Lessons 3 and 4

• C.R. composition: relative abundance of elements in the Earth (Solar System) and on the C.R..
• Propagation time of C.R. in our Galaxy.
• Measurement of the C.R. electromagnetic component: primary photons and electrons
• How to detect photons and electrons ?
• Main characteristics of apparatuses on atmospheric balloons, main results.
• The Compton Gamma Ray Observatory and its components: CGRO, BATSE, Comptel ed EGRET.
• Gamma Ray Bursts.
• CGRO Sky Map.
• ”FERMI”: the detector and and obtained results.
The all particle spectrum...
The all particle spectrum...

- Proton Larmor radius in Galactic B field:
  \[ r_L(p) \approx 10^{12} \left( \frac{pc}{\text{GeV}} \right) \left( \frac{3 \mu \text{G}}{B} \right) \text{ cm} \]

- Disc thickness: \( 5 \times 10^{20} \text{ cm} \)

\[ \rightarrow p_{\text{EG}} \approx 5 \times 10^{17} \text{ eV/c} \]
...and its individual components
The cosmic-ray observables

1. **Spectral energy distribution**: which physical mechanisms generate an almost featureless power law?

2. **Mass composition**: abundances + primaries/secondaries cosmic rays;

1. **Spatial distribution across the sky**: almost isotropic sky map of CRs.
The path to become a cosmic ray

ACCELERATION AT THE SOURCE
(and escape from the acceleration region)

PROPAGATION ACROSS THE GALAXY
i.e. interaction with local matter, radiation and magnetic fields

EXPERIMENTAL DETECTION
Space/ground based techniques
Primary and Secondary Cosmic Rays

CRs nuclei interact with matter of interstellar medium producing secondaries at expense of primaries changing its original composition.

Carbon is a primary CR.
Boron is a secondary produced by C and O on ISM.
A toy model for particle acceleration

Why \( dN / dE \) follows a "power law"?

The particle undergoes many successive processes of acceleration: in each one its energy increases by a quantity \( \Delta E = \xi E \) proportional to its own energy.

After a process: \( E_1 = E_0 + \xi E_0 = E_0(1+\xi) \),
after two processes: \( E_2 = E_1 + \xi E_1 = E_0 + \xi E_0 + \xi(E_0 + \xi E_0) = E_0 + 2\xi E_0 + \xi^2 E_0 = E_0(1+\xi)^2 \)

after \( n \) acceleration processes \( E_n = E_0(1+\xi)^n \). \hspace{1cm} (1)

If \( N_0 \) is the number of initial particles and \( N \) is the number of those that after an acceleration process are still confined in the acceleration region; we can define: \( \eta = N_1 / N_0 \) the probability of "confinement" for each interaction. After \( n \) acceleration processes the total probability of "confinement" is given by:

\[
P = \frac{N_n}{N_{n-1}} \frac{N_{n-1}}{N_{n-2}} \ldots \frac{N_2}{N_1} \frac{N_1}{N_0} = \frac{N_n}{N_0} = \eta^n \quad \text{da cui} \quad n \ln \eta = \ln \frac{N_n}{N_0} \hspace{1cm} (2)
\]

From (1) we derive \( \ln \frac{E_n}{E_0} = n \ln (1 + \xi) \) from which \( n = \frac{\ln \left( \frac{E_n}{E_0} \right)}{\ln (1+\xi)} \) substituting in (2) we obtain

\[
\frac{\ln \left( \frac{E_n}{E_0} \right)}{\ln (1+\xi)} \ln \eta = \ln \frac{N_n}{N_0} \quad \text{that we can rewrite} \quad (-s) \ln \left( \frac{E_n}{E_0} \right) = \ln \frac{N_n}{N_0} \quad \text{where} \quad s = (-\ln \eta) \ln (1 + \xi) > 0
\]

So we have \( \ln \left( \frac{E_n}{E_0} \right)^{-s} = \ln \left( \frac{E_0}{E_n} \right)^{s} = \ln \frac{N_n}{N_0} \) from which \( N_n = N(E) = N_0 \left( \frac{E_0}{E} \right)^{s} = \text{cost} \frac{1}{E^s} \)

Therefore we obtain the trend of the differential spectrum:

\[
\frac{dN}{dE} = \text{cost} \frac{1}{E^{s+1}} = \text{cost} E^{-(s+1)}.
\]
Energy Spectrum of various primary CR components

- It should be noted that the Boron has a more rapidly variable spectrum than the other elements: this is true for all the "secondary" nuclei, that are produced in the "spallation" reaction of heavier primary elements.
- The probability of producing a nucleus by "spallation" with energy $E$ decreases as $E$ increases: the spectrum of "secondary" is more rapidly decreasing than that of the primaries.

$$\frac{dN}{dE} = cost \, E^{-(s+1)}$$
Acceleration mechanisms of charged particles due to interactions with:

- "partially ionized gas (jet) clouds in motion"
- and / or
- "shock waves" they can easily explain a "power law" for the spectral trend in energy of the R.C. accelerated into astrophysical sources. Fermi mechanism of the first type:

\[
\frac{dN}{dE} \sim E^{-2}
\]

BUT the energy spectrum of charged C.R.s measured at the Earth follows the law:

\[
\frac{dN}{dE} \sim E^{-2.7}
\]
The propagation of C.R.s from the acceleration region to the Earth depends on the energy of the particles and/or on their ability to interact with the Galactic magnetic field and with the interstellar "medium" (Inter Stellar Medium, ISM).

We know that, on average, a C.R. with $E_{R.C.} > 1$ GeV can propagate over a distance of $5-10$ g/cm$^2$ of "equivalent hydrogen" between the time it was accelerated and when it is "observed" on Earth.

Note that the path $X$ [cm] of a particle in a medium can also be expressed as the "column density" $X \cdot \rho$ [g/cm$^2$], where $\rho$ [g/cm$^3$] is the mass density.
From astrophysical sources to the Earth (3)

For C.R.s with $E > 1$ GeV we note that the "amount of matter" encountered during the propagation (i.e. the total path, the propagation time in the galaxy) decreases with increasing energy: as energy increases, the probability of remaining confined decreases:

The dashed line is a theoretical model (the Nested Leaky Box Model) which explains several observations (the AMS positron fraction, the antiproton/proton ratio) by secondary production in cosmic ray propagation. The model is not explaining the highest energy bins.

Please note that on the horizontal axis is shown the “kinetic Energy per nucleon” !!!
Let’s recall that the ratio $B(E)/C(E)$ (Boron/Carbon for a given value of $E$) for the R.C. primary is an indicator of the # of spallation processes that occur during C.R. (mainly protons) propagation.

Recall also that in a "spallation" process

$$p + A \rightarrow p + A_1 + A_2$$

the kinetic energy per nucleon ($E_0$) remains unchanged:

$$E_A = E_{\text{tot}} = A \cdot E_0 \quad ; \quad E_{A1} = A_1 \cdot E_0 \quad ; \quad E_{A2} = A_2 \cdot E_0 \quad ; \quad E_{\text{tot}} = E_{A1} + E_{A2}$$

Several models have tried to describe the propagation of C.R.s in the galaxy: a thorough study of this phenomenon must consider the diffusion of C.R.s in the Galactic magnetic field.

"random" propagation in a chaotic magnetic field
If we define $N(E, \vec{x}, t)$ as the density of particles with energy $E$ present at time $t$ in the position $\vec{x}$, then "diffusion" can be expressed taking into account the fact that the particles propagate in space with speed $\vec{v}$ and that for each type of particle it can be defined a "free medium path" $\lambda_D$.

On the basis of these quantities we define the diffusion coefficient

$$D = \frac{1}{3} \lambda_D \nu \left[ \frac{L^2}{T} \right]$$

We can indicate with $\psi(x, E, t)$ the function that represents the flux of CRs with energy $E$ at time $t$ in an area of space (for one-dimensional time) identified with $x$ at time $t$. 
We defined $\psi = \psi(x,E,t)$ as a function that represents the flux of CRs with energy $E$ in position $x$ at time $t$.

Let $N_i(E, x, t)$ be the density of particles $i$ in a certain space and energy range

$$N_i(E, x, t) \equiv [\text{particles cm}^{-1} \text{ GeV}^{-1}]$$

The variation of $N_i(E, x, t)$ in the region identified by the quantities $dE, dx$ can be produced by:

- the diffusion of the particles, in the coordinate space, outside the volume considered
  $$\psi_x \equiv -D \frac{\partial N_i(E, x, t)}{\partial x}$$
  where $D$ is the diffusion coefficient $D \equiv [\text{cm}^2 \text{s}^{-1}]$

- a change in energy for any interaction process ($\Delta E < 0$) and/or acceleration ($\Delta E > 0$):
  $$\psi_E \equiv N_i \frac{dE}{dt}$$
The number of particles contained in the variable space \((dx, dE)\) indicated by the "green rectangle" is given by

\[ N_i(E, x, t)dxdE \]

and the variation at time \(t\) of this number can be expressed with:

\[
\frac{dN_i(E, x, t)}{dt} dxdE = [\psi_x(E, x, t) - \psi_{x+dx}(E, x + dx, t)]dE + \\
[\psi_E(E, x, t) - \psi_{E+dE}(E + dE, x, t)]dx + Q(E, x, t)dxdE
\]

where \(Q(E, x, t) \equiv \text{particles cm}^{-1}\text{GeV}^{-1}\text{s}^{-1}\) represents a "source of particles".

Therefore

\[
\frac{dN_i(E, x, t)}{dt} = - \frac{\partial \psi_x(E, x, t)}{\partial x} - \frac{\partial \psi_E(E, x, t)}{\partial E} + Q(E, x, t)
\]

recalling that

\[
\psi_x \equiv -D \frac{\partial N_i(E, x, t)}{\partial x}
\]

replacing

\[
\frac{dN_i(E, x, t)}{dt} = D \frac{\partial^2 N_i(E, x, t)}{\partial x^2} - \frac{\partial \psi_E(E, x, t)}{\partial E} + Q(E, x, t)
\]

which generalizing to three dimensions leads to

\[
\frac{dN_i(E, x, t)}{dt} = D\nabla^2 N_i(E, x, t) - \frac{\partial \psi_E(E, x, t)}{\partial E} + Q(E, x, t)
\]

where now both \(N\) and \(Q\) are normalized to the unitary volume of physical space.
From astrophysical sources to the Earth (9)

In conditions of equilibrium, neglecting the losses of energy (e.g. inelastic interactions) we have, for each type of particle

\[ \frac{\partial N_i(E, \bar{x}, t)}{\partial t} = 0 \]

Let’s recall that variations of \( N_i(E, x, t) \) are possible in presence of sources \( Q_i(E, \bar{x}, t) \), diffusion \( \nabla D(\bar{x}) \nabla N(E, \bar{x}, t) \) and interactions \( p \cdot N_i(E, \bar{x}, t) \) (where \( p \) = probability of interaction, \( p = n_{ISM}(\bar{x}) \cdot \sigma_{int} \))

\[ \frac{\partial N_i(E, \bar{x}, t)}{\partial t} = Q_i(E, \bar{x}, t) + \nabla D(\bar{x}) \nabla N_i(E, \bar{x}, t) - p \cdot N_i(E, \bar{x}, t) \]

To evaluate the effect of different terms during propagation we consider that:

"the particles that come out of the Galaxy (and therefore escape the magnetic field that tries to contain them) are lost".
A simple "model" used to represent the diffusion of R.C. primary from the acceleration zone to the Earth is the **Leaky Box Model**.

It starts from the hypothesis that the space is homogeneous:

\[ N_i(E, \vec{x}, t) \equiv N(E, t) \]

We also make the hypothesis of being able to neglect the interactions:

\[ \frac{\partial N_i(E, \vec{x}, t)}{\partial t} = Q_i(E, \vec{x}, t) + \nabla D(\vec{x}) \nabla N_i(E, \vec{x}, t) \]

the term describing, by unit of time, the disappearance of particles of type \( i \) due to the diffusion effect can be expressed by defining \( \tau(E) = \text{escape time from the Galaxy} \): in this way the probability of escape in the unit of time \( \sim 1/\tau_{\text{esc}}(E) \) and therefore

\[ \frac{\partial N_i(E, \vec{x}, t)}{\partial t} = Q_i(E, \vec{x}, t) - \frac{N_i(E, t)}{\tau_{\text{esc}}(E)} \]
The Leaky Box Model (2)

\[ \frac{dN_i}{dt}(E, x, t) = -\frac{N_i}{\tau_i^{\text{esc}}(E)} + Q(E, x, t) = 0 \]

Let’s assume the CR distribution is isotropic: \( N_i(E, x, t) = N_i(E, t) \)

Then in condition of equilibrium it holds: \( dN_i(E, t)/dt = 0 \)

\[ \rightarrow N_i(E, t) = N_i(E) \]

If all interaction processes were negligible, the particle spectrum is

\[ -\frac{N_i(E)}{\tau_i^{\text{esc}}(E)} + Q(E) = 0 \rightarrow N_i(E) = \tau_i^{\text{esc}}(E)Q(E) \]

Energy spectrum of nuclei \( i \)

Spectrum at the source

This assumption holds if the interaction length is much greater than the escape length from the Galaxy

\[ \lambda_{\text{int}}(E) = \frac{1}{\sigma_{\text{int}} n_{\text{ISM}}} \gg \gg \lambda_{\text{esc}}(E) = c\tau_{\text{esc}}(E) \]
A comparison among timescales

\[ \tau_{\text{INT}} = \frac{1}{c < n_{\text{ISM}} > \sigma_{\text{INT}}} \]

we know that \( < n_{\text{ISM}} > \approx 0.3 \text{cm}^{-3} \), \( \sigma_{\text{INT}} \approx 200 \text{mbarn} \)

\[ \tau_{\text{INT}} = \frac{1}{3 \cdot 10^{10} \cdot 0.3 \cdot 200 \cdot 10^{-27}} \text{ s} = \frac{10^{15}}{0.9 \cdot 2 \cdot 3.15 \cdot 10^7} \text{ years} = \frac{10^8}{5.67} = 1.76 \cdot 10^7 \text{ years} \]

As long as \( \tau_{\text{esc}} < \tau_{\text{INT}} \), we can consider acceptable the hypothesis of neglecting the interactions between particles: let’s verify this assumption in the diffusive conditions of our Galaxy

\[ \tau_{\text{esc}}(E) = \tau_{\text{diff}}(E) = \frac{h^2}{D(E)} \text{ in diffusive motion} \]

\( h \approx 300 \text{ pc} \) and \( D(E) \approx 10^{28} \left( \frac{E}{\text{GeV}} \right)^{1/3} \text{ cm}^2 / \text{s} \)

\[ \rightarrow \tau_{\text{esc}}(E) \approx 2 \times 10^6 \left( \frac{E}{\text{GeV}} \right)^{-1/3} \text{ years} \]
Now let’s assume that we have the nucleus $A$, for example Carbon, which after interaction with a proton undergoes a spallation, creating a nucleus of Boron: $p+A \rightarrow p+X+ B$.

Recall that with this process the energy per nucleon ($E_0$) remains unchanged. The number of $B$ nuclei produced due to the process is given by:

$$\left. \frac{\partial N_B(E)}{\partial t} \right|_{\text{Spall.}} = \frac{N_A(E)}{\tau_A} \cdot BR(A \rightarrow B) = \frac{N_A(E)}{\lambda_A c} \cdot BR(A \rightarrow B)$$

- $N_A(E)$: A interactions per unit of time
- $BR(A \rightarrow B)$: Probability for having $B$ from the spallation of $A$
Once measured the energy spectra $N_i(E, t)$ of many different elements the "primary elements" such as C or Fe can be distinguished from "secondary elements" such as B, Be which are produced by spallation processes.

For the Boron, therefore, we assume that the "source" is precisely the spallation process: 

$$p + C(E_0) \rightarrow p + X + B(E_0)$$

Let’s recall that with this process the energy per nucleon ($E_0$) remains unchanged so the spectra of B and C as a function of $E_0$ should have the same "slope".
The Leaky Box Model (5)

Let us consider the process of spallation $p+A(E_0) \rightarrow p+X+ B(E_0)$ and let’s the condition of stationarity

$$\frac{\partial N_B(E,t)}{\partial t} = 0 \rightarrow N_B(E,t) \equiv N_B(E)$$

0 = $\frac{\partial N_B(E)}{\partial t} = Q_B(E,t) - \frac{N_B(E,t)}{\tau_{esc}(E)} + \frac{N_A(E)}{\lambda_A^{INT}} \cdot BR(A \rightarrow B)$

Possible direct source of B

B Escape from the Galaxy

Spallation of A in B

For example, the B / C ratio as a function of the kinetic energy of the nucleons

$\frac{N_B(E)}{N_A(E)} = BR(A \rightarrow B) \cdot \frac{\tau_{esc}^B(E) \cdot c}{\lambda_A^{INT}} = BR(A \rightarrow B) \cdot \tau_{esc}^B(E) \cdot \frac{1}{\tau_A^{INT}}$
The Leaky Box Model (6)

\[
\frac{N_B(E)}{N_A(E)} = BR(A \rightarrow B) \cdot \tau_{\text{esc}}^B(E) \cdot \frac{c}{\lambda_A^{\text{INT}}} = BR(A \rightarrow B) \cdot \tau_{\text{esc}}^B(E) \cdot c \cdot \sigma_A^{\text{INT}} \cdot <n_{\text{ISM}}>
\]

\[
c < n_{\text{ISM}} > \tau_{\text{ESC}}(E) = c < \rho_{\text{ISM}} > \frac{\tau_{\text{ESC}}(E)}{m_p} = \frac{\ell_{\text{ESC}}[g/cm^2]}{m_p}
\]

From these measurements it is inferred that the trend of the \(\tau^{\text{esc}}(E)\) of Boron (and therefore of the "escape length" as a function of energy) follows a power law of the type

\[
\tau^{\text{esc}}(E) \sim E^{-0.6-0.65}
\]

\(\ell_{\text{escape}}(E) = c \cdot \tau^{\text{esc}}(E)\)

if expressed as

\[
c < n_{\text{ISM}} > \tau_{\text{ESC}}(E) = c < \rho_{\text{ISM}} > \frac{\tau_{\text{ESC}}(E)}{m_p} = \frac{\ell_{\text{ESC}}[g/cm^2]}{m_p}
\]

As a function of Rigidity

\[
R = \frac{pc}{Ze} = \frac{E}{Z}
\]

\[
\ell_{\text{esc}} = 34.1 \beta R^{-0.60} \text{ g cm}^{-2} \quad \text{for } R > 4.4 \text{ GV}
\]

\[
\ell_{\text{esc}} = 14.1 \beta \text{ g cm}^{-2} \quad \text{for } R < 4.4 \text{ GV}
\]

Fig. 4: B/C ratio from this work (filled circles), from HEAO-3-C2 experiment (open diamonds) and from two diffusion models (solid and dashed lines).
and the CR spectral index at the Earth

Therefore from the study of heavy nuclei coming from the "spallation" phenomena (e.g. $p+C \rightarrow B$) one has a $\tau_{\text{escape}}(E)$ measurements (or in an equivalent way of the "Escape Length(E)")

$$\tau_{\text{escape}}(E) \approx E^{-0.6}$$

The spectrum of C.R. accelerated to the source is generally described by a law:

$$\frac{dN}{dE} \approx E^{-\alpha}$$

with $\alpha = 2.0 \mid 2.2$

Therefore

$$\frac{dN}{dE} = Q(E) \cdot \tau_{\text{escape}}(E) \approx E^{-\alpha} \cdot E^{-0.6}$$

$$\frac{dN}{dE} \approx E^{-2.7}$$

in good agreement with the experimental observations.
Detection of C.R.s and of "cosmic" photons: preliminary considerations

- Earth's atmosphere is about 28 radiation lengths ($\chi_0$) and about 11 interaction lengths: it absorbs photons with energy $\gtrsim 30$ MeV
- The flux of gamma rays from astrophysical sources is faint and decreases significantly with energy

$$\frac{d\Phi}{dE} \approx \Phi_0 \cdot E_{TeV}^{-\gamma} \text{ cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{TeV}^{-1}$$

integrating in energy

$$\Phi(E > E_{\text{min}}) = \Phi_0 \cdot E_{TeV}^{-\gamma-1} \text{ cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$$

- e.g. for “Vela”, the more intense “gamma ray source”:

$$\Phi(E>100\text{MeV})=1.3x10^{-5} \text{ photins cm}^{-2} \text{ s}^{-1}$$

and the spectral index is $\gamma-1=-1.89$.

Therefore, if the detection area is about 1000 cm$^2$

$$N(E>100\text{MeV}) \rightarrow \text{about 1 } \gamma/\text{minute;} \quad N(E>2\text{GeV}) \rightarrow \text{about 1 } \gamma \text{ in 2 hours}$$

- The flux of charged cosmic rays (protons) is much greater than the gamma:

$$\frac{d\Phi}{dE} \approx 9 \times 10^{-6} E_{TeV}^{-2.76} \text{ cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{TeV}^{-1}$$
Astronomy with charged C.R. and with “cosmic” photons
Cosmic Photons and Hadrons Fluxes

Direct measurements (roughly)
nomenclature used in astrophysics of C.R.s

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<tr>
<th>Energy range</th>
<th>Name</th>
<th>Experimental Technique</th>
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<td>10-30 MeV</td>
<td>Medium</td>
<td>Satellite</td>
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<tr>
<td>30Mev-30Gev</td>
<td>High Energy (HE)</td>
<td>Satellite</td>
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<td>30 GeV - 30 TeV</td>
<td>Very High Energy (VHE)</td>
<td>Cerenkov Array (g.b.)</td>
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<td>Ground Based Array</td>
</tr>
<tr>
<td>30 Pev -&gt;</td>
<td>Extremely High Energy (EHE)</td>
<td>Ground Based</td>
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</tbody>
</table>
Detection of charged Cosmic Rays and cosmic photons. Preliminary considerations

The measurement of astrophysical photons on Earth possible for $E_{\gamma} < 1\text{-}10 \text{ GeV}$.

• The Earth's atmosphere corresponds to about $28 \chi_0$ (radiation lengths) and to about $11 \lambda_{\text{int}}$ (interaction lengths): it absorbs the HE photons. A direct measurement of ‘primary’ astrophysical photons above $100 \text{ GeV}$ nearly impossible on the Earth.

• H.E. gamma rays from astrophysical sources could be detected outside the atmosphere but their flux is very faint and decreases significantly with energy: difficult to collect a good statistics in a reasonable time
  – For example: from “Vela”, a “very active” gamma source the integral of photon flux with $E_{\gamma} > 100 \text{ MeV}$ is
    $$\Phi(E>100\text{MeV}) = 1.3 \times 10^{-5} \text{ photons cm}^{-2} \text{ s}^{-1}$$
    with a detection area $A \approx 1000 \text{ cm}^2$ we would collect in 1 minute $N_{\text{ev}} = 1.3 \times 10^{-5} \times 1000 \times 60 \leq 1 \text{ event}$
    knowing that the spectrum energy dependence is $dN/dE \propto E^{-1.89}$ we can calculate that to detect 1 photon with energy > $2\text{GeV}$ we should collect data for 2 hours

• The measurement of H.E. photons from astrophysical sources is difficult since the background (due to the flux of charged cosmic rays) is very high:

• The flux of charged cosmic rays is much more intense than the gamma one:

$$\frac{d\Phi}{dE} \approx 9 \times 10^{-6} E_{\text{TeV}}^{-2.76} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ TeV}^{-1}$$
Radiation length

- The radiation length ($X_0$): the characteristic length that describes the energy decay of a beam of electrons:

\[ X_0 = \frac{716.4 \text{ g cm}^{-2} A}{Z(Z+1) \ln(287/\sqrt{Z})} \]

- Distance over which the electron energy is reduced by a factor of 1/e due to radiation losses only

- Radiation loss is approx. independent of material when thickness expressed in terms of $X_0$

- Higher $Z$ materials have shorter radiation length
  - want high-$Z$ material for an EM calorimeter
  - want as little material as possible in front of calorimeter

- Example:
  lead: $\rho = 11.4 \text{ g/cm}^3$ so $X_0 = 5.5 \text{ mm}$

- The energy loss by bremsstrahlung is:

\[ -\frac{dE}{dx} = \frac{E}{X_0} \]
Energy loss of photons and EM showers

- High-energy photons predominately lose energy in matter by $e^+e^-$ pair production.
- The mean free path for pair production by a high-energy photon
  \[
  \lambda = \frac{9}{7} X_0
  \]
- Note for electrons $\lambda = X_0$
- But then we have high energy electrons... so the process repeats!
  - This is an electromagnetic shower!
  - An electromagnetic cascade as pair production and bremsstrahlung generate more electrons and photons with lower energy.
Energy loss of photons and EM showers

**Electrons**
- (bremsstrahlung)
- Radiation length $\chi_0$
- Critical energy $E_c$

**Photons**
- Photoelectric effect
- Compton diffusion
- $e^+ e^-$ pair production

$E_c \approx 610 MeV \div (Z+1.24)$

(Solids, liquids)
Energy loss of photons and EM showers

Total cross section for carbon and lead photons according to photon energy. The contributions of the different processes are shown:

- $\sigma_{\text{p.e.}}$: Atomic photoeffect (electron ejection, photon absorption)
- $\sigma_{\text{coherent}}$: Coherent scattering (Rayleigh scattering—atom neither ionized nor excited)
- $\sigma_{\text{incoherent}}$: Incoherent scattering (Compton scattering off an electron)
- $\kappa_n$: Pair production, nuclear field
- $\kappa_e$: Pair production, electron field
- $\sigma_{\text{nuc}}$: Photonuclear absorption (nuclear absorption, usually followed by emission of a neutron or other particle)
Simplified scheme for an EM shower development

Simplified model

@ $\Delta x_i$: $X_0$

$e \rightarrow e'\gamma$

$E' = E'' = \frac{E}{2}$

$\gamma \rightarrow e^+e^-$

the multiplication process stops if $E < E_c$

and since then the number of particles decreases

$E(t) \propto e^{-\kappa t}$

$N(t) = \text{particles at } \kappa = t \cdot X_0$:

Where $E_c$ is the Critical Energy for electrons
EM showers characteristics

- **longitudinal distribution**
  \[ \frac{dE}{dt} \propto t^\alpha e^{-\beta t} \]

- **Position of shower maximum**
  \[ t_{\text{max}} = 1.4 \ell_n \frac{E_0}{E_c} \]

- **Longitudinal containment**
  \[ t_{95\%} = t_{\text{max}} + 0.08Z + 9.6 \]

- **Lateral containment dominated by multiple scattering (+ photon propagation)**
  \[ \langle \theta_M \rangle = \frac{21}{\rho \beta} \sqrt{t} \]
  \[ r_{95\%} = 2R_M \]
  \[ R_M = \frac{21 \text{MeV}}{E_c} X_0 \text{ g/cm}^2 \]
  \[ R_M \propto \frac{X_0}{E_c} \propto \frac{A}{Z} \text{ per } Z \gg 1 \]

*Molière radius*
Interaction length and nuclear radiation length

**Nuclear interaction length**: is the mean path length required to reduce the numbers of relativistic charged particles by the factor $1/e$, or 0.368, as they pass through matter.

- Interactions of heavy particles with nuclei can also produce hadronic showers
- Described by the **nuclear interaction length**

$$\lambda_n \approx 35 \text{ g cm}^{-2} A^{1/3}$$

- For heavy (high Z) materials we see that the nuclear interaction length is a lot longer than the electromagnetic one, $\lambda_n > \chi_0$
- So hadronic showers start later than electromagnetic showers and are more diffuse
- Example: lead $\sim$ steel $= 17$ cm

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<thead>
<tr>
<th>material</th>
<th>$\chi_0$ (g/cm$^2$)</th>
<th>$\lambda_n$ (g/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{H}_2$</td>
<td>63</td>
<td>52.4</td>
</tr>
<tr>
<td>Al</td>
<td>24</td>
<td>106</td>
</tr>
<tr>
<td>Fe</td>
<td>13.8</td>
<td>132</td>
</tr>
<tr>
<td>Pb</td>
<td>6.3</td>
<td>193</td>
</tr>
</tbody>
</table>
Nuclear interaction length

- The interaction of energetic hadrons (charged or neutral) is determined by various nuclear processes:

  \[ \text{Multiplicity} \propto \ln(E) \]
  \[ P_+ < 1 \text{ GeV/c} \]

- Excitation and finally breakup of nucleus: nuclear fragments and production of secondary particles

- For high energies (> 1 GeV) the cross-sections depend only little on the energy and on the type of the incident particle (p, π, K, ...)

- Define in analogy to \( X_0 \) a hadronic interaction length \( \lambda_I \):

  \[ \lambda_I = \frac{A}{NA \sigma_{total}} \propto A^{1/3} \]
Detection of primary cosmic rays with $E \leq 100\text{GeV}$

A small apparatus (r~6cm, aperture ~22 cm$^2$ sr) carried by balloons (a few tens/hundred flight hours $\rightarrow$ more than $10^6$ events) can collect a discrete statistic in the energy region up to ~10 GeV:

- composition of cosmic rays (photons, protons, heavy nuclei, ...)
- Energy spectrum
- Matter/antimatter (identification of positrons, antiprotons, anti-helium, ...)

All particle spectrum

~ 10 particelle/s

scintillators, wire chambers, tracking apparatus, magnetic field, Cherenkov, ....

$20\text{cm}$

$100\text{cm}^2$