

TESTING CPT IN THE NEUTRAL KAON SYSTEM BY MEANS OF THE BELL-STEINBERGER RELATION

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Abstract

The possibility to test the basic assumptions of quantum field theories, and in particular the CPT theorem, by means of unitarity relations in the neutral kaon system (Bell-Steinberger relation) is reviewed. The present status of these tests and their future prospects are also briefly outlined.

1 Introduction

The three discrete symmetries of charge conjugation (C), parity (P) and time reversal (T) are known to be violated in nature, both separately and in any bilinear combination. Only CPT, namely the product of the three (in any order), seems to be an exact symmetry in nature. This fact is not surprising: exact CPT invariance is expected in any quantum field theory respecting the general hypotheses of Lorentz invariance, locality and unitarity [1]. For this reason, testing the validity of CPT invariance is equivalent to probe some of the most fundamental assumptions on which the present description of particle physics is based. Interestingly enough, these hypotheses are likely to be violated at very high energy scales, where the quantum effects of gravitational interactions cannot be ignored [2]. On the other hand, since we still miss a consistent theory of quantum gravity, it is hard to predict at which level CPT-violating effects may show up in experimentally accessible systems.

The neutral kaon system offers a unique possibility for phenomenological studies of CPT invariance. One of the most significant tests is the one obtained by means of the Bell-Steinberger (BS) relation [3]. This relation makes use of unitarity (or the conservation of probability) to connect a possible violation of CPT invariance in the time-evolution of the $K^0-\bar{K}^0$ system ($m_{K^0} \neq m_{\bar{K}^0}$ and/or $\Gamma_{K^0} \neq \Gamma_{\bar{K}^0}$) to the observable CP-violating interference of K_L and K_S decays into the same final state f . Because of the involvement of the unitarity hypothesis, the BS relation cannot be considered as model-independent test of CPT invariance. However, this does not diminish the role of this relation in testing the basic assumptions of quantum field theories (we recall that unitarity is also one of the main hypothesis of the CPT theorem).

2 Theoretical framework

Within the Wigner-Weisskopf approximation, the time evolution of the neutral kaon system is described by [4]

$$i\frac{\partial}{\partial t}\Psi(t) = H\Psi(t) = (M - \frac{i}{2}\Gamma)\Psi(t) , \quad (1)$$

where M and Γ are 2×2 time-independent Hermitian matrices and $\Psi(t)$ is a two-component state vector in the $K^0-\bar{K}^0$ space. Denoting by m_{ij} and Γ_{ij} the elements of M and Γ in the $K^0-\bar{K}^0$ basis, CPT invariance implies

$$m_{11} = m_{22} \quad (\text{or } m_{K^0} = m_{\bar{K}^0}) \quad \text{and} \quad \Gamma_{11} = \Gamma_{22} \quad (\text{or } \Gamma_{K^0} = \Gamma_{\bar{K}^0}) . \quad (2)$$

The eigenstates of eq. (1) can be written as

$$K_{S,L} = \frac{1}{\sqrt{2(1+|\epsilon_{S,L}|^2)}} [(1 + \epsilon_{S,L}) K^0 \pm (1 - \epsilon_{S,L}) \bar{K}^0] , \quad (3)$$

$$\begin{aligned} \epsilon_{S,L} &= \frac{-i\text{Im}(m_{12}) - \frac{1}{2}\text{Im}(\Gamma_{12}) \pm \frac{1}{2} [m_{\bar{K}^0} - m_{K^0} - \frac{i}{2} (\Gamma_{\bar{K}^0} - \Gamma_{K^0})]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2} \\ &= \epsilon \pm \delta , \end{aligned} \quad (4)$$

such that $\delta = 0$ in the limit of exact CPT invariance.

Unitarity allows us to express the four entries of Γ in terms of appropriate combination of kaon decay amplitudes:

$$\Gamma_{ij} = \sum_f \mathcal{A}_i(f)\mathcal{A}_j(f)^* , \quad (5)$$

where the sum runs over all the accessible final states. Using this decomposition in eq. (4) leads to the BS relation: a link between $\text{Re}(\epsilon)$, $\text{Im}(\delta)$ and the physical kaon decay amplitudes. In particular, without any expansion in the CPT-conserving parameters and neglecting only $\mathcal{O}(\epsilon)$ corrections to the coefficient of the CPT-violating parameter δ , we find

$$\left[\frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i \tan \phi_{\text{SW}} \right] \left[\frac{\text{Re}(\epsilon)}{1 + |\epsilon|^2} - i \text{Im}(\delta) \right] = \frac{1}{\Gamma_S - \Gamma_L} \sum_f \mathcal{A}_L(f) \mathcal{A}_S^*(f), \quad (6)$$

where $\phi_{\text{SW}} = \arctan[2(m_L - m_S)/(\Gamma_S - \Gamma_L)]$. We stress that, contrary to similar expressions which can be found in the literature, eq. (6) is exact and phase-convention independent in the exact CPT limit: an evidence for a non-vanishing $\text{Im}(\delta)$ resulting from this relation can only be attributed to violations of: i) CPT invariance; ii) unitarity; iii) the time independence of M and Γ in eq. (1).

The advantage of the neutral kaon system is that only few decay modes give significant contributions to the r.h.s. in eq. (6): in practice, only the $\pi\pi(\gamma)$, $\pi\pi\pi$ and $\pi\ell\nu$ modes turn out to be relevant up to the 10^{-7} level. The product of the corresponding decay amplitudes are conveniently expressed in terms of the α_i parameters defined below.

2.1 Two-pion modes

Starting from two pion states, we define

$$\alpha_i = \frac{1}{\Gamma_S} \langle \mathcal{A}_L(i) \mathcal{A}_S^*(i) \rangle = \eta_i \text{BR}(\text{K}_S \rightarrow i), \quad i = \pi^0\pi^0, \pi^+\pi^-(\gamma) \quad (7)$$

where $\pi^+\pi^-(\gamma)$ denotes the inclusive sum over bremsstrahlung photons, and $\langle \rangle$ indicates the appropriate phase-space integrals. By construction, the η_i appearing in eq. (7) can also be expressed in terms non-integrated amplitude ratios: $\eta_i = \mathcal{A}_L(i)/\mathcal{A}_S(i)$.

The contributions from $\pi^+\pi^-\gamma$ direct-emission (DE) amplitudes not included in the $\alpha_{\pi^+\pi^-(\gamma)}$ parameter are encoded in

$$\alpha_{\pi\pi\gamma\text{DE}} = \alpha_{\pi\pi\gamma\text{E1-S}} + \alpha_{\pi\pi\gamma\text{E1-L}} + \alpha_{\pi\pi\gamma\text{DE}\times\text{DE}}, \quad (8)$$

where

$$\begin{aligned}
\alpha_{\pi\pi\gamma_{\text{E1-S}}} + \alpha_{\pi\pi\gamma_{\text{E1-L}}} &= \\
&= \frac{1}{\Gamma_S} [\langle \mathcal{A}_L(\pi\pi\gamma) \mathcal{A}_S^*(\pi\pi\gamma_{\text{E1}}) \rangle + \langle \mathcal{A}_L(\pi\pi\gamma_{\text{E1}}) \mathcal{A}_S^*(\pi\pi\gamma) \rangle] \\
&= \Delta B(K_S \rightarrow \pi\pi\gamma_{\text{DE}}) \eta_{+-} + (\eta_{+-\gamma} - \eta_{+-}) \text{BR}(K_S \rightarrow \pi\pi\gamma)
\end{aligned} \tag{9}$$

Here $\mathcal{A}_{L,S}(\pi\pi\gamma)$ and $\mathcal{A}_{L,S}(\pi\pi\gamma_{\text{E1}})$ denote the leading bremsstrahlung and the electric-dipole DE amplitudes, respectively. Their interference cannot be trivially neglected. $\text{BR}(K_S \rightarrow \pi\pi\gamma)$ indicates the branching fraction for a real photon emission, with minimum photon-energy cut equivalent to the one used in the corresponding $\eta_{+-\gamma}$ measurement. $\Delta B(K_S \rightarrow \pi\pi\gamma_{\text{DE}}) = \text{BR}(K_S \rightarrow \pi\pi\gamma)^{\text{exp}} - \text{BR}(K_S \rightarrow \pi\pi\gamma)^{\text{th-IB}}$ is the deviation of the observed $K_S \rightarrow \pi\pi\gamma$ decay distribution from the one inferred from a pure bremsstrahlung spectrum.

We have generically denoted by $\alpha_{\pi\pi\gamma_{\text{DE}} \times \text{DE}}$ the contribution arising from the interference of two DE amplitudes (either electric or magnetic ones). Given the strong experimental suppression of DE amplitudes, this term turns out to be safely negligible up to the 10^{-8} level [5].

2.2 Three-pion modes

For the three pion states we define

$$\alpha_i = \frac{1}{\Gamma_S} \langle \mathcal{A}_L(i) \mathcal{A}_S^*(i) \rangle = \frac{\tau_{K_S}}{\tau_{K_L}} \eta_i^* \text{BR}(K_L \rightarrow i) \quad i = 3\pi^0, \pi^0\pi^+\pi^-(\gamma). \tag{10}$$

Note that in this case the amplitudes are not necessarily constant over the phase space. As a result, the η_i appearing in eq. (10) should be interpreted as appropriate Dalitz-plot averages. In practice, given the poor direct experimental information on η_{000} , in the neutral case it turns out to be more convenient to set a bound on $|\alpha_{\pi^0\pi^0\pi^0}|$ by means of the relation

$$|\alpha_{\pi^0\pi^0\pi^0}|^2 = \frac{\tau_{K_S}}{\tau_{K_L}} \text{BR}(K_L \rightarrow 3\pi^0) \text{BR}(K_S \rightarrow 3\pi^0). \tag{11}$$

2.3 Semileptonic channels

In the case of semileptonic channels, introducing the standard decomposition [6]

$$\begin{aligned}
\mathcal{A}(K^0 \rightarrow l^+ \nu \pi^-) &= A_0(1 - y) , \\
\mathcal{A}(K^0 \rightarrow l^- \nu \pi^+) &= A_0^*(1 + y^*)(x_+ - x_-)^* , \\
\mathcal{A}(\bar{K}^0 \rightarrow l^- \nu \pi^+) &= A_0^*(1 + y^*) , \\
\mathcal{A}(\bar{K}^0 \rightarrow l^+ \nu \pi^-) &= A_0(1 - y)(x_+ + x_-) ,
\end{aligned} \tag{12}$$

assuming lepton universality, and expanding to first non-trivial order in the small CP- and CPT-violating parameters, leads to

$$\begin{aligned}
\sum_{\pi l \nu} \langle \mathcal{A}_L(\pi l \nu) \mathcal{A}_S^*(\pi l \nu) \rangle &= 2\Gamma(K_L \rightarrow \pi l \nu) \{ \text{Re}(\epsilon) - \text{Re}(y) - i [\text{Im}(x_+) + \text{Im}(\delta)] \} \\
&= 2\Gamma(K_L \rightarrow \pi l \nu) \{ (A_S + A_L)/4 - i [\text{Im}(x_+) + \text{Im}(\delta)] \} .
\end{aligned} \tag{13}$$

The dependence of $\text{Re}(y)$ has been eliminated taking advantage of the relation $\text{Re}(\epsilon) - \text{Re}(y) = (A_S + A_L)/4$ [6], where $A_{L,S}$ are the observable semileptonic charge asymmetries. The parameter $\text{Im}(x_+)$ can be measured by appropriate time-dependent distributions [7], while $\text{Im}(\delta)$ is one of the two output of the BS relation. In order to get rid of the explicit $\text{Im}(\delta)$ dependence, it is convenient to define

$$\begin{aligned}
\alpha_{\pi l \nu} &= \frac{1}{\Gamma_S} \sum_{\pi l \nu} \langle \mathcal{A}_L(\pi l \nu) \mathcal{A}_S^*(\pi l \nu) \rangle + 2i \frac{\tau_{K_S}}{\tau_{K_L}} \text{BR}(K_L \rightarrow \pi l \nu) \text{Im}(\delta) \\
&= 2 \frac{\tau_{K_S}}{\tau_{K_L}} \text{BR}(K_L \rightarrow \pi l \nu) [(A_S + A_L)/4 - i \text{Im}(x_+)] .
\end{aligned} \tag{14}$$

2.4 Determination of $\text{Re}(\epsilon)$ and $\text{Im}(\delta)$

The α_i defined in eqs. (7), (10), (8), and (14) can be determined (or bounded) in terms of measurable quantities. Taking into account these definitions (in particular the non-standard expression of $\alpha_{\pi l \nu}$), the solution to the unitarity relation in eq. (6) is:

$$\begin{bmatrix} \frac{\text{Re}(\epsilon)}{1 + |\epsilon|^2} \\ \text{Im}(\delta) \end{bmatrix} = \frac{1}{N} \begin{bmatrix} 1 + \kappa(1 - 2b) & (1 - \kappa) \tan \phi_{\text{SW}} \\ (1 - \kappa) \tan \phi_{\text{SW}} & -(1 + \kappa) \end{bmatrix} \begin{bmatrix} \sum_i \text{Re}(\alpha_i) \\ \sum_i \text{Im}(\alpha_i) \end{bmatrix} , \tag{15}$$

where $\kappa = \tau_{K_S}/\tau_{K_L}$, $b = \text{BR}(K_L \rightarrow \pi\ell\nu)$, and

$$N = (1 + \kappa)^2 + (1 - \kappa)^2 \tan^2 \phi_{\text{SW}} - 2b\kappa(1 + \kappa). \quad (16)$$

As anticipated, a non-vanishing $\text{Im}(\delta)$ resulting from this relation would signal a major breakthrough in our understanding of fundamental interactions: $\text{Im}(\delta) \neq 0$ could be attributed either to violations of CPT symmetry, or to violations of unitarity (including apparent violations due to undetected final states), or to violations of the Wigner-Weisskopf approximation.

3 Present experimental status and future prospects

The experimental determination of the α_i has recently been reviewed and updated in Ref. [8], taking into account a series of new measurements of K_L and K_S branching ratios by KLOE in conjunction with previous results by other experiments. The complete updated list of inputs is summarized in Table 1. As far as $K \rightarrow \pi l\nu$ amplitudes are concerned, the KLOE measurement of the K_S charge asymmetry and the PDG average of the K_L asymmetry have been combined with the time-dependent measurement of K^0 and \overline{K}^0 semileptonic rates by CPLEAR [7]. This has allowed an improved determination of the various parameters appearing in $K \rightarrow \pi l\nu$ amplitudes (see Table 2), and in particular of $\text{Im}(x_+)$, which is the main source of uncertainty in $\alpha_{\pi l\nu}$.

A detailed discussion about the results for all the relevant α_i can be found in Ref. [8]. In Fig. 1 we show the two most representative examples, namely $\alpha_{\pi^+\pi^-}$ and $\alpha_{\pi l\nu}$. Putting all the ingredients together, the values of $\text{Re}(\epsilon)$ and $\text{Im}(\delta)$ obtained by means of the unitarity relation are (see Fig. 2):

$$\text{Re}(\epsilon) = (159.6 \pm 1.3) \times 10^{-5}, \quad \text{Im}(\delta) = (0.4 \pm 2.1) \times 10^{-5}. \quad (17)$$

Thanks to the new KLOE data, the error on $\text{Im}(\delta)$ is now completely dominated by $\pi\pi$ states, and in particular by the $K_L \rightarrow \pi^+\pi^-$ channel. The semileptonic term contributes only to about $\sim 10\%$ of the error on $\text{Im}(\delta)$.

The limits on $\text{Im}(\delta)$ and $\text{Re}(\delta)$, which are perfectly compatible with exact CPT invariance, can be translated into constraints on the K^0 - \overline{K}^0 mass and width differences by means of the relation

$$\delta = \frac{i(m_{K^0} - m_{\overline{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\overline{K}^0})}{\Gamma_S - \Gamma_L} \cos \phi_{\text{SW}} e^{i\phi_{\text{SW}}} [1 + O(\epsilon)]. \quad (18)$$

Table 1: Input values to the Bell-Steinberger relation used in Ref. [8]. For the KLOE averages see Ref. [15].

Observable	Value	Source
τ_{K_S}	0.08958 ± 0.00005 ns	PDG [9]
τ_{K_L}	50.84 ± 0.23 ns	KLOE average
$m_L - m_S$	$(5.290 \pm 0.016) \times 10^9$ s ⁻¹	PDG [9]
$\text{BR}(K_S \rightarrow \pi^+ \pi^-)$	0.69186 ± 0.00051	KLOE average
$\text{BR}(K_S \rightarrow \pi^0 \pi^0)$	0.30687 ± 0.00051	KLOE average
$\text{BR}(K_S \rightarrow \pi e \nu)$	$(11.77 \pm 0.15) \times 10^{-4}$	KLOE [10]
$\text{BR}(K_L \rightarrow \pi^+ \pi^-)$	$(1.933 \pm 0.021) \times 10^{-3}$	KLOE average
$\text{BR}(K_L \rightarrow \pi^0 \pi^0)$	$(0.848 \pm 0.010) \times 10^{-3}$	KLOE average
ϕ_{+-}	$(43.4 \pm 0.7)^\circ$	PDG [9]
ϕ_{00}	$(43.7 \pm 0.8)^\circ$	PDG [9]
$R_{S,\gamma}(E_\gamma > 20\text{MeV})$	$(0.710 \pm 0.016) \times 10^{-2}$	E731 [11]
$R_{S,\gamma}^{\text{th-IB}}(E_\gamma > 20\text{MeV})$	$(0.700 \pm 0.001) \times 10^{-2}$	KLOE MC [13]
$ \eta_{+-\gamma} $	$(2.359 \pm 0.074) \times 10^{-3}$	E773 [12]
$\phi_{+-\gamma}$	$(43.8 \pm 4.0)^\circ$	E773 [12]
$\text{BR}(K_L \rightarrow \pi^+ \pi^- \pi^0)$	0.1262 ± 0.0011	KLOE average
η_{+-0}	$((-2 \pm 7) + i(-2 \pm 9)) \times 10^{-3}$	CPLEAR [7]
$\text{BR}(K_L \rightarrow 3\pi^0)$	0.1996 ± 0.0021	KLOE average
$\text{BR}(K_S \rightarrow 3\pi^0)$	$< 1.5 \times 10^{-7}$ at 95% CL	KLOE [14]
ϕ_{000}	uniform from 0 to 2π	
$\text{BR}(K_L \rightarrow \pi \ell \nu)$	0.6709 ± 0.0017	KLOE average
$A_L + A_S$	$(0.5 \pm 1.0) \times 10^{-2}$	$K_{\ell 3}$ average
$\text{Im}(x_+)$	$(0.8 \pm 0.7) \times 10^{-2}$	$K_{\ell 3}$ average

The allowed region in the $(m_{K^0} - m_{\bar{K}^0}, (\Gamma_{K^0} - \Gamma_{\bar{K}^0}))$ plane is shown in the right panel of Fig. 2. The strong correlation reflects the high precision of $\text{Im}(\delta)$ compared to $\text{Re}(\delta)$.

Since the total decay widths are dominated by long-distance dynamics, in models where CPT invariance is a pure short-distance phenomenon it is useful to consider the limit $\Gamma_{K^0} = \Gamma_{\bar{K}^0}$. In this limit (i.e. neglecting CPT-violating effects in the decay amplitudes), the following bounds on the neutral kaon mass difference are obtained

$$-5.3 \times 10^{-19} \text{ GeV} < m_{K^0} - m_{\bar{K}^0} < 6.3 \times 10^{-19} \text{ GeV} \quad \text{at 95 \% CL.} \quad (19)$$

Table 2: Results of the combined (CPLEAR+KLOE+PDG) fit of $K \rightarrow \pi l \nu$ amplitudes [8].

Amplitude	Value	Correlation coefficients				
$\text{Re}(\delta)$	$(3.4 \pm 2.8) \times 10^{-4}$	1				
$\text{Im}(\delta)$	$(-1.0 \pm 0.7) \times 10^{-2}$	-0.27	1			
$\text{Re}(x_-)$	$(-0.07 \pm 0.25) \times 10^{-2}$	-0.23	-0.58	1		
$\text{Im}(x_+)$	$(0.8 \pm 0.7) \times 10^{-2}$	-0.35	-0.12	0.57	1	
$A_S + A_L$	$(0.5 \pm 1.0) \times 10^{-2}$	-0.12	-0.62	0.99	0.54	1

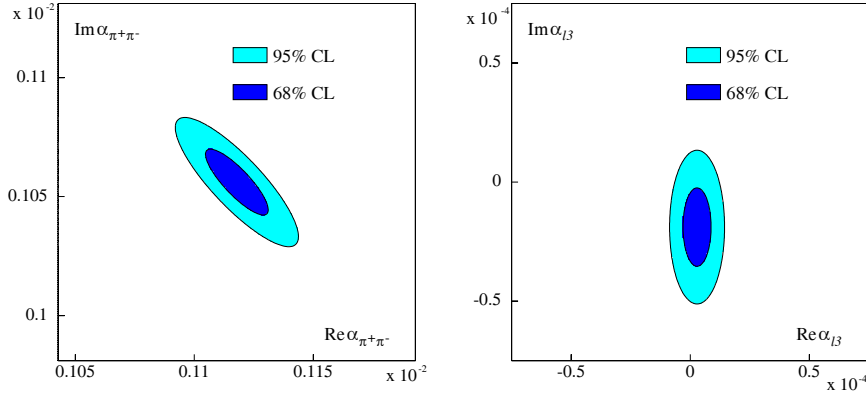


Figure 1: Determination of $\alpha_{\pi^+\pi^-}$ and $\alpha_{\pi l \nu}$ in the complex plane [8]. The two ellipses represent the 68% and the 95% CL contours.

As often emphasized in the literature, this limit provide a significant constraint on models where CPT violating effect scales linearly with the inverse of the Plank mass ($m_K^2/m_{\text{Planck}} \sim 10^{-19}$ GeV). While this fact should not be over emphasized (in several models the power behavior in m^2/m_{Planck} is not linear and the proportionality coefficient is far from unity), there is no doubt that this result is one of the most (if not the most) significant constraint on possible violations of CPT symmetry. It would therefore be very interesting trying to improve it in the future. To this purpose, the analysis of Ref. [8] demonstrates

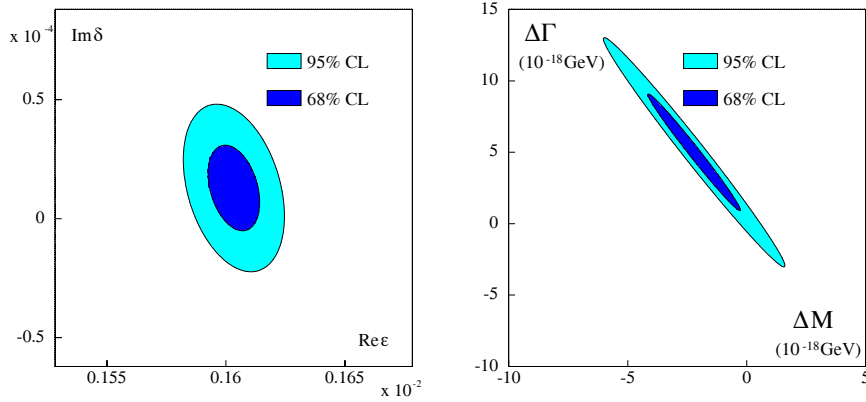


Figure 2: Left: allowed region at 68% and 95% C.L. in the $\text{Re}(\epsilon)$, $\text{Im}(\delta)$ plane. Right: allowed region at 68% and 95% C.L. in the $(m_{K^0} - m_{\bar{K}^0}) - (\Gamma_{K^0} - \Gamma_{\bar{K}^0})$ plane.

that this is possible with new high-precision interference measurements of the CP-violating phases of the $\pi\pi$ final states.

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