Unraveling the f_0 nature by connecting *KLOE* and *BABAR* data through analyticity

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Outline

1. Introduction, formulae and data
   - Basic knowledge on light scalar mesons
   - Amplitudes, cross sections and rates
   - *BaBar* cross section data
2. Strategy
   - Contributions to the tff's
   - Dispersion relations and χ^2
3. Results and new hypothesis
   - $q\overline{q}$ mesons
   - Properties of the $[qq][\overline{qq}]$ f_0 meson
   - $[qq][\overline{qq}]$ f_0: results

The $f_0(980)$ scalar meson

It is an isoscalar $J^{PC} = 0^{++}$ state

PDG estimate $\begin{cases} M_{f_0} = (980 \pm 10)\ MeV \\ \Gamma_{f_0} = (40 \div 100)\ MeV \end{cases}$

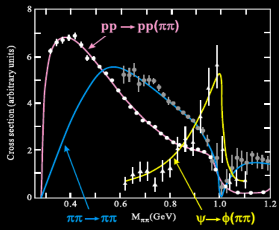
It has an isovector $a_0(980)$ with similar mass and width

PDG estimate $\begin{cases} M_{a_0} = (984.7 \pm 1.2)\ MeV \\ \Gamma_{a_0} = (50 \div 100)\ MeV \end{cases}$

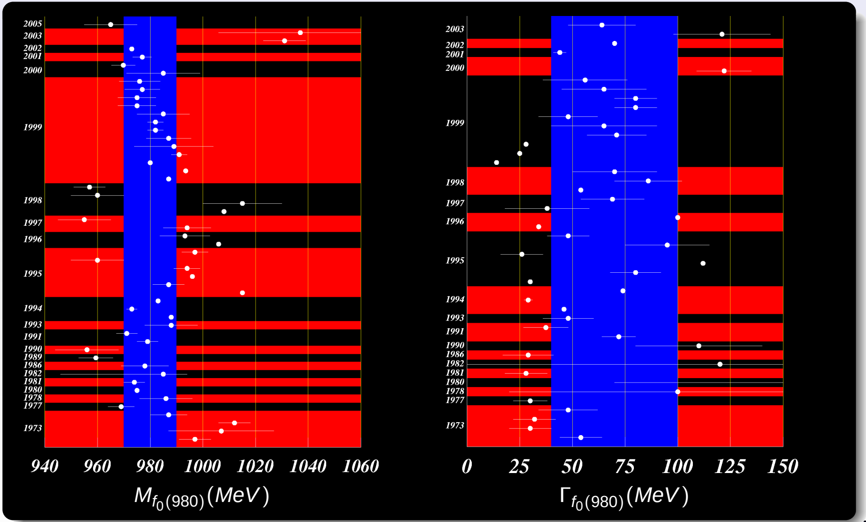
The **vacuum**, the **Higgs boson** and the expected **lowest-lying glueball** have the $f_0(980)$ quantum numbers

It is a bridge between light and strange quarks. It appears:

- as a peak in the $\pi\pi$ invariant mass distribution of strange particle decays;
- as a dip or a shoulder when produced from non-strange quarks.

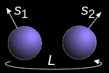


$f_0(980)$ mass and width time evolution



Long outstanding questions

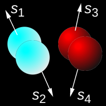
Is the $f_0(980)$ an $n^{2S+1}L_J = 1^3P_0$ element of the conventional scalar $q\bar{q}$ nonet of flavor $SU(3)$?



$S = 1, L = 1$

Is this meson a **multiquark**, $K\bar{K}$ or meson-meson bound states ?

Strong interest in recent literature
Maiani, Piccinini, Polosa, Riquer, Jaffe, Pennington, Close, Achasov,. . .



$S = 0, L = 0$

Is this meson the lower-lying **glueball** ?

Is it an **hybrid** $q\bar{q}g$ with a massive gluon component ?

In these cases

There exists a scalar $q\bar{q}$ nonet?

Where is it?

The $q\overline{q}$ scalar nonet

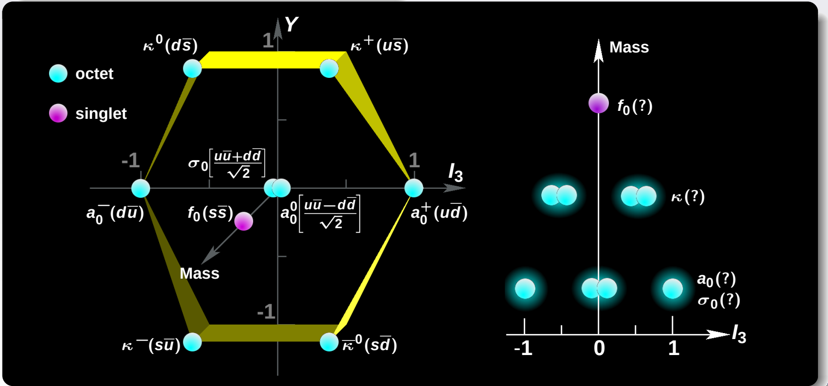
$SU(3)$ flavor nonet

The three light quarks u, d and s group into an octet and a singlet

$$q^3 \otimes \overline{q}^{\overline{3}} = [\overline{q}q]^1 \oplus [\overline{q}q]^8$$

Problems

- Mass degeneracy of f_0 and a_0
- σ and κ are broader than f_0 and a_0



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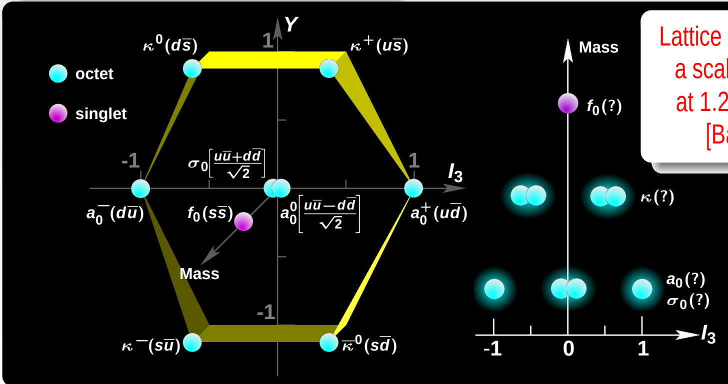


Lattice QCD predicts a scalar $q\bar{q}$ nonet at $1.2 \div 1.6$ *GeV* [Bali *et al.*]

The $qq\overline{qq}$ cryptoexotic scalar nonet

Exotica

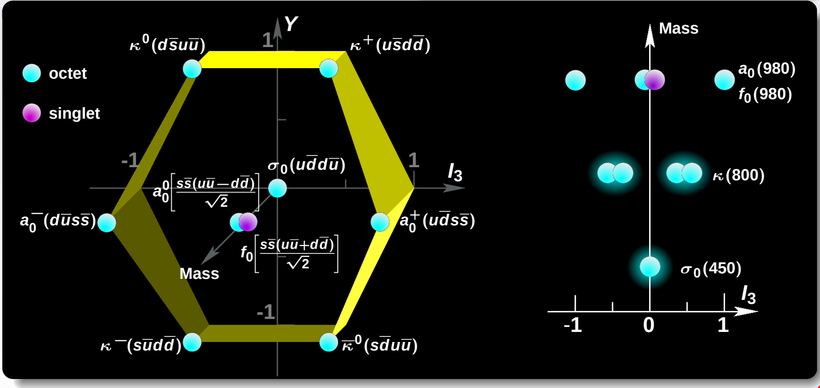
States $q\bar{q}q\bar{q}$ include exotics in **27**, **10** and $\overline{\mathbf{10}}$ representations of flavor $SU(3)$

Cryptoexotica $\quad Q \equiv |\{qq\}\bar{3}_c\bar{3}_f 0^+\rangle$

The light states are Q-dominated

$$\overline{Q}^{\bar{3}} \otimes Q^{3} = [\overline{Q}Q]^1 \oplus [\overline{Q}Q]^8$$

there are only non-exotic representations



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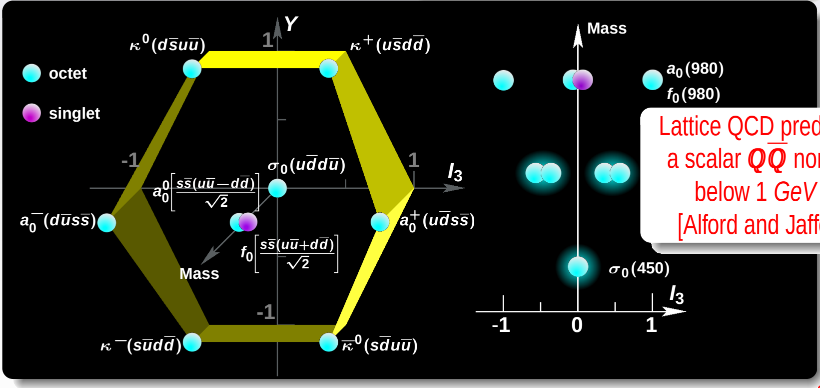
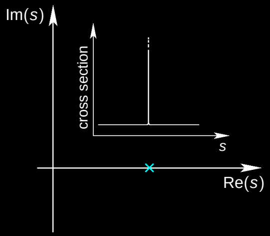


Lattice QCD predicts a scalar $Q\overline{Q}$ nonet below 1 *GeV* [Alford and Jaffe]

Ambiguity in studying scalar resonances

Breit-Wigner formula

	Propagator^{-1}	Fock space	Life time
$S(1000)$	$M_0^2 - s$	$s\bar{s}$	∞



Vector mesons $V = \rho, \omega, \phi, \ldots$

$V \rightarrow \left[P\overline{P}\right]_{\text{P-wave}}$: suppressed $\rightarrow$ threshold

$P\overline{P}$ is a little effect in the V Fock space

is unimportant in the V-resonance composition

Scalar mesons $S = f_0, a_0, \ldots$

$S \rightarrow \left[P\overline{P}\right]_{\text{S-wave}}$: no suppression

$P\overline{P}$ is a large component of the S Fock space

is crucial in the S-resonance composition

Ambiguity in studying scalar resonances

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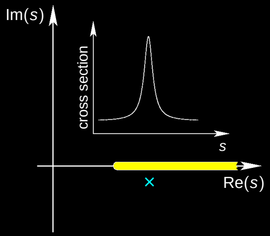
$S(1000)$	$M_0^2 - s$	$s\bar{s}$	∞
	$M_0^2 - s - \Pi_{\pi\pi}(s)$	$s\bar{s} + c_1\pi\pi$	$< \infty$



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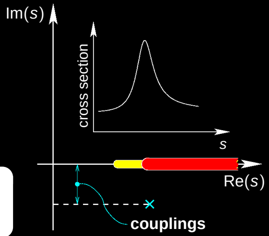
Breit-Wigner formula

	Propagator^{-1}	Fock space	Life time
$S(1000)$	$M_0^2 - s$	$s\bar{s}$	∞
	$M_0^2 - s - \Pi_{\pi\pi}(s)$	$s\bar{s} + c_1\pi\pi$	$< \infty$
	$M_0^2 - s - \Pi_{\text{tot}}(s)$	$s\bar{s} + c_1\pi\pi + \cdots$	$< \infty$

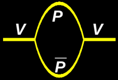


BW Distortion $\Longrightarrow$ **New approaches:** $BR = \int BW \longrightarrow$ **couplings**

Vector mesons $V = \rho, \omega, \phi, \ldots$

$V \to \left[P\bar{P}\right]_{\text{P-wave}}$: suppressed $\to$ threshold

$P\bar{P}$ is a little effect in the V Fock space



is unimportant in the V-resonance composition

Scalar mesons $S = f_0, a_0, \ldots$

$S \to \left[P\bar{P}\right]_{\text{S-wave}}$: no suppression

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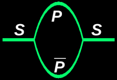


is crucial in the S-resonance composition

Collecting some information about the $f_0(980)$

$Z^0 \to X + f_0$

- Large $u\bar{u}$ and $d\bar{d}$ component
- No evidence of gluon content
- No information about the structure

Lattice QCD

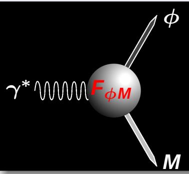
- Lower-lying glueball: $M_G \sim 1.6\ GeV$
- A $q\bar{q}$ P-wave scalar nonet is predicted at $1.2 \div 1.6\ GeV$
- Strong interacting $\boldsymbol{Q\overline{Q}}$ produce an S-wave scalar nonet below 1 GeV

The standard analysis of the $f_0(980)$ resonance is quite difficult. There are cuts on the complex s-plane that distort the scalar pole.

This Work

The study of the vertex $\phi f_0(980)\gamma$, described in terms of transition form factors, gives direct information about the f_0 structure avoiding the problem of the analysis of the scalar resonance.

The plan to study the vertex $\phi M \gamma$ with $M = \eta, f_0(980)$

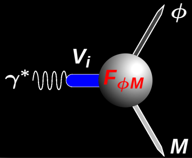


Data on the cross section for

$$e^+e^- \to \gamma^*(q) \to \phi M$$

give

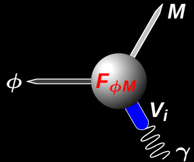
$$\left|F_{\phi M}(q^2)\right|^2 \quad q^2 \geq (M_\phi + M_M)^2$$



In the unphysical region $[(M_\phi - M_M)^2, (M_\phi + M_M)^2]$

$$F_{\phi M}(q^2) = \sum_{i=1}^{N_V} G_{ee}^{V_i} G_{\phi M}^{V_i} BW_i(q^2)$$

Number N_V and species of the intermediate V_i strongly depend on the quark structure of the meson M



$$\Gamma(\phi \to M\gamma) = \mathrm{DR}\left\{ \underset{i=1,\ldots,N_V}{G_{\phi M}^{V_i}} ; \sigma(\phi M); \underset{q^2 \to \infty}{F_{\phi M}(q^2)} \right\}$$

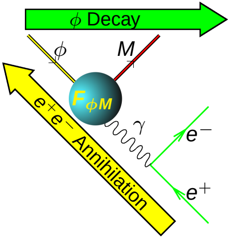
The asymptotic behaviour of $F_{\phi M}(q^2)$ is a function of the number of hadronic fields in the vertex $\phi M \gamma$

From annihilation cross section to radiative decay rate

$e^+e^- \longrightarrow \gamma^* \longrightarrow M\phi$

The cross section is:

$\sigma = \sigma_{\rm QED} \cdot |F_{\phi M}(q^2)|^2$



$\phi \longrightarrow \gamma^* M \longrightarrow e^+e^- M$

The diff. decay rate is:

$$\frac{d\Gamma}{dq^2} = \left[\frac{d\Gamma}{dq^2}\right]_{\rm QED} |F_{\phi M}(q^2)|^2$$

radiative decay ($q^2 = 0$):

$\Gamma = \Gamma_{\rm QED} \cdot |F_{\phi M}(0)|^2$

If we know the form factor $F_{\phi M}$ as a function of q^2, we can relate

BaBar $\sigma(e^+e^- \longrightarrow M\phi)$ and $\frac{d\Gamma(\phi \to e^+e^- M)}{dq^2}$ KLOE

Large values of radiative decay rate Γ $(q^2 = 0)$

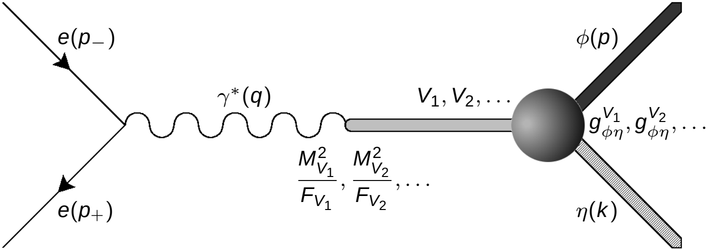
Unless drastic variation of the form factor

Large values of σ $[q^2 > (M_M + M_\phi)^2]$

Very large values of σ if $M_M \sim M_\phi$

Amplitude and $\phi\eta$ transition form factor

$e^+e^- \longrightarrow \phi\eta$

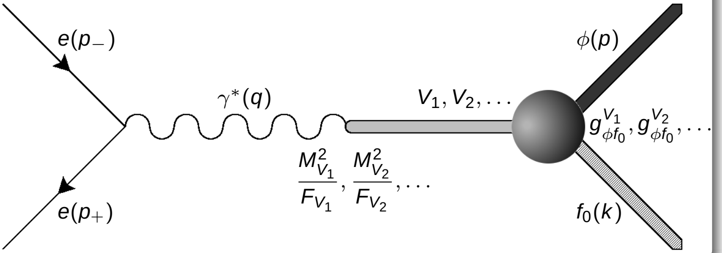


$$\mathcal{M} = \sum_j \frac{g^{V_j}_{\phi\eta}}{M_j^2 - q^2 - i\Gamma_j M_j} \frac{M^2_{V_j}}{F_{V_j}} \left[\varepsilon^{\mu\nu\alpha\beta}\epsilon_\nu p_\alpha q_\beta\right] \frac{1}{q^2} \left[-ie\overline{v}(p_+)\gamma_\mu u(p_-)\right]$$

$$F_{\phi\eta}(q^2) = \sum_j \frac{M_j}{eF_{V_j}} \frac{g^{V_j}_{\phi\eta}}{\Gamma_j} \frac{\Gamma_j M_j}{M_j^2 - q^2 - i\Gamma_j M_j}$$

Amplitude and ϕf_0 transition form factor

$e^+e^- \longrightarrow \phi f_0$

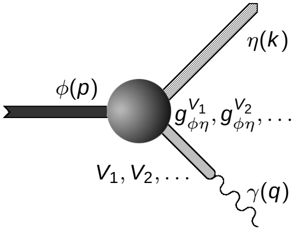


$$\mathcal{M} = \sum_j \frac{g^{V_j}_{\phi f_0}}{M_j^2 - q^2 - i\Gamma_j M_j} \frac{M^2_{V_j}}{F_{V_j}} \epsilon_\nu \left[p^\mu q^\nu - g^{\mu\nu}(pq)\right] \frac{1}{q^2} \left[-ie\overline{v}(p_+)\gamma_\mu u(p_-)\right]$$

$$F_{\phi f_0}(q^2) = \sum_j \frac{M_j}{eF_{V_j}} \frac{g^{V_j}_{\phi f_0}}{\Gamma_j} \frac{\Gamma_j M_j}{M_j^2 - q^2 - i\Gamma_j M_j}$$

Radiative decay amplitudes and rates

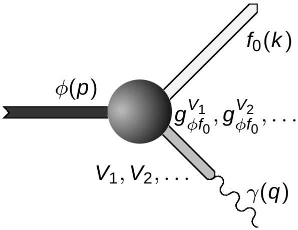
$\phi \longrightarrow \eta\gamma$



$$\mathcal{M} = F_{\phi\eta}(0)\varepsilon^{\mu\nu\rho\sigma}\epsilon_\mu(p)\epsilon_\nu(p)p_\rho q_\sigma$$

$$\Gamma(\phi \longrightarrow \eta\gamma) = \frac{\alpha}{3}\left[\frac{M_\phi^2 - M_\eta^2}{2M_\phi}\right]^3 F_{\phi\eta}(0)^2$$

$\phi \longrightarrow f_0\gamma$

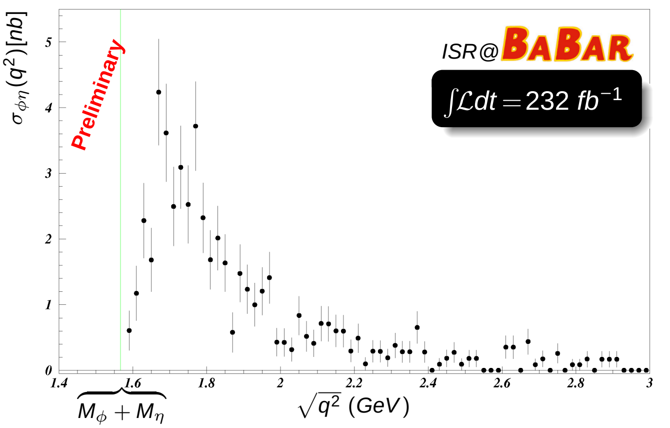


$$\mathcal{M} = F_{\phi f_0}(0)\Big[q^\mu p^\nu - g^{\mu\nu}(qp)\Big]\epsilon_\mu(p)\epsilon_\nu(p)$$

$$\Gamma(\phi \longrightarrow f_0\gamma) = \frac{\alpha}{3}\left[\frac{M_\phi^2 - M_{f_0}^2}{2M_\phi}\right]^3 F_{\phi f_0}(0)^2$$

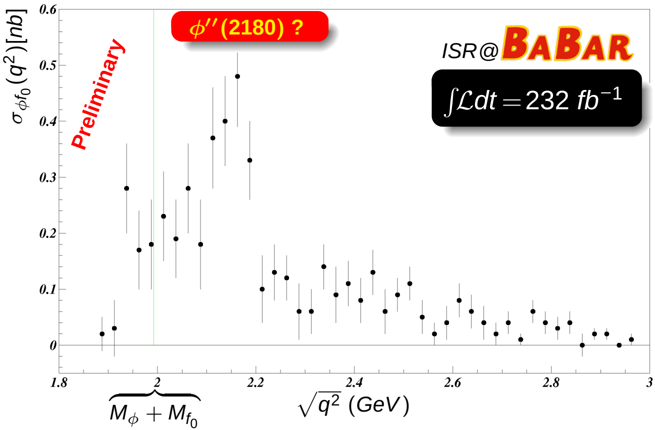
BaBar data: $\phi\eta$ cross section

$\sigma(e^+e^- \longrightarrow \phi\eta\gamma)$



BaBar data: ϕf_0 cross section

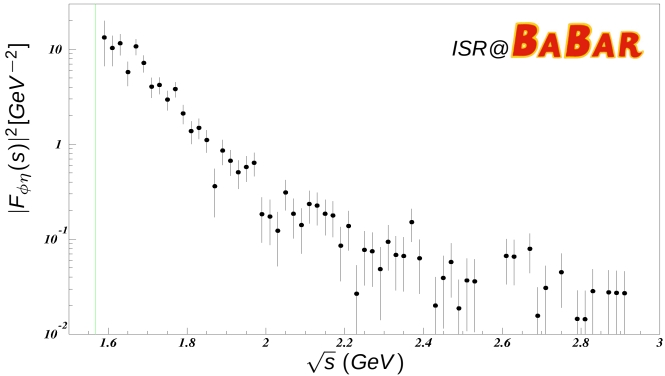
$\sigma(e^+e^- \to \phi f_0 \gamma)$



BABAR data: $\phi\eta$ transition form factor

$$\sigma_{\phi\eta}(s) = \frac{\pi}{6}\frac{\alpha^2}{s^3}\left(s+2m^2\right)\left\{\frac{\left[\left(s+M_\phi^2-M_\eta^2\right)^2-4M_\phi^2 s\right]^3}{s\left(s-4m^2\right)}\right\}^{\frac{1}{2}}\cdot\left|F_{\phi\eta}(s)\right|^2 \qquad s=q^2$$

$|F_{\phi\eta}(s)|^2$



BaBar data: ϕf_0 transition form factor

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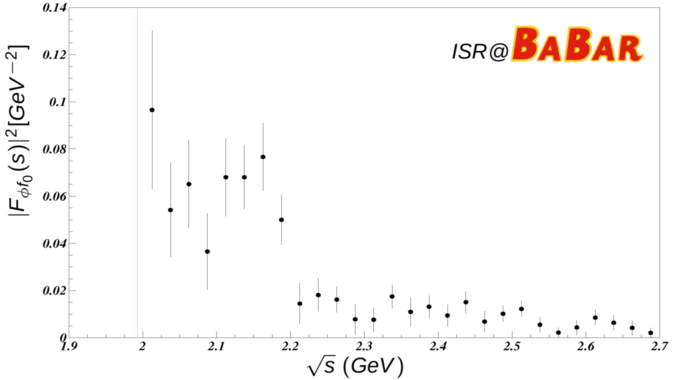
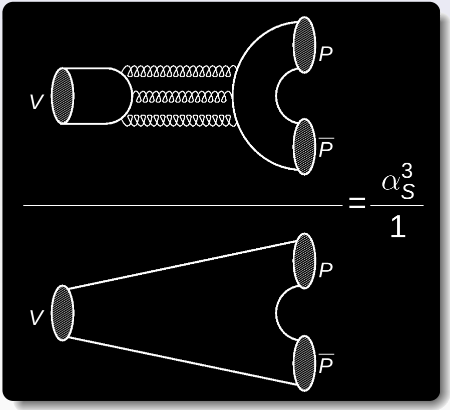
$|F_{\phi f_0}(s)|^2$



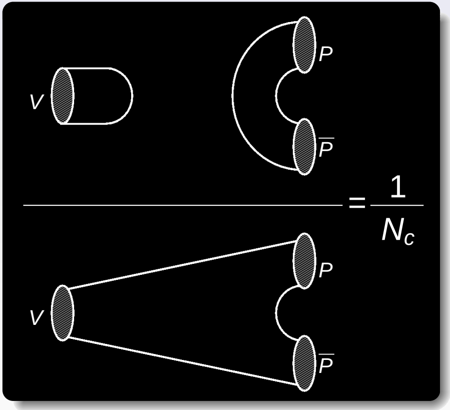
Okubo-Zweig-Iizuka (OZI) rule



Hard gluons

- Gluons carry color while mesons are colorless
  $\Rightarrow$ more than 1 gluon exchanged
- $C(V) = -1$
  $C(n_g \text{ gluons}) = (-1)^{n_g}$
  $\Rightarrow$ at least 3 gluons
- $E_V > 1\ GeV \Rightarrow \alpha_S < 1$
  $\Rightarrow$ suppression α_S^3

Okubo-Zweig-Iizuka (OZI) rule



Large N_c

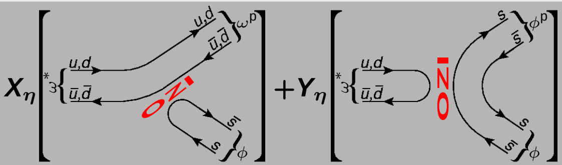
- Denominator
  Flavor connected process:
  $\Rightarrow \mathcal{O}\left(N_c^{-1}\right)$
- Numerator
  Flavor disconnected process with an extra quark loop:
  $\Rightarrow \mathcal{O}\left(N_c^{-2}\right)$

Contributions to the $\phi\eta$ transition form factor

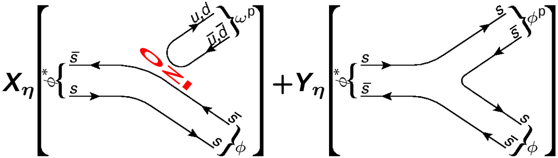
$\phi\eta$ final state: $I^G(J^{PC}) = 0^-(1^{--})$

$$|\phi\rangle = s\bar{s} \qquad |\eta\rangle = X_\eta|\omega^p\rangle + Y_\eta|\phi^p\rangle$$

$$|\omega^p\rangle = \left(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\right)_{\text{pseudoscalar}} \qquad |\phi^p\rangle = (s\bar{s})_{\text{pseudoscalar}} \qquad X_\eta^2 + Y_\eta^2 = 1$$



ω-family contributions are OZI suppressed



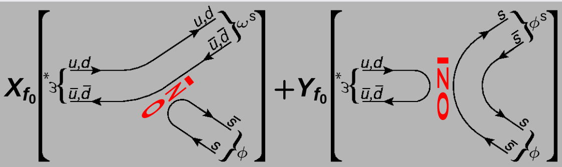
ϕ-family contributions are OZI allowed

Contributions to the $\phi f_0[q\bar{q}]$ transition form factor

ϕf_0 final state: $I^G(J^{PC}) = 0^-(1^{--})$

$$|\phi\rangle = s\bar{s} \qquad |f_0\rangle = X_{f_0}|\omega^s\rangle + Y_{f_0}|\phi^s\rangle$$

$$|\omega^s\rangle = \left(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\right)_{\text{scalar}} \qquad |\phi^s\rangle = (s\bar{s})_{\text{scalar}} \qquad X_{f_0}^2 + Y_{f_0}^2 = 1$$



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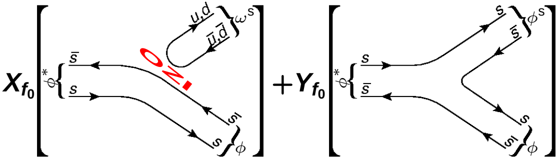
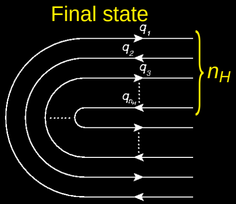


ϕ-family contributions are OZI allowed

Asymptotic behaviour

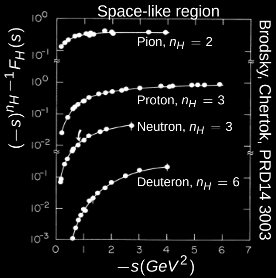
Nominal power law behaviour for hadronic form factor $F_H(s)$ as $s \to \infty$ [Brodsky, Lepage]



$$F_H(s) \propto \left(\frac{1}{s}\right)^{n_H - 1}$$

$$s \to \infty$$

n_H final hadronic fields



Helicity rule: hadronic helicity flip in $e^+e^- \to \gamma^* \to \phi M$ (spin$_M$=0)

- CM: $|\lambda_\phi - \lambda_M| = \left|\frac{\vec{p}\cdot\vec{S}_{\text{tot}}}{|\vec{p}|}\right| = 0, 1$
- Helicity conservation: $\lambda_\phi + \lambda_M = \lambda_\phi = 0$
- Only one d.o.f. (one transition form factor): $|\lambda_\phi| = 1$

$$F_H(s) \propto \left(\frac{1}{s}\right)^{n_H - 1 + n_\lambda}$$

$$n_\lambda = |\lambda_\phi - \lambda_M|$$

$F_{\phi\eta}(q^2)$ above the theoretical threshold $s_0 = (3M_\pi)^2$

Low energy behaviour of $F_{\phi\eta}(q^2)$ — 3 d.o.f.

$$F^{\rm VMD}_{\phi\eta}(q^2) = \frac{M_\phi}{eF_\phi}\,\frac{g^{\phi}_{\phi\eta}}{\Gamma_\phi}\,\frac{\Gamma_\phi M_\phi}{M_\phi^2 - q^2 - i\Gamma_\phi M_\phi} + \frac{M_{\phi'}}{eF_{\phi'}}\,\frac{g^{\phi'}_{\phi\eta}}{\Gamma_{\phi'}}\,\frac{\Gamma_{\phi'} M_{\phi'} e^{i\delta_\eta}}{M_{\phi'}^2 - q^2 - i\Gamma_{\phi'} M_{\phi'}}$$

Asymptotic behaviour ($q^2 \to \infty$)

$$F_{\phi\eta}(q^2) \propto \left(\frac{1}{q^2}\right)^{n_H+n_\lambda-1=2}$$

$n_H = 2$ final hadronic fields

$$n_\lambda = \begin{cases} 0 & \text{hadronic helicity conserved} \\ 1 & \text{otherwise} \end{cases}$$

$$F^{\rm th}_{\phi\eta}(s) = \begin{cases} F^{\rm VMD}_{\phi\eta}(s) & s_0 \le s \le s_{\rm asy} \\ F^{\rm VMD}_{\phi\eta}(s_{\rm asy})\left(\frac{s_{\rm asy}}{s}\right)^2 & s > s_{\rm asy} \end{cases}$$

$s \equiv q^2$

$s_0 = (3M_\pi)^2$

$s_{\rm asy} = (4\ GeV)^2$

$F_{\phi f_0}(q^2)$ above the theoretical threshold $s_0 = (3M_\pi)^2$

Low energy behaviour of $F_{\phi f_0}(q^2)$ **5 d.o.f.**

$$F^{\rm VMD}_{\phi f_0}(q^2) = BW_\phi(g^{\phi}_{\phi f_0}, q^2) + e^{i\delta_{f_0}} BW_{\phi'}(g^{\phi'}_{\phi f_0}, q^2) + e^{i\rho_{f_0}} BW_{\phi''}(g^{\phi''}_{\phi f_0}, q^2)$$

Three ϕ recurrences

Asymptotic behaviour ($q^2 \to \infty$)

$$F_{\phi f_0}(q^2) \propto \left(\frac{1}{q^2}\right)^{n_H + n_\lambda - 1 = 2}$$

$n_H = 2$ final hadronic fields

$$n_\lambda = \begin{cases} 0 & \text{hadronic helicity conserved} \\ 1 & \text{otherwise} \end{cases}$$

$$F^{\rm th}_{\phi f_0}(s) = \begin{cases} F^{\rm VMD}_{\phi f_0}(s) & s_0 \le s \le s_{\rm asy} \\ F^{\rm VMD}_{\phi f_0}(s_{\rm asy}) \left(\frac{s_{\rm asy}}{s}\right)^2 & s > s_{\rm asy} \end{cases}$$

$s \equiv q^2$

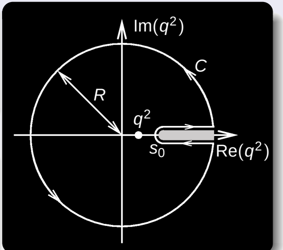
$s_0 = (3M_\pi)^2$

$s_{\rm asy} = (4\ GeV)^2$

Dispersion Relations

A form factor $f(q^2)$ is an analytic function on the q^2 complex plane with the cut $[s_0 = 9M_\pi^2, \infty)$

$$\boldsymbol{f(q^2) = |f(q^2)|e^{i\delta(q^2)}}$$



Dispersion relation for the imaginary part

$$f(q^2) = \lim_{R\to\infty} \frac{1}{2\pi i} \oint_C \frac{f(z)dz}{z-q^2} = \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\mathrm{Im} f(s)ds}{s-q^2}$$

Dispersion relation for the logarithm

Assuming no zeros on the physical sheet and using the function

$$\boldsymbol{\Phi(z) = \frac{\ln[f(z)]}{\sqrt{s_0 - z}}}$$

$q^2 < s_0$

$$\ln[f(q^2)] = \frac{\sqrt{s_0 - q^2}}{\pi} \int_{s_0}^{\infty} \frac{\ln|f(s)|ds}{(s-q^2)\sqrt{s-s_0}}$$

$q^2 \geq s_0$

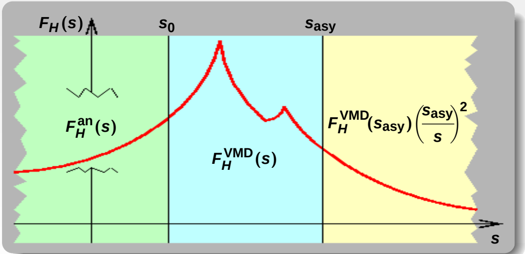
$$\delta(q^2) = -\frac{\sqrt{q^2 - s_0}}{\pi} \,\mathrm{Pr}\!\int_{s_0}^{\infty} \frac{\ln|f(s)|ds}{(s-q^2)\sqrt{s-s_0}}$$

Extension of the transition form factors below s_0

Analytic continuation via dispersion relations for the logarithm

$$\ln\left[F_H^{\rm an}(q^2)\right] = \frac{\sqrt{s_0 - q^2}}{\pi}\int_{s_0}^{\infty}\frac{\ln|F_H^{\rm th}(s)|ds}{(s-q^2)\sqrt{s-s_0}} \qquad q^2 < s_0$$

$$F_H(s) = \begin{cases} F_H^{\rm an}(s) & s < s_0 \\ F_H^{\rm VMD}(s) & s_0 \le s \le s_{\rm asy} \\ F_H^{\rm VMD}(s_{\rm asy})\left(\frac{s_{\rm asy}}{s}\right)^2 & s > s_{\rm asy} \end{cases}$$



χ^2

$$\chi^2_{\text{tot}} = \chi^2_{\text{exp}} + \tau \chi^2_{\text{th}}$$

The continuity of the derivative is a condition that have to be exactly verified

τ is chosen large enough to force the vanishing of the corresponding χ^2_{th}

Experimental contribution

$(M_\phi + M_{\eta(f_0)})^2 \le s \le [4(3)\ GeV]^2$

$$\chi^2_{\text{exp}} = \sum_{j=1}^{N} \left[\frac{F^{\text{th}}_{\phi\eta(f_0)}(s_j^{\text{exp}}) - F_j^{\text{exp}}}{\delta F_j^{\text{exp}}} \right]^2$$

Data: $\{s_j^{\text{exp}}, F_j^{\text{exp}} \pm \delta F_j^{\text{exp}}\}$

Theoretical contribution

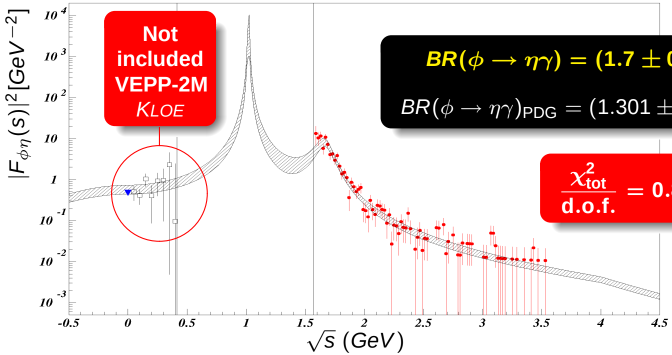
$s < s_0$ and $s > s_{\text{asy}}$

Super-convergence relation

$$\chi^2_{\text{th}} = \left[\frac{1}{F^{\text{an}}_{\phi\eta(f_0)}} \frac{dF^{\text{an}}_{\phi\eta(f_0)}}{ds}\bigg|_{s_0} - \lim_{\epsilon\to 0^+} \frac{1}{2\sqrt{\epsilon}} \int_{s_0}^{\infty} \frac{(2s_0 - s - \epsilon)\ \ln|F^{\text{th}}_{\phi\eta(f_0)}(s)|}{\sqrt{s-s_0}(s-s_0+\epsilon)^2} ds \right]^2$$

Reconstructed $\phi\eta$ transition form factor

$|F_{\phi\eta}(s)|^2$



Not included VEPP-2M KLOE

$$BR(\phi \to \eta\gamma) = (1.7 \pm 0.4)\%$$

$$BR(\phi \to \eta\gamma)_{\text{PDG}} = (1.301 \pm 0.024)\%$$

$$\frac{\chi^2_{\text{tot}}}{\text{d.o.f.}} = 0.83$$

Coupling constants

$$g^{\phi}_{\phi\eta} = 12 \pm 1 \ GeV^{-1}$$

$$g^{\phi'}_{\phi\eta} = 40 \pm 2 \ GeV^{-1}$$

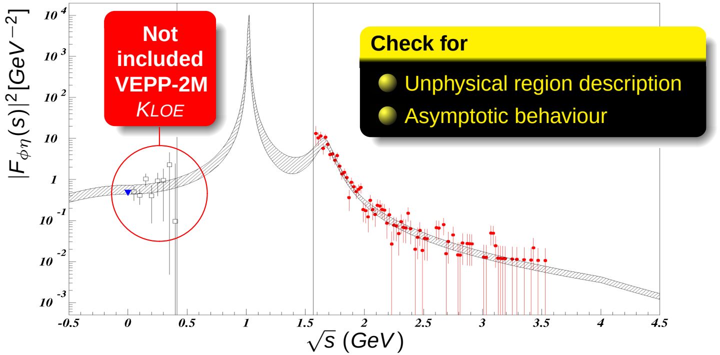
Resonances parameters (input)

$$M_\phi = 1019.5 \ MeV \qquad \Gamma_\phi = 4.3 \ MeV$$

$$M_{\phi'} = 1665 \ MeV \qquad \Gamma_{\phi'} = 159 \ MeV$$

Reconstructed $\phi\eta$ transition form factor

$|F_{\phi\eta}(s)|^2$



Coupling constants

$$g^{\phi}_{\phi\eta} = 12 \pm 1\ GeV^{-1}$$
$$g^{\phi'}_{\phi\eta} = 40 \pm 2\ GeV^{-1}$$

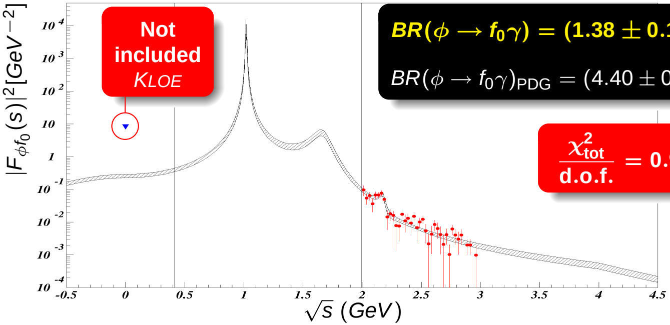
Resonances parameters (input)

$M_\phi = 1019.5\ MeV$ $\Gamma_\phi = 4.3\ MeV$

$M_{\phi'} = 1665\ MeV$ $\Gamma_{\phi'} = 159\ MeV$

Reconstructed ϕf_0 transition form factor

$|F_{\phi f_0}(s)|^2$



$$BR(\phi \longrightarrow f_0\gamma) = (1.38 \pm 0.17) \times 10^{-5}$$

$$BR(\phi \longrightarrow f_0\gamma)_{PDG} = (4.40 \pm 0.21) \times 10^{-4}$$

$$\frac{\chi^2_{tot}}{d.o.f.} = 0.95$$

Coupling constants

$$g^{\phi}_{\phi f_0} = 9 \pm 1\ GeV^{-1}$$
$$g^{\phi'}_{\phi f_0} = 36 \pm 5\ GeV^{-1}$$
$$g^{\phi''}_{\phi f_0} M_{\phi''}/F_{\phi''} = (6.3 \pm 0.3) \times 10^{-3}$$

Resonances parameters (input)

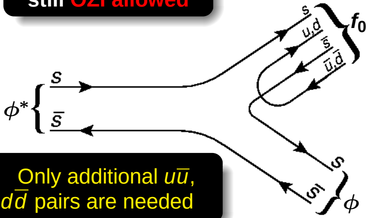
$M_{\phi} = 1019.5\ MeV$ $\quad \Gamma_{\phi} = 4.3\ MeV$

$M_{\phi'} = 1665\ MeV$ $\quad \Gamma_{\phi'} = 159\ MeV$

$M_{\phi''} = 2176\ MeV$ $\quad \Gamma_{\phi''} = 51\ MeV$

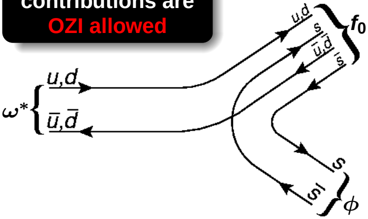
Contributions to the $\phi f_0[qq][\overline{qq}]$ transition form factor

ϕ-family contributions are still OZI allowed



Only additional $u\bar{u}$, $d\bar{d}$ pairs are needed

With a $[qq][\overline{qq}]$ f_0 ω-family contributions are OZI allowed



VMD contributions **7 d.o.f.**

$$F^{\mathrm{VMD}}_{\phi f_0}(s) = BW_{\phi}(g^{\phi}_{\phi f_0}, s) + e^{i\delta_{f_0}} BW_{\phi'}(g^{\phi'}_{\phi f_0}, s) + e^{i\rho_{f_0}} BW_{\phi''}(g^{\phi''}_{\phi f_0}, s) + e^{i\xi_{f_0}} BW_{\omega}(g^{\omega}_{\phi f_0}, s)$$

No ω' contribution is considered
In the same region ϕ' dominates

$$\frac{M_{\phi'}}{|F_{\phi'}|}\frac{|F_{\omega'}|}{M_{\omega'}} > 10$$

Asymptotic behaviour for $F_{\phi f_0}(s)$

$$F_{\phi f_0}(s) \propto \left(\frac{1}{s}\right)^{n_H+n_\lambda-1} = \left(\frac{1}{s}\right)^3$$

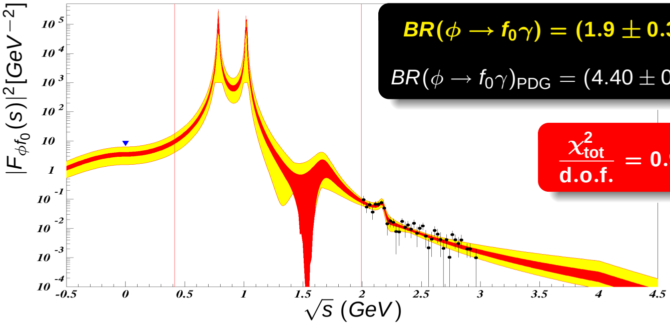
$n_H = 3$

$$F_{\phi f_0}(s) = \begin{cases} F^{\text{an}}_{\phi f_0}(s) & s < s_0 \\ F^{\text{VMD}}_{\phi f_0}(s) & s_0 \leq s \leq s_{\text{asy}} \\ F^{\text{VMD}}_{\phi f_0}(s_{\text{asy}}) \left(\frac{s_{\text{asy}}}{s}\right)^3 & s > s_{\text{asy}} \end{cases}$$

Without any additional low energy contribution (ω), the faster vanishing asymptotic behaviour should give a small value of the transition form factor at $s = 0$

Reconstructed ϕf_0 transition form factor with $f_0[qq][\overline{qq}]$

$|F_{\phi f_0}(s)|^2$



$$BR(\phi \longrightarrow f_0\gamma) = (1.9 \pm 0.3) \times 10^{-4}$$

$$BR(\phi \rightarrow f_0\gamma)_{\mathrm{PDG}} = (4.40 \pm 0.21) \times 10^{-4}$$

$$\frac{\chi^2_{\mathrm{tot}}}{\mathrm{d.o.f.}} = 0.90$$

$$g^{\omega}_{\phi f_0} = 76 \pm 7\ GeV^{-1}$$
$$g^{\phi}_{\phi f_0} = 34 \pm 4\ GeV^{-1}$$
$$g^{\phi'}_{\phi f_0} = 17 \pm 5\ GeV^{-1}$$
$$g^{\phi''}_{\phi f_0} M_{\phi''}/F_{\phi''} = (1.9 \pm 0.4) \cdot 10^{-3}$$

$M_\omega = 783.7\ MeV$	$\Gamma_\omega = 8.5\ MeV$
$M_\phi = 1019.5\ MeV$	$\Gamma_\phi = 4.3\ MeV$
$M_{\phi'} = 1665\ MeV$	$\Gamma_{\phi'} = 159\ MeV$
$M_{\phi''} = 2176\ MeV$	$\Gamma_{\phi''} = 51\ MeV$

Relative phases and VMD relation

Relative phases

$\xi(\omega) = (-178 \pm 3)^o$ $\quad$ $\delta(\phi') = (15 \pm 35)^o$ $\quad$ $\rho(\phi'') = (-218 \pm 20)^o$

VMD coupling constants relation

$$g^{\phi}_{f_0\gamma}\left[GeV^{-1}\right] = \sum_{V}^{\substack{\omega,\phi\\ \phi',\phi''}} \frac{g^V_{\phi f_0}}{eF_V} \simeq -\frac{250}{56} + \frac{113}{44} + \frac{55}{563} - \frac{6.4\times 10^{-3}}{2.176} \simeq -1.78$$

Reanalysis of *KLOE* data [PLB634, 148]

A reanalysis of *KLOE* data performed with a model independent procedure [Maiani-Isidori hep-ph/0603241] allows to extract directly the coupling $|g^{\phi}_{f_0\gamma}|$ instead of the Breit Wigner shape-dependent $BR(\phi \to f_0\gamma)$

KLOE
$|g^{\phi}_{f_0\gamma}| = (1.2 \div 2.0) GeV^{-1}$

Self-consistency check

$$BR(\phi \longrightarrow f_0\gamma) = \frac{\alpha}{3}\left(\frac{M_\phi^2 - M_{f_0}^2}{2M_\phi}\right)^3 \frac{|g^{\phi}_{f_0\gamma}|^2}{\Gamma_\phi} \simeq 2\times 10^{-4}$$

Duality

Asymptotic behaviour power law

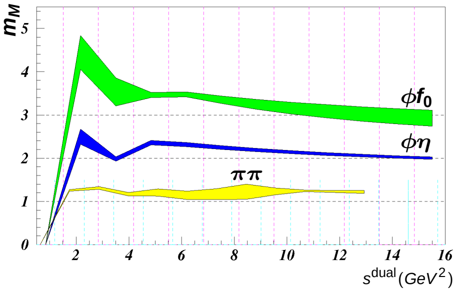
$$F_H^2(s) \propto \frac{1}{s^{2m}} \qquad m = n_H + n_\lambda - 1$$

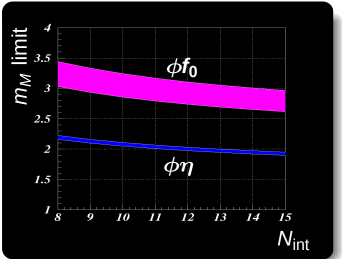
$$\lim_{s\to\infty} -\frac{\ln[F_H^2(s)/F_H^2(s_{\rm ref})]}{2\ln[s/s_{\rm ref}]} = m$$

$$m_M = -\frac{\ln(F_{M,j}^{2,\rm dual}) - \ln(F_{M,1}^{2,\rm dual})}{2[\ln(s_j^{\rm dual}) - \ln(s_1^{\rm dual})]}$$

$$F_{M,j}^{2,\rm dual} = \frac{1}{\Delta s}\int_{s_j^{\rm dual}-\Delta s/2}^{s_{j+1}^{\rm dual}+\Delta s/2} F_M^2(s)ds$$

$$s_j^{\rm dual} = s_0 + \left(j - \frac{1}{2}\right)\frac{\Delta s}{N_{\rm int}} \qquad j = 1, 2, \ldots, N_{\rm int}$$





Duality

Asymptotic behaviour power law

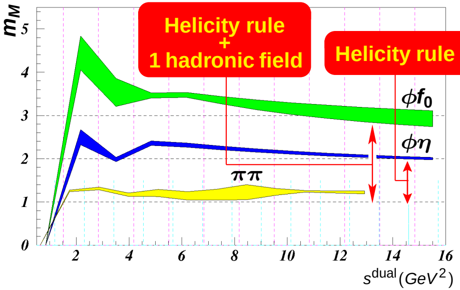
$$F_H^2(s) \propto \frac{1}{s^{2m}} \qquad m = n_H + n_\lambda - 1$$

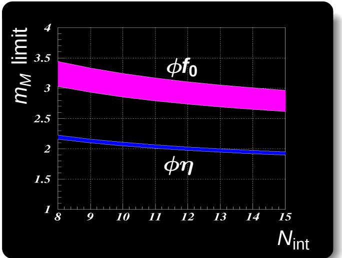
$$\lim_{s\to\infty} -\frac{\ln[F_H^2(s)/F_H^2(s_{\rm ref})]}{2\ln[s/s_{\rm ref}]} = m$$

$$m_M = -\frac{\ln(F_{M,j}^{2,{\rm dual}}) - \ln(F_{M,1}^{2,{\rm dual}})}{2[\ln(s_j^{\rm dual}) - \ln(s_1^{\rm dual})]}$$

$$F_{M,j}^{2,{\rm dual}} = \frac{1}{\Delta s}\int_{s_j^{\rm dual}-\Delta s/2}^{s_{j+1}^{\rm dual}+\Delta s/2} F_M^2(s)ds$$

$$s_j^{\rm dual} = s_0 + \left(j - \frac{1}{2}\right)\frac{\Delta s}{N_{\rm int}} \qquad j = 1, 2, \ldots, N_{\rm int}$$





Conclusions

- A general procedure, based on analyticity and annihilation cross section data, has been defined to obtain, for a generic ϕM transition form factor (where M is a pseudoscalar or a scalar light meson) an expression valid for all values of q^2.
- By means of this procedure, using as input the *BABAR* data for $\sigma(e^+e^- \to \phi\eta)$ and considering only the ϕ-like contributions to the transition form factor, we achieved an estimate of the branching ratio $BR(\phi \to \eta\gamma)$ in agreement with the PDG value.

- The same procedure, once repeated for the ϕf_0 transition form factor, with the *BABAR* data for $\sigma(e^+e^- \to \phi f_0)$ as input, and considering the f_0 as a $q\bar{q}$ bound state, gives a prediction for $BR(\phi \to f_0\gamma)$ 30 times lower than the PDG value.

- The procedure has been upgraded in order to account for a $[qq][\overline{qq}]$ f_0
  - a faster vanishing asymptotic behaviour ($\Rightarrow$ lower values at $q^2 = 0$);
  - additional contributions ($\Rightarrow$ enhancement at $q^2 = 0$).
- In this new framework, we get: $BR(\phi \to f_0\gamma) = (1.9 \pm 0.3) \cdot 10^{-4}$.
  The discrepancy with respect to the PDG value, is reduced to a factor of only 2.
  Agreement is obtained with a recent model-independent reanalysis of *KLOE* data.
- The power law asymptotic behaviour, that in light of the duality quark-hadron, appears also at low energy, confirms the 4-quark structure of the f_0 as explanation of the high value of $BR(\phi \to f_0\gamma)$ and the expected hadronic helicity flip.