CPT Violation and Decoherence in Quantum Gravity

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QUESTIONS

- **Are there theories which allow CPT breaking?**
- **How (un)likely is it that somebody finds CPT violation, and why?**
- **What formalism? How can we be sure of observing CPT Violation ? our current phenomenology is based on CPT invariance...**
- **No single "figure of merit" for CPT tests: Complex Phenomenology**
- **How should we compare various "figures of merit" of CPT tests**:
  Direct mass measurement, K^0-$\overline{K}^0$ mass difference a la CPLEAR, electron g-2, **antimatter factories spectroscopy**, cyclotron frequency comparison, **decoherence effects**, **EPR-modifications**, **...**

OUTLINE

- **WHAT IS CPT SYMMETRY.**
- **WHY CPT VIOLATION ?** Theoretical models and ideas, and generic order of magnitude estimates of expected effects: **Quantum Gravity Models violating Lorentz symmetry and/or quantum coherence**:

  (i) space-time foam,

  (ii) Standard Model Extension

  (iii) Loop Quantum Gravity/background independent formalism. Non-linear deformations of Lorentz symmetry (DSR) (?)
- **HOW CAN WE DETECT CPT VIOLATION?**

  (i) neutral mesons: KAONS, B-MESONS, entangled states in ϕ and B factories

  (ii) antihydrogen (precision spectroscopic tests on free and trapped molecules)

  (iii) Low energy atomic physics experiments.

  (iv) Ultra cold neutrons

  (v) Neutrino Physics

  (vi) Terrestrial & Extraterrestrial tests of Lorentz Invariance (modified dispersion relations of matter probes: GRB, AGN photons, Crab Nebula synchrotron-radiation constraint on electrons ...)

SOME THEORY

CPT THEOREM

C(harge) -P(arity=reflection) -T(ime reversal) INVARIANCE is a property of any quantum field theory in Flat space times which respects: (i) Locality, (ii) Unitarity and (iii) Lorentz Symmetry.

$$\Theta \mathcal{L}(x) \Theta^\dagger = \mathcal{L}(-x) \ ,$$
$$\Theta = CPT \ , \ \mathcal{L} = \mathcal{L}^\dagger \ \text{(Lagrangian)}$$

Theorem due to: Jost, Pauli (and John Bell).

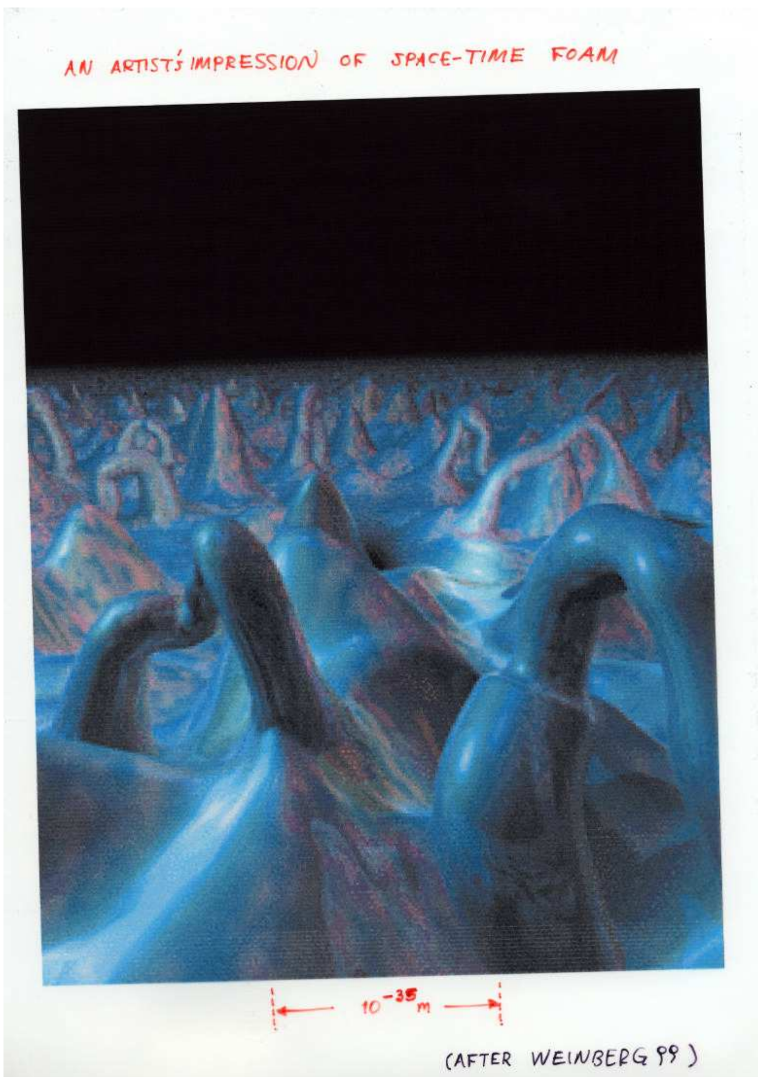
Jost proof uses covariance trnsf. properties of Wightman's functions (i.e. quantum-field-theoretic (off-shell) correlators of fields $< 0|\phi(x_1)\ldots\phi(x_n)|0 >$) under Lorentz group. (O. Greenberg, hep-ph/0309309)

Theories with HIGHLY CURVED SPACE TIMES , with space time boundaries of black-hole horizon type, may violate (ii) & (iii) and hence CPT.

E.g.: SPACE-TIME FOAMY SITUATIONS IN SOME QUANTUM GRAVITY MODELS.

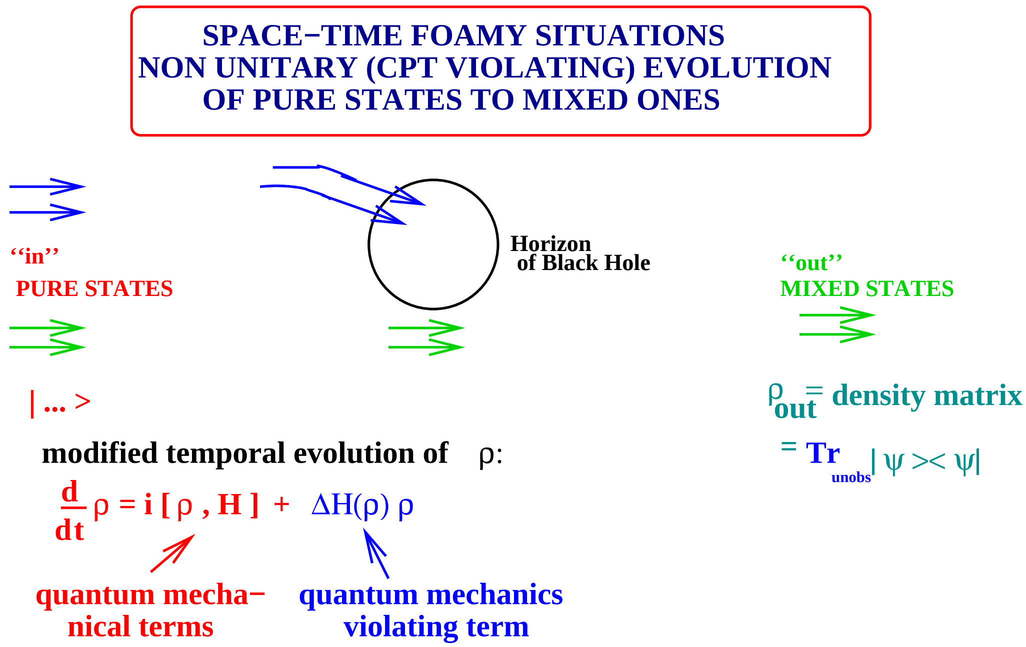
SPACE-TIME FOAM

Space-time MAY BE DISCRETE at scales $10^{-35}\ m$ (Planck) $\rightarrow$ LORENTZ VIOLATION (LV)? (and hence CPTV); also there may be ENVIRONMENT of GRAVITATIONAL d.o.f. INACCESSIBLE to low-energy experiments (non-propagating d.o.f., no scattering) $\rightarrow$ CPT VIOLATION (and may be LV)



FOAM AND UNITARITY VIOLATION

SPACE-TIME FOAM: Quantum Gravity SINGULAR Fluctuations (microscopic (Planck size) black holes etc) **MAY** imply: pure states $\longrightarrow$ mixed



$$\rho_{out} = \mathrm{Tr}_{\mathrm{unobs}}|out >< out| = \$\ \rho_{in},$$

$\$ \neq SS^\dagger$, $S = e^{iHt}$ =scattering matrix,

$\$$=**non invertible**, unitarity lost in effective theory.

BUT...HOLOGRAPHY can change the picture

(Strings in anti-de-Sitter space times (Maldacena, Witten), Hawking 2003- superposition of space-time topologies (Quantum Gravity) (but in Euclidean space time) may solve info-problem?: not quite sure (in QG) if the BH is there)

BUT NO PROOF AS YET ... OPEN ISSUE

SPACE-TIME FOAM and Intrinsic CPT Violation

A THEOREM BY R. WALD (1979): **If \$ $\ne S\ S^\dagger$, then CPT is violated, at least in its strong form.**

PROOF:

Suppose CPT is conserved, then there exists unitary, invertible opearator Θ : $\Theta\overline{\rho}_{in} = \rho_{out}$

$$\rho_{out} = \$\ \rho_{in} \to \Theta\overline{\rho}_{in} = \$\ \Theta^{-1}\overline{\rho}_{out} \to \overline{\rho}_{in} = \Theta^{-1}\$\ \Theta^{-1}\overline{\rho}_{out}.$$

But $\overline{\rho}_{out} = \$\overline{\rho}_{in}$, hence :

$$\overline{\rho}_{in} = \Theta^{-1}\$\Theta^{-1}\ \$\ \overline{\rho}_{in}$$

BUT THIS IMPLIES THAT \$ HAS AN INVERSE- $\Theta^{-1}\$\Theta^{-1}$, IMPOSSIBLE (information loss), hence CPT MUST BE VIOLATED (at least in its strong form).

NB: IT ALSO IMPLIES: $\Theta = \$\ \Theta^{-1}\ \$$ (fundamental relation for a full CPT invariance).

NB: My preferred way of CPTV by Quantum Gravity
Introduces fundamental arrow of time/microscopic time irreversibility

CPT SYMMETRY WITHOUT CPT SYMMETRY?

But....nature may be tricky: WEAK FORM OF CPT INVARIANCE might exist, such that the fundamental "arrow of time" does not show up in any experimental measurements (scattering experiments).

Probabilities for transition from ψ =initial pure state to ϕ =final state

$$P(\psi \to \phi) = P(\theta^{-1}\phi \to \theta\psi)$$

where θ: $\mathcal{H}_{\rm in} \to \mathcal{H}_{\rm out}$, $\mathcal{H}$= Hilbert state space,
$\Theta\rho = \theta\rho\theta^\dagger, \quad \theta^\dagger = -\theta^{-1}$ $(\rm anti - unitary)$.

In terms of superscattering matrix \$:

$$\$^\dagger = \Theta^{-1}\$\Theta^{-1}$$

Here, Θ is well defined on pure states, but \$ has no inverse, hence \$ $^\dagger \ne \$^{-1}$ (full CPT invariance: \$$= SS^\dagger$, $\$^\dagger = \$^{-1}$).

Supporting evidence for Weak CPT from Black-hole thermodynamics: *Although white holes do not exist (strong CPT violation), nevertheless the CPT reverse of the most probable way of forming a black hole is the most probable way a black hole will evaporate:* **the states resulting from black hole evaporation are precisely the CPT reverse of the initial states which collapse to form a black hole.**

COSMOLOGICAL CPTV?

(NM, hep-ph/0309221)

Recent Astrophysical Evidence for Dark Energy (acceleration of the Universe (SnIA), CMB anisotropies (WMAP...))

Best fit models of the Universe consistent with non-zero **cosmological constant** $\Lambda \neq 0$ (de Sitter)

Λ-universe will eternally accelerate, as it will enter in an inflationary phase again: $a(t) \sim e^{\sqrt{\Lambda/3}t}$, $t \to \infty$, there is **cosmological Horizon**.

Horizon implies incompatibility with S-matrix & decoherence: no proper definition of asymptotic state vectors, environment of d.o.f. crossing the horizon (c.f. dual picture of black hole, now observer is inside the horizon).

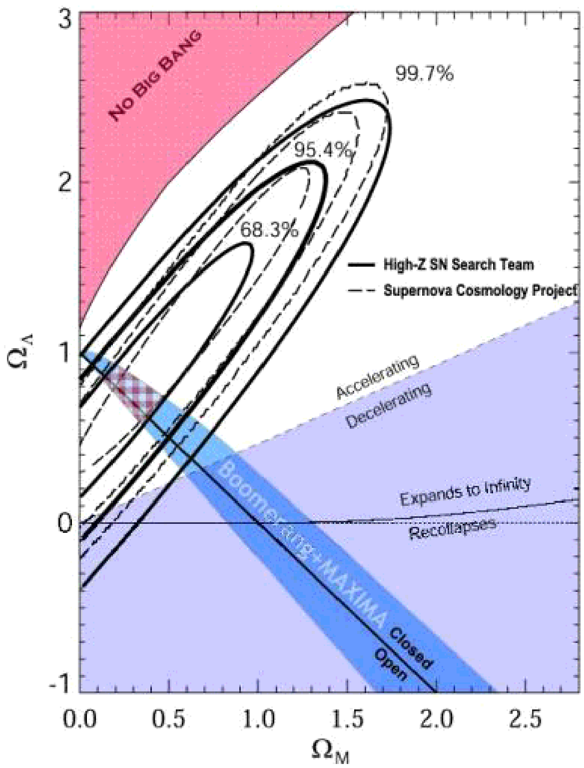
Theorem by Wald on \$-matrix and CPTV: CPT is violated due to $\Lambda > 0$ induced decoherence:

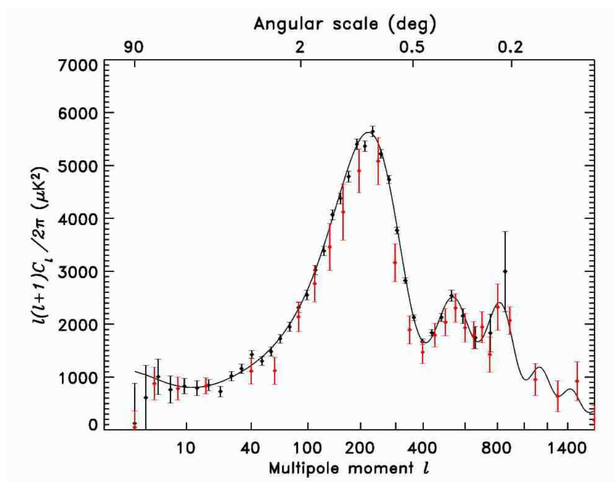
$$\partial_t \rho = i[\rho, H] + \frac{\Lambda}{M_P^3}[g_{\mu\nu}, [g^{\mu\nu}, \rho]]$$

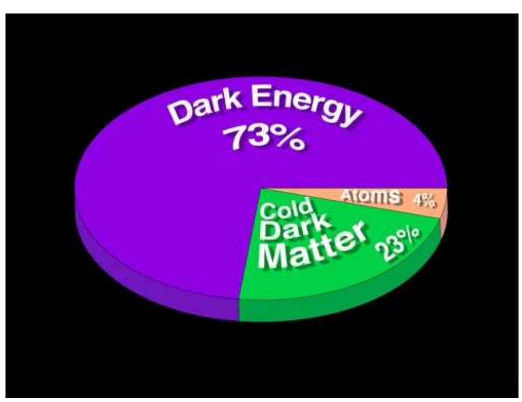
Tiny cosmological CPTV effects, but detected through Universe acceleration!

Evidence for Dark Energy

WMAP improved results on CMB: $\Omega_{\rm total} = 1.02 \pm 0.02$, high precision measurement of secondary (two more) acoustic peaks (c.f. new determination of Ω_b). Agreement with SnIa Data. Best Fit : $\Omega_\Lambda = 0.73$, $\Omega_{\rm Matter} = 0.27$







ORDER OF MAGNITUDE of CPTV

Tiny cosmological (global) CPTV effects may be much smaller than QG (local) space-time effects (foam etc).

Naively, Quantum Gravity (QG) has a dimensionful constant: $G_N \sim 1/M_P^2$, $M_P = 10^{19}$ GeV. Hence, CPT violating and decoherening effects may be expected to be suppressed by E^3/M_P^2 , where E is a typical energy scale of the low-energy probe. This would be hard to detect in neutral mesons, but neutrinos might be sensitive ! (e.g. modified dispersion relations (m.d.r.) for ultrahigh energy ν from GRB's (Ellis, NM, Nanopoulos, Volkov)) Also in some astrophysical cases, e.g. Crab Nebula or Vela pulsar synchrotron radiation constraints electron m.d.r. of this order (Jacobson, Liberati, Mattingly, Ellis, NM, Sakharov)

HOWEVER: RESUMMATION & OTHER EFFECTS in theoretical models may result in much larger effects of order: $\frac{E^2}{M_P}$.
(This happens, e.g., loop gravity, some stringy models of QG involving open string excitations ...)

SUCH LARGE EFFECTS ARE definitely ACCESSIBLE/FALSIFIABLE BY CURRENT AND IMMEDIATE FUTURE EXPERIMENTS.

FOAM DECOHERENCE: FORMALISM

Major approaches:

(i) Lindblad (linear) model-independent formalism (not specific to foam):

Requirements: (i) Energy conservation on average, (ii)(complete) positivity of ρ, (iii) monotonic entropy increase

Generic Decohering Lindblad Evolution:

$$\frac{\partial \rho_\mu}{\partial t} = \sum_{ij} h_i \rho_j f_{ij\mu} + \sum_\nu L_{\mu\nu}\rho_\mu \ ,$$
$$\mu, \nu = 0, \dots N^2 - 1, \quad i, j = 1, \dots N^2 - 1 \qquad (1)$$

for N-level systems, where h_i Hamiltonian terms.

Example for three generation neutrino oscillations: $N = 3$, f_{ijk} structure constants of SU(3).

Entropy increase requirement:

$$L_{0\mu} = L_{\mu 0} = 0 \ ,$$

$$L_{ij} = \frac{1}{4}\sum_{k,\ell,m} c_{l\ell}\left(-f_{i\ell m} f_{kmj} + f_{kim} f_{\ell mj}\right) \ ,$$

with c_{ij} a positive definite matrix (non-negative eigenvalues).

3-generation Lindblad Oscillation Probability

(Barenboim, NM, Sarben Sarkar, Waldron 2006)

$$P_{\nu_\alpha \to \nu_\beta}(t) = \mathrm{Tr}(\rho_{\nu_\beta}(t)\rho_{\nu_\alpha}) = \frac{1}{3} +$$

$$\frac{1}{2}\left\{\left[\rho_1^\alpha \rho_1^\beta \cos\left(\frac{|\Omega_{12}|t}{2}\right) + \left(\frac{\Delta L_{21}\rho_1^\alpha \rho_1^\beta}{|\Omega_{12}|}\right)\sin\left(\frac{|\Omega_{12}|t}{2}\right)\right] e^{(L_{11}+L_{22})\frac{t}{2}}\right.$$

$$+ \quad \left[\rho_4^\alpha \rho_4^\beta \cos\left(\frac{|\Omega_{13}|t}{2}\right) + \left(\frac{\Delta L_{54}\rho_4^\alpha \rho_4^\beta}{|\Omega_{13}|}\right)\sin\left(\frac{|\Omega_{13}|t}{2}\right)\right] e^{(L_{44}+L_{55})\frac{t}{2}}$$

$$+ \quad \left[\rho_6^\alpha \rho_6^\beta \cos\left(\frac{|\Omega_{23}|t}{2}\right) + \left(\frac{\Delta L_{76}\rho_6^\alpha \rho_6^\beta}{|\Omega_{23}|}\right)\sin\left(\frac{|\Omega_{23}|t}{2}\right)\right] e^{(L_{66}+L_{77})\frac{t}{2}}$$

$$+ \quad \left[\left(\rho_3^\alpha \rho_3^\beta + \rho_8^\alpha \rho_8^\beta\right)\cosh\left(\frac{\Omega_{38}t}{2}\right)\right.$$

$$+ \quad \left.\left(\frac{2L_{38}(\rho_3^\alpha \rho_8^\beta - \rho_8^\alpha \rho_3^\beta) + \Delta L_{83}\left(\rho_3^\alpha \rho_3^\beta - \rho_8^\alpha \rho_8^\beta\right)}{\Omega_{38}}\right)\sinh\left(\frac{\Omega_{38}t}{2}\right)\right]$$

$$\left. e^{(L_{33}+L_{88})\frac{t}{2}}\right\}$$

$\Delta L_{ij} \equiv L_{ii} - L_{jj}$, $\Omega_{12} = \sqrt{(L_{22}-L_{11})^2 - 4\left(\frac{\Delta m_{12}^2}{2p}\right)^2}$, $\Omega_{13} = \sqrt{(L_{44}-L_{55})^2 - 4\left(\frac{\Delta m_{13}^2}{2p}\right)^2}$, $\Omega_{23} = \sqrt{(L_{66}-L_{77})^2 - 4\left(\frac{\Delta m_{23}^2}{2p}\right)^2}$, $\Omega_{38} = \sqrt{(L_{33}-L_{88})^2 + 4L_{38}^2}$,

$$\rho_0^\alpha = \sqrt{\frac{2}{3}},\ \rho_1^\alpha 2Re(U_{\alpha 1}^* U_{\alpha 2}),\ \rho_2^\alpha - 2Im(U_{\alpha 1}^* U_{\alpha 2}),\ \rho_3^\alpha |U_{\alpha 1}|^2 - |U_{\alpha 2}|^2,$$

$$\rho_4^\alpha 2Re(U_{\alpha 1}^* U_{\alpha 3}),\ \rho_5^\alpha - 2Im(U_{\alpha 1}^* U_{\alpha 3}),\ \rho_6^\alpha 2Re(U_{\alpha 2}^* U_{\alpha 3}),$$

$$\rho_7^\alpha - 2Im(U_{\alpha 2}^* U_{\alpha 3}),\ \rho_8^\alpha \sqrt{\frac{1}{3}}\left(|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 - 2|U_{\alpha 3}|^2\right)$$

NB: Note the Lindblad $e^{-(\dots)t}$ suppression

FOAM DECOHERENCE: FORMALISM

BEYOND LINDBLAD

(ii) Non-critical Strings (possibly non-linear, specific to QG foam) (**Ellis, NM, Nanopoulos 1992)**:

$$\partial_t \rho = i[\rho, H] + :\beta^i < V_i V_j > [g^j, \rho] :,$$

where $< ... >$ hides non linearities, $g^i = g_{\mu\nu}, ...$ string backgrounds, $\beta^i = \sum_n C^i_{i_1 \dots i_n} g^{i_1} \dots g^{i_n}$, describes deviation from conformal invariance on the world sheet (foam effect). **Can include Lindblad as a special case**

(iii) Fokker-Planck equation for probability density P distributions with diffusion $\mathcal{D}$,

$$\partial_t P = \mathcal{D} \nabla^2 P + \nabla \cdot \mathcal{J}$$

diffeomorphism invariant, leading to non-linear Schrödinger equation (Doebner-Goldin) for matter wavefunction ψ in gravitational environment (no use of density matrices):

$$i\hbar \partial_t \Psi = -\frac{\hbar^2}{2m} \nabla^2 \Psi + i\mathcal{D}\hbar \left(\nabla^2 \Psi + \frac{|\nabla \Psi| 2}{|\Psi|^2} \Psi \right)$$

if foam-induced diffusion: $\mathcal{D} = O\left((E/M_P)^n\right)$.

BUT **supersymmetry** implies **linearity** in string-inspired models **(NM & Szabo 2001, NM 2004)**.

FOAM DECOHERENCE: FORMALISM

BEYOND LINDBLAD II

(iv) Stochastically fluctuating space times with metrics fluctuating along direction of motion (for simplicity) **(Sarben Sarkar, NM 2006)**

$$g^{\mu\nu} = \begin{pmatrix} -(a_1+1)^2 + a_2^2 & -a_3(a_1+1) + a_2(a_4+1) \\ -a_3(a_1+1) + a_2(a_4+1) & -a_3^2 + (a_4+1)^2 \end{pmatrix}.$$

with random variables $\langle a_i \rangle = 0$ and $\langle a_i a_j \rangle = \delta_{ij}\sigma_i$.

EXAMPLE: Two generation Dirac neutrinos with MSW interaction V (of unspecified origin, could be space-time foam effect) oscillation probability:

$$\langle e^{i(\omega_1-\omega_2)t}\rangle = e^{i\frac{\left(z_0^+ - z_0^-\right)t}{k}} e^{-\frac{1}{2}\left(-i\sigma_1 t\left(\frac{(m_1^2-m_2^2)}{k} + V\cos 2\theta\right)\right)} \times$$
$$e^{-\frac{1}{2}\left(\frac{i\sigma_2 t}{2}\left(\frac{(m_1^2-m_2^2)}{k} + V\cos 2\theta\right) - \frac{i\sigma_3 t}{2} V\cos 2\theta\right)} \times$$
$$e^{-\left(\frac{(m_1^2-m_2^2)^2}{2k^2}(9\sigma_1+\sigma_2+\sigma_3+\sigma_4) + \frac{2V\cos 2\theta(m_1^2-m_2^2)}{k}(12\sigma_1+2\sigma_2-2\sigma_3)\right)t^2}$$

where $\Upsilon = \frac{Vk}{m_1^2-m_2^2}$, $|\Upsilon| \ll 1$, and $k^2 \gg m_1^2, m_2^2$, and

$$z_0^+ = m_1^2 + \Upsilon(1+\cos 2\theta)(m_1^2-m_2^2) + \Upsilon^2(m_1^2-m_2^2)\sin^2 2\theta$$
$$z_0^- = m_2^2 + \Upsilon(1-\cos 2\theta)(m_1^2-m_2^2) - \Upsilon^2(m_1^2-m_2^2)\sin^2 2\theta.$$

NB: σ-modifications of oscil. period, $e^{-(\dots)t^2}$ suppression.

Uncertainty induced Decoherence

Gaussian Averaged ν-oscillations can produce Decoherence (T. Ohlsson, hep-ph/0012272)

Recall oscillation formula:

$$P_{\alpha\beta} = P_{\alpha\beta}(L,E) =$$
$$\delta_{\alpha\beta} - 4\sum_{a=1}^{n}\sum_{\beta=1,a<b}^{n} \mathrm{Re}\left(U^*_{\alpha a}U_{\beta a}U_{\alpha b}U^*_{\beta b}\right)\sin^2\left(\frac{\Delta m^2_{ab}L}{4E}\right) -$$
$$2\sum_{a=1}^{n}\sum_{b=1,a<b}^{n} \mathrm{Im}\left(U^*_{\alpha a}U_{\beta a}U_{\alpha b}U^*_{\beta b}\right)\sin\left(\frac{\Delta m^2_{ab}L}{2E}\right)$$

where $\alpha,\beta = e,\mu,\tau,...,\ a,b = 1,2,...n,$
$\Delta m^2_{ab} = m^2_a - m^2_b$

BUT...UNCERTAINTIES for E IN PRODUCTION OF ν-WAVE; Also: NOT WELL-DEFINED PROPAGATION LENGTH L :

$$\Delta E \neq 0, \qquad \Delta L \neq 0$$

Hence, have to AVERAGE Oscillation Probability P over L/E Dependence.

Gaussian Average Decoherence

GAUSSIAN AVERAGE: Approximate $\langle L/E \rangle \simeq \langle L \rangle / \langle E \rangle$

$$\langle P \rangle = \int_{-\infty}^{\infty} \mathrm{d}x \; P(x) \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\ell)^2}{2\sigma^2}}$$

$\ell \equiv \langle x \rangle$, $\sigma = \sqrt{\langle (x - \langle x \rangle)^2}$, $x = L/4E$.

AVERAGE $\langle P_{\alpha\beta} \rangle$:

$$\langle P_{\alpha\beta} \rangle = \delta_{\alpha\beta} -$$
$$2 \sum_{a=1}^{n} \sum_{\beta=1, a<b}^{n} \mathrm{Re} \left(U^*_{\alpha a} U_{\beta a} U_{\alpha b} U^*_{\beta b} \right) \left(1 - \cos(2\ell \Delta m^2_{ab}) e^{-2\sigma^2 (\Delta m^2_{ab})^2} \right)$$
$$-2 \sum_{a=1}^{n} \sum_{b=1, a<b}^{n} \mathrm{Im} \left(U^*_{\alpha a} U_{\beta a} U_{\alpha b} U^*_{\beta b} \right) \sin(2\ell \Delta m^2_{ab}) e^{-2\sigma^2 (\Delta m^2_{ab})^2}$$

NB: Damping factors due to σ (!)

EXAMPLE: TWO FLAVOURS

$$\langle P_{\alpha\beta} \rangle = \frac{1}{2} \sin^2 2\theta \left(1 - e^{-2\sigma^2 (\Delta m^2)^2} \cos(2\ell \Delta m^2) \right), \; \ell = \frac{\langle L \rangle}{4\langle E \rangle}$$

Bounds on σ (T. Ohlsson)

- Pessimistic: $\sigma \simeq \Delta x \simeq \Delta \frac{L}{4E} \leq \frac{\langle L \rangle}{4\langle E \rangle} \left(\frac{\Delta L}{\langle L \rangle} + \frac{\Delta E}{\langle E \rangle} \right)$
- Optimistic: $\sigma \leq \frac{\langle L \rangle}{4\langle E \rangle} \left([\frac{\Delta L}{\langle L \rangle}]^2 + [\frac{\Delta E}{\langle E \rangle}]^2 \right)^{1/2}$

Equivalence with Lindblad decoherence ($D_i^\dagger = D_i$) (Adler 2000)

$$\dot{\rho} = i[\rho, H] + \mathcal{D}[\rho],\ \mathcal{D}[\rho] = \sum_{i=1}^{n}[D_i, [D_i, \rho]].$$

Example: TWO FLAVOURS: One Decoherence Coefficient γ ($L = t$, $c = 1$):

$$P_{e\mu}(L, E) = \frac{1}{2}\sin^2 2\theta \left(1 - e^{-\gamma L}\cos(\frac{\Delta m^2 L}{2E})\right)$$

COMPARE WITH "FAKE" GAUSSIAN AVERAGE:

$$2\sigma^2(\Delta m^2)^2 = \gamma L \quad \rightarrow \quad \gamma = \frac{(\Delta m^2)^2}{8E^2} L r^2$$

with $\sigma = (L/4E)r$, $r = \frac{\Delta L}{L} + \frac{\Delta E}{E}$ (pessimistic), or $r = \sqrt{(\frac{\Delta L}{L})^2 + (\frac{\Delta E}{E})^2}$ (optimistic).

For atmospheric ν: $\sigma_{\rm atm} \sim 1.5 \times 10^3$ eV^2 (for $L \sim 12000\ Km$), $r \sim \mathcal{O}(1)$, hence

$$\gamma_{\rm atm, fake} < 10^{-24}\ {\rm GeV}$$

COMPARE WITH QG: (i) optimistic (Ellis, NM, Nanopoulos) : $\gamma_{QG} \sim E^2/M_{QG}$, (ii) pessimistic: (Adler) $\gamma_{QG} \sim \frac{(\Delta m^2)^2}{E^2 M_{QG}}$.

NB: In QG NO L Dependence, but $1/M_{QG}$ (in 4-dim $M_{QG} \sim M_P \sim 10^{19}$ GeV) CAN DISENTANGLE (!)

Quantum Gravity Uncertainties

NB: GAUSSIAN AVERAGE ALSO DUE TO QUANTUM-GRAVITY UNCERTAINTIES:

If ΔL is due to "Fuzziness" of space time due to quantum fluctuations, then (Van Dam, Ng, Ellis, NM, Nanopoulos)

$$\frac{\Delta L}{L}, \quad \frac{\Delta E}{E} \sim \beta \left(\frac{E}{M_{QG}}\right)^{\alpha},$$

α some positive integer, $\alpha \geq 1$, $\beta = \beta(L)$ some coefficient. In this case $r \sim \beta \left(\frac{E}{M_{QG}}\right)^{\alpha}$.

Then, from Gaussian Average we get for Decoherence:

$$\gamma \sim \frac{(\Delta m^2)^2}{8E^2} \beta \left(\frac{E}{M_{QG}}\right)^{\alpha} L$$

NB: modified E-dependence, but still $\propto L$ if β=const.

INTERESTING TO EXPLORE FURTHER...(c.f. below)

HOWEVER, IN GENERAL SUCH EFFECTS CAN BE DISENTANGLED FROM OTHER α, β, γ COEFFICIENTS OR STOCHASTIC-MEDIUM EFFECTS BY THEIR L DEPENDENCE...

Genuine vs "Fake" CPTV & Decoherence Effects

Important to distinguish: Intrinsic (genuine, due to QG) from Extrinsic ("fake") CPTV effects due to matter influences (e.g. K^0, $\overline{K}^0$ in regenerator, or neutrinos in matter media).

SOME NOMENCLATURE

Probability differences:
$P_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta), P_{\overline{\alpha}\overline{\beta}} = P(\overline{\nu_\alpha} \to \overline{\nu_\beta})$, Greek indices=flavour.

- (I) CP: $\Delta P^{\rm CP}_{\alpha\beta} = P_{\alpha\beta} - P_{\overline{\alpha}\overline{\beta}}$
- (II) T: $\Delta P^{\rm T}_{\alpha\beta} = P_{\alpha\beta} - P_{\beta\alpha}$
- (III) CPT: $\Delta P^{\rm CPT}_{\alpha\beta} = P_{\alpha\beta} - P_{\overline{\beta}\overline{\alpha}}$

Probability Conservation for 'fake' CPTV:
$\sum_{\alpha=e,\mu,\tau,\dots} \Delta P^{\rm CPT}_{\alpha\beta} = \sum_{\beta=e,\mu,\tau,\dots} \Delta P^{\rm CPT}_{\alpha\beta} = 0$ and $\Delta P^{\rm CPT}_{\alpha\beta} = -\Delta P^{\rm CPT}_{\overline{\beta}\overline{\alpha}}$ i.e. probability difference for $\overline{\nu}$ do not give further information. CONTRAST WITH GENUINE CPTV where $\Delta P^{\rm CPT}_{\alpha\beta} \neq \Delta P^{\rm CPT}_{\overline{\beta}\overline{\alpha}}$ due to different decoherence parameters between ν and $\overline{\nu}$ sectors.

L/E dependence of $\Delta P^{\rm CPT}$ due to matter would distinguish it from QG effects, where one might have enhancement with ν energy E .

Order of “Fake” CPTV

Experiment	CPT probability differences	
	Quantities	Numerical value
BNL NWG	$\Delta P_{\mu e}^{\mathrm{CPT}}$	0.010
BNL NWG	$\Delta P_{\mu e}^{\mathrm{CPT}}$	0.032
BooNE	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$6.6 \cdot 10^{-13}$
MiniBooNE	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$4.1 \cdot 10^{-14}$
CHOOZ	$\Delta P_{ee}^{\mathrm{CPT}}$	$-3.6 \cdot 10^{-5}$
ICARUS	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$4.0 \cdot 10^{-5}$
	$\Delta P_{\mu \tau}^{\mathrm{CPT}}$	$-3.8 \cdot 10^{-5}$
JHF-Kamioka	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$3.8 \cdot 10^{-3}$
	$\Delta P_{\mu \mu}^{\mathrm{CPT}}$	$-1.3 \cdot 10^{-4}$
K2K	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$1.0 \cdot 10^{-3}$
	$\Delta P_{\mu \mu}^{\mathrm{CPT}}$	$-5.3 \cdot 10^{-5}$

Experiment	CPT probability differences	
	Quantities	Numerical value
KamLAND	$\Delta P_{ee}^{\mathrm{CPT}}$	-0.033
LSND	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$4.8 \cdot 10^{-15}$
MINOS	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$1.9 \cdot 10^{-4}$
	$\Delta P_{\mu \mu}^{\mathrm{CPT}}$	$-1.1 \cdot 10^{-5}$
NuMI I	$\Delta P_{\mu e}^{\mathrm{CPT}}$	0.026
NuMI II	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$2.6 \cdot 10^{-3}$
NuTeV	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$1.6 \cdot 10^{-18}$
NuTeV	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$8.2 \cdot 10^{-20}$
OPERA	$\Delta P_{\mu \tau}^{\mathrm{CPT}}$	$-3.8 \cdot 10^{-5}$
Palo Verde	$\Delta P_{ee}^{\mathrm{CPT}}$	$-1.2 \cdot 10^{-5}$
Palo Verde	$\Delta P_{ee}^{\mathrm{CPT}}$	$-2.2 \cdot 10^{-5}$

Table 1: Extrinsic CPT pds for some past, present, and future long-baseline experiments (Jacobson-Ohlsson, hep-ph/0305064).

NB: Extrinsic CPTV negligible for future ν factories ($\sim 10^{-5}$), sensitive to genuine CPTV? (study for 2 cases: $L \sim 3000\ Km, 7000\ Km, hep-ph/0305064$)

CPTV PHENOMENOLOGY & TESTS

COMPLEX PHENOMENOLOGY OF CPT VIOLATION

LORENTZ & CPT VIOLATION IN THE HAMILTONIAN

- **Standard Model Extension (Kostelecky et al.)**
  See Lehnert's talk
- **Modified Dispersion Relations (GRB, neutrino oscillations, synchrotron radiation)**

Decoherence-CPTV TESTS

- **Neutral Mesons: Neutral Kaons, B-mesons, and Factories (entangled states).**
  Ultracold Neutrons
  Neutrinos

QG-DECOHERENCE & CPT:

NEUTRAL MESONS

QG Decoherence in neutral Kaons

Quantum Gravity (QG) may induce decoherence and oscillations $K^0 \to \overline{K}^0$ (Ellis, Hagelin, Nanopoulos, Srednicki, Lopez+NM).

$$\partial_t \rho = i[\rho, H] + \delta \not{H} \rho$$

where

$$H_{\alpha\beta} = \begin{pmatrix} -\Gamma & -\frac{1}{2}\delta\Gamma & -\mathrm{Im}\Gamma_{12} & -\mathrm{Re}\Gamma_{12} \\ -\frac{1}{2}\delta\Gamma & -\Gamma & -2\mathrm{Re}M_{12} & -2\mathrm{Im}M_{12} \\ -\mathrm{Im}\Gamma_{12} & 2\mathrm{Re}M_{12} & -\Gamma & -\delta M \\ -\mathrm{Re}\Gamma_{12} & -2\mathrm{Im}M_{12} & \delta M & -\Gamma \end{pmatrix}$$

and

$$\delta \not{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix}$$

positivity of ρ requires: $\alpha, \gamma > 0, \qquad \alpha\gamma > \beta^2$.

α, β, γ violate CPT (Wald : decoherence) & CP:

$CP = \sigma_3 \cos\theta + \sigma_2 \sin\theta, \qquad [\delta \not{H}_{\alpha\beta}, CP] \neq 0$

DECOHERENCE vs. CPTV IN QM

Should distinguish two types of CPT Violation (CPTV):

(i) CPTV within Quantum Mechanics:
$\delta M = m_{K^0} - m_{\overline{K}^0}$, $\delta\Gamma = \ldots$. This could be due to (spontaneous) Lorentz violation.

(ii) CPTV through decoherence α, β, γ (entanglement with QG 'environment').

Experimentally two types can be disentangled !

RELEVANT OBSERVABLES: $\langle O_i \rangle = \mathrm{Tr}\,[O_i \rho]$

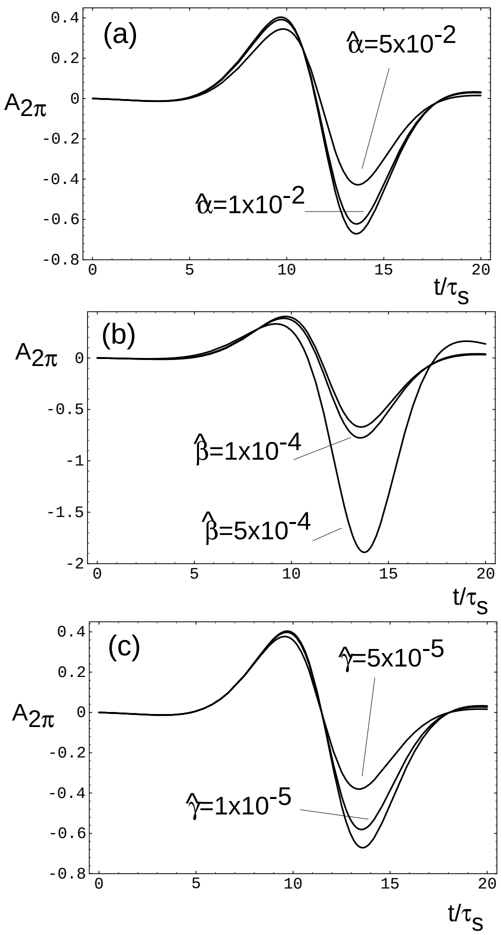
LOOK AT DECAY ASYMMETRIES for $K^0, \overline{K}^0$:

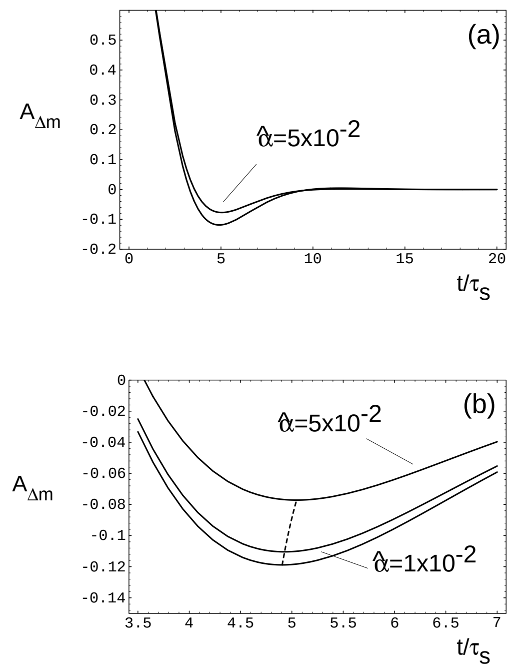
$$A(t) = \frac{R(\bar{K}^0_{t=0} \to \bar{f}) - R(K^0_{t=0} \to f)}{R(\bar{K}^0_{t=0} \to \bar{f}) + R(K^0_{t=0} \to f)}\,, \qquad (2)$$

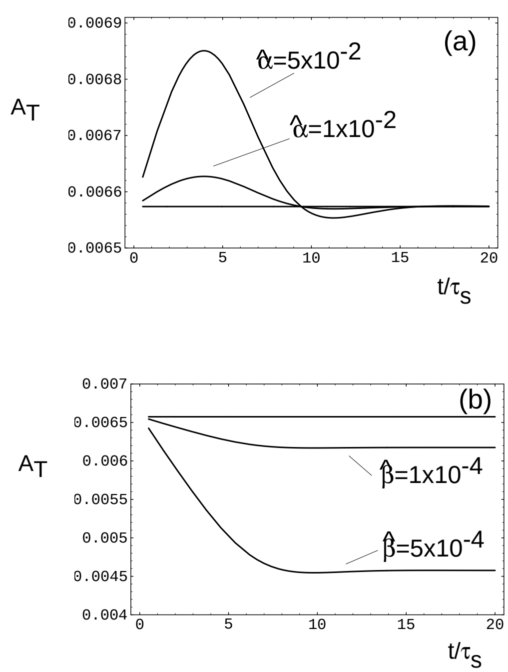
$R(K^0 \to f) \equiv \mathrm{Tr}\,[O_f \rho(t)]$ =decay rate into the final state f (pure K^0 at $t = 0$).

NEUTRAL KAON ASYMMETRIES: identical final states $f = \bar{f} = 2\pi$: $A_{2\pi}$, $A_{3\pi}$,
semileptonic: A_T (final states
$f = \pi^+ l^- \bar{\nu} \;\neq\; \bar{f} = \pi^- l^+ \nu$), A_{CPT}
($\overline{f} = \pi^+ l^- \bar{\nu},\; f = \pi^- l^+ \nu$), $A_{\Delta m}$.

NEUTRAL KAON ASYMMETRIES







INDICATIVE BOUNDS

Table 2: Compilation of indicative bounds on CPT-violating parameters and their source.

Source	Indicative bound
$R_{2\pi}, A_{2\pi}$	$\widehat{\alpha} < 5.0 \times 10^{-3}$
$R_{2\pi}, A_{2\pi}$	$\widehat{\beta} = (2.0 \pm 2.2) \times 10^{-5}$
$\lvert m_{K^0} - m_{\bar{K}^0} \rvert$	$\widehat{\beta} < 2.6 \times 10^{-5}$
$R_{2\pi}$	$\widehat{\gamma} \lesssim 5 \times 10^{-7}$
ζ	$\frac{\widehat{\gamma}}{2\lvert\epsilon\rvert^2} - \frac{2\widehat{\beta}}{\lvert\epsilon\rvert} \sin\phi = 0.03 \pm 0.02$
Positivity	$\widehat{\alpha} > \widehat{\beta}^2 / \widehat{\gamma}_{\rm max} \sim (10^3 \widehat{\beta})^2$

FROM CPLEAR MEASUREMENTS (PLB364 (1995) 239): $\alpha < 4.0 \times 10^{-17}$ GeV , $|\beta| < 2.3. \times 10^{-19}$ GeV , $\gamma < 3.7 \times 10^{-21}$ GeV

NB(1): Theoretically expected values (some models) $\alpha \, , \beta \, , \gamma = \mathcal{O}(\xi \frac{E^2}{M_P})$.

NB(2): $m_{K^0} - m_{\overline{K}^0} \sim 2|\beta|$

(at present $(m_{K^0} - m_{\overline{K}^0})/m_{K^0} < 7.5 \times 10^{-19}$)

ENTANGLED STATES (Neutral Mesons)

- Complete Positivity

  Different parametrization of Decoherence matrix for (entangled) mesons: (in α, β, γ framework: $\alpha = \gamma$, $\beta = 0$)

  c.f. Floreanini's talk.

- Novel (geunine) two-body effects: EPR correlation modification.

  c.f. Bernabeu's talk.

CPTV and EPR-correlations modification

(Bernabeu, NM and Papavassiliou, hep-ph/0310180 (PRL 92))

If CPT is broken, e.g. via Quantum Gravity (QG) effects on $\$ \ne SS^\dagger$, then: CPT operator Θ is ILL defined $\Rightarrow$ Antiparticle Hilbert Space INDEPENDENT OF particle Hilbert space.

Neutral mesons K^0 and $\overline{K}^0$ SHOULD NO LONGER be treated as IDENTICAL PARTICLES. This implies that the initial Entangled State in ϕ (B) factories $|i>$ can now be written (in terms of mass eigenstates):

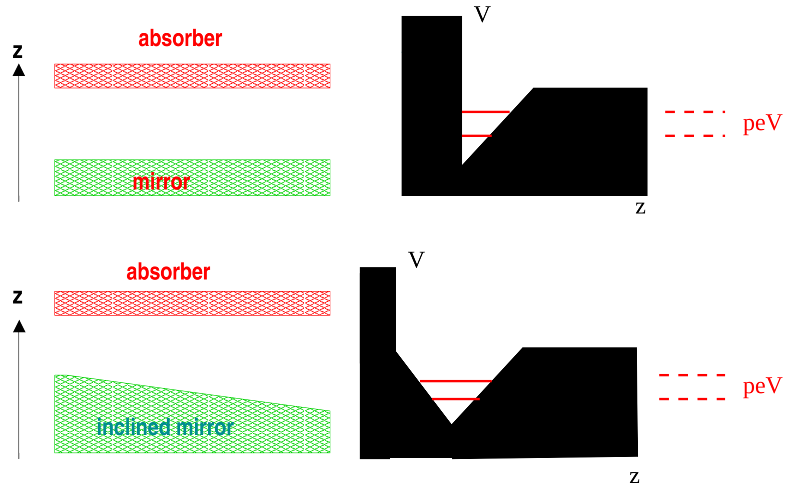
$$
\begin{aligned}
|i> &= C\Big[\Big(|K_S(\vec{k}), K_L(-\vec{k}) > - |K_L(\vec{k}), K_S(-\vec{k}) >\Big) \\
&+ \omega\Big(|K_S(\vec{k}), K_S(-\vec{k}) > - |K_L(\vec{k}), K_L(-\vec{k}) >\Big)\Big]
\end{aligned}
$$

NB! $K_S K_S$ or $K_L - K_L$ combinations, due to CPTV ω, important in decay channels. There is contamination of C(odd) state with C(even). Complex ω controls the amount of contamination by the "wrong" (C(even)) symmetry state.

Experimental Tests of ω-Effect in ϕ, B factories... in B-factories: ω-effect $\rightarrow$ demise of flavour tagging (Alvarez et al. (PLB607)) **Disentangle ω from non-unitary evolution and background effects**.

ULTRACOLD NEUTRONS

ULTRACOLD NEUTRONS



Inclined mirror ensures Parity invariance of QG modifications and hence formalism similar to neutral kaons. A few (two here) energy states (peV energy differences between levels) are inside the Earth' s potential well. Probability of finding neutrons in either state is:

$$\mathrm{Tr}(\rho' \varrho_{1,2}) = \frac{1}{2} \pm \frac{1}{2} e^{-\frac{\alpha+\gamma}{2}t} \sin(\Delta E t) \ , \qquad \delta E \simeq \mathrm{peV}$$

If Lorentz invariance is violated $\alpha, \gamma \simeq \frac{E_{\mathrm{kin}}^2}{M_P}$; if NOT, $\alpha, \gamma \simeq \frac{m_n^2}{M_P}$. $t \sim \mathrm{msec}$ Second case effect is much larger. However, at present no significant sensitivity.

QG-DECOHERENCE & NEUTRINOS

QG Decoherence and neutrino mixing

Quantum Gravity (QG) may induce oscillations between neutrino flavours independently of masses (Liu et al., 1997, Chang et al., 1998, Lisi et al., Benatti & Floreanini 2000).

$$\partial_t \rho = i[\rho, H] + \delta \not{H} \rho$$

where (Ellis, Hagelin, Nanopoulos, Srednicki 1984)

$$\delta \not{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -2\alpha & -2\beta & 0 \\ 0 & -2\beta & -2\gamma & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

for energy and lepton number conservation. and

$$\delta \not{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix}$$

if energy and lepton number violated, but flavour conserved (σ_1 Pauli matrix). Positivity of ρ requires: $\alpha, \gamma > 0, \qquad \alpha\gamma > \beta^2$. α, β, γ **violate CPT** (Ellis, NM, Nanopoulos 1992, Lopez + EMN 1995). **Decoherence affects (damps) OSCILLATION PROBABILITIES**

QG Decoherence and neutrino mixing

In some models of QG Decoherence, with complete positivity in ideal Markov environments

$\beta = 0, \alpha = \gamma > 0$ **(Benatti, Floreanini)**.

Theoretical Models Predictions vs. Experiment: **Optimistic: (Ellis, NM, Nanopoulos, ...)** $\gamma \sim \gamma_0 (\frac{E}{\mathrm{GeV}})^n, n = 0, 2, -1,$ $n = 2$ stringy QG, $n = -1$ ordinary matter effects.

Pessimistic: (Adler 2000) $\gamma \sim \frac{(\Delta m^2)^2}{E^2 M_{qg}}$**,**
($M_{qg} \sim M_P \sim 10^{19}$ **GeV).**

with E the neutrino energy.

From Atmosperic ν data $\rightarrow$ Bounds:

$n = 0$, $\gamma_0 < 3.5 \times 10^{-23}$ GeV

$n = 2$, $\gamma_0 < 0.9 \times 10^{-27}$ GeV (c.f. CPLEAR bound for Kaons: $\gamma < 10^{-21}$ GeV (PLB364 (1995) 239))

$n = -1$, $\gamma_0 < 2 \times 10^{-21}$ GeV.

NB: Tests on ν-mixing from Decoherence exhibit much greater sensitivity than neutral mesons. Very stringent limits from neutrinos from exaglactic sources (Supernovae, AGN), if QG induces lepton number violation and/or flavour oscillations:From SN1987a, using the observed constraint on the oscillation probability $P_{\nu_e \rightarrow \nu_{\mu,\tau}} < 0.2$: $\gamma < 10^{-40}$ GeV.

QG Decoherence and neutrino mixing

FITTING THE DATA (Lisi et al. PRL 85 (2000), 1166)

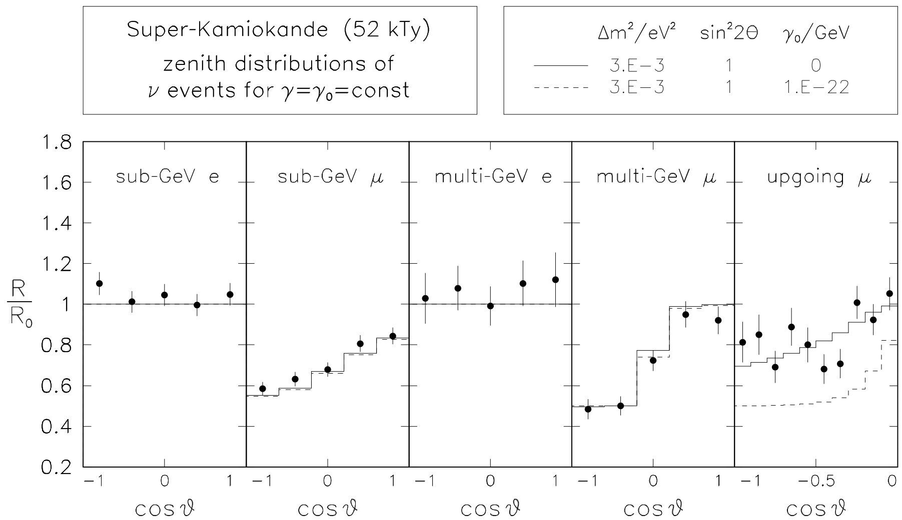


Figure 1: Effects of decoherence $(\gamma = \gamma_0 = \mathrm{const} \neq 0)$ on the distributions of lepton events as a function of the zenith angle ϑ

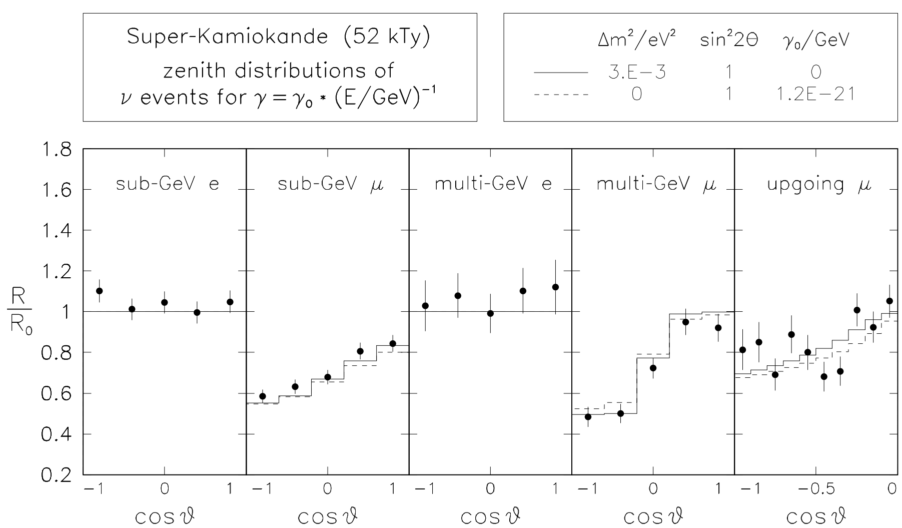


Figure 2: Best-fit scenarios for pure oscillations $(\gamma = 0)$ (solid line) and for pure decoherence with $\gamma \propto 1/E$ (dashed line).

Three ν Generations, Decoherence and LSND

Barenboim, Sarben Sarkar, Waldron, NM 2006

We managed to fit the 3-generation Lindblad probabilities preserving positivity and boundedness, with ALL data, including LSND and KamLand.

To fit spectral distortion KamLand requires for decoherence parameters: $L_{11} = L_{22}$, $L_{44} = L_{55}$,

$L_{66} = L_{77}$, $L_{33} = L_{88}, L_{38} = L_{83} = 0$,

$L_{33} = L_{66} = 0$, $L_{11} = L_{22} = L_{44} = L_{55} = -\frac{1.3 \cdot 10^{-2}}{L}$ we obtain excellent fits to the data, guaranteeing positivity:

NB: i.e. Oscillation-length independent damping exponents !.

CAN EXCLUDE SOME STOCHASTIC MODELS OF QUANTUM GRAVITY ALREADY !

Order of magnitude compatible with ordinary decoherence, due to energy uncertainties (Ohlsson)

$$\frac{\Delta E}{E} \sim 1.6 \cdot 10^{-1}$$

Puzzling aspect: NOT ALL decoherence exponents exhibit this modulation....

STILL FURTHER ANALYSIS NECESSARY, both theoretical and experimental...

FITTING ν DATA

Barenboim, Sarben Sarkar, Waldron, NM 2006

χ^2	decoherence	standard scenario
SK sub-GeV	38.0	38.2
SK Multi-GeV	11.7	11.2
Chooz	4.5	4.5
KamLAND	16.7	16.6
LSND	0.	6.8
TOTAL	70.9	77.3

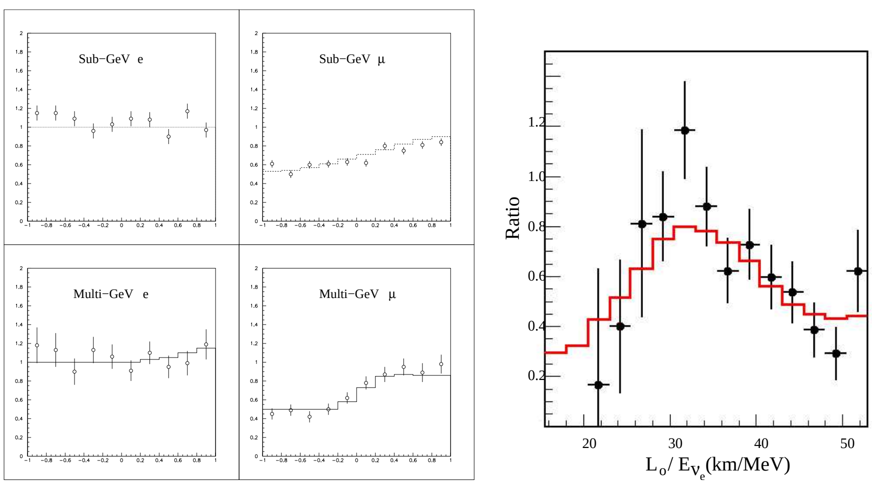


Figure 3: Left: Decoherence fit. Right: Ratio of the observed $\overline{\nu}_e$ spectrum to the expectation versus L_0/E for our decoherence model. The dots correspond to KamLAND data

CONCLUSIONS

CPTV may not be an academic issue, but a real feature of Quantum Gravity (QG).

Various ways for CPT breaking, in principle independent, e.g. decoherence and Lorentz Violation are independent effects. One may have Lorentz invariant decoherence in Quantum Gravity (Millburn).

Precision experiments in meson factories, will provide sensitive probes of QG-induced decoherence & CPT Violation, including NOVEL effects (ω-effect) exclusive to ENTANGLED states.

Neutrino Physics may provide a very useful guide in our quest for a theory of Quantum Gravity, in particular stringent constraints on CPT Violation. The scenario of three-generation antineutrino decoherence + mixing is still compatible with ALL ν data, including LSND and KamLAND; can exclude some stochastic QG models already.

What about Equivalence principle and QG?: are QG effects universal among particle species? ...

More work (Theory & Expt) to be done before conclusions are reached...