

ELEMENTI DI ANALISI MATEMATICA

FUNZIONI

- **DEFINIZIONE E RAPPRESENTAZIONE**
- **FUNZIONE LINEARE (RETTA)**
- **FUNZIONE ESPONENZIALE**
- **FUNZIONE LOGARITMICA**
- **RAPPRESENTAZIONE SEMILOGARITMICA**
- **FUNZIONI TRIGONOMETRICHE**

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FUNZIONI

funzioni ad una variabile indipendente

$$y = f(x)$$

x = variabile indipendente

y = variabile dipendente

grado della funzione:

massimo esponente della variabile x

esempio

$$y = 5x^4 - 7x^2 + 8x + 12 - \frac{3}{x}$$

funzione polinomiale di 4° grado



FUNZIONI

$y = a x + b$ **funzione lineare**

$y = A e^{kx}$ **funzione esponenziale**

$y = \text{sen } x$]

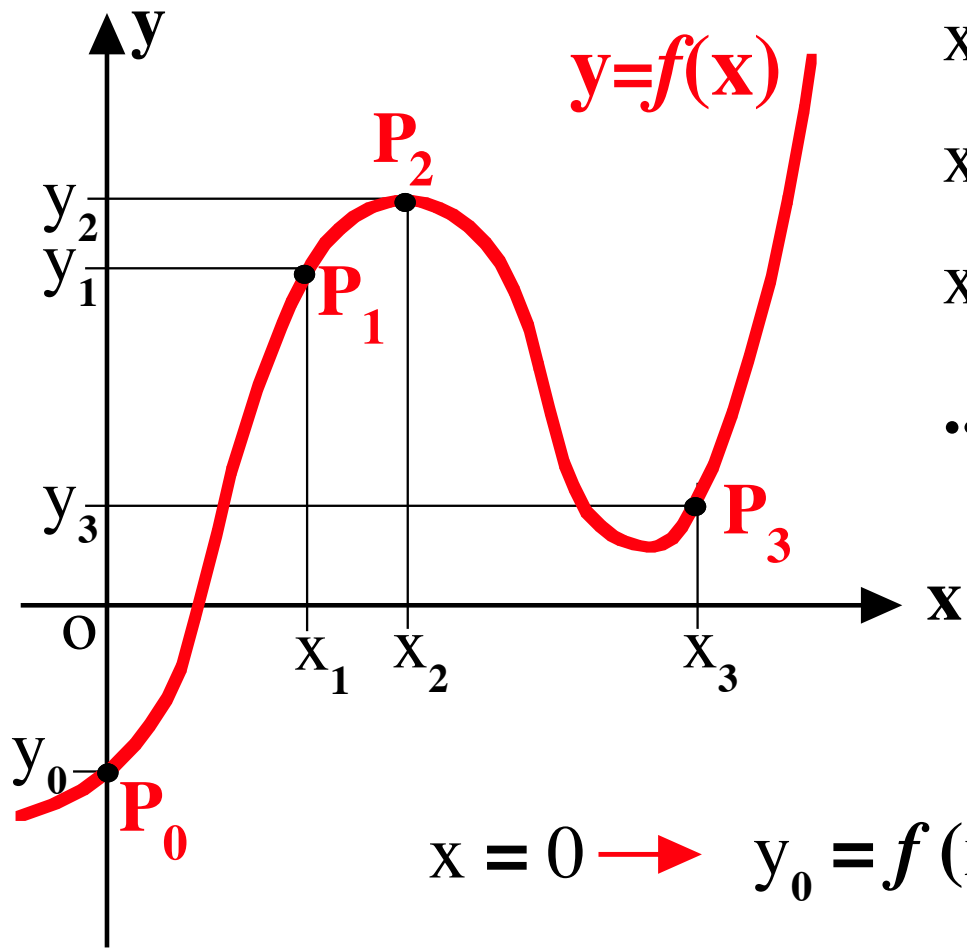
$y = \text{cos } x$]

funzioni a più variabili : $y = g(x,z)$



FUNZIONI

rappresentazione grafica: per punti



$$x = x_1 \rightarrow y_1 = f(x_1) \quad P_1(x_1, y_1)$$

$$x = x_2 \rightarrow y_2 = f(x_2) \quad P_2(x_2, y_2)$$

$$x = x_3 \rightarrow y_3 = f(x_3) \quad P_3(x_3, y_3)$$

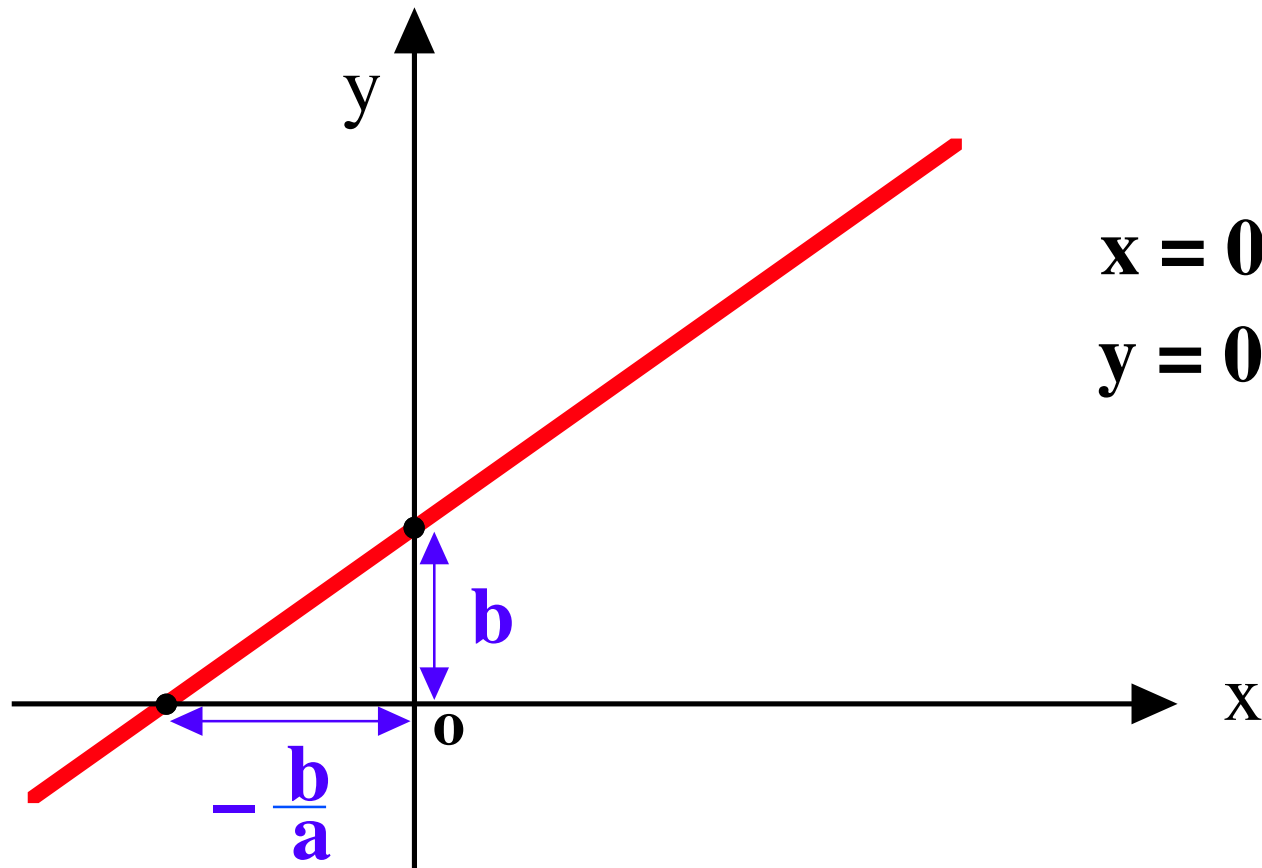
.....

$$x = 0 \rightarrow y_0 = f(x=0) \quad P_0(0, y_0)$$



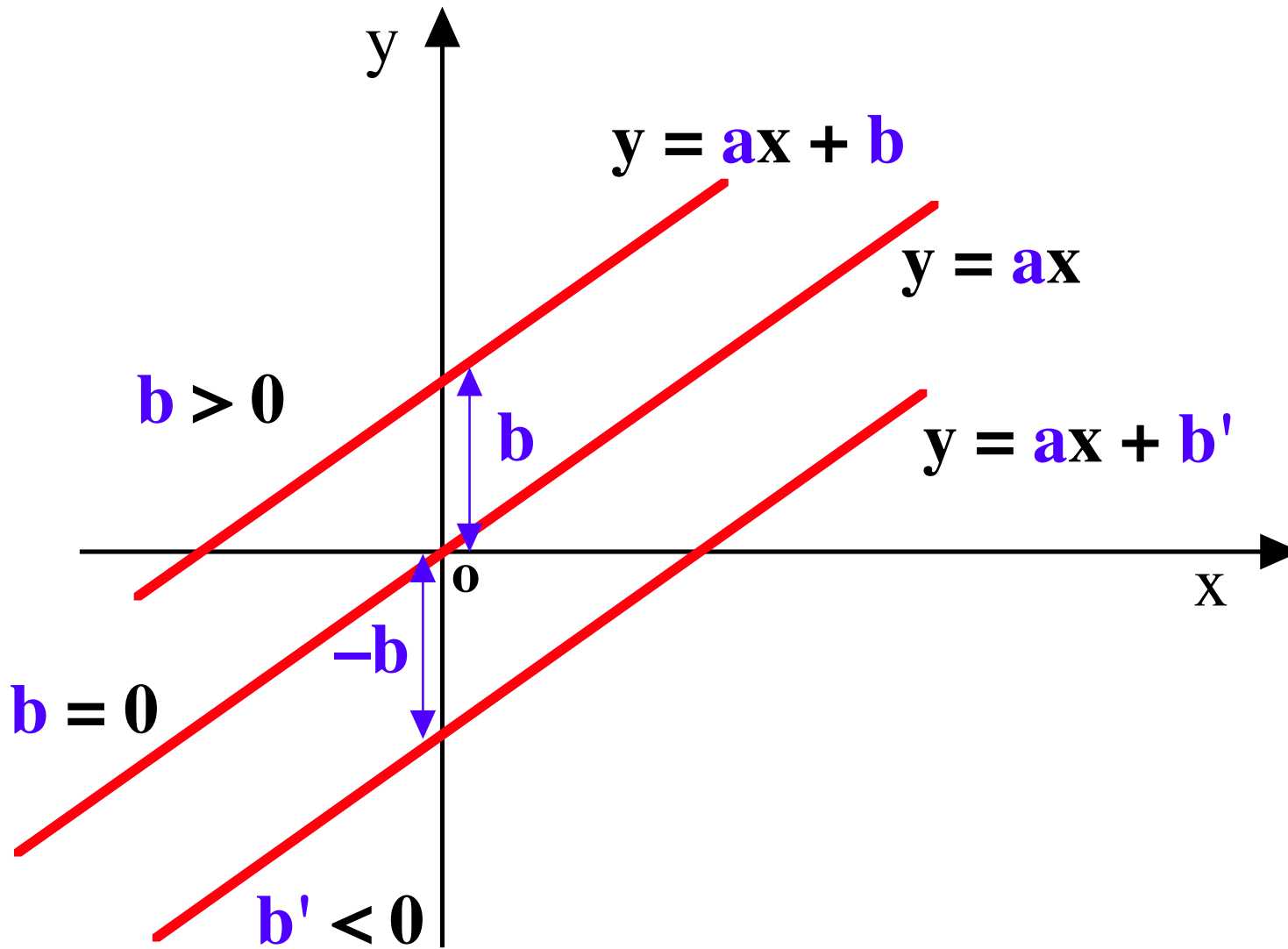
FUNZIONE LINEARE : LA RETTA

$$y = a x + b$$

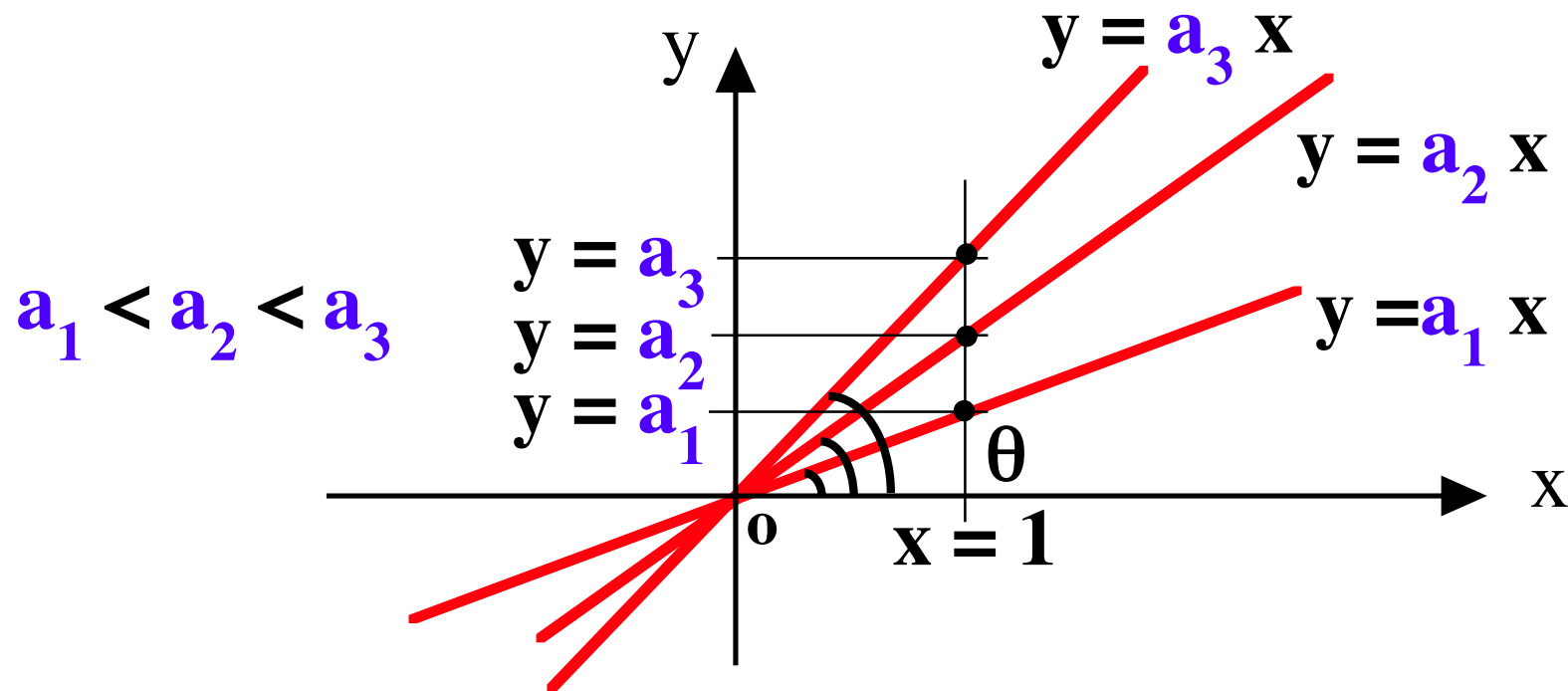


$$\begin{aligned} x = 0 &\rightarrow y = b \\ y = 0 &\rightarrow x = -\frac{b}{a} \end{aligned}$$

FUNZIONE LINEARE : LA RETTA



FUNZIONE LINEARE : LA RETTA

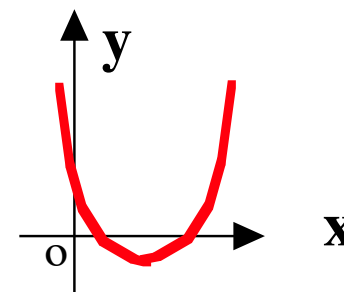


a = coefficiente angolare

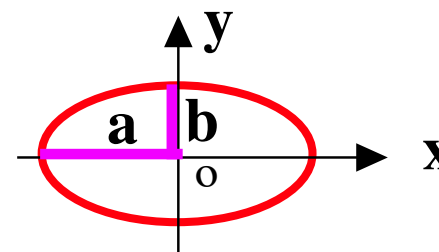
$$\theta = \operatorname{arctg} a$$

FUNZIONI QUADRATICHE

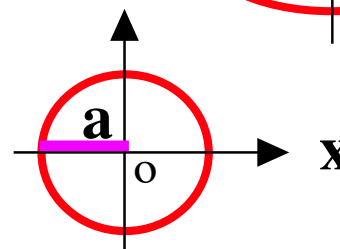
parabola : $y = a x^2 + b x + c$



ellisse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

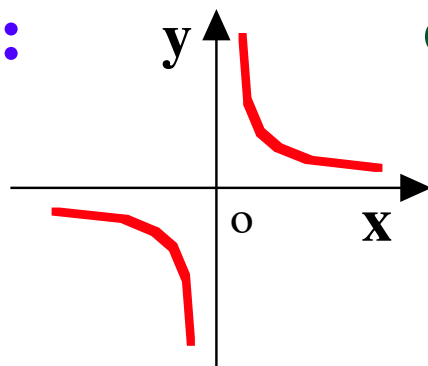


circonferenza : $x^2 + y^2 = a^2$

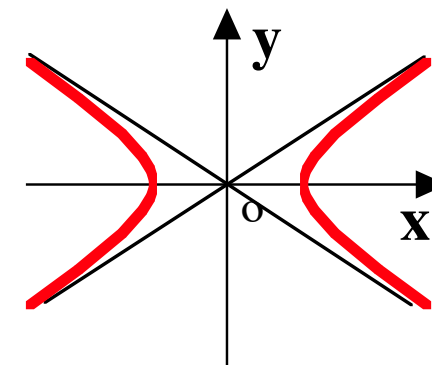


iperbole : oppure

$$y = \frac{k}{x}$$



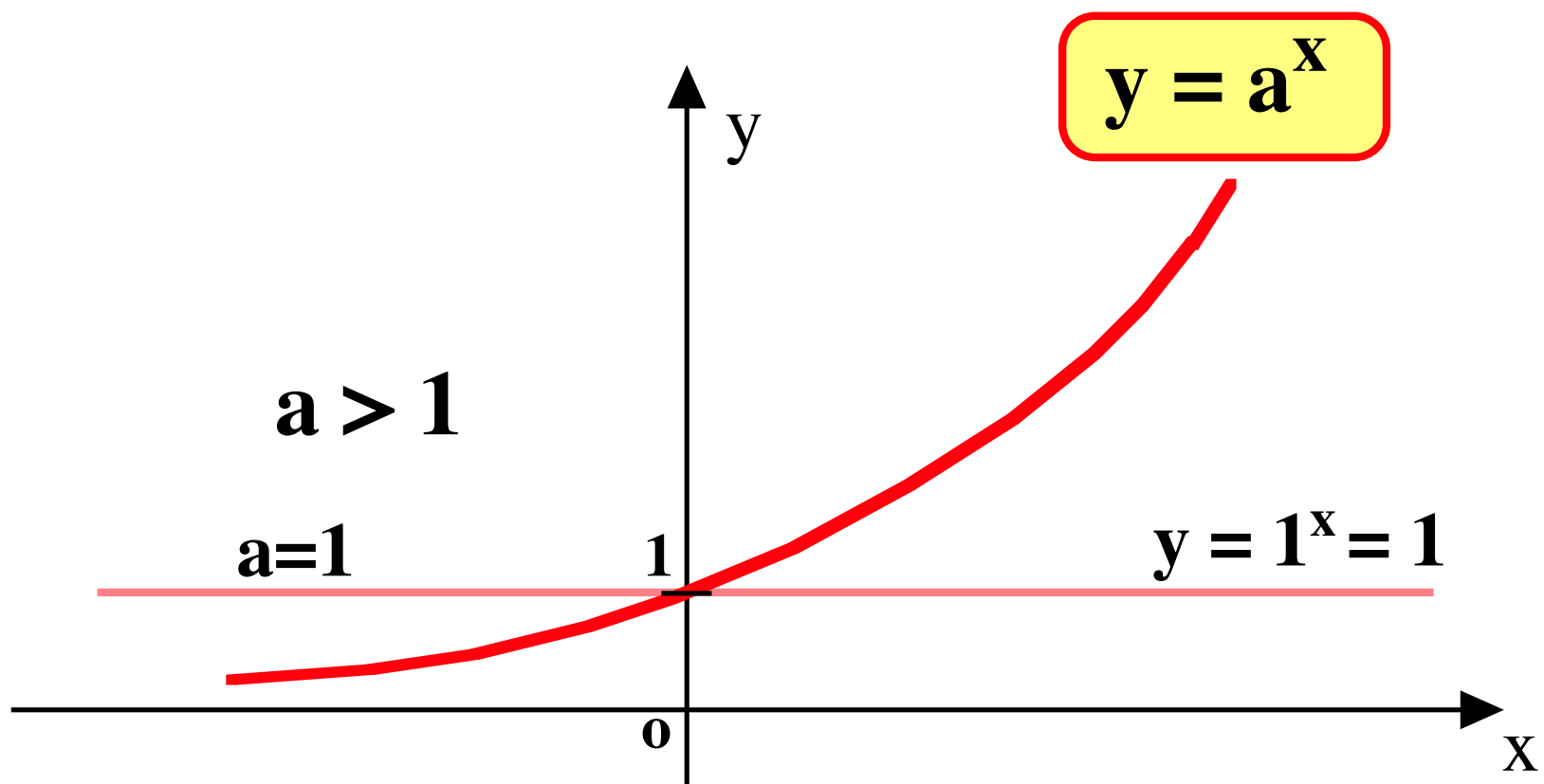
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



FUNZIONE ESPONENZIALE

funzione esponenziale con base **a**

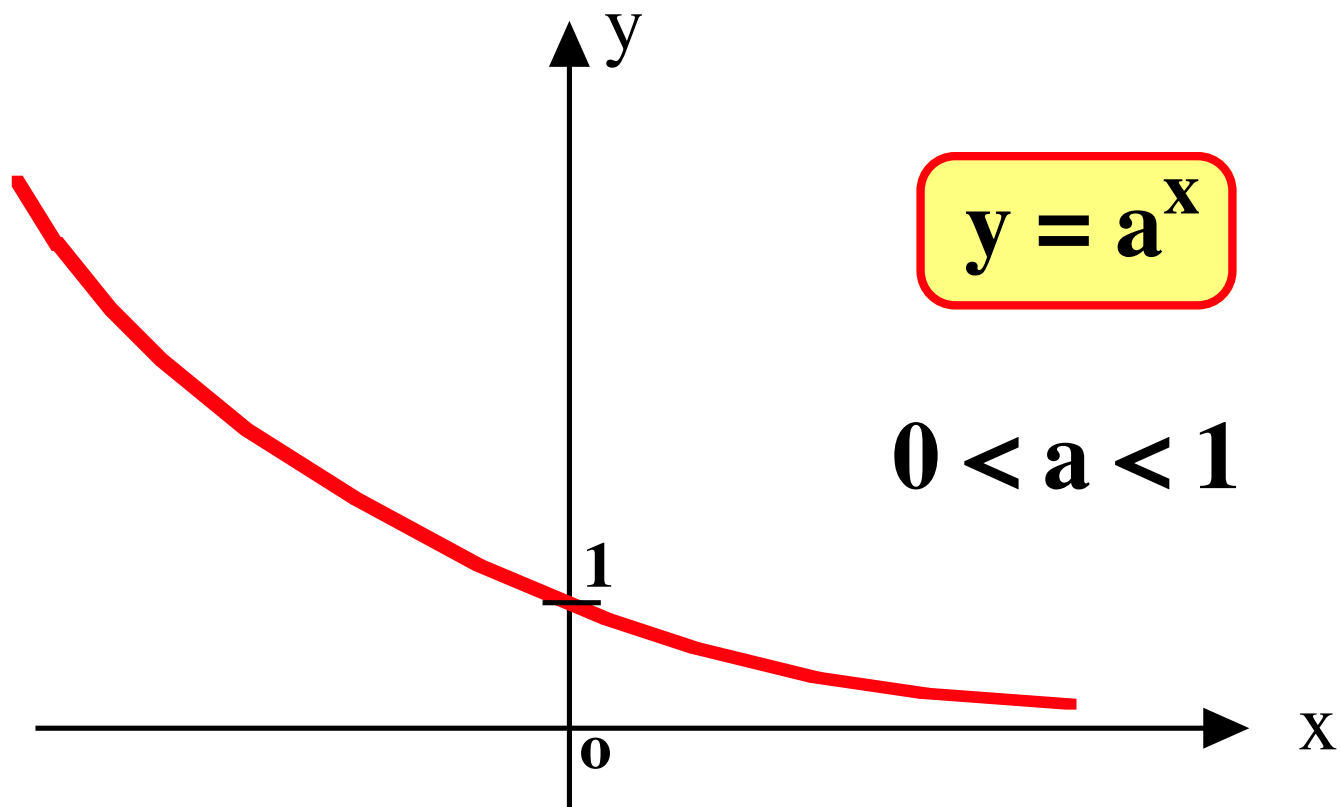
a = numero reale positivo maggiore di 1



FUNZIONE ESPONENZIALE

funzione esponenziale con base a

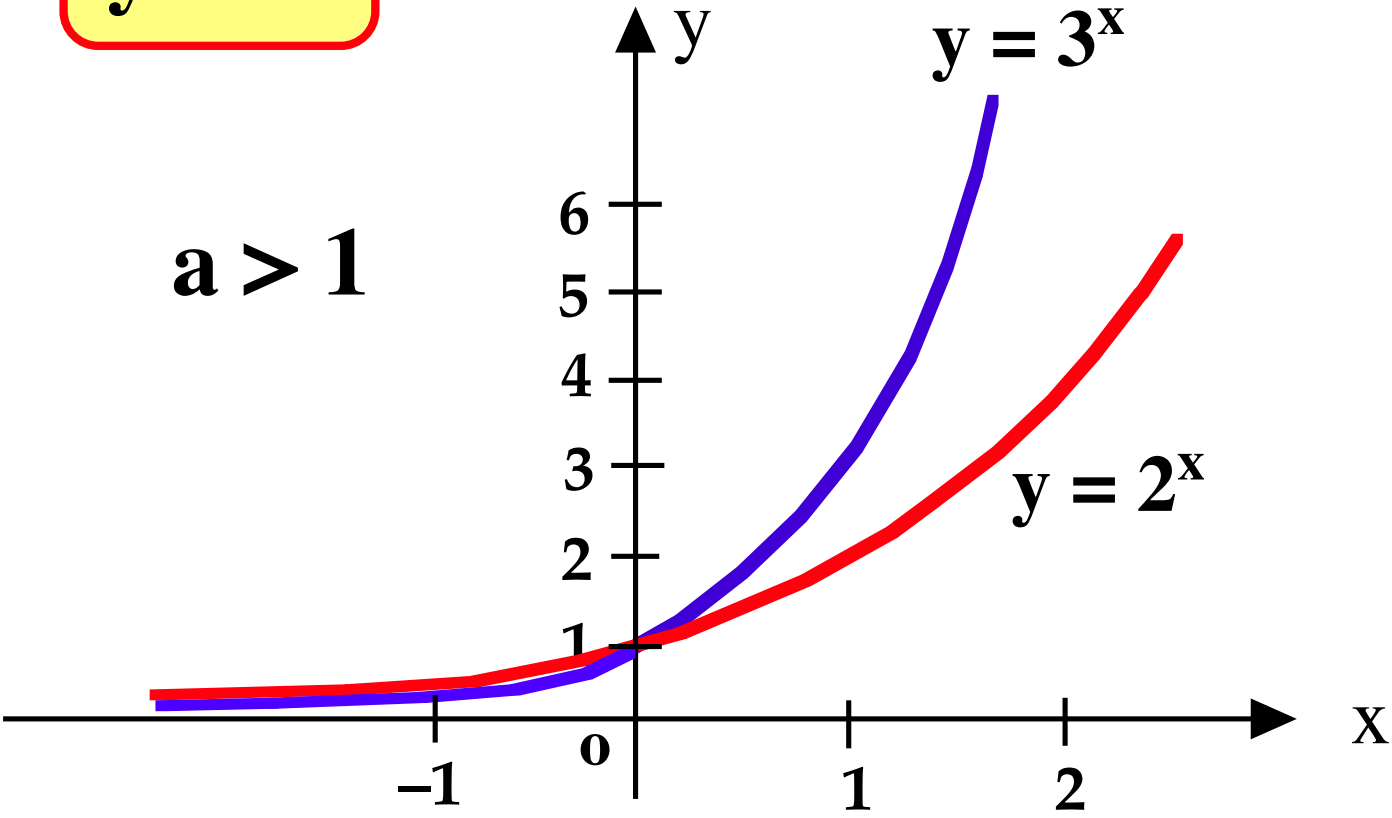
a = numero reale positivo compreso tra 0 e 1



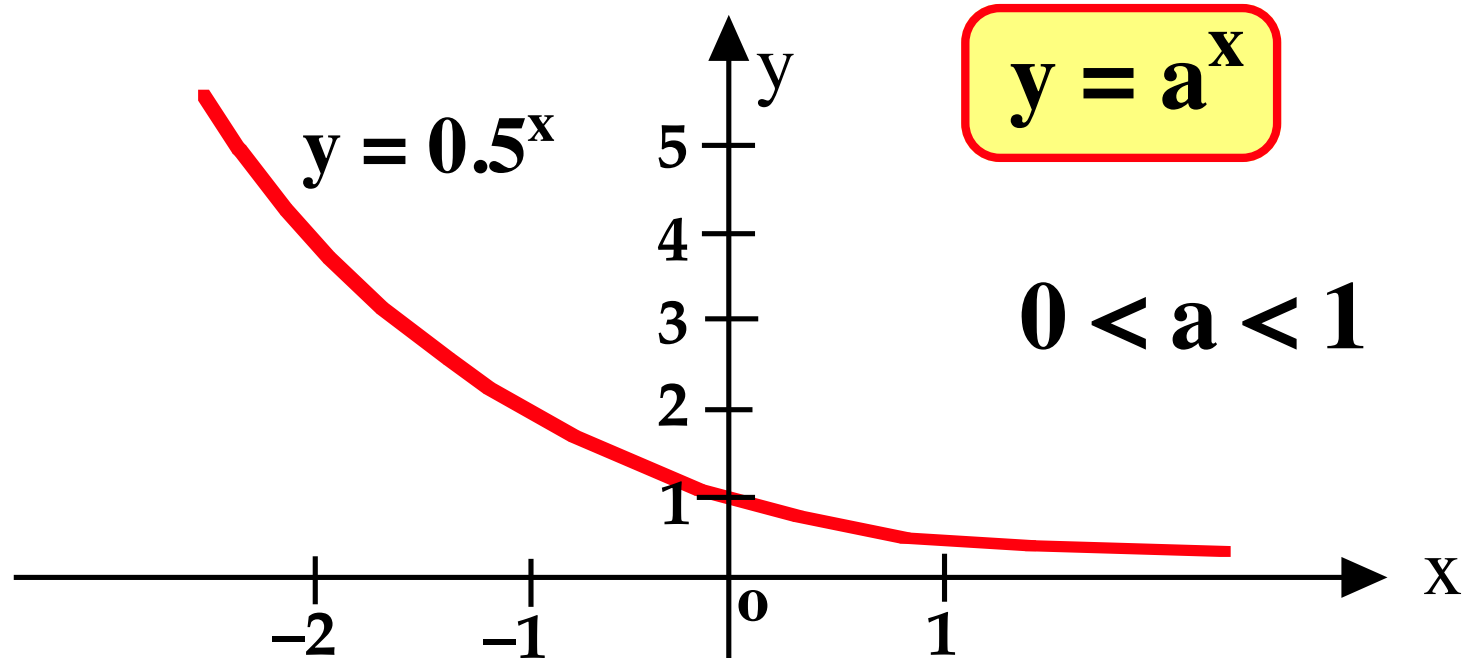
FUNZIONE ESPONENZIALE

$$y = a^x$$

$a > 1$



FUNZIONE ESPONENZIALE



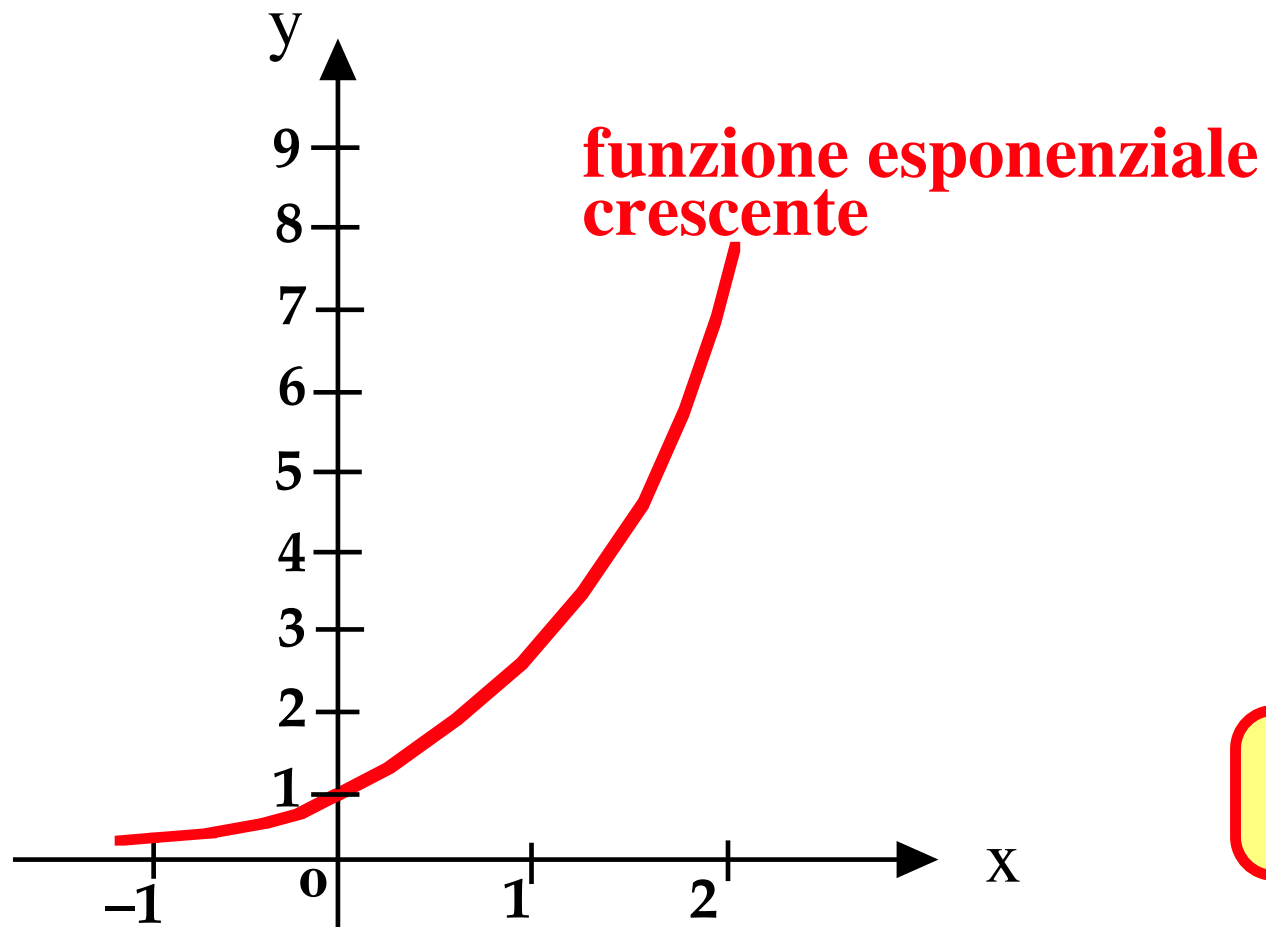
$$y = 0.5^x = \left(\frac{1}{2}\right)^x = \frac{1}{2^x} = 2^{-x}$$

$$y = a^x = b^{-x} \quad b = 1/a$$

FUNZIONE ESPONENZIALE

$$a \equiv e = 2.718\dots$$

$$y = e^x$$



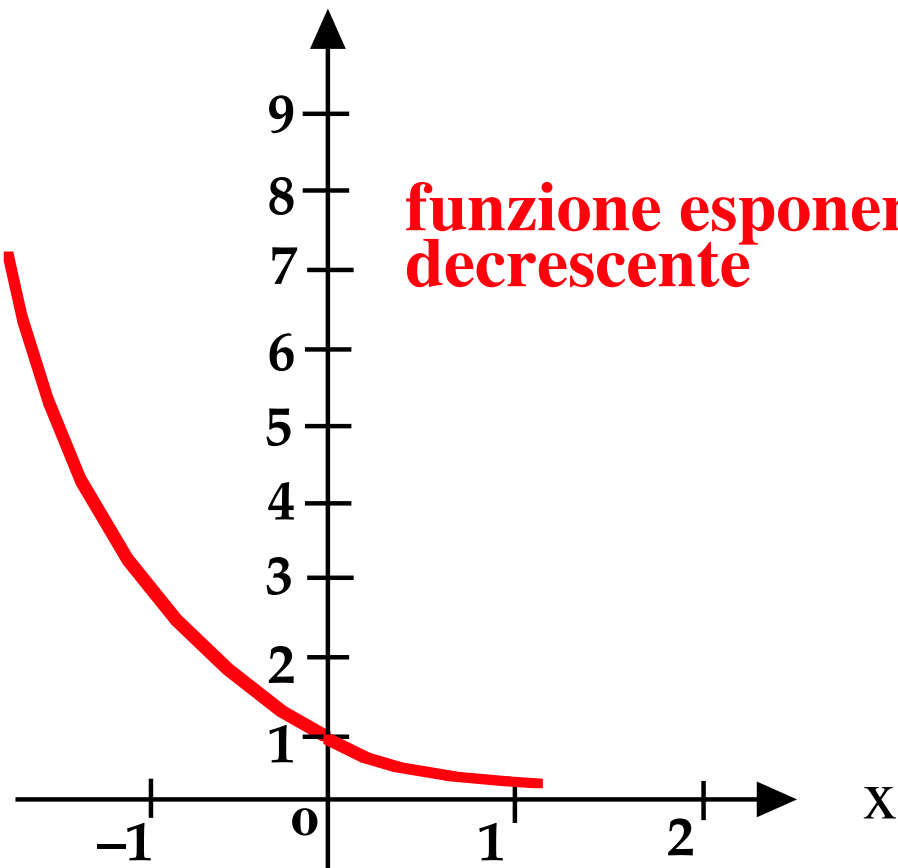
in generale

$$y = A e^{Bx}$$

FUNZIONE ESPONENZIALE

$e = 2.718\dots$

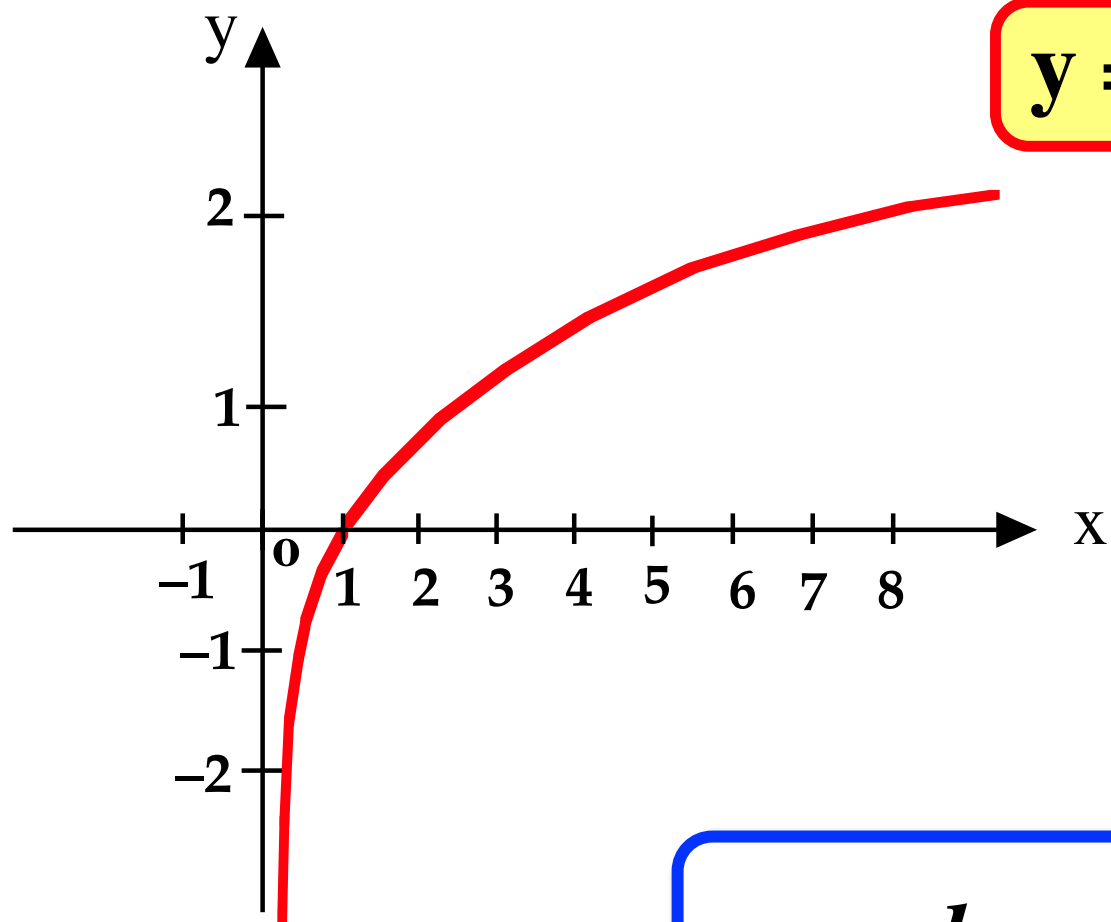
$$y = e^{-x}$$



in generale

$$y = A e^{-Bx}$$

FUNZIONE LOGARITMICA

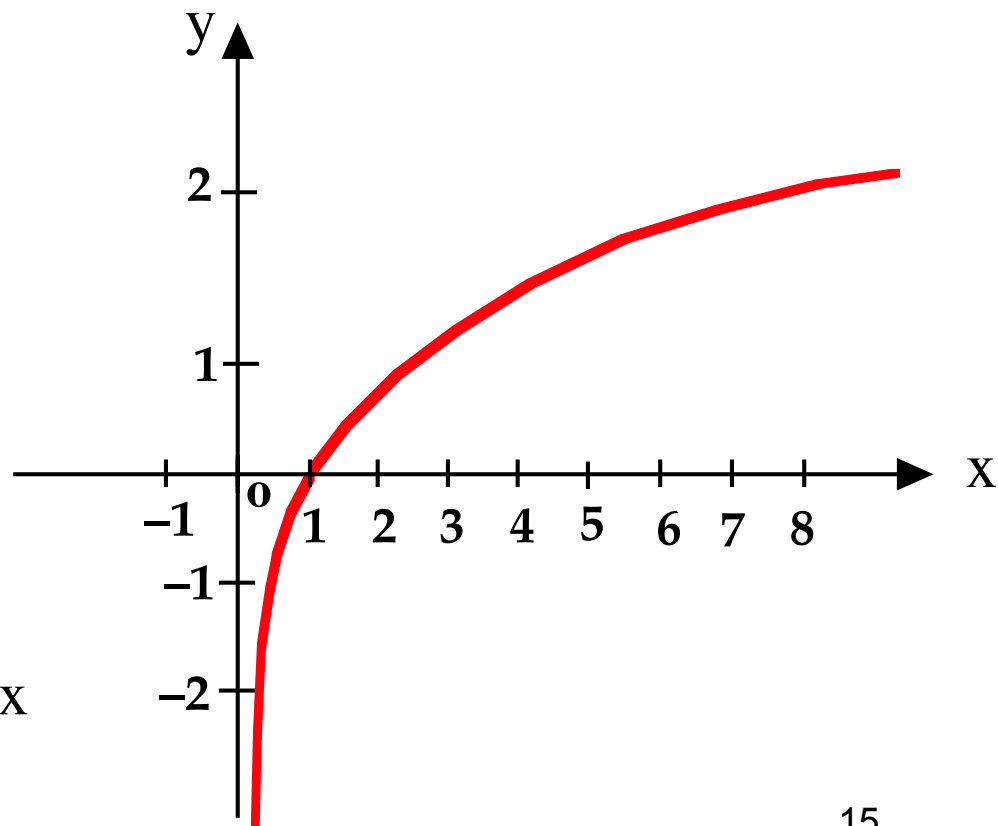
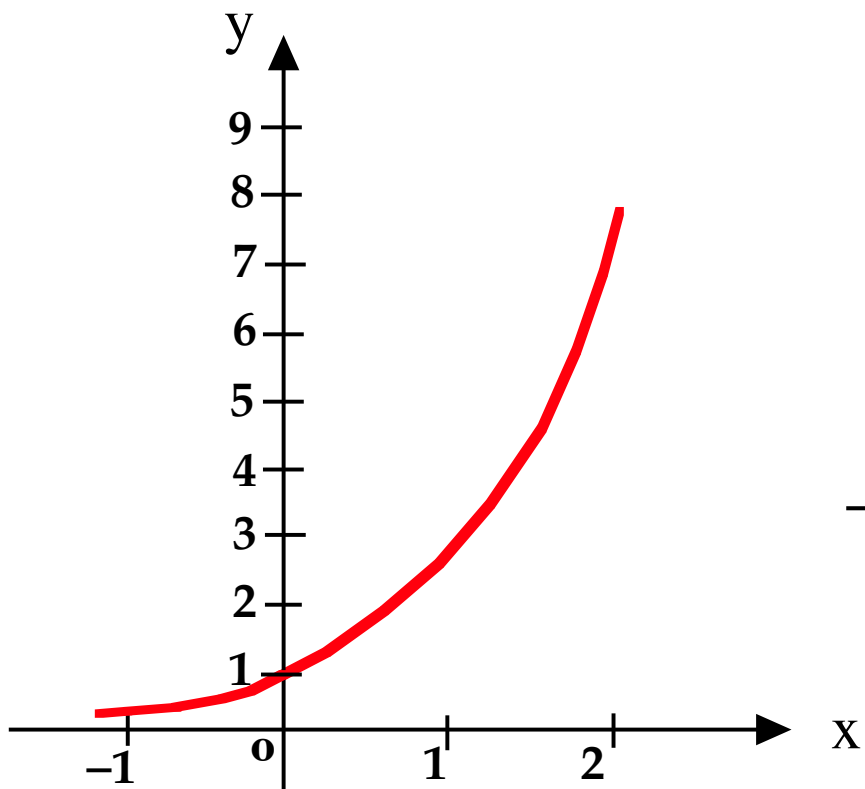


$$y = \ln x$$

$$y = \ln x \longrightarrow e^y = x$$

FUNZIONE LOGARITMICA

$$y = \ln x \longrightarrow e^y = x$$



RAPPRESENTAZIONE SEMI-LOGARITMICA

$$y = A e^{Bx}$$

rappresentazione grafica

carta semilogaritmica : $\begin{cases} Y = \text{Log } y \\ X = x \end{cases}$

$$Y = \text{Log } y = \text{Log } (A e^{Bx}) = \text{Log } A + \text{Log } (e^{Bx}) =$$

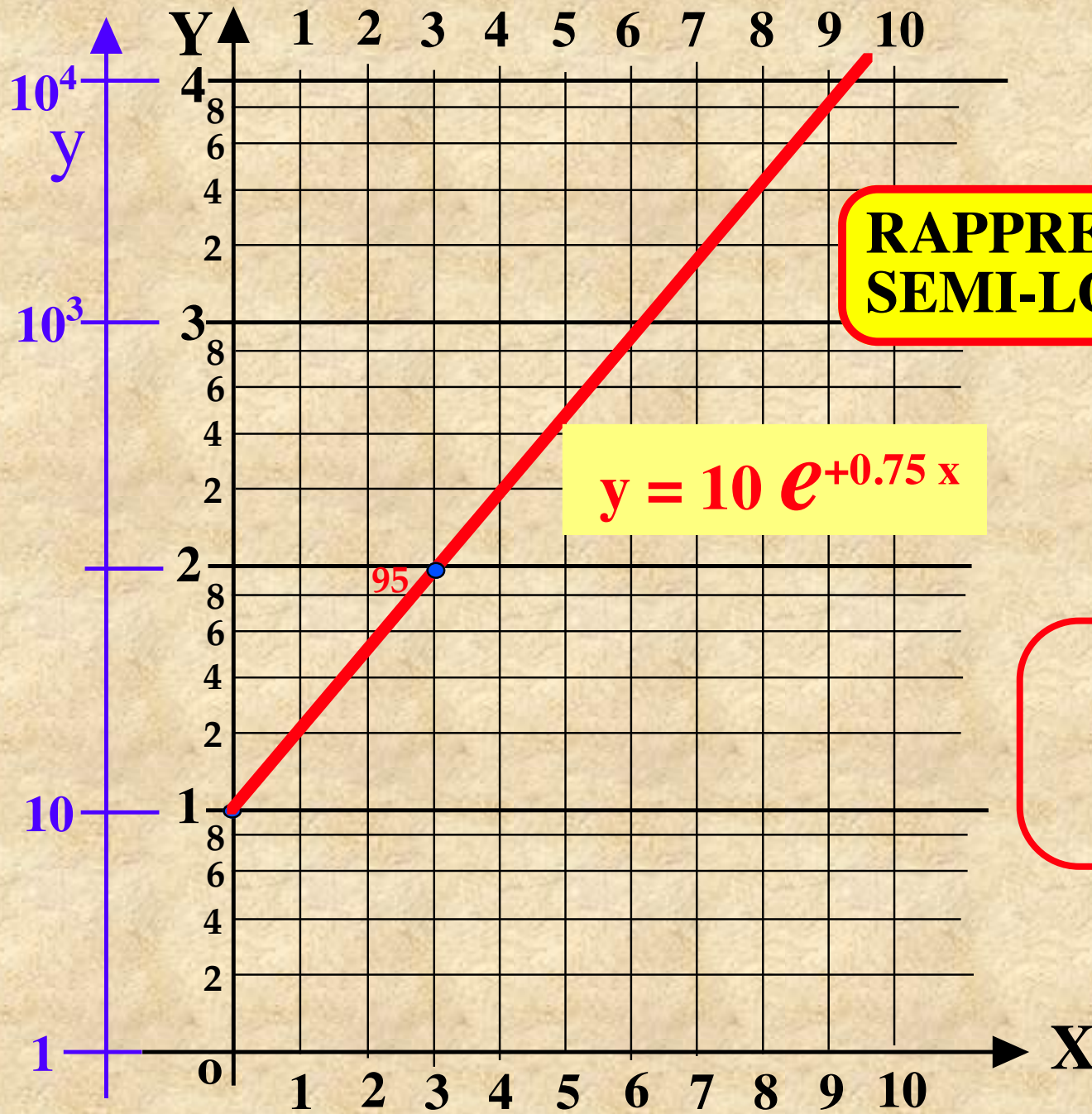
↓

$$= \text{Log } A + B x \text{Log } e =$$

$$Y = aX + b$$

funzione lineare (retta) $a = B \text{Log } e$ $b = \text{Log } A$

funzione esponenziale \longrightarrow retta

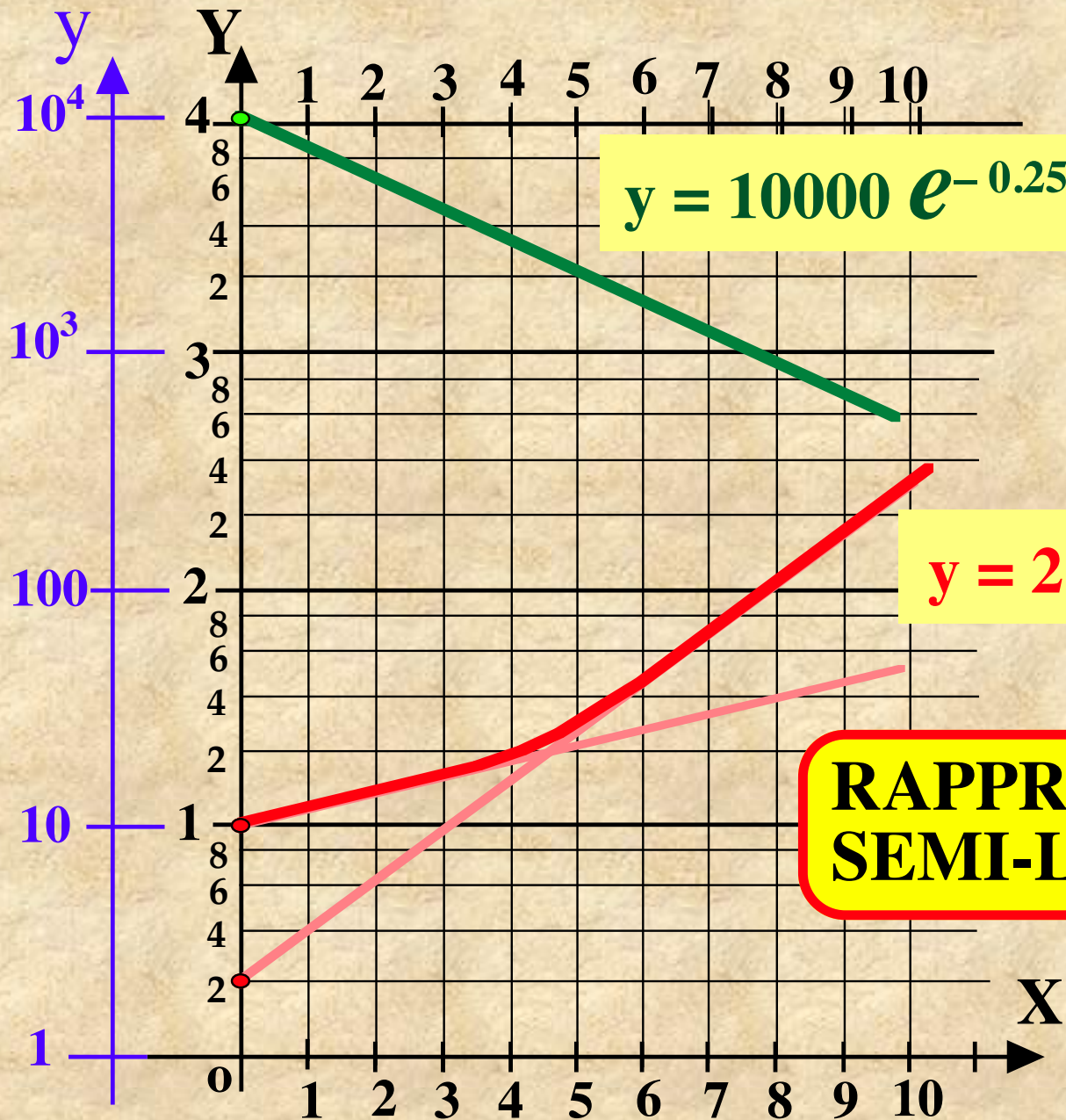


RAPPRESENTAZIONE SEMI-LOGARITMICA

$y = 10 e^{+0.75 x}$

$Y = \text{Log } y$
 $X = x$





$y = 10000 e^{-0.25 x}$

$Y = \text{Log } y$
 $X = x$

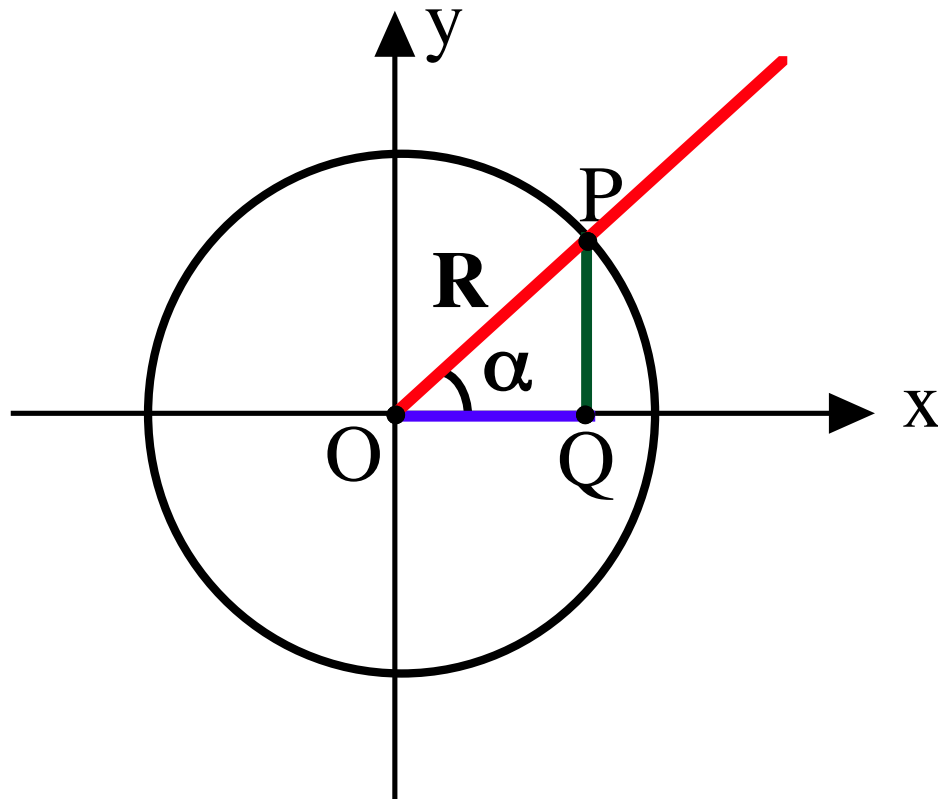
$y = 2 e^{0.5 x} + 10 e^{0.15 x}$

RAPPRESENTAZIONE SEMI-LOGARITMICA



FUNZIONI TRIGONOMETRICHE

funzioni seno e coseno



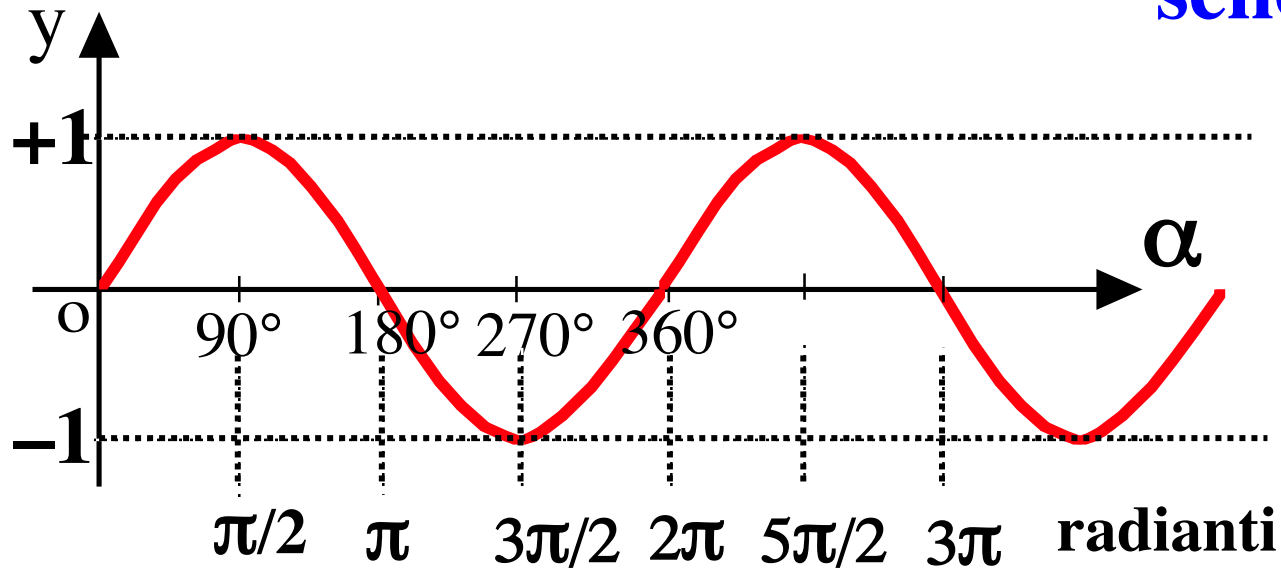
$$\text{sen } \alpha = \frac{\overline{PQ}}{\overline{OP}} = \frac{\overline{PQ}}{R}$$

$$\text{cos } \alpha = \frac{\overline{OQ}}{\overline{OP}} = \frac{\overline{OQ}}{R}$$

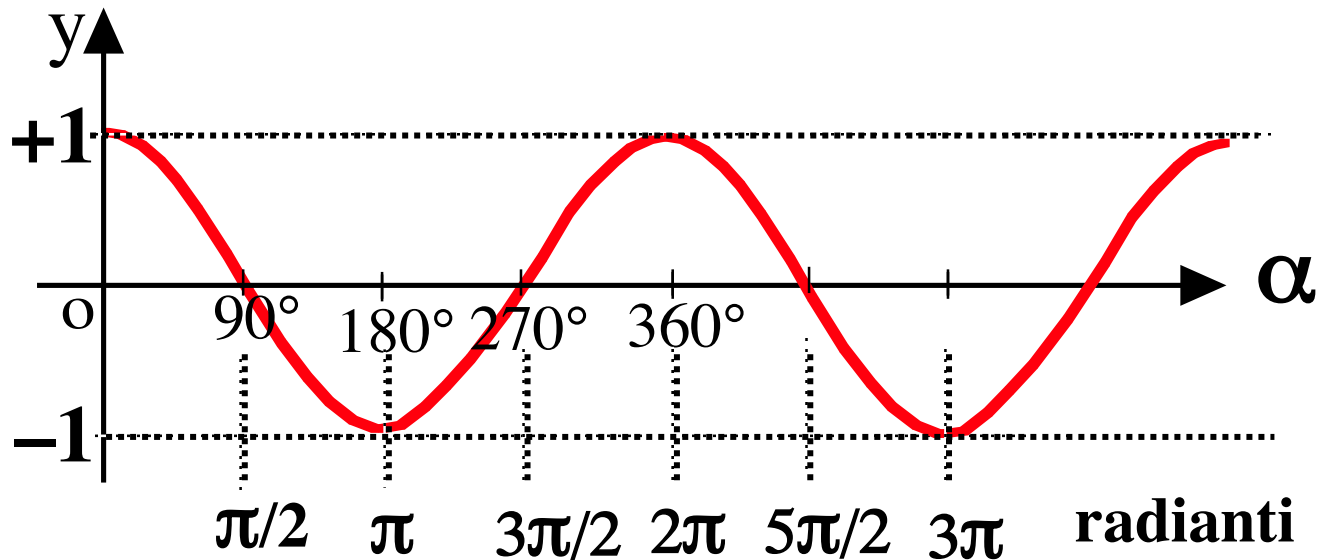
$$\frac{\overline{PQ}^2}{\overline{OP}^2} + \frac{\overline{OQ}^2}{\overline{OP}^2} = \frac{\overline{OP}^2}{\overline{OP}^2} = 1 = \text{sen}^2 \alpha + \text{cos}^2 \alpha$$

FUNZIONI TRIGONOMETRICHE

seno e coseno



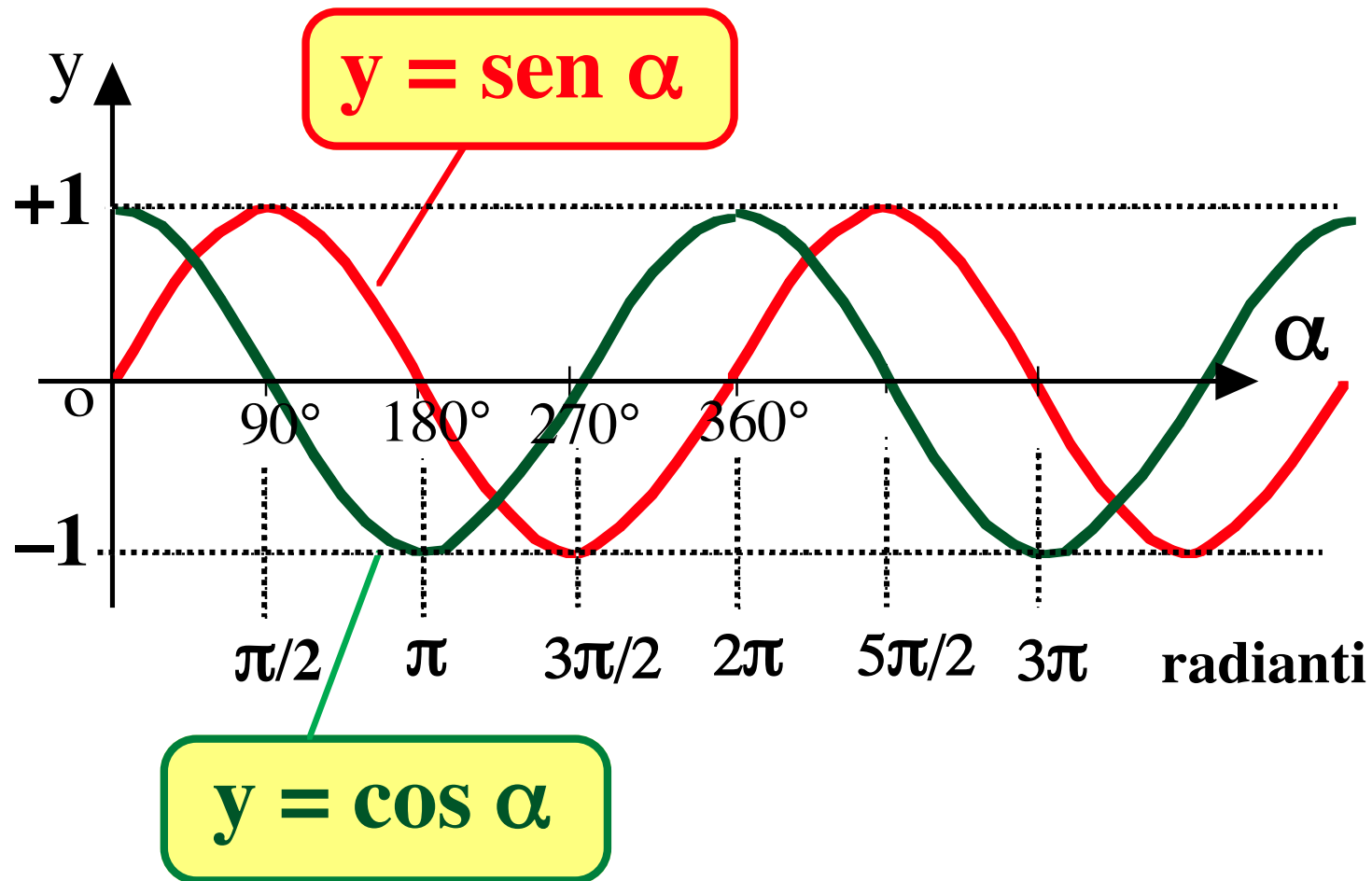
$$y = \sin \alpha$$



$$y = \cos \alpha$$

FUNZIONI TRIGONOMETRICHE

seno e coseno



FUNZIONI TRIGONOMETRICHE

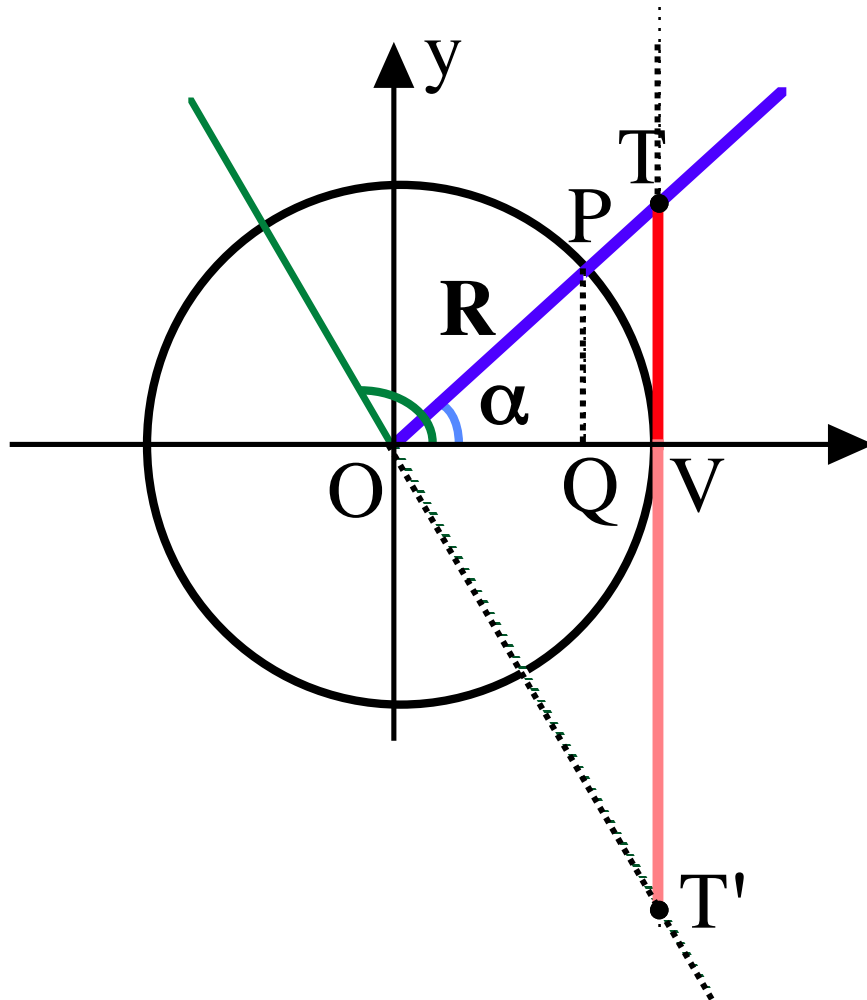
seno - coseno - tangente

α	sen α	cos α	tg $\alpha = \frac{\text{sen } \alpha}{\text{cos } \alpha}$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	∞



FUNZIONI TRIGONOMETRICHE

tangente



$$\operatorname{tg} \alpha = \frac{\overline{TV}}{\overline{OV}} = \frac{\overline{PQ}}{\overline{OQ}} = \frac{\operatorname{sen} \alpha}{\operatorname{cos} \alpha}$$

\parallel
 R

$$90^\circ < \alpha < 180^\circ$$



$$\operatorname{tg} \alpha = \frac{\overline{T'V}}{R} < 0$$

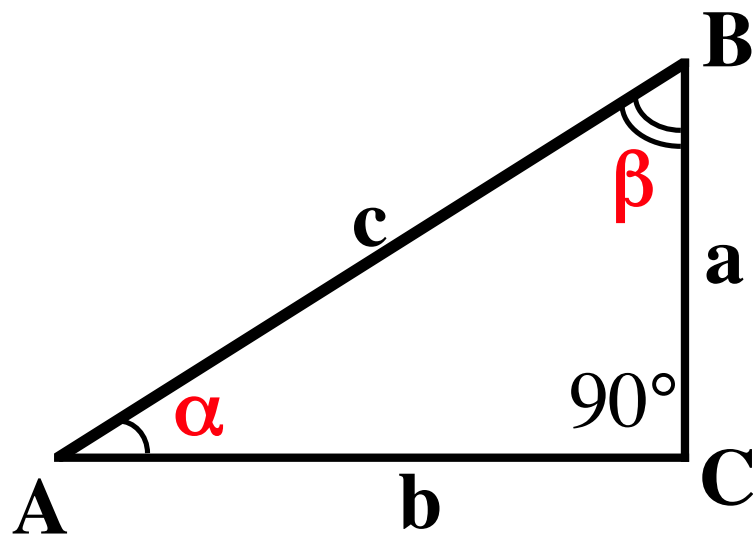
$$-\infty < \operatorname{tg} \alpha < +\infty$$



FUNZIONI TRIGONOMETRICHE

applicazioni

risoluzione triangolo rettangolo



- $a = c \operatorname{sen} \alpha = c \cos \beta$
- $b = c \cos \alpha = c \operatorname{sen} \beta$
- $\operatorname{tg} \alpha = \frac{a}{b}$
- $\operatorname{tg} \beta = \frac{b}{a}$
- $a^2 + b^2 = c^2$

FUNZIONI TRIGONOMETRICHE

applicazioni

relazioni trigonometriche utili

- $\sin^2 \alpha + \cos^2 \alpha = 1$
- $\cos(-\alpha) = \cos \alpha$
- $\frac{1}{\operatorname{tg} \alpha} = \operatorname{cotg} \alpha$
- $\frac{1}{\cos \alpha} = \sec \alpha$
- $\sin(-\alpha) = -\sin \alpha$
- $\operatorname{tg}(-\alpha) = -\operatorname{tg} \alpha$
- $\frac{1}{\sin \alpha} = \operatorname{cosec} \alpha$

- α molto piccolo:

$$\sin \alpha \approx \alpha$$

$$\cos \alpha \approx 1$$

$$\operatorname{tg} \alpha \approx \alpha$$

angoli α in radianti

FUNZIONI TRIGONOMETRICHE

applicazioni

relazioni trigonometriche utili

- **funzioni trigonometriche inverse:**

$$a = \text{sen } \alpha \longrightarrow \alpha = \text{arcsen } a$$

$$a = \text{cos } \alpha \longrightarrow \alpha = \text{arcos } a$$

$$a = \text{tg } \alpha \longrightarrow \alpha = \text{arctg } a$$

angoli α in radianti

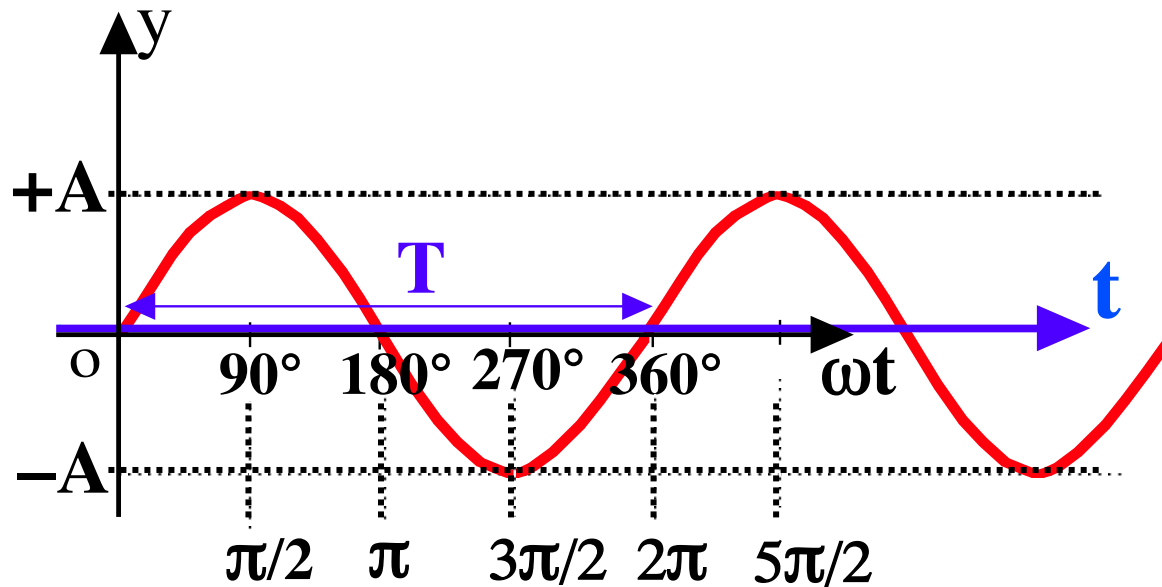


FUNZIONI TRIGONOMETRICHE

- applicazioni*
- moto armonico,
 - fenomeni ondulatori

$$y = A \text{ sen } \omega t$$

α



■ $\omega =$ pulsazione

■ $T =$ periodo

$$\omega(t+T) - \omega t = 2\pi \longrightarrow \omega T = 2\pi \longrightarrow \omega = \frac{2\pi}{T} = 2\pi \nu$$

$$\frac{1}{T} = \nu = \text{frequenza}$$

