OPEN QUANTUM DYNAMICS: COMPLETE POSITIVITY AND CORRELATED NEUTRAL KAONS

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Abstract

Neutral kaons can be treated as open systems, *i.e.* as subsystems immersed in an external environment, generated either by the fundamental dynamics of extended objects (strings and branes), or by matter fluctuations in a medium. New, non-standard phenomena are induced at low energies, producing irreversibility and dissipation, whose physical description requires however some care. Meson factories are suitable interferometric set-ups where these new effects can be experimentally studied with great accuracy.

1 Introduction

Standard quantum mechanics usually deals with closed physical systems, *i.e.* with systems that can be considered isolated from any external environment. The time-evolution of such systems is described by one-parameter groups of unitary operators, $U(t) = e^{-iHt}$, generated by the system hamiltonian H; they embody the reversible character of the dynamics.

This description is however just an approximation, valid when the action of the external world on the system under study can be considered vanishingly small. On the contrary, when a system $\mathcal{S}$ interacts with an environment $\mathcal{E}$ in a non-negligible way, it must be treated as an open quantum system, namely as a subsystem embedded within $\mathcal{E}$, exchanging with it energy and entropy, and whose time-evolution is irreversible.[1] Being closed, the total system $\mathcal{S} + \mathcal{E}$

[1] The literature on the theory of open quantum systems and their phenomenological applications is large. General reviews and monographs on the topic can be found in Refs.[1-17].

evolves in time with the unitary dynamics generated by the total hamiltonian $H_{\rm tot}$, that can always be written as:

$$H_{\rm tot} = H_{\mathcal{S}} + H_{\mathcal{E}} + H' \ . \tag{1}$$

In this decomposition, $H_{\mathcal{S}}$ is the hamiltonian driving the dynamics of $\mathcal{S}$ in absence of the environment, $H_{\mathcal{E}}$ describes the internal evolution of $\mathcal{E}$, while H' takes into account the interaction between subsystem and environment.

In many instances, one is interested in the dynamics of the subsystem $\mathcal{S}$ alone and not in the details of the $\mathcal{E}$ motion; one can then eliminate (*i.e.* sum over) the environment degrees of freedom. The resulting time-evolution for $\mathcal{S}$ turns out to be rather involved: it can not anymore be described in terms of a unitary evolution. Indeed, representing the states of $\mathcal{S}$ in terms of density matrices, the map transforming the initial state $\rho(0)$ into the final one $\rho(t)$ is given by:

$$\rho(0) \equiv {\rm Tr}_{\mathcal{E}}\left[\rho_{\mathcal{S}+\mathcal{E}}\right] \mapsto \rho(t) \equiv {\rm Tr}_{\mathcal{E}}\left[e^{-iH_{\rm tot}\, t}\, \rho_{\mathcal{S}+\mathcal{E}}\, e^{iH_{\rm tot}\, t}\right] \ , \tag{2}$$

where $\rho_{\mathcal{S}+\mathcal{E}}$ is the density matrix representing the initial state of the total system, while ${\rm Tr}_{\mathcal{E}}$ constitutes the trace operation over the environment degrees of freedom.[2] Due to the exchange of energy as well as entropy between $\mathcal{S}$ and $\mathcal{E}$, the evolution map $\rho(0) \to \rho(t)$ gives rise in general to nonlinearities and memory effects, and can not be described in closed form.

Nevertheless, the situation greatly simplifies when the interaction between subsystem and environment can be considered to be weak. In this case, physically plausible approximations lead to reduced dynamics $\rho(0) \to \rho(t) \equiv \gamma_t[\rho(0)]$ that involve only the $\mathcal{S}$ degrees of freedom: they are represented by linear maps γ_t, that are generated by *master equations*. Such reduced dynamics provides an effective description of how $\mathcal{E}$ affects the time-evolution of $\mathcal{S}$, and typically gives rise to dissipative and noisy effects.[1) – 9)]

However, not all time-dependent linear maps γ_t can represent suitable reduced dynamics: very basic physical requirements need to be satisfied. Although the dynamics is no longer reversible, forward in time composition should

[2] In absence of initial correlations between $\mathcal{S}$ and $\mathcal{E}$, a situation commonly encountered in many physical applications, it can be written in factrorized form: $\rho_{\mathcal{S}+\mathcal{E}} = \rho(0) \otimes \rho_{\mathcal{E}}$, where $\rho_{\mathcal{E}}$ is the density matrix representing the state of the environment.

be guaranteed: $\gamma_s \circ \gamma_t = \gamma_{t+s}$, for all positive times s, t; the one-parameter ($\equiv$ time) family of maps $\{\gamma_t\}$ is then said to be a semigroup. Further, γ_t should preserve probability and positivity.[1) – 7)] Indeed, in order to represent a physical state of the subsystem $\mathcal{S}$, a density matrix ρ must be a positive operator, since its eigenvalues have the meaning of probabilities; this is at the root of the statistical interpretation of quantum mechanics. The time evolution $\rho(0) \mapsto \rho(t) = \gamma_t[\rho(0)]$ must then preserve this fundamental property, and therefore map a positive initial $\rho(0)$ into a positive final $\rho(t)$. Such a property of the linear transformation γ_t is called *positivity*.

Apparently, positivity seems sufficient to assure the physical consistency of the reduced dynamics. In reality, the structure of quantum mechanics requires a more stringent requirement to be satisfied, that of *complete positivity*. [1) – 8)] This property guarantees the positivity not only of γ_t but also of the dynamics of a larger system built with two equal, mutually non interacting systems $\mathcal{S}$, immersed in the same external environment; as we shall see, such a situation is precisely that of correlated neutral kaons coming from the decay of a ϕ meson. The dynamics of this enlarged system is then described by $\mathit{\Gamma}_t = \gamma_t \otimes \gamma_t$. Positivity of $\mathit{\Gamma}_t$ means complete positivity of the map γ_t. [18)] It is important to note that this property is intimately related to entanglement, *i.e.* to the possibility that the initial state of the compound subsystem $\mathcal{S} + \mathcal{S}$ exihibits quantum correlations.[6), 19)]

A family of one-parameter linear maps γ_t that satisfy all the above mentioned properties, including complete positivity, forms a so-called *quantum dynamical semigroup*.[1) – 8)] In the regime of weak coupling between subsystem and environment, they represent the most general realization of a dissipative reduced dynamics compatible with the probabilistic interpretation of quantum mechanics.[20) – 26), 6)]

This description of the open quantum systems turns out to be very general. Although originally developed in the framework of quantum optics, [10) – 16)] it has been successfully applied to model very different situations, in atomic and molecular physics, quantum chemistry, solid state physics. [1) – 9), 17)] Further, it has been recently applied to the study of irreversibility and dissipation in the evolution of various particle systems, involving atoms, [27)] neutrons, [28), 29)] photons, [30) – 33)] neutrinos [34) – 36)] and in particular neutral mesons. [37) – 52)]

In standard treatments, these systems are usually considered as closed; once more, this is justified only in an idealized situation. Indeed, quantum gravity effects at Planck's scale or more in general, the dynamics of fundamental, extended objects (strings and branes) are expected to act as an effective environment, inducing non-standard, dissipative effects at low energies.[53) – 56), 44)] Similar effects are also produced when neutral kaons or neutrinos travel inside a fluctuating matter medium; in this case, due to the interactions between these elementary particles and the scattering matter centers, the medium plays the role of an environment producing noise and decoherence.[36), 52)]

These new phenomena are nevertheless expected to be very small; they are suppressed by at least one inverse power of the Planck mass, in the case of gravitational or stringy effects, while in presence of matter fluctuations they appear to be second order with respect to ordinary regeneration or oscillation effects. In spite of this, interesting bounds on some of the constants parametrizing the dissipative effects have been already obtained using available experimental data, and improvements are expected in the future.

In this respect, dedicated neutral meson experiments at meson factories, appear to be particularly promising. Indeed, as we shall see in the following, suitable neutral meson observables turn out to be particularly sensitive to the new, dissipative phenomena, so that their presence can be experimentally probed quite independently from other, non-standard effects.

2 Positivity and complete positivity

The evolution and decay of the neutral kaon system can be effectively modeled by means of a two-dimensional Hilbert space.[57) – 60)] A kaon state is then described by means of a 2×2 density matrix ρ, *i.e.* a positive hermitian operator (with real, nonnegative eigenvalues) and constant trace.

The evolution in time of the kaon system can then be formulated in terms of a linear *master equation* for ρ; it takes the general form:[1) – 7)]

$$\frac{\partial \rho(t)}{\partial t} = -iH_{\rm eff}\ \rho(t) + i\rho(t)\, H_{\rm eff}^\dagger + L[\rho(t)]\ . \tag{3}$$

The first two terms on the r.h.s. of this equation are the standard quantum mechanical ones: they contain the effective hamiltonian $H_{\rm eff} = M - i\Gamma/2$, which includes a non-hermitian part, characterizing the natural width of the kaon states.

The entries of this matrix can be expressed in terms of the complex parameters ϵ_S, ϵ_L, appearing in the eigenstates of $H_{\rm eff}$,

$$|K_S\rangle = \frac{1}{(1+|\epsilon_S|^2)^{1/2}}\big(|K_1\rangle+\epsilon_S|K_2\rangle\big)\ , \quad |K_L\rangle = \frac{1}{(1+|\epsilon_L|^2)^{1/2}}\big(\epsilon_L|K_1\rangle+|K_2\rangle\big)\ , \tag{4}$$

with $|K_{1,2}\rangle = (|K^0\rangle \pm |\overline{K^0}\rangle)/\sqrt{2}$, and the four real parameters m_S, γ_S and m_L, γ_L, the masses and widths of the states in (4), characterizing the eigenvalues of $H_{\rm eff}$: $\lambda_S = m_S - \frac{i}{2}\gamma_S$, $\lambda_L = m_L - \frac{i}{2}\gamma_L$. For later use, we introduce the following positive combinations: $\Delta\Gamma = \gamma_S - \gamma_L$, $\Delta m = m_L - m_S$, as well as the complex quantities $\Gamma_\pm = \Gamma \pm i\Delta m$ and $\Delta\Gamma_\pm = \Delta\Gamma \pm 2i\Delta m$, with $\Gamma = (\gamma_S + \gamma_L)/2$. One easily checks that CPT-invariance is broken when $\epsilon_S \neq \epsilon_L$, while a nonvanishing $\epsilon_S = \epsilon_L$ implies violation of CP symmetry.

On the other hand, the additional piece $L[\rho]$ in the evolution equation (3) encodes effects leading to dissipation and irreversibility: these are non-standard phenomena that in general give rise to further violations of CP and CPT symmetries.[61), 51)]

It should be stressed that in absence of the piece $L[\rho]$, pure states (*i.e.* states of the form $|\psi\rangle\langle\psi|$) are transformed by the evolution equation (3) back into pure states, even though probability is not conserved, a direct consequence of the presence of a non-hermitian part in the effective hamiltonian $H_{\rm eff}$. Only when the extra piece $L[\rho]$ is also present, $\rho(t)$ becomes less ordered in time due to a mixing-enhancing mechanism, producing possible loss of quantum coherence.

The explicit form of the linear map $L[\rho]$ can be uniquely fixed by taking into account the basic physical requirements that the complete time evolution, $\gamma_t : \rho(0) \mapsto \rho(t)$, generated by (3) needs to satisfy. As mentioned in the introductory remarks, in order to represent a physically consistent dynamical evolution, the one-parameter family of maps γ_t should obey the semigroup composition law, $\gamma_t[\rho(s)] = \rho(t+s)$, for t, $s \geq 0$, while transforming density matrices into density matrices, in particular preserving their positivity.

One can show that the semigroup request fixes $L[\rho]$ to be of Kossakowski-Lindblad form: [20)]

$$L[\rho] = \sum_{i,j=1}^{3} C_{ij}\Big[\sigma_j\rho\,\sigma_i - \frac{1}{2}\Big\{\sigma_i\sigma_j\ ,\ \rho\Big\}\Big]\ , \tag{5}$$

where σ_i, $i = 1,2,3$ are the Pauli matrices, while $[C_{ij}]$ is a 3×3 matrix, called the *Kossakowski matrix.* We shall consider dissipative evolutions for which the von Neumann entropy, $S = -\mathrm{Tr}[\rho \ln \rho]$, is increasing; this is a condition that is very well satisfied in usual phenomenological treatments of open systems consisting of elementary particles. In this case, the matrix $[C_{ij}]$ turns out to be real symmetric: its six entries parametrize the noise effects induced by the presence of the environment.[1), 6)]

The condition that the map γ_t generated by (3) preserve the positivity of the single kaon state ρ for all times gives further constraints on these real parameters. [29)] In order to explicitly show this, it is convenient to decompose the density matrix ρ along the Pauli matrices σ_j, $j = 1, 2, 3$, and the identity matrix σ_0, and represent ρ as a 4-dimensional ket-vector $|\rho\rangle$,

$$\rho = \begin{pmatrix} \rho_1 & \rho_3 \\ \rho_4 & \rho_2 \end{pmatrix} = \sum_{\mu=0}^{3} \rho^\mu \sigma_\mu \longmapsto |\rho\rangle = \begin{pmatrix} \rho^0 \\ \rho^1 \\ \rho^2 \\ \rho^3 \end{pmatrix} \tag{6}$$

$$\rho^0 = \frac{\rho_1 + \rho_2}{2}\ , \ \rho^1 = \frac{\rho_3 + \rho_4}{2}\ , \ \rho^2 = \frac{\rho_4 - \rho_3}{2i}\ , \ \rho^3 = \frac{\rho_1 - \rho_2}{2}\ . \tag{7}$$

Then, the action of the linear operator $L[\cdot]$ in (5) on the state ρ can be equivalently expressed as the action of a real symmetric 4×4 matrix $\big[L_{\mu\nu}\big]$ on the column vector $|\rho\rangle$. This matrix can be parametrized by six real constants a, b, c, α, β, and γ as follows:

$$\big[L_{\mu\nu}\big] = -2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & b & \alpha & \beta \\ 0 & c & \beta & \gamma \end{pmatrix}\ . \tag{8}$$

We know that any hamiltonian evolution preserves the positivity of the density matrices; then, one can limit the discussion to the contribution of the dissipative piece L only.[3] Since $\mathrm{Tr}(L[\rho]) = 0$, as easily checked from the expression in (5), we have $\rho^0(t) = \rho^0(0)$. Further, the positivity of the spectrum of ρ is preserved

[3]Let us indicate by ω_t the time-evolution generated by (3) in absence of the dissipative term L, and λ_t the one generated just by L in absence of the hamiltonian term. The complete evolution γ_t can then be expressed via the Lie-Trotter formula as: $\gamma_t = \lim_{n\to\infty}(\omega_{t/n} \circ \lambda_{t/n})^n$. [7)] Being the evolution ω_t positive, the positivity properties of γ_t are directly connected to those of λ_t.

at all times if and only if $\mathrm{Det}(\rho) = (\rho^0)^2 - \sum_{j=1}^3 (\rho^j)^2 \geq 0$. We now set $\rho \equiv \rho(0)$ and use

$$\frac{\mathrm{dDet}(\rho(t))}{\mathrm{d}t}\Big|_{t=0} = -2 \sum_{ij=1}^{3} [L\,]_{ij}\, \rho^i \rho^j\ . \tag{9}$$

If $|\rho\rangle = (\rho^0, \rho^1, \rho^2, \rho^3)$ is a pure state, $\mathrm{Det}(\rho) = 0$ and the right hand side of (9) cannot be negative, otherwise a negative eigenvalue would appear for $t > 0$. By varying ρ^j while keeping $\sum_j (\rho^j)^2 = (\rho^0)^2$, from $\sum_{ij=1}^3 [L\,]_{ij}\, \rho^i \rho^j \geq 0$ one gets that the real symmetric submatrix $[L\,]_{ij}$ must necessarily be positive, therefore that the following inequalities must be fulfilled,

$$\begin{cases} a \geq 0 \\ \alpha \geq 0 \\ \gamma \geq 0 \end{cases} ,\quad \begin{cases} a\alpha \geq b^2 \\ a\gamma \geq c^2 \\ \alpha\gamma \geq \beta^2 \end{cases} ,\quad \mathrm{Det}([L_{ij}]) \geq 0\ . \tag{10}$$

These conditions are also sufficient for preservation of positivity. In fact, since $-[L_{ij}] \geq 0$, we can write $-[L] = \mathcal{B}^2$, with $\mathcal{B}$ a symmetric 3×3 matrix. Then, the term in the right hand side of the equality in (9) is given by $\|\mathcal{B}|\rho\rangle\|^2$. Let us suppose $\mathrm{Det}[\rho(t')] < 0$, at time $t' > 0$; it follows that $\mathrm{Det}[\rho(t^*)] = 0$ at some time t^* such that $0 \leq t^* < t'$. Thus, $\mathcal{B}|\rho(t^*)\rangle\rangle = 0$, otherwise $\mathrm{Det}[\rho(t)] > 0$ for $t \geq t^*$; but this implies $|\dot\rho(t^*)\rangle\rangle = L\,|\rho(t^*)\rangle\rangle = -\mathcal{B}^2|\rho(t^*)\rangle\rangle = 0$. Therefore, for all $t > t^*$, $|\rho(t)\rangle\rangle = |\rho(t^*)\rangle\rangle$, and the dissipative dynamics generated by L is positivity-preserving.

Although the conditions (10) guarantee that the evolution $\rho(0) \mapsto \rho(t) \equiv \gamma_t[\rho(0)]$ of a single neutral kaon is physically consistent, more stringent constraints are needed in order to get a positive dissipative evolution when correlated kaons produced in a ϕ-meson decay are considered.[39), 41), 45)]

Since the ϕ-meson has spin 1, the two neutral spinless kaons produced in a ϕ-decay, and flying apart with opposite momenta in the meson ϕ rest-frame, are produced in an antisymmetric state:

$$|\psi_A\rangle = \frac{1}{\sqrt{2}}\Big(|K_1, -p\rangle \otimes |K_2, p\rangle - |K_2, -p\rangle \otimes |K_1, p\rangle\Big)\ . \tag{11}$$

Their corresponding density matrix $\rho_A \equiv |\psi_A\rangle\langle\psi_A|$ is antisymmetric in the spatial labels. By means of the projectors onto the CP eigenstates,

$$P_1 = |K_1\rangle\langle K_1|\ , \qquad P_2 = |K_2\rangle\langle K_2|\ , \tag{12}$$

and of the off-diagonal operators,

$$P_3 = |K_1\rangle\langle K_2| \ , \qquad P_4 = |K_2\rangle\langle K_1| \ , \tag{13}$$

we can write

$$\rho_A = \frac{1}{2}(P_1 \otimes P_2 + P_2 \otimes P_1 - P_3 \otimes P_4 - P_4 \otimes P_3) \ . \tag{14}$$

The time evolution of a system of two correlated neutral K-mesons, initially described by ρ_A, can be analyzed using the sigle K-meson dynamics so-far discussed. Indeed, as in standard quantum mechanics, it is natural to assume that, once produced in a ϕ decay, the kaons evolve in time each according to the map γ_t generated by (3), (5).[4]

Within this framework, the density matrix that describes a situation in which the first K-meson has evolved up to proper time t_1 and the second up to proper time t_2 is given by:

$$\begin{aligned}\rho_A(t_1, t_2) \equiv \big(\gamma_{t_1} \otimes \gamma_{t_2}\big)\big[\rho_A\big] = \frac{1}{2}\Big[&P_1(t_1) \otimes P_2(t_2)\\ + P_2(t_1) \otimes P_1(t_2) \ - P_3(t_1) \otimes P_4(t_2) - P_4(t_1) \otimes P_3(t_2)\Big]&\end{aligned} \tag{15}$$

where $P_i(t_1)$ and $P_i(t_2)$, $i = 1, 2, 3, 4$, represent the evolution according to (3) of the initial operators (12), (13), up to the time t_1 and t_2, respectively. In the following, we shall set $t_1 = t_2 = t$, and simply call $\rho_A(t) \equiv \rho_A(t, t)$ the evolution of (14) up to proper time t.

Consider then the state $|\psi_+\rangle$ which is as in (11) but with a plus sign between the two terms in parenthesis; it is an entangled state which is orthogonal to $|\psi_A\rangle$. The following quantity

$$\Delta(t) = \langle\psi_+|\rho_A(t)|\psi_+\rangle \ , \tag{16}$$

being a mean value, must be positive for all times. In particular, since $\Delta(0) = 0$, its time evolution must start at $t = 0$ with a positive derivative, otherwise

[4] We stress that this choice is the only natural possibility if one requires that after tracing over the degrees of freedom of one particle, the resulting dynamics for the remaining one be positive, of semigroup type and independent from the initial state of the other particle.

it would become negative as soon as $t > 0$. In other terms, presevation of positivity of the density matrix $\rho_A(t)$ implies the condition

$$\frac{d}{dt}\Delta(0) \equiv a + \alpha - \gamma \geq 0 \ . \tag{17}$$

By substituting for $|\psi_+\rangle$ the most general entangled state orthogonal to $|\psi_A\rangle$, one can show that the preservation of the positivity of the matrix $\rho_A(t)$ describing correlated kaons is equivalent to the following inequalities:

$$2R \equiv \alpha + \gamma - a \geq 0 \ , \qquad RS \geq b^2 \ , \tag{18}$$

$$2S \equiv a + \gamma - \alpha \geq 0 \ , \qquad RT \geq c^2 \ , \tag{19}$$

$$2T \equiv a + \alpha - \gamma \geq 0 \ , \qquad ST \geq \beta^2 \ , \tag{20}$$

$$RST \geq 2\, bc\beta + R\beta^2 + Sc^2 + Tb^2 \ , \tag{21}$$

that in turn are equivelent to the positivity of the Kossakowski matrix C_{ij} appearing in (5).

These constraints on the dissipative parameters a, b, c, α, β, γ are more stringent than those in (10); indeed, with the above conditions the master equation (3) generates not just a positive, but a *completely positive* evolution.[1) – 6)] This conclusion can be formalized in a Theorem: [18)] the dynamics $\gamma_t \otimes \gamma_t$, describing the dissipative evolution of correlated neutral kaons, is positive if and only if the single kaon dynamics γ_t is completely positive.

It should be stressed that it is the intimate structure of quantum mechanics, *i.e.* the existence of entangled states, that require any physically consistent dissipative dynamics to be completely positive. [6)] Any attempt to model noisy effects induced by a weakly coupled external environment via a positive, but not completely positive time evolution will unavoidably lead to unphysical results. As we shall see in the next section, such a conclusion can be experimentally exposed at a ϕ-factory.

3 Test of complete positivity at a ϕ-factory

As discussed in the previous section, a consistent statistical description of the initial single kaon density matrix ρ as a state requires the positivity of its eigenvalues that are interpreted as probabilities: for this description to hold for all times, the evolution map γ_t must be positive, *i.e.* it must preserve the positivity of the eigenvalues of $\rho(t)$, for any t.

On the other hand, complete positivity is a more stringent condition; it guarantees the positivity of the eigenvalues of density matrices describing states of correlated kaons, as those produced in ϕ-meson decays. We have seen that states of entangled, but not dynamically interacting kaons, evolve according to the factorized product $\gamma_t \otimes \gamma_t$ of the single-kaon dynamical maps. If γ_t is not completely positive, there are instances of correlated states that develop negative eigenvalues; in such cases, their statistical and physical interpretation is lost.

Therefore, the issue of complete positivity is not only theoretical, but can be given experimental relevance. Indeed, in the following we shall give explicit examples of experimentally accessible kaon observables, defined to be positive, that would return, in absence of complete positivity, negative mean values. [45]

Let us consider again the dissipative evolution of two initially correlated neutral kaons coming from the decay of a ϕ-meson, as given in (15), with $t_1 = t_2 = t$. Recalling the definitions (12), (13), the statistical description of $\rho_A(t) \equiv \rho_A(t,t)$ allows us to give a meaningful interpretation of the quantities

$$\mathcal{P}_{ij}(t) = \mathrm{Tr}[\rho_A(t) P_i \otimes P_j] \ , \quad i,j = 1,2 \ , \tag{22}$$

as the probabilities to have one kaon in the state $|K_i\rangle$ at proper time t, while the other is in the state $|K_j\rangle$ at the same proper time. When $i,j = 3,4$, the quantities $\mathcal{P}_{ij}(t)$ are complex and do not represent directly joint probabilities. However, as we shall see, they can still be obtained from data accessible to experiments.

On the basis of rough dimensional estimates,[54),55),44)] the parameters a, b, c, α, β and γ appearing in (8) are expected to be very small, since they are suppressed by at least one power of a very large energy scale, the one that characterizes the dynamics of fundamental objects (strings or branes). Assimilating this scale with the Planck mass M_P, one finds that the above dissipative parameters, being of dimension of energy, can be estimated to be at most of order $m_K^2/M_P \sim 10^{-19}$ GeV, with m_K the kaon mass. This value is roughly of the same order of magnitude of $\epsilon_S \Delta\Gamma$ and $\epsilon_L \Delta\Gamma$; therefore, in finding explicit solutions of the evolution equation (3) for the kaon density matrix $\rho(t)$ one can use an expansion in all these small parameters, and approximate expressions for the entries of $\rho(t)$ can be explicitly worked out.[38),41)]

Up to first order in all small parameters, one then finds:

$$\begin{aligned}
\mathcal{P}_{11}(t) &= \frac{\gamma}{\Delta\Gamma}\, e^{-2\Gamma t}\Big(1 - e^{-\Delta\Gamma t}\Big)\ , && (23)\\
\mathcal{P}_{12}(t) &= \frac{e^{-2\Gamma t}}{2}\ , && (24)\\
\mathcal{P}_{13}(t) &= 2\, e^{-2\Gamma t}\, \frac{c + i\beta}{\Delta\Gamma_+}\Big(1 - e^{-t\Delta\Gamma_+/2}\Big)\ , && (25)\\
\mathcal{P}_{22}(t) &= \frac{\gamma}{\Delta\Gamma}\, e^{-2\Gamma t}\Big(e^{\Delta\Gamma t} - 1\Big)\ , && (26)\\
\mathcal{P}_{23}(t) &= 2\, e^{-2\Gamma t}\, \frac{c + i\beta}{\Delta\Gamma_-}\Big(1 - e^{t\Delta\Gamma_-/2}\Big)\ , && (27)\\
\mathcal{P}_{33}(t) &= e^{-2\Gamma t}\, \frac{2b + i(\alpha - a)}{2\Delta m}\Big(1 - e^{-2it\Delta m}\Big)\ , && (28)\\
\mathcal{P}_{34}(t) &= -\frac{e^{-2\Gamma t}}{2}\Big(1 - 2(\alpha + a - \gamma)t\Big)\ . && (29)
\end{aligned}$$

The remaining quantities $\mathcal{P}_{ij}(t)$ can be derived from the previous expressions by using the following properties:

$$\begin{aligned}
&\mathcal{P}_{ij}(t) = \mathcal{P}_{ji}(t)\ , \qquad i,j = 1,2,3,4\ , && (30)\\
&\mathcal{P}_{i3}(t) = \mathcal{P}^*_{i4}(t)\ , \qquad i = 1,2\ , && (31)\\
&\mathcal{P}_{44}(t) = \mathcal{P}^*_{33}(t)\ . && (32)
\end{aligned}$$

Putting $a = b = c = \alpha = \beta = \gamma = 0$, one obtains the standard quantum mechanical effective description that evidentiates the singlet-like anti-correlation in $\rho_A(t)$: $\mathcal{P}_{ii}(t) \equiv 0$.

We emphasize that none of the above expressions contain the standard CP, CPT-violating parameters ϵ_S, ϵ_L. This fact makes possible, at least in principle, a direct determination of the non-standard parameters irrespectively of the values of ϵ_S, ϵ_L; one needs to fit the previous expressions of the quantities $\mathcal{P}_{ij}(t)$ with actual data from experiments at ϕ-factories.

To be more specific, we shall now explicitly show how the quantities $\mathcal{P}_{ij}$ can be directly related to frequency countings of decay events. First, notice that, given any single-kaon time-evolution $\rho \mapsto \rho(t)$, the matrix elements of the state $\rho(t)$ at time t can be measured by identifying appropriate orthogonal bases in the two-dimensional single kaon Hilbert space. The choice of the CP-eigenstates $|K_1\rangle$, $|K_2\rangle$ is rather suited to experimental tests. Indeed, since a two-pion state has the same CP eigenvalue of $|K_1\rangle$, the probability

$P_t(K_1) = \langle K_1|\rho(t)|K_1\rangle$ of having a kaon state K_1 at time t is directly related to the frequency of two-pion decays at time t. Possible direct CP violating effects, the only ones allowing $K_2 \to 2\pi$, can be safely neglected; they are proportional to the phenomenological parameter ε', that has been found to be very small. [62]

On the other hand, while the decay state $\pi^0\pi^0\pi^0$ has $CP = -1$, the state $\pi^+\pi^-\pi^0$ may have $CP = \pm 1$. Thus, the probability $P_t(K_2) = \langle K_2|\rho(t)|K_2\rangle$ to have a kaon state K_2 at time t is not as conveniently measured by counting the frequency of the three-pion decays. To avoid the difficulty, the following strangeness eigenstates can be used:

$$|K^0\rangle = \frac{|K_1\rangle + |K_2\rangle}{\sqrt{2}}\ , \qquad |\overline{K^0}\rangle = \frac{|K_1\rangle - |K_2\rangle}{\sqrt{2}}\ . \tag{33}$$

Then, the probabilities $P_t(K^0) = \langle K^0|\rho(t)|K^0\rangle$ and $P_t(\overline{K^0}) = \langle\overline{K^0}|\rho(t)|\overline{K^0}\rangle$, that the kaon state at time t be a K^0, respectively a $\overline{K^0}$, may be experimentally determined by counting the semileptonic decays $K^0 \mapsto \pi^-\ell^+\nu$, respectively $\overline{K^0} \mapsto \pi^+\ell^-\overline{\nu}$, the exchanged decays being forbidden by the $\Delta S = \Delta Q$ rule. (In the Standard Model, this selection rule is expected to be valid up to order 10^{-14}. [63]) Further, the probability $P_t(K_2) = \langle K_2|\rho(t)|K_2\rangle$ of having a kaon state K_2 at proper time t can be expressed as $P_t(K_2) = P_t(K^0) + P_t(\overline{K^0}) - P_t(K_1)$, by writing

$$|K_2\rangle\langle K_2| = |K^0\rangle\langle K^0| + |\overline{K^0}\rangle\langle\overline{K^0}| - |K_1\rangle\langle K_1|\ . \tag{34}$$

Hence, $P_t(K_2)$ can be measured by counting the frequencies of semileptonic decays and of decays into two pions.

In order to measure the off-diagonal elements $\langle K_1|\rho|K_2\rangle$, $\langle K_2|\rho|K_1\rangle$, we use the identity

$$|K^0\rangle\langle K^0| - |\overline{K^0}\rangle\langle\overline{K^0}| = |K_1\rangle\langle K_2| + |K_2\rangle\langle K_1|\ , \tag{35}$$

and extract $|K_1\rangle\langle K_2|$ from it. To do this, we need a third orthonormal basis of vectors whose projectors are measurable observables in actual experiments. An interesting possibility is based on the phenomenon of kaon-regeneration (see Refs.[64, 65] and references therein). The idea is to insert a slab of material across the neutral kaons path; the interactions of the K^0, $\overline{K^0}$ mesons with the nuclei of the material "rotate" in a known way the initial kaon states entering the regenerator into new ones. As initial states, consider the orthogonal vectors

$$|\widetilde{K}_S\rangle = \frac{|K_1\rangle - \eta^*|K_2\rangle}{\sqrt{1+|\eta|^2}}\ , \qquad |\widetilde{K}_L\rangle = \frac{\eta|K_1\rangle + |K_2\rangle}{\sqrt{1+|\eta|^2}}\ , \tag{36}$$

where η is a complex parameter which depends on the regenerating material. By carefully choosing the material and the tickness of the slab, one can tune the modulus and phase of η in such a way to completely suppress the $\widetilde{K}_L$ component and to regenerate the $\widetilde{K}_S$ state into a K_1, just outside the material slab. Thus, the probability $P_t(\widetilde{K}_S) = \langle\widetilde{K}_S|\rho(t)|\widetilde{K}_S\rangle$ that a kaon, impinging on a slab of regenerating material in a state $\rho(t)$ at time t, be a $\widetilde{K}_S$, can be measured by counting the decays into 2π just beyond the slab. Now, the projector onto the state $|\widetilde{K}_S\rangle$ reads

$$\begin{aligned} |\widetilde{K}_S\rangle\langle\widetilde{K}_S| \quad &= \quad \frac{1}{1+|\eta|^2}|K_1\rangle\langle K_1| + \frac{|\eta|^2}{1+|\eta|^2}|K_2\rangle\langle K_2| \\ &- \quad \frac{\eta}{1+|\eta|^2}|K_1\rangle\langle K_2| - \frac{\eta^*}{1+|\eta|^2}|K_2\rangle\langle K_1|\ . \end{aligned} \tag{37}$$

Then, from (33)–(35) and (37) it follows that

$$\begin{aligned} |K_1\rangle\langle K_2| \quad &= \quad \zeta_1\,|K_1\rangle\langle K_1| \,+\, \zeta_2\,|\widetilde{K}_S\rangle\langle\widetilde{K}_S| \\ &+ \quad \zeta_3\,|K^0\rangle\langle K^0| \,+\, \zeta_4\,|\overline{K^0}\rangle\langle\overline{K^0}|\ , \end{aligned} \tag{38}$$

where

$$\zeta_1 \quad = \quad \frac{1-|\eta|^2}{2i\mathcal{I}m(\eta)}\ , \qquad \zeta_2 = -\frac{1+|\eta|^2}{2i\mathcal{I}m(\eta)}\ , \tag{39}$$

$$\zeta_3 \quad = \quad \frac{|\eta|^2-\eta^*}{2i\mathcal{I}m(\eta)}\ , \qquad \zeta_4 = \frac{|\eta|^2+\eta^*}{2i\mathcal{I}m(\eta)}\ . \tag{40}$$

In this way, the determination of the off-diagonal elements of $\rho(t)$ amounts to counting the frequencies of decays into two pions with or without regeneration and the frequencies of semileptonic decays:

$$\begin{aligned} \langle K_1|\rho(t)|K_2\rangle \quad &= \quad \zeta_1\,P_t(K_1) + \zeta_2\,P_t(\widetilde{K}_S) \\ &+ \quad \zeta_3\,P_t(K^0) + \zeta_4\,P_t(\overline{K^0})\ . \end{aligned} \tag{41}$$

The application of these results to the case of correlated kaons is now straightforward. For sake of compactness, we identify the various kaon states with the projections Q_μ, $\mu = 1,2,3,4$, where:

$$Q_1 = |K_1\rangle\langle K_1| \ , \qquad Q_3 = |K^0\rangle\langle K^0| \ , \tag{42}$$

$$Q_2 = |\widetilde{K}_S\rangle\langle\widetilde{K}_S| \ , \qquad Q_4 = |\overline{K^0}\rangle\langle\overline{K^0}| \ . \tag{43}$$

As discussed, these operators can be measured by identifying 2π final states, in absence and presence of a regenerator (Q_1 and Q_2), and semileptonic decays (Q_3 and Q_4); the same holds for the projectors in (12) and (13), since:

$$P_1 = |K_1\rangle\langle K_1| \equiv Q_1 \ , \tag{44}$$

$$P_2 = |K_2\rangle\langle K_2| = Q_3 + Q_4 - Q_1 \ , \tag{45}$$

$$P_3 = |K_1\rangle\langle K_2| \equiv P_4^\dagger = \sum_{\mu=1}^{4} \zeta_\mu \, Q_\mu \ . \tag{46}$$

Further, we denote by $P_t(Q_\mu, Q_\nu)$ the probability that, at proper time t after a ϕ-decay, the two kaons be in the states identified by Q_μ and Q_ν, respectively. Then, the determination of the quantities $\mathcal{P}_{ij}(t)$ reduces to measuring joint probabilities, *i.e.* to counting frequencies of events of certain specificied types. Indeed, one explicitly finds:

$$\mathcal{P}_{11}(t) = P_t(Q_1, Q_1) \ , \tag{47}$$

$$\mathcal{P}_{12}(t) = P_t(Q_1, Q_3) + P_t(Q_1, Q_4) - P_t(Q_1, Q_1) \ , \tag{48}$$

$$\mathcal{P}_{13}(t) = \sum_{\mu=1}^{4} \zeta_\mu \, P_t(Q_1, Q_\mu) \ , \tag{49}$$

$$\mathcal{P}_{22}(t) = P_t(Q_1, Q_1) + P_t(Q_3, Q_3) + P_t(Q_4, Q_4) \\ +2\Big[P_t(Q_3, Q_4) - P_t(Q_1, Q_4) - P_t(Q_1, Q_4)\Big] \ , \tag{50}$$

$$\mathcal{P}_{23}(t) = \sum_{\mu=1}^{4} \zeta_\mu \, \Big[P_t(Q_3, Q_\mu) + P_t(Q_4, Q_\mu) - P_t(Q_1, Q_\mu)\Big] \ , \tag{51}$$

$$\mathcal{P}_{33}(t) = \sum_{\mu=1}^{4} \sum_{\nu=1}^{4} \zeta_\mu \, \zeta_\nu \, P_t(Q_\mu, Q_\nu) \ . \tag{52}$$

As a result of the previous analysis, the inconsistencies of models without complete positivity, besides being theoretically unsustainable, turn out to be experimentally exposable. Let P_{φ_+} and P_{ψ_+} project onto the correlated states

$$|\varphi_+\rangle = \frac{1}{\sqrt{2}}\Big(|K_1\rangle \otimes |K_1\rangle + |K_2\rangle \otimes |K_2\rangle\Big) \ , \tag{53}$$

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}\Big(|K_1\rangle\otimes|K_2\rangle + |K_2\rangle\otimes|K_1\rangle\Big)\ , \tag{54}$$

that are orthogonal to the state $|\psi_A\rangle$ in (11) produced in a ϕ decay. The averages of these two positive observables with respect to the state $\rho_A(t)$ read

$$\begin{aligned}
\Phi(t) &= \mathrm{Tr}[\rho_A(t)\,P_{\varphi_+}] \equiv \langle\varphi_+|\rho_A(t)|\varphi_+\rangle \\
&= \frac{1}{2}\big(\mathcal{P}_{11}(t)+\mathcal{P}_{22}(t)+\mathcal{P}_{33}(t)+\mathcal{P}_{44}(t)\big) \qquad (55)\\
\Psi(t) &= \mathrm{Tr}[\rho_A(t)\,P_{\psi_+}] \equiv \langle\psi_+|\rho_A(t)|\psi_+\rangle \\
&= \frac{1}{2}\big(\mathcal{P}_{12}(t)+\mathcal{P}_{21}(t)+\mathcal{P}_{34}(t)+\mathcal{P}_{43}(t)\big)\ , \qquad (56)
\end{aligned}$$

and, as explained before, can be directly obtained by measuring joint probabilities in experiments at ϕ-factories. On the other hand, (23)–(29) give, up to first order in the small parameters,

$$\begin{aligned}
\Phi(t) = e^{-2\Gamma t}\Bigg[&\frac{\gamma}{\Delta\Gamma}\sinh(t\Delta\Gamma) + \frac{b}{\Delta m}\Big(1-\cos(2t\Delta m)\Big) \\
&+\frac{a-\alpha}{2\Delta m}\sin(2t\Delta m)\Bigg]\ , \qquad (57)
\end{aligned}$$

$$\Psi(t) = e^{-2\Gamma t}\big(a+\alpha-\gamma\big)\,t\ . \tag{58}$$

Thus, $\Phi(0)=\Psi(0)=0$, whereas

$$\frac{\mathrm{d}\Phi(0)}{\mathrm{d}t} = a+\gamma-\alpha\ , \qquad \frac{\mathrm{d}\Psi(0)}{\mathrm{d}t} = a+\alpha-\gamma\ , \tag{59}$$

are both positive because of conditions (19) and (20). More in general, the mean values (55) and (56) are surely positive, for the complete positivity of the single-kaon dynamical maps γ_t implies $\rho(t)=\sum_j V_j(t)\,\rho\,V_j^\dagger(t)$,[1) − 6)] where the $V_j(t)$ are 2×2 matrices such that $\sum_j V_j^\dagger(t)V_j(t)$ is a bounded 2×2 matrix.[5] Then, the complete evolution $\rho_A\to\rho_A(t)=\sum_{i,j}[V_i(t)\otimes V_j(t)]\,\rho_A\,[V_i^\dagger(t)\otimes V_j^\dagger(t)]$ will never develop negative eigenvalues.

[5] Notice that, in absence of the extra contribution L in (3), the time evolution $\rho(t)$ is realized with a single matrix V, *i.e.* $j=1$, and $V_1(t)=e^{-iH_{\rm eff}\,t}$; in other words, in ordinary quantum mechanics the condition of complete positivity is trivially satisfied.

On the other hand, if the single-kaon dynamical map ω_t is not completely positive, inconsistencies may emerge. As an example, take the phenomenological models studied in Refs.[55, 56], where the non-standard parameters a, b, c are set to zero and $\alpha \neq \gamma$, $\alpha\gamma \geq \beta^2$. The corresponding dynamics is not completely positive: the inequalities (18)–(21) are in fact violated. In this case, one still has $\Phi(0) = \Psi(0) = 0$, but

$$\frac{\mathrm{d}\Phi(0)}{\mathrm{d}t} = \gamma - \alpha = -\frac{\mathrm{d}\Psi(0)}{\mathrm{d}t} \ . \tag{60}$$

Therefore, one of the mean values (55), (56) starts assuming negative values as soon as $t > 0$. The inconsistence is avoided only if $\alpha = \gamma$, which is a necessary condition for getting back the property of complete positivity. As explictly discussed above, planned set-ups at ϕ-factories can measure, at least in principle, the two mean values in (55) and (56) and therefore directly check the positivity of the two combinations in (59), thus clarifying also from the experimental point of view the need of complete positivity for the description of the dissipative dynamics of neutral kaons.

4 Tests of dissipative effects in kaon dynamics

From the discussion of the previous section, it is apparent that a physically consistent description of the dissipative dynamics of neutral kaons weakly coupled to an external environment can be realized only through the use of completely positive dynamical semigroups; these are generated by master equations of the form (3) and (5), with a positive Kossakowski matrix C. Indeed, only evolutions of this type satisfy the physical requirements that are at the basis of the statistical interpretation of quantum mechanics, so that the eigenvalues of the kaon density matrix can be correctly interpreted as probabilities. Modelling dissipative kaon evolutions with linear maps that are not completely positive will unavoidably lead to the appearence of negative values for some of those probabilities when correlated kaons are involved, thus spoiling the physical consistency of the whole treatment. Indeed, only completely positive time evolutions are compatible with the presence of entanglement. [6)]

In view of these considerations, it is apparent that the form (3), (5) of the kaon time-evolution is very general and quite independent from the detailed mechanism leading to the phenomena of noise and dissipation, that can be of gravitational, stringy or fluctuating medium origin. Indeed, the evolution of any quantum open system, immersed in a weakly coupled environment can be effectively modeled using quantum dynamical semigroups. In this respect, the equations (3), (5) should be regarded as phenomenological in nature, and therefore quite suitable to experimental test: any signal of non-vanishing value for some of the parameters in (8) would certainly attest in a model independent way the presence of non-standard, dissipative effects in kaon physics.

Physical observables of the neutral kaon system are associated with the decays of the kaons into suitable final state f, typically pion states, and semileptonic states. In the language of density matrices, these final decay states are described by suitable hermitian operators $\mathcal{O}_f$; taking the trace of these operators with $\rho(t)$, solution of the master equation (3), (5), allows computing the explicit time dependence of various experimentally relevant observables (*e.g.* see Refs.[38, 40, 41, 51, 52]).[6]

For instance, the operators $\mathcal{O}_{+-}$ and $\mathcal{O}_{00}$ that describe the $\pi^+\pi^-$ and $2\pi^0$ final states have the form:

$$\mathcal{O}_{+-} \sim \begin{bmatrix} 1 & r_{+-} \\ r^*_{+-} & |r_{+-}|^2 \end{bmatrix} , \qquad \mathcal{O}_{00} \sim \begin{bmatrix} 1 & r_{00} \\ r^*_{00} & |r_{00}|^2 \end{bmatrix} . \tag{61}$$

To lowest order in all small parameters, the complex constants r_{+-} and r_{00}, can be written as:

$$r_{+-} = \varepsilon - \epsilon_L + \varepsilon' , \qquad r_{00} = \varepsilon - \epsilon_L - 2\varepsilon' , \tag{62}$$

where ε and ε' are the familiar phenomenological parameters signaling direct CP and CPT violating effects.[57) – 60)] Similar results hold for the matrices $\mathcal{O}_{\pi^+\pi^-\pi^0}$, $\mathcal{O}_{3\pi^0}$, $\mathcal{O}_{\ell^-}$ and $\mathcal{O}_{\ell^+}$ that describe the decays into $\pi^+\pi^-\pi^0$, $3\pi^0$, $\pi^+\ell^-\bar{\nu}$ and $\pi^-\ell^+\nu$; for explicit expressions, see Refs.[39, 40, 50, 51].

With the help of these matrices, one can compute the time-dependence of various useful observables of the neutral kaon system, like decay rates and asymmetries. These quantities are accessible to the experiment, so that they can be used to obtain bounds on the dissipative effects, encoded in the parameters in (8); using the most recent available results on single kaon experi-

[6] Similar appraoches are employed also in Refs.[55, 56, 66, 67], where however, as discussed in the previous section, a physically inconsistent time evolution is adopted.

ments, [62),68),69)] one can indeed obtain estimates on some of them:

$$
\begin{aligned}
&a \leq 5.0 \times 10^{-17}\ \mathrm{GeV}\ ,\\
&c \leq 2.0 \times 10^{-17}\ \mathrm{GeV}\ ,\\
&\alpha \leq 6.0 \times 10^{-17}\ \mathrm{GeV}\ ,\\
&\beta \leq 1.0 \times 10^{-17}\ \mathrm{GeV}\ ,\\
&\gamma \leq 22.0 \times 10^{-20}\ \mathrm{GeV}\ .
\end{aligned}
\tag{63}
$$

Unfortunately, the precision of the present experimental results on single neutral kaons is not high enough, so that only rough upper bounds can be obtained. Although more complete and precise data will surely be available in the future, the most promising venues for studying the consequences of the dissipative dynamics in (3), (5) are certainly the experiments on correlated neutral kaons at ϕ-factories.

The typical observables that can be studied in such physical situations are double decay rates, *i.e.* the probabilities that a kaon decays into a final state f_1 at proper time t_1, while the other kaon decays into the final state f_2 at proper time t_2:

$$
\mathcal{G}(f_1,t_1;f_2,t_2) \equiv \mathrm{Tr}\Big[\Big(\mathcal{O}_{f_1}\otimes\mathcal{O}_{f_2}\Big)\rho_A(t_1,t_2)\Big]\ ; \tag{64}
$$

as before, the operators $\mathcal{O}_{f_1}$ and $\mathcal{O}_{f_2}$ are the 2×2 hermitian matrices describing the final decay states. By studying these observables in a high-luminosity ϕ-factory it will be possible to determine the values of the non-standard parameters a, b, c, α, β, γ.[7] For instance, the long time behaviour ($t \gg 1/\gamma_S$) of the three pion probability gives direct informations on the parameter γ:

$$
\mathcal{G}(\pi^+\pi^-\pi^0,t\,;\pi^+\pi^-\pi^0,t) \sim \frac{\gamma}{\Delta\Gamma}\ e^{-2\gamma_L t}\ . \tag{65}
$$

Similarly, the small time behaviour of the ratio of semileptonic probabilities,

$$
\frac{\mathcal{G}(\ell^\pm,t;\ell^\pm,t)}{\mathcal{G}(\ell^\pm,t;\ell^\mp,t)} \sim 2\ a\ t\ , \tag{66}
$$

[7]As observed before, notice that the time-behaviour of these decay rates is completely different from the one required by ordinary quantum mechanics, which for instance predicts $\mathcal{G}(f,t;f,t)\equiv 0$ for all final states f, due to the antisymmetry of the initial state ρ_A.

is sensitive to the parameter a. [41]

However, much of the analysis at ϕ-factories is carried out using integrated distributions at fixed time interval $t = t_1 - t_2$. [70] One then deals with single-time distributions, defined by:

$$\Gamma(f_1, f_2; t) = \int_0^\infty d\tau\ \mathcal{G}(f_1, \tau + t; f_2, \tau)\ , \qquad t > 0\ . \tag{67}$$

Starting with these integrated probabilities, one can form asymmetries that are sensitive to various parameters in the theory. A particularly interesting example is given by the following observable, involving two-pion final states,

$$\mathcal{A}_{\varepsilon'}(t) = \frac{\Gamma(\pi^+\pi^-, 2\pi^0; t) - \Gamma(2\pi^0, \pi^+\pi^-; t)}{\Gamma(\pi^+\pi^-, 2\pi^0; t) + \Gamma(2\pi^0, \pi^+\pi^-; t)}\ ; \tag{68}$$

it is used for the determination of the ratio ε'/ε. The clear advantage of using the asymmetry $\mathcal{A}_{\varepsilon'}$ to determine the value of ε'/ε in comparison to the familiar "double ratio" method, [62] is that, at least in principle, both real and imaginary part can be extracted from the time behaviour of (68). [70] Due to the presence of the dissipative parameters however, this appears to be much more problematic than in the standard case; a meaningful determination of ε'/ε is possible provided independent estimates on c, β and γ are obtained from the measure of other independent asymmetries. This is particularly evident if one looks at the large-time limit ($t \gg 1/\gamma_S$) of (68):

$$\mathcal{A}_{\varepsilon'}(t) \sim 3\,\mathcal{R}e\left(\frac{\varepsilon'}{\varepsilon}\right) \frac{|\varepsilon|^2 + 2\,\mathcal{R}e\big(\varepsilon\, C/\Delta\Gamma_+\big)}{|\varepsilon|^2 + \mathcal{D}} - 6\,\mathcal{I}m\left(\frac{\varepsilon'}{\varepsilon}\right) \frac{\mathcal{I}m\big(\varepsilon\, C/\Delta\Gamma_+\big)}{|\varepsilon|^2 + \mathcal{D}}\ , \tag{69}$$

where

$$\mathcal{D} = \frac{\gamma}{\Delta\Gamma} - 4\left|\frac{C}{\Delta\Gamma_+}\right|^2 + 4\,\mathcal{R}e\left(\frac{\varepsilon\, C}{\Delta\Gamma_+}\right) - 4\,\mathcal{R}e\left(\frac{\epsilon_L C}{\Delta\Gamma}\right)\ , \tag{70}$$

with $C = c + i\beta$; only when $c = \beta = \gamma = 0$, the expression in (69) reduces to the standard result: $\mathcal{A}_{\varepsilon'} \sim 3\,\mathcal{R}e(\varepsilon'/\varepsilon)$. Therefore, if the non-standard, dissipative parameters in (8) are found to be non-zero, even neglecting the contribution from the imaginary part, the actual value of $\mathcal{R}e(\varepsilon'/\varepsilon)$ could be significantly different from the measured value of the quantity $\mathcal{A}_{\varepsilon'}/3$. [46]

In conclusion, dissipative effects in the dynamics of both single and correlated neutral kaon systems could affect the precise determination of various relevant quantities in kaon physics; dedicated experiments, in particular those involving correlated kaons, will certainly provide stringent bounds on these dissipative effects, and possibly allow a definite clarification of the role played by complete positivity in open quantum system dynamics.

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