High Energy Neutrino Astrophysics – A. A. 2020-2021

(20 hours) – Preliminary program

- Introduction to Cosmic rays Physics.
- Complementarity between the study of Cosmic Rays events/properties and elementary particle physics at accelerators.
- Differential energy flux and mass composition of primary Cosmic Rays. Flux of "secondary C. R." due to the interaction of primary C. R.
- Transport equations of primary and secondary Cosmic Rays in the atmosphere. Development of hadronic and electromagnetic showers in the atmosphere.
- Ultra High Energy Cosmic Rays: the measurements and their implications.
- Propagation of Ultra High Energy Cosmic Rays in the Universe: the case for protons, photons, neutrinos, heavier nuclei. The Greisen-Zatsepin-Kuzmin cut-off.
- The origin of Ultra High Energy Cosmic Rays, the possible acceleration mechanisms, first and second order Fermi acceleration mechanism.
- Experimental techniques for the observation/study of primary Cosmic Ray fluxes (protons, photons, heavy nuclei, neutrinos) up to energies $>10^{22}$ eV: experiments in the space, in the atmosphere, at ground, deep underground.
- Open problems in particle and astroparticle physics: direct and indirect search for dark matter, matter-antimatter asymmetry, neutrino properties (DAMA, XENON, AMS, PAMELA, FERMI, DAMA, CUORE, IceCube, ANTARES, KM3NeT, ...)
- Astrophysics with High Energy photons: experimental techniques and results (HESS, MAGIC, VERITAS, CTA, LHAASO ...)
- Astrophysics with High Energy neutrinos (IceCube, ANTARES, KM3NeT, ...)
- Astrophysics with High Energy protons and nuclei ($E>10^{17}$ eV): experimental techniques and results: AGASA, HiReS, Telescope Array, The Pierre Auger Observatory, TUNKA.

Basic Bibliography

- **Multiple Messengers and Challenges in Astroparticle Physics**, Aloisio R., Coccia E., Vissani F., Capone A, Lipari P., et al, Springer, Cham. https://doi.org/10.1007/978-3-319-65425-6_4, (2018);
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- M. Spurio, **Particles and Astrophysics, astronomy and Astrophysics Library,** Spinger, 2014.
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- Donald Perkins, "**Particle Astrophysics**" (Oxford University Press)
- H. V. Klapdor-Kleingrothaus, K. Zuber , "**Particle Astrophysics**" (Institute of Physics Publishing Bristol and Philadelphia)
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- S. Yoshida and H. Dai, J. Phys. G: Nucl. Part. Phys. 24 (1998) 905
- F. Halzen and D. Hooper, Rep. Prog. Phys. 65 (2002) 1025
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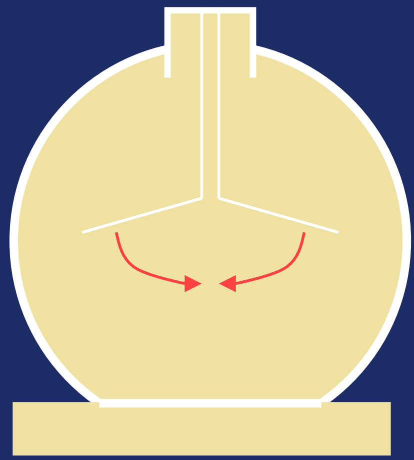
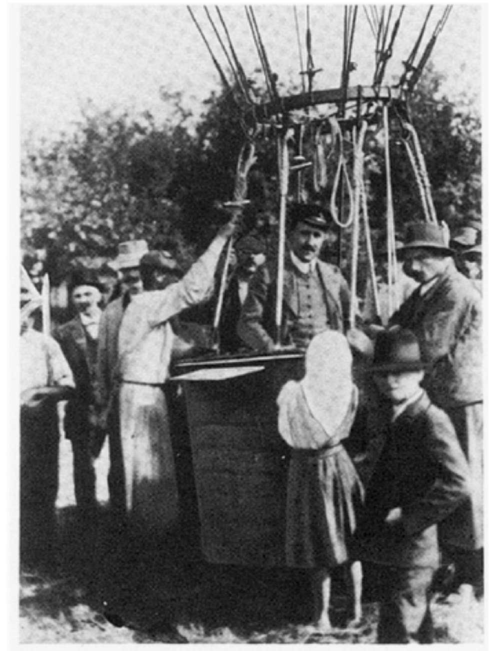
Lessons 1 and 2

- Description of the course program and of the bibliography.
- Primary Cosmic Rays (C.R.) intensity, energy spectrum and composition.
- A power law can describe the C.R. energy spectrum.
- Energy density in C.R., in the Microwave Cosmic Background Radiation (MCBR), in the Galactic Magnetic field.
- C.R. composition: relative abundance of elements in the Earth (Solar System) and on the C.R..
- Propagation time of C.R. in our Galaxy.
- The Leaky Box Model and the spectral index of CR at Earth.

The first evidence of the C.R. existence

The first instrument used to demonstrate the existence of C.R. was the "Gold-leaf electroscope" used by Victor Hess in 1911-1912:





Hess bei Ballonlandung (1912).

Fig. 2. Victor Hess (in the middle) and his crew in the balloon gondola after the landing in Pieskow.

From the observation that an electroscope, previously charged, loses its charge, it was already supposed the existence of a "radiation" originated "at Earth". He observed that the discharge speed of the electroscope was decreasing, while moving up, for a distance from the ground up to 1 km, then was increasing with the height (up to 5.3 km):

$\Rightarrow$ this radiation was not due to processes on the ground but was penetrating the atmosphere from outer space. His discovery was confirmed by R.A. Millikan in 1925, who gave the radiation the name "Cosmic Rays".

Cosmic Rays flux intensity

Let's define "directional intensity" of particles of a given type the number of particles that cross the surface dA, in the time dt within a solid angle $d\Omega$:

$$I(\theta,\varphi) = \frac{dN}{dA\, dt\, d\Omega}\ [part./cm^{-2}s^{-1}sr^{-1}]$$

Often we refer to the energy integrated flux intensity: $\int_{Emin}^{Emax} E\, \frac{dN}{dE} dE$

It is defined "integrated intensity" (or "omnidirectional") the quantity

$$\phi = \int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I(\theta,\varphi)cos(\theta)d\Omega \quad [part./cm^{-2}s^{-1}].$$

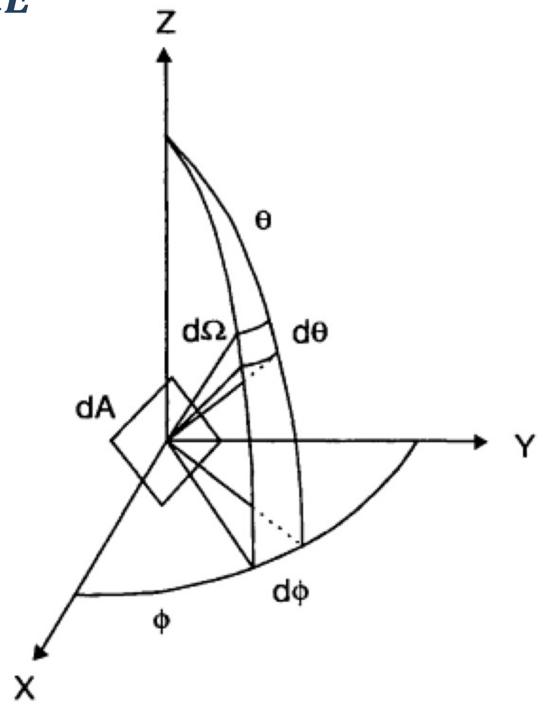
We also define $\bar{\phi}$ the number of particles that cross (down-going) the horizontal unit surface dA, in the unit time dt:

$$\bar{\phi} = \int_{\theta=0}^{\pi/2}\int_{\varphi=0}^{2\pi} I(\theta,\varphi)d\Omega = \int_{\theta=0}^{\pi/2}\int_{\varphi=0}^{2\pi} I(\theta,\varphi)sin(\theta)d\theta d\varphi =$$

that gives

$$= 2\pi \int_{\theta=0}^{\pi/2} I(\theta)sin(\theta)d\theta\ [part./cm^{-2}s^{-1}]$$

if the particles flux does is not varying with the azimuth angle.



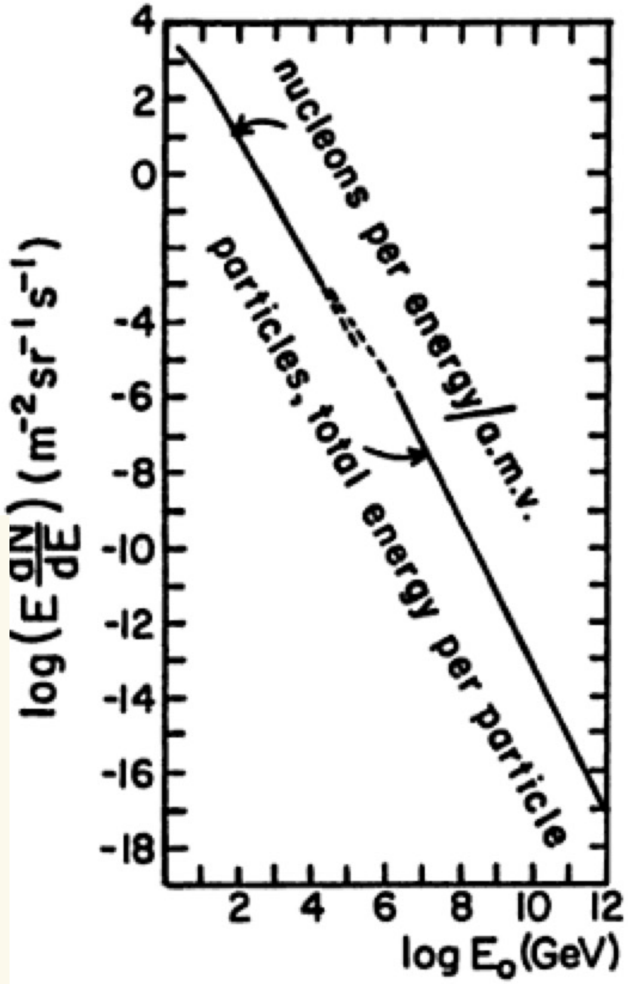
Cosmic Rays energy spectrum

The differential energy spectrum $\boldsymbol{dN/dE}$ has dimensions $[particles/(m^2\ s\ sr\ GeV)]$.

Given the high variability of $N(E)$, as a function of E, is often preferred the quantity $dN/d(logE)$.

But $\boldsymbol{d\log E = dE/E}$ then

$$\frac{dN}{d\log E} = E\frac{dN}{dE} = E\phi(E) \quad [particles/(m^2\ s\ sr)].$$

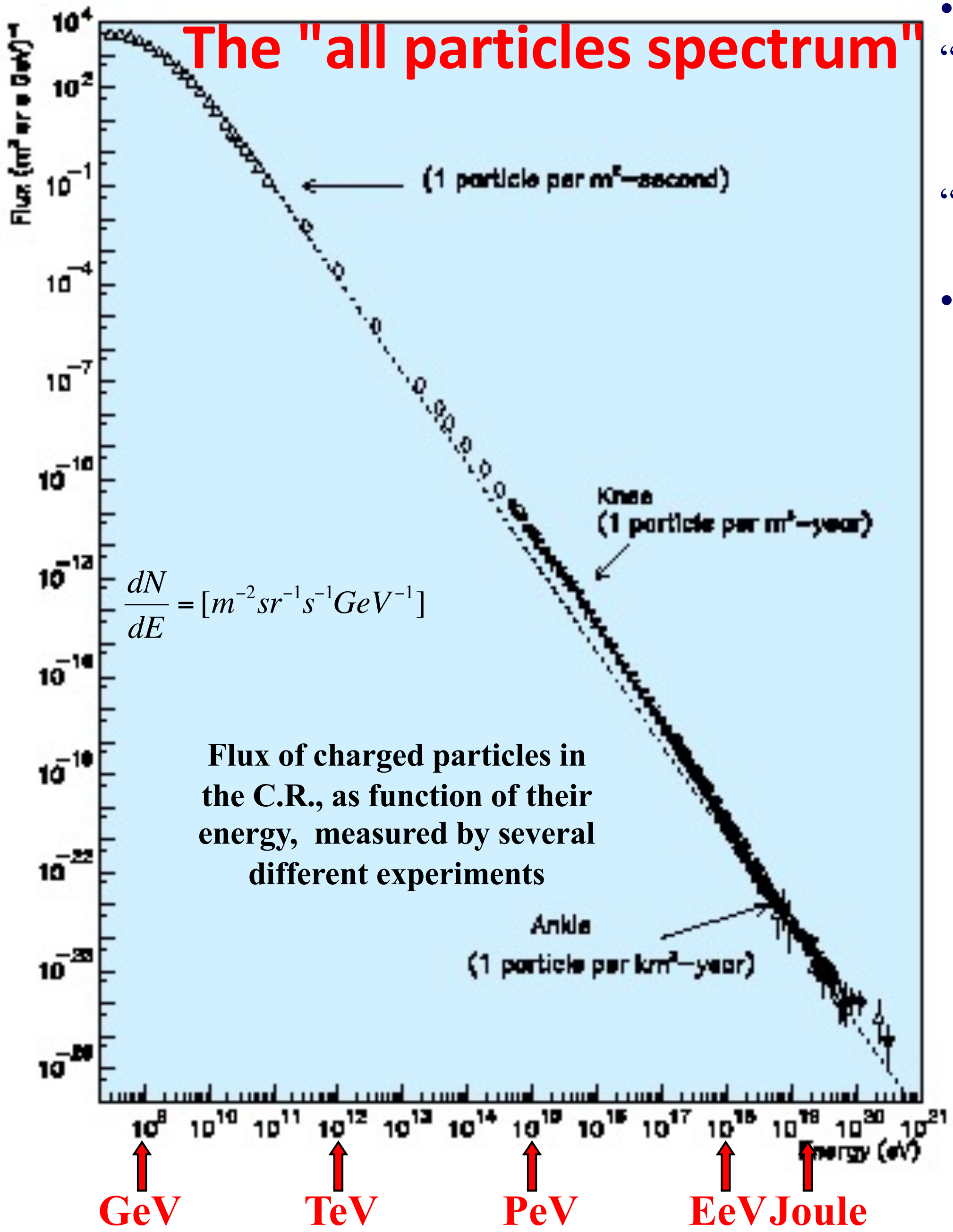


All C.R. spectra show show a fast decrease of the particles flux for increasing energy. As we will see the C.R. differential flux can be expressed by a power law :

$$\frac{dN}{dE} \propto E^{\alpha}$$

10 GeV < E < 1 PeV (10^{15} eV)	α= - 2.7
10 PeV < E < 1 EeV (10^{18} eV)	α= - 3.1
E > 10 EeV	α= - 2.6

Charged Cosmic Rays Energy spectrum



- ~ 1000 particles/(s·m²)

"primary" Cosmic Rays:

86% protons, 11% α particles, 1% heavier nuclei (up to Uranium), 2% electrons

"secondary" Cosmic Rays (or atmospheric C.R.):

e^+, e^-, p, p, light nuclei and anti-nuclei ?

- which is the origin of primary Cosmic Rays ?
  - Originated in the solar System ? Only a small fraction (at low energies, < 10 GeV), characterized by a strong variability, with time, due to violent phenomena in the Sun
  - Originated in the Galaxy ? Yes, a large fraction (>90%). The flux of these C.R. is also anti-correlated with the most intense solar activities
  - Extragalactic ? Yes, the main component of the most energetic part of the spectrum

Primary C.R. flux and composition, for $E_{CR} < 10^{14}$ eV, can be directly measured

in the space (satellites)

in the top part of the atmosphere (baloons)

For $E_{CR} > 10^{15}$ eV → indirect measurement

Extensive shower in the atmosphere

- using Cherenkov or Fluorescence det. in air,
- using detectors on the Earth ground

In underground laboratories (neutrino detection)

Different nuclei fluxes and energy spectra

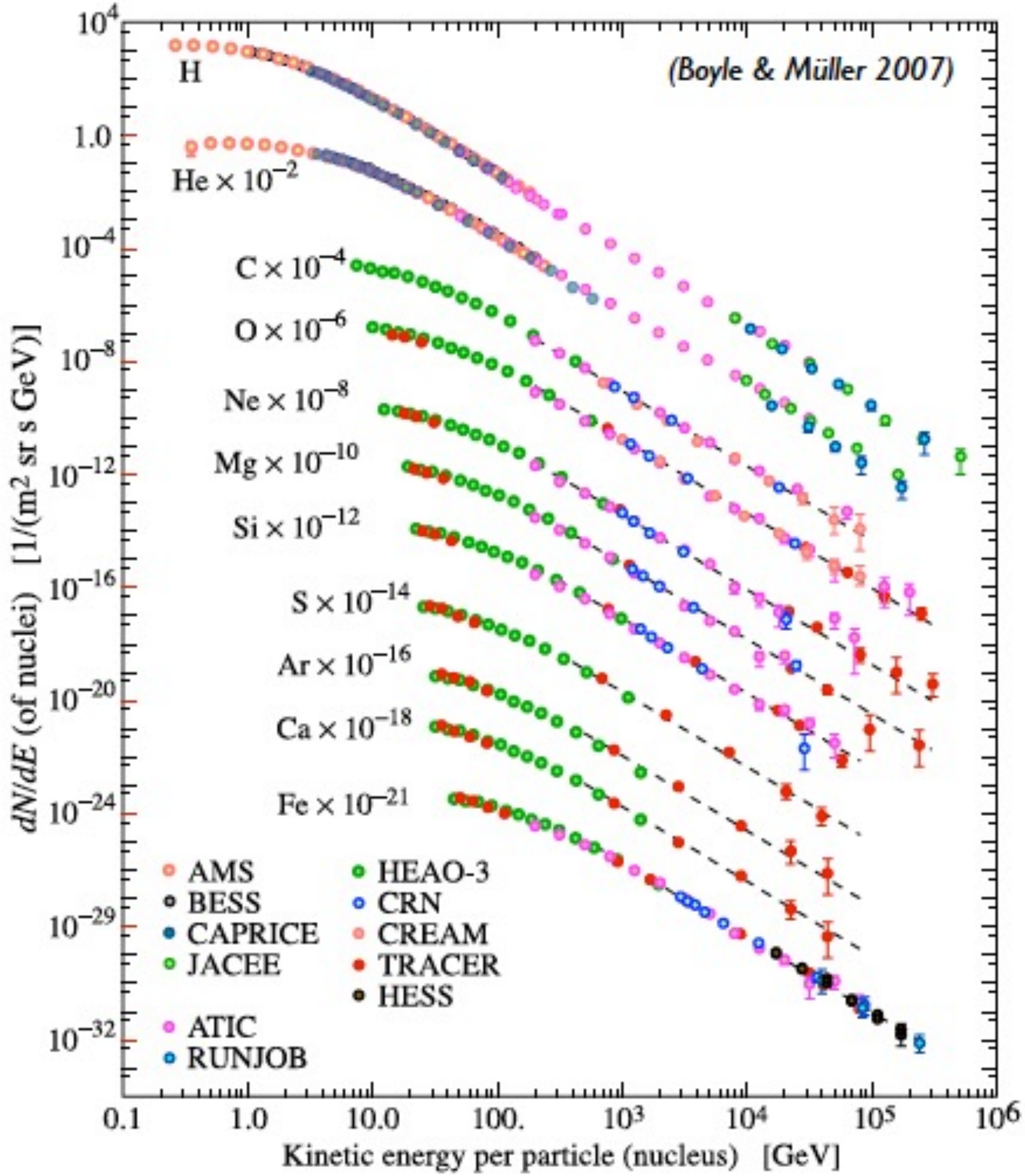
Power law also found for individual elements

Index of power law almost identical (heavier elements have slightly harder spectra)

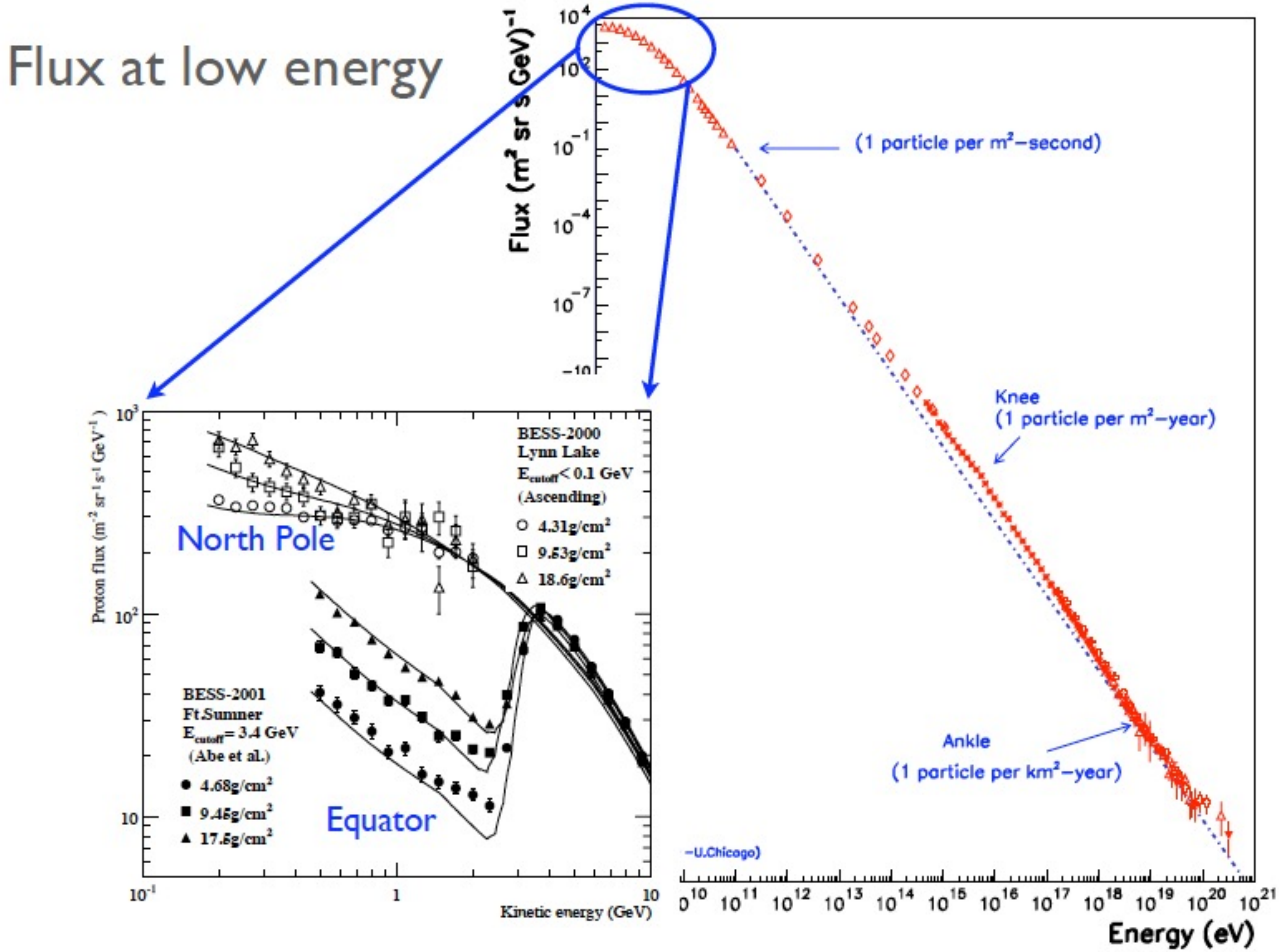
Relative abundance of nuclei

H : He : Z= 6-9 : 10-20 : 21-30

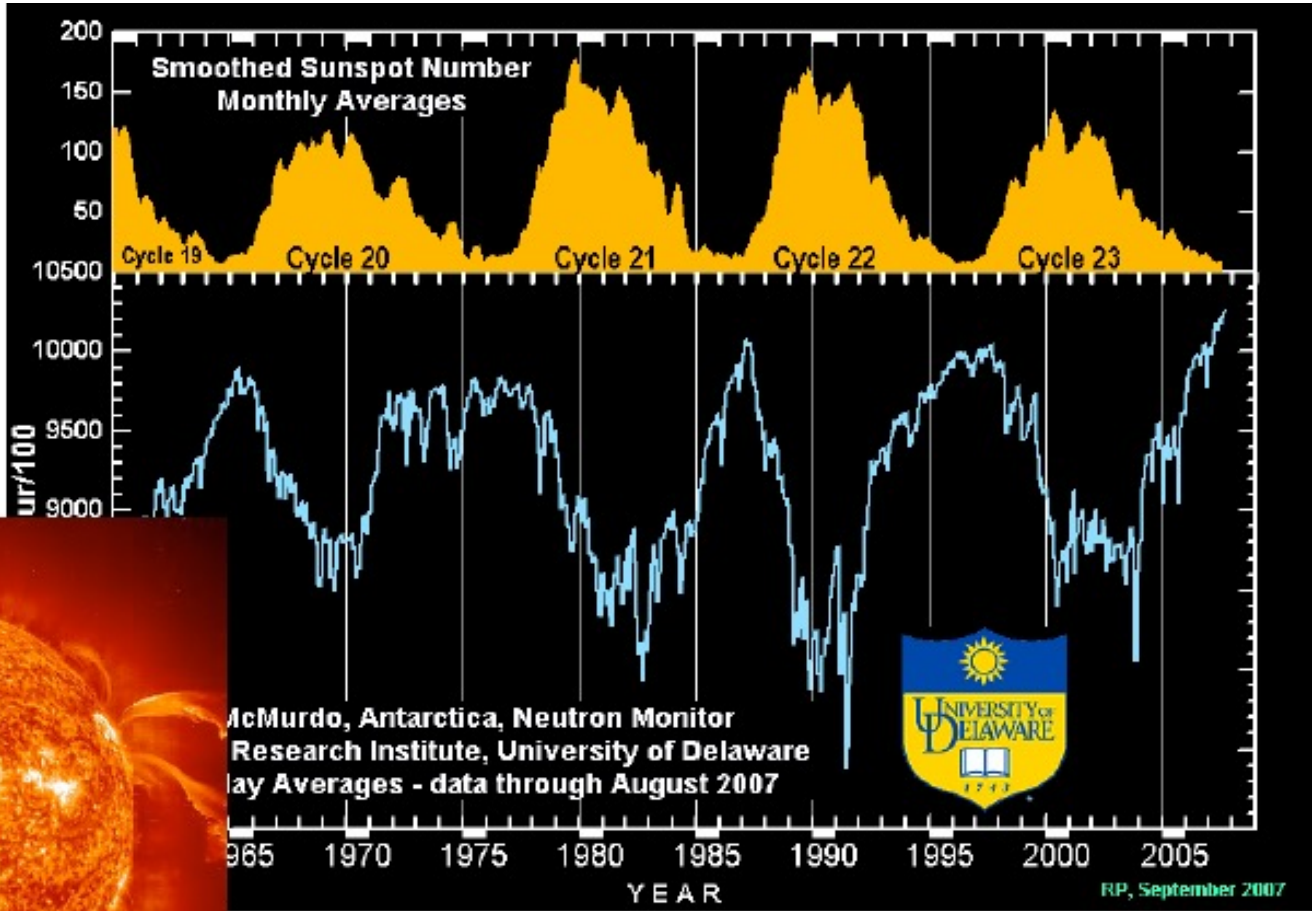
1 : 0.38 : 0.22 : 0.15 : 0.4



For E<10 GeV solar and Earth magnetic fields affect the C.R. spectra



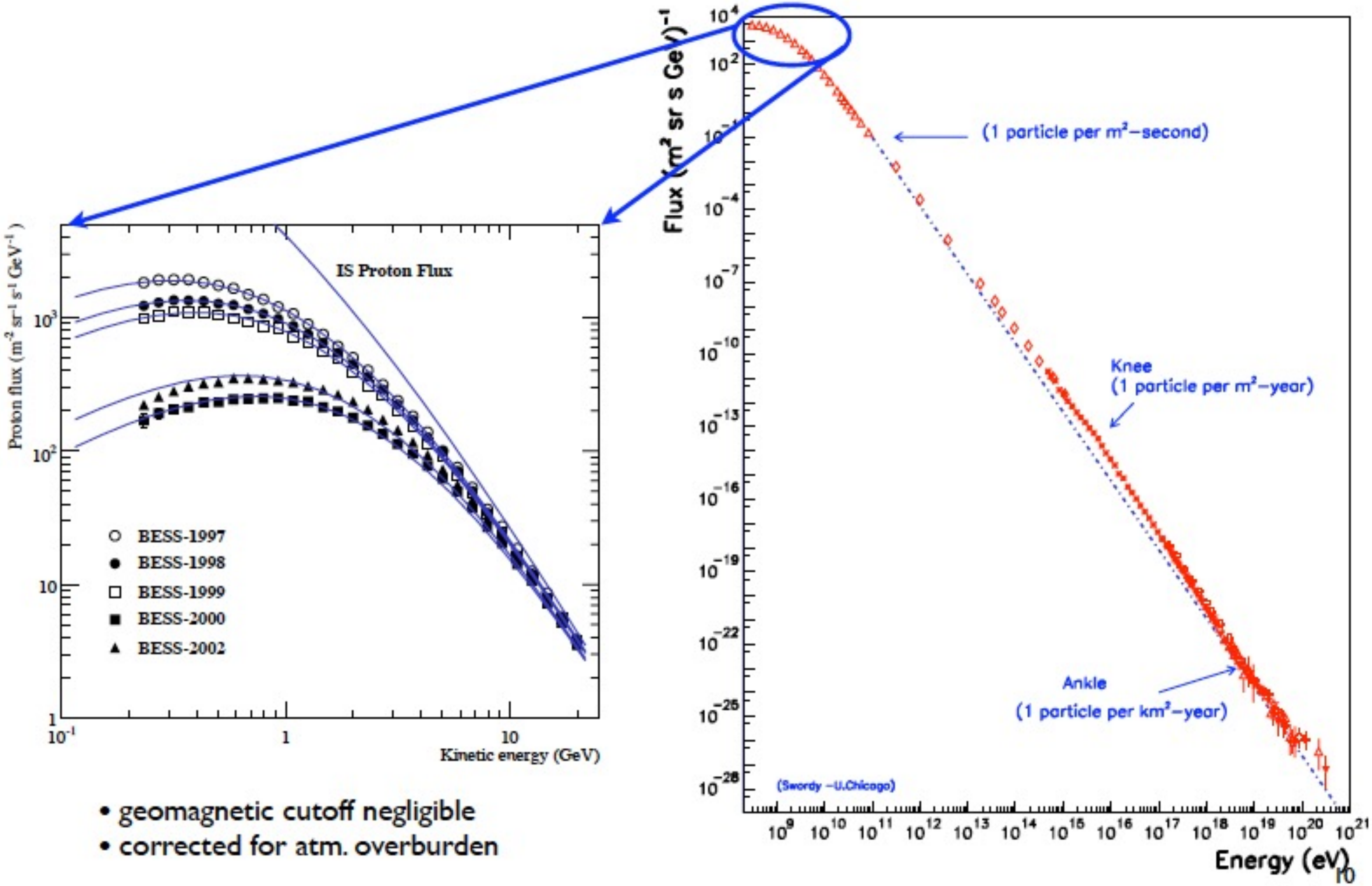
C.R. flux intensity and »solar activity» are anti-correlated: the solar magnetic field effect



Flux of cosmic rays at poles

Differential rotation of sun: reversal of mag. field every 11 years (full period 22 years)

”Seasonal" variation of C.R. at the Earth pole: Earth magnetic field effect



- geomagnetic cutoff negligible
- corrected for atm. overburden

- “primary” charged C.R. composition:
  86% protons, 11% α particles, 1% heavier nuclei (up to Uranium), 2% electrons

Particle energy can be evaluated from the measurement of their track deflection in a magnetic field $\vec{B}$.
Let’s recall: the Lorentz force that acts on a particle with charge ***e*** that moves with velocity $\vec{v}$ in the magnetic field $\vec{B}$ is given by:

$$\vec{F} = e\frac{\vec{v}}{c} \times \vec{B}$$

In the time ***δt*** this force gives to a particle with charge ***Ze*** that travel for the lenght ***L***, a transverse momentum **δp**:
(let’s assume β=1). $\delta p \approx Z\ e\ B\ \delta t = Z\ e\ B\ \frac{L}{c}$

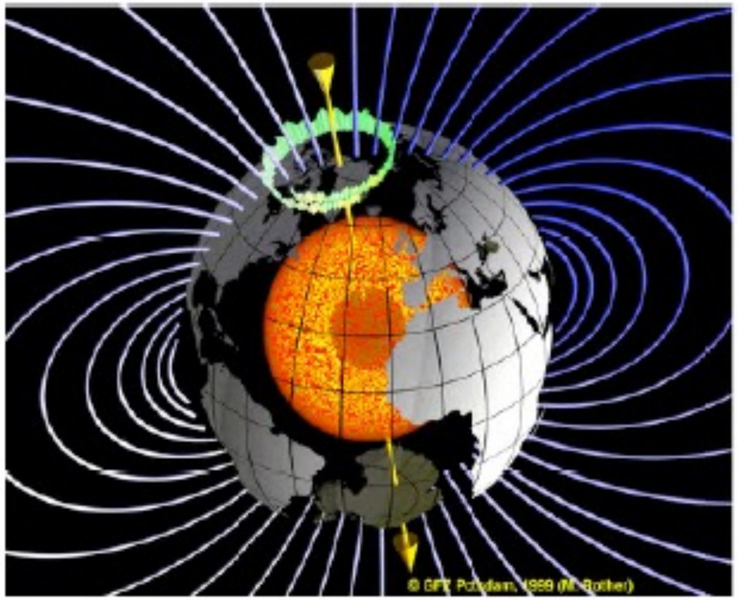
It follows $\frac{\delta p}{p} \approx Z\ e\ B\ \frac{L}{pc} \approx \frac{\delta x}{L}$ where $\delta x \approx Z\ e\ B\ \frac{L^2}{c}$ measures

the deviation of the particle trajectory (measurable in a spectrometer)
The fraction of particles indicated above, as relative contribution to the CR flux, are given for a given value of the particle:

Rigidity = pc/(Ze)

i.e. for particles that have the same probability to propagate through the atmosphere under the effect of the Earth magnetic field.

The geomagnetic "cut-off" and the East-West effect

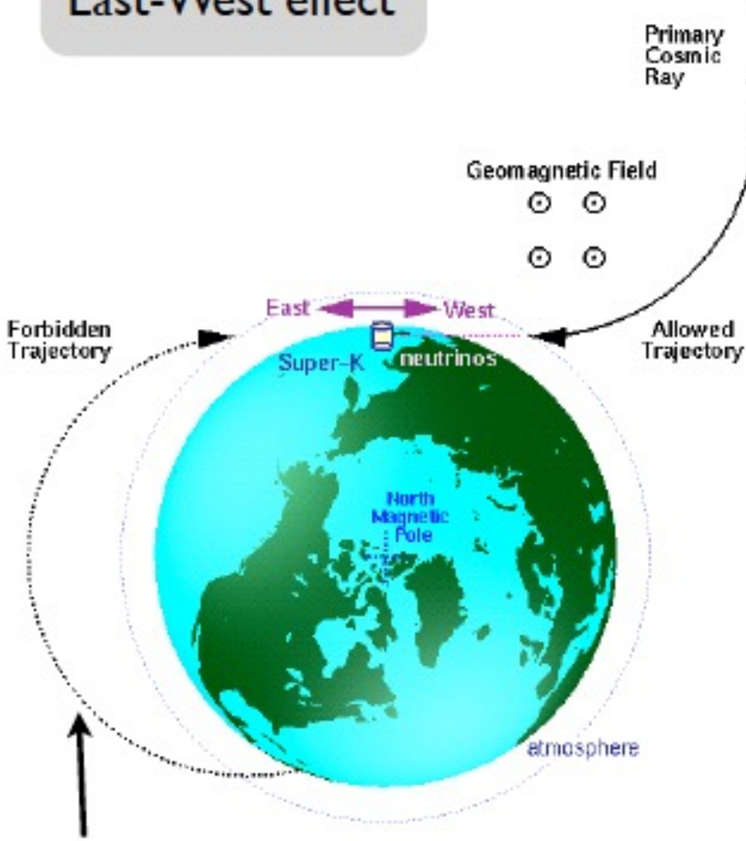


Earth's magnetic field

Vincinity of poles: B ≈ 60 μT
Equator: B ≈ 30 μT

$$R_L = 3 \times 10^3 \left(\frac{E}{\mathrm{GeV}}\right)\left(\frac{\mu\mathrm{T}}{ZB}\right) \mathrm{km}$$

East-West effect

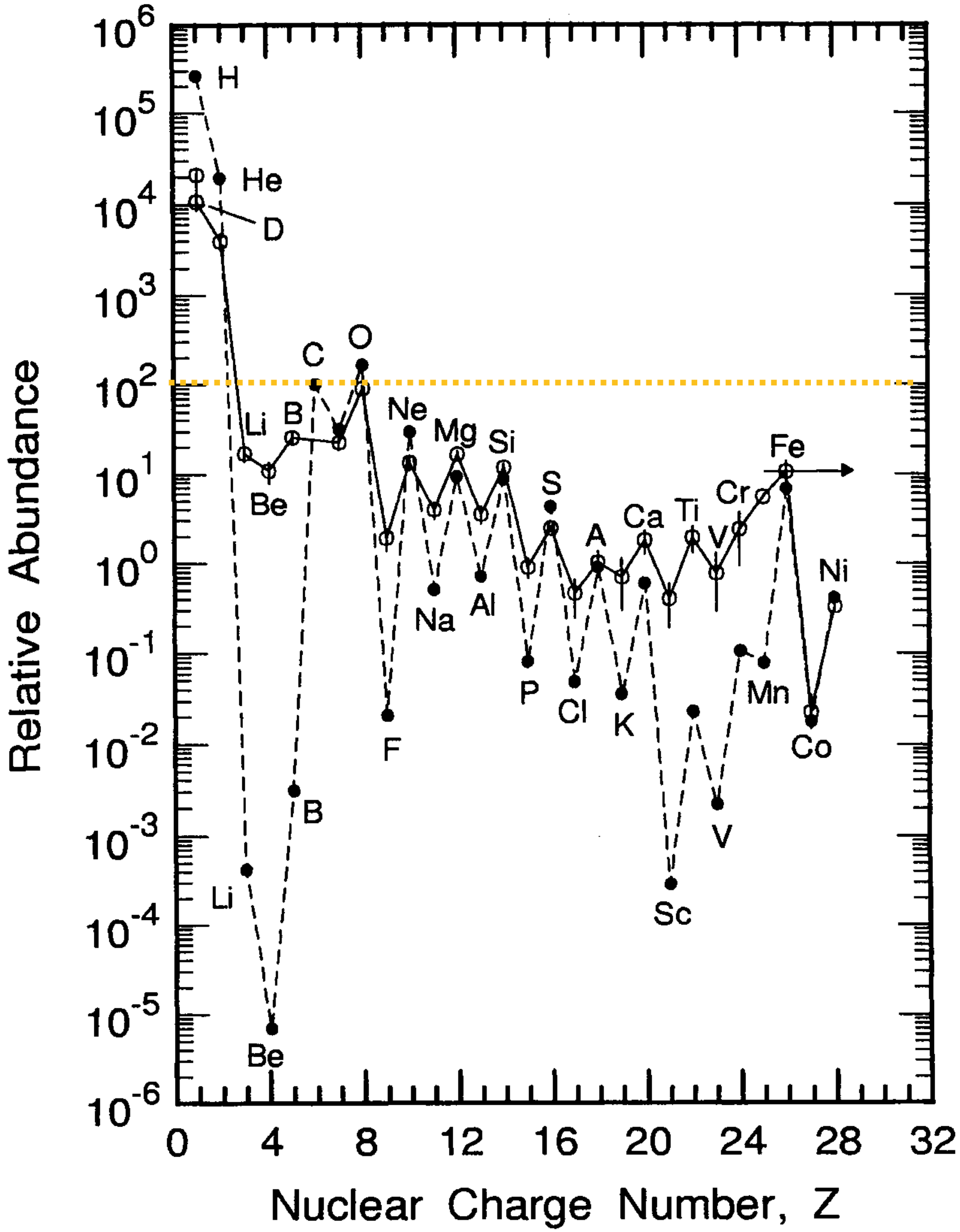


Radius of curvature
smaller than radius of Earth

Elements relative Abundance in primary C.R.

Relative elemental composition from hydrogen to nickel in the galactic cosmic radiation at > 1 GeV/nucleon kinetic energy arriving near the top of the Earth's atmosphere (o) compared to the solar system or "universal" abundances (●).

The data are **normalized to the cosmic ray carbon abundance**, set to 100 %, obtained with the satellite IMP-8 (Wefel, 1991; Simpson, 1983 and 1997).



Elements relative abundance in primary C.R.

Within C.R. heavy elements (Z>1) are present with higher relative abundance, relative to protons, if compared to the same quantity for the matter of the Solar System:

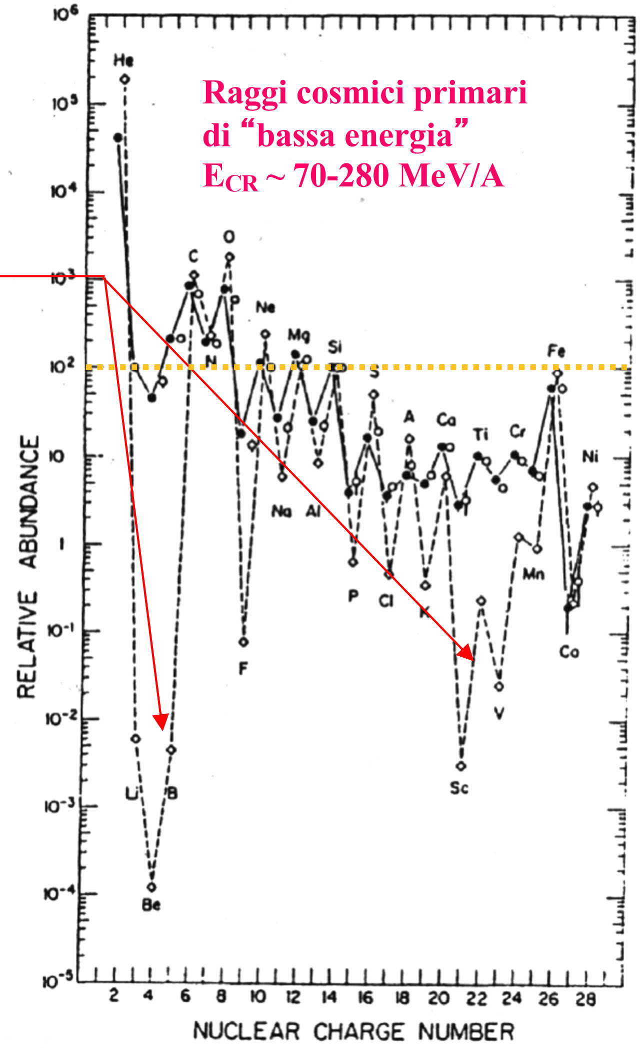
- **within C.R., with respect to what happens in the Solar System, the percentage of protons is reduced due to the higher ionization potential of H (if compared with heavier nuclei)**
- **within C.R. the elements: Li, Be, B, Sc, Ti, V, Cr, Mn are much more abundant than in the Solar System matter: these elements would be nearly absent in the final state of a stellar nucleosyntheses. Within C.R. These elements do exist as result of nuclear interaction of heavier elements present in C.R., i.e. Oxigen (C, O, Fe, …)) and Iron (Sc, V, Cr, Mn) with Inter-Stellar Medium (ISM). From this assumption we can derive information on the permanence time of C.R. in our Galaxy. Indeed we have to assume that nuclei slightly heavier (C, O, Fe, …) have the possibility to interact, i.e. that they cross an amount of matter $X = 5\text{-}10 g/cm^2$**
- **we know that the density of protons in the galactic disk is $\rho_p \sim 1$ proton/cm^3 $= 1.67 \cdot 10^{-24}$ g/cm^3**
- **then we can evaluate the length of the trajectory that C, O, Fe, … have to cross in order to explain the observed amount of B, Be, Sc, … . This length can be expressed in pc**

$$X / (m_p \cdot \rho_p) = 3 \cdot 10^{24} \text{ cm} \sim 1000 \text{kpc}$$

1pc=1AU/1 second of degree =$1.5 \cdot 10^{13}/4.85 \cdot 10^{-6}$

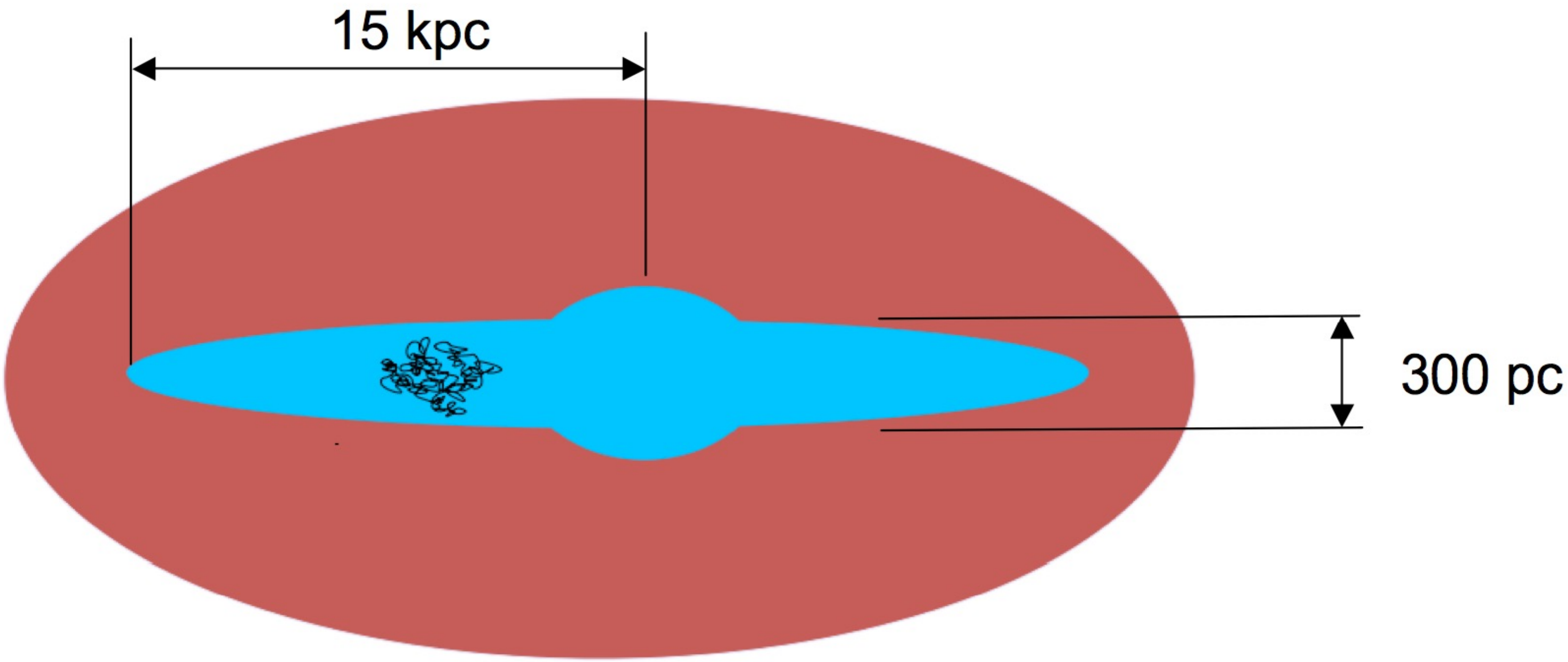
1pc=$3.1 \cdot 10^{18}$cm = 3.26 Ly

- **C.R., travelling at the light speed, remain trapped in our Galaxy per $\tau_{esc} \sim 3 \div 10$ millions of years $\sim 1 \div 3\ 10^{14}$ s**



Elements relative abundance in C.R. (He to Ni, continuous line) relative to Si (100%), compared to nuclei relative abundance in the Solar System (dashed line).

Cosmic Rays in our Galaxy



Let's recall: **$1\ pc \sim 3.1 \cdot 10^{18}\ cm$**

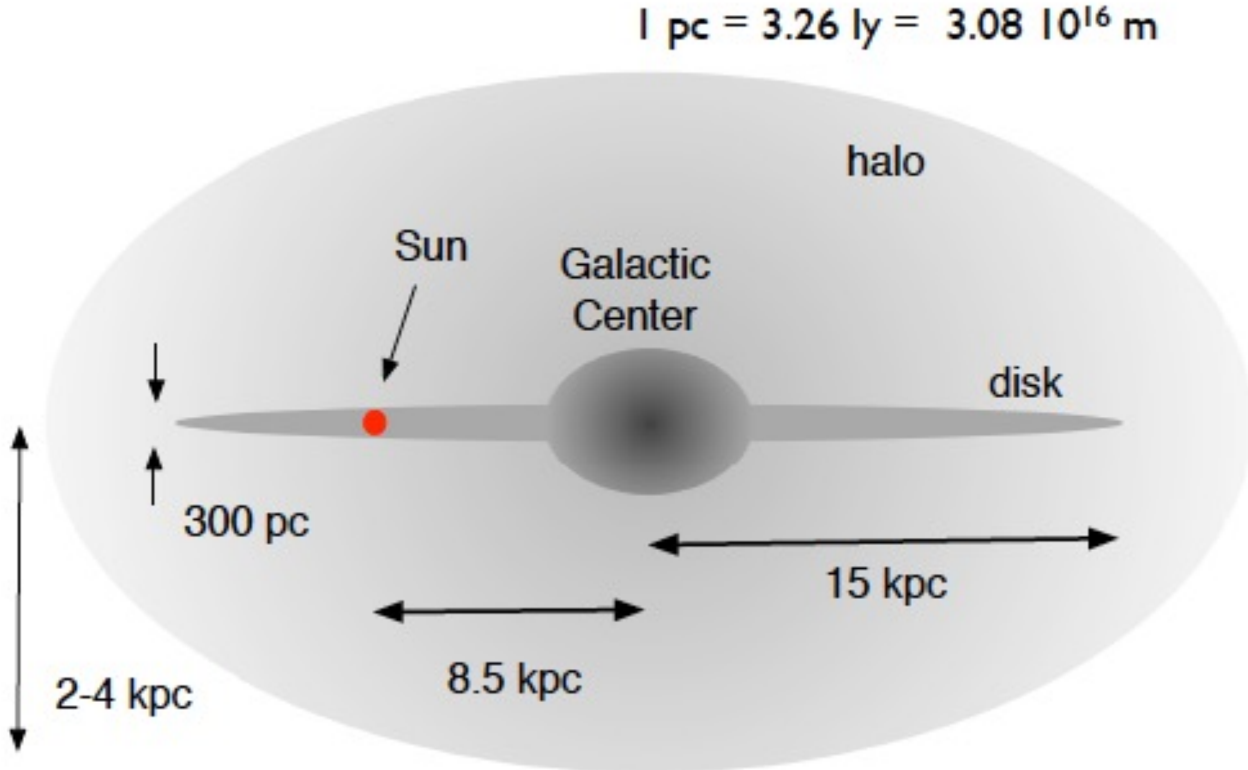
If we assume the Galaxy like a disk with radius **$r_{gal} = 15kpc \sim 45 \cdot 10^{21}\ cm$**

and height **$h_{gal} = 300pc \sim 10^{21}\ cm$** the galactic volume can be evaluated

as **$V_{disk} = \pi\ r_{gal}^2\ h_{gal} \sim 10^{67}\ cm^3$**

Our Galaxy and its galactic magnetic field

(Andromeda, M31)



$$R_L \simeq 1\,\mathrm{pc} \times \left(\frac{E}{10^{15}\,\mathrm{eV}}\right)\left(\frac{\mu\mathrm{G}}{ZB}\right)$$

Magnetic field not well known,
B = 3 μG = 30 nT close to Solar System

Diffusion: distance scales ~ (time)^2 → Extragalactic sources unlikely

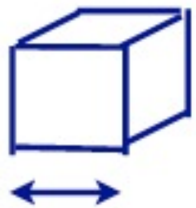
How much power is needed to justify the energy transported by C.R. ?

Assumption: entire galaxy homogeneously filled with cosmic rays

Density of particles for given flux

Isotropy $\int d\Omega = 4\pi$

$$\frac{dN}{dE\,dV} = \frac{4\pi}{c}\,\frac{dN}{dE\,d\Omega\,dA\,dt}$$



Total cosmic ray energy

$$E_{\rm tot} = \int dV \int dE\, E \cdot \frac{dN}{dE\,dV}$$

Mean escape time $\tau_{\rm esc} \approx 10^7$ *years*

C.R. energy density

Φ = C.R. flux that crosses the unit surface A

can be expressed using the "cosmic rays density"

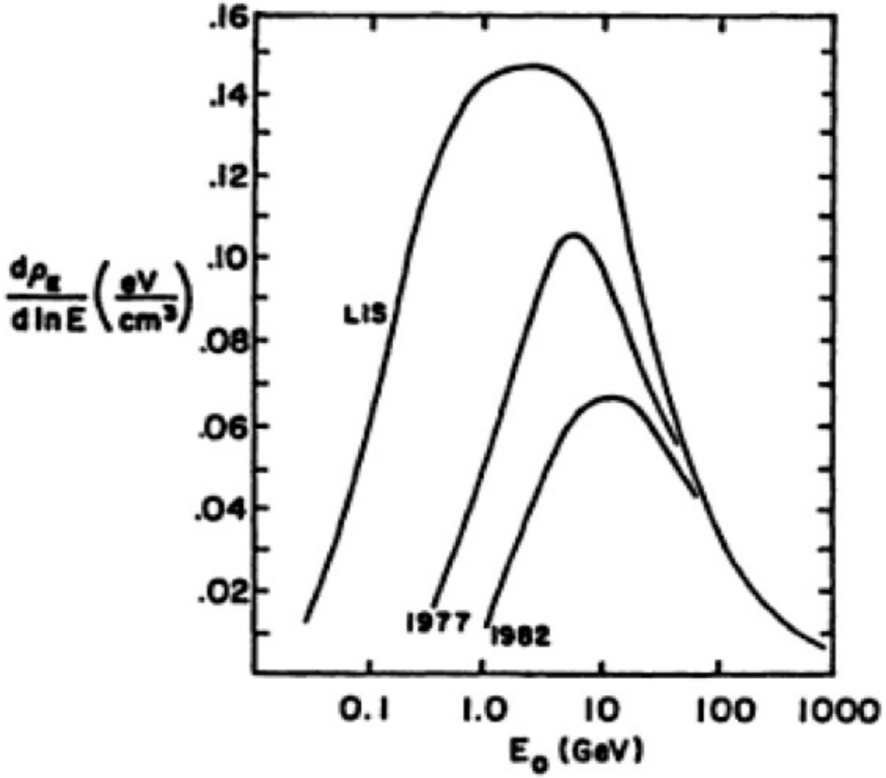
ρ_{RC}= number of particles/unità of volume

$$\Phi = \frac{\rho_{RC}\, A\, \beta c\, t}{4\pi\, A\, t}\left[\frac{particelle}{cm^2 s\, sr}\right] = \frac{\rho_{RC}\, \beta c}{4\pi}$$

In the same way we can define the energy density carried by C.R. as

$$\rho_E = \int_{Emin}^{\infty} \rho_{RC} E\, dE = 4\pi \int_{Emin}^{\infty} \frac{\Phi}{\beta c} E\, dE =$$

$$4\pi \int_{Emin}^{\infty} \frac{\frac{dN}{dE}}{\beta c} E\, dE = 4\pi \int_{Emin}^{\infty} \frac{E^2}{\beta c}\frac{dN}{dE}\, d\ln E$$



Measurement of the C.R. energy density performed in periods with different solar activity

LIS (Local Intestellar medium): protons ρ_E~0.83 eV/cm^3

Helium ρ_E~0.27 eV/cm^3

(remember 1 eV = 1.6 · 10^{-12} erg)

The median of the C.R. kinetic energy distribution in the Inter Stella Medium (ISM) is about 6 GeV: more than 90% of the C.R. kinetic energy carried by particles with E < 50GeV.

We know that the energy density carried by C.R. in the local (galactic) I.S.M. is

$$\rho_{E,RC} = \frac{4\pi}{c}\int E\frac{dN}{dE}dE \sim 10^{-12}\, erg/cm^3$$

Assuming this value for the energy density in C.R., the value

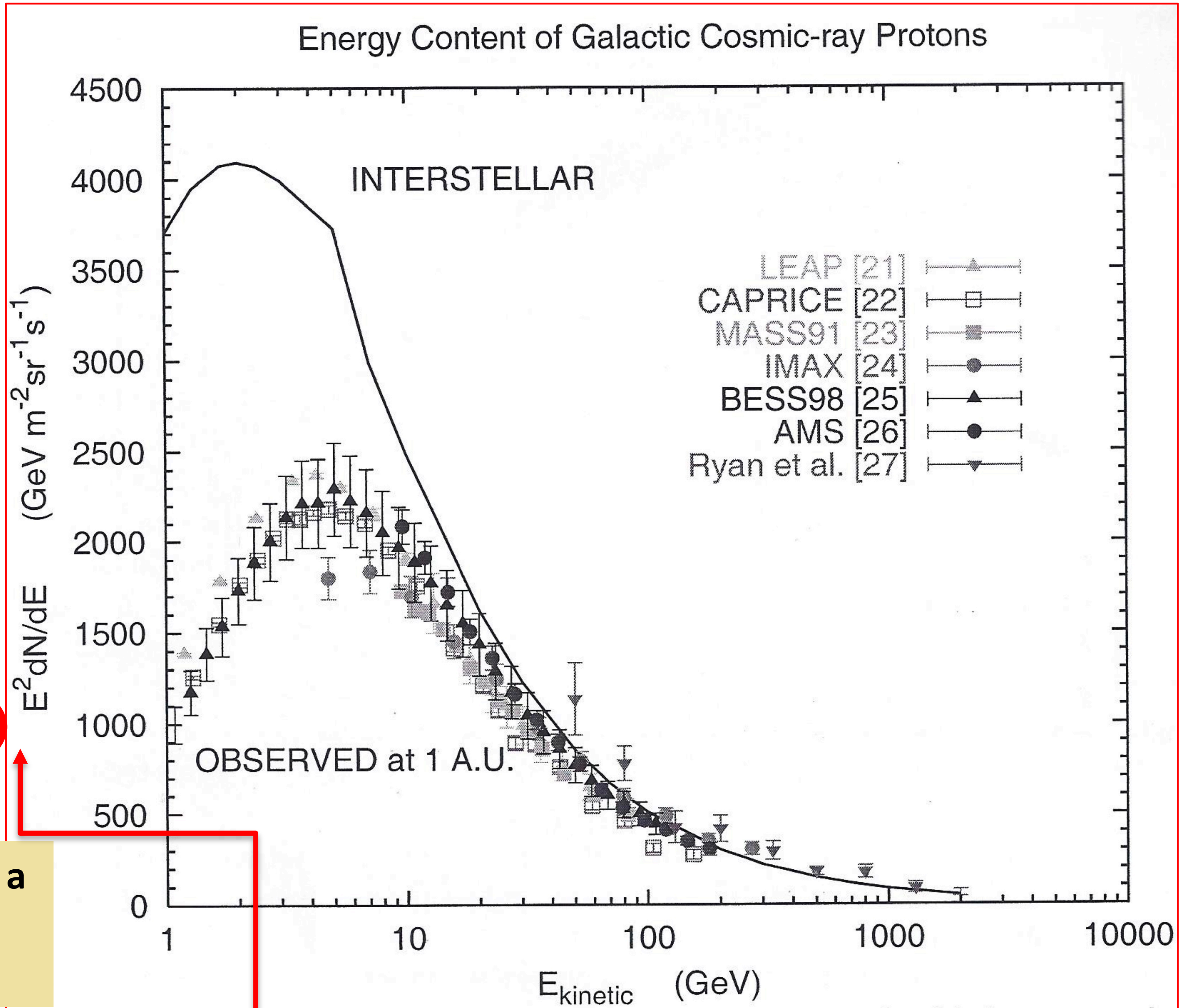
Vdisk =10^{67}cm^3

for the volume of our Galaxy and

τ_{esc} ~ 3•10^6 years ~ 10^{14} s

as the time of permanence of C.R. in the Galaxy, we can evaluate the amount of power needed to maintain such a phenomenon

$$\frac{\rho_{E,RC}V_{disk}}{\tau_{esc}} \sim \frac{10^{-12} * 10^{67}}{10^{14}} \sim 10^{41}\, erg/s$$



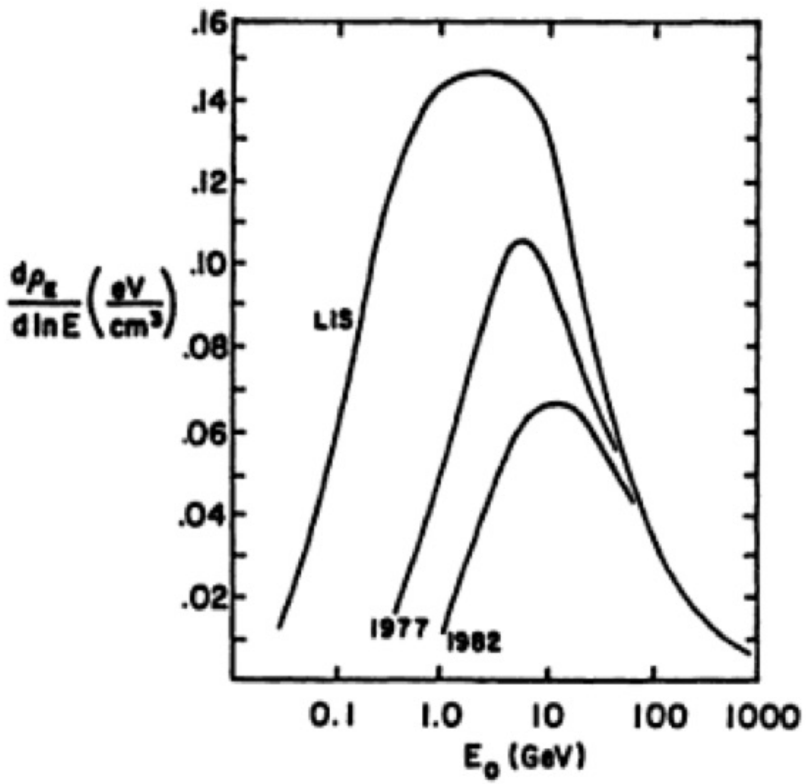
We know that a SuperNova releases a flux of particles that can carry a kinetic energy as high as a 10^{51} erg.

In our Galaxy we do expect 1 SN event each 30 years → this corresponds to an emitted "power" equivalent to 10^{51} erg / (30 •3,14•10^7) s =10^{42} erg/s (the two numbers are compatible assuming a 10% efficiency for the proton acceleration)

C.R. energy density

LIS (Local Intestellar medium):

protons ρ_E~0.83 eV/cm
Helium ρ_E~0.27 eV/cm^3



Densità di energia misurata per protoni è funzione dell'attività solare

How this energy density compares with the one associated to the galactic magnetic field B~3μGauss ???

$$(\rho_E)_B = \frac{1}{2}\frac{B^2}{\mu_0} = 0.21 \text{ eV/cm}^3$$

a calculation exercise …

A useful exercise: the calculation of the energy density associated with the Galactic magnetic field

$$(\rho_E)_B = \frac{1}{2}\frac{B^2}{\mu_0}, \quad \text{dove} \quad B = 3\ \mu G = 3 \cdot 10^{-10}\ T$$

Let's recall: $\mu_0 = 4\pi \cdot 10^{-7}\frac{N}{A^2}$, $1G = 10^{-4}T$, $1T = 1\ \frac{Wb}{m^2} = 1\ \frac{N}{A\,m}$,

therefore

$$(\rho_E)_B = \frac{1}{2}\frac{B^2}{\mu_0}\frac{Joule}{m^3} = \frac{1}{2}\frac{\left(3 \cdot 10^{-10}\right)^2}{12.56\ \cdot 10^{-7}}\frac{T^2}{NA^{-2}} = \frac{9 \cdot 10^{-20}}{25 \cdot 10^{-7}}\frac{N^2}{A^2\,m^2}\frac{1}{NA^{-2}}$$

$$= 36 \cdot 10^{-22} \cdot 10^7\frac{N}{m^2}$$

$$(\rho_E)_B = 36 \cdot 10^{-15}\frac{J}{m^3} = 36 \cdot 10^{-21}\frac{J}{cm^3}$$

then recalling that $1\ eV = \ 1.6 \cdot 10^{-19} J$ we have:

$$(\rho_E)_B = \frac{36 \cdot 10^{-21}}{1.6 \cdot 10^{-19}} = 0.21\frac{eV}{cm^3}$$

One other calculation …

A useful exercise: the calculation of the energy density associated with the Cosmic Microwave Background (CMBR) radiation field

Let's recall: the density of photons in the Universe is ~ 411 photons/cm^3

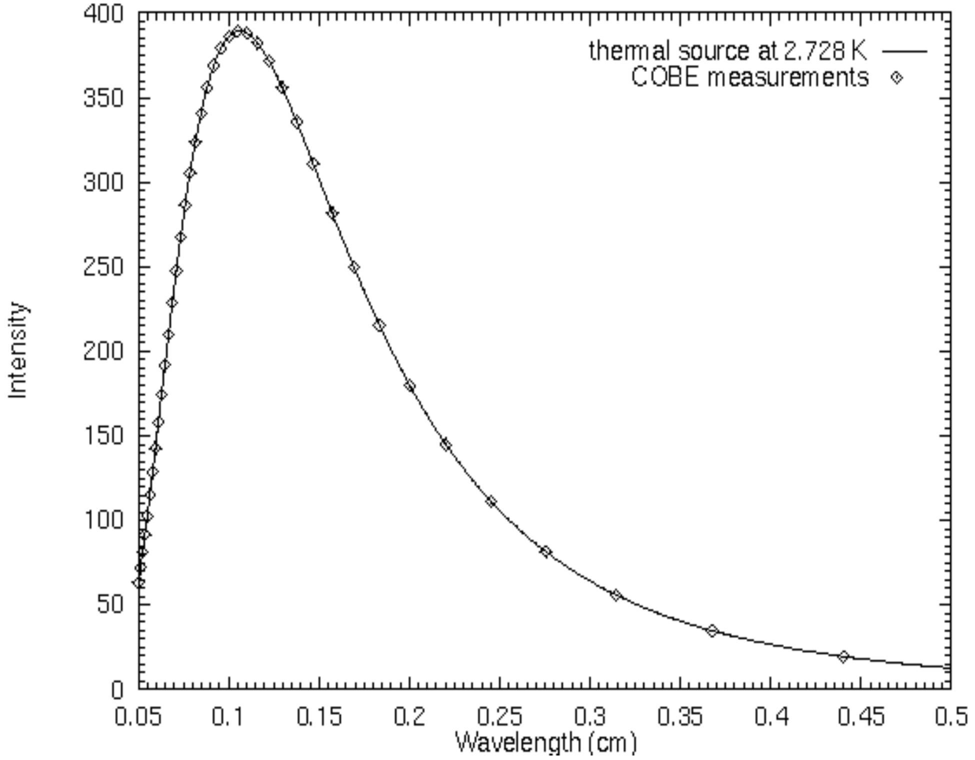
The CMBR can be represented as a black body with T=2.725 K

$kT=(8.617 \cdot 10^{-5}\, eV/K) \cdot 2.725\, K = 2.35 \cdot 10^{-4}\, eV$

But: $E_{peak} = 2.70 * k * 2.725 = 6.34 \cdot 10^{-4}\, eV$

$E_{mean} = 2.82 * k * 2.725 = 6.62 \cdot 10^{-4}\, eV$

let's use for these photons an energy in the high energy tail $E_g = 1.4 \cdot 10^{-3}\, eV$

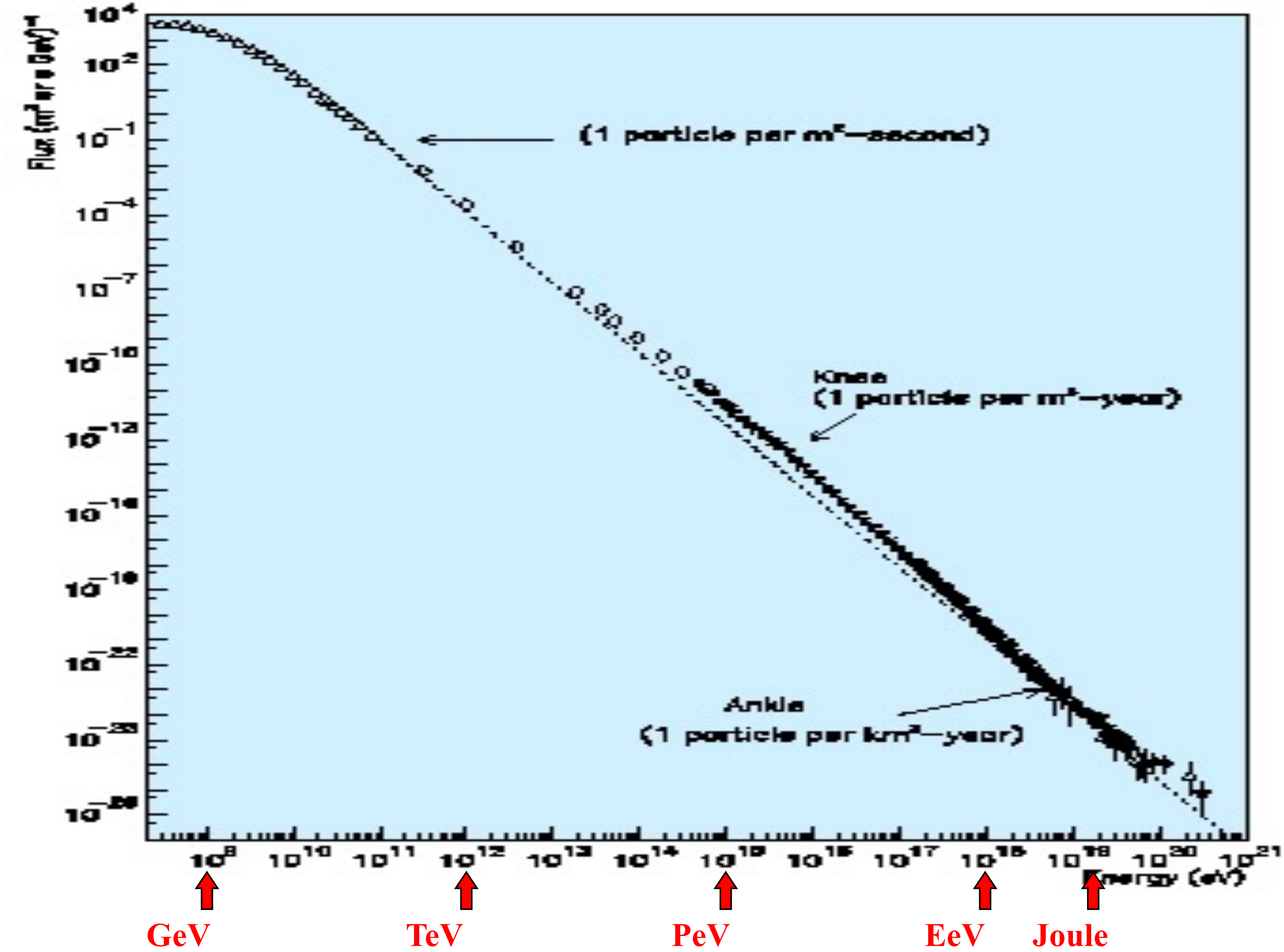


we have:

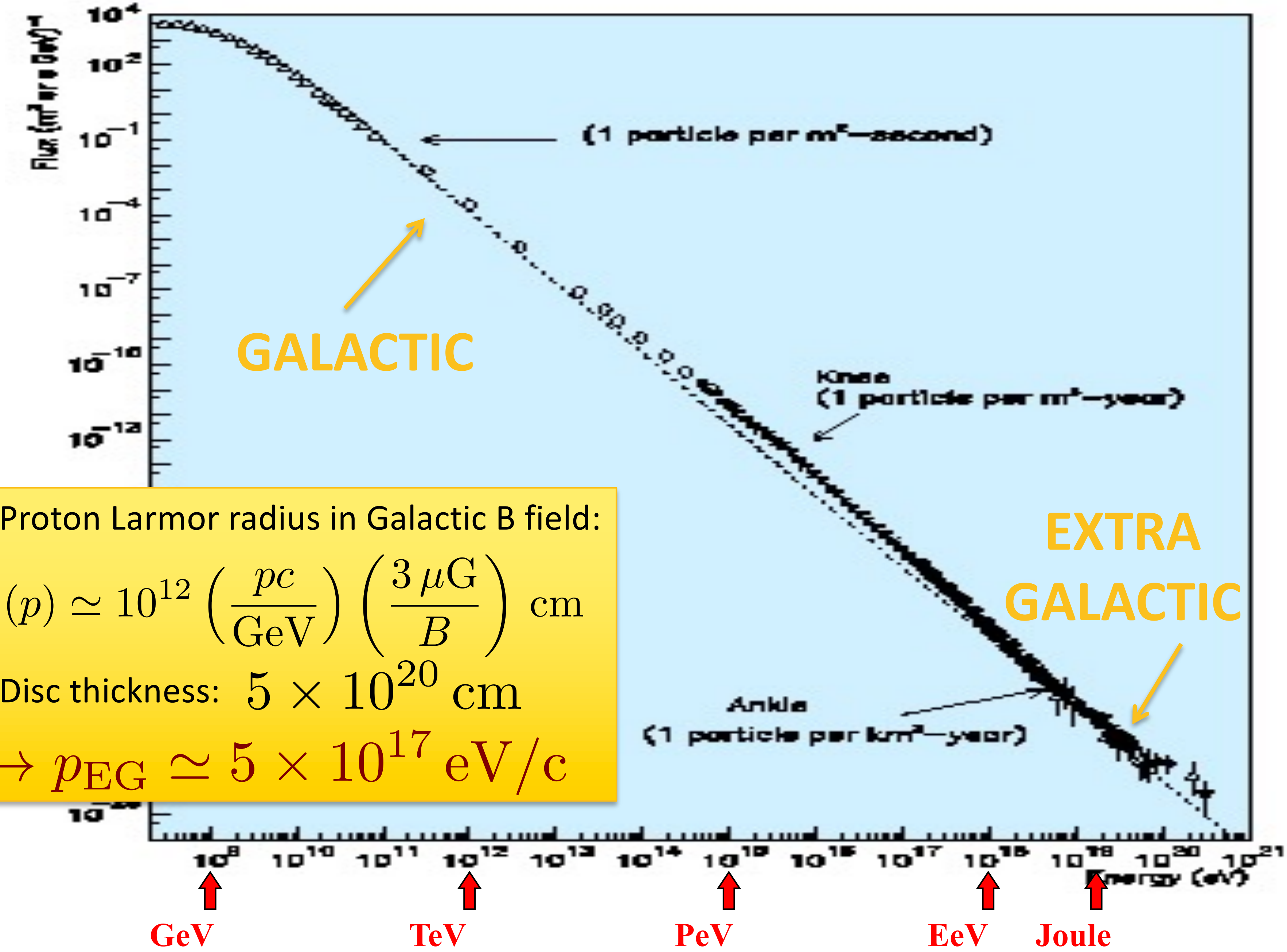
$$\langle \rho_{CMBR} \rangle = 411 * 1.4 * 10^{-3}\, \frac{eV}{cm^3} = 0.58\, \frac{eV}{cm^3}$$

- C.R. composition: relative abundance of elements in the Earth (Solar System) and on the C.R..
- Propagation time of C.R. in our Galaxy.
- Measurement of the C.R. electromagnetic component: primary photons and electrons
- How to detect photons and electrons ?
- Main characteristics of apparatuses on atmospheric balloons, main results.
- The Compton Gamma Ray Observatory and its components: CGRO, BATSE, Comptel ed EGRET.
- Gamma Ray Bursts.
- CGRO Sky Map.
- ”FERMI”: the detector and and obtained results.

The all particle spectrum…



The all particle spectrum…



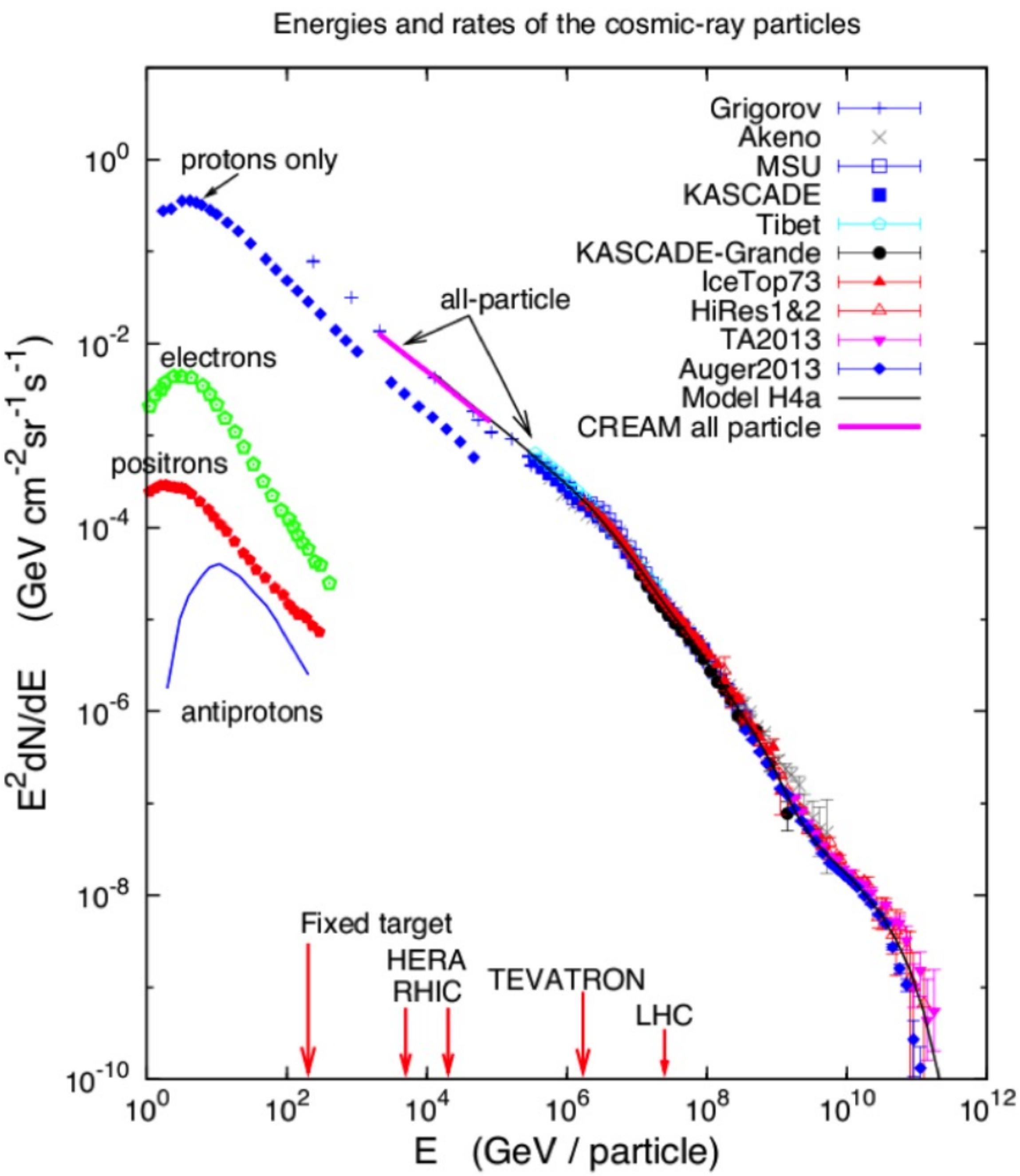
- Proton Larmor radius in Galactic B field:

$$r_L(p) \simeq 10^{12} \left(\frac{pc}{\mathrm{GeV}}\right) \left(\frac{3\,\mu\mathrm{G}}{B}\right) \mathrm{cm}$$

- Disc thickness: $5 \times 10^{20}\,\mathrm{cm}$

$$\longrightarrow p_{\mathrm{EG}} \simeq 5 \times 10^{17}\,\mathrm{eV/c}$$

…and its individual components



The cosmic-ray observables

1. **Spectral energy distribution**: which physical mechanisms generate an almost featureless power law?

2. **Mass composition**: abundances + primaries/secondaries cosmic rays;

1. **Spatial distribution across the sky**: almost isotropic sky map of CRs.

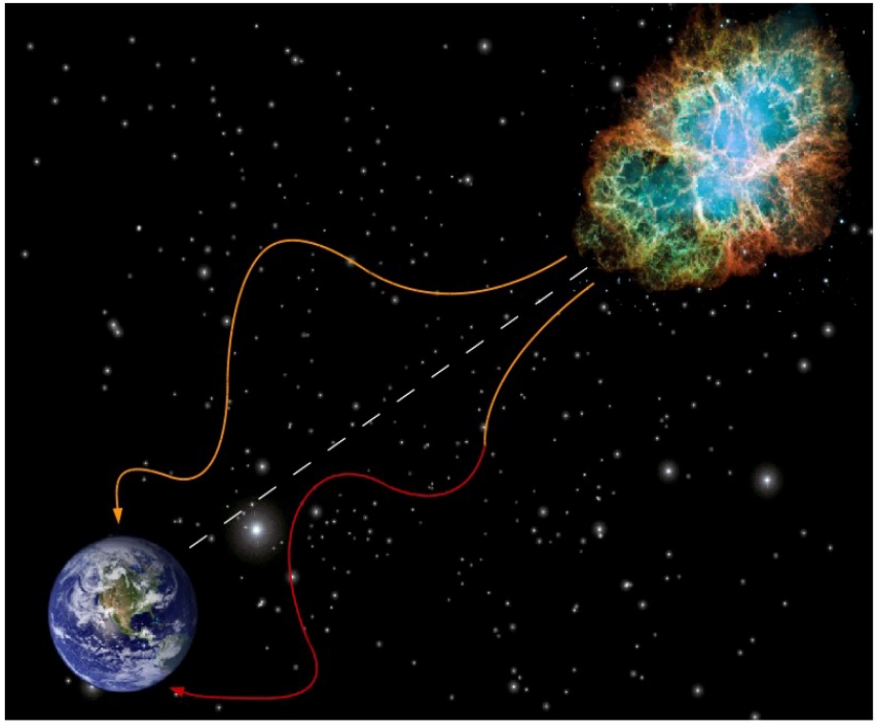
The path to become a cosmic ray

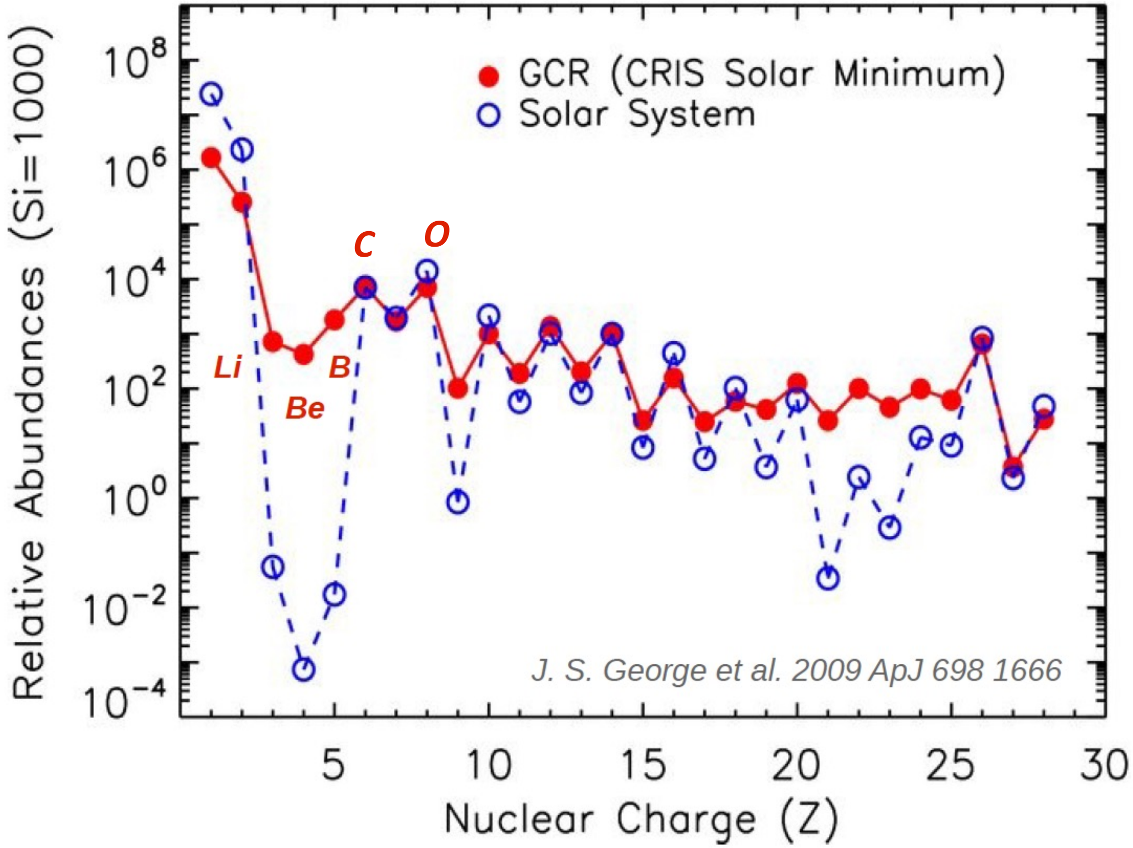
ACCELERATION AT THE SOURCE
(and escape from the acceleration region)

PROPAGATION ACROSS THE GALAXY
i.e. interaction with local matter,
radiation and magnetic fields

EXPERIMENTAL DETECTION
Space/ground based techniques

Primary and Secondary Cosmic Rays





CRs nuclei interact with matter of interstellar medium producing secondaries at expense of primaries changing its original composition.

Carbon is a primary CR.
Boron is a secondary produced by C and O on ISM.

A toy model for particle acceleration

Why dN / dE follows a "power law"?

The particle undergoes many successive processes of acceleration: in each one its energy increases by a quantity $\Delta E = \xi E$ proportional to its own energy.

After a process: $\mathbf{E_1 = E_0 + \xi E_0 = E_0(1+\xi)}$,

after two processes: $\mathbf{E_2} = E_1 + \xi E_1 = E_0 + \xi E_0 + \xi(E_0 + \xi E_0) = E_0 + 2\xi E_0 + \xi^2 E_0 = \mathbf{E_0(1+\xi)^2}$

after n acceleration processes $\mathbf{E_n = E_0(1+\xi)^n}$. (1)

If N_0 is the number of initial particles and N is the number of those that after an acceleration process are still confined in the acceleration region; we can define: $\boldsymbol{\eta = N_1/ N_0}$ the probability of "confinement" for each interaction. After n acceleration processes the total probability of "confinement" is given by:

$\mathbf{P} = (N_n/N_{n-1})\,(N_{n-1}/N_{n-2}) \ldots (N_2/N_1)\,(N_1/N_0) = \mathbf{N_n/ N_0 = \eta^n}$ da cui $n \ln \eta = \ln \frac{N_n}{N_0}$ (2)

From (1) we derive $\ln \frac{E_n}{E_0} = n \ln (1+\xi)$ from which $\boldsymbol{n} = \frac{\mathbf{ln}\left(\frac{E_n}{E_0}\right)}{\mathbf{ln}\,(1+\xi)}$ substituting in (2) we obtain

$\frac{\mathbf{ln}\left(\frac{E_n}{E_0}\right)}{\mathbf{ln}\,(1+\xi)} \ln \eta = \ln \frac{N_n}{N_0}$ that we can rewrite $(-\boldsymbol{s})\ \mathbf{ln}\left(\frac{E_n}{E_0}\right) = \ln \frac{N_n}{N_0}$ where $s = (-\ln \eta) \ln(\mathbf{1} + \boldsymbol{\xi}) > \mathbf{0}$

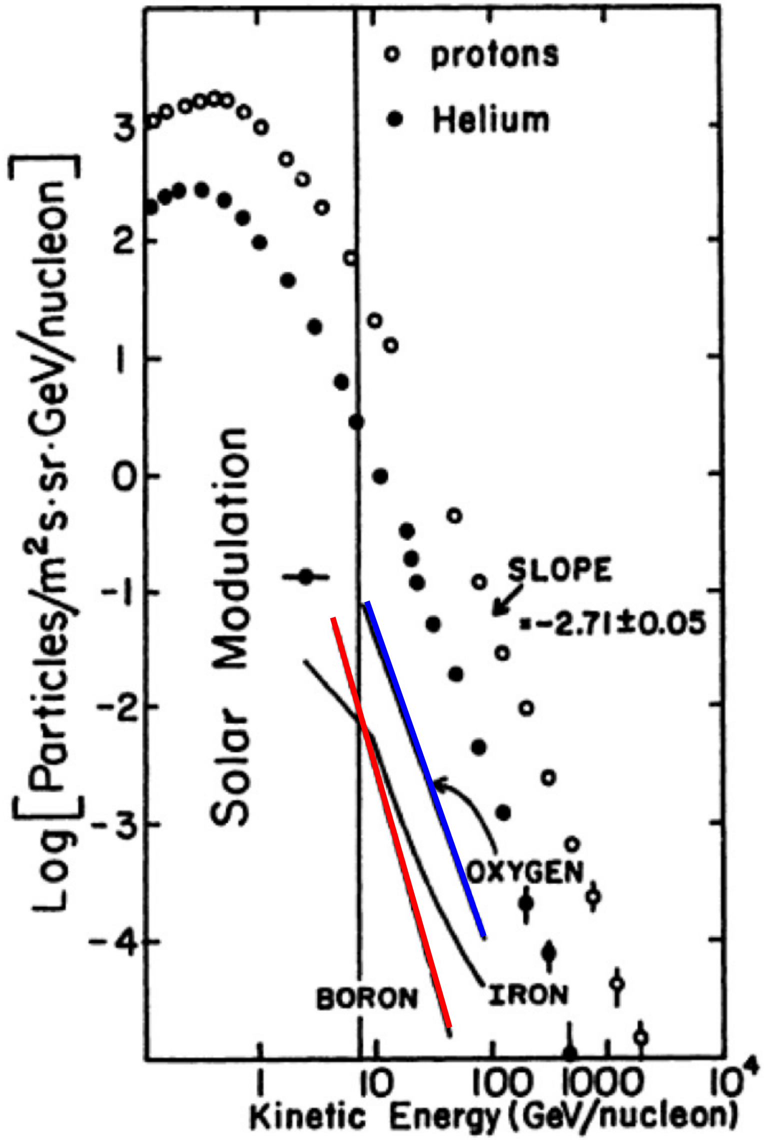
So we have $\mathbf{ln}\left(\frac{E_n}{E_0}\right)^{-s} = \ln \left(\frac{E_0}{E_n}\right)^{s} = \ln \frac{N_n}{N_0}$ from which $N_n = N(\boldsymbol{E}) = N_0 \left(\frac{E_0}{E}\right)^{s} = cost\,\frac{1}{E^s}$

therefore we obtain the trend of the differential spectrum: $\boldsymbol{\frac{dN}{dE} = cost\,\frac{1}{E^{s+1}} = cost\ E^{-(s+1)}}$.

Energy Spectrum of various primary CR components

- It should be noted that the Boron has a more rapidly variable spectrum than the other elements: this is true for all the **"secondary"** nuclei, that are produced in the "spallation" reaction of heavier primary elements.
- The probability of producing a nucleus by **"spallation"** with energy E decreases as E increases: the spectrum of "secondary" is more rapidly decreasing than that of the primaries.

$$\frac{dN}{dE} = cost\ E^{-(s+1)}$$



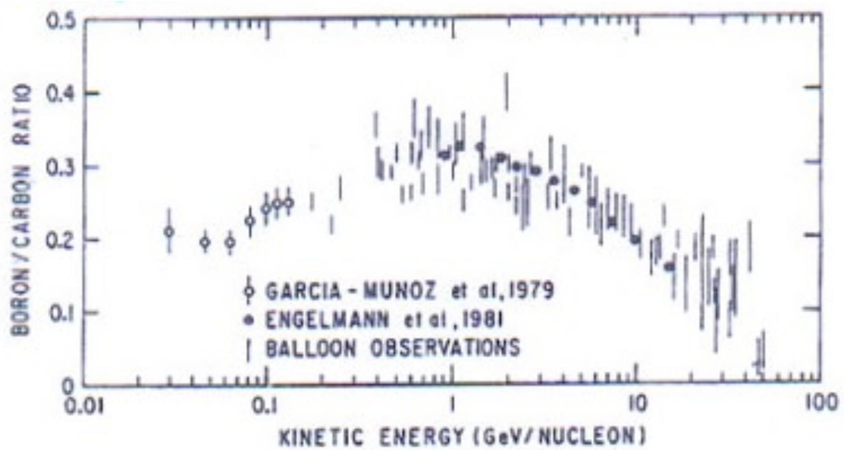


Figure 9.1: Summary of measurements of the ratio of boron to carbon from Ormes & Protheroe (1983). (Reprinted with permission from *The Astrophysical Journal.*)

From astrophysical sources to the Earth (1)

Acceleration mechanisms of charged particles due to interactions with:

- **"partially ionized gas (jet) clouds in motion"**

and / or

- **"shock waves" they can easily explain a "power law" for the spectral trend in energy of the R.C. accelerated into astrophysical sources. Fermi mechanism of the first type:**

$$\frac{dN}{dE} \sim E^{-2}$$

BUT the energy spectrum of charged C.R.s measured at the Earth follows the law:

$$\frac{dN}{dE} \sim E^{-2.7}$$

From astrophysical sources to the Earth (2)

The propagation of C.R.s from the acceleration region to the Earth

depends on the energy of the particles

and/or on their ability to interact with the

Galactic magnetic field

and with the

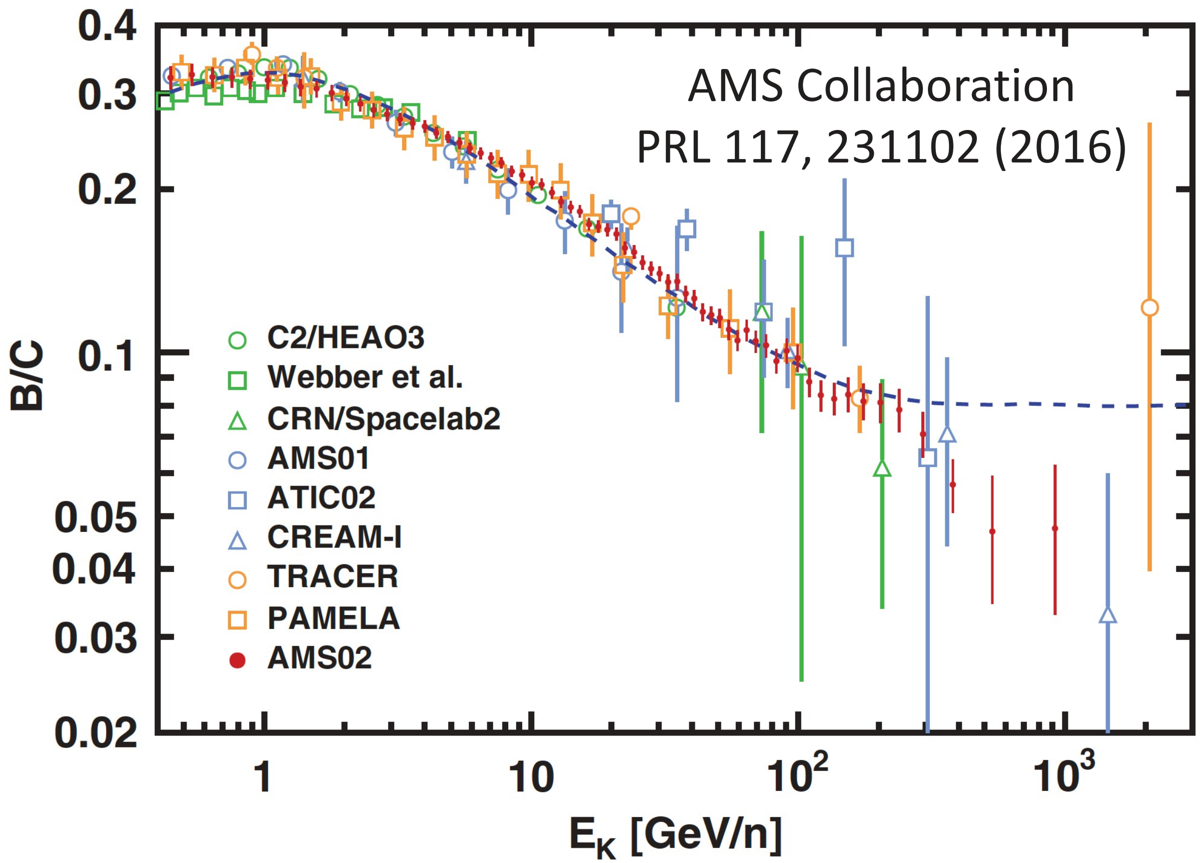
interstellar "medium" (Inter Stellar Medium, ISM).

We know that, on average, a C.R. with $E_{R.C.} > 1$ GeV can propagate over a distance of 5-10 g/cm^2 of "equivalent hydrogen" between the time it was accelerated and when it is "observed" on Earth.

Note that the path X [cm] of a particle in a medium can also be expressed as the "column density" $X \bullet \rho$ [g/cm^2], where ρ [g/cm^3] is the mass density

From astrophysical sources to the Earth (3)

For C.R.s with E> 1 GeV we note that the "amount of matter" encountered during the propagation (i.e. the total path, the propagation time in the galaxy) decreases with increasing energy: as energy increases, the probability of remaining confined decreases:



The dashed line is a theoretical model (the **Nested Leaky Box Model**) which explains several observations (the AMS positron fraction, the antiproton/proton ratio) by secondary production in cosmic ray propagation.
The model is not explaining the highest energy bins.

Please note that on the horizontal axis is shown the “kinetic Energy per nucleon” !!!

From astrophysical sources to the Earth (4)

Let's recall that the ratio B(E)/C(E) (Boron/Carbon for a given value of E) for the R.C. primary is an indicator of the # of spallation processes that occur during C.R. (mainly protons) propagation.

Recall also that in a "spallation" process

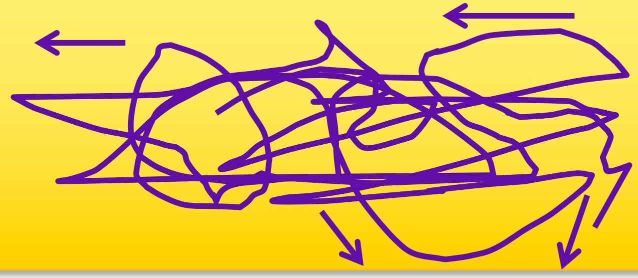
$$p + A \rightarrow p + A_1 + A_2$$

the kinetic energy per nucleon (E_0) remains unchanged:

$$E_A = E_{tot} = A \bullet E_0 \quad ; \quad E_{A1} = A_1 \bullet E_0 \quad ; \quad E_{A2} = A_2 \bullet E_0 \quad ; \; E_{tot} = E_{A1} + E_{A2}$$

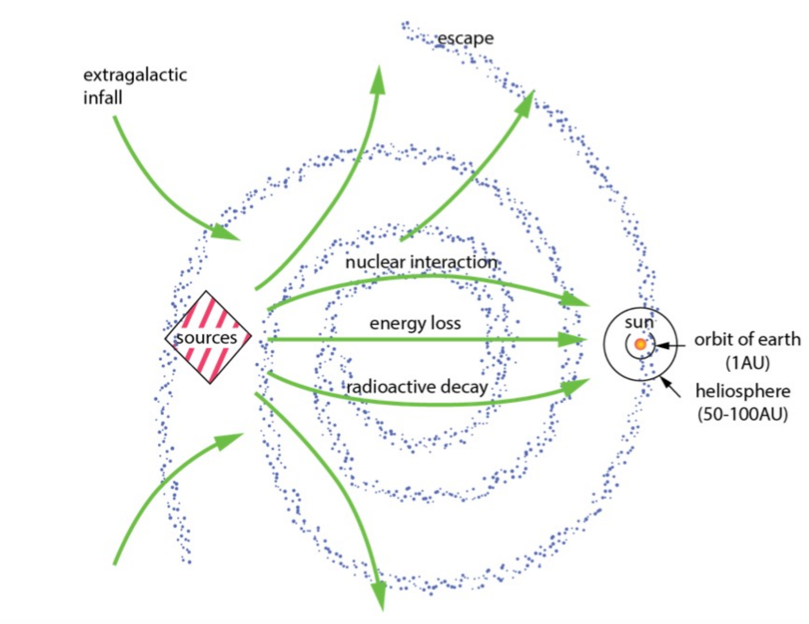
Several models have tried to describe the propagation of C.R.s in the galaxy: a thorough study of this phenomenon must consider the diffusion of C.R.s in the Galactic magnetic field

"random" propagation in a chaotic magnetic field



From astrophysical sources to the Earth (5)

If we define $N(E,\vec{x},t)$ as the density of particles with energy E present at time t in the position $\vec{x}$, then "diffusion" can be expressed taking into account the fact that the particles propagate in space with speed $\vec{v}$ and that for each type of particle it can be defined a "free medium path" λ_D.



On the basis of these quantities we define the diffusion coefficient

$$D = \frac{1}{3}\lambda_D v \left[\frac{L^2}{T}\right]$$

We can indicate with $\psi(x,E,t)$ the function that represents the flux of CRs with energy E at time t in an area of space (for two-dimensional time) identified with x at time t.

From astrophysical sources to the Earth (6)

We defined $\psi = \psi(x,E,t)$ as a function that represents the flux of CRs with energy E in position x at time t.

Let $N_i(E, x, t)$ be the density of particles ***i*** in a certain space and energy range

$$N_i(E,x,t) \equiv [particles\ cm^{-2}\ GeV^{-1}]$$

The variation of $N_i(E, x, t)$ in the region identified by the quantities dE, dx can be produced by:

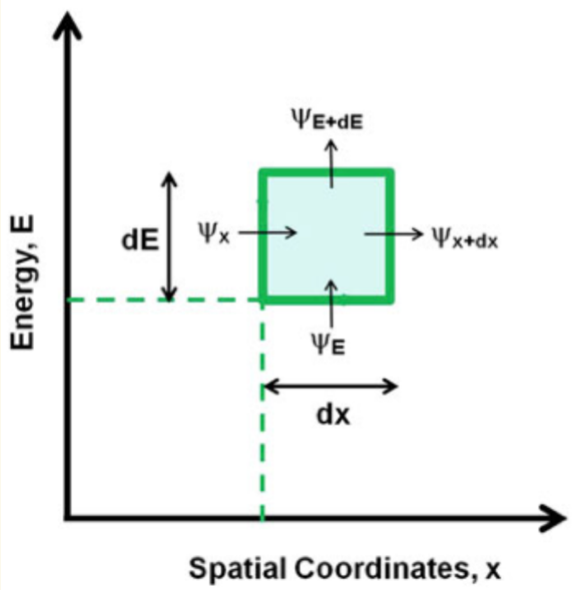
- the diffusion of the particles, **in the coordinate space**, outside the volume considered

$$\psi_x \equiv -D\frac{\partial N_i(E,x,t)}{\partial x}$$

  where D is the diffusion coefficient $D \equiv [cm^2 s^{-1}]$

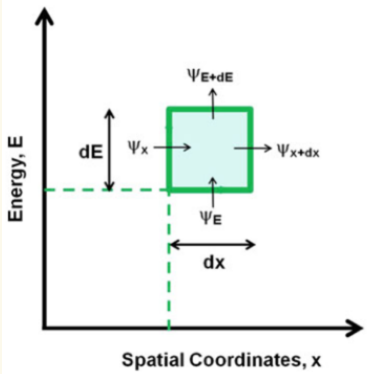
- a **change in energy** for any interaction process ($\Delta E < 0$) and/or acceleration ($\Delta E > 0$):

$$\psi_E \equiv N_i \frac{dE}{dt}$$



From astrophysical sources to the Earth (7)

The number of particles contained in the variable space (dx, dE) indicated by the "green rectangle" is given by $N_i(E,x,t)dxdE$ and **the variation at time t** of this number can be expressed with:



$$\frac{dN_i(E,x,t)}{dt}dxdE = [\psi_x(E,x,t) - \psi_{x+dx}(E,x+dx,t)]dE +$$

$$+[\psi_E(E,x,t) - \psi_{E+dE}(E+dE,x,t)]dx + Q(E,x,t)dxdE$$

where $Q(E,x,t) \equiv particles\ cm^{-1}GeV^{-2}s^{-1}$ represents a "**source of particles**".

Therefore $\frac{dN_i(E,x,t)}{dt} = -\frac{\partial\psi_x(E,x,t)}{\partial x} - \frac{\partial\psi_E(E,x,t)}{\partial E} + Q(E,x,t)$ recalling that

$\psi_x \equiv -D\frac{\partial N_i(E,x,t)}{\partial x}$, replacing → $\frac{dN_i(E,x,t)}{dt} = D\frac{\partial^2 N_i(E,x,t)}{\partial x^2} - \frac{\partial\psi_E(E,x,t)}{\partial E} + Q(E,x,t)$

which generalizing to three dimensions leads to

$$\frac{dN_i(E,x,t)}{dt} = D\nabla^2 N_i(E,x,t) - \frac{\partial\psi_E(E,x,t)}{\partial E} + Q(E,x,t)$$

where now both N and Q are normalized to the unitary volume of physical space.

From astrophysical sources to the Earth (8)

In conditions of equilibrium, neglecting the losses of energy (e.g. inelastic interactions) we have, for each type of particle

$$\frac{\partial N_i(E,\vec{x},t)}{\partial t} = 0$$

Let's recall that variations of N_i(E, x, t) are possible in presence of sources $Q_i(E,\vec{x},t)$, diffusion $\nabla D(\vec{x})\ \nabla N(E,\vec{x},t)$ and interactions $p \cdot N_i(E,\vec{x},t)$ (where p = probability of interaction, $p = n_{ISM}(\vec{x}) \cdot \sigma_{\text{int}}$)

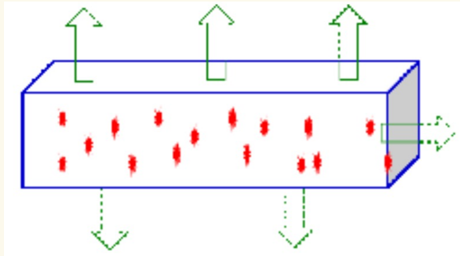
$$\frac{\partial N_i(E,\vec{x},t)}{\partial t} = Q_i(E,\vec{x},t) + \nabla D(\vec{x})\ \nabla N_i(E,\vec{x},t) - p \cdot N_i(E,\vec{x},t)$$

To evaluate the effect of different terms during propagation we consider that:

" the particles that come out of the Galaxy (and therefore escape the magnetic field that tries to contain them) are lost ".

The Leaky Box Model (1)

A simple "model" used to represent the diffusion of R.C. primary from the acceleration zone to the Earth is the Leaky Box Model.



It starts from the hypothesis that the space is homogeneous:

$$N_i(E,\vec{x},t) \equiv N(E,t)$$

We also make the hypothesis of being able to neglect the interactions:

$$\frac{\partial N_i(E,\vec{x},t)}{\partial t} = Q_i(E,\vec{x},t) + \nabla D(\vec{x})\ \nabla N_i(E,\vec{x},t)$$

the term describing, by unit of time, the disappearance of particles of type i due to the diffusion effect can be expressed by defining $\tau(E)$ = escape time from the Galaxy: in this way the probability of escape in the unit of time ~ $1/\tau^{esc}(E)$ and therefore

$$\frac{\partial N_i(E,\vec{x},t)}{\partial t} = Q_i(E,\vec{x},t) - \frac{N_i(E,t)}{\tau_i^{esc}(E)}$$

The Leaky Box Model (2)

$$\frac{dN_i}{dt}(E,x,t) = -\frac{N_i}{\tau_i^{\mathrm{esc}}(E)} + Q(E,x,t) = 0$$

Let's assume the CR distribution is isotropic: $N_i(E,x,t) = N_i(E,t)$

Then in condition of equilibrium it holds: $dN_i(E,t)/dt = 0$

$$\longrightarrow N_i(E,t) = N_i(E)$$

If all interaction processes were negligible, the particle spectrum is

$$-\frac{N_i(E)}{\tau_i^{\mathrm{esc}}(E)} + Q(E) = 0 \longrightarrow N_i(E) = \tau_i^{\mathrm{esc}}(E)Q(E)$$

Energy spectrum of nuclei *i*

Spectrum at the source

This assumption holds if the interaction length is much greater than the escape length from the Galaxy

$$\lambda_{\mathrm{int}}(E) = \frac{1}{\sigma_{\mathrm{int}} n_{\mathrm{ISM}}} \quad >> \quad \lambda_{\mathrm{esc}}(E) = c\tau_{\mathrm{esc}}(E)$$

A comparison among timescales

$$\tau_{INT} = \frac{1}{c < n_{ISM} > \sigma_{INT}}$$ we know that $< n_{ISM} > \approx 0.3 cm^{-3}$, $\sigma_{INT} \approx 200 mbarn$

$$\tau_{INT} = \frac{1}{3 \cdot 10^{10} \cdot 0,3 \cdot 200 \cdot 10^{-27}} \text{ s } = \frac{10^{15}}{0,9 \cdot 2} \frac{1}{3,15 \cdot 10^{7}} \text{ years } = \frac{10^{8}}{5,67} = 1.76 \cdot 10^{7} \text{ years}$$

As long as $\tau_{esc} < \tau_{INT}$, we can consider acceptable the hypothesis of neglecting the interactions between particles: let's verify this assumption in the diffusive conditions of our Galaxy

$$\tau_{\rm esc}(E) = \tau_{\rm diff}(E) = \frac{h^2}{D(E)}$$ in diffusive motion

$$h \simeq 300\,{\rm pc}$$ and $$D(E) \simeq 10^{28} \left(\frac{E}{\rm GeV}\right)^{1/3} {\rm cm^2/s}$$

$$\longrightarrow \tau_{\rm esc}(E) \simeq 2 \times 10^6 \left(\frac{E}{GeV}\right)^{-1/3} \text{years}$$

The Leaky Box Model (3)

Now let's assume that we have the nucleus A, for example Carbon, which after interaction with a proton undergoes a spallation, creating a nucleus of Boron: p+A →p+X+ B.

Recall that with this process the energy per nucleon (E_0) remains unchanged. The number of B nuclei produced due to the process is given by

$$\left.\frac{\partial N_B(E)}{\partial t}\right|_{Spall.} = \frac{N_A(E)}{\tau_A^{INT}} BR(A \to B) = \frac{N_A(E)}{\lambda_A^{INT}/c} \cdot BR(A \to B)$$

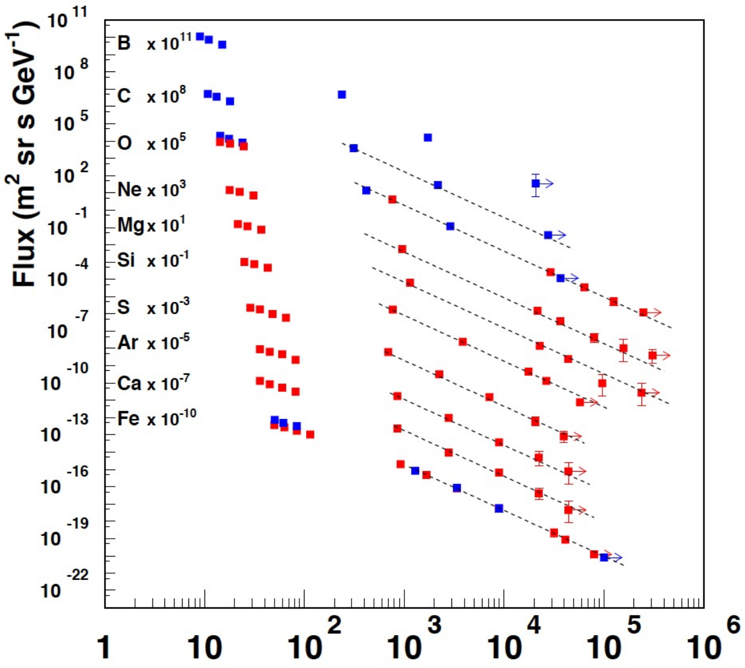
A interactions per unit of time

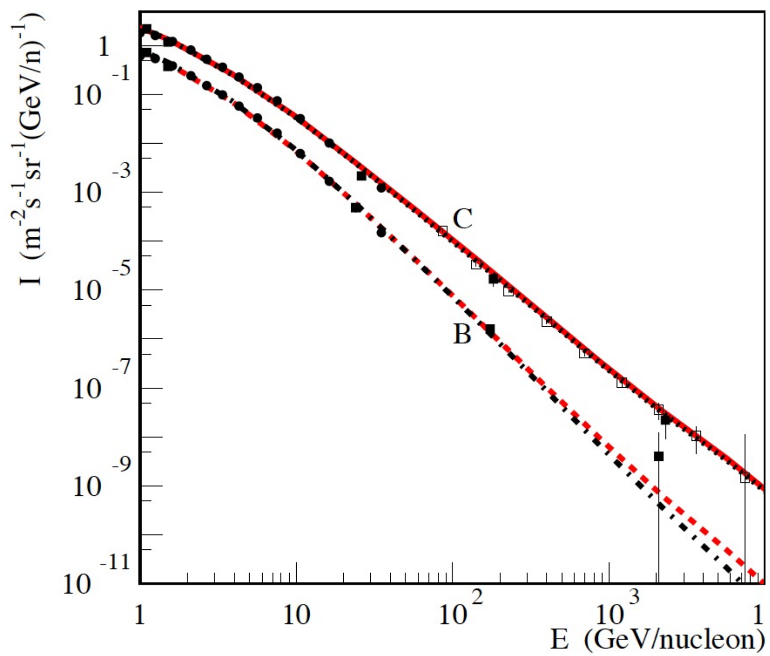
Probability for having B from the spallation of A

The Leaky Box Model (4)

Once measured the energy spectra $N_i(E, t)$ of many different elements the ”primary elements" such as C or Fe can be distinguished from "secondary elements" such as B, Be which are produced by spallation processes.

For the Boron, therefore, we assume that the "source" is precisely the spallation process: $p+C(E_0)\rightarrow p+X+B(E_0)$
Let’s recall that with this process the energy per nucleon (E_0) remains unchanged so the spectra of B and C as a function of E_0 should have the same "slope".





The Leaky Box Model (5)

Let us consider the process of spallation p+A(E_0)→p+X+ B(E_0) and

let's the condition of stationarity $\frac{\partial N_B(E,t)}{\partial t} = 0 \rightarrow N_B(E,t) \equiv N_B(E)$

$$0 = \frac{\partial N_B(E)}{\partial t} = \cancel{Q_B(E,t)} - \frac{N_B(E,t)}{\tau_{esc}^{B}(E)} + \frac{N_A(E)}{\lambda_A^{INT}/c} \cdot BR(A \rightarrow B)$$

Possible direct source of B

B Escape from the Galaxy

Spallation of A in B

$$0 = -\frac{N_B(E)}{\tau_{esc}^{B}(E)} + \frac{N_A(E)}{\lambda_A^{INT}/c} \cdot BR(A \rightarrow B) \quad \rightarrow \quad \frac{N_B(E)}{\tau_{esc}^{B}(E)} = \frac{N_A(E)}{\lambda_A^{INT}/c} \cdot BR(A \rightarrow B)$$

$$\frac{N_B(E)}{N_A(E)} = BR(A \rightarrow B) \cdot \tau_{esc}^{B}(E) \cdot \frac{c}{\lambda_A^{INT}} = BR(A \rightarrow B) \cdot \tau_{esc}^{B}(E) \cdot \frac{1}{\tau_A^{INT}}$$

For example, the B / C ratio as a function of the kinetic energy of the nucleons

The Leaky Box Model (6)

$$\frac{N_B(E)}{N_A(E)} = BR(A \to B) \cdot \tau^B_{esc}(E) \cdot \frac{c}{\lambda^{INT}_A} = BR(A \to B) \cdot \tau^B_{esc}(E) \cdot c \cdot \sigma^A_{INT} \cdot < n_{ISM} >$$

$$c < n_{ISM} > \tau_{ESC}(E) = c \frac{< \rho_{ISM} >}{m_P} \tau_{ESC}(E) = \frac{\ell_{ESC}[g/cm^2]}{m_P}$$

From these measurements it is inferred that the trend of the $\tau^{esc}(E)$ of Boron (and therefore of the "escape length" as a function of energy) follows a power law of the type $\tau^{esc}(E) \sim E^{-0.6-0.65}$

$\ell_{escape}(E) = c \cdot \tau^{esc}(E)$

if expressed as

$$c < n_{ISM} > \tau_{ESC}(E) = c \frac{< \rho_{ISM} >}{m_p} \tau_{ESC}(E) = \frac{\ell_{ESC}(E)[g/cm^2]}{m_p}$$

As a function of Rigidity $R = \frac{pc}{Ze} = \frac{E}{Z}$

$$\ell_{esc} = 34.1\ \beta R^{-0.60}\ g\ cm^{-2} \qquad for\ R > 4.4\ GV$$

$$\ell_{esc} = 14.1\ \beta\ g\ cm^{-2} \qquad for\ R < 4.4\ GV$$

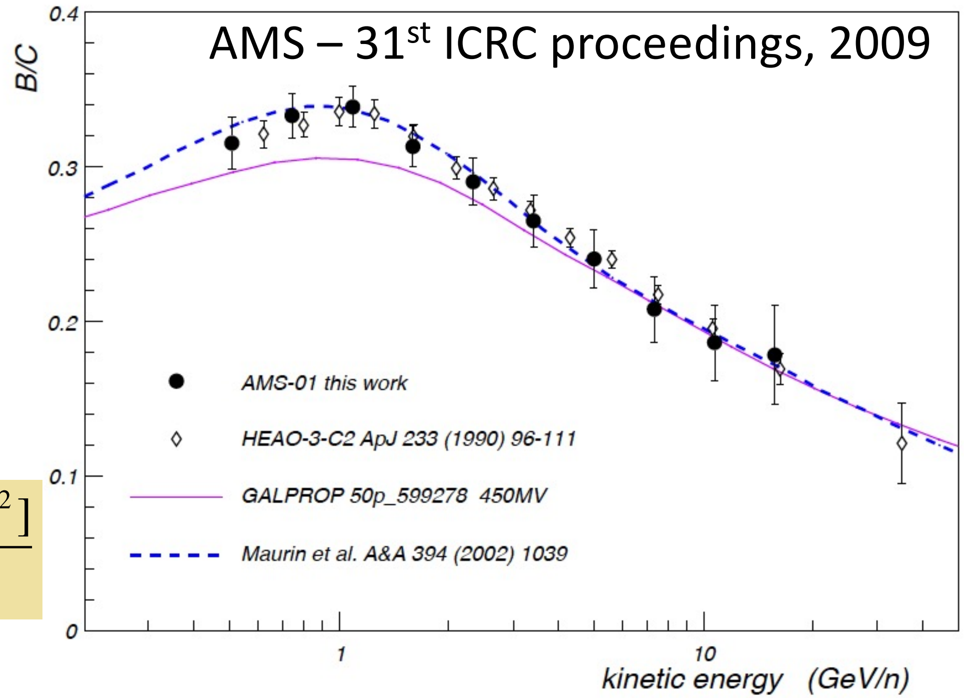


Fig. 4: B/C ratio from this work (filled circles), from HEAO-3-C2 experiment (open diamonds) and from two diffusion models (solid and dashed lines).

and the CR spectral index at the Earth

Therefore from the study of heavy nuclei coming from the "spallation" phenomena (e.g. $p+C \rightarrow B$) one has a τ_{escape} (E) measurements (or in an equivalent way of the "Escape Length(E)")

$$\tau_{escape}(E) \approx E^{-0.6}$$

The spectrum of C.R. accelerated to the source is generally described by a law:

$$dN/dE \approx E^{-\alpha}$$

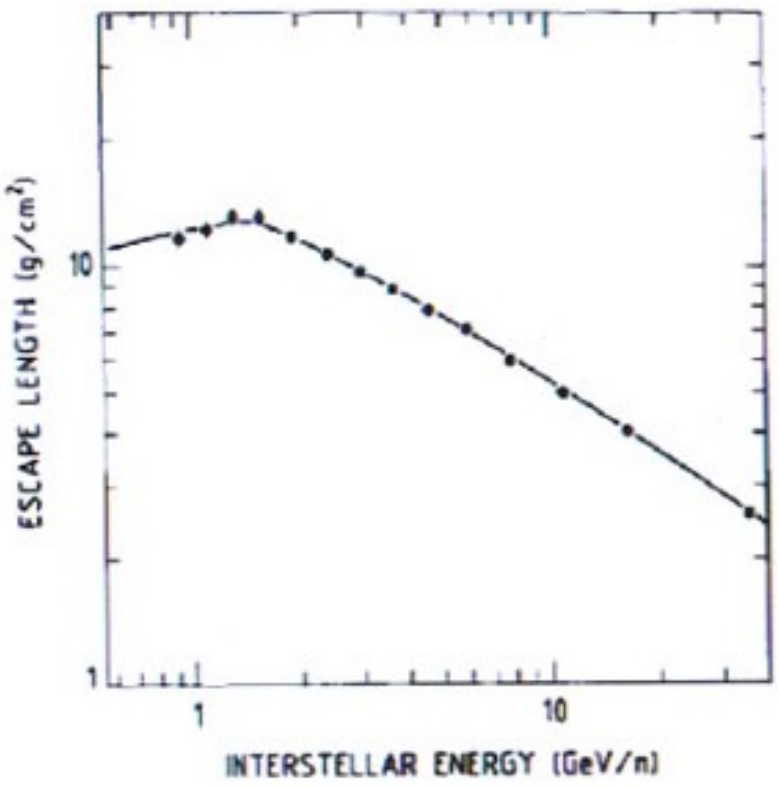
with $\alpha = 2.0 \mid 2.2$

Therefore

$$dN/dE = Q(E) \cdot \tau_{escape}(E) \approx E^{-\alpha} \cdot E^{-0.6}$$

$$dN/dE \approx E^{-2.7}$$

in good agreement with the experimental observations.



Escape Length (g cm^{-2})