On the 2-body problem in quantum gravity

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Giovanni AMELINO-CAMELIA
Univ. of Rome "LA SAPIENZA"

- **some introductory remarks on the "QG problem"**

- **General reasons of interest in the description of n-particle systems in Quantum Gravity**

- **The illustrative example of noncommutative geometry**

Where is the problem?

- **Quantum Mechanics and GR are very successful in their respective domains**

- **Nearly impossible to gain experimental access to the domain when both must be taken into account, but the "QM+GR theory" predicts that this "common domain" exists (this domain includes, e.g., early stages of evolution of Universe)**

- **Quantum Mechanics limits measurability of pairs of observables. It never limits the measurability of a single observable. BUT combining the Heisenberg Uncertainty Principle with Einstein's Equivalence Principle various arguments suggest that certain observables (distance between two events?) could never be measured sharply**

How should "Quantum Gravity" look like?

- **Quantum Mechanics works well with dimensionless coupling constants (needed for perturbative renormalizability). The GR description of gravity involves a dimensionful coupling constant**

- **Quantum Mechanics works well when it can assume a given background spacetime. In the GR description of gravity a crucial ingredient is the background independence (diffeomorphism invariance)**

- **Quantum Mechanics describes most things in terms of noncommuting observables with discrete spectrum. The GR description of gravity is based on a classical commutative continuous picture of spacetime**

Some proposals (must all be viewed as mere speculations, without any support in data, but perhaps capturing some aspects of the nature of the quantum-gravity problem)

String Theory: key issue is the dimensionful coupling constant. Keep a classical continuous background spacetime, but replace fundamental point particles with fundamental extended objects in such a way that the dimensionful quantity really is the characteristic size of the objects, while the true coupling constant is dimensionless. Background independence will somehow (how?) emerge at some point in the development of the theory

Loop Quantum Gravity: key issue is background independence. Dimensionful coupling somehow (how?) will not be a problem. Spacetime discreteness emerges naturally

Noncommutative geometry: key issue is noncommutativity, even of spacetime itself. Allows straightforward introduction of an uncertainty principle for the measurability of the distance between two events, but the problems associated with background independence and the dimensionful coupling will have to somehow find a cure as we develop the theories

“QG problem” still completely open, but we have strong “theoretical evidence” that Planck scale is characteristic scale (modulo large extra dim….)

Planck-scale effects can be “striking”

- small extra dimensions
- violations of EP
- violations of Poincarè/Lorentz symmetry
- violations of CPT symmetry…

but these effects are always very small because $E_P = 10^{28}$ eV is much greater than energies accessible to us

Most of the relevant phenomenology focuses on one-particle states (e.g. laws of propagation of a particle)

General observations for the two-particle state in quantum gravity

- **2+1D QG is a topological theory!!!**
  Chern-Simons theory
  2-particle states are not described by elements of the space of
  tensor products of one-particle states
  Not surprising that such theories could exist in 2+1D: the winding of world lines is meaningful in 2+1D

- **Some approaches to 3+1D QG, notably some formulations of LoopQG, reflect rather strongly the properties of the 2+1D limit (topological theory plus nontopological terms)**

- **Recent approaches to quantum mechanics in non-Minkowski backgrounds:**
  quantum field theory in deSitter spacetime is problematic....rather than assuming deSitter symmetries it appears to help if one assumes "q-deSittersymmetries", But then something analogous to kappaMinkowski occurs

Problematic aspects of two-particle states in noncommutative geometry
The example of "kappa-Minkowski"

The idea of NCgeometry can be viewed in analogy with the noncommutative geometry of (x,p) phase space for ordinaryQM. Let us consider the specific example

$$[x_j, t] = i\lambda x_j \qquad [x_j, x_m] = 0$$

- **Some recent papers advocate a role in String Theory (nonconstant backgrounds…) for "Lie-algebra noncommutative spacetimes" of which kappa-Minkowski is an example** $[X_\mu, X_\nu] = E_{NC}^{-1} C^\alpha_{\mu\nu} X_\alpha$

- **Key role in the analysis of theories in kappa-Minkowski played by the Fourier transform which is essentially standard (see, e.g., Madore+Schraml+Schupp+Wess, EPJC16,161)**

$$f(x) = \int d^4k \left(\varphi(k) e^{ikx} e^{ik_0 t} \right)$$

Any f(x), function of kappa-Minkowski coordinates, can be written as the Fourier transform of a (commutative) φ(k)

Notice the alternative $$f(x) = \int d^4k \left(\varphi(k) e^{ikx + ik_0 t} \right)$$ **(an ambiguity?)**

- **Translation generators in kappa-Minkowski:**

$$P_\mu \left(e^{ikx} e^{ik_0 t} \right) = k_\mu \left(e^{ikx} e^{ik_0 t} \right)$$ < **classical action**

Note that

$$\left(e^{ikx} e^{ik_0 t} \right) \left(e^{iKx} e^{iK_0 t} \right) = \left(e^{i(k + e^{\lambda k_0} K)x} e^{i(k_0 + K_0)t} \right)$$

then $P_\mu \left(e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t} \right) = P_\mu \left(e^{i(k + e^{\lambda k_0} K)x} e^{i(k_0 + K_0)t} \right)$

$$= \left(k_\mu + e^{-\lambda k_0} K_\mu \right) \left(e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t} \right)$$

$$= \left[P_\mu \left(e^{ikx} e^{ik_0 t} \right) \right] \left(e^{iKx} e^{iK_0 t} \right) + \left[e^{-\lambda P_0} \left(e^{ikx} e^{ik_0 t} \right) \right] P_\mu \left(e^{iKx} e^{iK_0 t} \right)$$

Nontrivial coproduct!!

Translations are not really classical in kappa-Minkowski

IS THE ENERGY OF THE TWO-PARTICLE STATE NOT SYMMETRIC UNDER PARTICLE EXCHANGE??

- **Rotation generators in kappa-Minkowski:**

$$R_j\left(e^{ikx}e^{iEt}\right)=i\varepsilon_{jlm}\left(x_l P_m - x_m P_l\right)\left(e^{ikx}e^{iEt}\right)$$ **<📫classical actio**

then📫☢ $$R_j\left(e^{ikx}e^{iEt}e^{i\mathrm{K}x}e^{i\Omega t}\right)=R_j\left(e^{i(k+e^{\lambda E}\mathrm{K})x}e^{i(E+\Omega)t}\right)$$

$$=\left[R_j\left(e^{ikx}e^{iEt}\right)\right]\left(e^{i\mathrm{K}x}e^{i\Omega t}\right)+\left(e^{ikx}e^{iEt}\right)R_j\left(e^{i\mathrm{K}x}e^{i\Omega t}\right)$$

Trivial coproduct!!
Rotations are really classical in k-M

- **Boosts in kappa-Minkowski:**

$$N_j\left(e^{ikx}e^{iEt}\right)=\left[x_j\frac{(1-e^{2\lambda P_0})}{2\lambda}-x_0P_j-\frac{\lambda}{2}x_jP_lP_l+\lambda x_lP_lP_j\right]\left(e^{ikx}e^{iEt}\right)$$

Modified action needed for consistency with Hopf algebra structure….
IF one adopted unmodified (classical) action then the would-be coproduct requires operators external to the algebra…
Modification of boosts was expected since commutators involve a length scale…
With this modified action the coproduct is OK (can be expressed in terms of P,R,N)
Note that:

$$\left[N_j,P_l\right]=-i\lambda P_lP_j+i\delta_{jl}\left(\frac{\lambda}{2}P_mP_m+\frac{(1-e^{2\lambda P_0})}{2\lambda}\right)$$

and the "mass Casimir" for these deformed transformations is

$$\cosh(\lambda m)=\cosh(\lambda E)-\frac{\lambda^2}{2}e^{\lambda E}P^2$$

For one-particle states most of the familiar equations can be generalized to the kappaMinkowski case

Klein-Gordon equation:

③ • $$\Phi(x) \equiv \frac{\left[1 - \cosh\left(\lambda \mathrm{P}_0\right)\right] - \mathrm{e}^{\lambda \mathrm{P}_0} \vec{\mathrm{P}}^{\,2}}{\lambda^2} \Phi(x) = \frac{\left[1 - \cosh\left(\lambda \mathrm{m}\right)\right]}{\lambda^2} \Phi(x)$$

Dirac equation:

$$\left[iD_\mu(P)\gamma^\mu + \frac{\sinh(\lambda m)}{2\lambda} \mathrm{I} \right] \Psi(x) = 0$$

usual γ !!!

$$where \quad D_0(P) = \left[e^{\lambda P_0} - \cosh(\lambda m) \right] \frac{i}{\lambda} \text{ and } D_j(P) = iP_j e^{\lambda P_0}$$

BUT the description of multi-particle states is still not well understood

The possible "ordering ambiguity", mentioned earlier, must still be fully analyzed, and one could think it has a role

But in Roma1 we are presently following a path based on the properties of the "differential calculus"

Translations and differential calculus:

$$x_\mu \to x'_\mu = x_\mu + dx_\mu$$

$$f(x) \to f'(x) = f(x) + idx_\mu P_\mu f(x) \equiv f(x) + df(x)$$

$$[dx_0, x_\mu] = 0; [dx_j, x_l] = 0; [dx_j, x_0] = i\lambda dx_j$$

$$N.B.: d(fg) = (fg)' - fg = (df)g + f(dg)$$

When this is done we should have testable predictions for two-particle states