

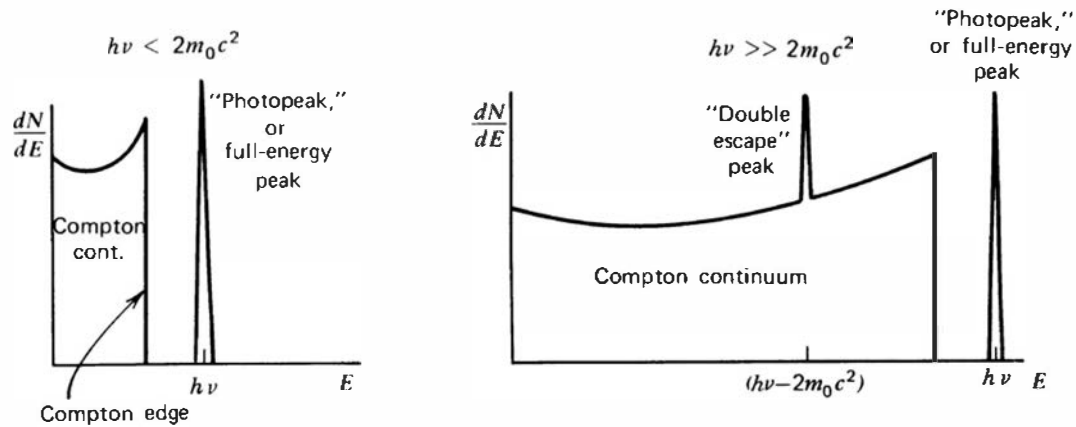
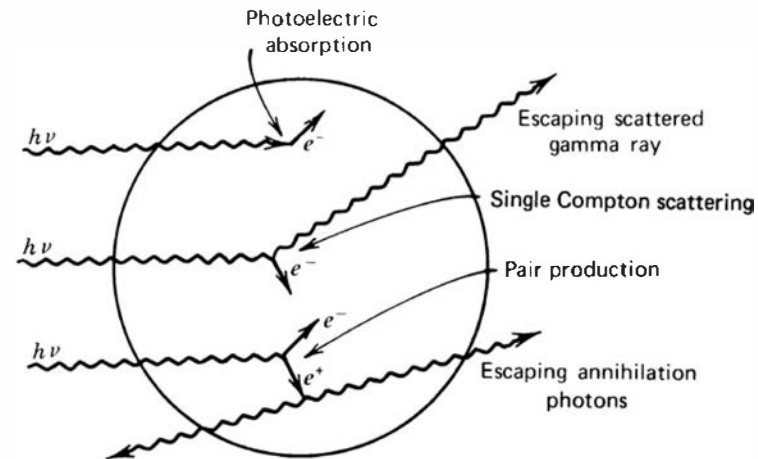


Figure 10.2 The "small detector" extreme in gamma-ray spectroscopy. The processes of photoelectric absorption and single Compton scattering give rise to the low-energy spectrum at the left. At higher energies, the pair production process adds a double escape peak shown in the spectrum at the right.

The energy of the scattered gamma ray $h\nu'$ in terms of its scattering angle θ is given by

$$h\nu' = \frac{h\nu}{1 + (h\nu/m_0c^2)(1 - \cos \theta)} \quad (10.1)$$

where m_0c^2 is the rest mass energy of the electron (0.511 MeV). The kinetic energy of the recoil electron is therefore

$$E_{e^-} = h\nu - h\nu' = h\nu \left(\frac{(h\nu/m_0c^2)(1 - \cos \theta)}{1 + (h\nu/m_0c^2)(1 - \cos \theta)} \right) \quad (10.2)$$

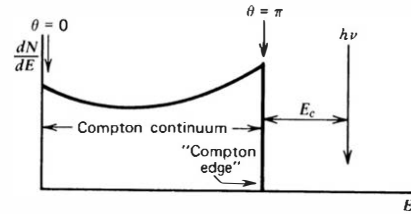
Two extreme cases can be identified:

1. A grazing angle scattering, or one in which $\theta \cong 0$. In this case, Eqs. (10.1) and (10.2) predict that $h\nu' \cong h\nu$ and $E_{e^-} \cong 0$. In this extreme, the recoil Compton electron has very little energy and the scattered gamma ray has nearly the same energy as the incident gamma ray.
2. A head-on collision in which $\theta = \pi$. In this extreme, the incident gamma ray is backscattered toward its direction of origin, whereas the electron recoils along the direction of incidence. This extreme represents the maximum energy that can be transferred to an electron in a single Compton interaction. Equations (10.1) and (10.2) yield for this case

$$h\nu'|_{\theta=\pi} = \frac{h\nu}{1 + 2h\nu/m_0c^2} \quad (10.3)$$

$$E_{e^-}|_{\theta=\pi} = h\nu \left(\frac{2h\nu/m_0c^2}{1 + 2h\nu/m_0c^2} \right) \quad (10.4)$$

In normal circumstances, all scattering angles will occur in the detector. Therefore, a continuum of energies can be transferred to the electron, ranging from zero up to the maximum predicted by Eq. (10.4). Figure 10.1 shows the shape of the distribution of Compton recoil electrons predicted by the Klein–Nishina cross section (Chapter 2) for several different values of the incident gamma-ray energy. For any one specific gamma-ray energy, the electron energy distribution has the general shape shown in the sketch below.



The gap between the maximum Compton recoil electron energy and the incident gamma-ray energy is given by

$$E_C \equiv h\nu - E_{e^-}|_{\theta=\pi} = \frac{h\nu}{1 + 2h\nu/m_0c^2} \quad (10.5)$$

In the limit that the incident gamma-ray energy is large, or $h\nu \gg m_0c^2/2$, this energy difference tends toward a constant value given by

$$E_C \cong \frac{m_0c^2}{2} (= 0.256 \text{ MeV}) \quad (10.6)$$

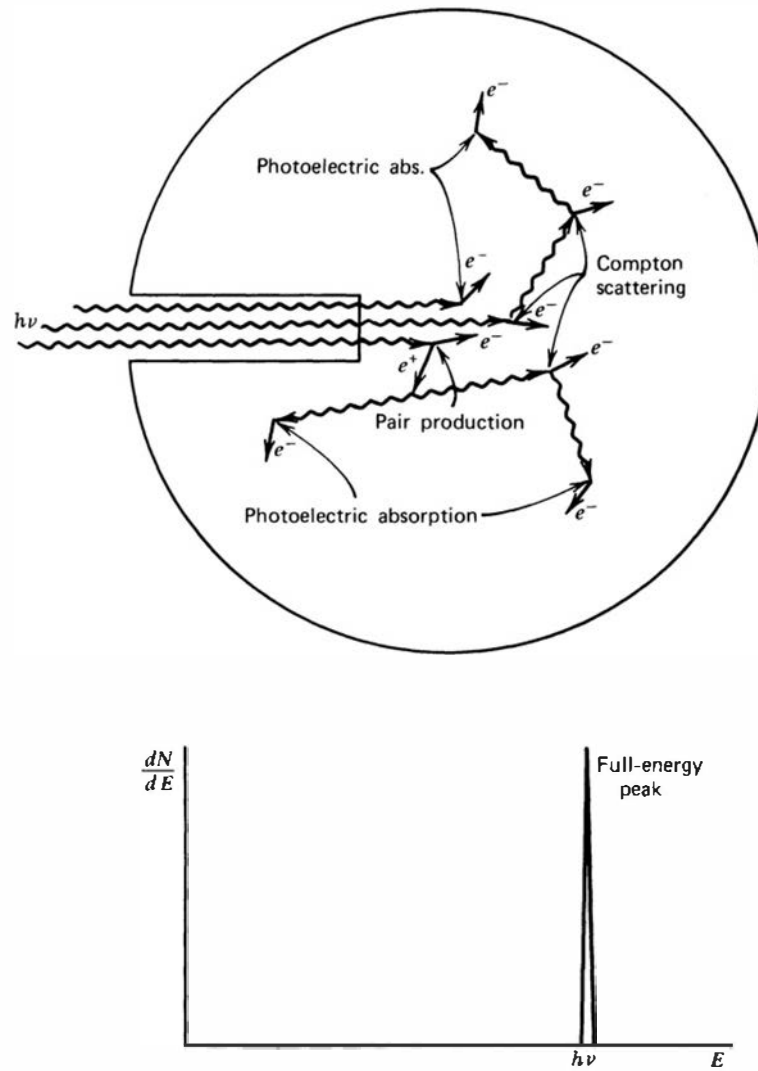


Figure 10.3 The “large detector” extreme in gamma-ray spectroscopy. All gamma-ray photons, no matter how complex their mode of interaction, ultimately deposit all their energy in the detector. Some representative histories are shown at the top.

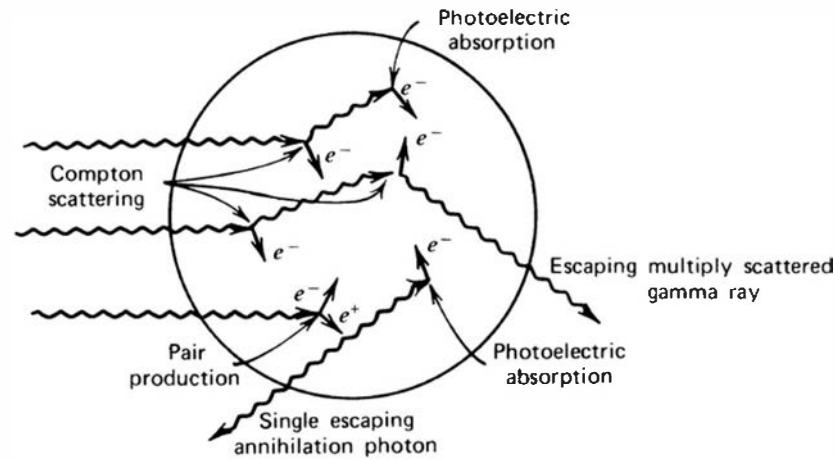
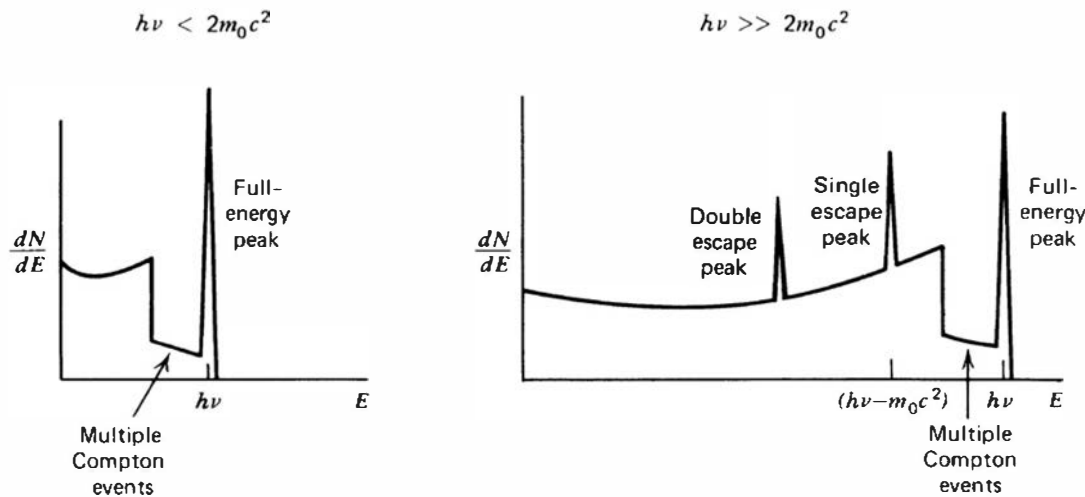
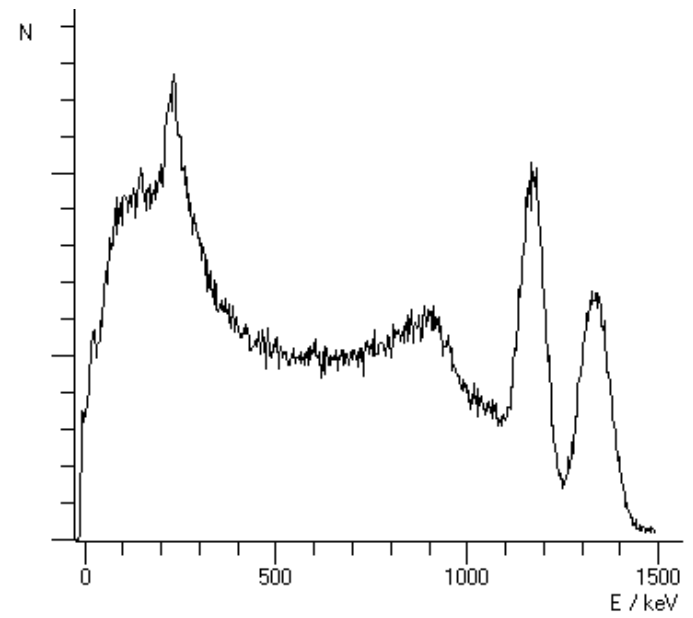
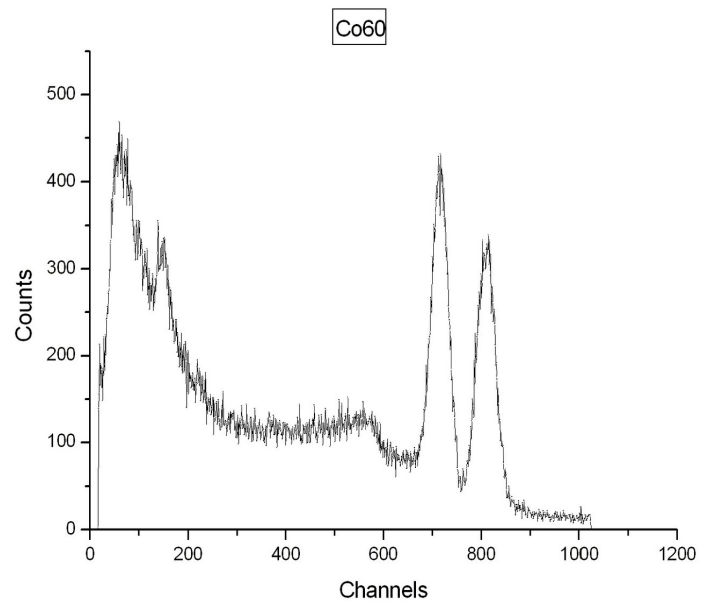


Figure 10.4 The case of intermediate detector size in gamma-ray spectroscopy. In addition to the continuum from single Compton scattering and the full-energy peak, the spectrum at the left shows the influence of multiple Compton events followed by photon escape. The full-energy peak also contains some histories that began with Compton scattering. At the right, the single escape peak corresponds to initial pair production interactions in which only one annihilation photon leaves the detector without further interaction. A double escape peak as illustrated in Fig. 10.2 will also be present due to those pair production events in which both annihilation photons escape.





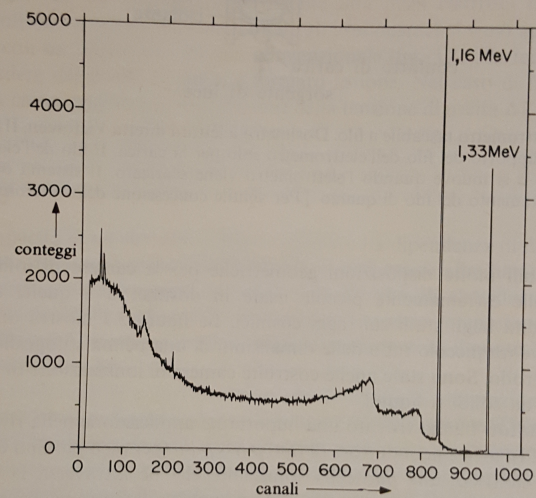
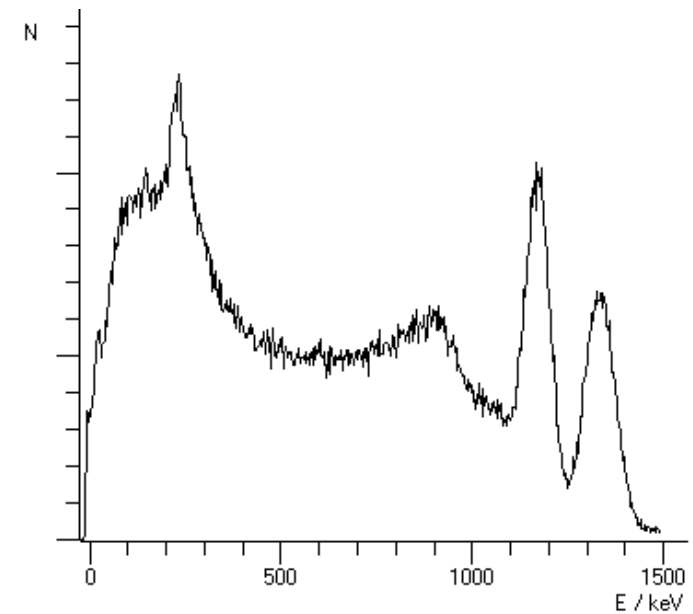
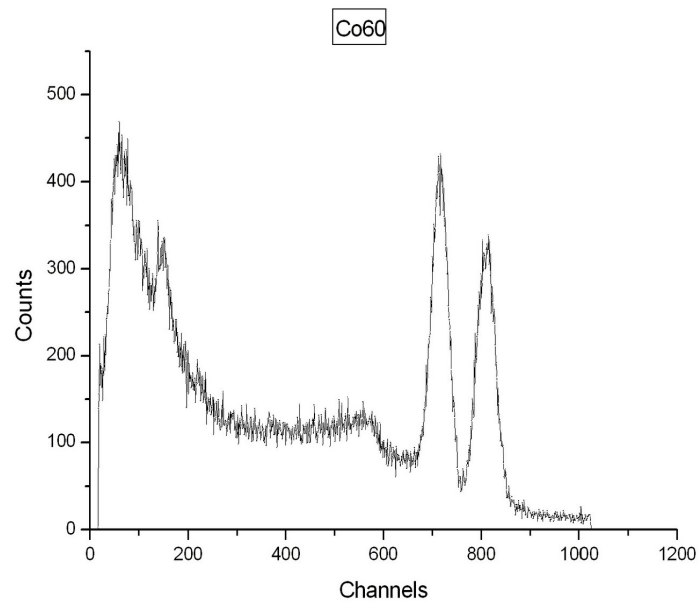
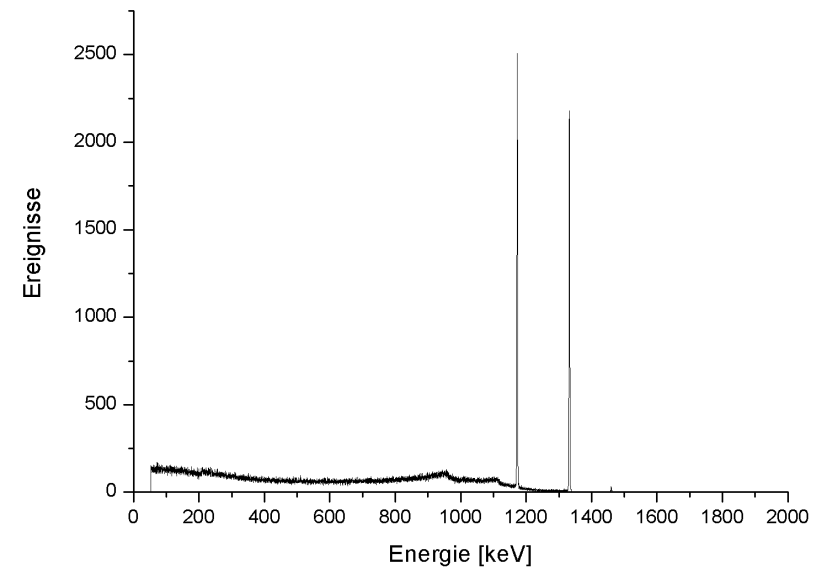


Figura 3.5. Spettro dei raggi gamma del ^{60}Co ottenuto con un rivelatore al germanio. Si notino i due picchi dell'energia totale dovuti ai raggi gamma di 1,16 MeV e di 1,33 MeV, e la distribuzione Compton ad energie inferiori a quelle dei picchi. Le discese nette nella curva ai canali 680 e 790 sono i cosiddetti salti Compton corrispondenti alla massima energia ceduta nell'urto agli elettroni dai raggi gamma di 1,16 MeV e 1,33 MeV. [Goulding e Stone, *Science*, **170**, 280, 1970].



$$\dot{N}_i^{teo} = L' \int_{k_i}^{k_{i+1}} dk \int_0^{\infty} \frac{d\sigma}{dk'} g(k - k', c) dk'$$

$$L' = AL$$

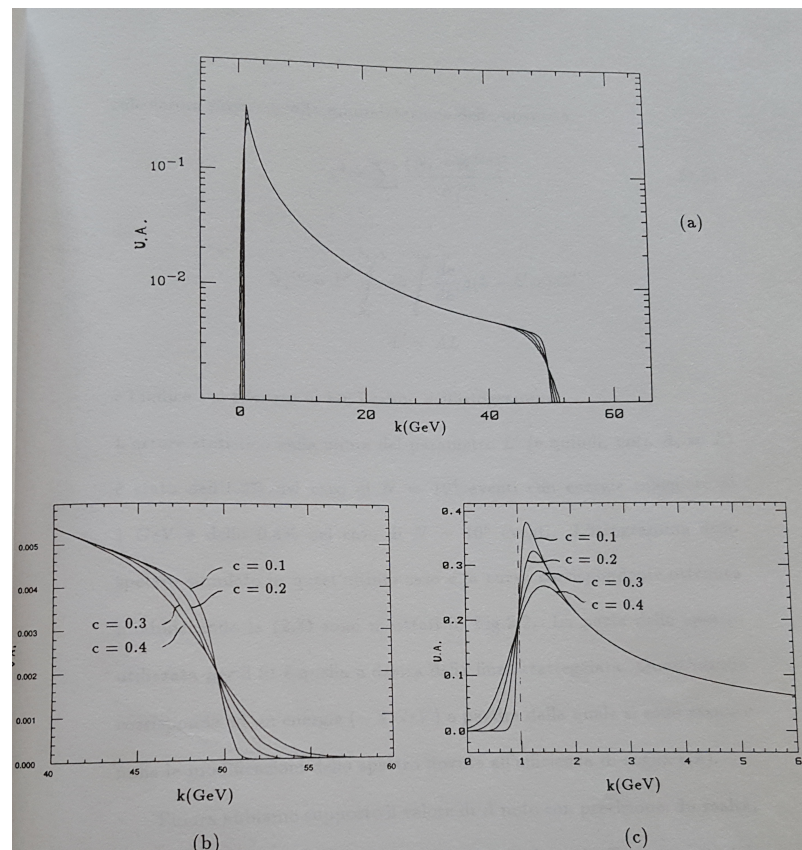
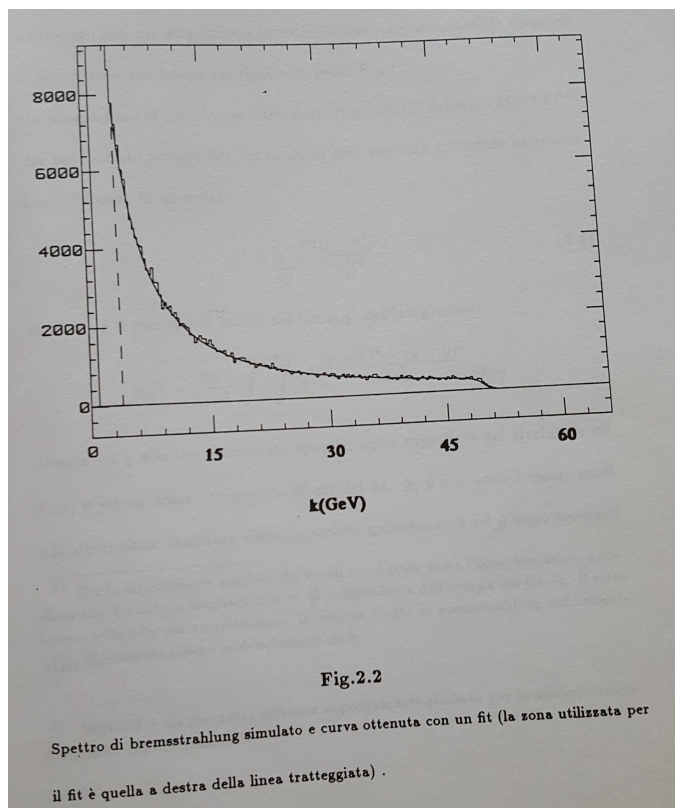
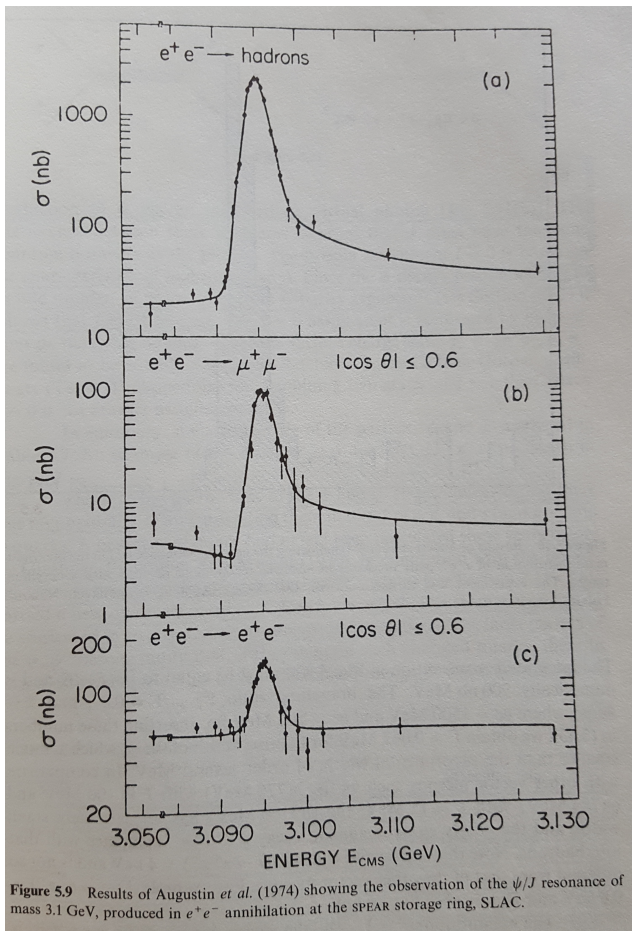


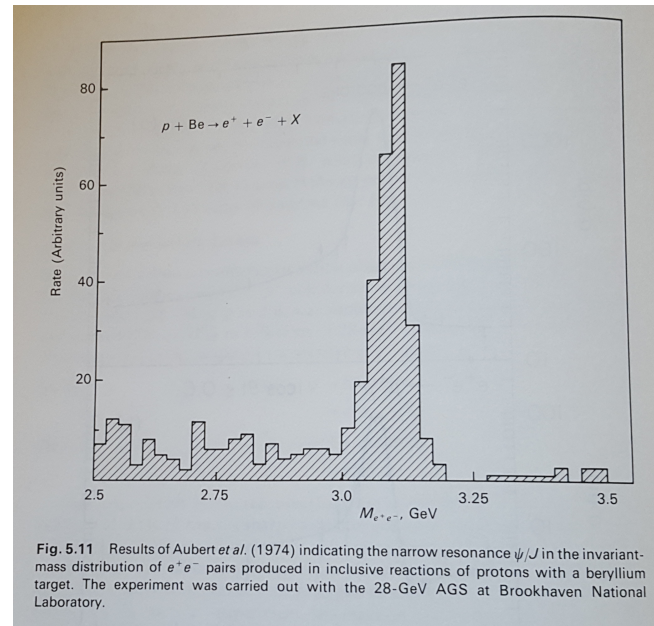
Fig.2.1

Spettro di bremsstrahlung singolo (a) ricavato dall'eq.(2.2) con un'efficienza di rivelazione a gradino ($k_{min} = 1 \text{ GeV}$) e con una funzione di risoluzione gaussiana nei casi $c=0.1, 0.2, 0.3$ e 0.4 . Sono mostrati ingrandimenti del grafico nella zona di alta (b) e bassa (c) energia.



SLAC: $e^+e^- \rightarrow \psi \rightarrow \text{hadrons}$
 $\rightarrow e^+e^-, \mu^+\mu^-$

BNL: $p + \text{Be} \rightarrow \psi/J + \text{anything.}$
 $\rightarrow e^+e^-$



$J/\psi(1S)$

$$I^G(J^{PC}) = 0^-(1^{--})$$

$$\text{Mass } m = 3096.900 \pm 0.006 \text{ MeV}$$

$$\text{Full width } \Gamma = 92.9 \pm 2.8 \text{ keV} \quad (S = 1.1)$$

$$\Gamma_{ee} = 5.55 \pm 0.14 \pm 0.02 \text{ keV}$$

$$\sigma_E(\text{BNL}) \sim 25 \text{ MeV}$$

$$\sigma_E(\text{SLAC}) \sim 2 \text{ MeV}$$

$J/\psi(1S)$ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level (MeV/c)
hadrons	$(87.7 \pm 0.5) \%$	—
virtual $\gamma \rightarrow$ hadrons	$(13.50 \pm 0.30) \%$	—
ggg	$(64.1 \pm 1.0) \%$	—
γgg	$(8.8 \pm 1.1) \%$	—
e^+e^-	$(5.971 \pm 0.032) \%$	1548
$e^+e^- \gamma$	$(8.8 \pm 1.4) \times 10^{-3}$	1548
$\mu^+\mu^-$	$(5.961 \pm 0.033) \%$	1545

Resolution effect:

Smearing of spectrum structures,

i.e. enlarging peaks, smoothing sharp edges, filling holes or gaps

Exercise:

Evaluate numerically the convolution integral and the effect of an arbitrary resolution on a simple sharp distribution like a step-function + a narrow peak.

Spettro del prodotto di due segnali [\(Convoluzione Calcolo con MATHCAD\)](#)

Abbiamo visto che lo spettro di due segnali sommati [$s_1(t) + s_2(t)$] è dato dalla combinazione dei rispettivi spettri. Diversamente lo spettro del prodotto di due segnali $u(t)$, $v(t)$ **non** è dato dal prodotto dei loro spettri.

Cerchiamo quindi a cosa equivale lo spettro del prodotto.

Siano $u(t) \leftrightarrow U(\omega)$ $v(t) \leftrightarrow V(\omega)$ i segnali con i rispettivi spettri e $s(t) = u(t)v(t)$ il segnale prodotto allora lo spettro sarà:

$$S(\omega) = \int_{-\infty}^{+\infty} u(t)v(t)e^{-i\omega t} dt$$

Se mettiamo al posto di $v(t)$ la sua antitrasformata: $v(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} V(\xi)e^{i\xi t} d\xi$

otteniamo:

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} u(t) \left[\int_{-\infty}^{+\infty} V(\xi)e^{i\xi t} d\xi \right] e^{-i\omega t} dt$$

Invertendo l'ordine d'integrazione otteniamo:

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} V(\xi) \left[\int_{-\infty}^{+\infty} u(t)e^{-i(\omega-\xi)t} dt \right] d\xi$$

Possiamo perciò riscrivere lo spettro nel seguente modo:

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} V(\xi)U(\omega-\xi)d\xi$$

L'integrale $\int_{-\infty}^{+\infty} V(\xi)U(\omega-\xi)d\xi$ è noto come *integrale di convoluzione delle funzioni V e U* e

viene indicato, simbolicamente: $V(\omega)*U(\omega)$

Possiamo allora concludere che **lo spettro del prodotto ordinario di due segnali è uguale, a meno di un fattore $1/2\pi$, al prodotto di convoluzione degli spettri.**

$$s(t) = u(t)v(t)$$

$$\underline{S}(\omega) = V(\omega)*U(\omega)$$

Ovviamente $V(\omega)*U(\omega) = U(\omega)*V(\omega)$

Si può anche dimostrare il teorema inverso:

•Se lo spettro di un segnale può essere rappresentato come prodotto ordinario di due spettri

$$S(\omega) = S_1(\omega) S_2(\omega) \text{ tali che: } S_1(\omega) \leftrightarrow s_1(t) \quad S_2(\omega) \leftrightarrow s_2(t)$$

allora il segnale $s(t) \leftrightarrow S(\omega)$ è la convoluzione di $s_1(t)$ e $s_2(t)$ nel dominio del tempo.

$$S_1(\omega)S_2(\omega) \leftrightarrow \int_{-\infty}^{+\infty} s_1(t-\xi)s_2(t)d\xi$$

Una interessante conseguenza di questo è il teorema della convoluzione. Esso afferma che la **trasformata di Fourier del prodotto di convoluzione è equivalente al prodotto ordinario delle trasformate**. Per dimostrarlo riscriviamo in modo sintetico, indicando con $\mathfrak{F}$ la trasformata di Fourier, il teorema già dimostrato: se $s(t) = u(t)v(t)$

$$\mathfrak{F}[s(t)] = \mathfrak{F}[u(t)v(t)] = U(\omega)*V(\omega) = S(\omega)$$

se ora facciamo la trasformata (anti) dell'espressione precedente, otteniamo:

$$s(t) = u(t)v(t) = \mathfrak{F}^{-1}[U(\omega) * V(\omega)] = \mathfrak{F}^{-1}[S(\omega)]$$

se ora al posto delle due funzioni del tempo sostituiamo le loro trasformate otteniamo

$$\mathfrak{F}^{-1}[U(\omega)]\mathfrak{F}^{-1}[V(\omega)] = \mathfrak{F}^{-1}[U(\omega) * V(\omega)] \text{ cvd}$$

è ovviamente possibile dimostrare, ed è valido, il teorema inverso:

$$\mathfrak{F}[u(t)]\mathfrak{F}[v(t)] = \mathfrak{F}[u(t) * v(t)]$$

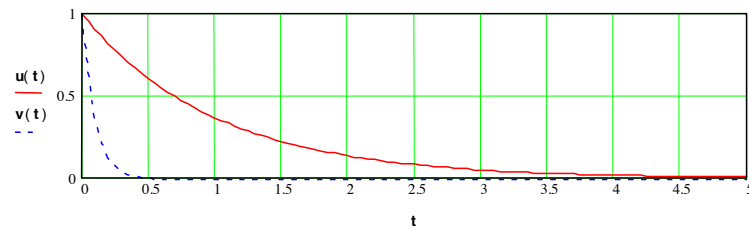
L'importanza di questo teorema è enorme in quanto molti processi fisici si presentano come convoluzione di altri e quindi tramite l'applicazione di questo teorema è possibile risalire ai processi primari. Questo metodo è noto come deconvoluzione (unfolding) delle componenti.

Convoluzione tra due segnali esponenziali con modello matematico): $s(t)=\exp(-t)$; $v(t)=\exp(-kt)$

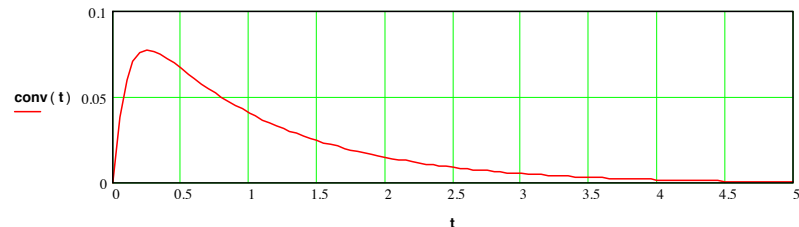
(Calcolo)

(Video-Clip)

$$u(t) := \exp(-t) \quad k := 10 \quad v(t) := \exp(-k \cdot t) \quad t := 0, .05..5 \quad a := 0, .05..5$$



$$\text{conv}(t) := \int_0^t \exp(-\lambda) \cdot \exp(-(t-\lambda) \cdot k) d\lambda \quad \text{conv}(t) := \frac{(\exp(-t) - \exp(-k \cdot t))}{(-1 + k)}$$



$$\text{spetroconv}(\omega) := \int_0^\infty \frac{(\exp(-t) - \exp(-k \cdot t))}{(k-1)} \cdot \exp(-i \cdot \omega \cdot t) dt$$

$$\text{spetroconv}(\omega) := \left| \frac{i}{((-i + \omega) \cdot (k + i \cdot \omega))} \right| \quad \text{spetroconv}(\omega) := \frac{1}{(\sqrt{1 + \omega^2} \cdot \sqrt{k^2 + \omega^2})}$$

$$U(\omega) := \int_0^\infty \exp(-t) \cdot \exp(-i \cdot \omega \cdot t) dt \quad U(\omega) := \left| \frac{-i}{(-i + \omega)} \right| \quad U(\omega) := \frac{1}{\sqrt{1 + \omega^2}}$$

$$V(\omega) := \int_0^\infty \exp(-k \cdot t) \cdot \exp(-i \cdot \omega \cdot t) dt \quad V(\omega) := \left| \frac{1}{(k + i \cdot \omega)} \right| \quad V(\omega) := \frac{1}{\sqrt{k^2 + \omega^2}}$$

$$UV(\omega) := \frac{1}{(\sqrt{1 + \omega^2} \cdot \sqrt{k^2 + \omega^2})}$$

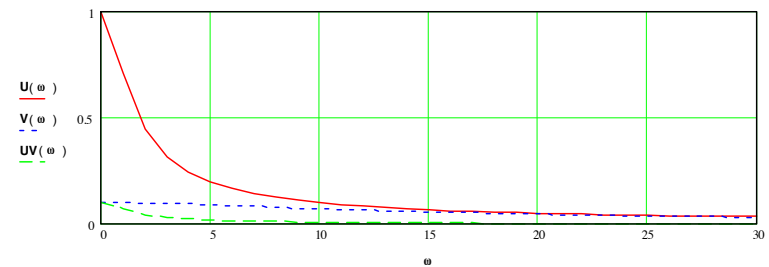
Dividendo lo spettro del prodotto di convoluzione per lo spettro di uno dei due segnali è possibile ottenere lo spettro dell'altro segnale e quindi, antitrasformando, il segnale incognito.

Questo procedimento, molto importante per lo studio dei sistemi e dei segnali, è noto come deconvoluzione (unfolding). Spesso dal punto di vista sperimentale si procede eccitando un sistema con un segnale noto e si misura, acquisendo dati relativi alla risposta del sistema, la funzione convoluzione dell'eccitazione e dell'operatore del sistema (incognito), quindi si procede alla sua determinazione attraverso il metodo di deconvoluzione che viene eseguito numericamente per mezzo della FFT (Fast Fourier Transform).

$$Y(\omega) := \frac{i}{((-i + \omega) \cdot (k + i \cdot \omega))} \quad Y(\omega) := \frac{1}{(k + i \cdot \omega)} \quad \text{la cui antitrasformata è}$$

$$v(t) := \frac{1}{2 \cdot \pi} \int_0^\infty \frac{1}{(k + i \cdot \omega)} \cdot \exp(i \cdot \omega \cdot t) d\omega$$

$$\omega := 0..3 \cdot 10^4$$



$$\Delta = x_t - x_m = (x_t - \mu) + (\mu - x_m) = \delta + \delta_m$$

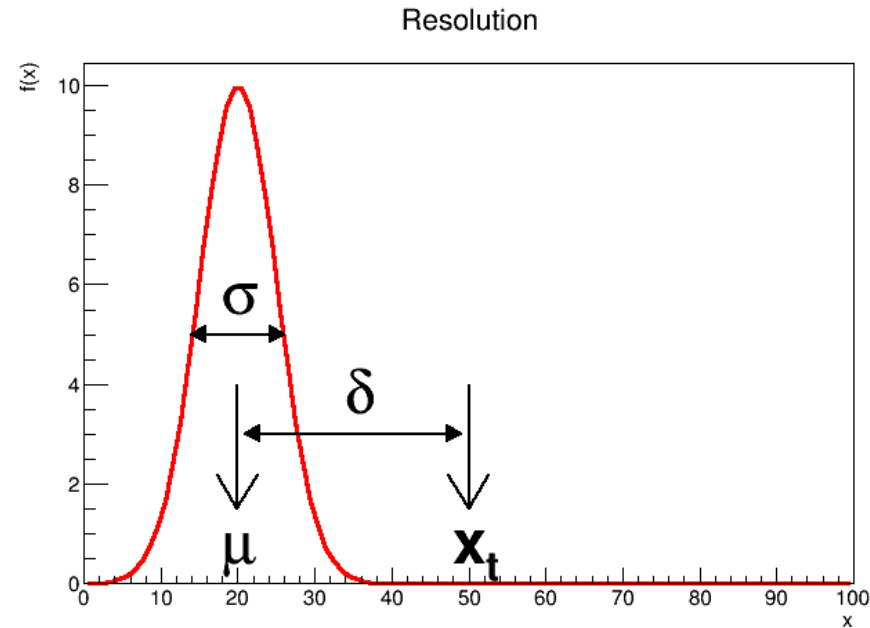


FIGURE 3. Example of a gaussian response function (red) with μ and σ and comparison with the true value x_t . δ is the systematic error. The outcome of the measurement x_m is distributed according to the apparatus response function. The length of the double arrow indicating the σ is actually the function FWHM (full width half maximum) that in case of a gaussian function is $\text{FWHM} = 2.36 \times \sigma$.

Counting

- So we do collisions at a given $\sqrt{s}$. What do we actually measure ?
- We “count” the number of times a final state is obtained. This frequency is somehow related to the probability of that final state and so it allows to measure the cross-section/decay width/branching ratios
- Connection btw probability and frequency:
 - Population $\rightarrow$ probability
 - Sample $\rightarrow$ frequency
- Sampling fluctuations

Random Variables – Outline - I

- Concept of PDF
 - Meaning and connection to actual probabilities
 - Discrete vs. real variables
 - Single vs. multiple variables: factorization
- Definitions/properties
 - Physical dimension, positivity, normalization
 - Momenta $\rightarrow$ “functional”
 - Mean, variance, standard deviation, skewness, kurtosis
 - Covariance matrix
 - Propagation

Random variables - II

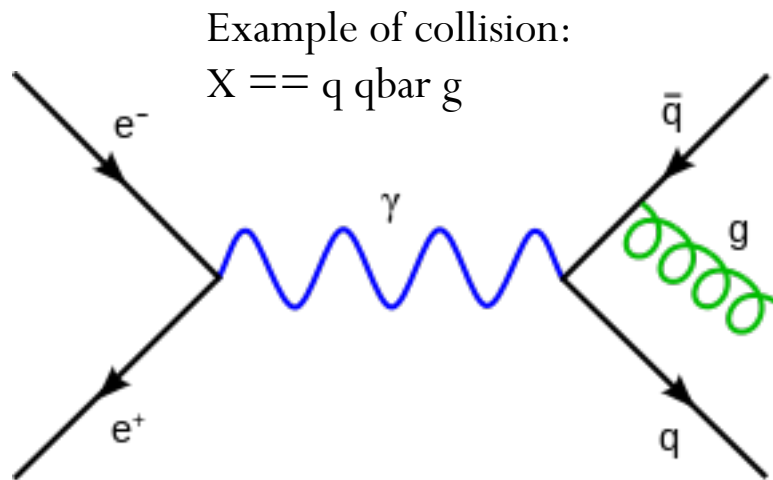
- The average and the RMS: two particular and interesting random variables, functions of random variables
- Few random variables that allow to have good statistical models of typical situations in experimental physics:
 - Binomial
 - Poissonian
 - Exponential
 - Gaussian
 - χ^2
- BUT: up to here only “populations”
- $\Rightarrow$ Statistical inference

“Logic” of an EPP experiment - I

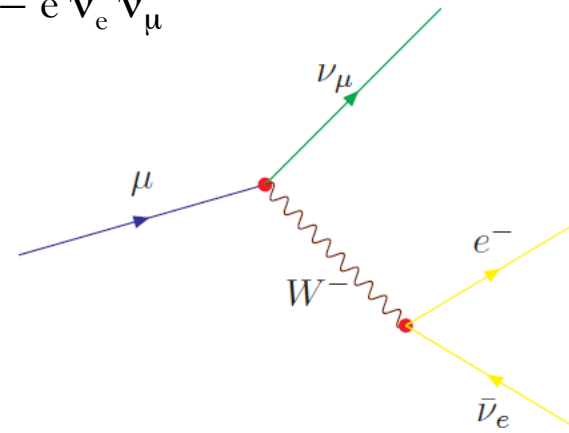
- Collision or decay: $\rightarrow$ **process to look at**
 - **Initial state** (proj. + target) OR (decaying particle);
 - **Final state** X = all particles produced
- Quadri-momentum conservation should always be at work
- In principle there is no need to measure ALL final state particles: a final state could be: $\rightarrow \mu^+ \mu^- + X$ (“inclusive” search)
- Possible final states:
 - $a + b \rightarrow a + b$: **elastic collision** (e.g. $pp \rightarrow pp$)
 - $a + b \rightarrow X$: **inelastic collision** (e.g. $pp \rightarrow pp\pi^0$)
- The experimentalist should set-up an experimental procedure to select the final state he/she searches. First of all he should be able **to count the number N_X of final states X .**

Why count ? – I

- Why count ?
- Because QFT based models allow to predict quantities (like *cross-sections*, *decay widths* and *branching ratios*, see later) that are proportional to “*how probable is*” a given final state.



Example of decay:
 $X \Rightarrow e \nu_e \nu_\mu$



Why count ? – II

- Given a collision or a decaying particle you have several possibilities, several different final states.
- So: if I have produced N initial states (either $a+b$ collisions or decaying particles), and out of them n times I observe the final state I am looking for, I can access this probability that should be $\approx n/N$
- Let's introduce the concept of **Event**:
 - The collection of all the particles of the final state from a single collision.
 - It is a collection of particles with their quadri-momenta.
 - Be careful not to overlap particles from different collisions.