Open quantum dynamics: complete positivity and correlated neutral kaons

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Outline

- Treating neutral kaons as an isolated quantum system is just an approximation
- When the interaction with the external environment is weak, they evolve in time with a Markovian (memoryless), completely positive dynamics
- Noise and decoherence are the typical results of such quantum dynamics
- These effects can be experimentally probed

Quantum Mechanics

In standard university courses, the state of a quantum system is said to be described by a wave function $|\Psi\rangle$

For any observable $\mathcal{O} = \mathcal{O}^\dagger$, the theory gives its mean value

$$\langle \mathcal{O} \rangle \equiv \langle \Psi | \mathcal{O} | \Psi \rangle$$

Dynamics is dictated by the Schroedinger equation:

$$\boxed{\partial_t |\Psi_t\rangle = -i\, H\, |\Psi_t\rangle}$$

through the unitary evolution

$$|\Psi_t\rangle = U_t\; |\Psi_0\rangle\;, \qquad U_t = e^{-i\,H\,t}$$

Alternatively, pure states can be described by projectors

$$P = |\Psi\rangle\langle\Psi| \ , \qquad P^2 = P$$

Mean values of observables can be obtained through a trace operation

$$\boxed{\langle\mathcal{O}\rangle = \mathrm{Tr}\big[\mathcal{O}\, P\big] \equiv \langle\Psi|\mathcal{O}|\Psi\rangle}$$

while the unitay evolution in time becomes

$$P_t = |\Psi_t\rangle\langle\Psi_t| = U_t \ P_0 \ U_{-t}$$

Wave functions and projectors are suitable when the knowledge on the state of a quantum system is complete

When this is lacking, one has to deal with statistical mixtures

Quantum states

More in general, states of a quantum system are described by density matrices

$$\rho = \sum_i \lambda_i \, P_i \ , \qquad P_i = |\psi_i\rangle\langle\psi_i| \ , \quad \lambda_i \geq 0 \ , \quad \mathrm{Tr}[\rho] = \sum_i \lambda_i = 1$$

i.e. by positive, normalized operators

Observable mean values are given by

$$\boxed{\langle \mathcal{O} \rangle_\rho \equiv \mathrm{Tr}\left[\rho \, \mathcal{O}\right] = \sum_i \lambda_i \, \langle \psi_i | \mathcal{O} | \psi_i \rangle}$$

In particular, the probability of finding the system in a given state

$$\mathrm{Tr}\left[\rho \, |\phi\rangle\langle\phi|\right] \equiv \langle\phi|\rho|\phi\rangle = \sum_i \lambda_i |\langle\phi|\psi_i\rangle|^2 \geq 0$$

This constitutes the bulk of the statistical interpretation of quantum mechanics!

Unitary Quantum Evolution

The time evolution of density matrices is generated by the Liouville - von Neumann equation

$$\boxed{\frac{\partial}{\partial t}\,\rho(t) = -i\big[H,\,\rho(t)\big]}$$

whose formal solution is given by

$$\rho(0) \mapsto \rho(t) = U_t\;\rho(0)\;U_{-t} = e^{-iHt}\,\rho(0)\,e^{iHt}$$

States of single kaons are described by 2x2 density matrices, while for correlated ones, one needs 4x4 density matrices

Open Quantum Systems

An open system can be in general represented as a subsystem S immersed in an external environment E

The total system $S+E$ evolves unitarily with the total Hamiltonian

$$H_{\rm tot} = H_S + H_E + gH_{\rm int}$$

The dynamics of the subsystem alone can then be obtained by eliminating the environment degrees of freedom

$$\boxed{\rho(0) \mapsto \rho(t) = {\rm Tr}_E\left[e^{-iH_{\rm tot}\,t}\left(\rho(0)\otimes\rho_E\right)e^{iH_{\rm tot}\,t}\right]}$$

The resulting evolution equation for $\rho(t)$ is very complicated with memory effects

When the interaction between subsystem and environment is weak, all memory effects disappear, and the evolution for $\rho(t)$ takes the general form

$$\boxed{\frac{\partial}{\partial t}\,\rho(t) = -i\big[H,\ \rho(t)\big] + D[\rho(t)] \equiv L[\rho(t)]}$$

The additional term is a linear map, such that:

- it preserves probability: $\mathrm{Tr}\big(D[\rho]\big) = 0$
- it generates irreversibility
- it induces noise and decoherence

Physical consistency imposes further conditions on $D[\rho]$, since the positivity of $\rho(t)$ must be preserved for all times

Single Kaon Dynamics

Single kaon states are described by

$$\rho = \begin{pmatrix} r_1 & r_3 \\ r_3^* & r_2 \end{pmatrix} \;, \qquad r_1 + r_2 = 1 \;, \qquad \rho \geq 0 \quad \Rightarrow \quad \mathrm{Det}[\rho] \geq 0$$

It results convenient to decompose this matrix along the Pauli matrices and the unit matrix

$$\boxed{\rho = \frac{1}{2}\left(\mathbf{1} + \vec{\rho} \cdot \vec{\sigma}\right)}\,, \qquad \rho_i^* = \rho_i \;, \qquad 4\,\mathrm{Det}[\rho] = 1 - |\vec{\rho}|^2 \geq 0$$

so that pure states are on the surface of the Bloch sphere: $|\vec{\rho}| = 1$

Its (entropy increase) time evolution $\rho(0) \mapsto \rho(t) = \Gamma_t[\rho(0)]$
can then be written as

$$\boxed{\frac{\partial}{\partial t}\vec{\rho}(t) = \left[\mathcal{H} + \mathcal{D}\right]\vec{\rho}(t)}$$

The matrix $\mathcal{H}$ represents the Hamiltonian piece

$$H = \vec{\omega}\cdot\vec{\sigma}\ , \qquad \mathcal{H}_{ij} = -2\,\epsilon_{ijk}\,\omega_k$$

while the dissipative contribution can be described in general by the real, symmetric matrix

$$\boxed{\mathcal{D} = -2\begin{pmatrix} a & b & c \\ b & \alpha & \beta \\ c & \beta & \gamma \end{pmatrix}}$$

Positivity Condition

Assume the initial state $\vec{\rho} \equiv \vec{\rho}(0)$ to be pure: $|\vec{\rho}\,| = 1$, $\quad \mathrm{Det}[\rho(0)] = 0$

The positivity of the density matrix then requires

$$\frac{d}{dt}\mathrm{Det}[\rho(t)]\Big|_{t=0} = -2\sum_{i,j=1}^{3} \rho_i\,\mathcal{D}_{ij}\,\rho_j \geq 0$$

In other terms, the dissipative dynamics $\varGamma_t$ is positive if and only if

$$\begin{pmatrix} a & b & c \\ b & \alpha & \beta \\ c & \beta & \gamma \end{pmatrix} \geq 0$$

The structure of Quantum Mechanics (i.e. the presence of entangled states) requires more!

Correlated Kaons

The state of the two neutral kaons that come from the decay of a spin-one Φ meson is entangled

$$\boxed{|\psi_-\rangle = \frac{1}{\sqrt{2}}\Big(|K^0,-p\rangle \otimes |\overline{K^0},p\rangle - |\overline{K^0},-p\rangle \otimes |K^0,p\rangle\Big)}$$

Being a pure state, the corresponding density matrix is a projection

$$\rho_{\psi_-} = |\psi_-\rangle\langle\psi_-|$$

and since the two kaons are independent, it evolves with a product dynamics

$$\rho_{\psi_-}(0) \mapsto \rho_{\psi_-}(t) = \Big(\Gamma_t \otimes \Gamma_t\Big)[\rho_{\psi_-}(0)]$$

Consider then the following average:

$$\boxed{\Delta(t) = \langle\psi_+|\rho_{\psi_-}(t)|\psi_+\rangle} \qquad \Delta(0) = 0$$

The time evolution must satisfy the condition $\Delta(t) \geq 0$, and in particular

$$\frac{d}{dt}\Delta(0) \equiv a + \alpha - \gamma \geq 0$$

The complete set of conditions read:

$$\boxed{\begin{array}{ll} 2R \equiv \alpha + \gamma - a \geq 0 & RS \geq b^2 \\ 2S \equiv a + \gamma - \alpha \geq 0 & RT \geq c^2 \\ 2T \equiv a + \alpha - \gamma \geq 0 & ST \geq \beta^2 \\ \multicolumn{2}{l}{RST \geq 2\,bc\beta + R\beta^2 + Sc^2 + Tb^2} \end{array}}$$

The presence of entangled states requires any quantum dynamics to be completely positive!

Complete Positivity

Indeed, for any quantum evolution in finite dimensions, one has

Theorem: $\mathit{\Gamma}_t \otimes \mathit{\Gamma}_t$ is positive $\Leftrightarrow$ $\mathit{\Gamma}_t$ is completely positive

The property of complete positivity fixes the form of the time evolution:

$$\mathit{\Gamma}_t \; : \; \rho(0) \mapsto \rho(t) = \sum_k V_k(t) \; \rho(0) \; V_k^\dagger(t)$$

with $V_k(t)$ bounded operators; it generalize the standard unitary evolution, which is completely positive:

$$k = 1 \; , \qquad\qquad V_1(t) \equiv U_t = e^{-itH}$$

Dissipative Single Kaon Dynamics

The most general, physically consistent time evolution $\mathit{\Gamma}_t$ incorporating dissipative and decoherence is generated by an equation in Kossakowski - Lindblad form:

$$\boxed{\begin{aligned}\frac{\partial}{\partial t}\rho(t) &= -iH_{\text{eff}}\;\rho(t) + i\rho(t)\;H_{\text{eff}}^{\dagger} \\ &\qquad + \frac{1}{2}\sum_{ij=1}^{3} C_{ij}\left[2\,\sigma_j\,\rho(t)\,\sigma_i - \left\{\sigma_i\sigma_j\,,\rho(t)\right\}\right]\end{aligned}}$$

with $C_{ij} = C_{ij}^{\dagger}$ and

$$\boxed{C_{ij} \geq 0}$$

Physical Motivations

- Quantum Gravity: quantum fluctuations at Planck scale can lead to loss of quantum coherence

- Dynamics of Extended Objects: they can generate dissipation at low energies

- Extra Dimensions: possible energy leakage into the bulk due to gravity effects will inject noise in our brane world

- Kaons in Medium: matter fluctuations lead to noise and decoherence

Order of Magnitude

The dissipative parameters are expected to be small:

$$\boxed{C_{ij} \simeq \frac{m_k^2}{M_F}}$$

For gravitationally induced quantum dissipative effects:

$$M_F \simeq M_{\mathrm{Planck}}$$

so that at most

$$\boxed{C_{ij} \simeq 10^{-19}\ \mathrm{GeV}}$$

Meson Factories

The six, real dissipative parameters in C_{ij} modify in a distinctive way the form of meson observables (double decay rates):

$$\boxed{\mathcal{P}(f_1,\tau_1;f_2,\tau_2) \equiv \mathrm{Tr}\Big[\Big(\mathcal{O}_{f_1}\otimes\mathcal{O}_{f_2}\Big)\rho_{\psi_-}(\tau_1,\tau_2)\Big]}$$

For instance:

$$\mathcal{P}(\pi^+\pi^-\pi^0,\tau\,;\pi^+\pi^-\pi^0,\tau) \sim \frac{\gamma}{\Delta\Gamma}\; e^{-2\gamma_L\tau}$$

$$\frac{\mathcal{P}(\ell^\pm,\tau;\ell^\pm,\tau)}{\mathcal{P}(\ell^\pm,\tau;\ell^\mp,\tau)} \sim 2\; a\; \tau$$

and of single-time distributions:

$$\boxed{\mathit{\Pi}(f_1, f_2; \tau) = \int_0^\infty dt\, \mathcal{P}(f_1, t+\tau; f_2, t)\ , \qquad \tau > 0}$$

For instance:

$$\mathcal{A}_{\varepsilon'}(\tau) = \frac{\mathit{\Pi}(\pi^+\pi^-, 2\pi^0; \tau) - \mathit{\Pi}(2\pi^0, \pi^+\pi^-; \tau)}{\mathit{\Pi}(\pi^+\pi^-, 2\pi^0; \tau) + \mathit{\Pi}(2\pi^0, \pi^+\pi^-; \tau)}$$

so that:

$$\boxed{\mathcal{A}_{\varepsilon'}(\tau) \sim 3\,\mathcal{R}e\left(\frac{\varepsilon'}{\varepsilon}\right)\, \frac{|\varepsilon|^2 + 2\,\mathcal{R}e\big(\varepsilon\, C/\Delta\Gamma_+\big)}{|\varepsilon|^2 + D} - 6\,\mathcal{I}m\left(\frac{\varepsilon'}{\varepsilon}\right)\, \frac{\mathcal{I}m\big(\varepsilon\, C/\Delta\Gamma_+\big)}{|\varepsilon|^2 + D}}$$

Summary

- Open quantum systems are subsystem in weak interaction with large environments

- The reduced dynamics of the subsystem is described by a quantum dynamical semigroup, i.e. a one-parameter family of completely positive maps

- This paradigm is very general and can be applied to describe the time evolution of neutral mesons, typically leading to dissipation and loss of quantum coherence

- Meson factories are the suitable interferometric set-ups needed to experimentally probe these non-standard effects