η-η' mixing at LNF: status and prospects

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- *Introduction*

  ⋆ mixing of mass eigenstates

octet-singlet basis

quark-flavour basis

1 mixing angle

$$|\eta\rangle = \cos\theta_P|\eta_8\rangle - \sin\theta_P|\eta_0\rangle$$
$$|\eta'\rangle = \sin\theta_P|\eta_8\rangle + \cos\theta_P|\eta_0\rangle$$

with
$$|\eta_8\rangle = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$
$$|\eta_0\rangle = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$$|\eta\rangle = \cos\phi_P|\eta_q\rangle - \sin\phi_P|\eta_s\rangle$$
$$|\eta'\rangle = \sin\phi_P|\eta_q\rangle + \cos\phi_P|\eta_s\rangle$$

and
$$|\eta_q\rangle = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$
$$|\eta_s\rangle = s\bar{s}$$

$$\theta_P = \phi_P - \arctan\sqrt{2} \simeq \phi_P - 54.7^\circ$$

Assumptions:

- no energy dependence
- $\Gamma_{\eta,\eta'} \ll m_{\eta,\eta'}$
- no mixing with other pseudoscalars (π^0, η_c, glueballs)

- *Introduction*
  - ⋆ mixing of decay constants

octet-singlet basis

$$\langle 0|A^a_\mu|P(p)\rangle = i f^a_P p_\mu$$

with $A^a_\mu = \bar{q}\gamma_\mu\gamma_5\frac{\lambda^a}{\sqrt{2}}q$

f^a_P $(a = 8, 0; P = \eta, \eta')$

$$\begin{pmatrix} f^8_\eta & f^0_\eta \\ f^8_{\eta'} & f^0_{\eta'} \end{pmatrix} = \begin{pmatrix} f_8 \cos\theta_8 & -f_0 \sin\theta_0 \\ f_8 \sin\theta_8 & f_0 \cos\theta_0 \end{pmatrix}$$

2 mixing angles

2 decay constants

quark-flavour basis

$$\langle 0|A^i_\mu|P(p)\rangle = i f^i_P p_\mu$$

with

$$A^q_\mu = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)$$

and $A^s_\mu = \bar{s}\gamma_\mu\gamma_5 s$

f^i_P $(i = q, s; P = \eta, \eta')$

$$\begin{pmatrix} f^q_\eta & f^s_\eta \\ f^q_{\eta'} & f^s_{\eta'} \end{pmatrix} = \begin{pmatrix} f_q \cos\phi_q & -f_s \sin\phi_s \\ f_q \sin\phi_q & f_s \cos\phi_s \end{pmatrix}$$

- *Introduction*

  ⋆ mixing of decay constants

Large Nc ChPT = ChPT + η_0 in a combined perturbative expansion

octet-singlet basis:

$$f_8^2 = \frac{4f_K^2 - f_\pi^2}{3}\,, \qquad f_0^2 = \frac{2f_K^2 + f_\pi^2}{3} + f_\pi^2 \Lambda_1\,,$$

$$f_8 f_0 \sin(\theta_8 - \theta_0) = -\frac{2\sqrt{2}}{3}(f_K^2 - f_\pi^2)\,,$$

OZI rule violating parameter

quark-flavour basis:

$$f_q^2 = f_\pi^2 + \frac{2}{3} f_\pi^2 \Lambda_1\,, \qquad f_s^2 = 2f_K^2 - f_\pi^2 + \frac{1}{3} f_\pi^2 \Lambda_1\,,$$

$$f_q f_s \sin(\phi_q - \phi_s) = \frac{\sqrt{2}}{3} f_\pi^2 \Lambda_1\,,$$

Approximate relations valid for $\phi_q = \phi_s \equiv \phi$

$$f_8 = \sqrt{\frac{1}{3} f_q^2 + \frac{2}{3} f_s^2}\,, \qquad \theta_8 = \phi - \arctan\left(\frac{\sqrt{2} f_s}{f_q}\right)$$

$$f_0 = \sqrt{\frac{2}{3} f_q^2 + \frac{1}{3} f_s^2}\,, \qquad \theta_0 = \phi - \arctan\left(\frac{\sqrt{2} f_q}{f_s}\right)$$

SU(3) breaking effect

- *Present status*

phenomenology EPJC 7, 271 (1999)

SU(3) breaking through $m_s/\bar{m}$

$$\mathcal{L}_{VP\gamma} = g\,\epsilon_{\mu\nu\alpha\beta}\,\partial^\mu A^\nu\, tr(Q(\partial^\alpha V^\beta P + P\partial^\alpha V^\beta))$$

(3.4±0.2)°

$$R_\phi \equiv \frac{\Gamma(\phi\to\eta'\gamma)}{\Gamma(\phi\to\eta\gamma)} = \cot\varphi_P^2\left(1-\frac{m_s}{\bar{m}}\frac{\tan\varphi_V}{\sin 2\varphi_P}\right)^2\left(\frac{p_{\eta'}}{p_\eta}\right)^3$$

1.24±0.07

experiment PLB 541, 45 (2002)

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Physics Letters B 541 (2002) 45–51

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Measurement of $\Gamma(\phi\to\eta'\gamma)/\Gamma(\phi\to\eta\gamma)$ and the pseudoscalar mixing angle

KLOE Collaboration

$$\varphi_P = (41.8^{+1.9}_{-1.6})^\circ$$

most precise determination from a single experiment !

- *Present status*

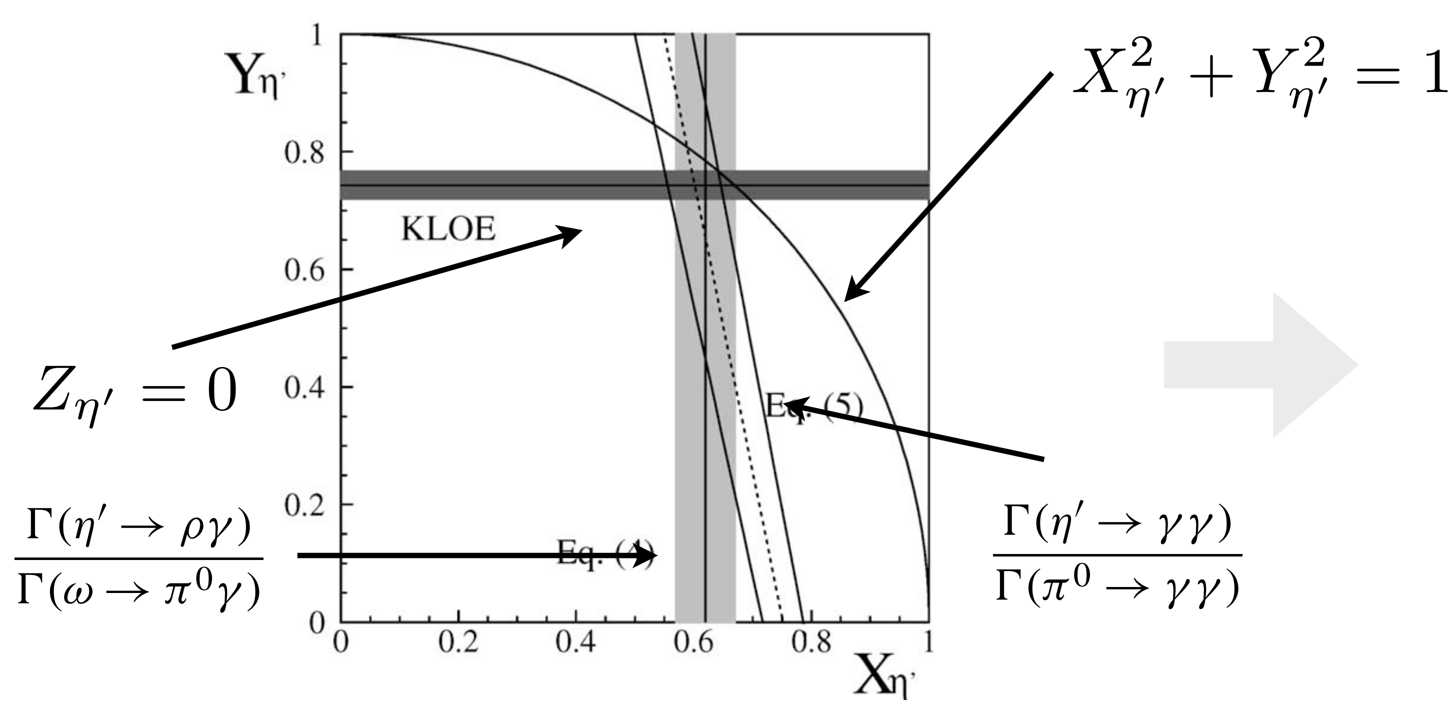
  ★ mixing with gluonium

theory PRD 27, 1101 (1983)

$$|\eta\rangle = X_\eta |u\bar{u} + d\bar{d}\rangle/\sqrt{2} + Y_\eta |s\bar{s}\rangle + Z_\eta |\mathrm{glue}\rangle,$$
$$|\eta'\rangle = X_{\eta'} |u\bar{u} + d\bar{d}\rangle/\sqrt{2} + Y_{\eta'} |s\bar{s}\rangle + Z_{\eta'} |\mathrm{glue}\rangle$$

A gluonium component of η' → $Z^2_{\eta'} > 0$ or $X^2_{\eta'} + Y^2_{\eta'} < 1$

experiment PLB 541, 45 (2002)



KLOE results:

$$Z^2_{\eta'} = 0.06^{+0.09}_{-0.06}$$

Gluonium fraction below 15% !

- *Future prospects*

  ⋆ to measure as precise as possible the η-η' mixing angle

  better theoretical determinations of:

$$\phi \longrightarrow \pi^0 \eta \gamma$$

$$\eta \longrightarrow \pi^+ \pi^- \pi^0 / \pi^0 \pi^0 \pi^0$$

$$\eta \longrightarrow \pi^0 \gamma \gamma$$

  to be compared with future KLOE measurements

  ⋆ to establish the gluon admixture of the η and η'

  ⋆ to help measuring the mixing parameters in the two mixing angle scenario

- *Study of the η-η' system in the two mixing angle scheme*

R. E., J.-M. Frère, JHEP 06, 029 (2005)

Motivation:

- To perform an updated phenomenological analysis of various decay processes using the two mixing angle description of the η-η' system
- The analysis will serve us to check the validity of the two mixing angle scheme and its improvement over the standard one angle picture
- The analysis also tests the sensitivity to the mixing angle schemes: octet-singlet basis vs. quark-flavour basis

⋆ Notation

The decay constants of the η-η' system in the octet-singlet basis f_P^a $(a = 8, 0; P = \eta, \eta')$ are defined as

$$\langle 0|A_\mu^a|P(p)\rangle = i f_P^a p_\mu \ ,$$

The decay constants are parameterized in terms of f_8, f_0 and θ_8, θ_0 as

$$\begin{pmatrix} f_\eta^8 & f_\eta^0 \\ f_{\eta'}^8 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8 \cos\theta_8 & -f_0 \sin\theta_0 \\ f_8 \sin\theta_8 & f_0 \cos\theta_0 \end{pmatrix} .$$

- *Study of the η-η' system in the two mixing angle scheme*

η,η'→γγ decays

The interpolating fields η and η' are related with the axial-vector currents

$$\eta(x)=\frac{1}{m_\eta^2}\frac{f_{\eta'}^0\partial^\mu A_\mu^8(x)-f_{\eta'}^8\partial^\mu A_\mu^0(x)}{f_{\eta'}^0 f_\eta^8-f_{\eta'}^8 f_\eta^0}\ ,\qquad \eta'(x)=\frac{1}{m_{\eta'}^2}\frac{f_\eta^0\partial^\mu A_\mu^8(x)-f_\eta^8\partial^\mu A_\mu^0(x)}{f_\eta^0 f_{\eta'}^8-f_\eta^8 f_{\eta'}^0}\ .$$

This leads to

$$\Gamma(\eta\to\gamma\gamma)=\frac{\alpha^2 m_\eta^3}{96\pi^3}\left(\frac{f_{\eta'}^0-2\sqrt{2}f_{\eta'}^8}{f_{\eta'}^0 f_\eta^8-f_{\eta'}^8 f_\eta^0}\right)^2=\frac{\alpha^2 m_\eta^3}{96\pi^3}\left(\frac{c\theta_0/f_8-2\sqrt{2}s\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}\right)^2\ ,$$

$$\Gamma(\eta'\to\gamma\gamma)=\frac{\alpha^2 m_{\eta'}^3}{96\pi^3}\left(\frac{f_\eta^0-2\sqrt{2}f_\eta^8}{f_\eta^0 f_{\eta'}^8-f_\eta^8 f_{\eta'}^0}\right)^2=\frac{\alpha^2 m_{\eta'}^3}{96\pi^3}\left(\frac{s\theta_0/f_8+2\sqrt{2}c\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}\right)^2\ .$$

VPγ decays

We extend our analysis to the couplings of the radiative decays $V\to(\eta,\eta')\gamma$ and $\eta'\to V\gamma$ with $V=\rho,\omega,\phi$.
The form factors $F_{VP\gamma}(0,0)$ are fixed by the AVV triangle anomaly

Using their analytic properties

$$F_{VP\gamma}(0,0)=\frac{f_V}{m_V}g_{VP\gamma}+\cdots\qquad \text{(Vector Meson Dominance)}$$

where the vertex couplings $g_{VP\gamma}$ are the on-shell V-P electromagnetic form factors

$$\langle P(p_P)|J_\mu^{\rm EM}|V(p_V,\lambda)\rangle|_{(p_V-p_P)^2=0}=-g_{VP\gamma}\epsilon_{\mu\nu\alpha\beta}p_P^\nu p_V^\alpha\varepsilon_V^\beta(\lambda)\ .$$

• *Study of the η-η' system in the two mixing angle scheme*

V	P	$g_{VP\gamma}$ (th.)		$g_{VP\gamma}$ (exp.)
ρ	η	$\frac{\sqrt{3}m_\rho}{4\pi^2 f_\rho}\frac{c\theta_0/f_8-\sqrt{2}s\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(1.48\pm 0.08)$ GeV^{-1}	(1.59 ± 0.11) GeV^{-1}
ρ	η'	$\frac{\sqrt{3}m_\rho}{4\pi^2 f_\rho}\frac{s\theta_0/f_8+\sqrt{2}c\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(1.23\pm 0.08)$ GeV^{-1}	(1.35 ± 0.06) GeV^{-1}
ω	η	$\frac{m_\omega}{2\sqrt{2}\pi^2 f_\omega}\frac{(c\theta_V-s\theta_V/\sqrt{2})c\theta_0/f_8-s\theta_V s\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(0.57\pm 0.04)$ GeV^{-1}	(0.46 ± 0.02) GeV^{-1}
ω	η'	$\frac{m_\omega}{2\sqrt{2}\pi^2 f_\omega}\frac{(c\theta_V-s\theta_V/\sqrt{2})s\theta_0/f_8+s\theta_V c\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(0.56\pm 0.04)$ GeV^{-1}	(0.46 ± 0.03) GeV^{-1}
ϕ	η	$-\frac{m_\phi}{2\sqrt{2}\pi^2 f_\phi}\frac{(s\theta_V+c\theta_V/\sqrt{2})c\theta_0/f_8+c\theta_V s\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(-0.76\pm 0.04)$ GeV^{-1}	(-0.690 ± 0.008) GeV^{-1}
ϕ	η'	$-\frac{m_\phi}{2\sqrt{2}\pi^2 f_\phi}\frac{(s\theta_V+c\theta_V/\sqrt{2})s\theta_0/f_8-c\theta_V c\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(0.86\pm 0.05)$ GeV^{-1}	(0.71 ± 0.04) GeV^{-1}

Table 1: Theoretical and experimental values of the on-shell V-(η,η') electromagnetic vertex couplings in the octet-singlet η-η' mixing angle scheme. For $g_{VP\gamma}$ (th.) we give the experimental errors coming from the decay constants $f_{P,V}$ and the mixing angle values θ_8 and θ_0. We use $\theta_V=(38.7\pm 0.2)^\circ$ for the ϕ-ω mixing angle. Experimental values are taken from the Particle Data Group Coll., Phys. Lett. B **592** (2004) 1.

- quite remarkable agreement except for the $\omega\eta\gamma$ and $\omega\eta'\gamma$ couplings
  $\omega\eta\gamma$: (0.53 ± 0.05) GeV^{-1} (PDG'02) $\to$ (0.46 ± 0.02) GeV^{-1} (PDG'04) because of $e^+e^-\to\eta\gamma$ exclusion
  $\omega\eta'\gamma$: rather sensitive to θ_V, for instance $\theta_V=35.3^\circ \Longrightarrow$ 10% reduction
- for $f_8=1.28 f_\pi$: $g_{\phi\eta\gamma}=(-0.80\pm 0.04)$ GeV^{-1} and $g_{\phi\eta'\gamma}=(0.91\pm 0.06)$ GeV^{-1}
- fixing $\theta_8=\theta_0\equiv\theta$ implies an increase of $\chi^2/\mathrm{d.o.f.}$ by a factor of 3, and $g_{\phi\eta'\gamma}=(1.20\pm 0.06)$ GeV^{-1}, in clear contradiction with data.

• *Study of the η-η' system in the two mixing angle scheme*

⋆ Results

	Assumptions	Results	χ^2/d.o.f.	Assumptions	Results	χ^2/d.o.f.
octet-singlet basis	θ_8 and θ_0 free $f_8 = 1.28 f_\pi$	$\theta_8 = (-22.2 \pm 1.4)^\circ$ $\theta_0 = (-5.5 \pm 2.3)^\circ$ $f_0 = (1.25 \pm 0.04) f_\pi$	40.5/5	$\theta_8 = \theta_0 \equiv \theta$ $f_8 = 1.28 f_\pi$	$\theta = (-18.9 \pm 1.2)^\circ$ $f_0 = (1.11 \pm 0.03) f_\pi$	73.7/6
	θ_8 and θ_0 free f_8 free	$\theta_8 = (-23.8 \pm 1.4)^\circ$ $\theta_0 = (-1.1 \pm 2.3)^\circ$ $f_8 = (1.51 \pm 0.05) f_\pi$ $f_0 = (1.32 \pm 0.05) f_\pi$	17.9/4	$\theta_8 = \theta_0 \equiv \theta$ f_8 free	$\theta = (-18.2 \pm 1.2)^\circ$ $f_8 = (1.39 \pm 0.04) f_\pi$ $f_0 = (1.13 \pm 0.03) f_\pi$	66.9/5
quark-flavour basis	ϕ_q and ϕ_s free $f_q = f_\pi$	$\phi_q = (42.7 \pm 2.0)^\circ$ $\phi_s = (41.6 \pm 1.3)^\circ$ $f_s = (1.69 \pm 0.07) f_\pi$	32.6/5	$\phi_q = \phi_s \equiv \phi$ $f_q = f_\pi$	$\phi = (41.8 \pm 1.2)^\circ$ $f_s = (1.68 \pm 0.07) f_\pi$	32.8/6
	ϕ_q and ϕ_s free f_q free	$\phi_q = (41.6 \pm 2.3)^\circ$ $\phi_s = (41.5 \pm 1.4)^\circ$ $f_q = (1.09 \pm 0.03) f_\pi$ $f_s = (1.68 \pm 0.07) f_\pi$	17.9/4	$\phi_q = \phi_s \equiv \phi$ f_q free	$\phi = (41.5 \pm 1.2)^\circ$ $f_q = (1.09 \pm 0.03) f_\pi$ $f_s = (1.68 \pm 0.07) f_\pi$	17.9/5

Table 1: Results for the η-η' mixing parameters in the octet-singlet basis *(upper part)* and the quark-flavour basis *(down part)* of the two mixing angle scheme *(left)* and the one mixing angle scheme *(right)*.

- in the octet-singlet basis a two mixing angle scheme is needed to describe experimental data in a better way;
- in the quark-flavour basis a one mixing angle description of data is enough at the current experimental accuracy.

- *Study of the η-η' system in the two mixing angle scheme*

  ⋆ Discussion about the mixing parameters

Our best results for the mixing parameters are

$$f_8 = (1.51 \pm 0.05) f_\pi \ , \qquad \theta_8 = (-23.8 \pm 1.4)^\circ \ ,$$
$$f_0 = (1.29 \pm 0.04) f_\pi \ , \qquad \theta_0 = (-2.4 \pm 1.9)^\circ \ ,$$

in the octet-singlet basis, and

$$f_q = (1.09 \pm 0.03) f_\pi \ , \qquad \phi_q = (39.9 \pm 1.3)^\circ \ ,$$
$$f_s = (1.66 \pm 0.06) f_\pi \ , \qquad \phi_s = (41.4 \pm 1.4)^\circ \ ,$$

in the quark-flavour basis.

At the present accuracy, our results satisfy the approximate relations

$$f_8 = \sqrt{1/3 f_q^2 + 2/3 f_s^2} \ , \qquad \theta_8 = \phi - \arctan(\sqrt{2} f_s / f_q) \ ,$$
$$f_0 = \sqrt{2/3 f_q^2 + 1/3 f_s^2} \ , \qquad \theta_0 = \phi - \arctan(\sqrt{2} f_q / f_s) \ .$$

For comparison:

Large N_c χPT (H. Leutwyler, Nucl. Phys. Proc. Suppl. **64** (1998) 223):

$f_8 = 1.28 f_\pi$, $\theta_8 = -20.5^\circ$, $f_0 \simeq 1.25$, and $\theta_0 \simeq -4^\circ$

phenomenological analysis (T. Feldmann *et. al.*, Phys. Rev. D **58** (1998) 114006):

$f_q = (1.07 \pm 0.02) f_\pi$, $f_s = (1.34 \pm 0.06) f_\pi$ and $\phi = (39.3 \pm 1.0)^\circ$

- *Study of the η-η' system in the two mixing angle scheme*

  ⋆ Discussion about the mixing parameters

Large N_c χPT

H. Leutwyler, Nucl. Phys. Proc. Suppl. **64** (1998) 223
R. Kaiser and H. Leutwyler, arXiv:hep-ph/9806336

octet-singlet basis

$$f_8^2 = \frac{4f_K^2 - f_\pi^2}{3} \ ,$$
$$f_0^2 = \frac{2f_K^2 + f_\pi^2}{3} + f_\pi^2 \Lambda_1 \ ,$$
$$f_8 f_0 \sin(\theta_8 - \theta_0) = -\frac{2\sqrt{2}}{3}(f_K^2 - f_\pi^2) \ .$$

quark-flavour basis

$$f_q^2 = f_\pi^2 + \frac{2}{3} f_\pi^2 \Lambda_1 \ ,$$
$$f_s^2 = 2f_K^2 - f_\pi^2 + \frac{1}{3} f_\pi^2 \Lambda_1 \ ,$$
$$f_q f_s \sin(\phi_q - \phi_s) = \frac{\sqrt{2}}{3} f_\pi^2 \Lambda_1 \ .$$

Our best results for the mixing parameters can be used to check the experimental consistency of the former Large N_c χPT equations.

- our fitted values for f_8 and $f_0 \Longrightarrow \theta_8 - \theta_0 = (-13.7 \pm 0.6)^\circ$, to be compared with our prediction $\theta_8 - \theta_0 = (-21.4 \pm 2.4)^\circ$
- our fitted value for $f_0 \Longrightarrow \Lambda_1 = 0.34 \pm 0.10$ for the OZI-rule violating parameter Λ_1
- the fitted values for $f_{q,s}$ and $\phi_{q,s} \Longrightarrow \Lambda_1 = 0.32 \pm 0.10$, 2.34 ± 0.60 and -0.10 ± 0.13

The wide range of different values for Λ_1 as well as the disagreements obtained for $\theta_8 - \theta_0$ and f_8 seem to indicate a possible discrepancy with the Large N_c χPT framework.

- *Conclusions*

  - ⋆ best measurement of the η-η' mixing angle from a single experiment $\varphi_P = (41.8^{+1.9}_{-1.6})^\circ$

  - ⋆ precise measurement of the gluon admixture of the η' (Gluonium fraction below 15%) $Z^2_{\eta'} = 0.06^{+0.09}_{-0.06}$

  - ⋆ good prospects for helping in the measure of the mixing parameters in the two mixing angle scheme

  - ⋆ *Study of the η-η' system in the two mixing angle scheme*

    - ✔ we have shown that a two mixing angle description in the octet-singlet basis is required in order to achieve good agreement with experimental data

    - ✔ on the contrary, in the quark-flavour basis and with the present experimental accuracy a one mixing angle description of the processes is still enough to reach agreement

    - ✔ this behaviour gives experimental support to the fact that the difference of the two mixing angles in the octet-singlet basis is a $SU(3)$-breaking effect while in the quark-flavour basis is a OZI-rule violating effect which appears to be smaller