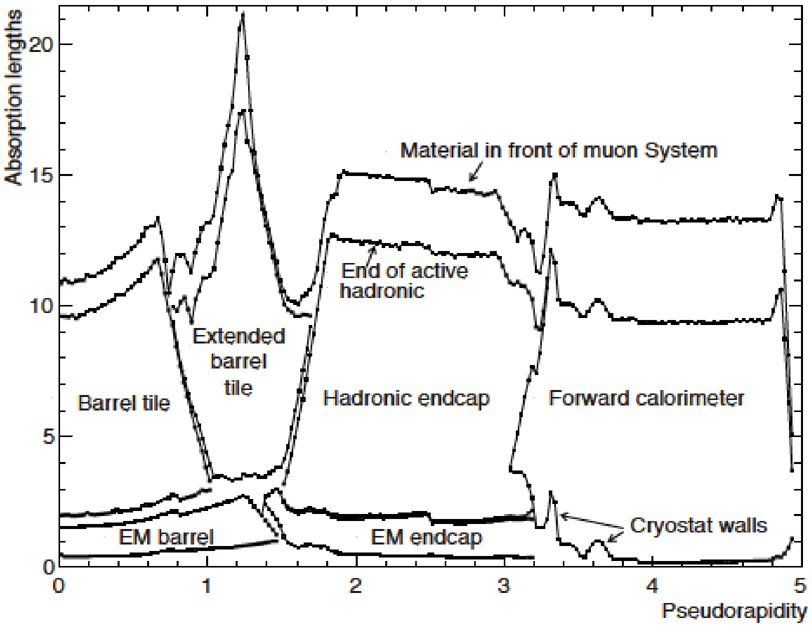
| Parameter | Main physics criteria | Performance desired | Performance actual | Comments |
|---|---|---|---|---|
| **Momentum measurement** | | | | |
| $\Delta p_T/p_T$ at 20 GeV | $H \to ZZ^* \to 4l$ | 1–2% | ~2.5% | Muon spectrometer only limited by energy loss and multiple scattering |
| | | | ~1.6% | Combined with inner tracker |
| $\Delta p_T/p_T$ at 75 GeV | $H \to ZZ \to 4l$ (MSSM) | 1–2% | ~2.4% | Muon spectrometer only limited by energy loss and multiple scattering |
| | | | ~2.0% | Combined with inner tracker |
| $\Delta p_T/p_T$ at 1000 GeV | $Z' \to \mu\mu$ | few % | ~11% | Resolution limited by cost–performance optimization; charge determination is the driving criterion |
| Rapidity coverage | Above processes | ~3 | 2.7 | Limited by system integration and shielding |
| **Trigger** | | | | |
| Low-$p_T$ threshold | b physics and CP violation (event rate) | ~ 5 GeV | 6 GeV | Limited by muon energy loss for triggering behind the calorimeter and by hadron decays in flight |
| High-$p_T$ threshold | $H \to ZZ^* \to 4\mu$ (event rate) | 20 GeV | 20 GeV | Background-dependent, tunable |
| Rapidity coverage | $b \to \mu x$ (event rate) | ~ 2.7 | 2.4 | Limited by low-$p_T$ rate and accidentals |
| | $H \to ZZ^* \to 4\mu$ | ~2.5 | 2.4 | Single-muon trigger at high $p_T$ provides good trigger efficiency |
| | $Z' \to \mu\mu$ (asymmetry) | ~2.0 | 2.4 | Single-muon trigger at high $p_T$ provides good trigger efficiency |
| Bunch crossing identification | Event matching | $\sigma < 5$ ns | $\sigma < 4$ ns | |

# Muon identification

- Muon identification is based on the absorption of other paricles producing EM and Hadronic showers in the calorimeters
- In ATLAS >~10 interaction lengths provide shower containement [<95% of energy] for pions up to approx 100 GeV
- Simulations provide the number and momentum of charge particles “leaking” from showers



# Magnetic field configuration: two different choices

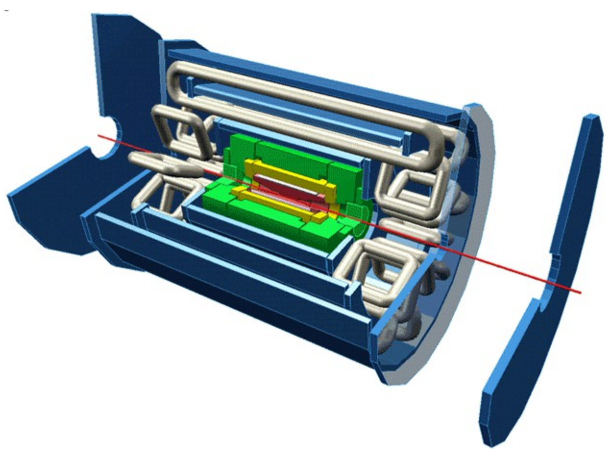
**ATLAS:**

- thin solenoid inside EM CALI, B~2T
- muon system in large air-core toroidal field
- “smaller” inner tracker
- precise stand-alone muon momentum measurement in the MS

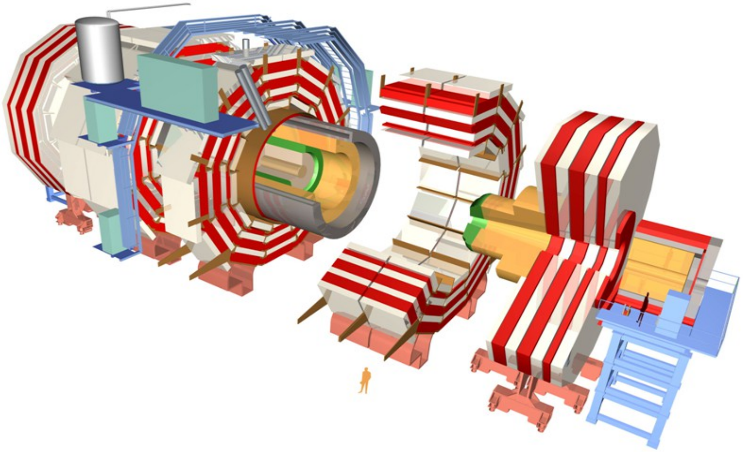
**CMS:**

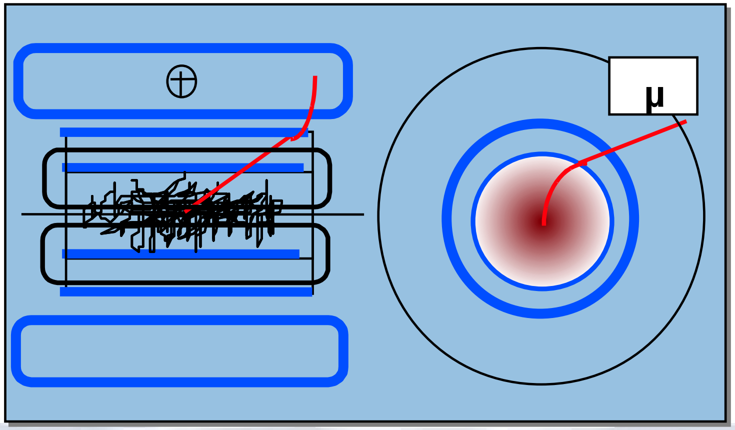
- large solenoid outside calorimeters, B~4 T
- muon system in the iron yoke for magnetic field return
- large inner tracker

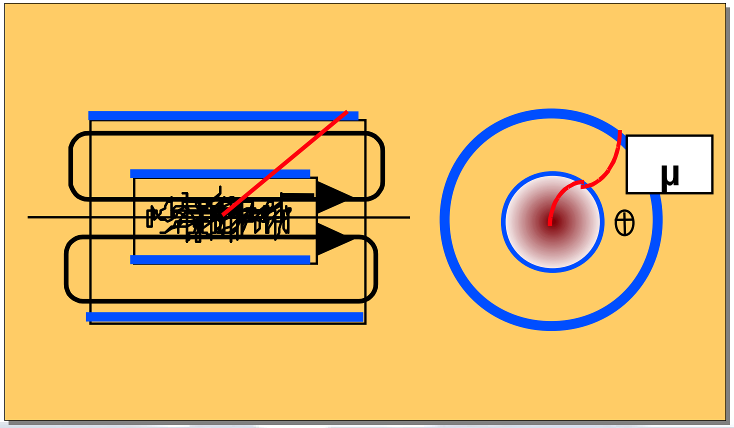
**ATLAS A T**oroidal **L**HC **A**pparatus



**CMS C**ompact **M**uon **S**olenoid

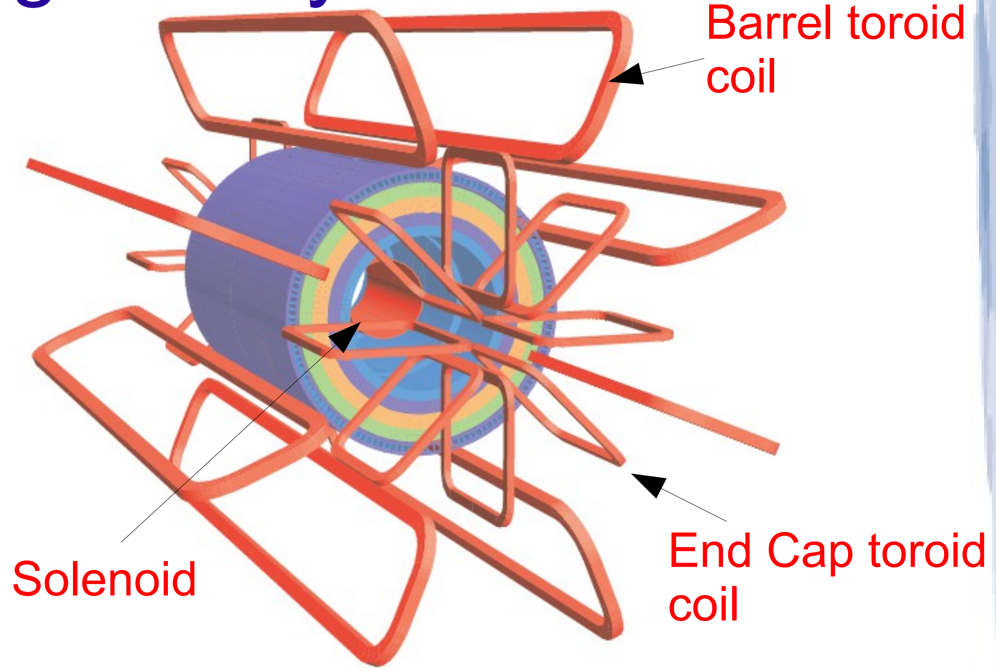


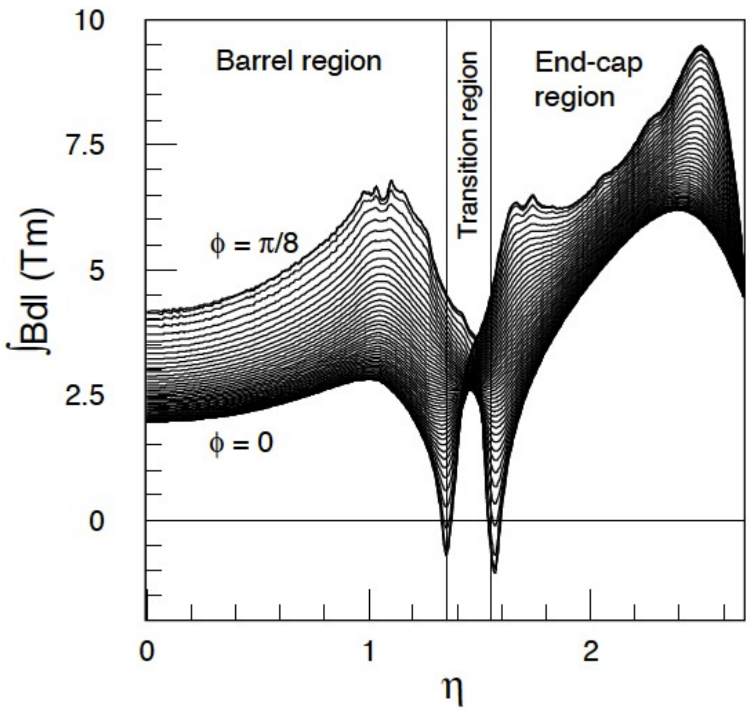


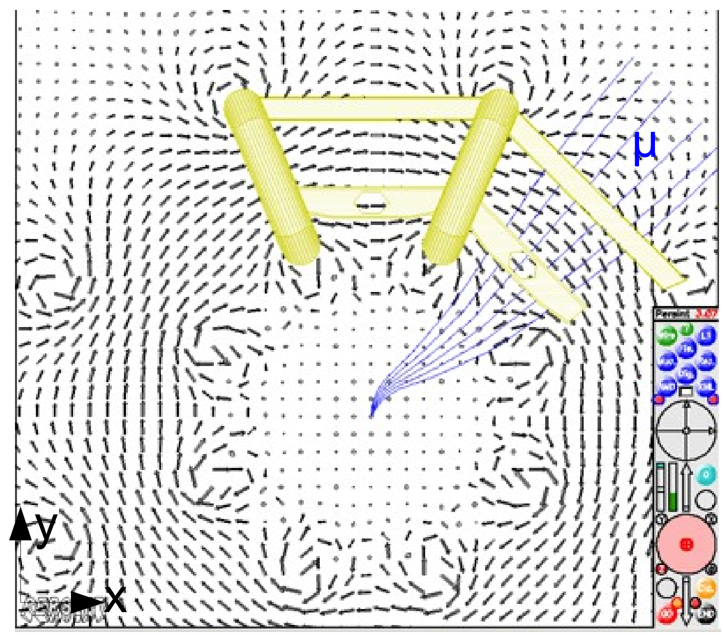



# The ATLAS magnetic system

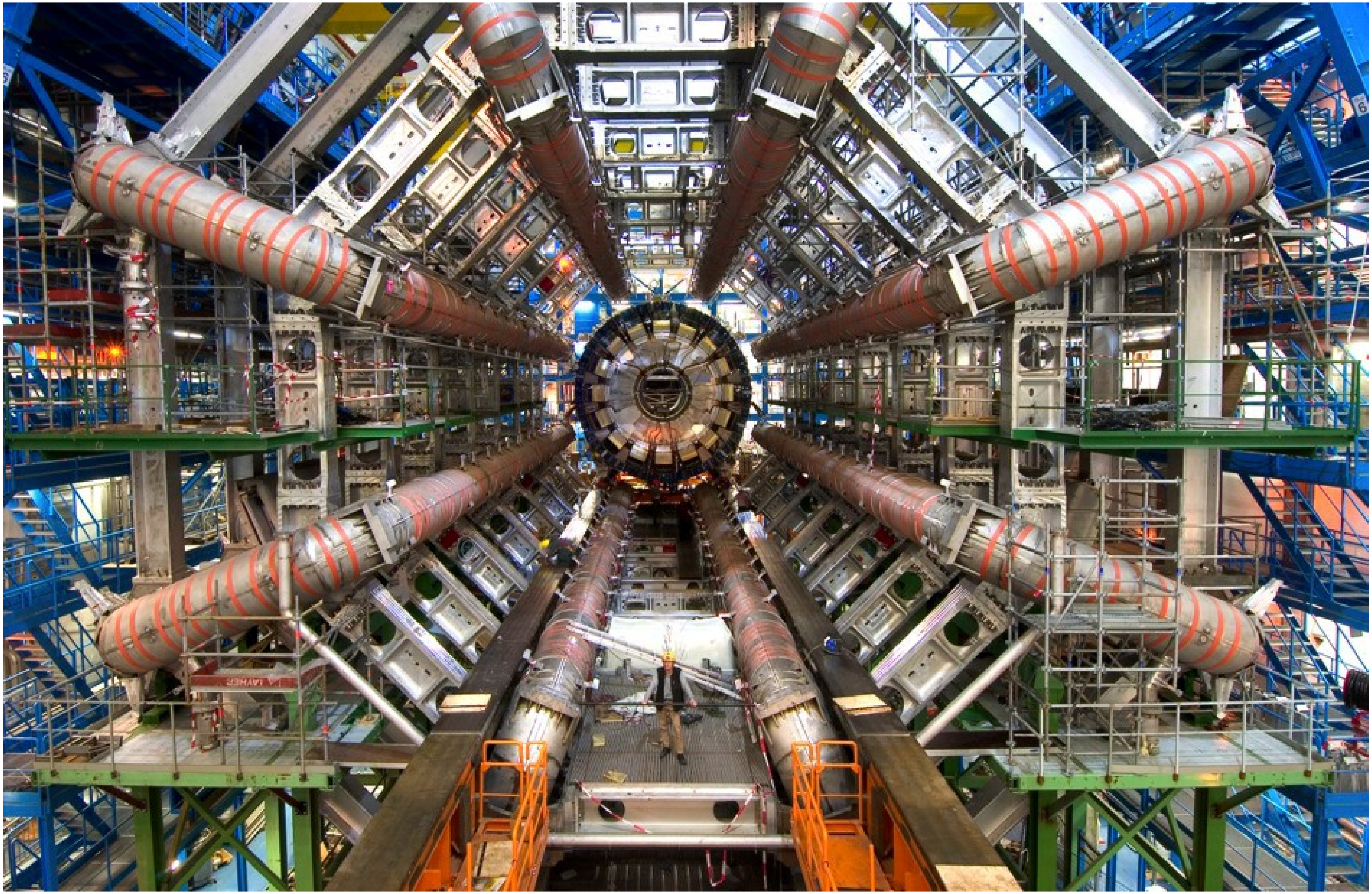
- Central barrel solenoid B=2T, R=1 m
- Barrel toroid (8 coils)
- Two End-Cap toroids (8 coils)
- MS: bending in η, straight tracks in φ
- Complex field configuration due to "few" coils and Barrel/End Cap transition
- Field integral seen by a muon in MS: $\int B\,dl = 2.5 : 10$ Tm



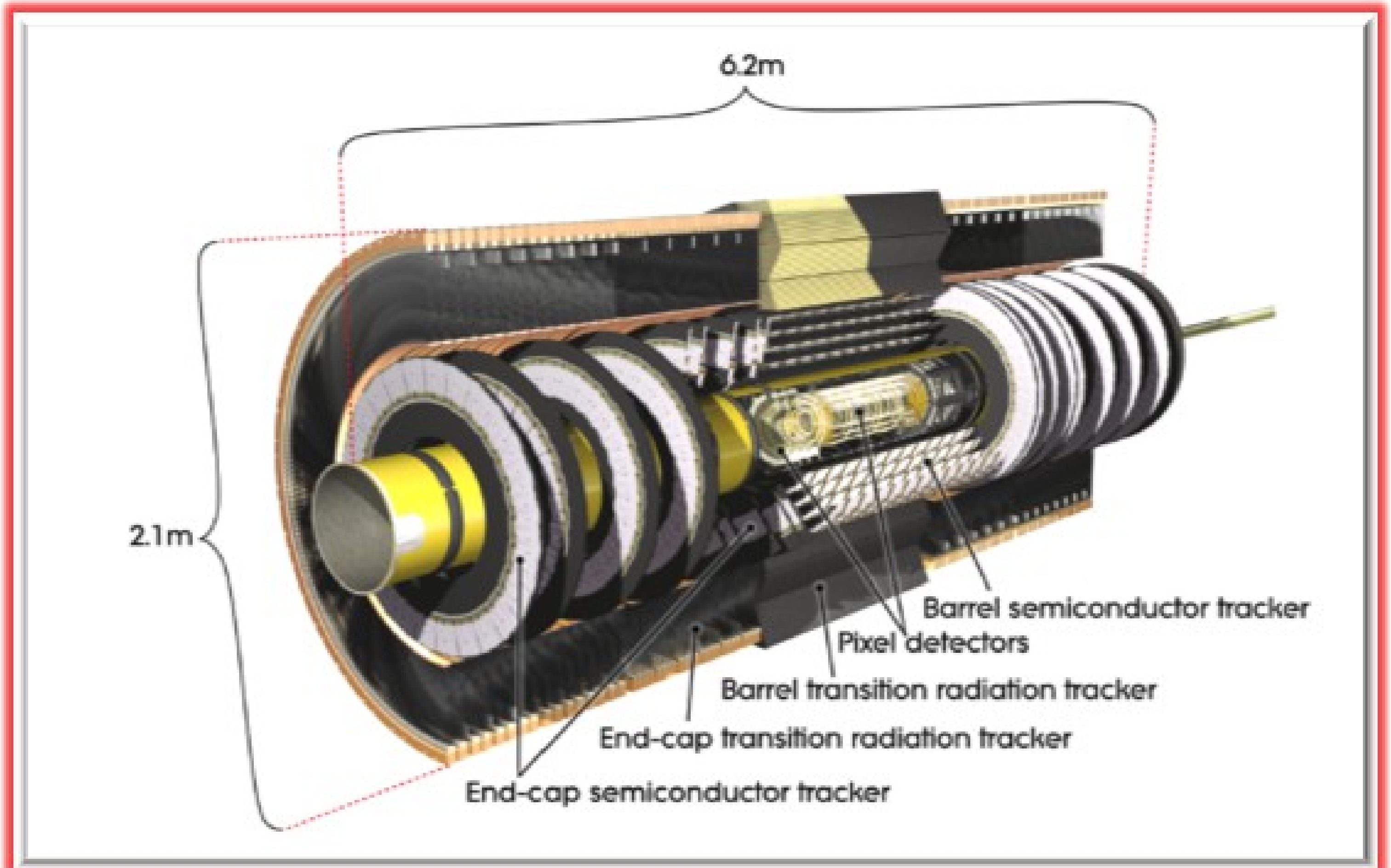



# The Barrel toroid coils during construction



# The Inner Detector (ID)

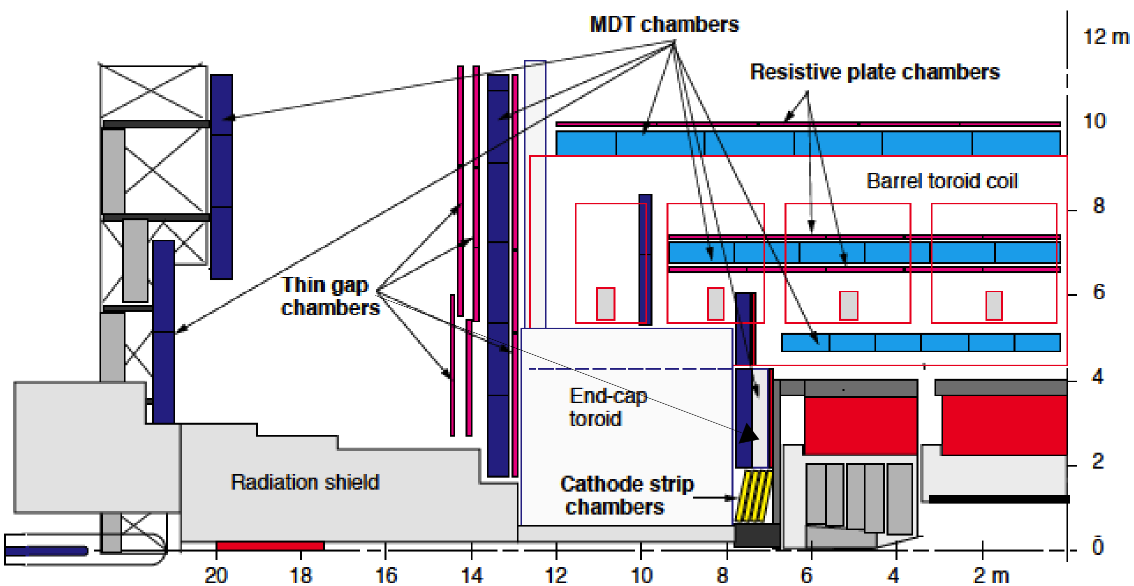


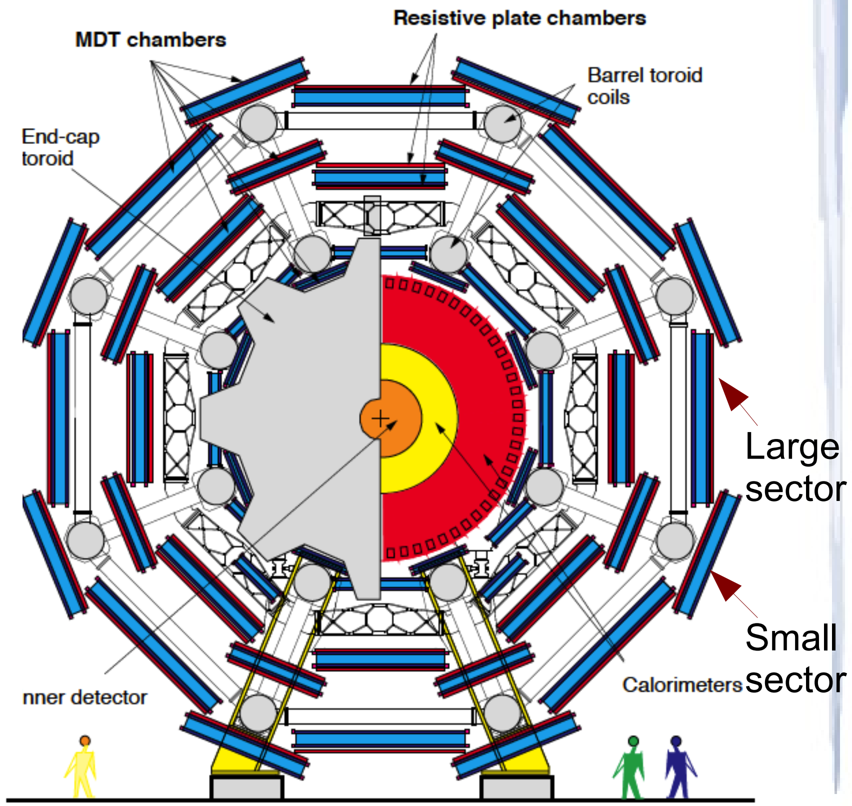

- Pixel: 3 layers 2D - Hit resolution bending plane: 10 μm (NOW 4 Layers with the IBL)
- Silicon Strips: (SCT) 4-9 layers 2D (stereo strips) - Hit resol bending plane: 17 μm
- Straw Tubes (TRT): up to 160 planes -1D Hit resol bending plane: 130 μm
- Magnetic Field : Solenoid 2 T, Angular Coverage $\eta$<2.5

# The muon system (MS)

- Three layers of “precision” chambers
  for precise measurement in the bending plane
  - MDT (monitored drift tubes)
  - CSC (cahode strip chambers) inner layer |η|>2

- 3(4) layers of “trigger” chambers
  for triggering and φ coordinate
  - RPC (resistive plate chambers) in barrel
  - TGC (Thing gap chambers) in endcaps



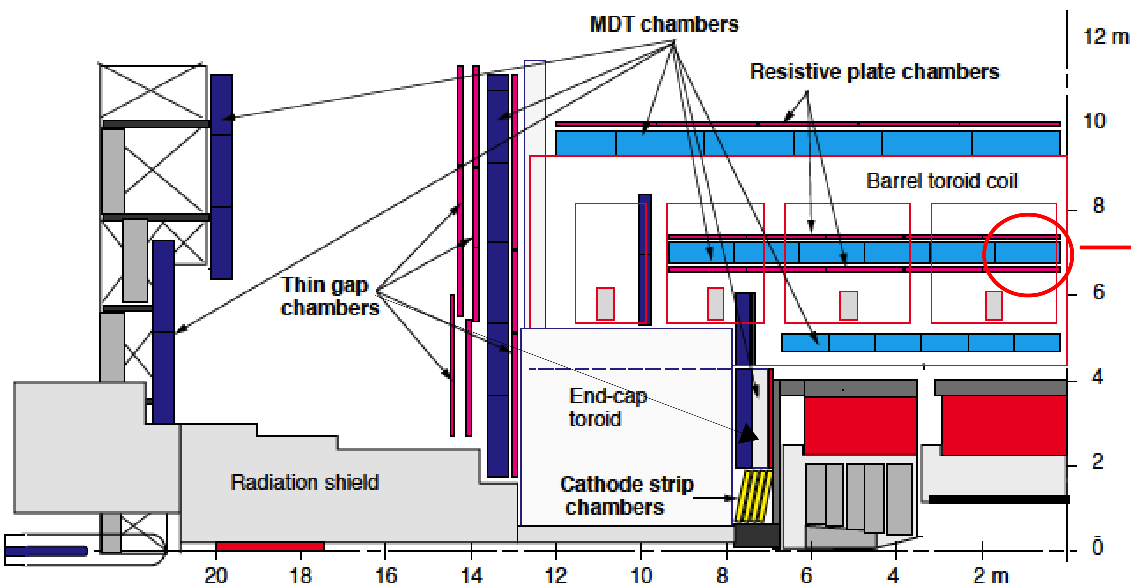
# The muon system (MS)

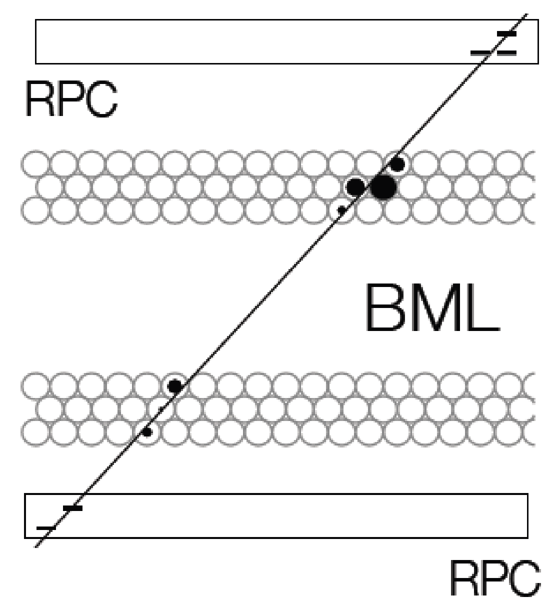
- Three layers of "precision" chambers
  for precise measurement in the bending plane
  - MDT (monitored drift tubes)
  - CSC (cahode strip chambers) inner layer $|\eta|>2$

- 3(4) layers of fast "trigger" chambers
  for trigger and $\varphi$ coordinate
  - RPC (resistive plate chambers) in barrel
  - TGC (Thing gap chambers) in endcaps

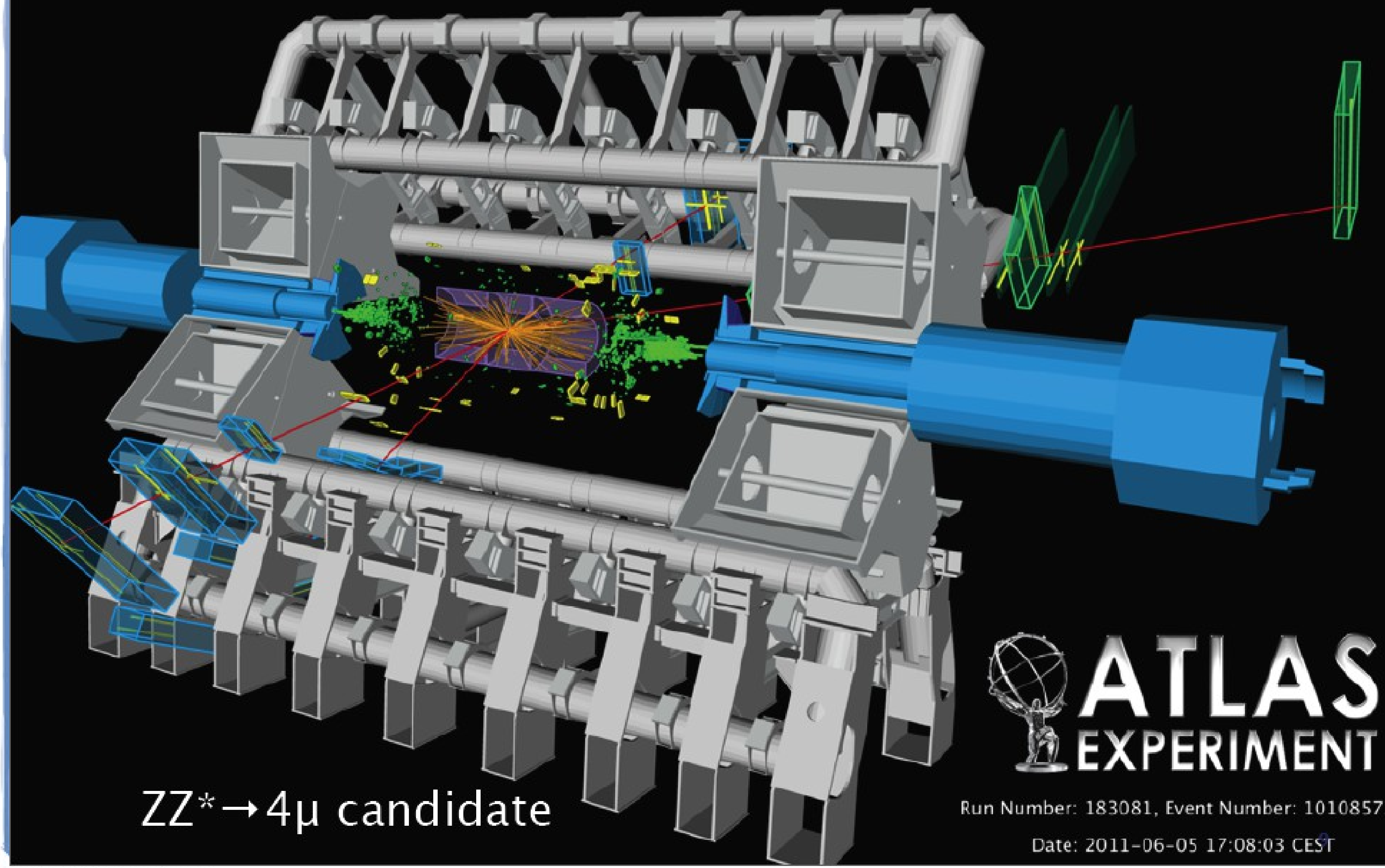
Total hits along track:
~ 20 precision hits
~ 6 (barrel) to 12 (endcap) trigger hits

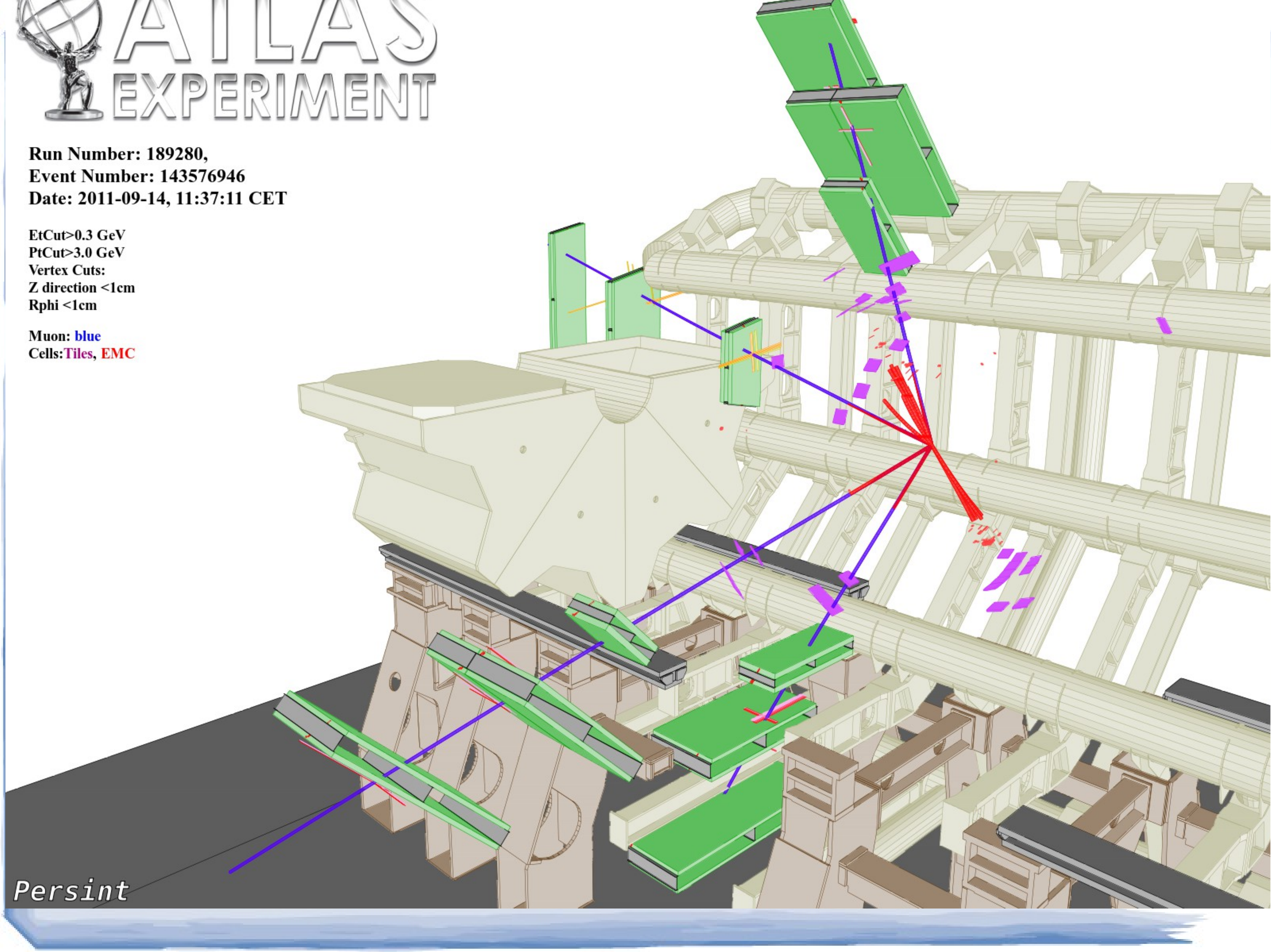
Example:
"Barrel Middle Large" station:
3+3 precision and 2+2 trigger points

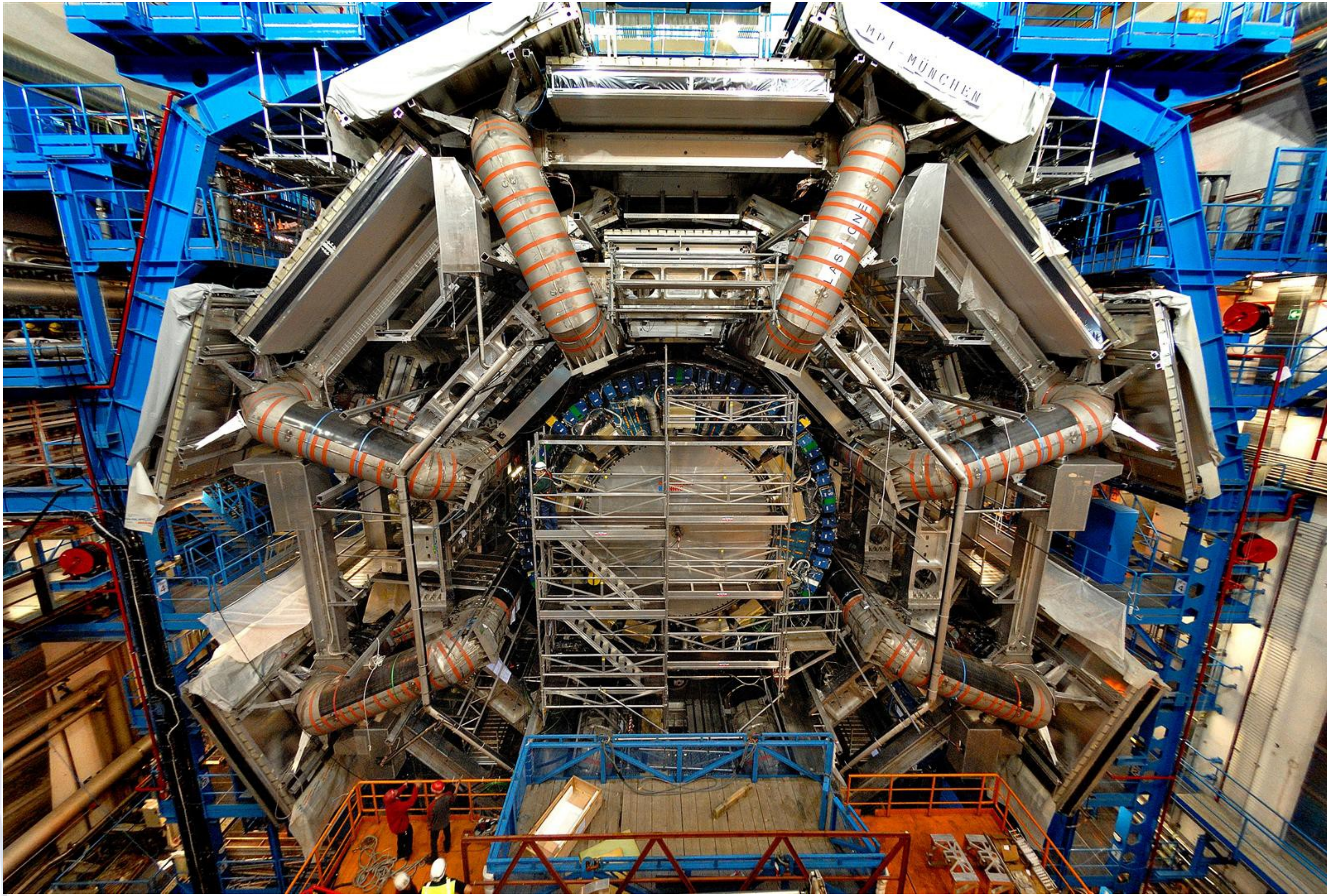


# In Reality ? ... a bit more complicated




ATLAS EXPERIMENT
Run Number: 189280,
Event Number: 143576946
Date: 2011-09-14, 11:37:11 CET
EtCut>0.3 GeV
PtCut>3.0 GeV
Vertex Cuts:
Z direction <1cm
Rphi <1cm
Muon: blue
Cells:Tiles, EMC
Persint


MPI-MÜNCHEN

# MDTs

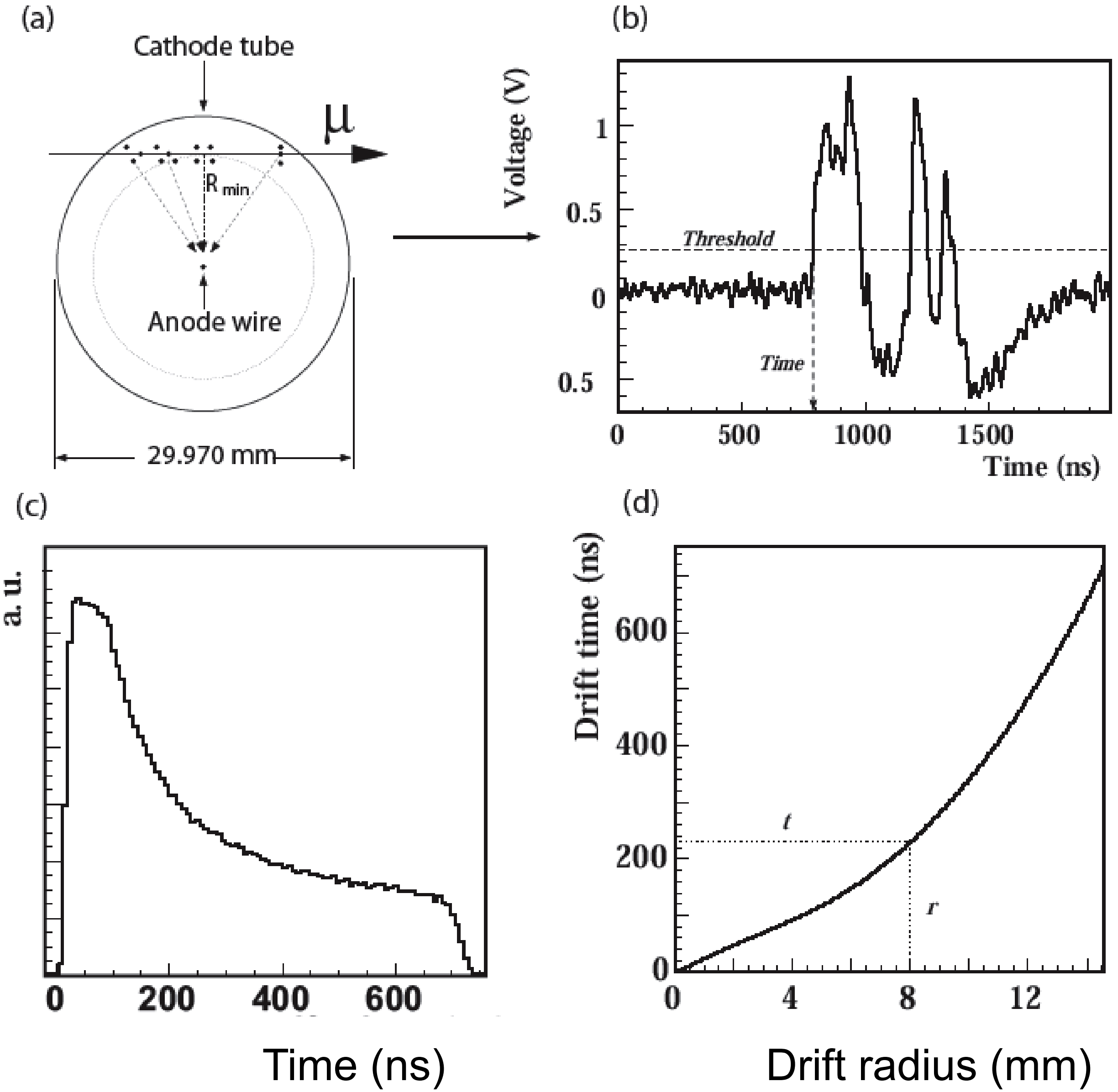
Drift tubes:
- d=30 mm, wire d=50 μm
- P=3 bar (abs)
- Ar-$N_2$-$CH_4$ (91%/4%/5%)
- HV=3270 V

The time of first electron gives is converted into a “drift radius” using the known r(t) relation.

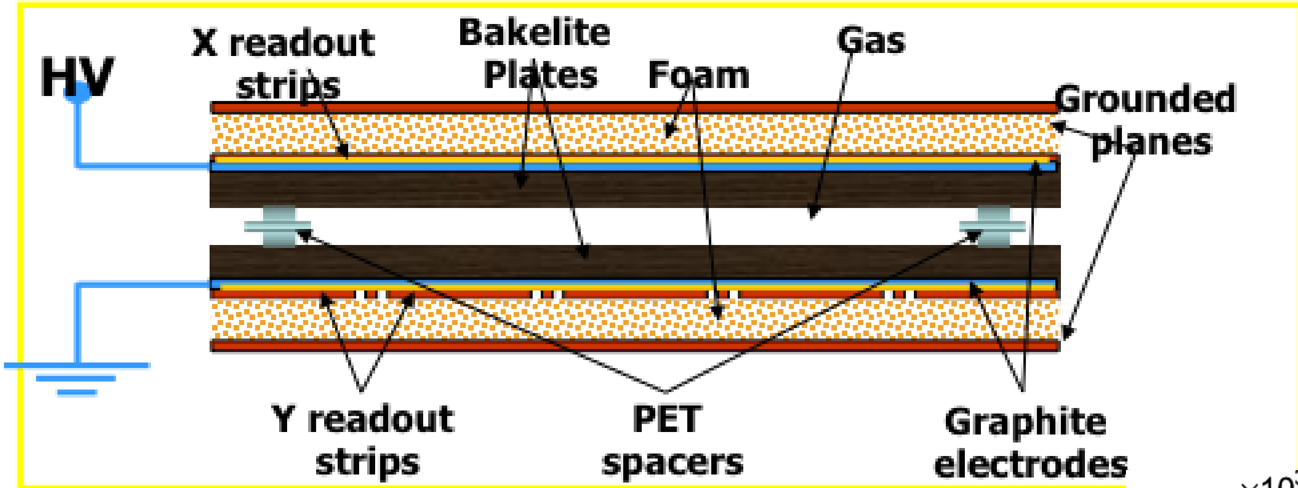
Max drift time ~700 ns

Space resolution ~80 μm (*)

NB:
* knowing the “start” time, and the position of the muon along the tube



# Trigger chambers example: RPCs




**Operating Conditions**

($E_{gas}$~ 5 KV/mm)

Gas: $C_2H_2F_4$ 95% - $C_4H_{10}$ 4.5% - $SF_6$ 0.5% ;

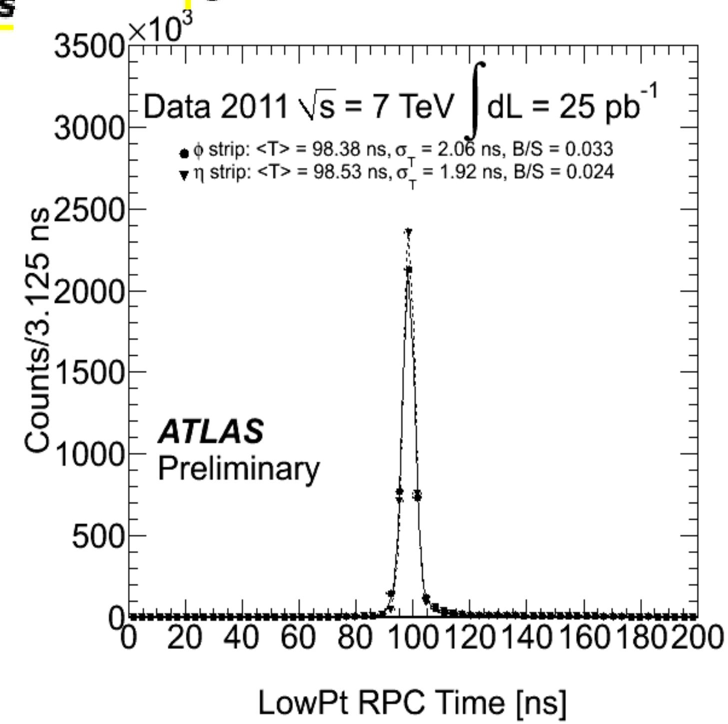
$\rho_{bakelite}$ ~ $2 \times 10^{10}$ Ωcm ;

Gas Gap d = 2 mm ;

Graphite coated HV electrodes

Cu read out strips 30 mm pitch

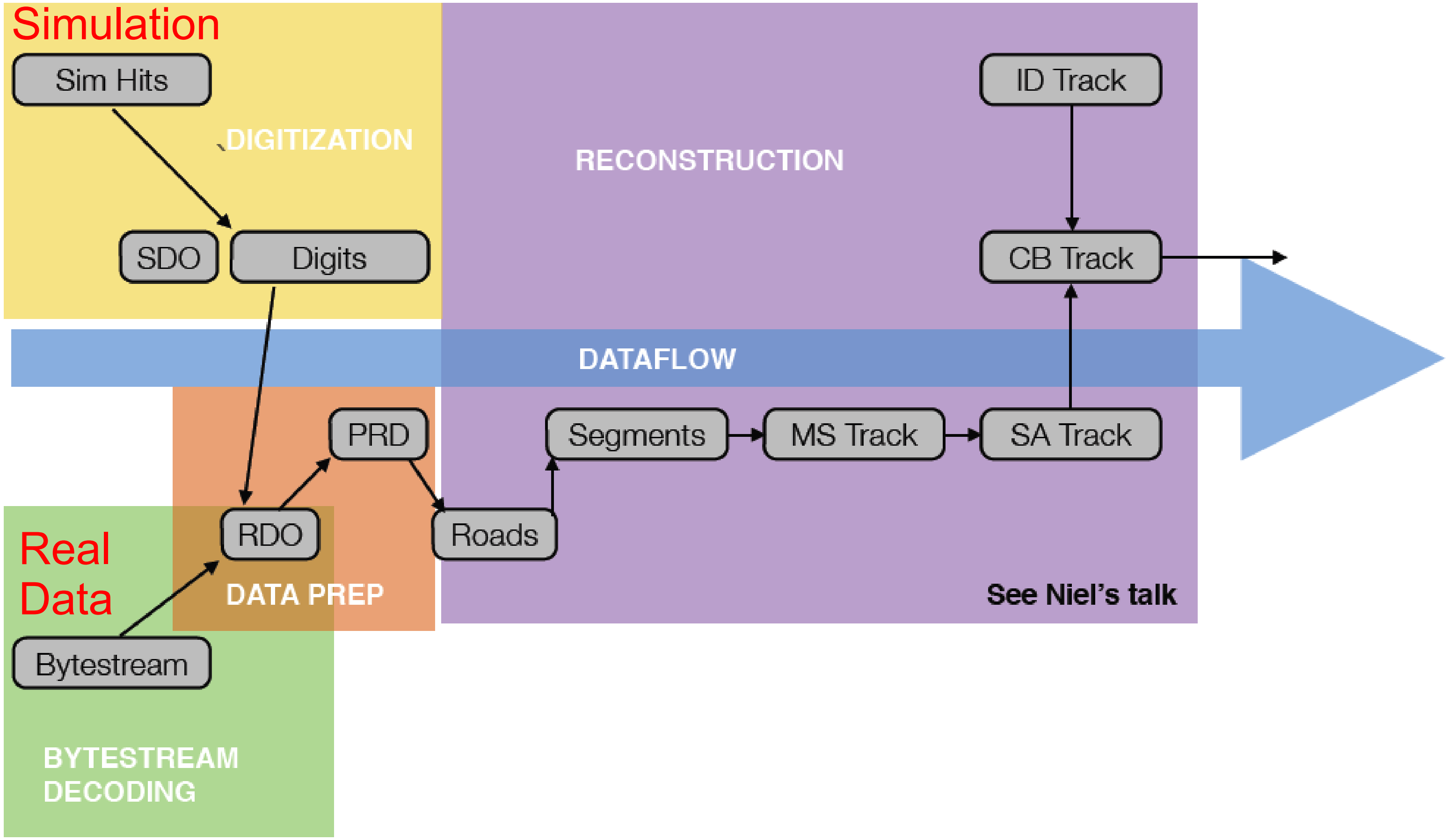
- Readout strips in η and φ
- pitch ~3 cm
- time resolution ~ 2 ns



# Track Reconstruction

- Once hits are produced in the detectors
  The ATLAS reconstruction program should
  - identify the muons with high efficiency and purity
  - reconstruct muon parameters (charge, momentum, direction)

- Online version to be run in the trigger should also be fast

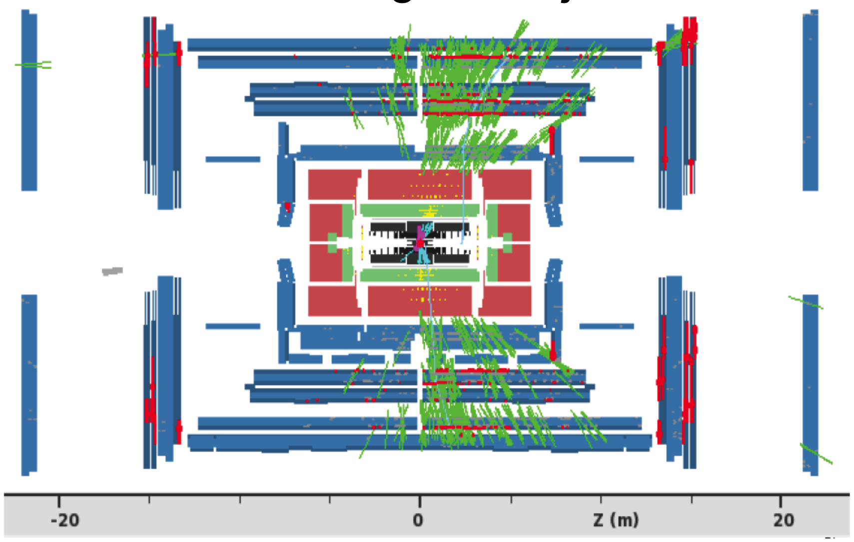


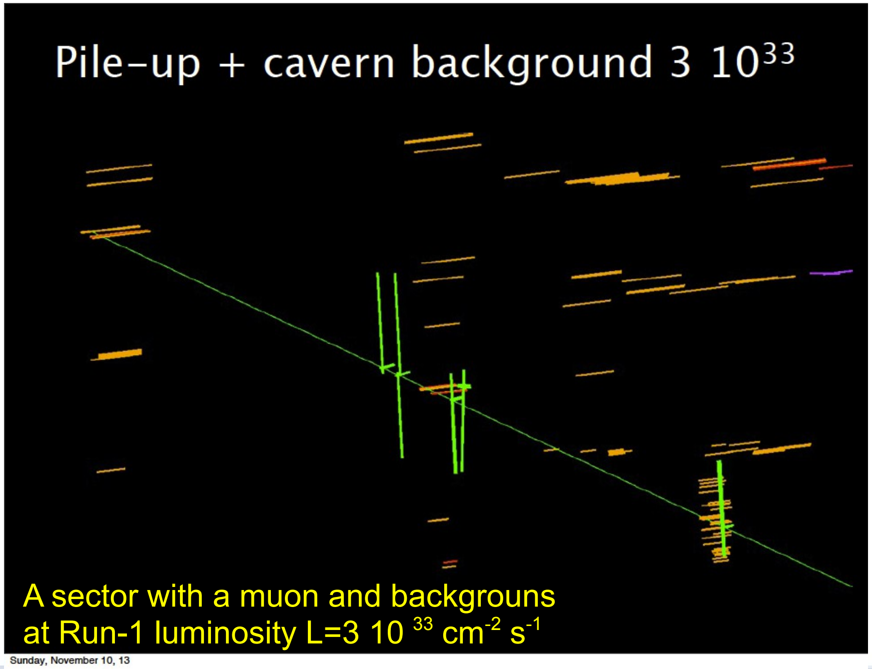
# Pattern recognition

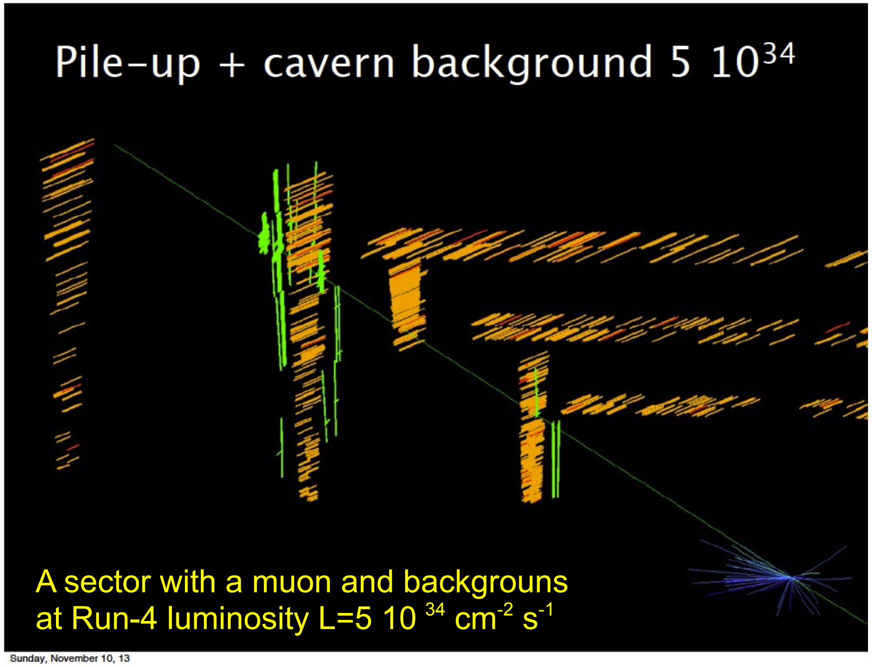
The MS is filled by hits,
not only muons but also :
- Tails of hadronic showers
- Neutron and photons from hadronic int. including a long-lifetime component from slow neutrons (cavern background)
- electronic noise

Not possible to try all hit combinations
(CPU time would diverge)
Need to find patterns from charged tracks

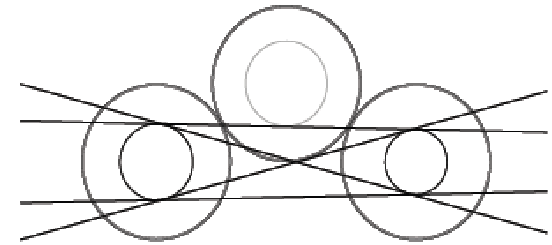
High-Et Dijet event

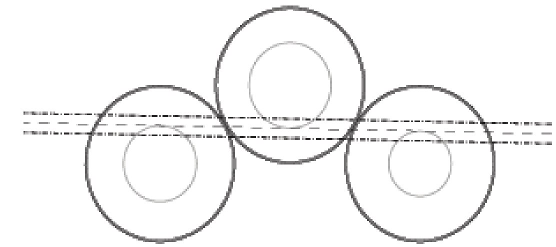


# Pattern recognition





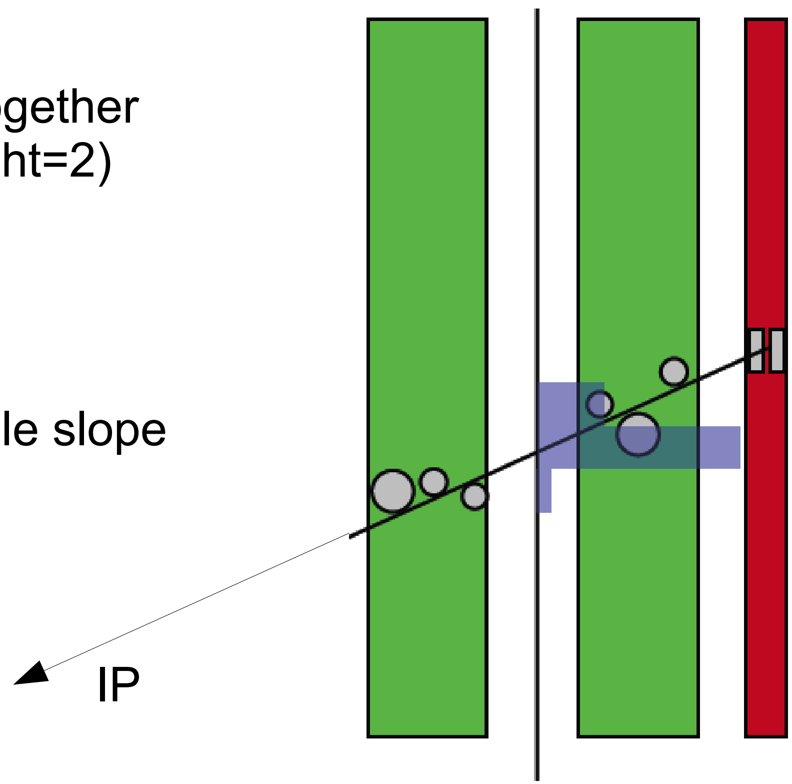
3 tubes MDT  multilayer :

- combining tangents to drift circles gives many possibilities
- Need to consider that tubes are very efficient and accept only the possibilities with more hits

- Simple histogramming technique:
  - project hits along the direction pointing to the interaction point (IP)
  - Select cases with Nhits>=5
  - Can use trigger and precision chambers together with different weights (e.g. trigger hits weight=2)

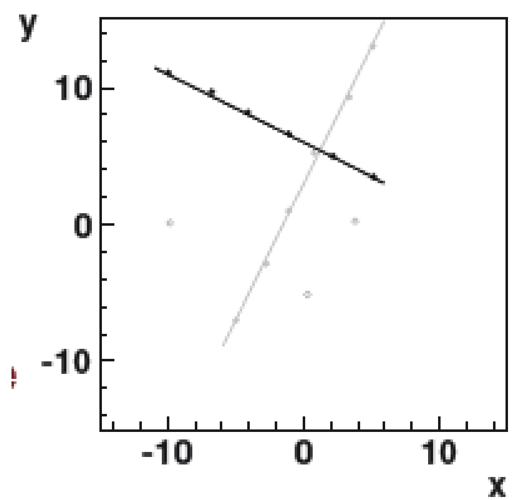
  => very fast, linear with num of hits
  => works only for straight tracks from IP

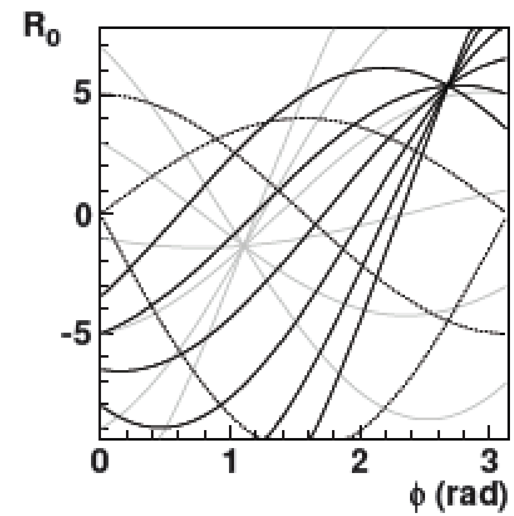
- Extension :
  do different histograms, one for each possible slope
  => Hough Transform

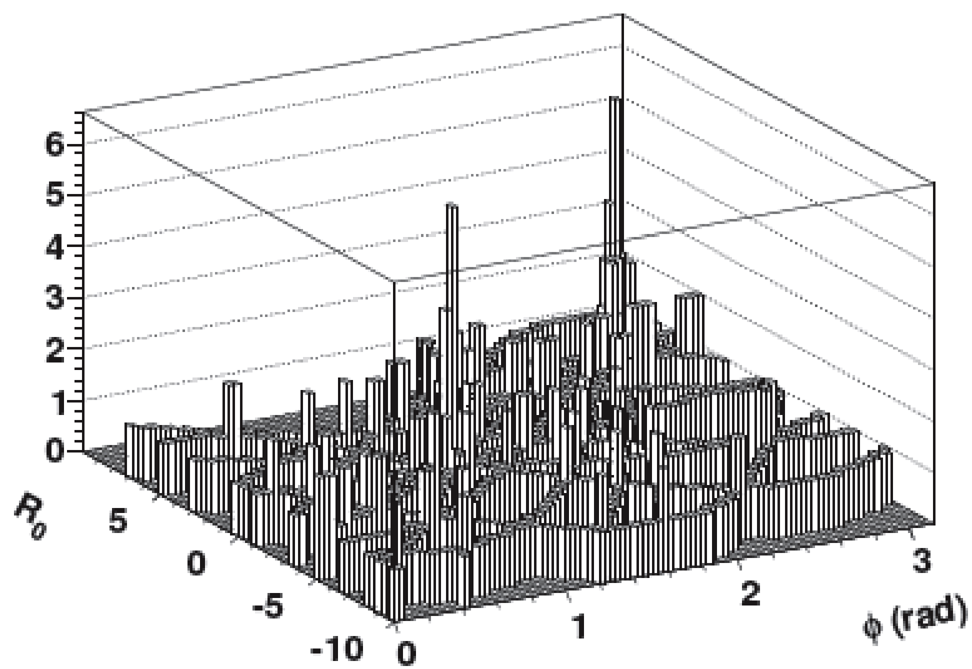


# Hough transform

- Each point in x,y belongs to a family of of straight  lines identified by slope Φ and intercept $R_{0,}$ i.e. it is represented by a curve in the $R_0$ Φ plane
- The curves in  $R_0$ Φ from points on the same segment cross at the same $R_0$ Φ point
- fill histograms in $R_0$ Φ
- select maxima
- Very simple and general approach



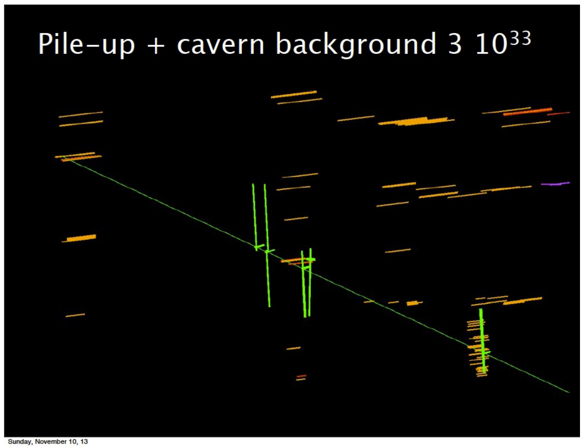



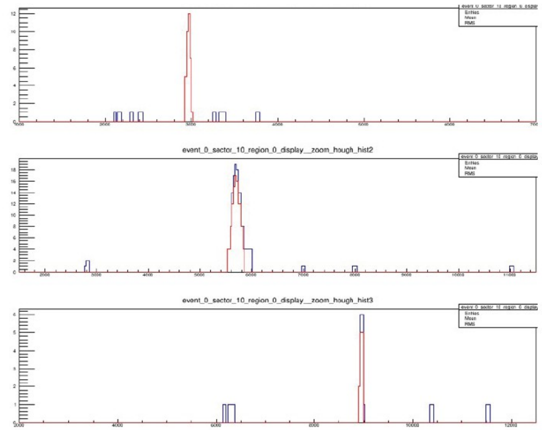
# Hough transform, example

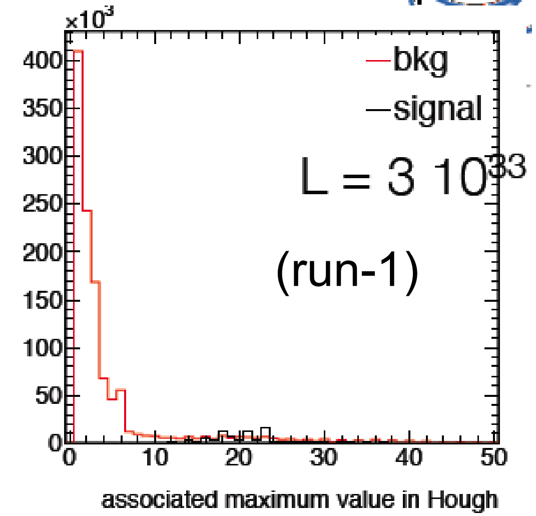
Hits in a sector

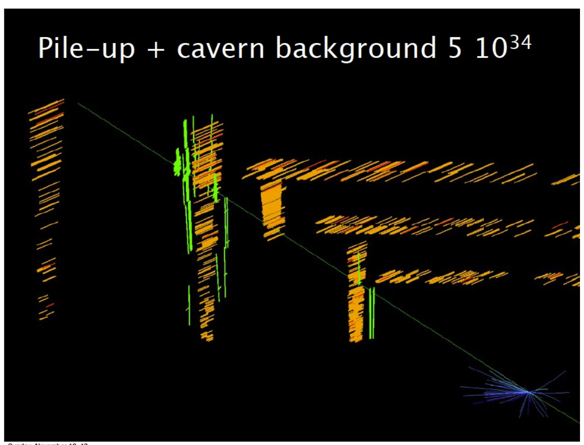
Histograms along $R_0$,fixed $\Phi$

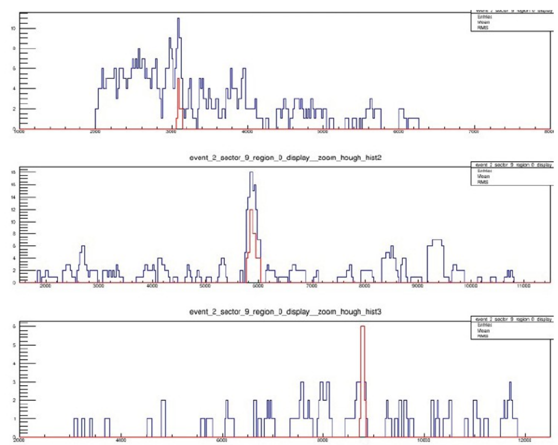
Distribution of hits per bin

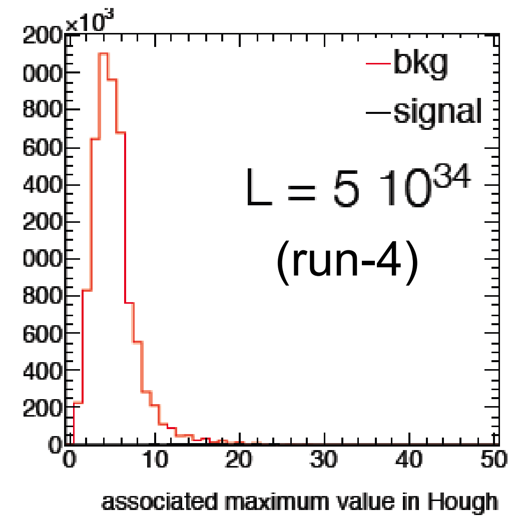




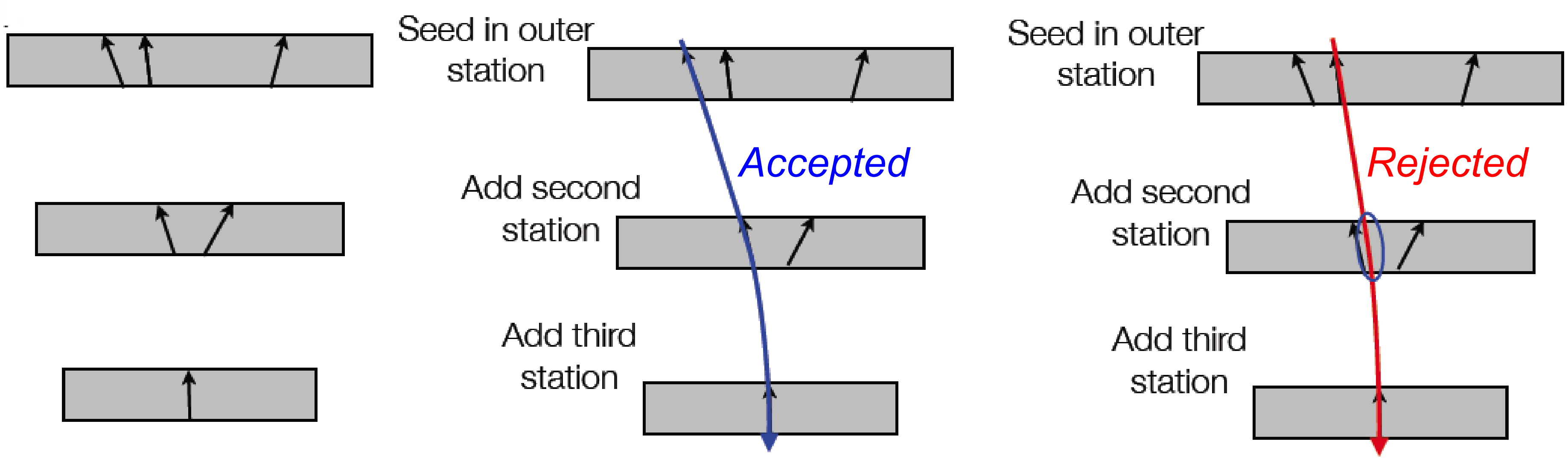






Good signal/bkg separation for Run-1/2 backgrounds
Will need to add more constraint for Run-2/3

# Pattern recognition in the full MS

- Segments found in different chambers are combined starting from outer layers and following the the track trajectory inward
- Combinations with common segments are removed based on number of holes
- Finally we are left with “MS-only” tracks



# Momentum from sagitta measurement

Momentum component perpendicular to B
is related to local curvature R by

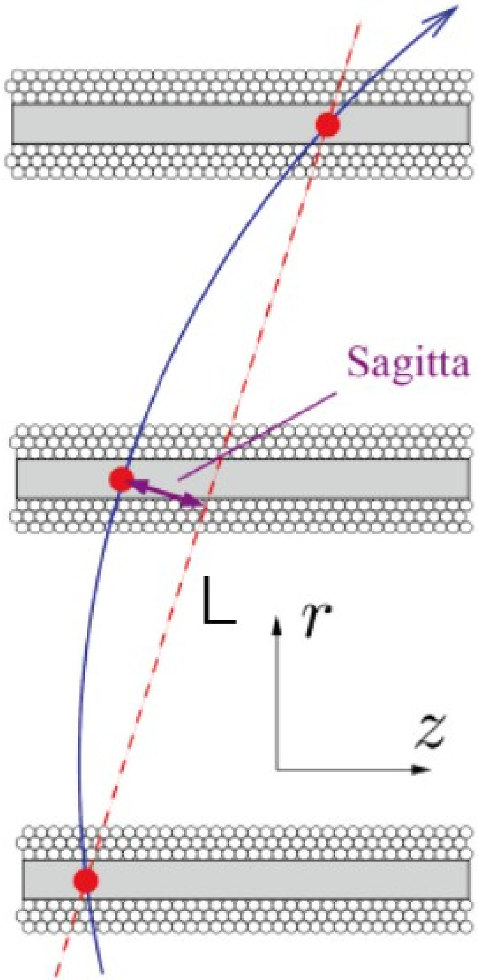
$p_T \sim= k\, q\, B\, R$

$k \sim= 0.3$ GeV/T/m

- With three points in a magnetic field
  we can measure the muon momentum from the
  sagitta S:

$$q/p \sim= 8\, S \;/\; (k\, B\, L^2)$$

- Typically 1 TeV correspond to S~1 mm
- In practice the B field is not homogeneous,
  there are more than 3 measurements, we need
  to extract the track parameters from a fit



# Track fit in the MS

- A track is characterised by 5 parameters,
- e.g. choosing as a reference surface the cylinder with radius $r_0$ corresponding to the MS entrance :

$$V(r_0) = (q/p, z(r_0), \varphi(r_0), dz/dr(r_0), d\varphi/dr(r_0))$$

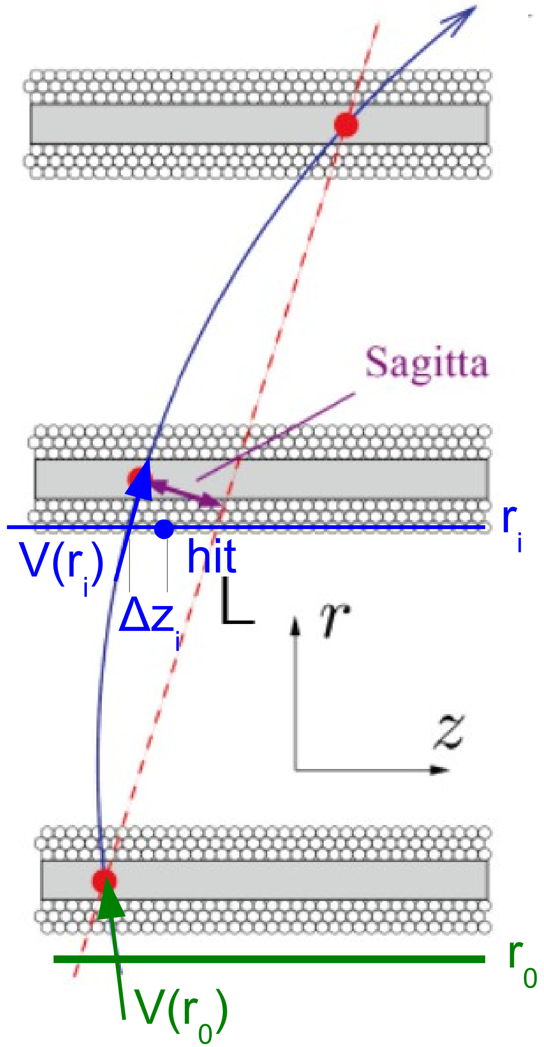
- Given $V(r_0)$ it is possible to extrapolate the track to any $i^{th}$ detector layer using a precise numerical transport code, together with a precise map of the B field and of detector positions:

$$V(r_0) \Rightarrow V(r_i)$$

  the track covariance matrix is propagated as well.

- A global χ2 can be calculated from the residuals $\Delta z_i$ between extrapolated track at $r_i$ and the actual $i^{th}$ measurements.

- A global χ2 minimization gives the best parameters at MS entrance

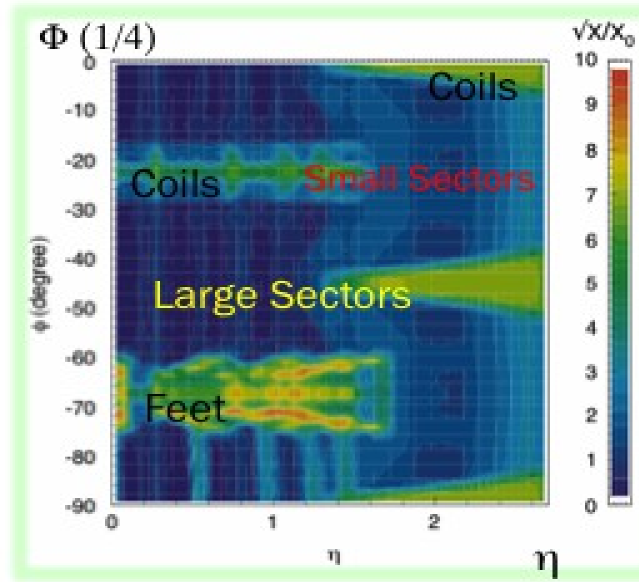


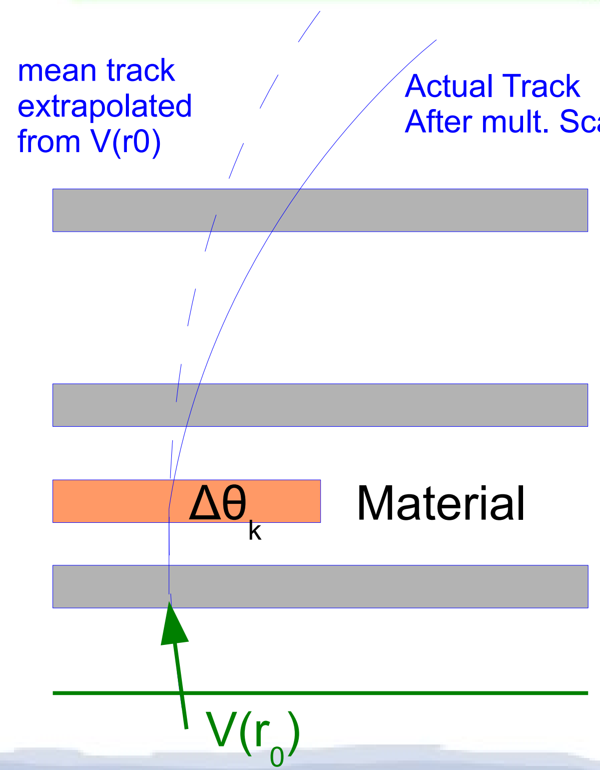
# Multiple scatterig in the MS

- The previous picture is complicated by multiple scattering: the MS contains many radiation lengths of support structures and detectors
- Multiple scattering: RMS angular deflection from material:

$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{\beta c p}\, z\, \sqrt{x/X_0}\Big[1 + 0.038\ln(x/X_0)\Big]$$

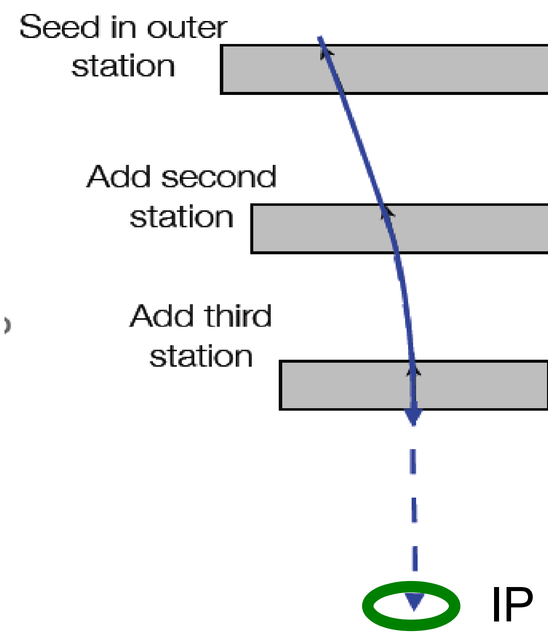
- Multiple scattering is a stochastic phenomenon that introduces irreversibility into track transport
- Additional “kink” parameters in global fit that allow a deflection ($\theta_k$ $\varphi_k$) at reference scattering planes.
  They are treated as nuisance parameter in the fit giving a penality if the deflection is too large. $\Delta(\chi 2)_k = (\theta_k/\theta_{rms,k})^2 + (\varphi_k/\theta_{rms,k})^2$
- 
- At this point we have the best track at the MS entrance

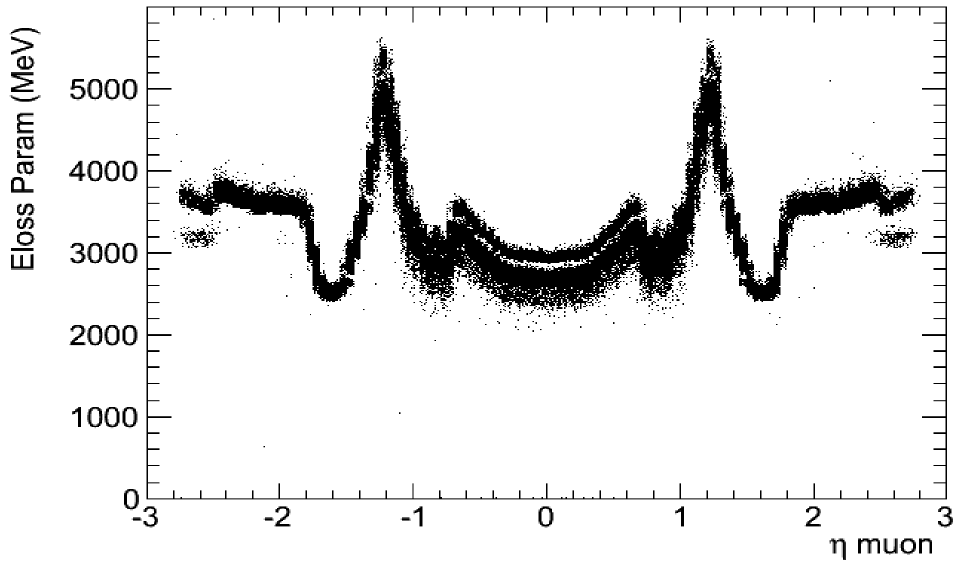


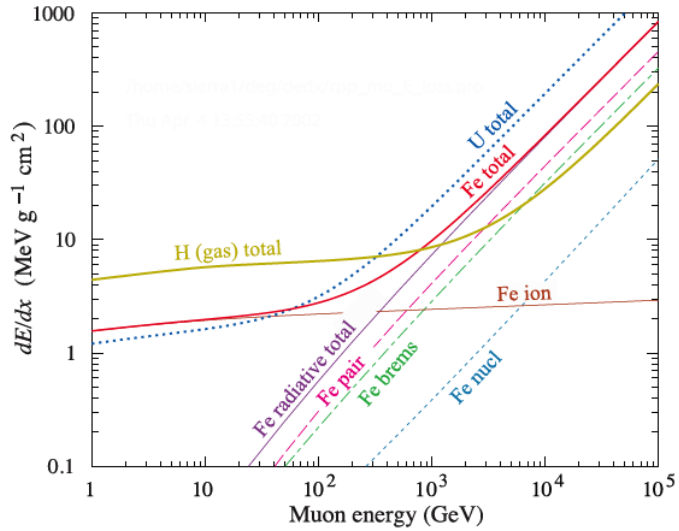



# Extrapolation to the IP: energy loss

- What we are really interested in are the muon parameters at the IP: need to back-extrapolate through the calorimeters
- Energy loss : approx. 3 GeV (eta dependent)
- Landau tail: large fluctuations
- Use a combination of parametrization and CAL mesurement to estimate energy loss
- CAL measurement used only for isolated muons and for large losses
- Final "Muon Extrapolated" fit including the IP connstraint gives the track parameters at the IP

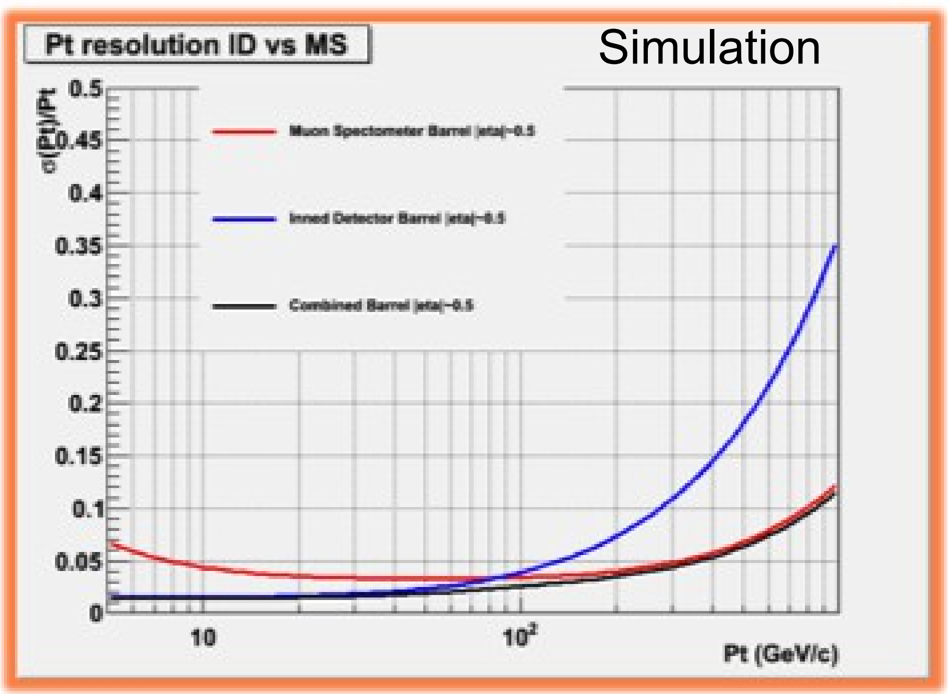


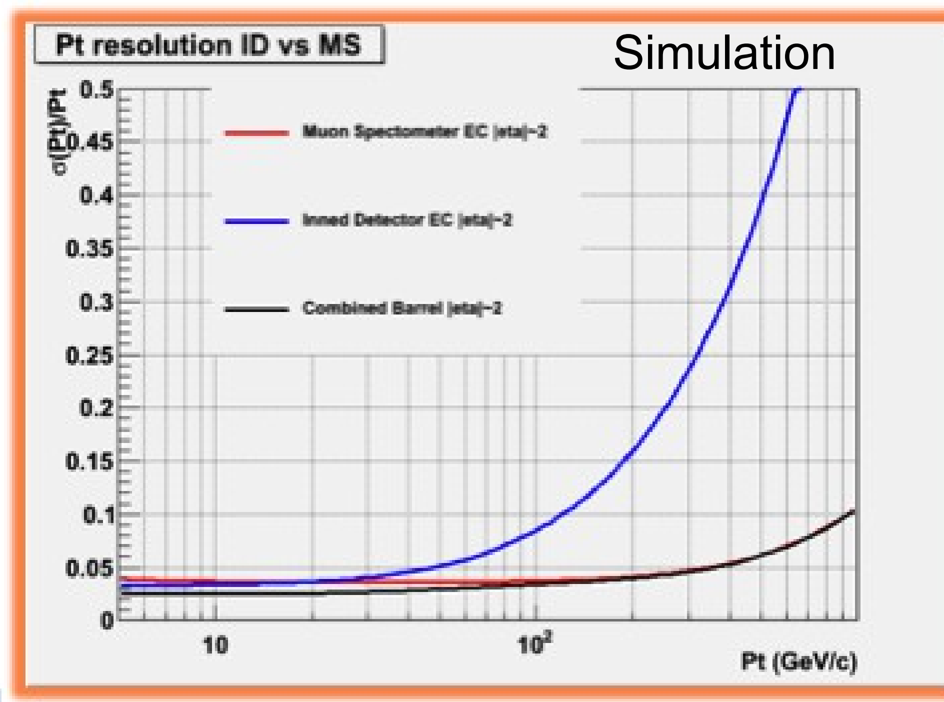



# ID-MS combined muons

- Outside-In reconstruction: “Muon-Extrapolated” tracks are matched with Inner Detector (ID) tracks to form “Combined Muons”, a full combined fit of ID and MS hits is performed to obtain the final parameters.
- MS dominates the CB measurement for pT>80 (20) GeV, depending on η
- Inside-Out reconstruction: start from ID tracks and add hits in the MS allows to recover acceptance for low-quality muons with few hits in the MS

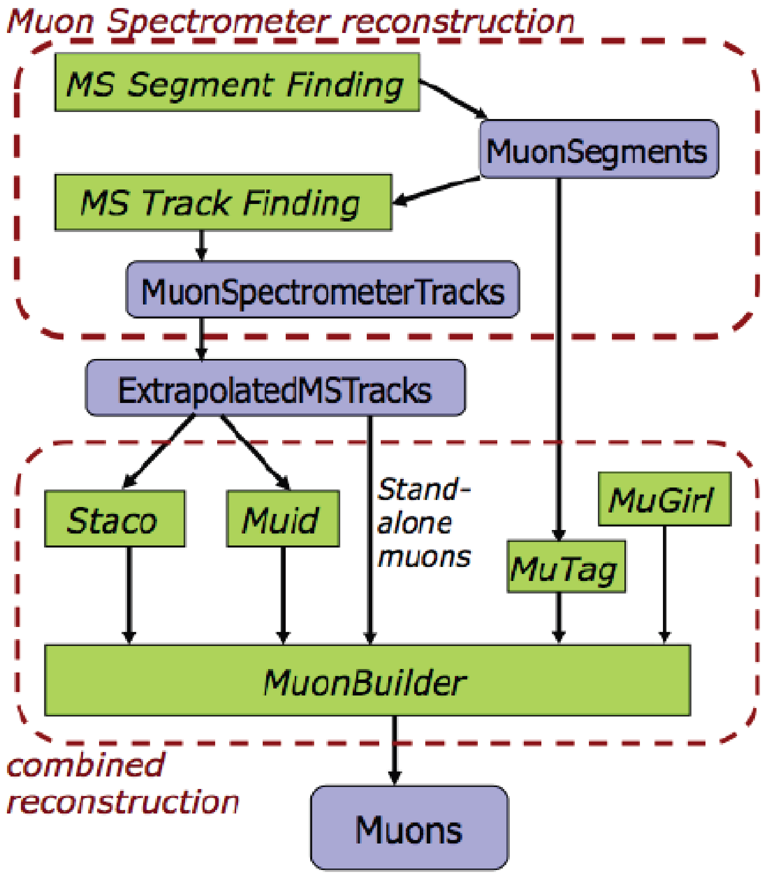


# Reconstruction output

Final Muon “collection” for analysis

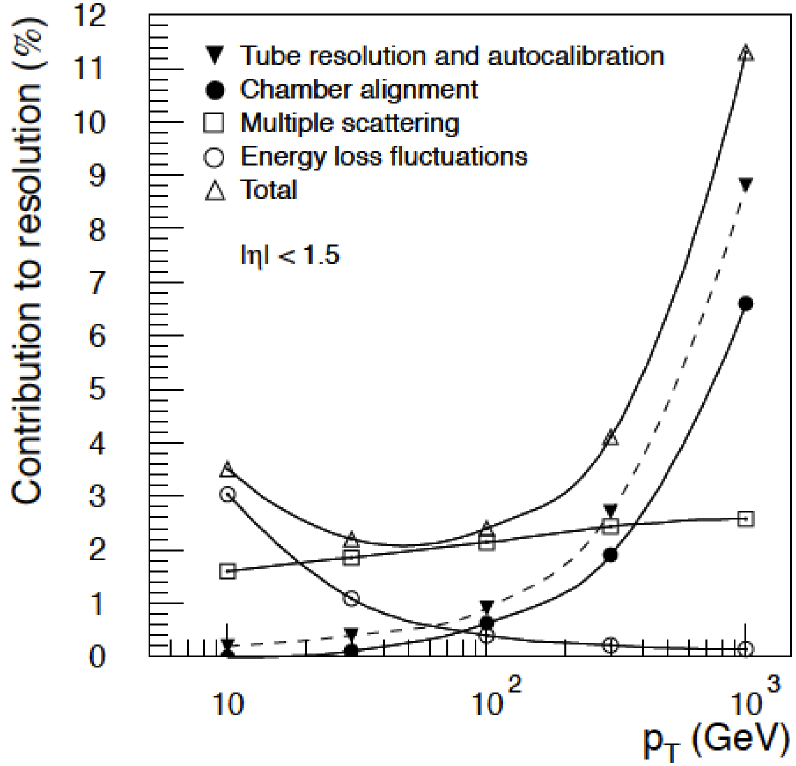
- Outside-in CB muons (best quality): two possible combination algorithms: Muid (track refit) or Staco (statistical combination)
- Inside-out muons: (two algorithms Mugirl, MuTag)
- Stand-Alone Extrapolated muons (recover ID failures, |η|>2.5)

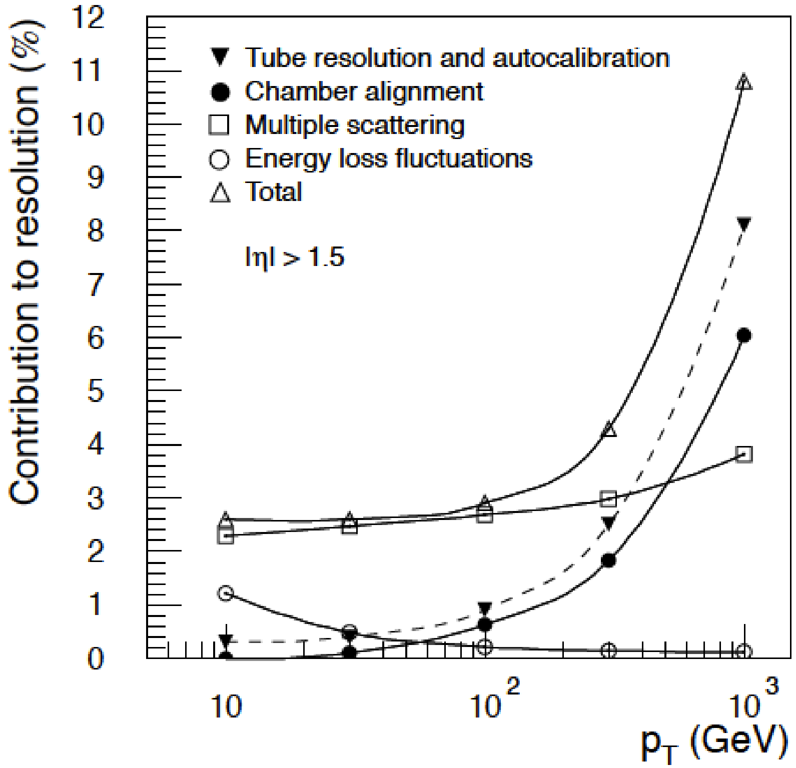


# Momentum resolution of the MS

Main contribution to momentum error:
- Error on hit measurements (e.g. uncertainty on sagitta): $\Delta p/p \sim k_2 p$
- Multiple scattering : $\Delta p/p \sim k_1$
- Energy loss fluctuations : $\Delta p/p \sim k_0/p$

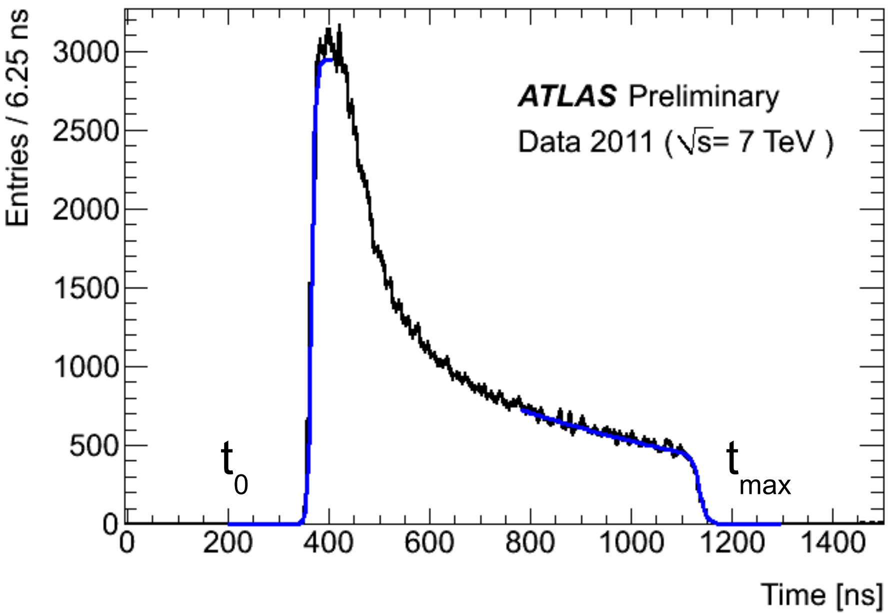
For $p_T$<100 GeV multiple scattering dominates ($k_1$ =2-2.5%)
For $p_T$>100 GeV the "intrinsic term" ($k_2$ ~10%/TeV)
Energy loss fluctuations relevant at low p ($k_0$ ~ 250 MeV)

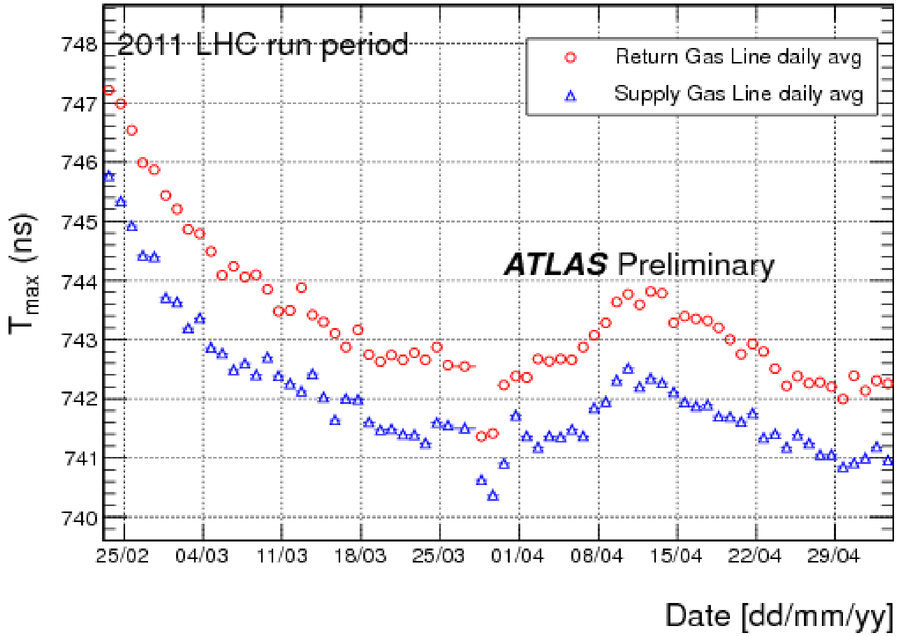


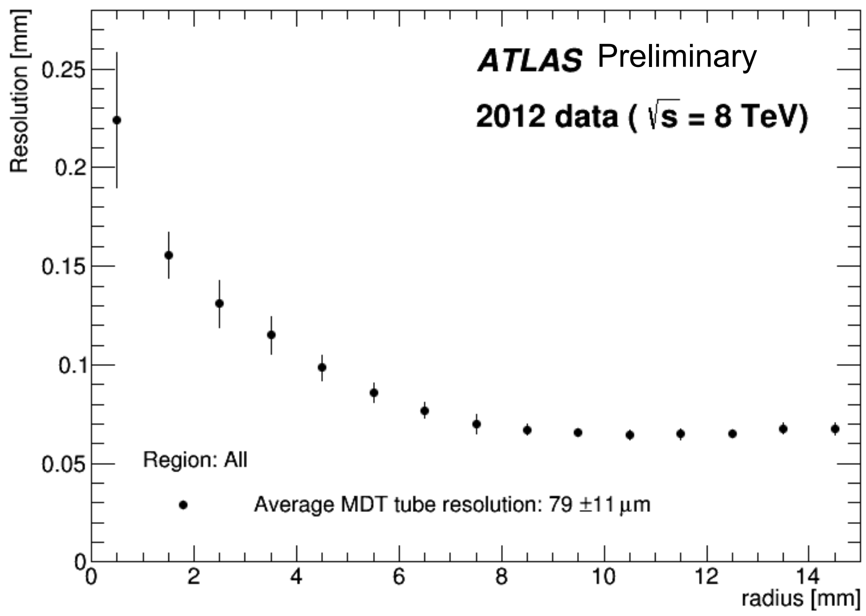



# MDT r(t) calibration

- To arrive to the nominal hit resolution r(t) relation needs to be calibrated, In particular:
  - $t_0$ tube-by-tube variations
  - $t_{max}$: drift velocity variations due to gas temperature, pressure, composition
- Final hit resolution ~80 μm



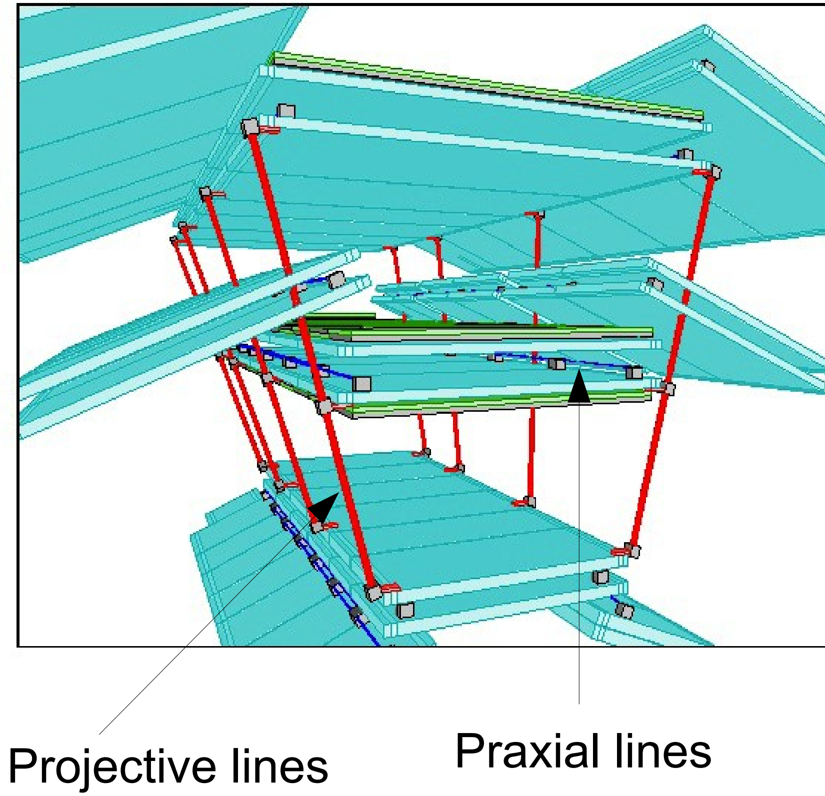



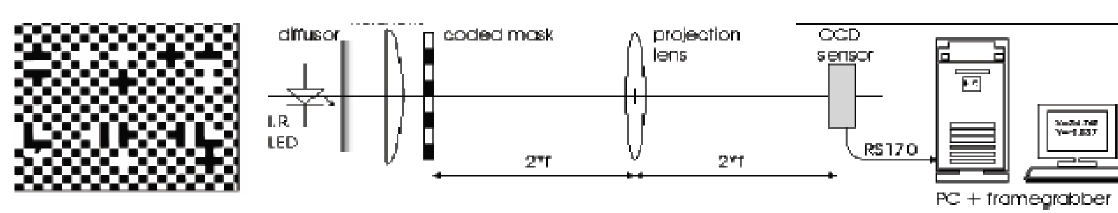



# Alignment System

- The “intrinsic” term of p resolution has two components:
  - hit resolution
  - knowledge of detector position: alignment
- The MDT chambers are constructed as precision objects: wires can be located inside a chamber within few tens of μm
- Location and orientation of MDT chambers in ATLAS not trivial: we aim at precision <50 μm over distances of O(~10 m)
- “Absolute” alignment based on tracks (next page)
- Optical alignment system used to
  - follow the relative displacements between different alignment runs
  - Constraint “weak modes”

Barrel optical alignment



# Track-based alignment

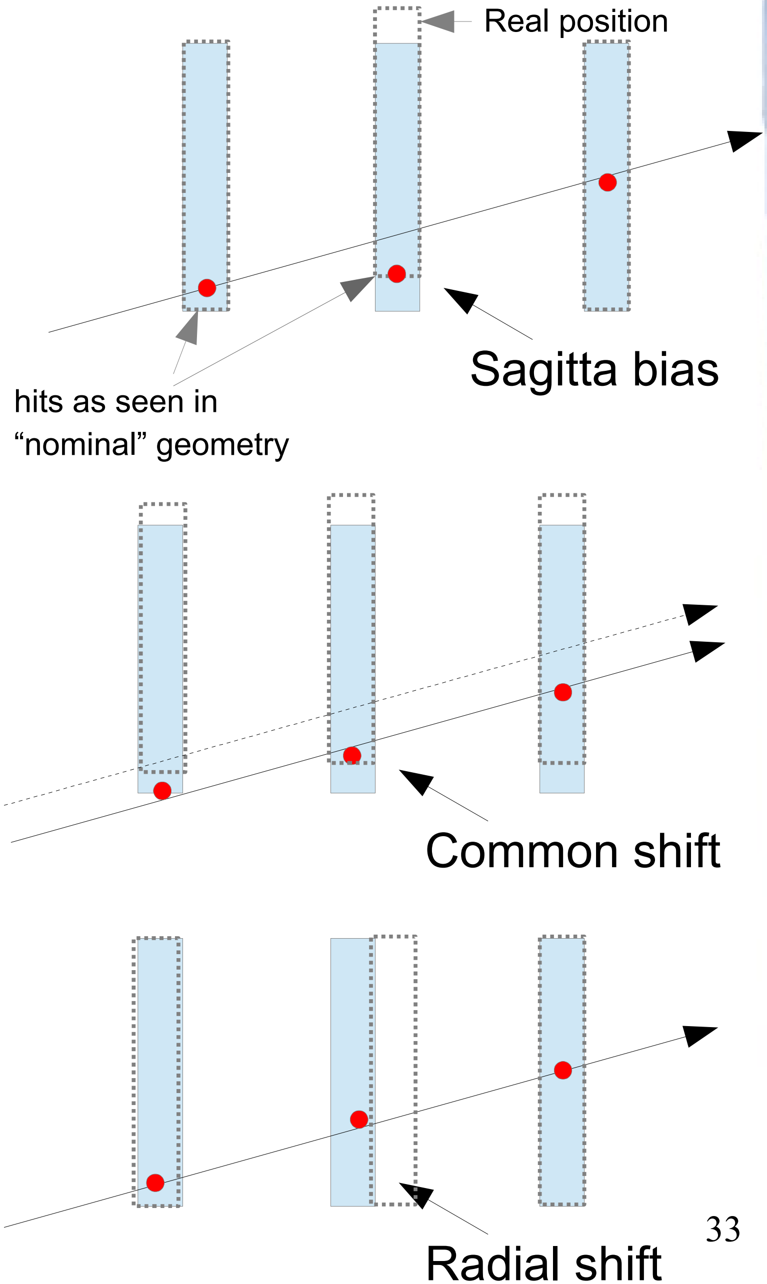
- "Absolute" alignment is performed in special runs with the toroidal magnetic field off
  - all tracks are approx. straight lines
    no sensitivity to knowledge of B field or material
  - ID (solenoid) allows to select high-pT tracks
    to reduce multiple scattering

- Cosmic rays are used for the Barrel
- Special collision runs for End-Caps (expensive !)

- In practice only sensitive to sagitta bias

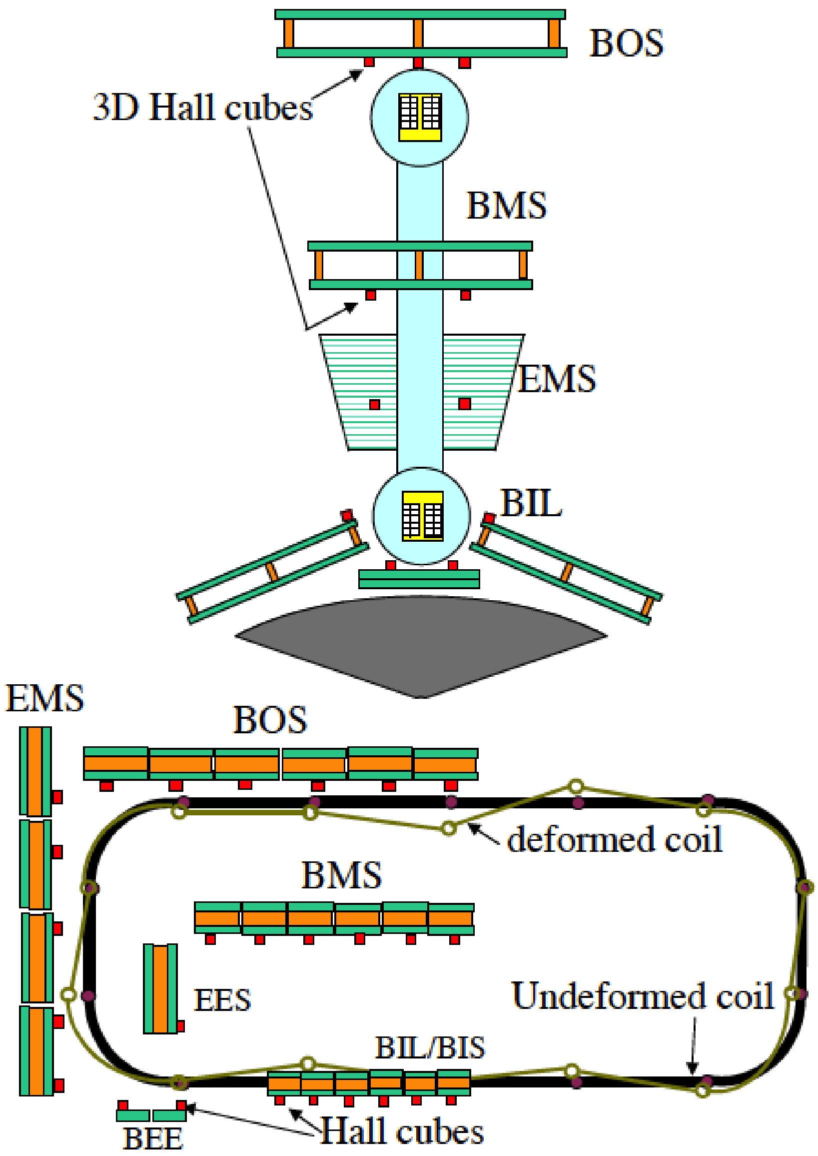
- weak modes: common shifts + radial distortions partially recovered from overlaps between sectors and cosmic rays crossing different sectors

- Current precision on sagitta bias ~40 μm RMS using ~50M events from collisions with toroid off



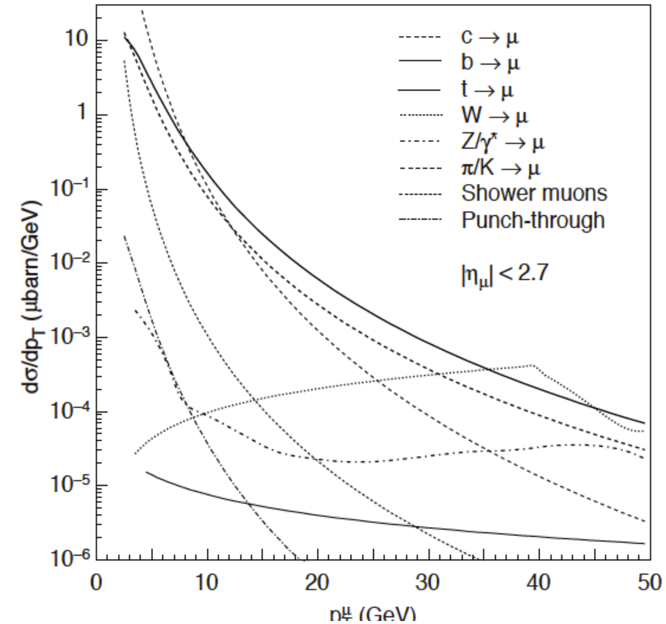
# Magnetic field measurement

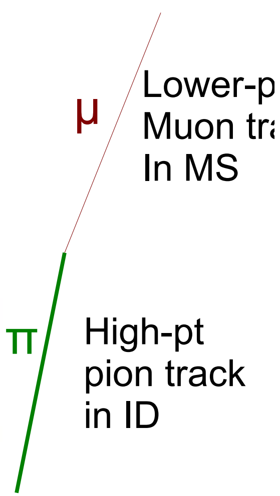
- The B field integral (actually ∫ B L dL !) should be known precisely to avoid momentum scale biases
- B field maps are made with numerical codes based on the Biot-Savart law, plus non-linear perturbations from ferromagnetic materials (e.g. calorimeters, iron supports of the calorimeters, iron inside concrete walls, cranes etc.)
- The currents in the coils are known precisely, the actual shape of the superconductor coil inside the cryogenic vessel is not so well known
- 3D Hall probes measure B on each chamber => coil shape is fitted to get the best model/measurements agreement
- B known to ~$3*10^{-3}$ T

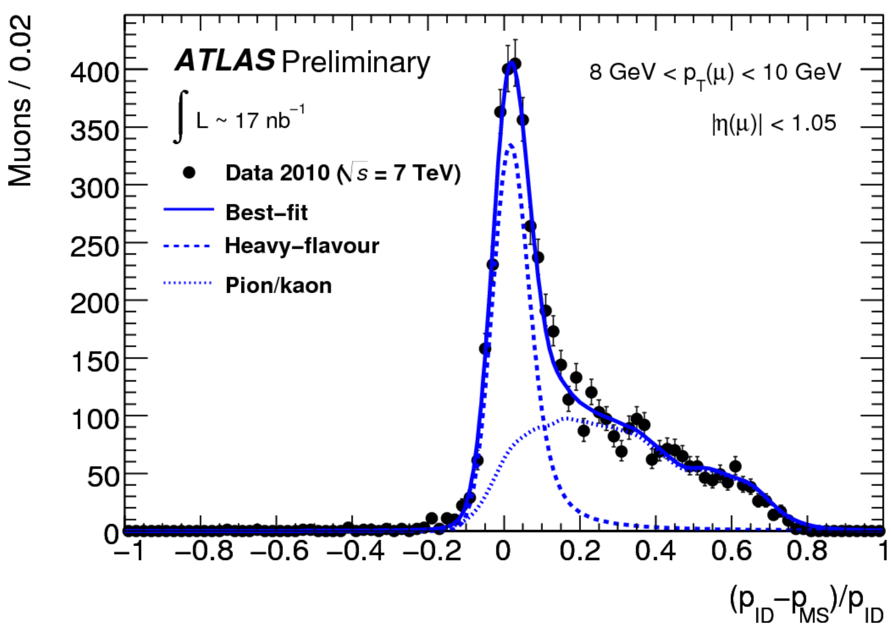


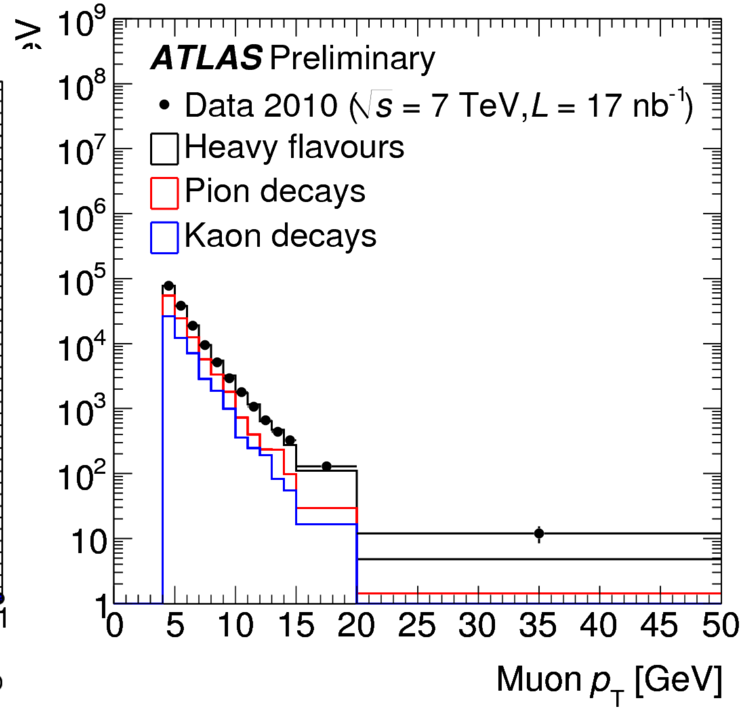
# Backgrounds from π, K decays

- Muons in the MS originate from
  - π/K decays
  - heavy quark decays
  - ( Z,W decays )
- π/K decays can be removed with tighter cuts on ID-MS momentum difference
- Probability that a π/K is identifed as a muon (pT=20 GeV) : ~0.2%, ~0.1% with tight cut on ID-MS momentum matching



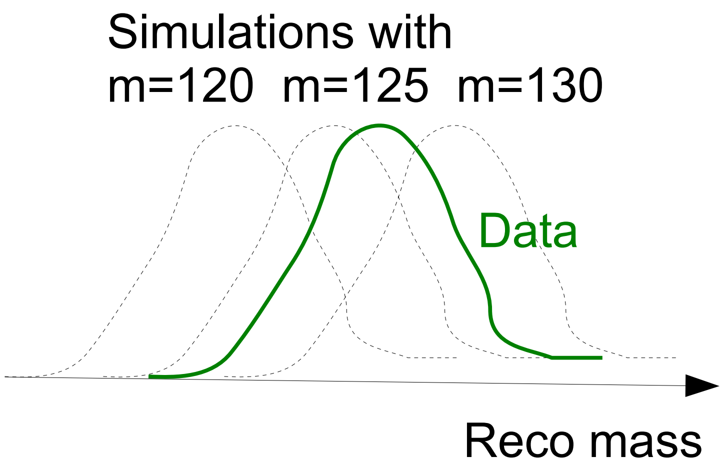
# Measurement of performance

- Main Performance parameters to be measured in data
  - Efficiency
  - Momentum resolution and scale
- In physics analyses MC simulations are used to unfold the detector response
- The differences in efficiency and momentum resolution/scale are compared between data and simulation

  => corrections are applied to MC simulation to give the best description of the data

Example Higgs mass:

*Not so important that reco mass is correct*
*But that Simulations reproduces the data*



# Efficiency: Tag and Probe method

- Need a sample of “unbiased” muons
- “Tag and probe” method : select Z->μμ by
  - Tag: isolated high-pT CB muon
  - Probe: ID track making the correct invariant mass once combined with the Tag

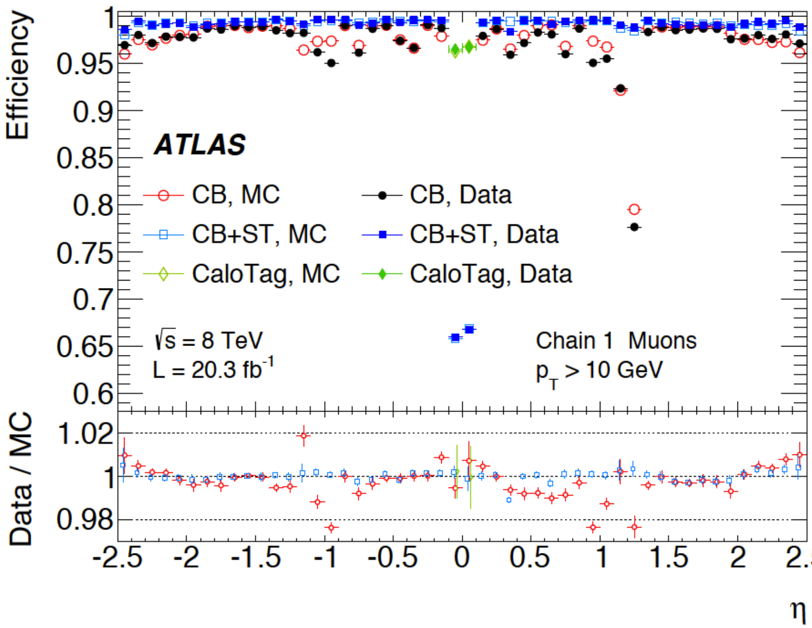
- Use the probe to check combined muon efficiency (given an ID track):
  P( CB | ID ) =
  N(probes matched to CB)/ N(probes)

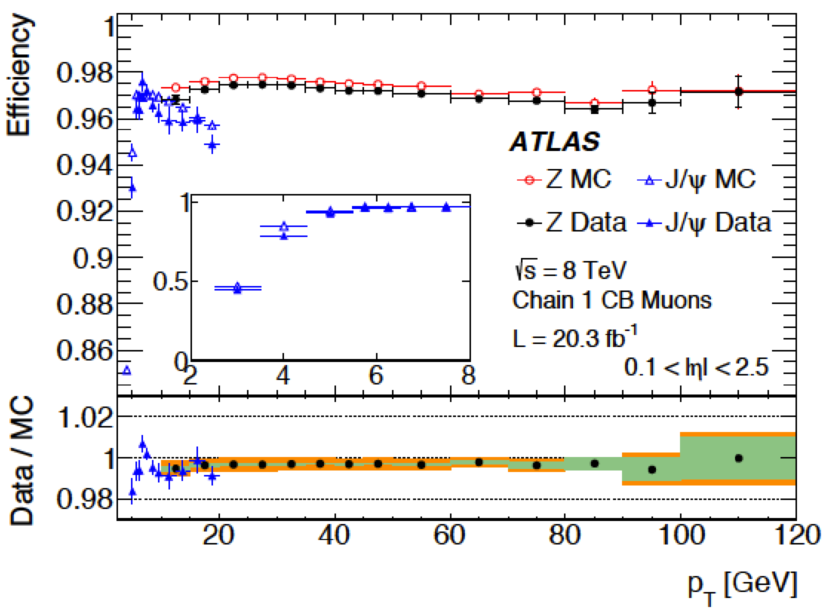
- The approach can be inverted using MS muons as probes to check the ID efficiency (TP approximation):

  eff(CB) = P( CB | ID )  P( ID | true-μ)
  ~= P( CB | ID)  P( ID | ME )

- Requirement that Calorimeter deposit associated to ID probes is compatible with a muon (CaloTag) to reduce the remaining backgrounds

- J/ψ decays used for low-$p_T$



# Efficiency: uncertainties and Scale Factors

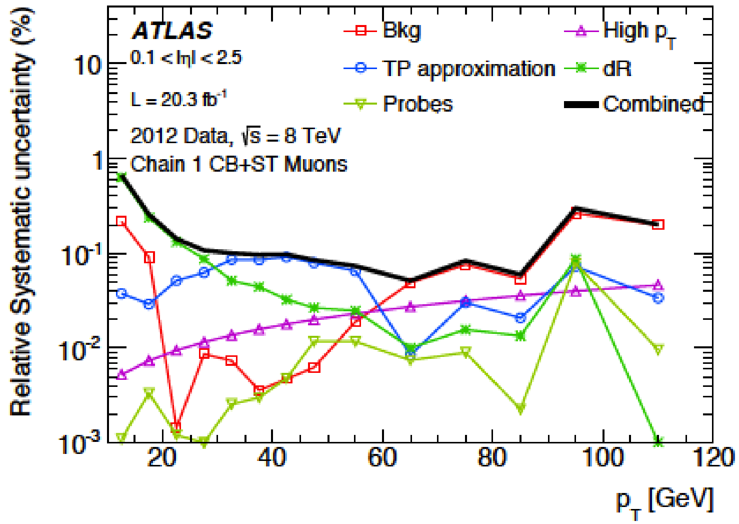
- Main systematics uncertainties from

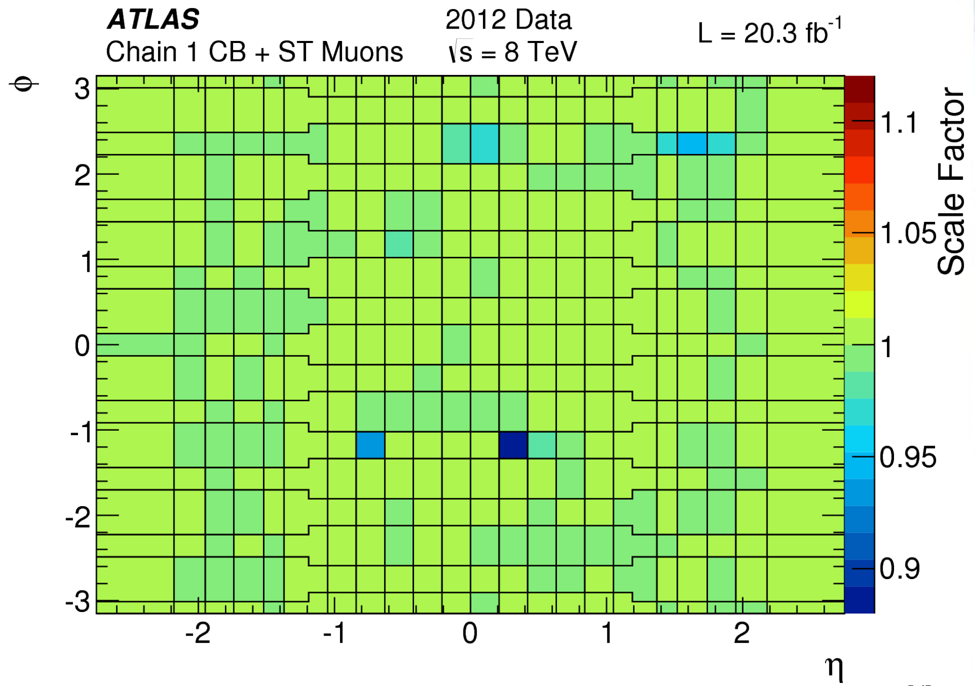
  - "TP approximation" estimted comparing measured and true efficiency in MC

  - backgrounds at large $p_T$ estimated from same-sign dimuons and MC

- Scale factors for physics analysis: η-φ maps of eff(Data)/eff(MC) to be used to correct MC in physics analyses

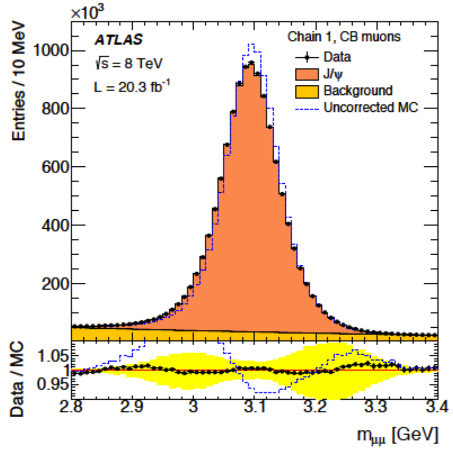
- Data/MC differences in general within 1% Few differences due to problematic chambers, or to low efficiency of trigger chambers.

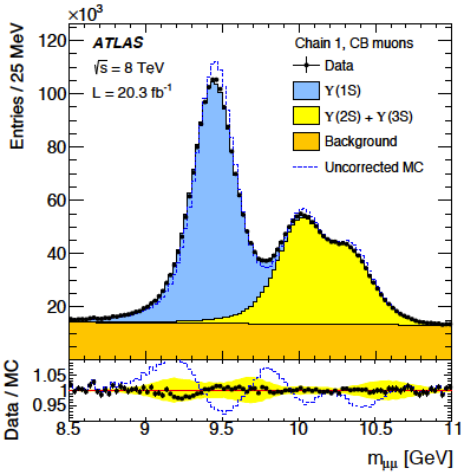


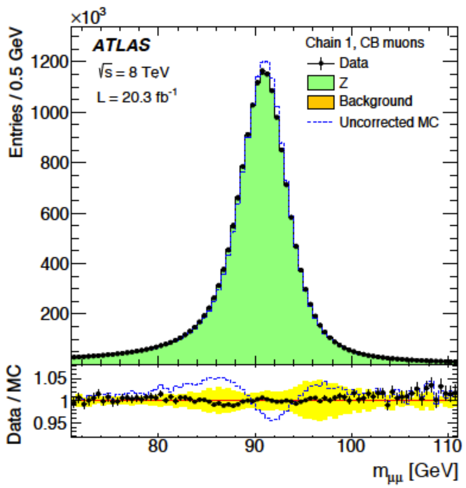



# Momentum corrections

- Resonances of well known mass are used to “calibrate” the momentum response.
- MC corrections (in η bins):
  Scale: $p_T \rightarrow s_0 + p_T\,(1 + s_1)$
  Resolution: add random smearing terms to $p_T$ of sigma $\Delta r_0$ , $\Delta r_1 p_T$ , $\Delta r_2 p_T^2$
- The best parameters are obtained by comparing data and smeared MC distributions for a set of invariant mass distributions from J/ψ and Z samples.
- Same apporach repeated for ID and MS measurements separately. Then comined to obtain the correction for CB muons

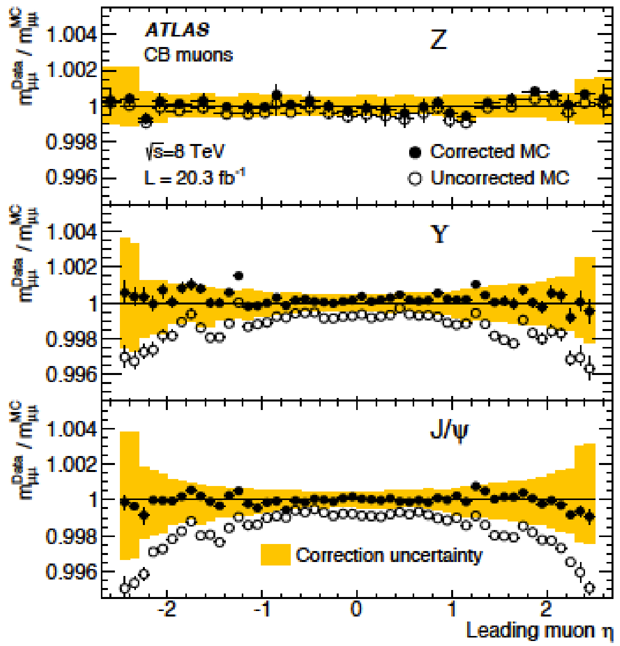


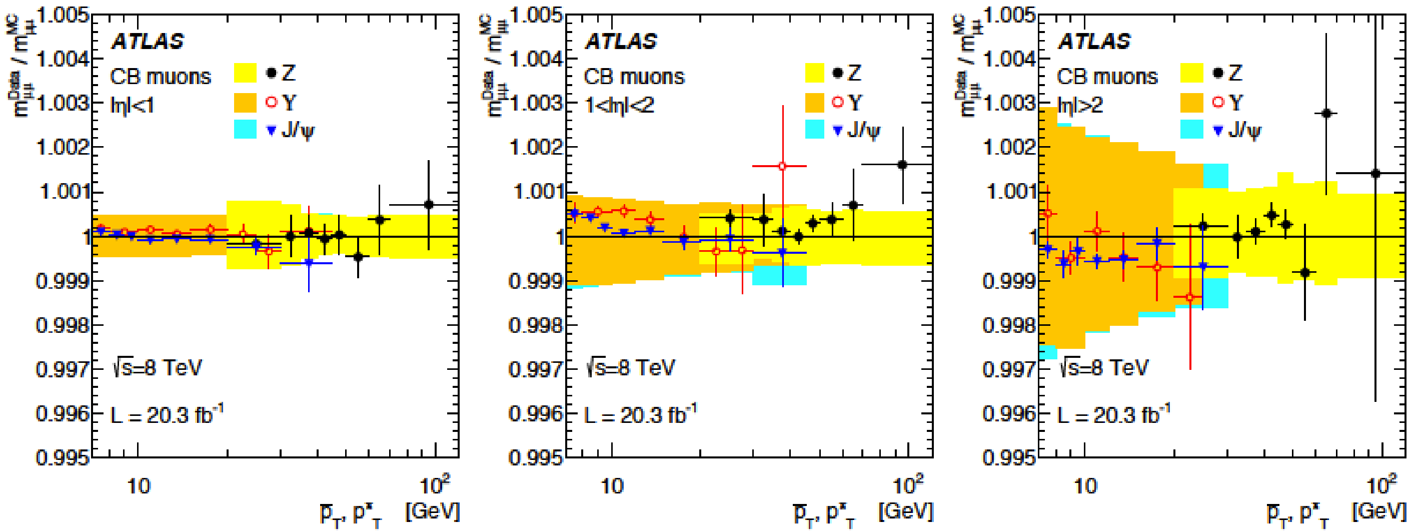



# Momentum scale: results

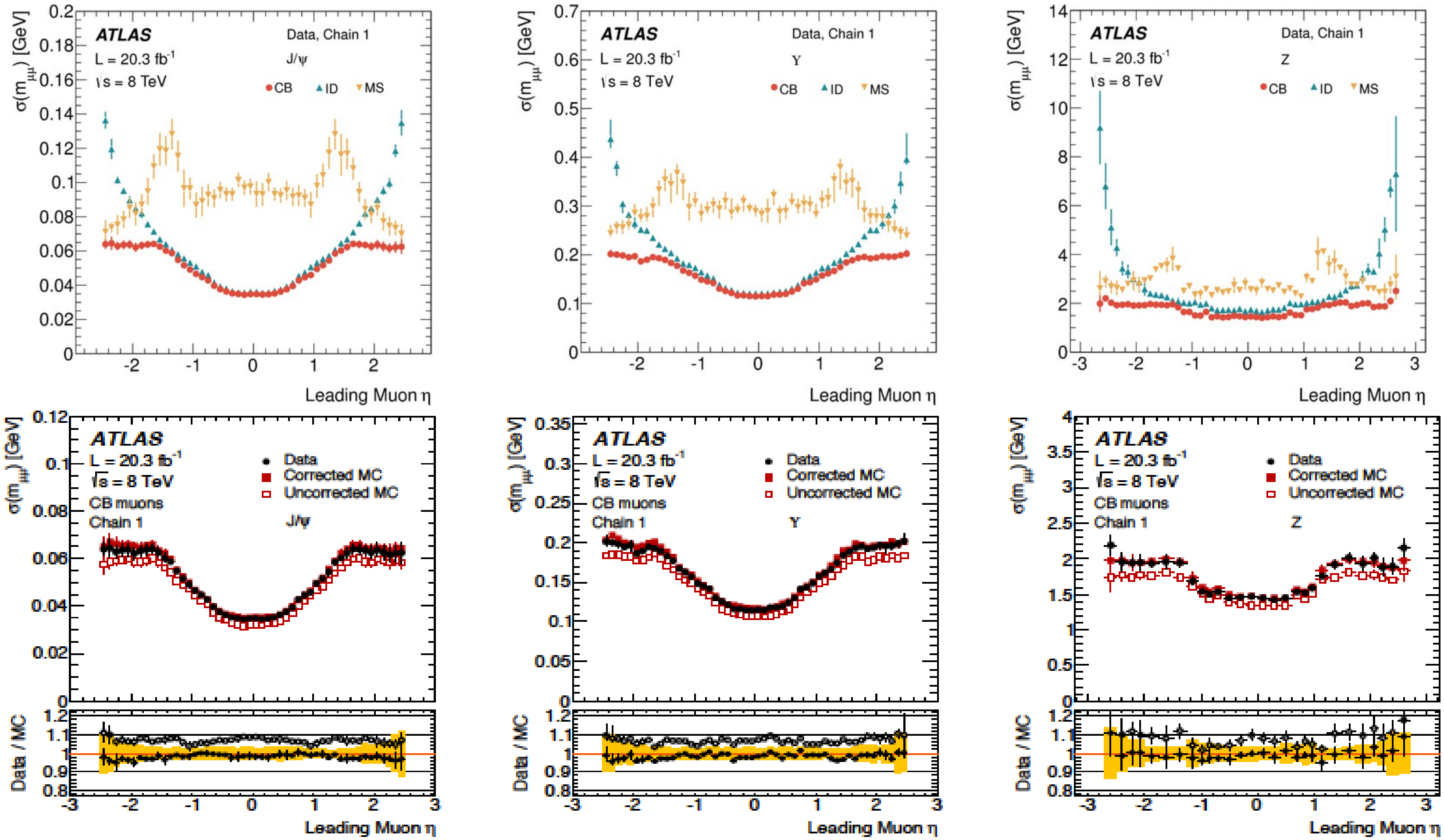
- Momentum scale in data wrt ideal MC :
  - 0.1% offset in ID scale
  - bias at low-pt (E-loss)
- Corrected MC agrees with data within uncertainties (<0.1% from fit syst + stat)



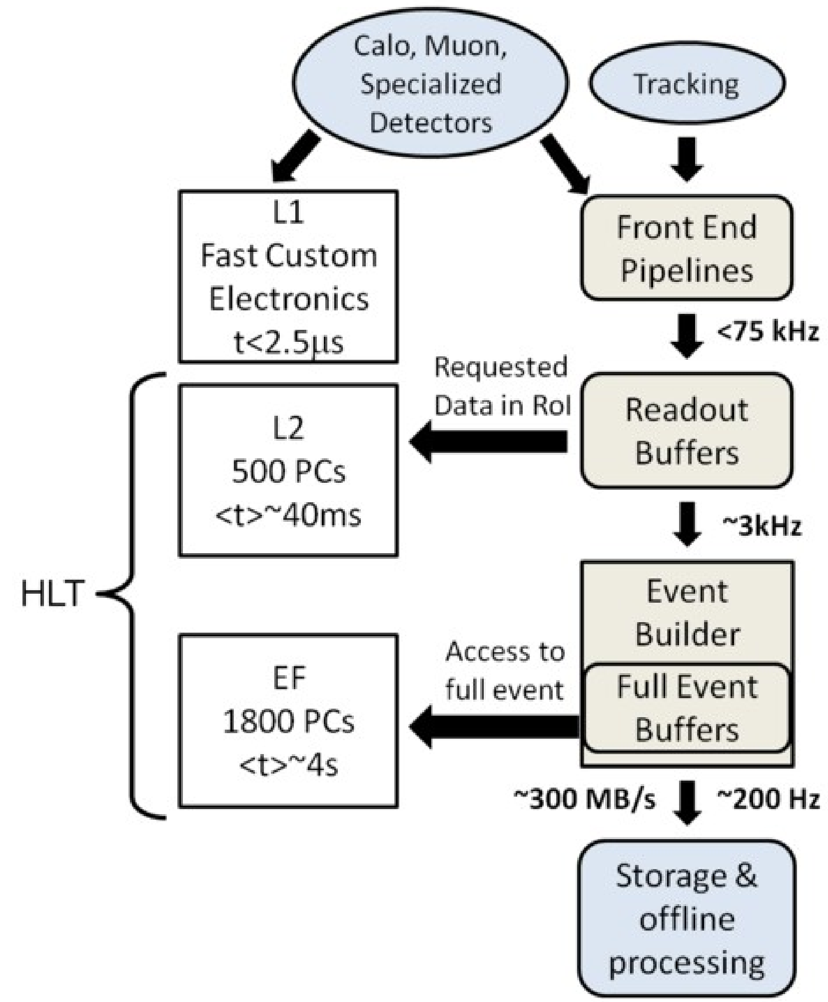
# Momentum resolution: results

- Good data/MC agreement after correction
- Mass resolution $\sigma(m)/m \sim 1/\sqrt{2}\ \sigma(p)/p$
- At the Z: $\sigma(m)/m$ ~1.5 to 2%.



# ATLAS Trigger

- In Run-2 ATLAS has a two-level trigger:
- Level 1, hardware,
  Input 40 MHz  =>  Output 100 kHz
  Time to take a decision (latency) < 2.6  μs
- High-Level Trigger (HLT) sofware on a computer farm
  Input 100 kHz => output ~1 kHz
- In Run-1 the HLT was further divided in level-2 (reading only partial data) and Event Filter (EF)



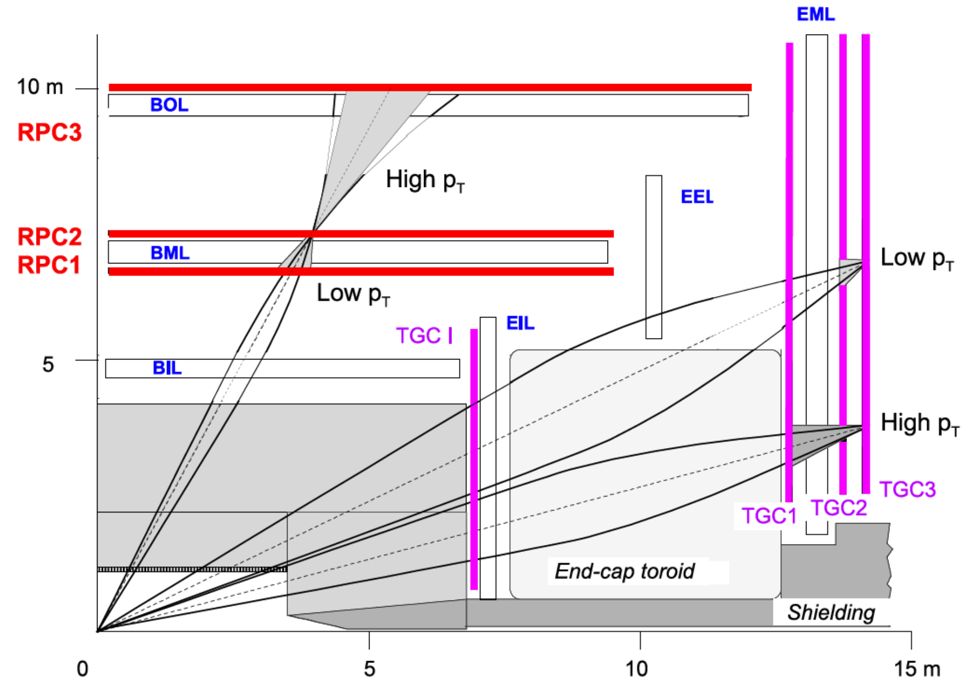
# L1 Muon Trigger

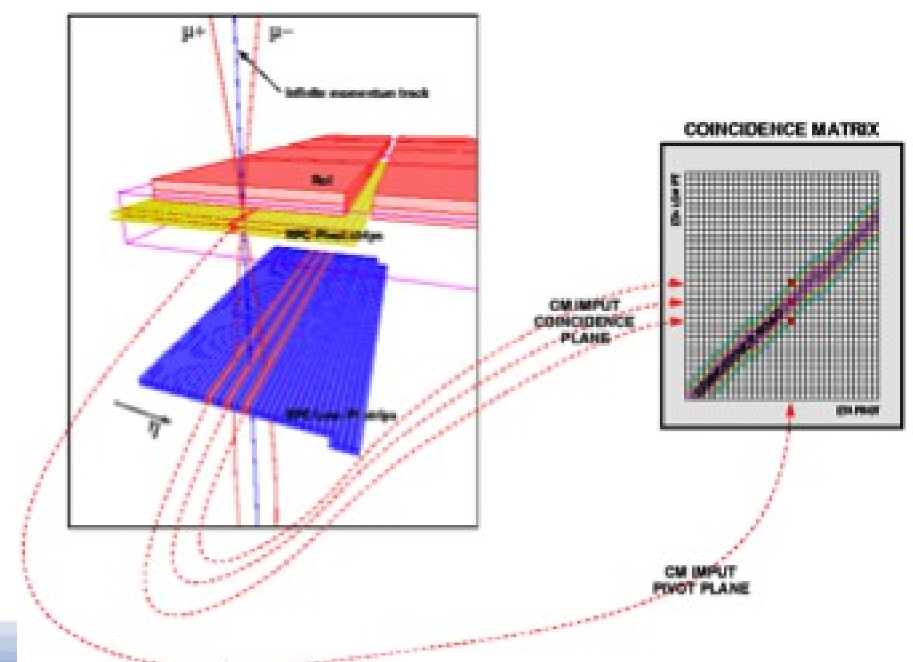
- Level-1 muon Trigger

- Barrel:
  low-pt: (4-10 GeV) two-stations
  High-pt: (11-20 GeV) three stations

- Endcap:
  - two-stations (4 GeV)
  - three stations (6-20 GeV)
  From Run-2 additional coincidence on TGC-Inner

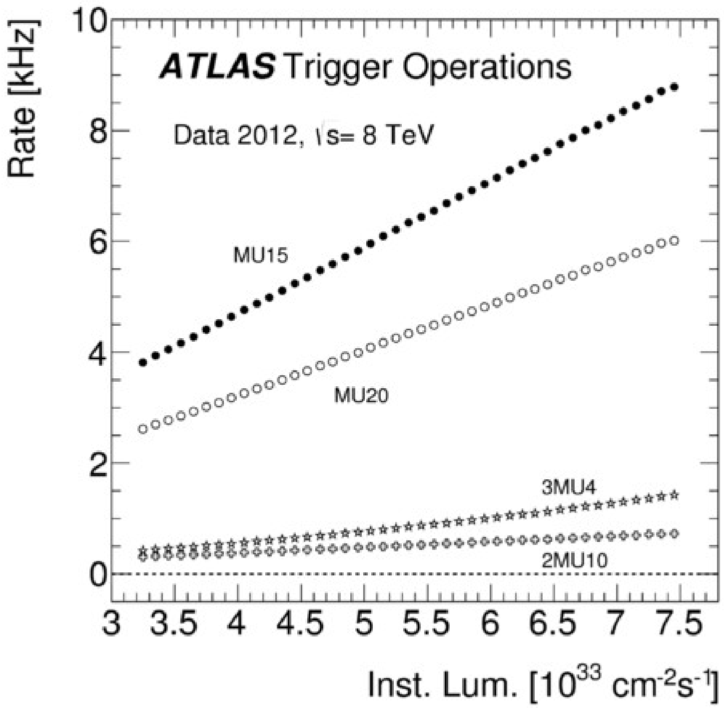
- Barrel (Roma-1):
  - “coincidence matrix” ASIC performs programamble space and time coincidence betwen pivot and confirm planes
  - $\eta$ and $\varphi$ matrices are combined by the “Pad” board

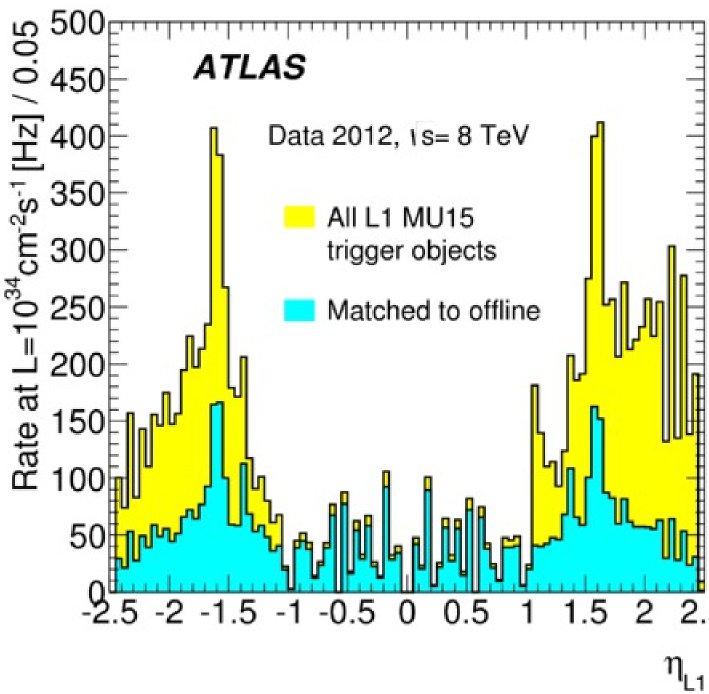


# Trigger: rates

- L1 rate allocated by ATLAS for single muons is ~20 kHz (out of 100 kHz total)
- In Run-1 pT>15 GeV threshold, well within 10kHz
- In Run-2 expected ~20 kHz with pT>20 GeV (factor ~2 for increased luminosity)
- Most rate from the Endcaps, mainly charged tracks (protons) from secondary interactions downstream of the IP
- Partly reduced in Run-2 with further coincidence with inner plane
- Dimuon triggers 2MU10 (plus 2MU4, 2MU6 for B physics)



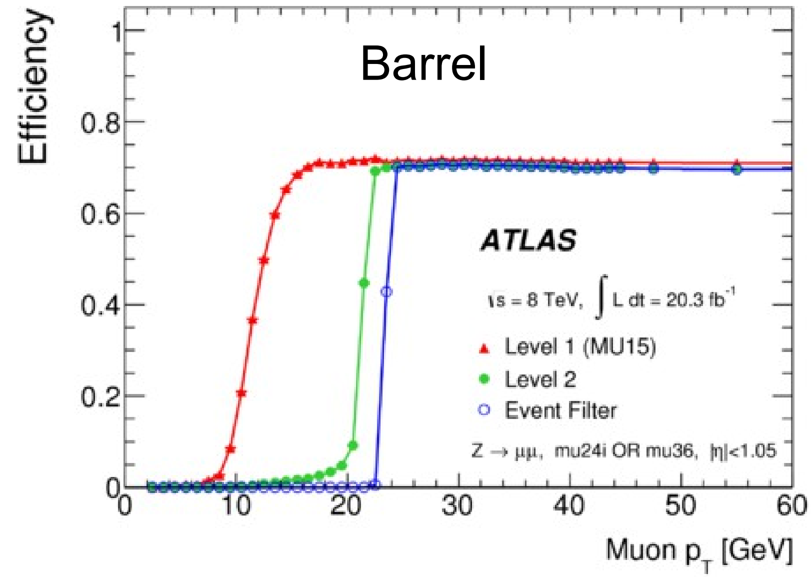



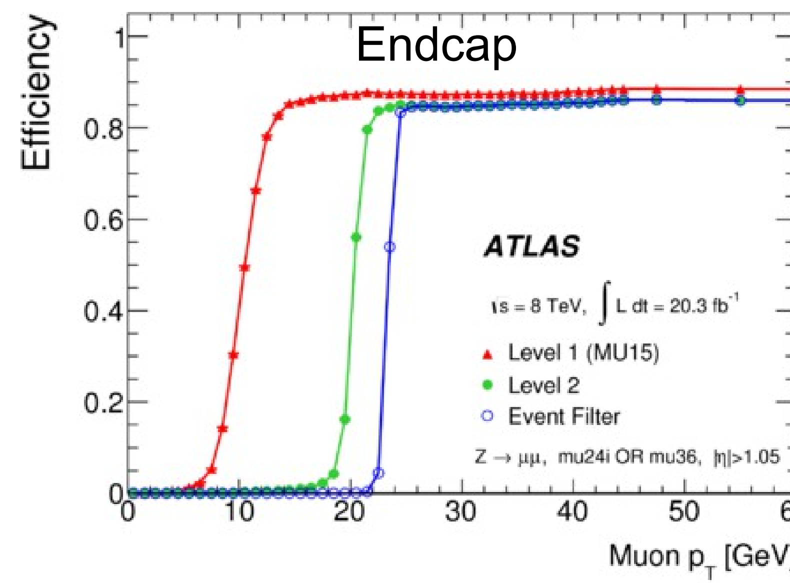
# Efficiency and thresholds

Efficiency “turn-on” curves for
- L1 : $p_T$>15 GeV
- HLT : $p_T$>24 GeV

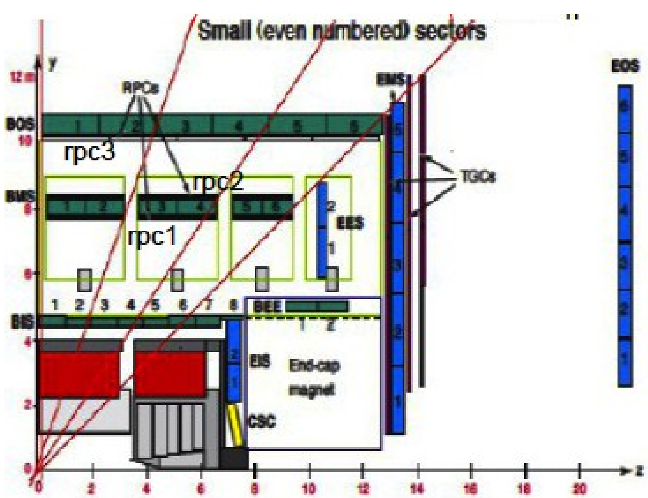
Barrel ~70%:
due to large acceptance holes for coils support and atlas structures and calorimeter services

Endcap: ~90%




Holes
In barrel
3-station
trigger

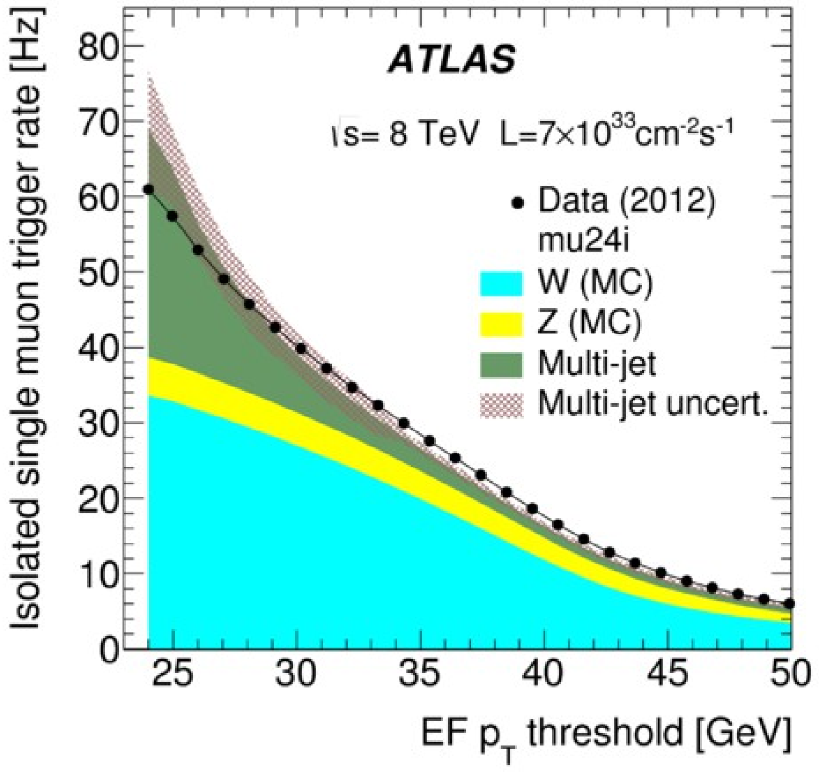


# Summary

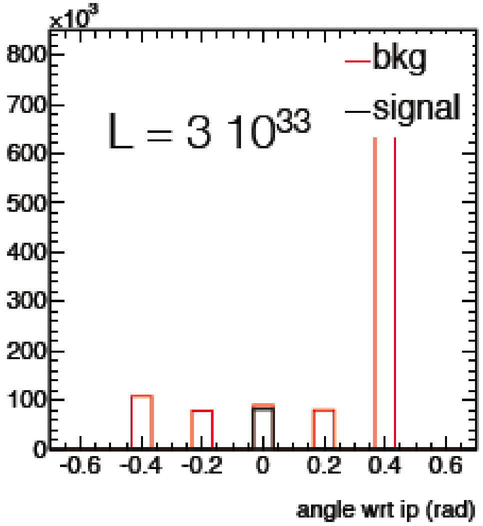
- Overall design
- Track reconstruction
- Performance measurements
- Trigger
- Outlook

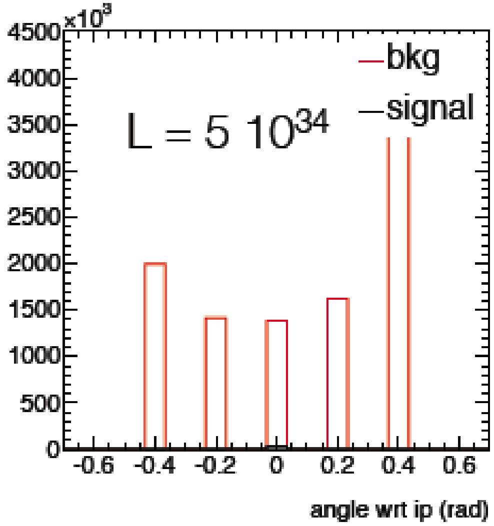
# Backup slides

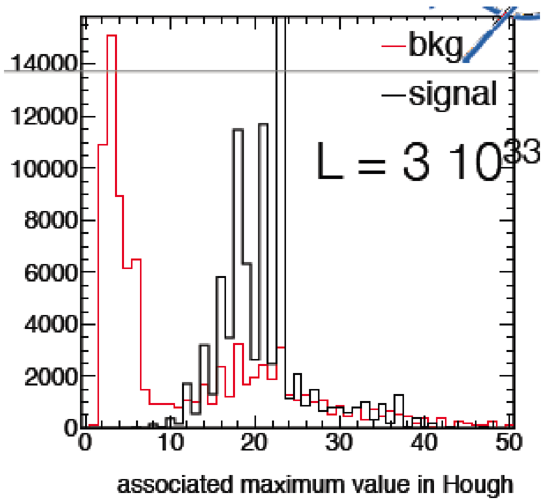
# HLT : rates

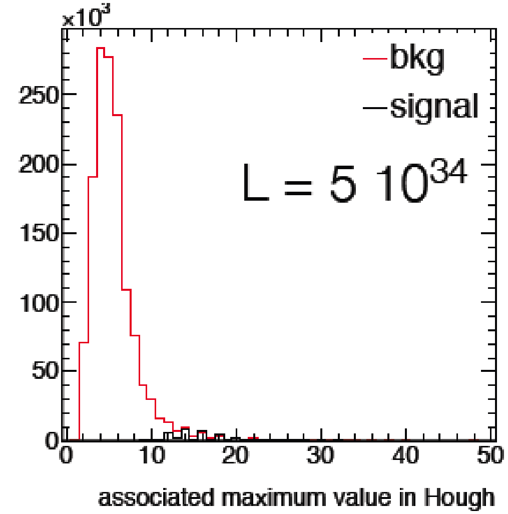

ATLAS
√s= 8 TeV L=7×10^33cm^-2s^-1
Data (2012)
mu24i
W (MC)
Z (MC)
Multi-jet
Multi-jet uncert.
Isolated single muon trigger rate [Hz]
EF p_T threshold [GeV]
0
10
20
30
40
50
60
70
80
25
30
35
40
45
50

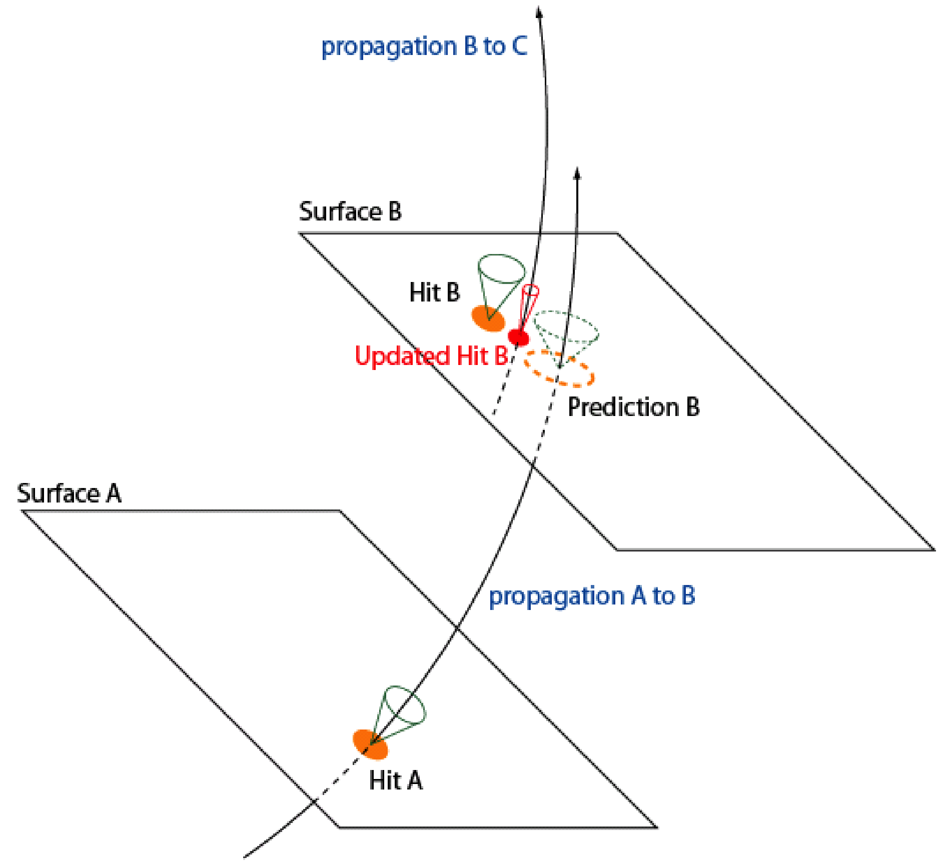
# Hough transform, selecting only pointing segments

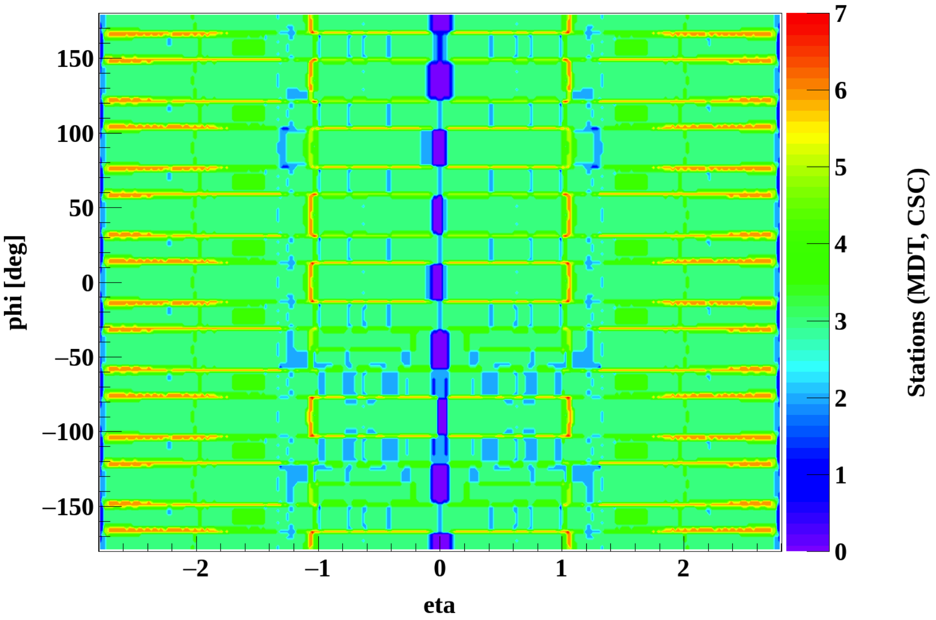


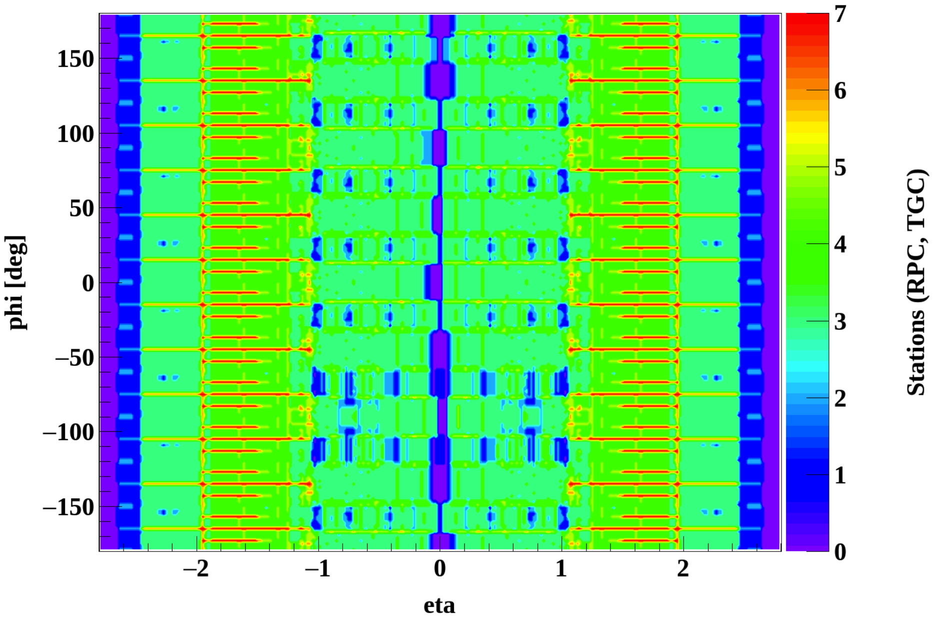




propagation B to C
Surface B
Hit B
Updated Hit B
Prediction B
Surface A
propagation A to B
Hit A



150
100
50
0
−50
−100
−150
phi [deg]
−2
−1
0
1
2
eta
7
6
5
4
3
2
1
0
Stations (MDT, CSC)

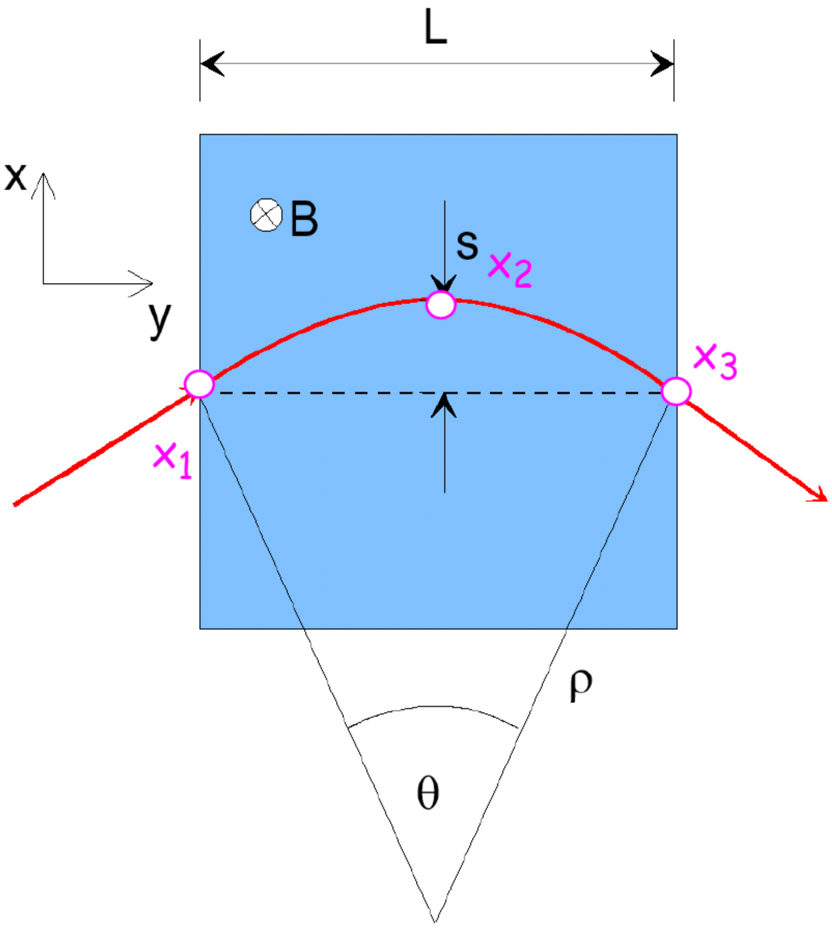



150
100
50
0
−50
−100
−150
phi [deg]
−2
−1
0
1
2
eta
7
6
5
4
3
2
1
0
Stations (RPC, TGC)

Momentum is determined by measurement of track curvature $\kappa = 1/\rho$ in B field:

Measure sagitta $s$ of the track. For the momentum component transverse to B field:

$$p_T = qB\rho$$




Units: $p_T[\mathrm{GeV}] = 0.3B[\mathrm{T}]\rho[\mathrm{m}]$

$$\frac{L/2}{\rho} = \sin\frac{\theta}{2} \approx \frac{\theta}{2} \text{ (for small } \theta) \Rightarrow \theta \approx \frac{L}{\rho} = \frac{0.3B \cdot L}{p_T}$$

$$s = \rho\left(1 - \cos\frac{\theta}{2}\right) \approx \rho\left(1 - \left(1 - \frac{1}{2}\frac{\theta^2}{4}\right)\right) = \rho\frac{\theta^2}{8} \approx \frac{0.3}{8}\frac{L^2 B}{p_T}$$

For the simple case of three measurements:

$s = x_2 - (x_1 + x_3)/2 \Rightarrow ds = dx_2 - dx_1/2 - dx_3/2$

with $\sigma_x \approx dx_i$ uncorrelated error of single measurement:

$$\sigma_s^2 = \sigma_x^2 + \frac{\sigma_x^2}{4} \cdot 2 = \frac{3}{2}\sigma_x^2$$