Experimental Elementary Particle Physics: problems with solutions

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1 Problems

The problems presented here are discussed during the lectures of the Experimental Elementary Particle Physics course of the last year of Laurea Magistrale in Physics at Sapienza. To solve these problems, access at the Particle Data Group web page (http://pdg.lbl.gov) is needed. Other relevant informations can be obtained by consulting the course slides (http://www.roma1.infn.it/people/bini/corsoFSPE1516.html).

1. A charged kaon (K^+) beam is produced with a rate of 1.2×10^2 Hz. Our detector takes data for Δt = 24 hours and aims to count the total number of decays $K^+ \to e^+\nu_e$. The efficiency of our detector for this final state is ϵ=63.2% with negligible uncertainty. Evaluate the minimum value of the rejection power needed for the $K^+ \to \mu^+\nu_\mu$ decay if we want to maintain the uncertainty on N($K^+ \to e^+\nu_e$) below 15% (neglect other possible backgrounds and the uncertainties on background).

2. In an e^+e^- experiment at a center of mass energy $\sqrt{s}$=1.5 GeV, we aim to count the number of $e^+e^- \to K^+K^-$ final states. At the end of the experiment, after the selection, we get N_{cand}=136. We estimate the background to be N_b=13.2±0.9. The selection efficiency is obtained by selecting 5922 events from a sample of 10^4 Montecarlo simulated $e^+e^- \to K^+K^-$ final states. Calculate N($e^+e^- \to K^+K^-$) with its uncertainty. What is the dominant contribution to the uncertainty ? How many st.dev. is the signal from 0 ?

3. We have designed an event selection chain based on the simulation in such a way that at the end of the selection 25% of the selected events are *signal* events and 75% are *background* events. How many total candidates do we need to collect in order to observe the signal with at least 5 st.dev. significance ?

4. The expected rate of neutrinos interacting in our detector is 0.23×10^{-2} evts/day, and the average efficiency for the detection of such interactions is 43.2%. Evaluate the probability to detect at least a neutrino in the first 24h, in the first year and in the first 10 years of operation.

5. The most updated values of the parameter $\mu = \sigma/\sigma_{SM}$ for the Higgs boson from ATLAS for the three main decay channels (in 2014) were:

$$\mu_{\gamma\gamma} = 1.55 \pm 0.30$$

$$\mu_{ZZ} = 1.43 \pm 0.37$$

$$\mu_{WW} = 0.99 \pm 0.29$$

   Evaluate the compatibility among the three independent ATLAS results and calculate the best overall estimate of μ from ATLAS. Then evaluate the compatibility with the SM expectation (μ=1).

6. In the 2011+2012 LHC dataset (corresponding to about 25 fb^{-1}), a sample of 2.24×10^5 $t\bar{t}$ events has been collected. We know that $\sigma(pp \to t\bar{t} + X)$ is 177 $\pm$ 5 pb. How large was the efficiency for $t\bar{t}$ events assuming no background ?

7. We perform a cross-section measurement and obtain the following values: N_{cand} =128, N_b = 14 $\pm$ 2, ϵ = 0.523 $\pm$ 0.002, L_{int} = 2.43 pb^{-1} $\pm$ 1.8 %: calculate the resulting cross-section with its uncertainty. In case this is a measurement of $e^+e^- \to \pi^+\pi^-$ at $\sqrt{s} = 1$ GeV, determine the value of the pion time-like form factor with its uncertainty. The formula relating the cross-section to the form factor $F_\pi(s)$ is the following:

$$\sigma(s) = \frac{\pi\alpha^2}{3s}\beta_\pi^3|F_\pi(s)|^2$$

8. The Higgs boson production at a linear collider happens mainly through the reaction $e^+e^- \to ZH$. If M_H = 125 GeV, and the cross-section $\sigma(e^+e^- \to ZH, \sqrt{s} =$ 300GeV)= 220 fb, which value of luminosity do we need to get $O(10^6)$ events in 1 year of data taking ? How many final states with two muons and two photons from the $Z \to \mu^+\mu^-$ and $H \to \gamma\gamma$ simultaneous decays do we get in the same period ? Evaluate the maximum and minimum photon energies from the Higgs.

9. Consider the reaction $e^+e^- \to K^+K^-$ at a Φ-factory. Which fraction of events have at least one kaon decaying within a sphere of R = 20 cm ? In which fraction of events both kaons decay within the same sphere ?

10. The SM expected semi-leptonic K_S charge asymmetry is 3×10^{-3}. At Dafne we expect to produce a sample of 1.2×10^9 tagged K_Ss. If the BR($K_S \to \pi e\nu$)=BR($K_S \to \pi^+e^-\overline{\nu}$)+BR($K_S \to \pi^-e^+\nu$)=$6.95\times10^{-4}$ which error can we reach on the asymmetry ?

11. Which average instantaneous luminosity is required to improve by a factor 3 such an uncertainty in one year of data taking (assuming a duty cicle of 50% and a tagging efficiency of 30%) ? [$\sigma(e^+e^- \to \phi)$= 5 μb at the ϕ peak].

12. We want to set-up a trigger to detect $Z \to \mu^+\mu^-$ decays in pp collisions at LHC. We have a low threshold (LT, p_T >4 GeV) and a high threshold (HT, $p_T > 20$ GeV) single muon triggers. The efficiencies of the two triggers for the muons coming from Z decays are ϵ(LT)=89.2%, ϵ(HT)=62.1%. Determine the efficiencies for triggering on Z decays in the two configurations: (1) LT1 AND LT2, (2) HT1 OR HT2 .

13. The fraction of K_L produced in e^+e^- collisions at the ϕ peak interacting in the KLOE calorimeter is approximately 5%. Determine the K_L-lead cross-section, using the following assumptions: The KLOE calorimeter is a single lead spherical layer 12 cm thick; the inner surface of the KLOE calorimeter is 2 m away from the e^+e^- interaction region.

14. Consider the decay $\phi \to \eta\gamma$ in the center of mass frame of the ϕ. Calculate the energy of the photon and the maximum and minimum energy of the photons in case the η decays in $\gamma\gamma$. We want to identify this decay looking at the inclusive radiative photon spectrum from a sample of 10^6 ϕ produced at rest. If we know that the combinatorial photon spectrum in the energy region between 300 and 400 MeV is almost flat with a number of events equal to 300 evts/MeV/$10^4\phi$, determine the energy resolution required to observe with enough significance the searched decay.

15. Consider the parameters of the three accelerators:

    - LHC: protons, R = 4.3 km, E_{max} = 7 TeV, T_{BC} = 25 ns;
    - LEP: electrons, R = 4.3 km, E_{max} = 100 GeV, T_{BC} = 22 μs;
    - DAFNE: electrons, R = 15 m, E_{max} = 500 MeV, T_{BC} = 2.7 ns;

    Evaluate for each accelerator the following quantities: the revolution frequency f; the number of bunches n_b; the minimum value of the magnetic field B_{min} required to hold the particles in orbit. From the luminosity and current profile plots shown as examples in the course slides, determine for DAFNE and LHC, the products $\sigma_x \times \sigma_y$

16. Design a pp machine at $\sqrt{s}$ = 40 TeV and $L = 10^{36}$ cm^{-2}s^{-1}. Which values of σ_x and σ_y are needed ? The following limits have to be respected:

    - B < 5T
    - N_1, $N_2 < 10^{11}$/bunch
    - $T_{BC} > 10$ ns

17. Evaluate the maximum $\sqrt{s_{NN}}$ that can be obtained at LHC for Cu-Cu and Pb-Pb collisions respectively.

18. Evaluate the value of $\sqrt{s_{NN}}$ for Au-Au collisions if the energy of the Au ions is 10.5 TeV. In case these collisions are done at RHIC for which value of the luminosity the pile-up becomes of order 1 ? (RHIC circumference = 3.834 km, n_b=111)

19. Consider the Higgs production (M_H =125 GeV) at a pp collider at $\sqrt{s}$ = 14 TeV. Evaluate the interval in rapidity y and the minimum value of x for direct Higgs production.

20. The VLHC program (Very Large Hadron Collider) proposes proton-proton collisions at a center of mass energy between 40 and 50 TeV and a luminosity larger than $10^{35}\text{cm}^{-2}\text{s}^{-1}$, in a ring with a radius of 17.5 km. The project requires a time between bunch crossings not smaller than 25 ns (as it is for LHC). How many bunches can be put ? If we know that the total proton-proton cross-section at this energy is about 100 mb, evaluate the average value of the pile-up. Finally evaluate the minimum value of x and the maximum value of y for the production of an Higgs boson (M_H=125 GeV) and of a second exotic Higgs boson having a mass of 5 TeV.

21. Estimate the space resolution needed to discriminate the charge of 1 TeV muons with 3 detector layers in a B=1 T magnetic field with an overall lever arm of 5 m.

22. Estimate the time resolution needed to discriminate between muons and electrons of the same momentum, 500 MeV/c with two detectors at a distance of 3 m.

23. Define the thickness (in cm) of a lead absorber for:

    - E=10 GeV photons
    - E=10 GeV muons
    - E=10 GeV protons

24. Estimate the mass resolution required to observe a signal of J/ψ production if the number of expected candidates is S=54 and the background per unit of mass is b = 13 MeV^{-1}.

25. A high intensity pulsed proton beam is directed onto a target. Downstream the target a magnet system sweeps away all the charged particles so that only neutral particles reach the experimental region, namely photons and neutrons in the kinetic energy range between 5 and 100 MeV. The detector is located 5 m from the target and measures the Time of Flight of photons and neutrons. Draw schematically the arrival time distribution of all the particles. If the repetition rate of the proton beam is 10 MHz, determine the kinetic energy of the neutrons that can be confused with the photons.

26. We study antiproton annihilations at rest in an hydrogen target and we want to discriminate the two processes $p\overline{p} \to \pi^+\pi^-$ and $p\overline{p} \to K^+K^-$. Calculate the momenta of the pions and of the kaons and estimate the ratio of the rates of the two processes (assuming only phase-space). Compare two possible systems to discriminate between the two final states: one based on 3 stations in a 0.3 T magnetic field and one based

on a time of flight counter. In both cases a radius of L = 2 m is available. Determine the space resolution in the first case and the time resolution in the second case needed to obtain an acceptable separation.

27. A detector aims to observe the $e^+e^- \to n\overline{n}$ process in e^+e^- collisions at $\sqrt{s} = 2$ GeV. The strategy consists in identifying $\overline{n}$ annihilations through the measurement of the velocity of the particle. The detector is an alluminium sphere of radius R=1 m and thickness 3 cm where the $\overline{n}$ annihilates, surrounded by tracking and ToF detectors that measure respectively the annihilation position and time and hence its velocity. We know that:

    - the $e^+e^- \to n\overline{n}$ cross-section at 2 GeV is equal to 1 nb;
    - the $\overline{n}$ -aluminium annihilation cross-section is = 720 mb;
    - we have collected an integrated luminosity of 0.54 pb^{-1};
    - a combinatorial background of 3800 evts per unit of β is expected in this data sample;
    - the annihilation point resolution is $\sigma(r)$=1.5 cm.

    Estimate the ToF resolution required to observe the $n\overline{n}$ signal.

28. We want to measure the ep elastic cross-section with high precision. For this purpose we design an *ep symmetric* collider with a luminosity of $10^{35}\text{cm}^{-2}\text{s}^{-1}$. The project of the electron ring should respect the following limits: B$\leq$ 0.2 T; beam currents I < 1 A. Moreover the detector requires $T_{BC} > 50$ ns.

    Calculate the proton and electron momenta for the $s = 25$ GeV^2 maximum center of mass energy configuration;

    define possible values of the ring radius R, of the number of electrons per bunch N_e and of the dissipated power, and, assuming $N_e = N_p$, evaluate the product $\sigma_x \times \sigma_y$;

    evaluate the rate of elastic collisions at $s = 25$ GeV^2 if $\sigma(ep \to ep) = 1.2\ \text{nb}/E_e(\text{GeV})$, where E_e is the energy of electrons in an equivalent fixed target experiment.

29. Consider the two decays $\phi \to \eta\gamma$ and $\phi \to \pi^0\gamma$ at a Φ-factory. To distinguish between the two decays we set a cut at a photon energy of 430 MeV. Using the energy resolution of the KLOE calorimeter determine which is the probability that a $\pi^0\gamma$ event is wrongly identified as $\eta\gamma$ event and viceversa. Which is the fraction of $\eta\gamma$ events in the selected sample of $\pi^0\gamma$?

30. We study the charge exchange reaction $\pi^-p \to \pi^0 n$ with a fixed target experiment. We use a negative pion beam of energies between 1 and 10 GeV and a liquid hydrogen target 5 m thick. The charge exchange cross-section in the energy range of the experiment is about 1 μb. Evaluate the negative pion rate needed to get a sample

of about 10^{10} events in 1 month of data taking; define a possible detector design to identify the $\pi^0 n$ final states and reject the large background with charged hadrons; define a method to discriminate between photons and neutrons.

31. We perform a measurement of the cross-section for $e^+e^- \to W^+W^-$ at $\sqrt{s}$=200 GeV at LEP. At the end of the selection we have the following: N_{cand} = 1590; N_b=640 (evaluated from side-bands); ϵ = 0.246 ± 0.015; L=253 pb^{-1} ±1% Calculate the value of the measured cross-section with its total uncertainty.

32. In the first period of LHC operation the average instantaneous luminosity was 10^{32} cm^{-2} s^{-1} and the time between the bunch crossing was T_{BC}=50 ns. Calculate in that regime the probability to have more than one interaction in a single bunch-crossing. If the gaussian sigma in the longitudinal direction of each bunch was σ_Z = 9 cm, evaluate the probability that in case of 2 interactions the distance between the two vertices was lower than 1 cm.

33. Consider the AMS detector: which space resolution is required on each tracker plane if we want discriminate electrons from positrons up to p = 200 GeV at the 3σ level ? (B=0.15 T)

34. A collider detector is limited in the polar angles 45< θ <135^o. Evaluate the acceptance for isotropic particles, and for particles produced according to the $sin^2\theta$ law.

35. A reactor emits neutrinos isotropically. The flux is $2\times10^{18}\nu$/s/sr. We install a neutrino detector based on water at a distance of 1.25 km from the reactor with the purpose to measure the flux from the reactor at this distance and see if it is significantly reduced with respect to the expectation. The neutrino interaction cross-section is 10^{-42} cm^2 and our detector efficiency is 50%. How much water shall we use to be sensitive to a flux decrease of at least 10% in one year of data taking ?

36. We have a monochromatic neutral kaon beam with p=3.5 GeV. At z=0 (corresponding to the position of the production target) the neutral kaon beam rate is r=10^{-2} Hz and the beam is composed by an equal quantity of K_S and K_L.

    - Evaluate the fraction f_S of K_S among all the kaon not yet decayed after a flight distance Δl_1=20cm.
    - Evaluate at which flight distance Δl_2 the K_S fraction is reduced to less than 10^{-6}.

    We build a detector to study the decays that take place after Δl_2 and we want to select the CP-violating decay $K_L \to \pi^+\pi^-$. We prepare two alternative selection procedures, named S1 and S2 characterized by the following performance: ϵ_1=0.71

R_1=300; ϵ_2=0.33, R_2=3000 where R_1 and R_2 are the rejection factors with respect to the main background, the decay $K_L \to \pi\mu\nu$.

- Evaluate the signal significativity and the signal purity that we obtain using for each of the two selection procedures on a data sample taken in 1 year of continuous data taking.

37. At HL-LHC ($\sqrt{s}$=14 TeV) we plan to constraint the proton pdfs using the Drell-Yan process $pp \to \mu^+\mu^- X$. Evaluate the minimum value of the Bjorken variable x and of the rapidity y that can be obtained if the trigger of the experiment is limited to dimuon invariant masses $M(\mu\mu)$ >6 GeV.

38. We plan to measure the $e^+e^- \to$ hadrons cross-section in the center of mass energy range between $\sqrt{s}$=1 and $\sqrt{s}$=3 GeV at steps of 50 MeV with a statistical uncertainty below 1% per point. Which instantaneus luminosity do we need if we want to complete this program in 1 month of data taking ?

2 Solutions

1. We have a number of signal events S given by $S = Rate(K^+) \times \Delta t \times BR(K^+ \rightarrow e^+\nu_e) \times \epsilon = 104$. The requirement on the statistical uncertainty to be below 15% means that:

$$\sigma(S) = \sigma(N - B) = \sqrt{N} = \sqrt{S+B} < 0.15 \times S$$

that translates in the following inequality for B:

$$B < 0.15^2 \times S^2 - S = 139$$

On the other hand the number of background events B is $B = Rate(K^+) \times \Delta t \times BR(K^+ \rightarrow \mu^+\nu_e)/R = 6.6 \times 10^6/R$ from which we translate the inequality on B to an inequality on R: $R > 4.7 \times 10^4$.

2. We have to combine all the informations according to the known formulas: $\epsilon = 0.592 \pm 0.005$, $\sigma(N)/N = 0.095$. So, we get: $N(e^+e^- \rightarrow K^+K^-) = 207 \pm 20$, where the dominant contribution is clearly the one from the candidate events statistics. The signal is anyhow more than 10 st.dev. away from 0, so it is very well defined.

3. The data of this problem allow to estimate the *purity* of the candidate sample, but not the signal *significativity*. Infact the purity scales linearly $S/(S+B)$=0.25 so that it is independent on the total number of events in the candidate sample, while the significativity scales as $S/\sqrt{S+B}$ so that it depends on the total number $N = S+B$ of events in the candidate sample. So we have to impose:

$$\frac{N \times 0.25}{\sqrt{N}} > 5$$

that finally gives $N > 400$ evts. If we select a sample of 400 evts and we evaluate that 300 of them are background (without any uncertainty on it apart from statistical fluctuations), we get 100 ± 20 that is a 5 st.dev. significant signal.

4. This is a typical Poisson problem. We have to evaluate the λ values for the three proposed periods of operation.

$$\begin{aligned}
\lambda_{1d} &= 0.23 \times 10^{-2} \times 0.432 = 0.00099 \\
\lambda_{1y} &= 0.00099 \times 365 = 0.363 \\
\lambda_{10ys} &= 0.363 \times 10 = 3.63
\end{aligned}$$

with these values of λ we can get the probabilities by simply evaluating the probability to have no events at all $P(0) = e^{-\lambda}$ and then doing:

$$P(>0) = 1 - P(0)$$

We get: 0.1% (1 day), 34% (1 year), 98% (10 years)

5. First of all we have to evaluate the weighted average among the three measurements with its uncertainty: $\overline{\mu} = 1.30 \pm 0.18$. This is the best estimate of the combined value of μ provided the two measurements are consistent.

   To check the consistency we can do a χ^2 test. We test the hypothesis that the three measurements come from the same true value. We have 2 degrees of freedom since we use the weighted average. We get: $\chi^2 = 1.94$. Clearly the test is passed, so that the best value of μ is the one reported above.

   To estimate the compatibility with $\mu = 1$ (the SM value) we can do a simple gaussian hypothesis test. In doing this we are assuming that the uncertainties are gaussian. This could be an approximate assumption, but it normally works. So we build as sample statistics the variable $Z = (\mu - 1)/\sigma(\mu)$=1.67. This value of Z corresponds to 1.67 st.dev. that is the probability to have an equal or larger deviation from 0 (in absolute value) is about 9%. We accept the hypothesis of compatibility with 1. Of course more data will be needed to see if this tiny discrepancy gets significant or is absorbed.

6. Here we have to invert the formula of the cross-section to get the efficiency:

$$\epsilon = \frac{2.24 \times 10^5}{177 \times 25 \times 10^3} = 5.1\%$$

7. First we evaluate the resulting cross-section with the usual formulas and we get (the uncertainty turns out to be dominated by the event counting statistics):

$$\sigma(e^+e^- \to \pi^+\pi^-) = 90 \pm 9 \,\mathrm{pb}$$

   To obtain the form factor value we have to invert the formula proposed there. We know π, s and α, we only need to evaluate β_π. The latter is determined by s and the pion mass given the two-body character of the final state. It is $\beta_\pi = 0.963$.

$$|F(s)| = \sqrt{\frac{3s\sigma(s)}{\pi\alpha^2\beta_\pi^3}}$$

   It is important to take care of the dimensions in this formula. Infact in the square root we have a dimensional quantity that will result in a number with units $\mathrm{GeV}^2\times$ pb. To transform it in an adimensional quantity we have to divide by the usual factor $(\hbar c)^2 = (197\mathrm{MeV} \times \mathrm{fm})^2$. For the uncertainty we have to apply the error propagation rule, and consider only the error coming from $\sigma(s)$.

$$|F(s)| = 0.0681 \pm 0.0034$$

8. Assuming $\epsilon = 1$ for this kind of events and assuming that we run for the whole year we need an average instantaneous luminosity of 1.4×10^{35} cm^{-2} s^{-1}. The branching ratio BR($Z \to \mu^+\mu^-$)=3.37% from PDG. The BR($H \to \gamma\gamma$) is not directly measured up to date, but is estimated to be 2×10^{-3} according to the SM. So we have only 67 evts with a muon pair and a gamma pair. The Higgs is produced monochromatic in this case with the same momentum of the Z: p = 104 GeV. The maximum and minimum energy of the photons are 147 and 27 GeV respectively.

9. We need to know the mean decay path of the kaons. This is simplified by the fact that the two kaons are monochromatic, $E(K^{\pm}) = M(\phi)/2$ =510 MeV, $p = \sqrt{E(K)^2 - M^2} = 128$ MeV $\beta\gamma = p/M = 0.259$. The decay length is: $l_K = \tau\beta\gamma c$=96 cm. The fraction of kaons decaying within a 20 cm sphere is:

$$f_{<20} = 1 - e^{-20/l_K} = 0.188$$

   Since each kaon decays independently from the other we can evaluate the OR and AND fractions by combining the probabilities:

$$\begin{aligned} f_{OR} &= 2f_{<20} - f_{<20}^2 = 34\% \\ f_{AND} &= f_{<20}^2 = 3.5\% \end{aligned}$$

10. The error on the asymmetry is $\sigma(\mathcal{A}) = 1/\sqrt{N} \times \sqrt{1 - \mathcal{A}^2}$, with N the total number of events and $\mathcal{A}$ the asymmetry. In this case the expected value of the asymmetry is very small so that it can be neglected in the error formula, and the statistical error on the asymmetry is simply $1/\sqrt{N}$. We evaluate N as the product of the number of tagged K_Ss times the semileptonic branching ratio, $N = 8.3 \times 10^5$ and hence the asymmetry can be obtained with an error of 0.11%, approximately 1/3 of the expected asymmetry.

11. To reduce the statistical uncertainty by a factor 3 we need to increase by a factor 9 the number of events. So we need:

$$\sigma(e^+e^- \to \phi)BR(\phi \to K_S K_L)BR(K_S \to \pi e\nu)\epsilon_{tag}\epsilon_{d.c.}\Delta t L > 9 \times 8.3 \times 10^5$$

   and we extract the luminosity $L > 1.4 \times 10^{33}$ cm^{-2} s^{-1}.

12. We assume non correlation so we apply the formulas for OR and AND. ϵ(LT1ANDLt2) = $\epsilon(LT)^2 = 79.6\%$. ϵ(HT1ORHT2) = $2\epsilon(HT) - \epsilon(HT)^2 = 85.6\%$.

13. This is essentially a fixed target experiment where the target is provided by the KLOE calorimeter and the projectiles are provided by the K_L coming from the ϕ not decayed before the calorimeter. We call ϕ_0 the flux of K_L from the Dafne interaction

region, $f_{int} = 5\%$ the fraction of K_L interacting, $\delta x = 12$ cm the target thickness and σ_{K_L-lead} the cross-section we have to determine. Moreover l_L is the K_L decay length $l_L = 3.44$ m. The master formula can be written as:

$$f_{int}\phi_0 = \sigma_{K_L-lead}\frac{\rho\delta x e^{-2/l_L}\phi_0}{Am_N}$$

where the flux ϕ_0 cancels so that it is not required for the cross-section estimate. We get $\sigma_{K_L-lead} = 227$ mb.

14. First from kinematics we get the following: $E(\gamma_{rad}) = 362.5$ MeV, $E(\gamma_{min}) = 273$ MeV and $E(\gamma_{max}) = 509$ MeV. The number of signal events in the sample of $N = 10^6$ ϕ decays is, assuming $\epsilon = 1$, $S = N \times BR(\phi \to \eta\gamma) = 1.31 \times 10^4$. On the other hand in the photon energy range 300÷400 MeV we have $b = 3 \times 10^4$ evts/MeV. We have to impose a significance larger than 5:

$$\frac{S}{\sqrt{S + 6\sigma b}} > 5$$

from which we get $\sigma < 38$ MeV. This corresponds to an energy resolution better than $6.3\%/\sqrt{E(GeV)}$.

15. Results:

- LHC: $f = 1.1 \times 10^4$ Hz; $n_b = 3600$; $B_{min} = 5.4$ T
- LEP: $f = 1.1 \times 10^4$ Hz; $n_b = 4$; $B_{min} = 0.08$ T
- DAFNE: $f = 3.2$ MHz; $n_b = 116$; $B_{min} = 0.11$ T

Slide 145 shows the profile of the LHC beam intensities together with the value of the luminosity for a typical 2012 run. From the plot we get: $N_1 = N_2 = 1.88 \times 10^{14}$ and $L = 39.9(\mu bs)^{-1}$. We can assume the number of bunches equal to the nominal value evaluated above ($n_b = 3600$). So we have:

$$\sigma_x \times \sigma_y = \frac{f n_b N_1 N_2}{4\pi L} = 2.18 \times 10^{-6}\text{cm}^2$$

For Dafne we have to rely on slide 144. In this case the currents are given directly in mA so that we have to divide them by the electron charge to get the number of particles per bunch and use the same formula given above. We get: $\sigma_x \times \sigma_y = 70 \times 10^{-6}$ cm^2.

In case of round beams we have transverse beam radius of 15 μm (LHC) and 84 μm (Dafne).

16. The requirement on the maximum magnetic field translates in a lower limit on the radius R=13.3 km. This determines the revolution frequency $f = 3.6$ kHz that, together with T_{BC} limits $n_b < 27800$. We invert the luminosity formula to get the product $\sigma_x \times \sigma_y$ that should be below 8×10^{-8}. Assuming a "round beam" it corresponds to a beam transverse radius of 3 μm.

17. In the LHC ring a particle of charge Z has a momentum $p = 0.3ZBR$ so that $p_{Cu} = 203$ TeV and $p_{Pb} = 574$ TeV. The nucleon-nucleon center of mass energy $\sqrt{s_{NN}} = 2E_N = 2p/A$ so that it is 6.38 TeV for Cu-Cu collisions and 5.54 TeV for Pb-Pb collisions.

18. The energy per nucleon is equal to 10.5/A = 53.2 GeV so that $\sqrt{s_{NN}} = 106.2$ GeV. Then we assume that the Au-Au total cross-section scales as $A^{2/3}$. The revolution frequency is $f = 12$ kHz so that to maintain the pile-up to order 1 we have the following constraint on the luminosity

$$L < \frac{f n_b}{\sigma(pp) A^{2/3}} = 4 \times 10^{29} \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

that for a heavy ion collider is quite large.

19. The product $x_1 x_2$ of the Bjorken variables for the two partons producing the Higgs should be equal to the ratio (m_H^2/s) so that, if one of the two partons has $x_1 = 1$ the other is limited to be $x_2 = (m_H^2/s) = 8 \times 10^{-5}$. The rapidity y depends on the unbalancing between the two partons the maximum will be $y = ln(x_1/x_2)/2 = 4.72$.

20. The maximum number of bunches is $n_b = 1/fT_{BC} = 14650$ and the corresponding pile-up is 275, very high. The minimum x and the maximum y are respectively: $x_{min} = 6.2 \times 10^{-6}$, $y_{max} = 6$ for the "standard" Higgs and $x_{min} = 10^{-2}$, $y_{max} = 2.3$ for a non-standard Higgs with a mass of 5 TeV.

21. To discriminate the charge of the muons at 5 st.dev. level, we need to achieve a relative momentum resolution better than 20%. We neglect the contribution of the multiple scattering and assume that the momentum of the particle is transverse to the magnetic field direction. We get $\sigma < 150\mu$m.

22. The time difference between 500 MeV electrons and muons in a beam line of 3 m is 210 ps (10.21 ns muons and 10 ns electrons). To discriminate at 5 st.dev. level we need a resolution of 210/5=42 ps, very challenging for any detector.

23. For a 10 GeV photon a reasonable containment is obtained with a number of radiation lengths of 20÷25, that is $\sim$ 14 cm of lead. For 10 GeV muons, extrapolating from the PDG plot we get R/M=70000 g/cm^2/GeV that corresponds to R=6.4 m. Finally for a 10 GeV proton, we have to consider two effects: the nuclear interactions and the

ionization losses. The first effect is accounted by the nuclear interaction length λ_I, $5\times\lambda_I$ being normally enough to contain an hadronic shower. For lead it corresponds to about 90 cm. The range of a 10 GeV proton in lead is similar to the one of the muon, around 6 m, so that the first effect dominates.

24. We use the approximate estimate $S/\sqrt{S + 6\sigma b} > 5$. From this requirement we get $\sigma < 0.8$ MeV that is a really stringent requirement much better than a permit mass resolution.

25. All the photons reach the detector at the same time giving rise to a narrow pulse every 100 ns. The delay of the neutrons with respect to the photons depends on the neutron energy and is in the range between 22 ns (for100 MeV neutrons) and 144 ns (for 5 MeV neutrons). In order to find the kinetic energy of the neutrons that have a delay of 100 ns (so that they arrive together with the photons of the subsequent bunch), we have to impose:
$$\frac{l}{\beta c} - \frac{l}{c} = 100 \text{ ns}$$
from which we get $\beta = 0.143$. This corresponds to a kinetic energy $E_K = m_n/\sqrt{1-\beta^2} - m_n = 13.4$ MeV.

26. Since the annihilation is at rest, the center of mass energy of the reaction is $\sqrt{s} = 2m_p$ neglecting the motion of the interacting particles. Pions and kaons emerge with moment $p_\pi = 928$ MeV and $p_K = 797$ MeV respectively. The available phase space volume is proportional to the momentum of the emerging particles, so that the expected ratio of pion to kaon rate is $R = 1.16$, that is pion pairs are 16% more frequent than kaon pair. Any discrepancy from this ratio would indicate some effect in the matrix element.

    Using the magnetic field we need to discriminate between 797 and 928 MeV so that we need at least a resolution of $\sigma(p)/p < (928-797)/797/5 = 3.2\%$. This corresponds to $\sigma_x < 1.5$ mm very easy to reach. Using the time of flight method we have to evaluate the time of flight difference between the two particles: $t_\pi = 6.74$ ns and $t_K = 7.85$ ns, so we need to discriminate a time difference of at least 1.11/5=0.222 ns. Even this resolution is achievable.

27. In the data sample we have $L\sigma(e^+e^- \to n\overline{n})$=540 $n\overline{n}$ pairs and a number of signal events S given by:
$$S = L\sigma(e^+e^- \to n\overline{n})\frac{\rho_{Al}\delta x}{A_{Al}m_N}\sigma(\overline{n} - Al) = 70$$
Each $\overline{n}$ has a β of 0.343 and takes a time of $T_{\overline{n}} = R/\beta c$=9.7 ns to reach the aluminum sphere. In order to observe the $n\overline{n}$ signal at more than 5 st.dev. level we need a $\sigma(\beta)$

less than 0.0055, that is less than 1.6% relative uncertainty. The β resolution is related to the ToF resolution through the:

$$(\frac{\sigma(\beta)}{\beta})^2 = (\frac{\sigma(r)}{r})^2 + (\frac{\sigma(T)}{T})^2$$

but since $\sigma(r)/r = 1.5\%$ in order to match the requirement on β we need a somehow negligible resolution on T, $\sigma(T) < 0.6\% \times T = 57$ ps. Again this is a challenging requirement. Either increasing R or improving the space resolution would make the requirement more realistic.

28. In a *symmetric* collider the laboratory reference frame coincides with the center of mass reference frame. This implies that the two particles have the same momentum p with opposite direction. If we want that $s = 25$ GeV^2, and considering the proton mass, we get $p = 2.41$ GeV. Given the limits posed in the design, we get that the radius of the ring has to be larger than 40 m, the number of bunches should be lower than 16.7 and the number of electrons per bunch should be less than 3×10^{11}. The product $\sigma_x \times \sigma_y$ should be less than 1.4×10^{-6} cm^2 that, for a round beam means about 12 μm radius.

    In a fixed target experiment we need electrons of 12.8 GeV to get the same value of s. The rate is given by $L\sigma(ep \to ep) = 9.3$ Hz.

29. The two reactions are characterized by monochromatic photons of energies $E_\gamma = 501$ MeV (for $\pi^0\gamma$) and $E_\gamma = 363$ MeV (for $\eta\gamma$). We assume gaussian distributions for the two signals with widths obtained by the KLOE calorimeter resolution: $\sigma(E) = 38$ MeV at 501 MeV and $\sigma(E)$=32 MeV at 363 MeV. The misidentification probabilities are, respectively 1.83% (η considered a π^0) and 3.07% (π^0 considered a η). Applying the Bayes theorem with prior given by the respective branching ratios we get $P(\eta/E_\gamma > 430$ MeV)=15.9%.

30. The number of charge exchange events obtained in 1 month of data taking is given by:

$$N(\Delta t) = \frac{R(\pi)\rho\delta x\sigma\Delta t}{m_p}$$

    so that

$$R(\pi) = \frac{N(\Delta t)m_p}{\rho\delta x\sigma\Delta t} = 180\,\text{MHz}$$

    The final state is completely neutral. In this case a magnetic field can be unuseful, but on the other hand a tracking system can be used to veto all events with at least one charged particle. After the tracking system a calorimeter with good timing properties can be used to identify photons and neutrons and separate them through time of flight.

31. Putting together all the numbers we get: $\sigma = (15.3 \pm 1.1)$ pb.

32. The amount of pile-up can be evaluated by the ratio of the interaction rate and the collision rates. Assuming a cross-section of 100 mb we get an average pile-up of 0.5. This has the meaning of a poissonian λ so that, to get the probability of more than one interaction we have:

$$P(>1) = 1 - P(0) - P(1) = 1 - e^{-0.5} - 0.5e^{-0.5} = 0.09$$

In case of two interactions, assuming that the positions along Z of the two interactions are uncorrelated, the distribution of the difference between the two is a gaussian with a width $\sigma(Z_1 - Z_2) = \sqrt{2}\sigma_Z$ =12.7 cm. For such a guassian $P(|Z_1 - Z_2| < 1$ cm)=6.4%.

33. The AMS tracker has 4 planes in 2 m. By applying the formula for the momentum resolution and extracting the value of σ_x we get: $\sigma_x <$32 μm.

34. To get the acceptance A we have just to do a couple of integrals. The only point is to consider the solid angle here:

$$\begin{aligned} A &= \frac{\int_{45^o}^{135^o} d\phi dcos\theta}{\int_0^{180} d\phi dcos\theta} = 70.7\% \\ A &= \frac{\int_{45^o}^{135^o} d\phi(1 - cos^2\theta) dcos\theta}{\int_0^{180} d\phi(1 - cos^2\theta) dcos\theta} = 88.6\% \end{aligned}$$

In the second case the acceptance is larger and this is not surprising, since the $sin^2\theta$ distributions gives rise to more events in the transverse direction.

35. If the detector has a thickness δx and a transverse surface $S = \delta\omega R^2$ (with $\delta\omega$ the solid angle fraction seen by the reactor center), the water mass is $M = \rho\delta x\delta\omega R^2$. The number of events collected in 1 year can be written as:

$$N = \frac{\phi\Delta t\delta\omega\rho\delta x}{Am_N}\epsilon\sigma = \frac{\phi\Delta t M}{R^2 Am_N}\epsilon\sigma$$

independent on the solid angle fraction and on the shape of the detector. Actually it depends only on the water mass and on the distance of the detector from the reactor. Since we want to be sensitive to a 10% variation, we need an uncertainty on the number of events at least below 2% (5 st.dev.) so we need at least 2500 events. For the mass we get: $M > 37 \times 10^6$ g that is 37 tons.

36. The decay lengths are respectively λ_S = 18.8 cm and λ_L =108 cm, so that after Δl_1 = 20 cm we have f_S = 25.7%, and we need at least Δl_2 = 260 cm to reduce it below 10^{-6}.

The selection S1 gives a purity of 60.8% and a significativity of 11; the selection S2 gives a purity of 87.8% and a significativity of 9.3.

37. The minimum value of x is 1.8×10^{-7} and the range in y is between -7.75 and +7.75.

38. The $e^+e^- \to$ hadrons cross-section in the energy range between 1 and 3 GeV can be estimated as the product of R and the $e^+e^- \to \mu^+\mu^-$ cross-sections. It results to be between 220 and 24 nb, the latter being the value at 3 GeV. We need at least 10^4 events per point, so we concentrate to the point with lower cross-section assuming for simplicity to use the same amount of time for all the points. We have to do 40 points in one month of data taking, so we can use 6.5×10^4 s per point. The required luminosity should be $L > 6.4 \times 10^{30}$ cm^{-2} s^{-1}.