Interferometric methods applied to neutral kaon pairs at a $\phi$-factory offer unique possibilities to perform fundamental tests of discrete symmetries, as well as of the basic principles of quantum mechanics. In this paper a general review on neutral kaon interferometry at a $\phi$-factory is given. The most recent results obtained by the KLOE experiment at the DAΦNE $e^+e^-$ collider, the Frascati $\phi$-factory, are reviewed. A recent proposal for continuing the KLOE physics program (KLOE-2) at an upgraded DAΦNE machine is discussed in this context.

1 Introduction

The neutral kaon doublet is one of the most intriguing systems in nature. During its time evolution a neutral kaon oscillates between its particle and antiparticle states with a beat frequency $\Delta m \approx 5.3 \times 10^9 \text{s}^{-1}$, where $\Delta m$ is the small mass difference between the exponentially decaying states $K_L$ and $K_S$. The fortunate coincidence that $\Delta m$ is about half the decay width of $K_S$ makes possible to observe a variety of intricate interference phenomena in the production and decay of neutral kaons. In turn, such observations enable us to test the linear superposition principle of quantum mechanics, the interplay of different conservation laws and the validity of various symmetry principles.

A unique feature of a $\phi$-factory is the production of neutral kaon pairs in a pure quantum state with the consequent possibility to study quantum interference effects, and to have pure monochromatic tagged $K_S$ and $K_L$ beams. Besides the possibility to measure to high accuracy most, if not all, of the properties of the kaon system, the correlation between the two kaons could open up new horizons in the study of discrete symmetries and of the basic principles of quantum mechanics. For instance a possible violation of the $CPT$
symmetry\(^1\) (where \(C\) is charge conjugation, \(P\) is parity, and \(T\) is time reversal) could manifest in conjunction with tiny modifications of the initial correlation, decoherence effects, or Lorentz symmetry violations, which, in turn, might be justified in a quantum theory of gravity. At a \(\phi\)-factory the sensitivity to some observable effects can reach the level of the interesting Planck’s scale region, i.e. \(\mathcal{O}(m_K^2/M_{Planck}) \sim 2 \times 10^{-20}\) GeV, which is a very remarkable level of accuracy, presently unreachable in other similar systems (e.g. the B meson system). Moreover recent theoretical studies demonstrated that entangled neutral kaons at a \(\phi\)-factory are suitable to test the foundations of quantum mechanics, such as Bohr’s complementarity principle, the quantum erasure and marking concepts, and the coherence of states over macroscopic distances, while for the more classical test using Bell’s inequalities, new ideas have been put forward. Therefore neutral kaon interferometry constitutes a powerful tool and a very attractive opportunity to be fully exploited at a \(\phi\)-factory.

This paper is organized as follows: a brief introduction on the neutral kaon system is given in Sects. 2 and 3; the basic concepts of neutral kaon interferometry and the description of the most important “standard” tests on discrete symmetries that can be performed at a \(\phi\)-factory are reviewed in Sect. 4; a brief introduction is given on possible tests of quantum mechanics (Sect. 5), decoherence and \(CPT\) violation effects that could be induced in a quantum gravity framework (Sect. 6), and \(CPT\) and Lorentz symmetry violation effects (Sect. 7). Detailed reviews on these subjects can be found in the other contributions of this handbook. The most recent results of the KLOE experiment at DAΦNE, the Frascati \(\phi\)-factory, are reviewed in Sect. 8; finally, the improved sensitivities and prospects for the proposed KLOE-2 experiment are discussed in Sect. 9.

\(^1\)The \(CPT\) theorem\(^1, 2, 3, 4\) ensures that exact \(CPT\) invariance holds for any quantum field theory assuming (1) Lorentz invariance, (2) Locality, and (3) Unitarity. Testing the validity of \(CPT\) invariance therefore probes the most fundamental assumptions of our present understanding of particles and their interactions.
2 The neutral kaon system

The time evolution of a neutral kaon that is initially a generic superposition of $K^0$ and $\bar{K}^0$,

$$|K(0)⟩ = a(0)|K^0⟩ + b(0)|\bar{K}^0⟩,$$

(1)
can be described by the state vector

$$|K(t)⟩ = a(t)|K^0⟩ + b(t)|\bar{K}^0⟩ + \sum_j c_j(t)|f_j⟩,$$

(2)
where $t$ is the time in the kaon rest frame, $f_j$'s with $\{j = 1, 2, \ldots\}$ represent all possible decay final states, and $a(t)$, $b(t)$, and $c_j(t)$ are time dependent functions. In the Wigner-Weisskopf approximation 5), which is valid for times larger than the typical strong interaction formation time, the functions $a(t)$ and $b(t)$, describing the time evolution of the state in the $\{K^0, \bar{K}^0\}$ sub-space, obey the Schrödinger-like equation

$$i \frac{∂}{∂t} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = H \begin{pmatrix} a(t) \\ b(t) \end{pmatrix},$$

(3)
where the effective Hamiltonian $H$ is a $2 \times 2$ complex, not Hermitian, and time independent matrix. It can be decomposed in terms of its hermitian and anti-hermitian parts

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = M - \frac{i}{2} \Gamma = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix},$$

(4)
where $M$ and $\Gamma$ are two hermitian matrices with positive eigenvalues, usually called mass and decay matrices, and indices 1 and 2 stand for $K^0$ and $\bar{K}^0$, respectively.

The true Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{wk}$, where $\mathcal{H}_0$ governs the strong and electromagnetic interactions and conserve strangeness ($\mathcal{H}_0|K^0⟩ = M_0|K^0⟩$, $\mathcal{H}_0|\bar{K}^0⟩ = M_0|\bar{K}^0⟩$, $S|K^0⟩ = |K^0⟩$, $S|\bar{K}^0⟩ = -|\bar{K}^0⟩$), while $\mathcal{H}_{wk}$ is a small perturbation governing weak interactions and not conserving strangeness, is related to the effective Hamiltonian $H$ as follows:

$$M_{ij} = M_0δ_{ij} + ⟨i|\mathcal{H}_{wk}|j⟩ + \mathcal{P} \sum_f \left( \frac{⟨i|\mathcal{H}_{wk}|f⟩⟨f|\mathcal{H}_{wk}|j⟩}{M_0 - E_f} \right)$$

(5)
\[ \Gamma_{ij} = 2\pi \sum_f \langle i | \mathcal{H}_{wk} | f \rangle \langle f | \mathcal{H}_{wk} | j \rangle \delta(M_0 - E_f) \]  

(6)

where \( i, j = 1, 2 \), \( \mathcal{P} \) stands for the principal part, and the intermediate states \( f \) correspond to virtual (\( \mathcal{M} \)) or real (\( \Gamma \)) decay channels.

The matrix \( \mathbf{H} \) is characterized by eight independent real parameters; seven of them are observables, while one phase is arbitrary and unphysical. In fact the flavor symmetry of the strong interaction leaves the freedom to redefine the relative phase of \( |K^0\rangle \) and \( |\bar{K}^0\rangle \) states:

\[
|K^0\rangle \rightarrow e^{i\vartheta} |K^0\rangle \\
|\bar{K}^0\rangle \rightarrow e^{-i\vartheta} |\bar{K}^0\rangle,
\]

(7)

implying that the off-diagonal elements of \( \mathbf{H} \) depend on the arbitrary phase \( \vartheta \)

\[
H_{12} \rightarrow e^{-2i\vartheta} H_{12} \\
H_{21} \rightarrow e^{2i\vartheta} H_{21}.
\]

(8)

Thus expressions which depend on \( \vartheta \) are not suited to represent experimental results, unless \( \vartheta \) is fixed to a definite value by convention. However the diagonal elements of \( \mathbf{H} \), the product of the off-diagonal elements, their absolute values, the trace of \( \mathbf{H} \), its determinant and eigenvalues are all phase convention independent quantities.

The conservation of discrete symmetries constrains the matrix elements of \( \mathbf{H} \), and the following phase-invariant conditions hold\(^2\):

\[
H_{11} = H_{22} \quad \text{for } CPT \text{ conservation,} \quad (9) \\
|H_{12}| = |H_{21}| \quad \text{for } T \text{ conservation,} \quad (10) \\
H_{11} = H_{22} \text{ and } |H_{12}| = |H_{21}| \quad \text{for } CP \text{ conservation.} \quad (11)
\]

The eigenvalues of \( \mathbf{H} \) are

\[
\lambda_S = m_S - i\Gamma_S/2 \\
\lambda_L = m_L - i\Gamma_L/2,
\]

(12)

\(^2\)For a general review on discrete symmetries in the neutral kaon system see Refs. 6, 7, 8, 9, 10, 11).
where $m_{S,L}$ and $\Gamma_{S,L}$ are the masses and widths of the physical states, respectively. It is also useful to define the differences

$$\Delta m = m_L - m_S > 0 \quad \Delta \Gamma = \Gamma_S - \Gamma_L > 0$$

and the so called superweak phase

$$\tan \phi_{SW} = \frac{2\Delta m}{\Delta \Gamma}.$$  \hspace{1cm} (14)

The physical states that diagonalize $H$ are the short- and long-lived states; they evolve in time as pure exponentials

$$|K_S(t)\rangle = e^{-i\lambda_S t}|K_S\rangle$$
$$|K_L(t)\rangle = e^{-i\lambda_L t}|K_L\rangle,$$  \hspace{1cm} (15)

and are usually written as:

$$|K_S\rangle = \frac{1}{\sqrt{2(1 + |\epsilon_S|^2)}}\{(|1 + \epsilon_S|)K^0 + (1 - \epsilon_S)|\bar{K}^0\}$$
$$|K_L\rangle = \frac{1}{\sqrt{2(1 + |\epsilon_L|^2)}}\{(|1 + \epsilon_L|)K^0 - (1 - \epsilon_L)|\bar{K}^0\},$$  \hspace{1cm} (16)

where $\epsilon_{S,L}$ are two small complex parameters describing the CP impurity in the physical states; one can equivalently define the parameters

$$\bar{\epsilon} \equiv (\epsilon_S + \epsilon_L)/2, \quad \delta \equiv (\epsilon_S - \epsilon_L)/2.$$  \hspace{1cm} (17)

Ignoring negligible quadratic terms, they can be expressed in terms of the elements of $H$ as:

$$\bar{\epsilon} = \frac{H_{12} - H_{21}}{2(\lambda_S - \lambda_L)} = \frac{-i\Delta M_{12} - \frac{1}{2}i\Delta \Gamma_{12}}{\Delta m + i(\Delta \Gamma)/2},$$
$$\delta = \frac{H_{11} - H_{22}}{2(\lambda_S - \lambda_L)} = \frac{1}{2}\left(\frac{\Delta (M_{22} - M_{11}) - \frac{7}{2}(\Delta \Gamma_{22} - \Gamma_{11})}{\Delta m + i(\Delta \Gamma)/2}\right).$$  \hspace{1cm} (18)

It is worth noting that the parameter $\bar{\epsilon}$ is phase convention dependent. The arbitrariness in the choice of the phase $\theta$ can be conveniently used to have either $\arg(\Gamma_{12}) = 0$ (in this case $\bar{\epsilon} = |\bar{\epsilon}|e^{i\phi_{SW}}$), or the phase of some decay amplitude such that $\arg(\Gamma_{12}) \ll 1$ (as in the Wu-Yang phase convention 12).
In this case it can be shown \(^9, 11, 13, 14\) that the real part of \(\bar{\epsilon}\) does not depend on \(\text{arg}(\Gamma_{12})\), and the following relation holds\(^3\):

\[
\frac{|H_{12}|^2 - |H_{21}|^2}{|H_{12}|^2 + |H_{21}|^2} \approx 4\Re\bar{\epsilon}.
\] (20)

Then it is easy to show that

- \(\delta \neq 0\) implies CPT violation;
- \(\Re\bar{\epsilon} \neq 0\) implies \(T\) violation;
- \(\Re\bar{\epsilon} \neq 0\) or \(\delta \neq 0\) implies \(CP\) violation.

The effective Hamiltonian \(H\) can thus be expressed in terms of the following 7 observable quantities: 4 being in the complex eigenvalues \(\lambda_{S,L}\), 2 in the complex parameter \(\delta\), and 1 in the real part of \(\epsilon\).

3 Correlated kaons

The correlations between the decay modes of a system consisting of a \(K\bar{K}\) pair produced in nucleon-antinucleon annihilation were first considered in 1958 by Goldhaber, Lee and Yang \(^{15}\). Neutral kaon pairs can also be produced in the strong decay of some scalar, vector, or tensor unflavored neutral mesons, e.g. \(f_0\), \(\phi\), or \(f'_2\), with definite \(J^{PC} = 0^{++}, 1^{--}, 2^{++}\) quantum numbers. In such a case only the two following zero strangeness states need to be considered:

\[
|K^0(+\vec{p})\rangle|\bar{K}^0(-\vec{p})\rangle
\]

\[
|\bar{K}^0(+\vec{p})\rangle|K^0(-\vec{p})\rangle
\] (21)

where the kaon momentum \(+\vec{p}\) (or \(-\vec{p}\)) is specified in the decaying meson rest frame. Neutral kaons are spinless bosons and the physical \(K^0\bar{K}^0\) state is required to be symmetric under the combined operation of charge conjugation \(C\) and permutation of space coordinates \(P\), i.e. \(CP = +1\). For an arbitrary and well defined orbital angular momentum \(L\), the system is an eigenstate of \(C\) with eigenvalue \((-1)^L\). Hence, for the decay of scalar or tensor mesons into

\(^3\)Always neglecting \(|\bar{\epsilon}|^2 \ll 1\) and \(|\delta|^2 \ll 1\).
$K^0\bar{K}^0$, one has $L = 0, 2$ ($C = P = +1$) and necessarily the following symmetric combination of states (21):

$$|i\rangle = \frac{1}{\sqrt{2}}\{|K^0(\bar{p})\rangle|\bar{K}^0(-\bar{p})\rangle + |\bar{K}^0(\bar{p})\rangle|K^0(-\bar{p})\rangle\}$$

$$= \frac{1}{\sqrt{2}}\{[|K_S(\bar{p})\rangle|K_S(-\bar{p})\rangle - |K_L(\bar{p})\rangle|K_L(-\bar{p})\rangle]$$

$$-2\delta[|K_S(\bar{p})\rangle|K_L(-\bar{p})\rangle + |K_L(\bar{p})\rangle|K_S(-\bar{p})\rangle]\} \quad (22)$$

while, for the decay of vector mesons, one has $L = 1$ ($C = P = -1$) and the antisymmetric combination:

$$|i\rangle = \frac{1}{\sqrt{2}}\{|K^0(\bar{p})\rangle|\bar{K}^0(-\bar{p})\rangle - |\bar{K}^0(\bar{p})\rangle|K^0(-\bar{p})\rangle\}$$

$$= \frac{N}{\sqrt{2}}\{|K_S(\bar{p})\rangle|K_L(-\bar{p})\rangle - |K_L(\bar{p})\rangle|K_S(-\bar{p})\rangle\} \quad (23)$$

where

$$N = \sqrt{(1 + |\epsilon_S|^2)(1 + |\epsilon_L|^2)} \approx 1 \quad (24)$$

is a normalization factor.

It is worth noting that:

- for identical spinless bosons, Bose statistics forbids states with odd angular momentum; hence in the case $L = 1$ terms like $K_SK_S$ or $K_LK_L$ (or $K^0\bar{K}^0$, etc.) cannot appear; this is also true for simultaneous kaon states at any time in the evolution of the system after production; the state results totally antisymmetric, and eq.(23) is exact regardless of any CP or CPT violation in the neutral kaon system (apart the case of a possible CPT violation in which Bose statistics does not apply, as described in Ref. 16, 17);

- in the case $L = 0, 2$ terms of the order $\epsilon^2$ and $\delta^2$ have been neglected in eq.(22) but the effect of possible CPT violation has been included, leading to the appearance of $K_SK_L$ and $K_LK_S$ terms.

4 Kaon interferometry at a $\phi$-factory

Neutral kaon pairs in the antisymmetric state (23) are ideally and copiously produced at a $\phi$-factory ($J^{PC} = 1^{--}$ for the $\phi$ meson) in the reaction $e^+e^- \rightarrow$
\( \phi \rightarrow K^0 \bar{K}^0 \). According to quantum mechanics, one can evaluate the decay amplitude for state (23) into final states \( f_1 \) and \( f_2 \) produced in the \(+\bar{p}\) and \(-\bar{p}\) directions at kaon proper times \( t_1 \) and \( t_2 \), respectively:

\[
A(f_1, t_1; f_2, t_2) = \frac{N}{\sqrt{2}} \{ \langle f_1|T|K_S(t_1)\rangle \langle f_2|T|K_L(t_2)\rangle \\
- \langle f_1|T|K_L(t_1)\rangle \langle f_2|T|K_S(t_2)\rangle \}
\]

\[
= \frac{N}{\sqrt{2}} \{ \langle f_1|T|K_S\rangle \langle f_2|T|K_L\rangle e^{-i\lambda_s t_1} e^{-i\lambda_L t_2} \\
- \langle f_1|T|K_L\rangle \langle f_2|T|K_S\rangle e^{-i\lambda_L t_1} e^{-i\lambda_s t_2} \} .
\] (25)

The double differential decay rate into final states \( f_1 \) and \( f_2 \) at proper times \( t_1 \) and \( t_2 \) can be readily computed from eq.(25):

\[
I(f_1, t_1; f_2, t_2) = C_{12} \{ |\eta_1|^2 e^{-\Gamma_L t_1 - \Gamma_S t_2} + |\eta_2|^2 e^{-\Gamma_S t_1 - \Gamma_L t_2} \\
- 2|\eta_1||\eta_2| e^{-\frac{(\Gamma_s + \Gamma_L)}{2}(t_1 + t_2)} \cos[\Delta m(t_1 - t_2) + \phi_2 - \phi_1] \}
\] (26)

where

\[
\eta_i \equiv |\eta_i| e^{i\phi_i} = \frac{\langle f_i|T|K_L\rangle}{\langle f_i|T|K_S\rangle},
\] (27)

\[
C_{12} = \frac{|N|^2}{2} \langle f_1|T|K_S\rangle \langle f_2|T|K_S\rangle^2 ,
\]

and a proper account of phase-space integrals is implicitly assumed. After integration in \((t_1 + t_2)\), at fixed difference of time \( \Delta t = t_1 - t_2 \), the following distribution is obtained, sometimes simpler to manipulate and compare to data:

\[
I(f_1, f_2; \Delta t \geq 0) = \frac{C_{12}}{\Gamma_S + \Gamma_L} \{ |\eta_1|^2 e^{-\Gamma_L \Delta t} + |\eta_2|^2 e^{-\Gamma_S \Delta t} \\
- 2|\eta_1||\eta_2| e^{-\frac{(\Gamma_s + \Gamma_L)}{2} \Delta t} \cos[\Delta m \Delta t + \phi_2 - \phi_1] \}
\] (28)

valid for \( \Delta t \geq 0 \), while for \( \Delta t < 0 \) the substitutions \( \Delta t \rightarrow |\Delta t| \) and \( 1 \leftrightarrow 2 \) have to be applied.

Both eqs.(26) and (28) show a time interference term (in the second line of their expressions) giving rise to a characteristic correlation between the two kaon decays. It can be exploited to study the neutral kaon system and discrete symmetries. In fact the decay amplitude ratios \( \eta_i \) defined in eq.(27), as well as the kinematical properties of neutral kaons, i.e. \( \Gamma_S, \Gamma_L \) and \( \Delta m \), can be
evaluated by measuring the distribution (28) with different choices of final states \( f_1 \) and \( f_2 \). From these measurements several parameters describing the neutral kaon system can be extracted.

In general two kind of asymmetries can be constructed from eq.(28); the first one can be obtained by considering eq.(28) for positive and negative \( \Delta t \)'s:

\[
A(\Delta t) = \frac{I(f_1, f_2; \Delta t > 0) - I(f_1, f_2; \Delta t < 0)}{I(f_1, f_2; \Delta t > 0) + I(f_1, f_2; \Delta t < 0)},
\]

(29)

for \( |\Delta t| \gg \tau_S \) (where \( \tau_{S,L} = 1/\Gamma_{S,L} \) is the \( K_{S,L} \) lifetime) it becomes:

\[
A(\Delta t \gg \tau_S) \simeq |\eta_1|^2 - |\eta_2|^2,\]

(30)

while for \( |\Delta t| < 5\tau_S \) it depends on the complex ratio \( \eta_2/\eta_1 \), and therefore from the phase difference \( \phi_2 - \phi_1 \).

The second asymmetry can be defined by considering three different final states \( f_1, f_2, \) and \( f_3 \):

\[
A_{f_1, f_2}(\Delta t) = \frac{I(f_1, f_3; \Delta t) - I(f_2, f_3; \Delta t)}{I(f_1, f_3; \Delta t) + I(f_2, f_3; \Delta t)},
\]

(31)

For large positive \( \Delta t \) one obtains:

\[
A_{f_1, f_2}(\Delta t \gg \tau_S) \simeq \frac{\Gamma(K_L \to f_1) - \Gamma(K_L \to f_2)}{\Gamma(K_L \to f_1) + \Gamma(K_L \to f_2)},
\]

(32)

while for large negative \( \Delta t \) one has:

\[
A_{f_1, f_2}(\Delta t \ll -\tau_S) \simeq \frac{\Gamma(K_S \to f_1) - \Gamma(K_S \to f_2)}{\Gamma(K_S \to f_1) + \Gamma(K_S \to f_2)}.
\]

(33)

For \( |\Delta t| < 5\tau_S \) the asymmetry (31) depends on the ratios \( \eta_1/\eta_3 \) and \( \eta_2/\eta_3 \).

4.1 Decays into two charged and two neutral pions

The parameter \( \epsilon'/\epsilon \) signaling direct \( CP \) violation \( 18, 6 \) in \( K \to \pi\pi \) decays can be measured with the choice \( f_1 = \pi^+\pi^- \) and \( f_2 = 2\pi^0 \); in this case the corresponding \( \eta_i \) parameters are defined as follows:

\[
\eta_{++} \equiv |\eta_{++}|e^{i\phi_{++}} = \epsilon + \epsilon',
\]

\[
\eta_{00} \equiv |\eta_{00}|e^{i\phi_{00}} = \epsilon - 2\epsilon'.
\]

(34)
where\(^4\):

\[
\epsilon = \bar{\epsilon} - \delta + i \frac{\Im A_0}{\Re A_0} + \frac{\Im B_0}{\Re A_0},
\]

and the decay amplitudes of \(K^0\) and \(\bar{K}^0\) into a \(\pi\pi\) final state of definite isospin \(I = 0, 2\) are written as

\[
\langle \pi\pi; I|T|K^0 \rangle = (A_I + B_I) e^{i\delta_I},
\]

\[
\langle \pi\pi; I|T|\bar{K}^0 \rangle = (A_I^* - B_I^*) e^{i\delta_I},
\]

with \(\delta_I\) the \(\pi\pi\) strong interaction phase shift for channel of total isospin \(I\).

Here \(A_I\) \((B_I)\) describe the \(CPT\)-conserving \((CPT\)-violating\) part of \(\pi\pi\) decay amplitudes (see Refs. 6, 19, 18) for a detailed discussion).

The distribution (28) in the case of \(f_1 = \pi^+\pi^-\) and \(f_2 = 2\pi^0\) is shown in Fig.1 (where the effect of \(\epsilon' / \epsilon \neq 0\) is emphasized). One can construct an asymmetry of the kind of eq.(29):

\[
A_{\epsilon' / \epsilon}(|\Delta t|) = \frac{I \left( \pi^+\pi^-, \pi^0\pi^0; \Delta t > 0 \right) - I \left( \pi^+\pi^-, \pi^0\pi^0; \Delta t < 0 \right)}{I \left( \pi^+\pi^-, \pi^0\pi^0; \Delta t > 0 \right) + I \left( \pi^+\pi^-, \pi^0\pi^0; \Delta t < 0 \right)}
= A_R(|\Delta t|) \Re \left( \frac{\epsilon'}{\epsilon} \right) - A_I(|\Delta t|) \Im \left( \frac{\epsilon'}{\epsilon} \right)
\]

where terms proportional to \(\left( \frac{\epsilon'}{\epsilon} \right)^2\) have been neglected in the last equality, and

\[
A_R(|\Delta t|) = \frac{3}{e^{-\Gamma_E |\Delta t|} + e^{-\Gamma_S |\Delta t|} - 2 e^{-\frac{t_{\epsilon + \epsilon}}{2} |\Delta t|} \cos(\Delta m |\Delta t|)} \]

\[
A_I(|\Delta t|) = \frac{2 e^{-\frac{t_{\epsilon + \epsilon}}{2} |\Delta t|} \sin(\Delta m |\Delta t|)}{e^{-\Gamma_E |\Delta t|} + e^{-\Gamma_S |\Delta t|} - 2 e^{-\frac{t_{\epsilon + \epsilon}}{2} |\Delta t|} \cos(\Delta m |\Delta t|)}
\]

The asymmetry is sensitive to \(\Im (\epsilon' / \epsilon)\) for \(|\Delta t| \leq 5 \tau_S\), while for \(|\Delta t| \gg \tau_S\) tends to \(3 \Re (\epsilon' / \epsilon)\).

\(^4\)It is worth remarking that \(\epsilon\) and \(\epsilon'\) are measurable quantities independent of any phase convention.
4.2 Double semileptonic decays

The semileptonic decay amplitudes can be parametrized as follows 6):

\[ \langle \pi^- l^+ \nu | T | K^0 \rangle = a + b , \quad \langle \pi^+ l^- \bar{\nu} | T | K^0 \rangle = a^* - b^* \]
\[ \langle \pi^+ l^- \bar{\nu} | T | K^0 \rangle = c + d , \quad \langle \pi^- l^+ \nu | T | K^0 \rangle = c^* - d^* \]  \hspace{1cm} (40)

where \( a, b, c, d \) are complex quantities; \( CPT \) invariance implies \( b = d = 0 \), \( \Delta S = \Delta Q \) rule implies \( c = d = 0 \), \( T \) invariance implies \( \Im a = \Im b = \Im c = \Im d = 0 \), while \( CP \) invariance implies \( \Re a = \Re b = \Re c = \Re d = 0 \). Then three measurable parameters can be defined:

\[ x_+ = \frac{c^*}{a} , \quad x_- = -\frac{d^*}{a} ; \]  \hspace{1cm} (41)

\( x_+ (x_-) \) describes the violation of the \( \Delta S = \Delta Q \) rule in \( CPT \) conserving (violating) decay amplitudes, while \( y \) parametrizes \( CPT \) violation for \( \Delta S = \)
ΔQ transitions. Then the semileptonic charge asymmetries for $K_S$ and $K_L$ states can be expressed as

$$A_S = \frac{\Gamma(K_S \rightarrow \pi^- l^+ \nu) - \Gamma(K_S \rightarrow \pi^+ l^- \bar{\nu})}{\Gamma(K_S \rightarrow \pi^- l^+ \nu) + \Gamma(K_S \rightarrow \pi^+ l^- \bar{\nu})} = 2 \Re \bar{\epsilon} + 2 \Re \delta - 2 \Re y + 2 \Re x_-, \quad (42)$$

and

$$A_L = \frac{\Gamma(K_L \rightarrow \pi^- l^+ \nu) - \Gamma(K_L \rightarrow \pi^+ l^- \bar{\nu})}{\Gamma(K_L \rightarrow \pi^- l^+ \nu) + \Gamma(K_L \rightarrow \pi^+ l^- \bar{\nu})} = 2 \Re \bar{\epsilon} - 2 \Re \delta - 2 \Re y - 2 \Re x-. \quad (43)$$

With the choice $f_1 = \pi^- l^+ \nu$ and $f_2 = \pi^+ l^- \bar{\nu}$, the corresponding $\eta_i$ parameters are

$$\eta_+ \simeq 1 - 2\delta - 2x_+ - 2x_-$$
$$\eta_- \simeq -1 - 2\delta + 2x_+^* - 2x_-^*. \quad (44)$$

The decay intensity (28) in this case is shown in Fig.2 (where a possible effect of $\delta \neq 0$ is emphasized). The following asymmetry can be constructed:

$$A_{CPT}(\Delta t) = \frac{I(\pi^- l^+ \nu, \pi l^- \bar{\nu}; \Delta t > 0) - I(\pi^- l^+ \nu, \pi^+ l^- \bar{\nu}; \Delta t < 0)}{I(\pi^- l^+ \nu, \pi l^- \bar{\nu}; \Delta t > 0) + I(\pi^- l^+ \nu, \pi^+ l^- \bar{\nu}; \Delta t < 0)} = -\frac{4}{3} \{A_R(\Delta t)|\delta_R + A_I(\Delta t)|\delta_I\} \times \frac{\cosh(\Delta \Gamma |\Delta t|/2) - \cos(\Delta m |\Delta t|)}{\cosh(\Delta \Gamma |\Delta t|/2) + \cos(\Delta m |\Delta t|)} \quad (45)$$

that is sensitive to CPT and/or $\Delta S = \Delta Q$ rule violations. In fact for $|\Delta t| \gg \tau_S$ it tends to $-4\delta_R$, where $\delta_R = \Re \delta + \Re x_-$, while for $|\Delta t| \leq 5\tau_S$ it is sensitive to $\delta_I = 3\delta + 3x_+$.  

4.3 Semileptonic and two pion decays

The decay intensity (28) in the cases of $f_1 = \pi^- l^+ \nu$, $f_2 = \pi^+ l^- \bar{\nu}$, and $f_3 = \pi\pi$ is shown in Fig.3; an asymmetry of the kind of eq.(31) can be constructed

$$A_{l+l^-}(\Delta t) = \frac{I(\pi^- l^+ \nu, \pi \pi; \Delta t) - I(\pi^+ l^- \bar{\nu}, \pi \pi; \Delta t)}{I(\pi^- l^+ \nu, \pi \pi; \Delta t) + I(\pi^+ l^- \bar{\nu}, \pi \pi; \Delta t)}, \quad (46)$$
that at large positive times $\Delta t \gg \tau_S$ coincides with the $K_L$ semileptonic asymmetry $A_L$ given in eq.(43), while for short times it is sensitive to $|\eta_{\pi\pi}|$ and $\phi_{\pi\pi}$.

4.4 Decays into identical final states

In the case of $f_1 = f_2 = \pi^+\pi^-$ the dependence on the $\eta_{\pi\pi}$ parameter factorizes out, and the shape of distribution (28) is sensitive only to the kinematical quantities $\Gamma_S$, $\Gamma_L$ and $\Delta m$, as shown in Fig.4. The same holds for any choice of identical final states, i.e. with $f_1 = f_2$.

More detailed reviews on this subject can be found in Refs. 18, 6, 19, 20).
5 Entanglement and neutral kaons

As mentioned above the interference term in eqs.(26) and (28) gives rise to a characteristic correlation between the two kaon decays. For instance, a complete destructive interference prevents the two kaons from decaying into the same final state \( f \) at the same time \( t \), i.e.:

\[
I(f, t; f, t) = 0
\]  \hspace{1cm} (47)

for any \( f \) and \( t \) (as it can be also noticed in Fig.(4) for \( |\Delta t| = 0 \)). This is a consequence of the antisymmetry of state (23). From an intuitive point of view, once produced, the two kaons can be viewed as two freely propagating independent particles. However even though the two decays can be regarded as separated space-like events (the kaons are produced with opposite momentum in the \( \phi \) meson rest frame), it is like the kaon flying in the \(+\vec{p}\) direction.
cannot “freely” decay into a certain final state $f$ at a certain proper time $t$, but its behaviour depends on what the other kaon flying in the opposite $-\vec{p}$ direction does. This kind of correlation (entanglement) for neutral kaon pairs was emphasized already in 1960 by Lee and Yang \cite{21}, and later on by several authors \cite{22,23,24}. It cannot be simply explained in terms of conservation laws, and is of the type first pointed out by Einstein, Podolsky and Rosen (EPR) in their famous paper \cite{25}.

This feature of the initial state (23) has long reaching consequences in terms of potentialities of the neutral kaon system in testing fundamental aspects of quantum mechanics. This can be easily understood by recognizing that the quantum number strangeness $\pm 1$ for a neutral kaon can play the same role of spin up or down along a chosen direction. Then, the correlations implied
by the state (23) for a kaon pair lead to a quite straightforward formal analogy with the system of spin 1/2 particles in the singlet state. Therefore, kaon pairs produced at a φ-factory might be suitable for a significant test of Bell’s inequality, as it is discussed in detail in the contributions of Berthmann and Hiesmayr 26), and Bramon, Escribano and Garbarino 27) (see also the contribution of Go 28)), or for the study of Bohr’s complementarity principle with an interesting implementation of the quantum erasure concepts, as described in the contribution of Bramon, Garbarino and Hiesmayr 29).

6 Decoherence and CPT violation

6.1 Furry’s hypothesis and a simple decoherence model

Most of the key features of the entangled state (23) resides in its non-separability. It has been suggested that the state soon after the φ-meson decay, spontaneously factorizes to an equally weighted statistical mixture of states $|K_S\rangle|K_L\rangle$ and $|K_L\rangle|K_S\rangle$, (commonly known as Furry’s hypothesis 30)). In this case the characteristic quantum interference term would disappear from expressions (26) and (28); to be more specific, this means that the calculation of intensity $I(f_1,t_1; f_2,t_2)$ is no more given by eq.(26), as in orthodox quantum mechanics, but is given by the incoherent sum:

$$I(f_1,t_1; f_2,t_2)_{\text{Furry}} = \frac{|N|^2}{2} \left( |\langle f_1|T|K_S(t_1)\rangle\langle f_2|T|K_L(t_2)\rangle|^2 + |\langle f_1|T|K_L(t_1)\rangle\langle f_2|T|K_S(t_2)\rangle|^2 \right).$$

(48)

One of the most direct way to search for such deviations from quantum mechanics 31) is to introduce a decoherence parameter $\zeta_{SL}$, and a factor $(1 - \zeta)$ multiplying the interference term in eq.(26):

$$I(f_1,t_1; f_2,t_2; \zeta) = C_{12} \left( |\eta_1|^2 e^{-\Gamma_L t_1 - \Gamma_S t_2} + |\eta_2|^2 e^{-\Gamma_S t_1 - \Gamma_L t_2} - 2(1 - \zeta) |\eta_1||\eta_2| e^{-\frac{\Gamma_S + \Gamma_L}{2} (t_1 + t_2)} \cos[\Delta m (t_1 - t_2) + \phi_2 - \phi_1] \right).$$

(49)

The case $\zeta = 0$ corresponds to the usual orthodox quantum theory, while for $\zeta = 1$ the case of spontaneous factorization of state, as in eq.(48), is obtained, i.e. total decoherence. Different $\zeta$ values correspond to intermediate situations between these two. However, the state could also spontaneously factorize into another mixture of states, e.g. $|K^0\rangle|\bar{K}^0\rangle$ and $|\bar{K}^0\rangle|K^0\rangle$, giving rise to a different
decay intensity expression. As pointed out in Ref. 32), in general the definition of \( \zeta \) depends on the basis in which is written the initial state (23) because the interference term changes with the basis (obviously in the *orthodox* quantum theory the final result does not depend on the basis choice). For a generic basis \(|K_\alpha\rangle, |K_\beta\rangle\), distribution (26) is modified as follows:

\[
I(f_1, t_1; f_2, t_2; \zeta_{\alpha\beta}) = \frac{|N'|^2}{2} \left[ |\langle f_1 | T | K_\alpha(t_1) \rangle \langle f_2 | T | K_\beta(t_2) \rangle|^2 + |\langle f_1 | T | K_\beta(t_1) \rangle \langle f_2 | T | K_\alpha(t_2) \rangle|^2 - 2(1 - \zeta_{\alpha\beta}) \Re[\langle f_1 | T | K_\beta(t_1) \rangle \langle f_2 | T | K_\alpha(t_2) \rangle^* \langle f_1 | T | K_\alpha(t_1) \rangle \langle f_2 | T | K_\beta(t_2) \rangle^*] \right],
\]

(50)
defining the basis dependent decoherence parameter \( \zeta_{\alpha\beta} \).

6.2 A general approach to decoherence

In general decoherence is the time evolution of a pure state into an incoherent mixture of states. The density matrix formalism correctly treats pure and mixed states in a unique consistent framework. According to quantum mechanics, the time evolution of the density matrix \( \rho \) of a system is given by the Liouville - von Neumann equation:

\[
\frac{d\rho}{dt} = -i[H, \rho]
\]

(51)

Decoherence can be introduced at a more fundamental level than inserting by hand the parameter \( \zeta \), by suitably modifying eq.(51). Very general modifications have been proposed in Ref. 33, 34, 35, 36, 37, 38) for single kaon and correlated pair of kaon systems. In the broad framework of open quantum systems, neutral kaons can be modeled as being small subsystems in weak interaction with large environments. The reduced dynamics for the subsystem is obtained by tracing over the environment degrees of freedom, and the time evolution is assumed to be described by a completely positive dynamical map. A detailed review on this subject can be found in the contribution of Benatti and Floreanini 39).

6.3 Decoherence and CPT violation due to quantum gravity effects

The decoherence mechanism can be made more specific in the case it is induced by quantum gravity effects. In fact one of the main open problem in quantum
gravity is related to what is commonly known as the black hole information-loss paradox. In 1976 Hawking showed \(^40\) that the formation and evaporation of black holes, as described in the semiclassical approximation, appear to transform pure states near the event horizon of black holes into mixed states. This corresponds to a loss of information about the initial state, in striking conflict with quantum mechanics and its unitarity description.

At a microscopic level, in a quantum gravity picture, space-time might be subjected to inherent non-trivial quantum metric and topology fluctuations at the Planck scale (~ \(10^{-33}\) cm), called generically space-time foam, with associated microscopic event horizons. As further suggested by Hawking himself\(^41\), this space-time structure, might induce a pure state to evolve into a mixed one, i.e. decoherence of apparently isolated matter systems. This decoherence, in turn, necessarily implies, by means of a theorem \(^42\), CPT violation, in the sense that the quantum mechanical operator generating CPT transformations cannot be consistently defined.

The information-loss paradox generated a lively debate during the last decades with no generally accepted solution. Even the recent proposed solutions in favor of no-loss and preservation of information do not completely solve the problem, some aspects of which still remaining a puzzle (see for instance Refs. \(^43\), \(^44\), \(^45\)). It seems therefore extremely interesting to put experimental limits at the level of the Planck’s scale region on possible decoherence effects.

The above mentioned decoherence mechanism lead Ellis and coworkers\(^46\) to formulate a model in which a single kaon is described by a density matrix \(\rho\) that obeys a modified Liouville-von Neumann equation:

\[
\frac{d\rho}{dt} = -i\mathbf{H}\rho + i\rho\mathbf{H}^\dagger + i\delta\mathbf{H}\rho
\]  

(52)

where now \(\mathbf{H} = \mathbf{M} - i\Gamma/2\) is the usual neutral kaon effective Hamiltonian, and the extra term \(\delta\mathbf{H}\) would induce decoherence in the system. Taking as orthonormal basis for \(\rho\) the states \(|K_1\rangle = \frac{1}{\sqrt{2}} [ |K^0\rangle + |\bar{K}^0\rangle]\) and \(|K_2\rangle = \frac{1}{\sqrt{2}} [ |K^0\rangle - |\bar{K}^0\rangle]\), and expanding \(\rho\) in terms of Pauli spin matrices \(\sigma_i\) and the identity \(\sigma_0\), i.e. \(\rho = \rho_\mu \sigma_\mu\), the extra term can be represented by a \(4 \times 4\) matrix
\( \delta H_{\mu\nu} (\mu, \nu = 0, 1, 2, 3) \) acting on a column vector with \( \rho_{\mu} \) as components:

\[
\delta H_{\mu\nu} = -2 \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \alpha & \beta \\
0 & 0 & \beta & \gamma
\end{pmatrix}
\]

(53)

where \( \alpha, \beta \) and \( \gamma \) are three new real parameters, which violate CPT symmetry and quantum mechanics, and satisfy the inequalities \( \alpha, \gamma > 0 \) and \( \alpha \gamma > \beta^2 \) (see Refs. 46, 47). They have mass dimension and are guessed to be at most of \( O(m_K^2/M_{\text{Planck}}) \sim 2 \times 10^{-20} \text{GeV} \), where \( M_{\text{Planck}} = \sqrt{G_N} = 1.22 \times 10^{19} \text{GeV} \) is the Planck mass.

The formalism described above is for single kaons. Its extension to the correlated kaon pair (23) has been described in Refs. 48, 17.

It is worth noting that the assumption of complete positivity 34, 35) introduces additional constraints on these three parameters, i.e. \( \alpha = \gamma \) and \( \beta = 0 \), reducing them to only one independent parameter.

As discussed above, in a quantum gravity framework inducing decoherence, the CPT operator is ill-defined. This consideration lead Bernabeu, Mavromatos and Papavassiliou 16, 17) to investigate intriguing consequences in correlated neutral kaon states. In fact the resulting loss of particle-antiparticle identity could induce a breakdown of the correlation of state (23) imposed by Bose statistics. As a result the initial entangled state (23) can be parametrized in general as:

\[
|i\rangle = \frac{1}{\sqrt{2}} \left\{ (|K_0^0\rangle|\bar{K}_0^0\rangle - |\bar{K}_0^0\rangle|K_0^0\rangle) + \omega (|K_0^0\rangle|\bar{K}_0^0\rangle + |\bar{K}_0^0\rangle|K_0^0\rangle) \right\}
\]

\[
\propto \left\{ (|K_S^0\rangle|K_L^0\rangle - |K_L^0\rangle|K_S^0\rangle) + \omega (|K_S^0\rangle|K_S^0\rangle - |K_L^0\rangle|K_L^0\rangle) \right\}
\]

(54)

where \( \omega \) is a complex parameter describing a completely novel CPT violation phenomenon, not included in previous analyses. Its order of magnitude might be at most \( |\omega| \sim \left[ \frac{(m_K^2/M_{\text{Planck}})}{\Delta \Gamma} \right]^{1/2} \sim 10^{-3} \), with \( \Delta \Gamma = \Gamma_S - \Gamma_L \).

From eq.(54) it is evident that the best decay channel to look for such CPT violation effects is the one with \( f_1 = f_2 = \pi^+\pi^- \): in fact in this case the leading \( K_S^0K_L^0 \) terms are CP suppressed while the new CPT violating \( K_S^0K_S^0 \) term is not.

A general review on the theoretical motivations for possible CPT violation induced by quantum gravity in the neutral kaon system can be found in
the contribution of Bernabeu, Ellis, Mavromatos, Nanopoulos and Papavassiliou 49); a review on decoherence models in this framework can be found in the contribution of Sarkar 50), while general considerations on quantum gravity phenomenology with a special focus on correlated states can be found in the contribution of Amelino-Camelia, Arzano and Marcianò 51).

7 CPT violation and Lorentz symmetry breaking

CPT invariance holds for any realistic Lorentz-invariant quantum field theory. However a very general theoretical possibility for CPT violation is based on spontaneous breaking of Lorentz symmetry, as developed by Kostelecky 52, 53, 54), which appears to be compatible with the basic tenets of quantum field theory and retains the property of gauge invariance and renormalizability (Standard Model Extension - SME). A detailed review on this subject can be found in the contribution of Lehnert 55). Here, after a brief introduction, some measurement methods at a \( \phi \)-factory are discussed.

In SME for neutral kaons, CPT manifests to lowest order only in the CPT violation parameter \( \delta \) (e.g. \( B_I, y \) and \( x_- \) vanish at first order), and exhibits a dependence on the 4-momentum of the kaon:

\[
\delta \approx i \sin \phi_{SW} e^{i\phi_{SW}} \gamma_K (\Delta a_0 - \beta_K^* \cdot \Delta \vec{a}) / \Delta m \tag{55}
\]

where \( \gamma_K \) and \( \beta_K^* \) are the kaon boost factor and velocity in the observer frame, and \( \Delta a_\mu \) are four CPT- and Lorentz-violating coefficients for the two valence quarks in the kaon.

The implications of the momentum dependence in the CPT violation parameter can be substantial, as it is evident in eq.(55). The analysis of experimental data requires a particular care in considering meson boost, momentum orientation, and possible diurnal effects arising from the rotation of the Earth relative to the constant vector \( \Delta \vec{a} \), in order to avoid cancellations of the CPT violation effects.

Following Ref. 53), the time dependence arising from the rotation of the Earth can be explicitly displayed in eq.(55) by choosing a three-dimensional basis \( (\hat{X}, \hat{Y}, \hat{Z}) \) in a non-rotating frame, with the \( \hat{Z} \) axis along the Earth’s rotation axis, and a basis \( (\hat{x}, \hat{y}, \hat{z}) \) for the rotating (laboratory) frame (see Fig.5). The CPT violating parameter \( \delta \) may then be expressed as:
Figure 5: Basis ($\hat{x}, \hat{y}, \hat{z}$) for the rotating frame, and basis ($\hat{X}, \hat{Y}, \hat{Z}$) for the fixed non-rotating frame. The laboratory frame precesses around the Earth’s rotation axis $\hat{Z}$ at the sidereal frequency $\Omega$; $\chi$ is the angle between the axes $\hat{Z}$ and $\hat{z}$.

\[
\delta(\vec{p}, t) = \frac{i \sin \phi_{SW} e^{i \phi_{SW}}}{\Delta m} \gamma_K \{ \Delta a_0 + \beta_K \Delta a_Z (\cos \theta \cos \chi - \sin \theta \cos \phi \sin \chi) \\
- \beta_K \Delta a_X \sin \theta \sin \phi \sin \Omega t \\
+ \beta_K \Delta a_Y (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) \cos \Omega t \\
+ \beta_K \Delta a_Y (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) \sin \Omega t \\
+ \beta_K \Delta a_Y \sin \theta \sin \phi \cos \Omega t \} \tag{56}
\]

where $\Omega$ is the Earth’s sidereal frequency, $\cos \chi = \hat{z} \cdot \hat{Z}$, and $\theta$ and $\phi$ are the conventional polar and azimuthal angles defined in the laboratory frame about the $\hat{z}$ axis.

The sensitivity to the four $\Delta a_\mu$ parameters can be very different for fixed target and collider experiments, showing complementary features \cite{53}. At a fixed target experiment usually the kaon momentum direction is fixed, while $|\vec{p}|$ might vary within a certain interval. On the contrary, at a $\phi$-factory kaons are emitted with the characteristic $p$-wave angular distribution $dN/d\Omega \propto \sin^2 \theta$,.
while $|\vec{p}|$ is fixed. Assuming a symmetric decay distribution in the azimuthal angle $\phi$, and an integration on this variable, the following expression is obtained for $\delta$:

$$\delta = \frac{1}{2\pi} \int_0^{2\pi} \delta(\vec{p}, t) d\phi$$

$$= \frac{i \sin \phi_{SW} e^{i\phi_{SW}}}{\Delta m} \gamma_K \{ \Delta a_0 + \beta_K \Delta a_Z \cos \theta \cos \chi$$

$$+ \beta_K (\Delta a_Y \sin \chi \cos \theta \sin \Omega t + \Delta a_X \sin \chi \cos \theta \cos \Omega t) \} , \quad (57)$$

displaying different angular and time dependences of the various terms proportional to $\Delta a_\mu$.

### 7.1 Measurement of $\Delta a_0$ at a $\phi$-factory

The $\Delta a_0$ parameter can be measured through the difference of the semileptonic charge asymmetries for $K_S$ and $K_L$, given in eqs.(42) and (43), by performing the measurement of each asymmetry with a symmetric integration over the polar angle $\theta$, thus averaging to zero any possible contribution from the terms proportional to $\cos \theta$ in eq.(57). Then one obtains that the difference $(A_S - A_L)$ is proportional to $\Delta a_0$, i.e.:

$$A_S - A_L \simeq \left[ 4 \Re \left( i \sin \phi_{SW} e^{i\phi_{SW}} \right) \gamma_K \right] \Delta a_0 . \quad (58)$$

An alternative method to measure $\Delta a_0$ consists in exploiting the correlation between the two kaons in double semileptonic decays $\phi \rightarrow K_S K_L \rightarrow \pi^+ \ell^- \bar{\nu}, \pi^- \ell^+ \nu$ with opposite lepton charges. The two kaons are practically emitted back-to-back, and terms proportional to $\cos \theta$ have opposite sign for the two kaons; $\Delta a_0$ can be evaluated through the asymmetry (45), which for large $\Delta t$ becomes:

$$A_{CPT} (|\Delta t| \gg \tau_S) \simeq - \left[ 4 \Re \left( i \sin \phi_{SW} e^{i\phi_{SW}} \right) \gamma_K \right] \Delta a_0 . \quad (59)$$

---

5 Apart small variations due to the small $\phi$ meson momentum in the laboratory frame.

6 This simplifying assumption will be maintained throughout the following; however small non-symmetric $\phi$ angle effects could be easily included in the formulas without significantly modifying the main conclusions below.
The above two methods are largely independent and could be useful for systematics cross-checks.

7.2 Measurement of $\Delta a_Z$ at a $\phi$-factory

The $\Delta a_Z$ parameter can be measured through the $A_L$ asymmetry measured separately for $K_L$'s emitted in the forward ($\cos \theta > 0$) and backward ($\cos \theta < 0$) direction; assuming data have been uniformly taken as a function of sidereal time, thus averaging to zero any possible contribution from the terms proportional to $\cos \Omega t$ and $\sin \Omega t$ in eq.(57) (otherwise a proper $t$-dependent analysis has to be performed), one has:

$$\Delta A_L \equiv A_L(\cos \theta > 0) - A_L(\cos \theta < 0) \simeq - \left[ 4 \Re \left( i \sin \phi_{SW} e^{i \phi_{SW}} \right) \frac{\beta_K \gamma_K \cos \langle \cos \theta \rangle}{\Delta m} \right] \Delta a_Z$$  \hspace{1cm} (60)

where $\langle \cos \theta \rangle$ is a proper average of $\cos \theta$ over the forward (backward) hemisphere.

Also for the measurement of $\Delta a_Z$ an alternative and independent method exists, based on neutral kaon interferometry with $\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^-, \pi^+ \pi^-$ decays. In this case the intensity $I(\pi^+ \pi^- (+), \pi^+ \pi^- (-); \Delta t)$ can be measured, where the two identical final states are distinguished by their forward or backward emission (the symbols + and − represent $\cos \theta > 0$ and $\cos \theta < 0$, respectively), and the following asymmetry evaluated:

$$A(|\Delta t|) = \frac{I(\pi^+ \pi^- (+), \pi^+ \pi^- (-); \Delta t > 0) - I(\pi^+ \pi^- (+), \pi^+ \pi^- (-); \Delta t < 0)}{I(\pi^+ \pi^- (+), \pi^+ \pi^- (-); \Delta t > 0) + I(\pi^+ \pi^- (+), \pi^+ \pi^- (-); \Delta t < 0)}$$  \hspace{1cm} (61)

To first order in small quantities, the above asymmetry for $|\Delta t| \gg \tau_S$ tends to zero, because $\epsilon$ and $\delta$ are $90^\circ$ out of phase (see Ref. 18)):

$$A(|\Delta t| \gg \tau_S) \simeq -2 \Re \left( \frac{\delta}{\epsilon} \right) \sim 0$$  \hspace{1cm} (62)

while for $|\Delta t| \leq 5 \tau_S$ it is sensitive to $\Im (\delta/\epsilon)$, and therefore to $\Delta a_Z$:

$$A(0 < |\Delta t| < 5 \tau_S) \propto \Im \left( \frac{\delta}{\epsilon} \right) \simeq \left[ \frac{\sin \phi_{SW} \beta K \gamma_K \cos \langle \cos \theta \rangle}{\Delta m |\epsilon|} \right] \Delta a_Z .$$  \hspace{1cm} (63)

Also in this case the two methods could be used for cross-checks.
7.3 Measurement of $\Delta a_X$, $\Delta a_Y$ at a $\phi$-factory

The $\Delta a_X$, $\Delta a_Y$ and $\Delta a_Z$ parameters can be all simultaneously measured by performing a proper sidereal time dependent analysis of asymmetries in eqs.(60) and (61).

8 The KLOE experiment at DAΦNE

DAΦNE, the Frascati $\phi$-factory 56), is an $e^+e^-$ collider working at a center of mass energy of $\sqrt{s} \sim 1020$ MeV, corresponding to the peak of the $\phi$ resonance. The $\phi$ production cross section is $\sim 3 \mu$b; the main $\phi$ decays and branching ratios are listed in tab. 1. The beams collide at the interaction point (IP)

| Table 1: Main decay channels and branching fractions of the $\phi$ meson |
|-----------------|-----------------|
| Decay channel   | Branching fraction (% units) |
| $\phi \rightarrow K^+K^-$   | 49.1 |
| $\phi \rightarrow K^0\bar{K}^0$ | 34.0 |
| $\phi \rightarrow \rho\pi, \pi^+\pi^-\pi^0$ | 15.4 |
| $\phi \rightarrow \eta\gamma$ | 1.3 |

with a crossing angle $\theta_x \simeq 25$ mrad, therefore $\phi$'s are produced with a small momentum of $\sim 12.5$ MeV in the horizontal plane. The beams collide with a frequency up to 370 MHz corresponding to a bunch crossing period of $T_{\text{bunch}} = 2.7$ ns and a maximum number of circulating bunches of 120. The KLOE interaction region is equipped with three low-$\beta$ quadrupoles, which reduce the beam-size in the vertical ($y$) direction. The typical sizes of the beam are $\sigma_x = 0.2$ cm; $\sigma_y = 20$ $\mu$m; $\sigma_z = 3$ cm. The maximum peak luminosity reached during KLOE data taking is $L \simeq 1.4 \times 10^{32}$ cm$^{-2}$ s$^{-1}$.

The KLOE detector consists mainly of a large volume drift chamber surrounded by an electromagnetic calorimeter. A superconducting coil around the calorimeter provides a 0.52 T solenoidal magnetic field.

The fine sampling lead-scintillating fiber calorimeter 57) consists of a barrel and two end-caps, and has solid angle coverage of 98%. Photon energies and arrival times are measured with resolutions $\sigma_E/E = 5.7%/\sqrt{E(\text{GeV})}$ and $\sigma_t = 54\text{ps}/\sqrt{E(\text{GeV})} \oplus 50\text{ps}$, respectively. Photon entry points are determined with an accuracy $\sigma_z \sim 1$ cm/$\sqrt{E(\text{GeV})}$ along the fibers and $\sigma_\perp \sim 1$ cm in the transverse direction.
The tracking detector is a 4 m diameter and 3.3 m long cylindrical drift chamber \(^{58}\) with a total of \(~ 52000\) wires, of which \(~ 12000\) are sense wires. In order to minimize multiple scattering and \(K_L\) regeneration and to maximize detection efficiency of low energy photons, the chamber works with a helium based gas mixture and its walls are made of light materials (mostly carbon fiber composites). The momentum resolution for tracks produced at large polar angle is \(\sigma_p/p \leq 0.4\%\). Vertices are reconstructed with a resolution of \(~ 3\) mm.

Kaon regeneration in the beam pipe is a non negligible disturbance. The beam pipe is spherical around the interaction point, with a radius of 10 cm. The walls of the beam pipe, 500 \(\mu m\) thick, are made of a 62%-beryllium/38%-aluminum alloy. A beryllium cylindrical tube of 4.4 cm radius and 50 \(\mu m\) thick, coaxial with the beam, provides electrical continuity.

KLOE completed the data taking in March 2006 with a total integrated luminosity of \(~ 2.5\) fb\(^{-1}\), corresponding to \(~ 7.5 \times 10^9\) \(\phi\)-mesons produced.

8.1 Decoherence and CPT symmetry tests

The quantum interference between the two kaon decays in the CP violating channel \(\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^- \pi^+ \pi^-\) has been observed for the first time by KLOE \(^{59}\). A data sample corresponding to \(~ 380\) pb\(^{-1}\) has been analysed; the selection of the signal requires two vertices, each with two opposite curvature tracks inside the drift chamber, with an invariant mass and total momentum compatible with the two neutral kaon decays. The experimental resolution on the time difference \(|\Delta t|\) in the case of \(\pi^+ \pi^-\) decays can be improved exploiting the good momentum resolution of the KLOE detector \(^{60}\) and the closed kinematics of the event. After a kinematic fit, a resolution \(\sigma_{|\Delta t|} \sim 0.9 \tau_S\) is obtained. The measured \(I(\pi^+ \pi^-; |\Delta t|)\) distribution as a function of \(|\Delta t|\) can be fitted with the expression given in eq.(28). After having included resolution and detection efficiency effects, having taken into account the background due to coherent and incoherent \(K_S\)-regeneration on the beam pipe wall, the small contamination of non-resonant \(e^+ e^- \rightarrow \pi^+ \pi^- \pi^+ \pi^-\) events, and keeping fixed in the fit \(\Gamma_S\) and \(\Gamma_L\) to the PDG \(^{61}\) values, \(\Delta m\) can be evaluated. The fit result is \(\Delta m = (5.61 \pm 0.33) \times 10^9\) s\(^{-1}\), which is compatible with the more precise value given by the PDG: \(\Delta m = (5.290 \pm 0.015) \times 10^9\) s\(^{-1}\).

A similar analysis can be done on the same data sample, by fixing \(\Delta m\) to the PDG value, using the modified expression given in eq.(50) after integration...
Figure 6: Fit of the $I(\pi^+\pi^-,\pi^+\pi^-;|\Delta t|)$ distribution. The black points are the experimental data, while the histogram is the fit result in the case of $\zeta_{SL}$ determination. The uncertainty arising from the detection efficiency evaluation is shown as the hatched area. The peak at $|\Delta t| \sim 17\tau_S$ is due to coherent and incoherent $K_S$-regeneration on the spherical beam pipe.

in ($t_1 + t_2$), and leaving the decoherence parameter $\zeta$ as a free parameter in the fit. The results in the two main basis, \{\!\!\{K_S, K_L\!\!\}\} and \{\!\!\{K^0, \bar{K^0}\!\!\}\}, are

\[
\begin{align*}
\zeta_{SL} &= 0.018 \pm 0.040_{\text{stat}} \pm 0.007_{\text{syst}} \\
\zeta_{0\bar{0}} &= (1.0 \pm 2.1_{\text{stat}} \pm 0.4_{\text{syst}}) \times 10^{-6}, 
\end{align*}
\]

compatible with the quantum mechanics prediction, i.e. $\zeta_{SL} = \zeta_{0\bar{0}} = 0$, and no decoherence effects. As an example, the fit of the $|\Delta t|$ distribution used to determine $\zeta_{SL}$ is shown in Fig.6.

The result on $\zeta_{0\bar{0}}$ has a high accuracy, of $O(10^{-6})$, due to the $CP$ suppression present in the specific $f_1 = f_2 = \pi\pi$ decay channel which makes the function (50) very sensitive to $\zeta_{0\bar{0}}$ deviations from zero. This result improves
by five orders of magnitude the previous limit obtained by Bertlmann and co-
workers [32] in a re-analysis of CPLEAR data [62] (a review of the CPLEAR
results can be found in the contribution of Go [28]). It can also be compared to
a similar result recently obtained in the B meson system [63], where an accuracy
of $\mathcal{O}(10^{-2})$ can be reached.

Another analysis based on the same data constrains the parameters $\alpha$, $\beta$ and $\gamma$ related to possible decoherence effects induced by quantum gravity, as discussed above. The theoretical expression of the $I(\pi^+\pi^-,\pi^+\pi^-;|\Delta t|)$ distribution including these effects can be found in Refs. [48, 17]. The KLOE preliminary results are [64]:

$$\begin{align*}
\alpha &= \left(-10^{+41}_{-31\text{stat}} \pm 9_{\text{syst}}\right) \times 10^{-17} \text{GeV} \\
\beta &= \left(3.7^{+6.9}_{-9.2\text{stat}} \pm 1.8_{\text{syst}}\right) \times 10^{-19} \text{GeV} \\
\gamma &= \left(-0.5^{+5.8}_{-5.1\text{stat}} \pm 1.2_{\text{syst}}\right) \times 10^{-21} \text{GeV}
\end{align*}$$

In the simplifying hypothesis of complete positivity, i.e. $\alpha = \gamma$ and $\beta = 0$, the KLOE result is [59]:

$$\gamma = \left(1.3^{+2.8}_{-2.4} \pm 0.4\right) \times 10^{-21} \text{GeV} ,$$

These results can be compared to the ones obtained by the CPLEAR collabora-
tion, studying single neutral kaon decays to $\pi^+\pi^-$ and $\pi\nu\bar{\nu}$ final states [65]:

$$\begin{align*}
\alpha &= \left(-0.5 \pm 2.8\right) \times 10^{-17} \text{GeV} \\
\beta &= \left(2.5 \pm 2.3\right) \times 10^{-19} \text{GeV} \\
\gamma &= \left(1.1 \pm 2.5\right) \times 10^{-21} \text{GeV} .
\end{align*}$$

All results are compatible with no $CPT$ violation, while the sensitivity ap-
proaches the interesting level of $\mathcal{O}(10^{-20}\text{GeV})$.

The uncertainties on the KLOE measurements of the $\zeta$, $\alpha$, $\beta$, $\gamma$, and $\omega$ parameters should improve by more than a factor two with the analysis of the full KLOE data sample of 2.5 fb$^{-1}$.

As discussed above $CPT$ violation effects might also induce a breakdown of the correlation of state (23), as given in eq. (54). A similar analysis performed on the same KLOE data as before, including in the fit the modified initial state (54), yields the first measurement of the complex parameter $\omega$ [59]:

$$\Re(\omega) = \left(1.1^{+8.7}_{-5.3} \pm 0.9\right) \times 10^{-4} \quad \Im(\omega) = \left(3.4^{+4.8}_{-5.0} \pm 0.6\right) \times 10^{-4} ;$$
compatible with no \( CPT \) violation, and with an accuracy that already reaches the interesting Planck’s scale region.

8.2 \( CPT \) symmetry tests with \( K_S \rightarrow \pi e\nu \) decays

For \( t_1 \gg t_2, \tau_S \) (or \( t_2 \gg t_1, \tau_S \)), the amplitude (25) factorizes, and everything behaves like the initial state were an incoherent mixture of states \( |K_S\rangle|K_L\rangle \) and \( |K_L\rangle|K_S\rangle \). Hence the detection of a kaon at large times tags a \( |K_S\rangle \) state in the opposite direction. This is a unique feature at a \( \phi \)-factory, not possible at fixed target experiments, that can be exploited to select a pure \( K_S \) beam.

At KLOE a \( K_S \) is tagged by identifying the interaction of the \( K_L \) in the calorimeter (\( K_L \)-crash). In fact about 50\% of the produced \( K_L \)'s in \( \phi \rightarrow K_S K_L \) events reach the calorimeter before decaying; their associated interactions are identified by a high energy, neutral and delayed deposit in the calorimeter, i.e. not associated to any charged track in the event, and delayed of \( \sim 30 \) ns (as \( \beta_K \sim 0.22 \)) with respect to a photon coming from the interaction region. Pure \( K_S \) samples have been selected exploiting this tagging technique. In particular \( K_S \rightarrow \pi e\nu \) decays are selected requiring a \( K_L \)-crash and two tracks forming a vertex close to the IP, and associated with two energy deposits in the calorimeter. Pions and electrons are recognized using a time-of-flight technique. The number of signal events is normalized to the number of \( K_S \rightarrow \pi^+\pi^- \) in the same data set. Then the first measurement of the \( K_S \) semileptonic charge asymmetry has been performed \( (66) \):

\[
A_S = (1.5 \pm 9.6_{\text{stat}} \pm 2.9_{\text{syst}}) \times 10^{-3}.
\]

The uncertainty on \( A_S \) can be reduced at the level of \( \approx 3 \times 10^{-3} \) with the analysis of the full data sample of 2.5 fb\(^{-1}\).

From the sum and the difference of the \( K_S \) and \( K_L \) semileptonic charge asymmetries one can test \( CPT \) conservation. Using the values of \( A_L, \Re \delta, \) and \( \Re \bar{\epsilon} \) from other experiments \( (61) \), the real part of the \( CPT \) violating and \( \Delta S = \Delta Q \) violating (conserving) parameter \( x_\pm \) (\( y \)) in semileptonic decay amplitudes (see eqs.(42) and (43)), can be evaluated \( (66) \):

\[
\Re x_+ = \frac{A_S - A_L}{4} - \Re \delta = (-0.8 \pm 2.5) \times 10^{-3}
\]

\[
\Re y = \Re \bar{\epsilon} - \frac{A_S + A_L}{4} = (0.4 \pm 2.5) \times 10^{-3}.
\]

(67)
8.3 CPT symmetry test from unitarity

The unitarity relation, originally derived by Bell and Steinberger [67],

$$
\left( \frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i \tan \phi_{SW} \right) \left[ \frac{\Re \bar{\epsilon}}{1 + |\bar{\epsilon}|^2} - i \Im \delta \right] = \frac{1}{\Gamma_S - \Gamma_L} \sum_f A^*(K_S \to f) A(K_L \to f) \equiv \sum_f \alpha_f
$$

(68)

can be used to bound \(\Im \delta\), after having provided all the \(\alpha_i\) parameters, \(\Gamma_S\), \(\Gamma_L\), and \(\phi_{SW}\) as inputs. A detailed review on this subject is given in the contribution of Isidori [68].

Using KLOE measurements, PDG [61] values, and a combined fit of KLOE and CPLEAR data, the following result is obtained [69]:

$$
\Re \bar{\epsilon} = (159.6 \pm 1.3) \times 10^{-5}, \quad \Im \delta = (0.4 \pm 2.1) \times 10^{-5},
$$

(69)

the main limiting factor of this result being the uncertainty on the phase \(\phi_{\pi^-}\) of the \(\eta_{\pi^+\pi^-}\) parameter entering in \(\alpha_{\pi^+\pi^-}\).

The limits on \(\Im(\delta)\) and \(\Re(\delta)\) can be used (see eq.(19)) to constrain the mass and width difference between \(K^0\) and \(\bar{K}^0\). In the limit \(\Gamma_1 = \Gamma_2\), i.e. neglecting CPT-violating effects in the decay amplitudes, the best bound on the neutral kaon mass difference is obtained:

$$
-5.3 \times 10^{-19} \text{ GeV} < M_{11} - M_{22} < 6.3 \times 10^{-19} \text{ GeV} \quad \text{at 95 \% CL .}
$$

(70)

8.4 Lorentz and CPT symmetries tests

From the measured value [66] of \(A_S\) and a preliminary evaluation of \(A_L\) by KLOE, the difference \(A_S - A_L = (-2 \pm 10) \times 10^{-3}\), and a first preliminary evaluation of the \(\Delta a_0\) parameter can be obtained [71]:

$$
\Delta a_0 = (0.4 \pm 1.8) \times 10^{-17} \text{ GeV}.
$$

(71)

With the analysis of the full KLOE data sample \((L = 2.5 \text{ fb}^{-1})\) a statistical sensitivity \(\delta(\Delta a_0) \sim 7 \times 10^{-18} \text{ GeV}\) could be reached. In the case of the method based on double semileptonic decays (see eq.(45) ), the analysis of the full data sample could yield a sensitivity \(\delta(\Delta a_0) \sim 1 \times 10^{-17} \text{ GeV}\).

An analysis has been performed on the same sample \((L = 380 \text{ pb}^{-1})\) of \(\phi \to K_S K_L \to \pi^+\pi^-, \pi^+\pi^-\) events used for the measurement of decoherence...
parameters, exploiting the method based on eq. (61). It yields a first preliminary evaluation of $\Delta a_Z$:

$$\Delta a_Z = (-1 \pm 4) \times 10^{-17} \text{ GeV}.$$  (72)

With the analysis of 2.5 fb$^{-1}$ a statistical sensitivity $\delta(\Delta a_Z) \sim 2 \times 10^{-17}$ GeV could be reached. In the case of the method based on eq. (60), the analysis of the full data sample could yield a sensitivity $\delta(\Delta a_Z) \sim 3 \times 10^{-17}$ GeV.

The same level of accuracy could also be reached on the $\Delta a_X$ and $\Delta a_Y$ parameters by means of a proper sidereal time dependent analysis. However in this case the sensitivity would not be competitive with a preliminary measurement performed by the KTeV collaboration based on the search of sidereal time variation of the phase $\phi_{+-,t}$ that constrains $\Delta a_X$ and $\Delta a_Y$ to less than $9.2 \times 10^{-22}$ GeV at 90% C.L.

The $\Delta a_\mu$ parameters have also been recently constrained in the B-meson system with an accuracy of $O(10^{-12}\text{GeV})$.

9 The KLOE-2 program

A proposal has been recently submitted for a physics program to be carried out with an upgraded KLOE detector, KLOE-2, at a new Frascati $e^+e^-$ collider, which is expected to deliver an integrated luminosity of the order of 50 fb$^{-1}$ at the $\phi(1020)$ peak. The high luminosity is necessary to reach significant sensitivities in the tests discussed above by means of neutral kaon interferometry.

As discussed above, the decay mode $\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ is very rich in physics. In general all decoherence effects show a deviation from the quantum mechanical prediction (47). Hence the reconstruction of events in the region at $\Delta t \approx 0$, i.e. with vertices near the IP, is crucial for precise determination of the parameters related to CPT violation and to the decoherence. The vertex resolution affects the $I(\pi^+ \pi^-, \pi^+ \pi^-; |\Delta t|)$ distribution precisely in that region, as shown in Fig. 7, and its impact on the decoherence parameter measurements has to be carefully evaluated. In fact, the resolution has two main effects: (1) to reduce the statistical sensitivity of the fit to the parameters; (2) to introduce a source of systematic uncertainties. In Figs. 8, 9 the statistical uncertainty on several decoherence and CPT-violating parameters is shown as a function of the integrated luminosity for the case $\sigma_{|\Delta t|} \approx \tau_S$ (present KLOE
resolution), and for $\sigma_{|\Delta t|} \approx 0.25 \tau_S$. As it can be seen in the last case an improvement of about a factor two could be achieved. Therefore the addition of a vertex detector between the spherical beam pipe and the drift chamber, improving the vertex resolution in that region in order to have $\sigma_{|\Delta t|} \approx 0.25 \tau_S$, is the major upgrade of the KLOE detector that has been considered in the KLOE-2 proposal. The KLOE-2 physics program concerning interferometry measurements is summarized in table 2, where the KLOE-2 statistical sensitivities to the main parameters that can be extracted from the experimental time distributions $I(f_1, f_2; \Delta t)$ with different choices of final states $f_i$, are listed in the hypothesis of an integrated luminosity $L = 50 \text{ fb}^{-1}$, and compared to the best presently published measurements.
Table 2: KLOE-2 statistical sensitivities on several parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>Best published meas.</th>
<th>KLOE-2 (50 fb$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_S \to \pi e\nu$</td>
<td>$A_S$</td>
<td></td>
<td>$(1.5 \pm 11) \times 10^{-3}$</td>
<td>$\pm 1 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>$\pi l\nu$</td>
<td>$A_L$</td>
<td>$(3322 \pm 58 \pm 47) \times 10^{-6}$</td>
<td>$\pm 25 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>$\pi^0\pi^0$</td>
<td>$\Re \mathcal{L}'$</td>
<td>$(1.66 \pm 0.26) \times 10^{-3}$</td>
<td>$\pm 0.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>$\pi^0\pi^0$</td>
<td>$\Im \mathcal{L}'$</td>
<td>$(1.2 \pm 2.3) \times 10^{-3}$</td>
<td>$\pm 3 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\pi^+l^-\bar{\nu}$</td>
<td>$\pi^-l^+\nu$</td>
<td>$(\Re \delta + \Re x_-)$</td>
<td>$\Re \delta = (0.29 \pm 0.27) \times 10^{-3}$</td>
<td>$\pm 0.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\pi^+l^-\bar{\nu}$</td>
<td>$\pi^-l^+\nu$</td>
<td>$(\Im \delta + \Im x_+)$</td>
<td>$\Im \delta = (0.4 \pm 0.7) \times 10^{-2}$</td>
<td>$\pm 3 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\pi_+\pi^-$</td>
<td>$\pi_+\pi^-$</td>
<td>$\Delta m$</td>
<td>$5.288 \pm 0.043 \times 10^9 s^{-1}$</td>
<td>$\pm 0.03 \times 10^9 s^{-1}$</td>
</tr>
<tr>
<td>$\pi_+\pi^-$</td>
<td>$\pi_+\pi^-$</td>
<td>$\zeta_{SL}$</td>
<td>$(1.8 \pm 4.1) \times 10^{-2}$</td>
<td>$\pm 0.2 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\pi_+\pi^-$</td>
<td>$\pi_+\pi^-$</td>
<td>$\zeta_{00}$</td>
<td>$(1.0 \pm 2.1) \times 10^{-6}$</td>
<td>$\pm 0.1 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\pi_+\pi^-$</td>
<td>$\pi_+\pi^-$</td>
<td>$\alpha$</td>
<td>$(-0.5 \pm 2.8) \times 10^{-17} \text{ GeV}$</td>
<td>$\pm 2 \times 10^{-17} \text{ GeV}$</td>
</tr>
<tr>
<td>$\pi_+\pi^-$</td>
<td>$\pi_+\pi^-$</td>
<td>$\beta$</td>
<td>$(2.5 \pm 2.3) \times 10^{-19} \text{ GeV}$</td>
<td>$\pm 0.1 \times 10^{-19} \text{ GeV}$</td>
</tr>
<tr>
<td>$\pi_+\pi^-$</td>
<td>$\pi_+\pi^-$</td>
<td>$\gamma$</td>
<td>$(1.1 \pm 2.5) \times 10^{-21} \text{ GeV}$</td>
<td>$\pm 0.2 \times 10^{-21} \text{ GeV}$</td>
</tr>
<tr>
<td>$\pi_+\pi^-$</td>
<td>$\pi_+\pi^-$</td>
<td>$\Re \omega$</td>
<td>$(1.1_{-8.7}^{+5.3} \pm 0.9) \times 10^{-4}$</td>
<td>$\pm 2 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\pi_+\pi^-$</td>
<td>$\pi_+\pi^-$</td>
<td>$\Im \omega$</td>
<td>$(3.4_{-5.0}^{+4.8} \pm 0.6) \times 10^{-4}$</td>
<td>$\pm 2 \times 10^{-5}$</td>
</tr>
<tr>
<td>$K_{S,L} \to \pi e\nu$</td>
<td>$\pi^+l^-\bar{\nu}$</td>
<td>$\pi^-l^+\nu$</td>
<td>$\Delta a_0$</td>
<td>$\pm 2 \times 10^{-18} \text{ GeV}$</td>
</tr>
<tr>
<td>$\pi_+\pi^-$</td>
<td>$\pi l\nu$</td>
<td>$\Delta a_Z$</td>
<td>$\pm 5 \times 10^{-18} \text{ GeV}$</td>
<td></td>
</tr>
<tr>
<td>$\pi_+\pi^-$</td>
<td>$\pi^+\pi^-$</td>
<td>$\Delta a_X, \Delta a_Y$</td>
<td>$\pm 3 \times 10^{-18} \text{ GeV}$</td>
<td></td>
</tr>
<tr>
<td>$\pi_+\pi^-$</td>
<td>$\pi^+\pi^-$</td>
<td>$\Delta a_X, \Delta a_Y$</td>
<td>$\pm 5 \times 10^{-18} \text{ GeV}$</td>
<td></td>
</tr>
<tr>
<td>$\pi_+\pi^-$</td>
<td>$\pi^+\pi^-$</td>
<td>$\Delta a_X, \Delta a_Y$</td>
<td>$\pm 3 \times 10^{-18} \text{ GeV}$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 8: The statistical sensitivity to the $\zeta_{SL}$, $\zeta_{00}$ and $\Re \omega$ parameters with the present KLOE resolution $\sigma_{|\Delta t|} \approx \tau_S$ (open circles), with an improved resolution $\sigma_{|\Delta t|} \approx 0.25 \tau_S$ (full circles).

10 Conclusions

A $\phi$-factory represents a unique opportunity to study the neutral kaon system, and the related fundamental discrete symmetries. It is also an ideal place to investigate the entanglement and correlation properties of the produced $K^0\bar{K}^0$ pairs, as well as CPT violation effects that might be induced by quantum gravity effects.

The KLOE experiment concluded the data taking at the beginning of 2006 with a total integrated luminosity of $\sim 2.5 \text{ fb}^{-1}$. Several parameters related to possible CPT violations in conjunction with decoherence or Lorentz symmetry violations, have been measured, some of them for the first time, and with a
Figure 9: The statistical sensitivity to the parameters $\alpha$, $\beta$, $\gamma$ with the present KLOE resolution $\sigma_{|\Delta t|} \approx \tau_S$ (open circles), and with an improved resolution $\sigma_{|\Delta t|} \approx 0.25 \tau_S$ (full circles); the horizontal lines represent the CPLEAR results.

precision reaching the interesting Planck’s scale region; with the analysis of the full KLOE data sample further improvements are expected on all results.

The search for such CPT violation effects and precision tests of quantum mechanics by means of neutral kaon interferometry constitute one of the main physics issues of the KLOE-2 proposal. With an integrated luminosity of about 50 fb$^{-1}$, significant improvements in a variety of observables involving different final states are expected.
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