Theoretical bounds on the Higgs boson mass

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The problem: a (renormalizable) theory of weak interactions

History

- Fermi theory of ß-decay (34):

contact interactions between two currents (prototype of modern effective theories)

$$\mathcal{L}_F = -\frac{G_\beta}{\sqrt{2}} J^\dagger_{(h)\,\mu} J^\mu_{(l)} + h.c. = -\frac{G_\beta}{\sqrt{2}} [\bar{p}(x)\gamma_\mu n(x)][\bar{e}(x)\gamma^\mu \nu(x)] + h.c.$$

- Parity nonconservation (56-57); V-A law (58); CVC hypothesis ($G_ß \sim G_\mu$) (58)

$$\mathcal{L}_F = -\frac{G_\beta}{\sqrt{2}} [\bar{p}(x)\gamma_\mu (1-\lambda\gamma_5) n(x)][\bar{e}(x)(\gamma^\mu (1-\gamma_5)\nu(x)] + h.c. \qquad (\lambda \simeq 1.27)$$

- Quark hypothesis (60); Cabibbo theory (63);

$$\begin{aligned}
\mathcal{L}_{eff} &= -\frac{G_\mu}{\sqrt{2}} J^\dagger_\lambda J^\lambda \\
J^\lambda &= J^\lambda_{(h)} + J^\lambda_{(l)} \\
J^\lambda_{(l)} &= \bar{\nu}_e \gamma^\lambda (1-\gamma_5) e + \bar{\nu}_\mu \gamma^\lambda (1-\gamma_5)\mu \\
J^\lambda_{(h)} &= \cos\theta_c\, \bar{u}\gamma^\lambda (1-\gamma_5) d + \sin\theta_c\, \bar{u}\gamma^\lambda (1-\gamma_5) s
\end{aligned}$$

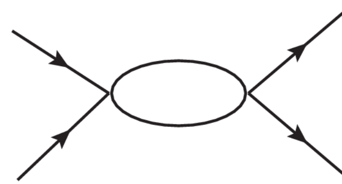
Θ_c : Cabibbo angle

$$G_\beta = G_\mu \cos\theta_c \simeq 0.98$$

Fermi theory (or any effective):

- Not renormalizable

$$[\mathcal{L}] = 4,\ [\psi] = 3/2 \Rightarrow [G_\mu] = -2$$



$$\sim \int \frac{d^4k}{k^2}$$

- Violate unitarity :
  Ex.: $\nu_\mu(k_1) + e^-(p_1) \to \nu_e(p_2) + \mu^-(k_2)$

$$\mathcal{M} = -i\frac{G_\mu}{\sqrt{2}}\bar{u}(\mu)\gamma^\lambda(1-\gamma_5)u(\nu_\mu)\bar{u}(\nu_e)\gamma_\lambda(1-\gamma_5)u(e)$$

$$|\bar{\mathcal{M}}|^2 = \frac{G_\mu^2}{2}\mathrm{Tr}\left[\not{k}_2\gamma_\mu(1-\gamma_5)\not{k}_1\gamma_\nu(1-\gamma_5)\right]\frac{1}{2}\mathrm{Tr}\left[\not{p}_2\gamma^\mu(1-\gamma_5)\not{p}_1\gamma^\nu(1-\gamma_5)\right] = \frac{G_\mu^2}{2}32s^2$$

$$d\sigma = |\bar{\mathcal{M}}|^2\frac{1}{4s}\frac{1}{(4\pi)^2}d\Omega$$

$$\sigma = \frac{G_\mu^2\, s}{\pi}$$

But optical theorem tells us the total cross section is related to the amplitude for elastic scattering in the forward direction

$$\sigma_T(\nu_\mu, e^- \to anything) = \frac{1}{s}\mathrm{Im}\mathcal{A}(s, J, \theta = 0)$$

$$\mathcal{A}(s, l, \theta) = 16\pi\sum_{l=o}^{\infty}(2l+1)P_l(\cos\theta)e^{i\delta_l}\sin\delta_l$$

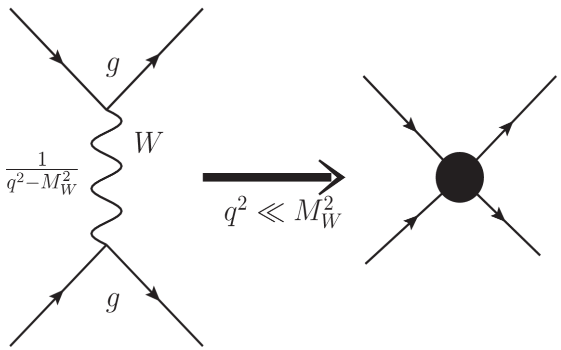
Spinless particle

$$\sigma \le \sigma_T \le \frac{O(\pi)}{s} \Rightarrow s \le \frac{O(\pi)}{G_\mu}$$

Intermediate Vector Boson theory (IVB)

The contact interaction between currents is the result of the exchange of a heavy charged vector boson

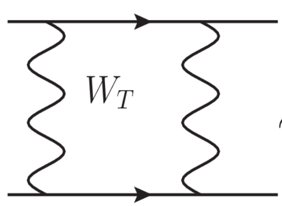
$$\mathcal{L}_{int} = -g J^{\mu}_{-} W^{+}_{\mu} + h.c. \qquad J^{\mu}_{-} \equiv J^{\mu}_{(l)} + J^{\mu}_{(h)}$$



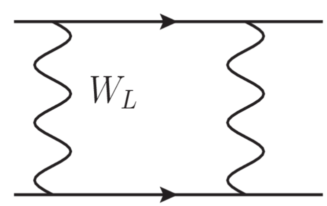
$$G_{\mu}/\sqrt{2} = g^2/M_W^2$$

[g]=0 but theory not renormalizable; problem stays in the longitudinal part of the vector boson propagator

$$i\Delta_W^{\mu\nu}(k) = -i\frac{g^{\mu\nu} - k^{\mu}k^{\nu}/M_W^2}{k^2 - M_W^2 + i\epsilon}$$



$$\sim \int \frac{d^4k}{k^6}$$



$$\sim \int \frac{d^4k}{k^2}$$

Similarly we expect unitarity problem in processes with longitudinal W's like

$$e^{+} + e^{-} \rightarrow W^{+} + W^{-}$$

The solution: a gauge theory

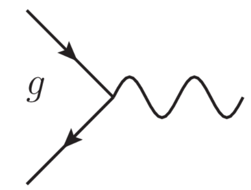
Promote the IVB to be the carrier of a gauge interaction as described by a gauge Lagrangian $\mathcal{L}_g$. To any vector boson V^A_μ there is an associated generator T^A of the gauge group G forming a closed algebra

$$[T^A, T^B] = i f^{ABC} T^C, \qquad f^{ABC}$$ Structure constants of G

$$\begin{aligned} \mathcal{L}_g &= -\frac{1}{4}\sum_{A=1}^{N} F^A_{\mu\nu} F^{A\,\mu\nu} + i\bar{\Psi}\not{D}\Psi + |D_\mu\phi|^2 \\ F^A_{\mu\nu} &= \partial_\mu V^A_\nu - \partial_\nu V^A_\mu + g f^{ABC} V^B_\mu V^C_\nu \\ D_\mu &= \partial_\mu - ig\sum_{A=1}^{N} V^A_\mu T^A \end{aligned}$$

Gauge symmetry dictates the Interactions of V^A_μ

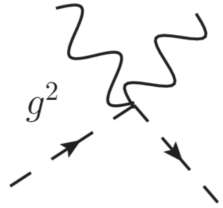
V^A_μ interact with matter fields via currents $\sum_{A=1}^{N} J^A_\mu V^{A\mu}$

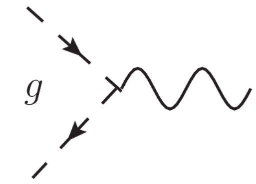


$$J^A_\mu(\Psi) = \bar{\Psi}\gamma_\mu T^A \Psi, \; J^A_\mu(\phi) = \phi^\dagger T^A \partial_\mu \phi - \partial_\mu \phi^\dagger T^A \phi$$

For scalars there Is also a “sea-gull” term

$$\sum_{A,B=1}^{N} \phi^\dagger T^A T^B \phi V^{A\dagger}_\mu V^{B\mu}$$





Fermions and scalars are arranged in representation of G. For massless fermions the l.h. and r.h. components can be given different transformation properties under the Symmetry

$$\bar{\Psi} i \not{D} \Psi = \bar{\Psi}_L i \not{D} \Psi_L + \bar{\Psi}_R i \not{D} \Psi_R$$

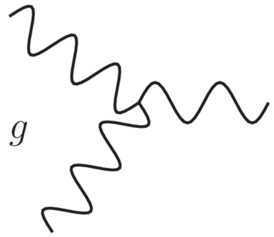
$$\Psi_L = \frac{1-\gamma_5}{2}\Psi \qquad \bar{\Psi}_L = \Psi_L^\dagger \gamma_0 = \bar{\Psi}\frac{1+\gamma_5}{2}$$
$$\Psi_R = \frac{1+\gamma_5}{2}\Psi \qquad \bar{\Psi}_R = \Psi_R^\dagger \gamma_0 = \bar{\Psi}\frac{1-\gamma_5}{2}$$

(Ψ Dirac field)

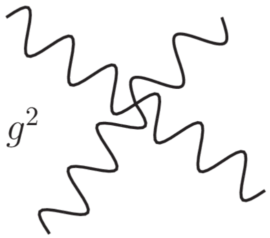
Mass terms break the symmetry if l.h. and r.h. fermions have different symmetry transformations

$$m\bar{\Psi}\Psi = m\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L$$

Non Abelian group: N generators, $f^{ABC} \neq 0$
Gauge symmetry gives trilinear and quadrilinear self- interactions of V^A_μ



derivative

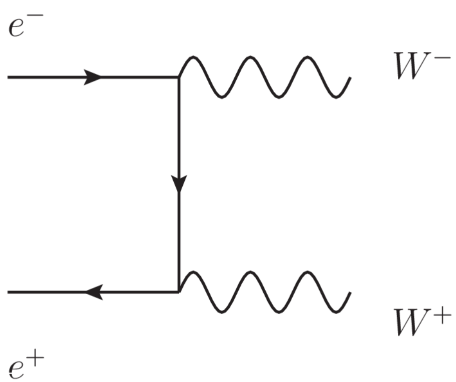


contact

Abelian group: U(1) (N=1, f^{ABC} =0)
QED: $T^1 = Q$, g =e, no self-interactions between photons

Gauge symmetry does not allow an explicit mass term $m \; V^A_\mu \; V^{A\mu}$

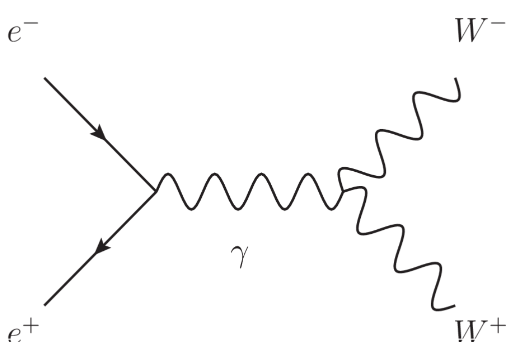
In the IVB we expected the $e^+ + e^- \rightarrow W^+ + W^-$ cross-section to raise with s (the C.M. energy) when the W's are longitudinally polarized

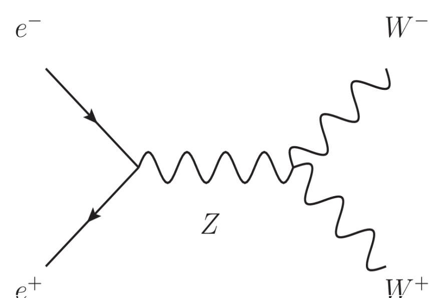


IVB

But in our gauge theory we have two extra contributions from

$$-\frac{1}{4}\sum_{A=1}^{3} F^A_{\mu\nu}F^{A\,\mu\nu} \Rightarrow 2\frac{1}{2} g\epsilon^{ABC}\partial_\mu W^A_\nu W^B_\mu W^C_\nu \Rightarrow g\epsilon^{123}\underbrace{\partial_\mu W^1_\nu W^2_\mu}_{W^\pm}\underbrace{W^3_\nu}_{\gamma\; Z}$$





These two diagrams cancel the bad high energy behavior of the neutrino exchange diagram

The solution: a gauge theory coupled to a system that exhibits spontaneous symmetry breaking (SSB)

SSB: The Lagrangian of the theory respects a symmetry, but the vacuum state breaks it.

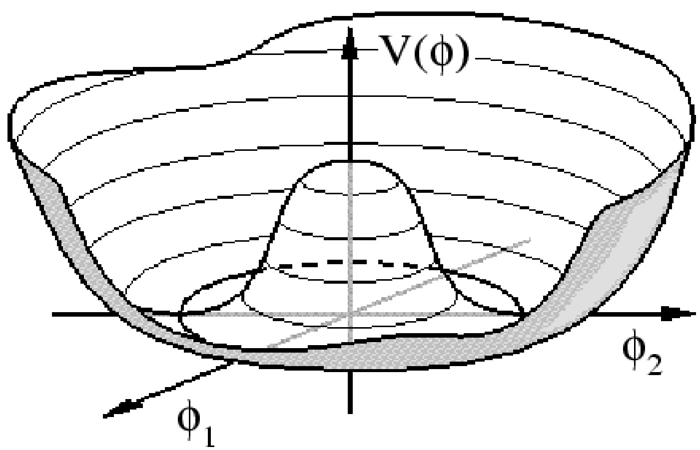
Important: If the symmetric lagrangian is renormalizable, after SSB the theory is still renormalizable

Goldstone model: Single complex scalar field $\phi \equiv \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$

$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - V(\phi), \quad V(\phi) = m^2|\phi|^2 + \lambda|\phi|^4$ (renormalizable interactions)

$\mathcal{L}$ symmetric under $\phi \to \phi' = \phi\, e^{i\theta}$ (global rotational symmetry)

If $m^2 < 0, \lambda > 0$



The potential has an infinite number of equivalent minima

for $|\phi|^2 = -\frac{m^2}{2\lambda}$

The system will choose one specific minimum, breaking the global rotational symmetry

We can expand the scalar field around a *real* vacuum expectation value (vev)

$$\phi \equiv \frac{1}{\sqrt{2}}\left[v + H(x) + iG(x)\right], \quad v = \sqrt{-\frac{m^2}{\lambda}}$$

At the minimum of the scalar potential (= the vacuum state) we have $\langle\phi\rangle = \frac{v}{\sqrt{2}}$

Up to an irrelevant constant, the scalar potential becomes

$$\begin{aligned} V \; = \; & (m^2 v + \lambda v^3)\, H + \frac{1}{2}\,(m^2 + 3\lambda v^2)\, H^2 + \frac{1}{2}\,(m^2 + \lambda v^2)\, G^2 \\ & + \lambda v\, H(H^2 + G^2) + \frac{\lambda}{4}\,(H^2 + G^2)^2 \end{aligned}$$

Inserting the value of v the linear term vanishes, and the masses of the scalars become

$$m_H^2 = -2\, m^2 = 2\,\lambda\, v^2\,, \qquad m_G^2 = 0$$

G is the *Goldstone boson* associated with the spontaneous breaking of the global symmetry

In general: the number of Goldstone boson is related to the number of broken generators of the symmetry

Broken generator: it does not annihilate the vacuum

The Higgs mechanism

Simplest U(1) model

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + |\underbrace{(\partial_\mu - ieA_\mu)}_{D_\mu}\phi|^2 - (m^2)|\phi|^2 - \lambda|\phi|^4 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \qquad m^2 < 0$$

$$\phi \equiv \frac{1}{\sqrt{2}}[v + h(x) + i\chi(x)], \quad v = \sqrt{-\frac{m^2}{\lambda}} \qquad <\phi>= \frac{v}{\sqrt{2}}$$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}\left[(\partial_\mu h)^2 + (\partial_\mu \chi)^2\right] + \frac{1}{2}(2\lambda v^2)h^2 + \frac{1}{2}e^2v^2A^\mu A_\mu + evA^\mu\partial_\mu\chi + \ldots$$

X field is not physical, it can be eliminated via a (field-dependent) gauge transformation

Original Lagrangian invariant under:

$$\phi \rightarrow \phi' = \phi\, exp[-i\epsilon(x)]$$
$$A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\epsilon(x)$$

Shift the field Φ and write it in polar coordinates: $\phi(x) = \frac{1}{\sqrt{2}}(h(x) + v)\, exp[+i\frac{\chi(x)}{v}]$

via a gauge transformation I can eliminate χ $\left(\epsilon(x) = \frac{\chi(x)}{v}\right)$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}e^2v^2A^\mu A_\mu + \frac{1}{2}e^2h^2A^\mu A_\mu + e^2hvA^\mu A_\mu + \mathcal{L}(h)$$

No χ, A_μ massive (3 d.o..f.); χ eaten by A_μ

SU(2)xU(1): (Weinberg 67, Salam 68)

SSB via an Higgs doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_{1/2}$

$$\mathcal{L} = \mathcal{L}_{symm} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yuk}$$

$$\mathcal{L}_{Higgs} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \underbrace{\left[(m^2)\Phi^\dagger\Phi + \lambda \left(\Phi^\dagger\Phi\right)^2\right]}_{V(\Phi^\dagger\Phi)}$$

Renormalizable interaction

$$\mathcal{L}_{Yuk} = -Y_d \bar{\psi}_L \Phi \psi_R^d - Y_u \bar{\psi}_L \tilde{\Phi} \psi_R^u + h.c., \quad \psi_L = \begin{pmatrix} \psi^u \\ \psi^d \end{pmatrix}_L, \quad \tilde{\Phi} = i\tau_2 \Phi^*$$

if $< \Phi >= \begin{pmatrix} o \\ \frac{v}{\sqrt{2}} \end{pmatrix}$

Shift **Φ** and write it in terms of 4 real fields, h, χ_1,χ_2,χ_3 as

$$\Phi(x) = exp[i\tau \cdot \chi(x)/v] \begin{pmatrix} 0 \\ \frac{h(x)+v}{\sqrt{2}} \end{pmatrix}$$

via gauge transformation I can eliminate χ (unitary gauge)

Q = T_3 +Y annihilates the vacuum $Q\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \left[\begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}\right]\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

3 broken generators, 3 χ's eaten: 3 massive vector boson, one massless:

SU(2)xU(1) → $U(1)_{em}$

Gauge boson masses:

$$D_\mu \Phi = \left[\partial_\mu - i\frac{g}{\sqrt{2}}\left(\tau^- W^+ - \tau^+ W^-\right) - igT^3 W^3_\mu - ig' Y B_\mu\right]\Phi, \quad \Phi \rightarrow <\Phi>$$

$$M_W^2 = \frac{g^2 v^2}{4}; \quad M_Z^2 = \frac{g^2 v^2}{4\cos^2\theta_W} \Rightarrow \rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

Only if the Higgs fields are singlets or doublets

If there are several Higgses in generic representation (T, T_3)

$$\rho = \frac{\sum_{\Phi_a} \frac{1}{2} < T^+ T^- + T^- T^+ >|_{<\Phi_a>} v^2_{\Phi_a}}{\sum_{\Phi_a} 2 < (T^3)^2 >|_{<\Phi_a>} v^2_{\Phi_a}} == \frac{\sum_{\Phi_a} \left[T(T+1) - (T^3)^2\right]_{\Phi_a} v^2_{\Phi_a}}{\sum_{\Phi_a} 2\left[(T^3)^2\right]_{\Phi_a} v^2_{\Phi_a}}$$

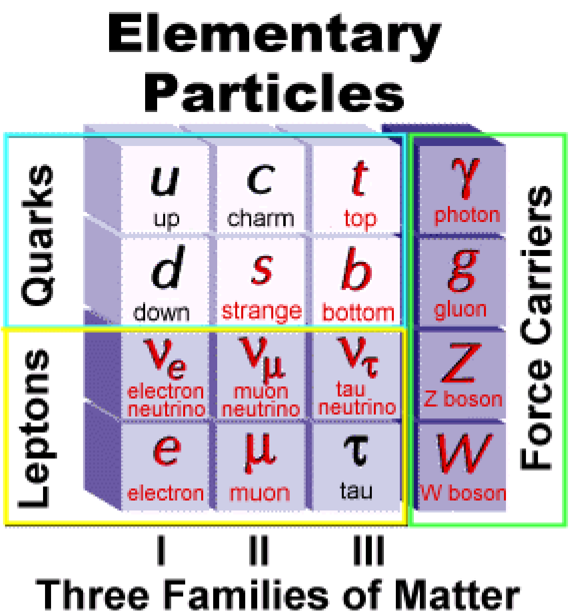
We must have at least one Higgs doublet to give mass to the fermions:

doublet, doublet, singlet

$$\begin{aligned}
\mathcal{L}_{Yuk} &= -Y_d \bar{\psi}_L \Phi \psi_R^d - Y_u \bar{\psi}_L \tilde{\Phi} \Psi_R^u + h.c. \\
& -Y_d \frac{v}{\sqrt{2}}\left(\bar{\psi}_L^d \psi_R^d + \bar{\psi}_R^d \psi_L^d\right) - Y_u \frac{v}{\sqrt{2}}\left(\bar{\psi}_L^u \psi_R^u + \bar{\psi}_R^u \psi_L^u\right) \qquad \Phi \Rightarrow <\Phi> \\
\Rightarrow \quad & m_u = \frac{Y_u v}{\sqrt{2}}, \qquad m_d = \frac{Y_d v}{\sqrt{2}}
\end{aligned}$$

The Standard Model

Strong, electromagnetic and weak interactions (not gravity) are described by a *renormalizable* Quantum Field Theory based on the principle of local gauge invariance with gauge symmetry group $SU(3)_c \times SU(2)_W \times U(1)_Y$ spontaneously broken to $SU(3)_c \times U(1)_{em}$. The quanta of the gauge fields (W,Z) acquire mass via the Higgs mechanism. The left-over of the EWSB process is (at least) a spin 0 particle, the Higgs particle, whose coupling to gauge bosons and to fermions is determined by their masses.



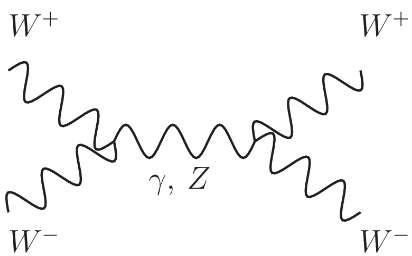
"The Higgs mechanism is just a reincarnation of the Comunist Party: it controls the masses"
Anonymous

Tree-level (unitarity) bound on the Higgs

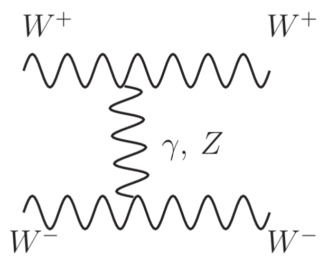
W-W scattering with longitudinal polarized W's

$$\epsilon_L^\mu \sim \frac{1}{m_W} p^\mu$$

W four-momentum



A



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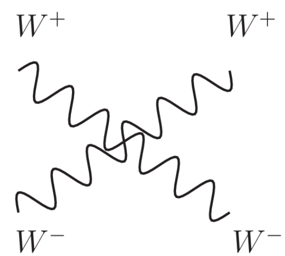
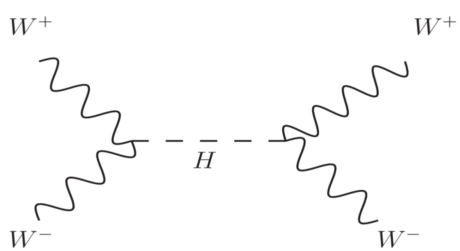


C

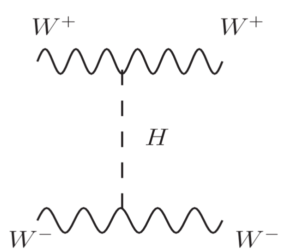
$$A + B = i \frac{g^2}{m_W^4} \left\{ -\frac{t^2 + s^2}{4} - s\,t + \frac{5}{4} m_W^2 (s + t) \right\}$$

$$A + B + C = i \frac{g^2}{m_W^2} \frac{s + t}{4}$$

growing with energy



D



E

$$A + \cdots + E = -i\sqrt{2}\, G_\mu m_H^2 \left(\frac{t}{t - m_H^2} + \frac{s}{s - m_H^2} \right)$$

does not grow with energy if the Higgs is not too heavy

$$\mathcal{A}(s, J, \theta) = 16\pi \sum_{j=0}^{\infty} (2j + 1)\, d^J(\theta) M^J \Rightarrow \left| M^J \right| \leq 1$$

$$M^{J=0} \sim -\frac{G_\mu\, m_H^2}{4\pi\sqrt{2}}, \qquad \left| M^{J=0} \right| \leq 1 \Rightarrow m_H^2 \leq \frac{4\pi\sqrt{2}}{G_\mu} \sim (10^3)^2$$

The effective potential

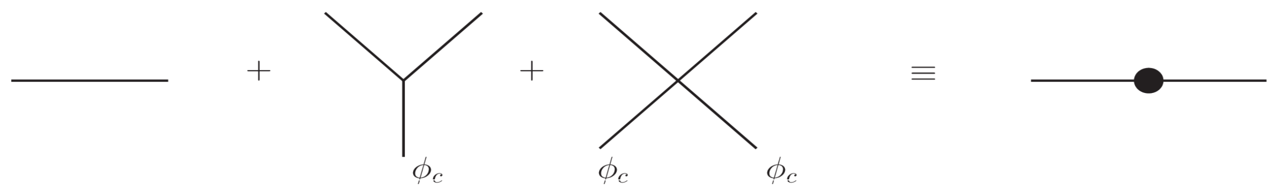
Single real field*:*

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi), \qquad V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

The minimum of V(Φ) gives the vacuum expectation value (vev) of Φ “at the classical level” (the state of lowest energy)

V(Φ) gives the lowest-order interactions (proper vertices, 1PI Green's functions at p^2 =0) after SSB and shifting the field by the vev

$V(\phi)$



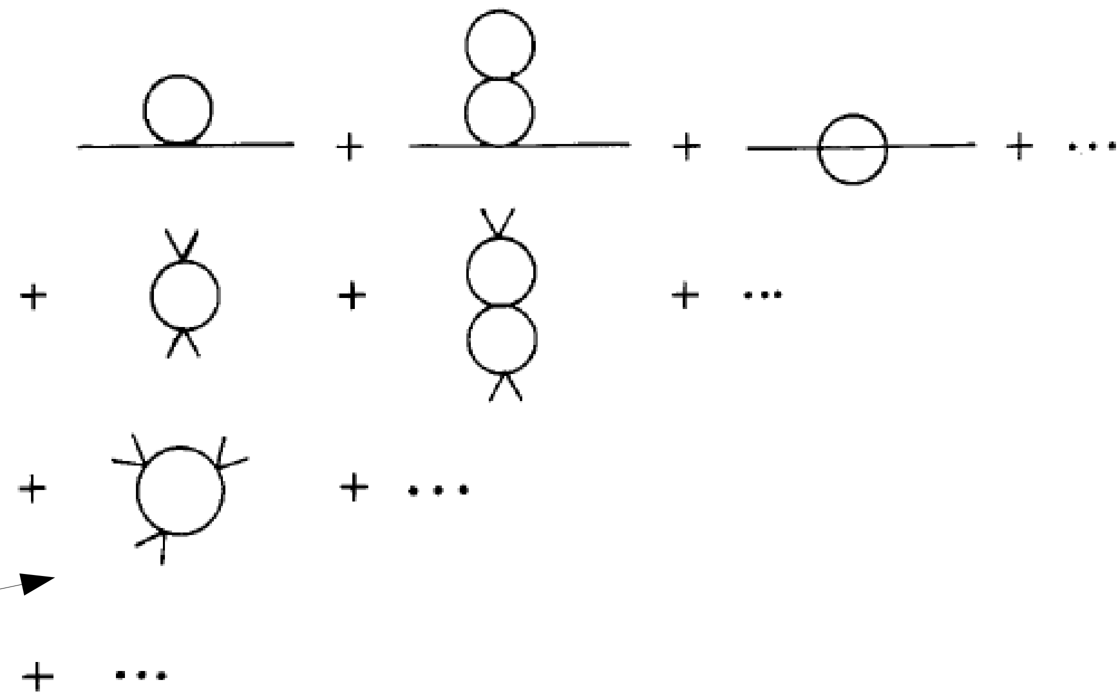
Quantum corrections will create new interactions . We are going to get interactions with 5,6,... external fields and the structure of the potential will be modified.

The vev of Φ including quantum corrections will be given by a new function, Veff (Φ_c), the effective potential that will agree with the classical potential energy to lowest order in perturbation theory

$$V_{eff}(\phi_c) = V(\phi_c) + V^{rad.}(\phi_c)$$

Infinite sum of Feynman diagrams

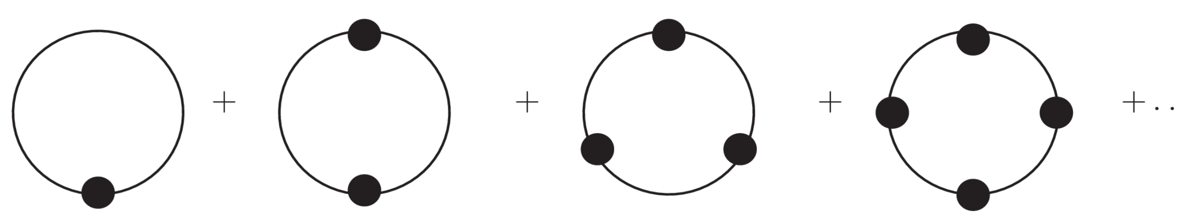
$$V(\phi_c) = \lambda \frac{\phi_c^n}{n!}$$



$$i \int \frac{d^4k}{(2\pi)^4} \frac{1}{2r} \left(\frac{\lambda \phi_c^{n-2}}{(n-2)!} \frac{1}{k^2 + i\epsilon} \right)^r$$

n=4, r=3

$$V_{eff}(\phi_c) = \lambda \frac{\phi_c^n}{n!} + i \int \frac{d^4k}{(2\pi)^4} \sum_{r=1}^{\infty} \frac{1}{2r} \left(\frac{\lambda \phi_c^{n-2}}{(n-2)!} \frac{1}{k^2 + i\epsilon} \right)^r = \lambda \frac{\phi_c^n}{n!} + \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \ln \left(1 + \frac{\lambda \phi_c^{n-2}}{(n-2)!\, k^2} \right)$$



$$V_{eff}(\phi_c) = V(\phi_c) + \frac{1}{2} \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \ln \left(1 + \frac{V''(\phi_c)}{k^2} \right) = V(\phi_c) + \frac{\Lambda^2}{32\pi^2} V'' + \frac{(V'')^2}{64\pi^2} \left[\ln \left(\frac{V''}{\Lambda^2} \right) - \frac{1}{2} \right]$$

$$V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

$$V_{eff}(\phi) = \frac{m^2}{2}\phi_c^2 + \frac{\lambda}{4}\phi_c^4 + \frac{(m^2+3\lambda\phi_c^2)^2}{64\pi^2}\ln(m^2+3\lambda\phi_c^2) + A(\Lambda)\phi_c^2 + B(\Lambda)\phi_c^4$$

Divergent factors, A, B, can be reabsorbed in the definition of the renormalized parameters, m^2_R, λ_R. The result will depend on the scale of the subtraction point, μ, (or in dimensional regularization on 't Hooft mass)

$$V_{eff}(\phi) = \frac{m^2}{2}\phi_c^2 + \frac{\lambda}{4}\phi_c^4 + \frac{(m^2+3\lambda\phi_c^2)^2}{64\pi^2}\ln\frac{m^2+3\lambda\phi_c^2}{\mu^2}$$

$m^2 = 0$

$$V(\phi) = \frac{\lambda}{4}\phi^4, \qquad \phi = 0 \;\; min$$

$$V_{eff}(\phi_c) = \frac{\lambda}{4}\phi_c^4 + \frac{9\lambda^2\phi_c^4}{64\pi^2}\ln\frac{\phi_c^2}{\mu^2},, \qquad \frac{dV_{eff}}{d\phi_c} = 0, \;\; \phi = 0 \;\; max, \; \lambda\ln\frac{\phi_c}{\mu} \sim -\frac{8}{9}\pi^2 \to \; min$$

Minimum occurs when $\lambda\ln\frac{\phi_c}{\mu} \sim \mathcal{O}(1)$ but higher loops contribute to V_{eff} as

$$\lambda\left(\lambda\ln\frac{\phi_c}{\mu}\right)^n$$

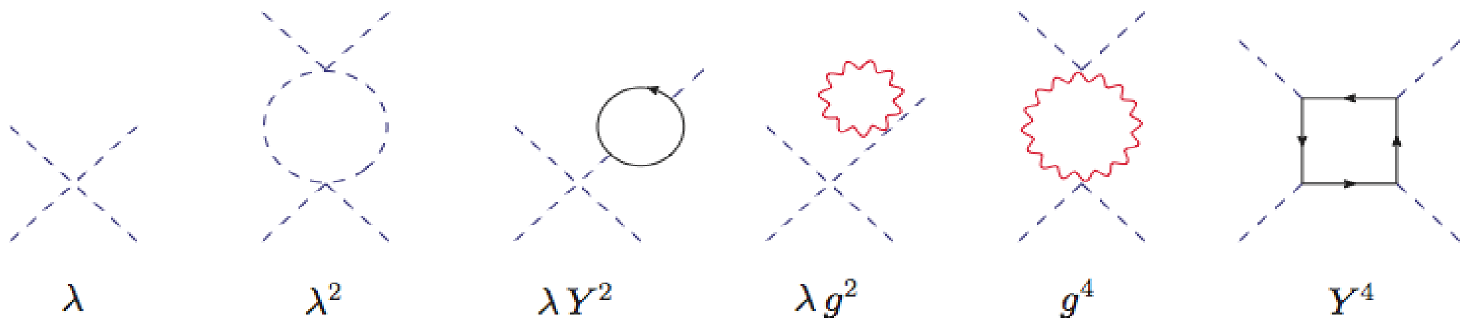
We have to resum the logs using the RGE.

RGE

$$V^{class}(\phi) = -\frac{1}{2}m^2\phi^2 + \lambda\phi^4 \longrightarrow V^{\text{eff}} \approx -\frac{1}{2}m^2(\mu)\phi^2(\mu) + \lambda(\mu)\phi^4(\mu) \sim \lambda(\mu)\phi^4(\mu)$$

$\phi \sim \mu \gg v$

λ runs



$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2}\left[+24\lambda^2 + \lambda\left(4N_cY_t - 9g^2 - 3g'^2\right) - 2N_cY_t^4 + \frac{9}{8}g^4 + \frac{3}{8}g'^4 + \frac{3}{4}g^2g'^2 + \ldots\right]$$

B<0 at the weak scale

$\lambda \sim 0 \to M_H \sim 0$

$$V_{eff}^{1l} = -\frac{1}{2}m^2(M)\phi^2 + \lambda(M)\phi^4 + \frac{B}{32\pi^2}\phi^4 \ln\frac{\phi^2}{M^2}$$

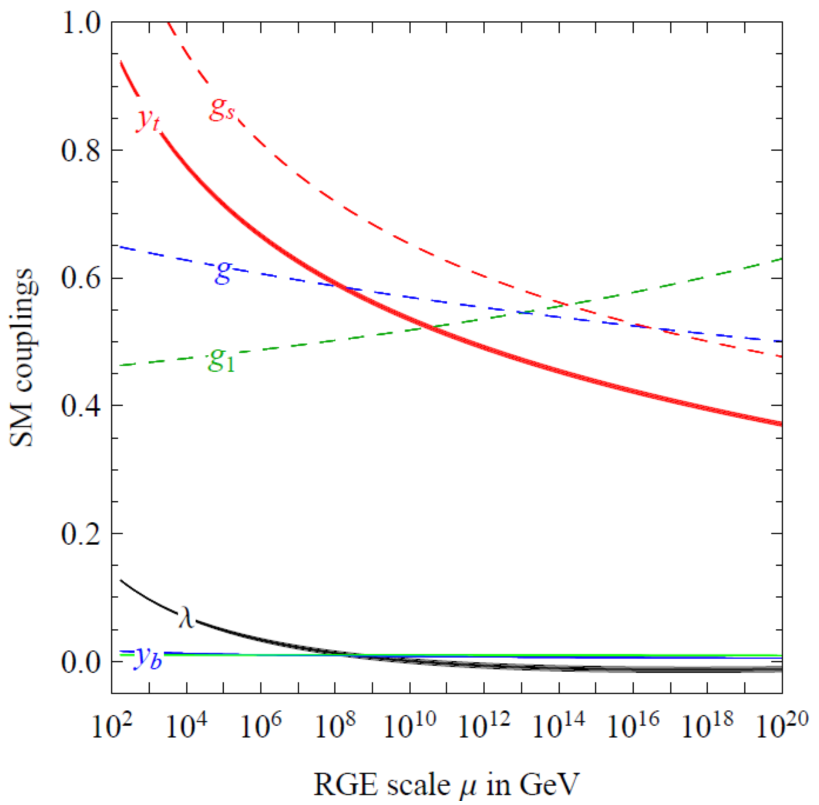
If B were constant at large values of Φ the potential would become negative and unbounded.
But B runs

Various possibilities:

- B is negative at the weak scale but not large enough to make B negative at a large scale such that the potential can become negative.
  SM vacuum is stable

- B is very negative at the weak scale and stays negative till the Planck scale
  SM vacuum is unstable
  N.P. should appear below the Planck scale to rescue our lives

- B is sufficient negative at the weak scale that the potential will become negative at a certain scale. However, increasing more the scale B turns positive. The potential develops a second deeper minimum at a large scale
  SM is unstable, but ….



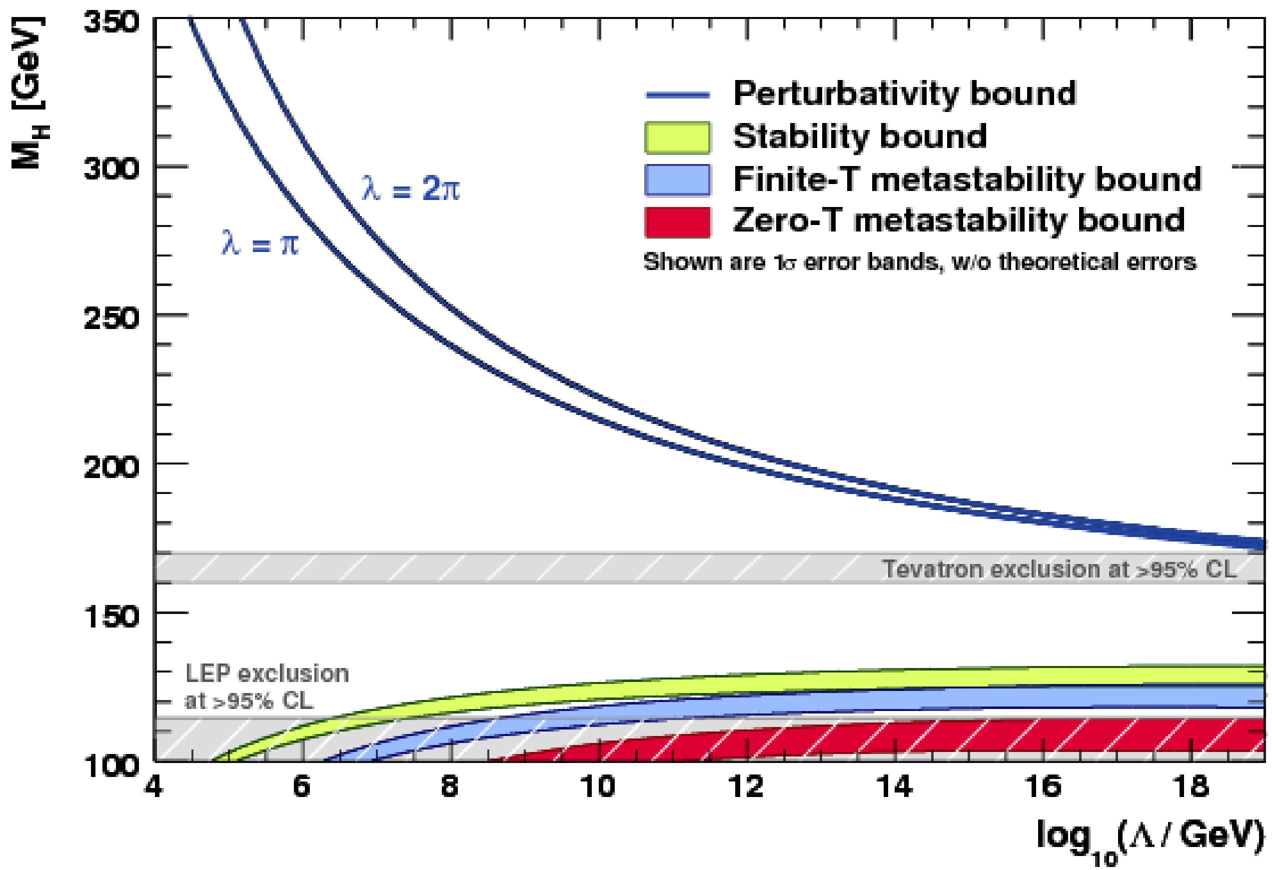
B ~ 0, M_H large

$$V^{1l}_{eff} \sim \lambda(M)\phi^4 + \frac{3\lambda^2}{4\pi^2}\phi^4 \ln\frac{\phi^2}{M^2} \Rightarrow V^{RGE}_{eff} = \frac{\lambda\phi^4}{1-\frac{3\lambda}{4\pi^2}\ln\frac{\phi^2}{M^2}}$$

Landau pole
At large Φ perturbativity is lost

Question: which values of the Higgs mass ensure vacuum stability and perburbativity up to the Planck scale ?

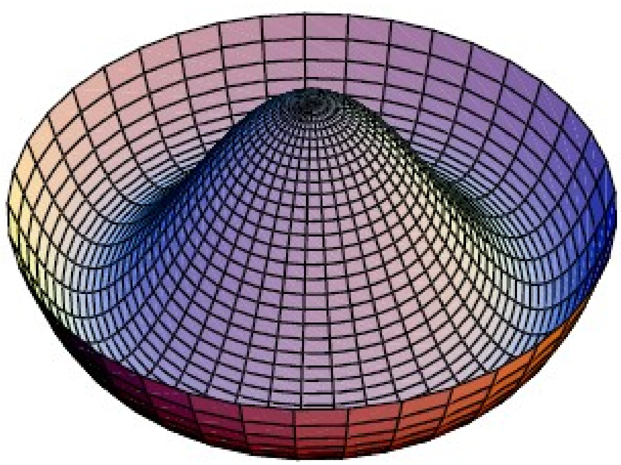
Answer: find when $\lambda = 0$ ($\sim V_{eff} = 0$) or when λ becomes large given the initial values for the couplings obtained form the experimental results ($M_t = 173.2 \rightarrow Y_t(M_t)$....)



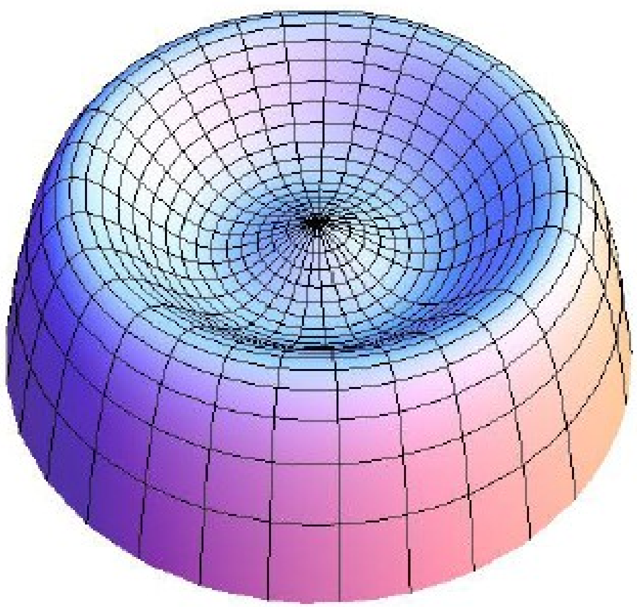
Ellis et al. 09

$M_H \sim$ 125-126 GeV: $-Y_t^4$ wins: $\lambda(M_t) \sim 0.14$ runs towards smaller values and can eventually become negative. If so the potential is either unbounded from below or can develop a second (deeper) minimun at large field values

Illustrative



$\rightarrow$

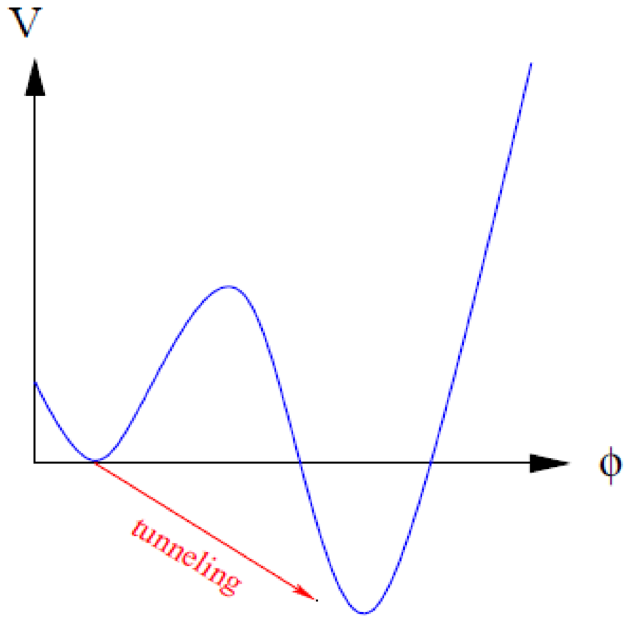


If your mexican hat turns out to be a dog bowl you have a problem...

from A. Strumia

The problem

There is a transition probability between the false and true vacua



It is really a problem ?

It is a problem that must be cured via the appearance of New Physics at a scale below that where the potential become unstable ONLY if the transition probability is smaller than the life of the universe.

Metastability condition: if λ becomes negative provided it remains small in absolute magnitude the SM vacuum is unstable but sufficiently long-lived compared to the age of the Universe

Vacuum stability at NNLO

- Two-loop effective potential (complete) — Ford, Jack, Jones 92,97; Martin (02)

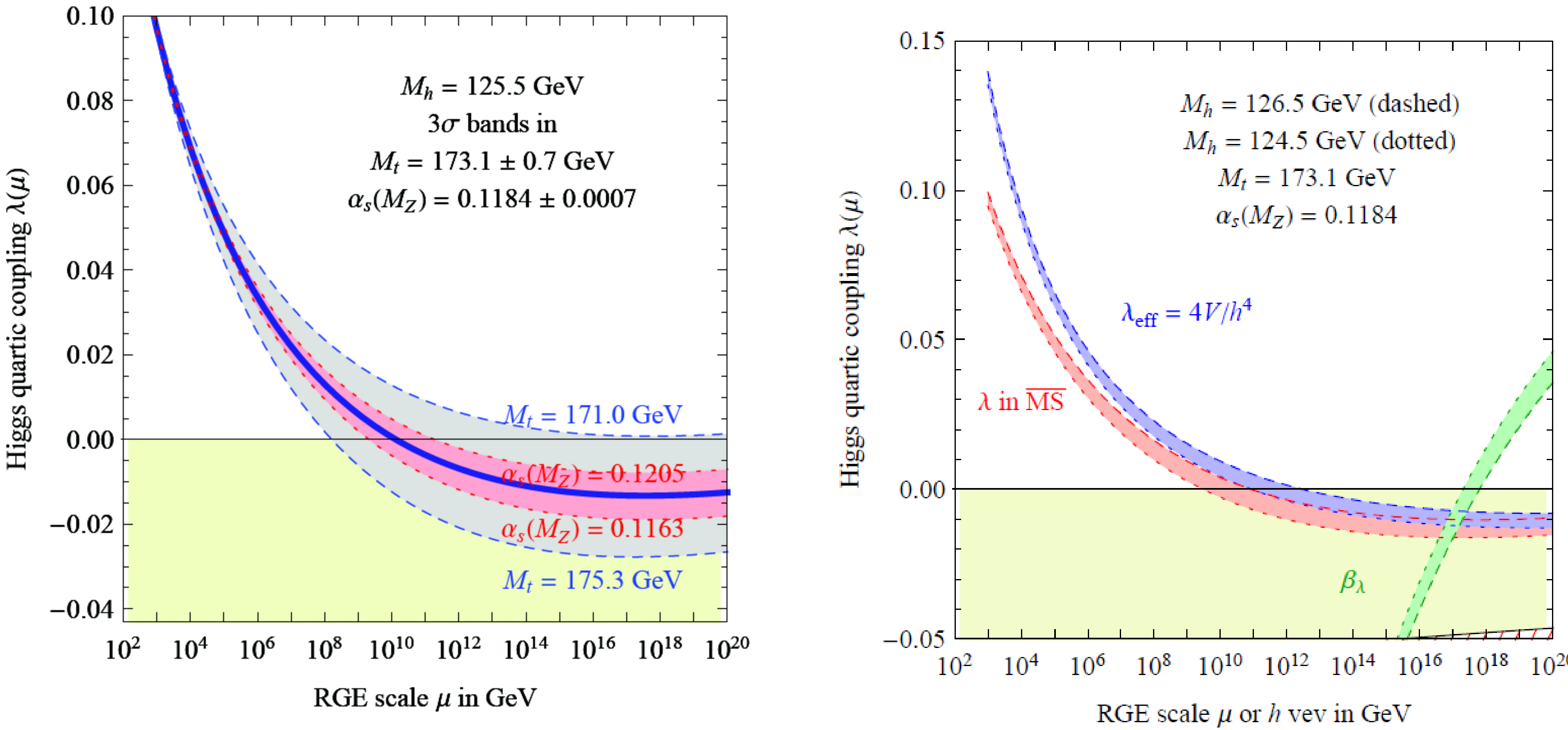
- Three-loop beta functions
  - gauge — Mihaila, Salomon, Steinhauser (12)
  - Yukawa, Higgs — Chetyrkin, Zoller (12, 13)

- Two-loop threshold corrections at the weak scale
  - λ: Yuk x QCD — Bezrukov et al. (12)
  - Yuk x QCD
  - SM gaugeless — Di Vita, Elias-Miro', Espinosa, Giudice Isidori, Strumia, G.D. (12)

Dominant theory uncertainty on the Higgs mass value that ensures vacuum stability still comes from the residual missing two-loop threshold corrections for λ at the weak scale

$$\frac{G_\mu}{\sqrt{2}} = \frac{1}{2v_0^2}(1+\Delta r_0)$$

$$\begin{aligned}\lambda(\mu) &= \frac{G_\mu}{\sqrt{2}} M_h^2 - \delta\lambda^{(1)} - \delta\lambda^{(2)} \\ \delta\lambda^{(2)} &= \frac{G_\mu}{\sqrt{2}} M_h^2 \left\{ \Delta r_0^{(2)} + \frac{1}{M_h^2}\left[\frac{T^{(2)}}{v_{\rm ren}} + {\rm Re}\,\Pi_{hh}^{(2)}(M_h^2)\right] \right. \\ &\left. - \frac{\Delta r_0^{(1)}}{M_h^2}\left[M_h^2 \Delta r_0^{(1)} + \frac{3\,T^{(1)}}{2\,v_{\rm ren}} + {\rm Re}\,\Pi_{hh}^{(1)}(M_h^2)\right] \right\}\end{aligned}$$

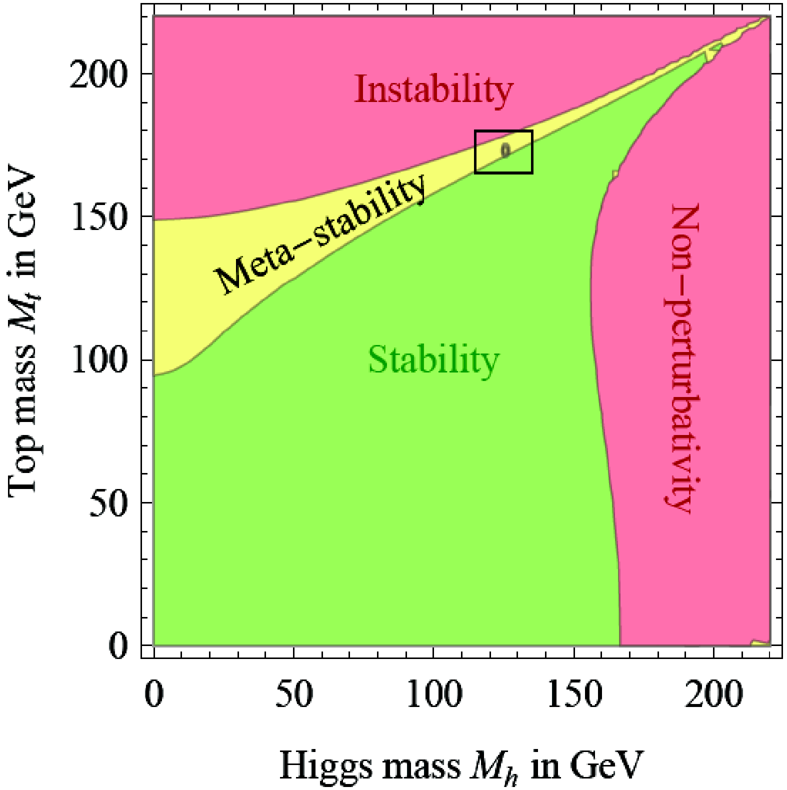


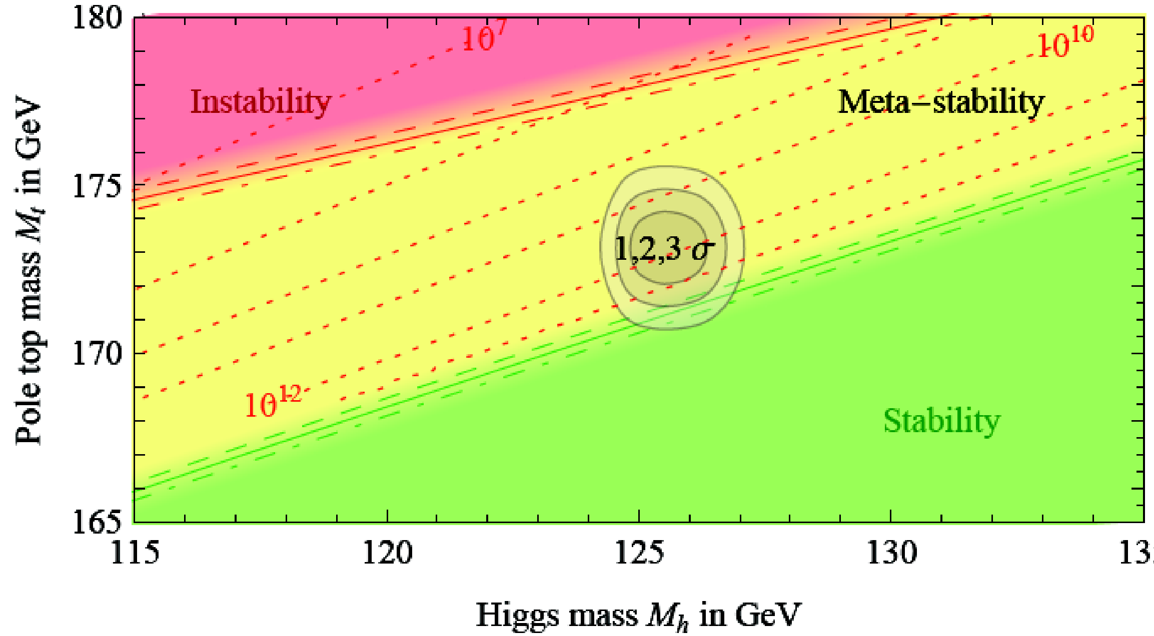
Full stability is lost at $\Lambda \sim 10^{11}$ GeV. but λ never becomes too negative

$$\lambda(M_{Pl.}) = -0.0144 + 0.0028\left(\frac{M_h}{\text{GeV}} - 125\right) \pm 0.0047_{M_t} \pm 0.0018_{\alpha_s(M_Z)} \pm 0.0028_{\rm th}$$

Both λ and β_λ are very close to zero around the Planck mass

Are they vanishing there?





$$M_h\,[\mathrm{GeV}] > 129.4 + 1.4\left(\frac{M_t\,[\mathrm{GeV}] - 173.1}{0.7}\right) - 0.5\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 1.0_{\mathrm{th}}$$

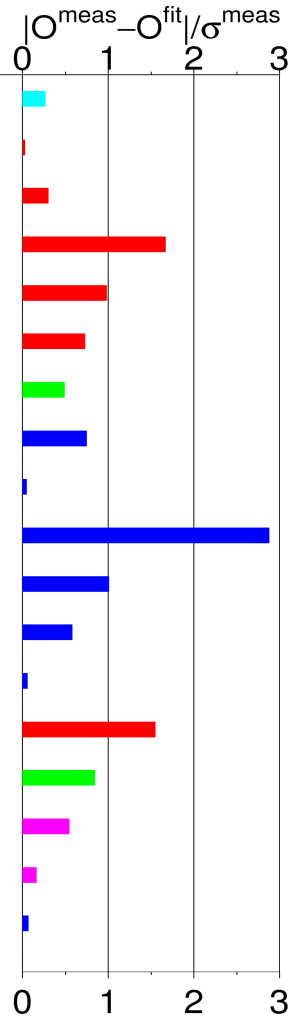
We live in a metastable universe close to the border with the stability region.
If the top pole mass would be ~ 171 GeV we were in the stable region.
Is the Tevatron number really the “pole” (what is?) mass?
Monte Carlo are used to reconstruct the top pole mass form its decays products that contain jets, missing energy and initial state radiation.

$M_t^{\overline{MS}}$ can be extracted form total production cross section and the corresponding pole mass is consistent with the standard value albeit with a larger error

SM Fit

One can make a global fit including "all" possible measurements and using the radiatively corrected predictions for the various observable. The latter, besides α, G_{μ}, M_Z and lepton masses depend upon: $m_t, \Delta\alpha_{had}^{(5)}, \alpha_s(M_Z), M_H$

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^0 [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26

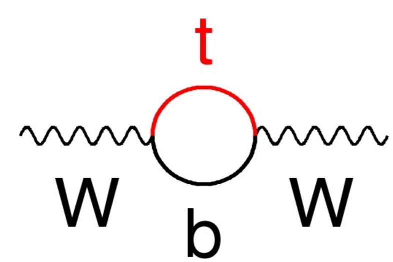


March 2012

Precision $5 - 1 \times 10^{-3}$ or better

Sensitivity to quantum effects

Predictions for M_t, M_W, M_H



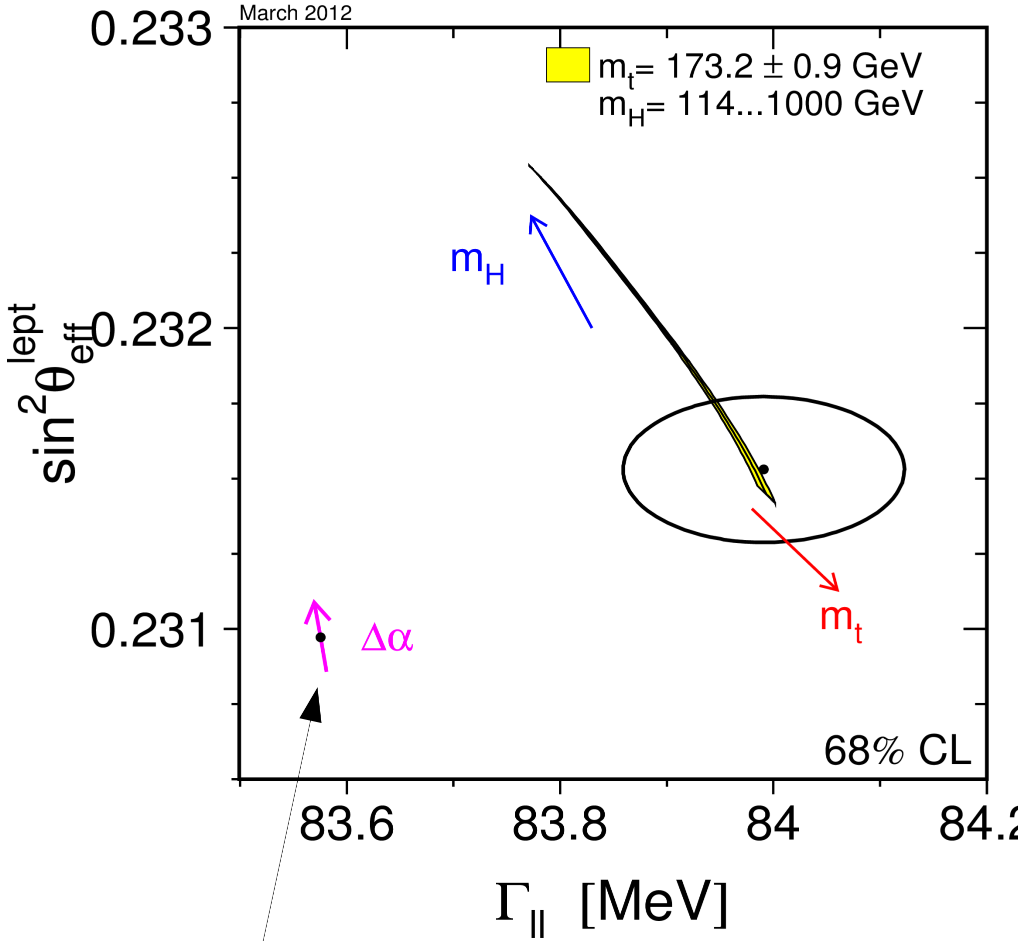
$\sim \alpha \frac{M_t^2}{M_W^2}$

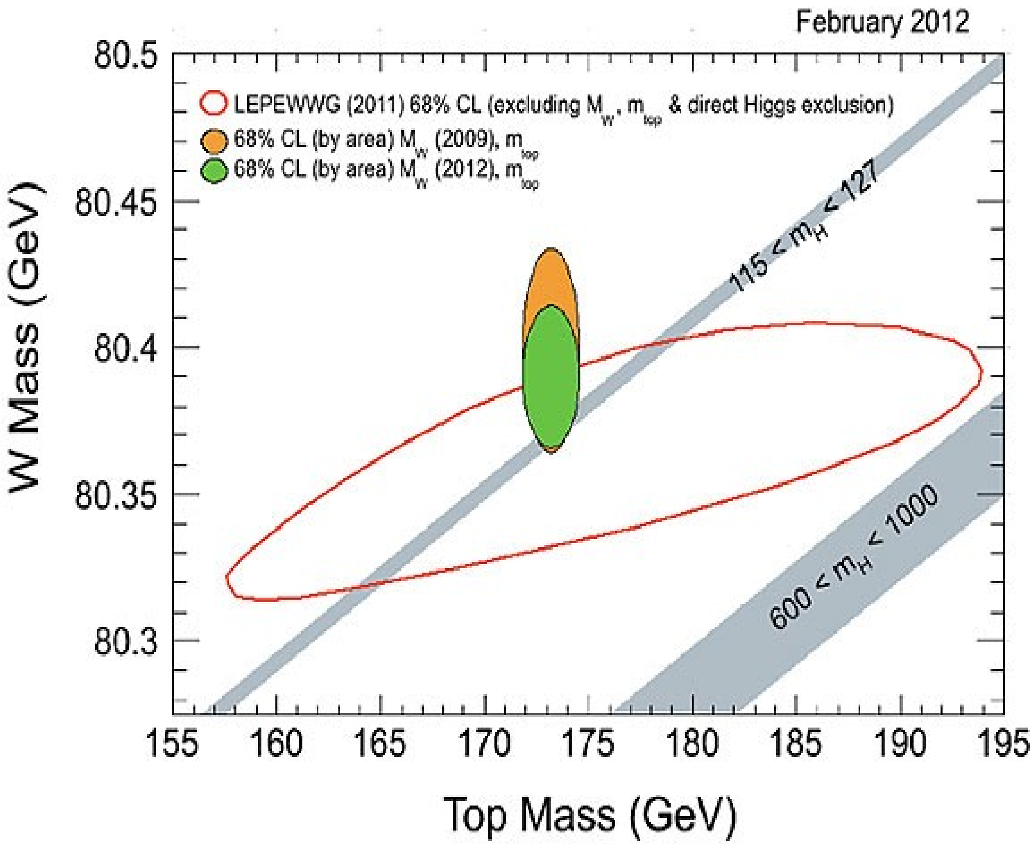
$\sim \alpha \log \frac{M_h^2}{M_Z^2}$

Very weak sensitivity to M_H, without the Value of M_t we cannot predict it.

Purely EW corrections established

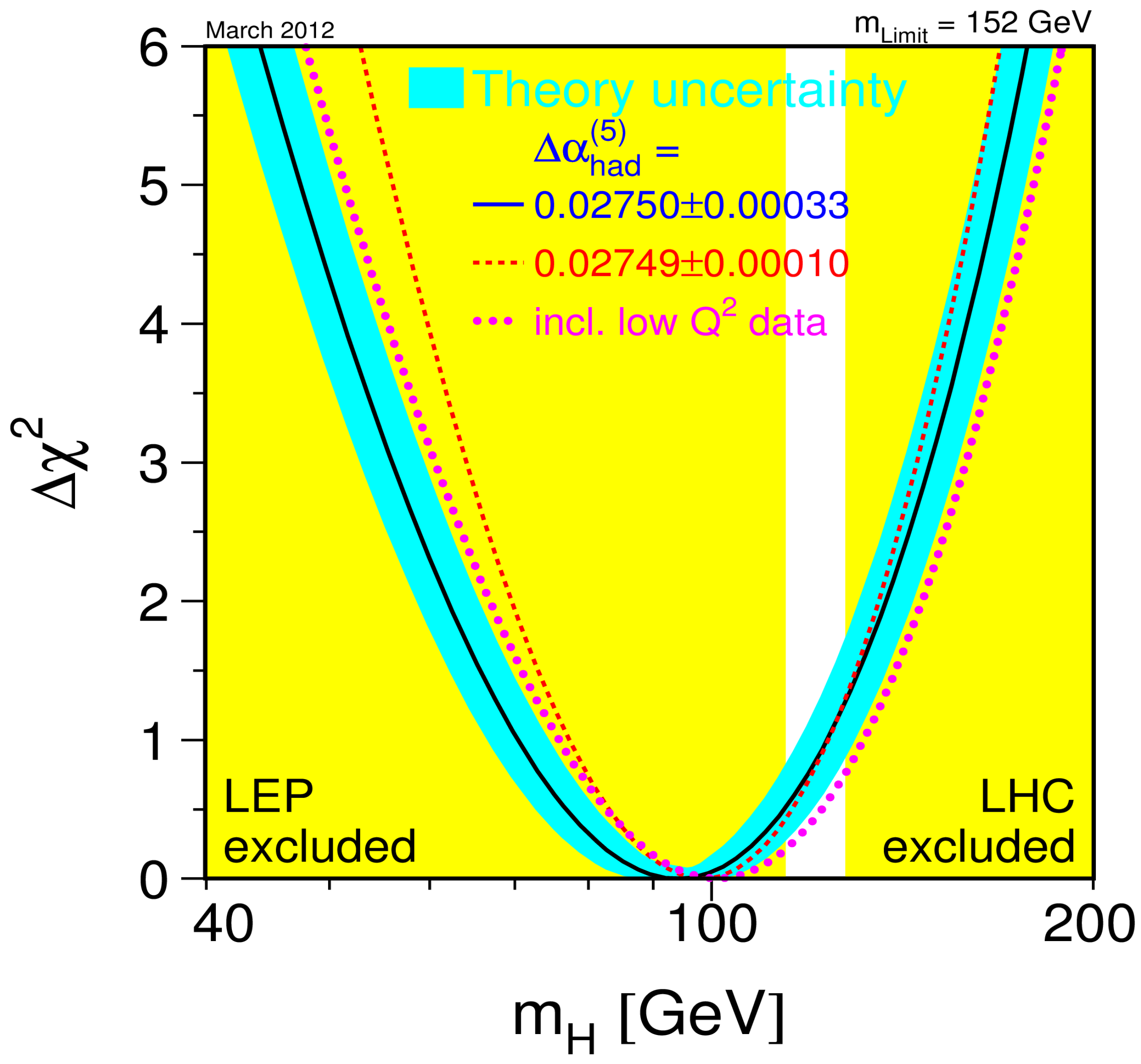
indirect vs. direct $M_t,\ M_W$ determination





only QED corrections

Global fit to M_H



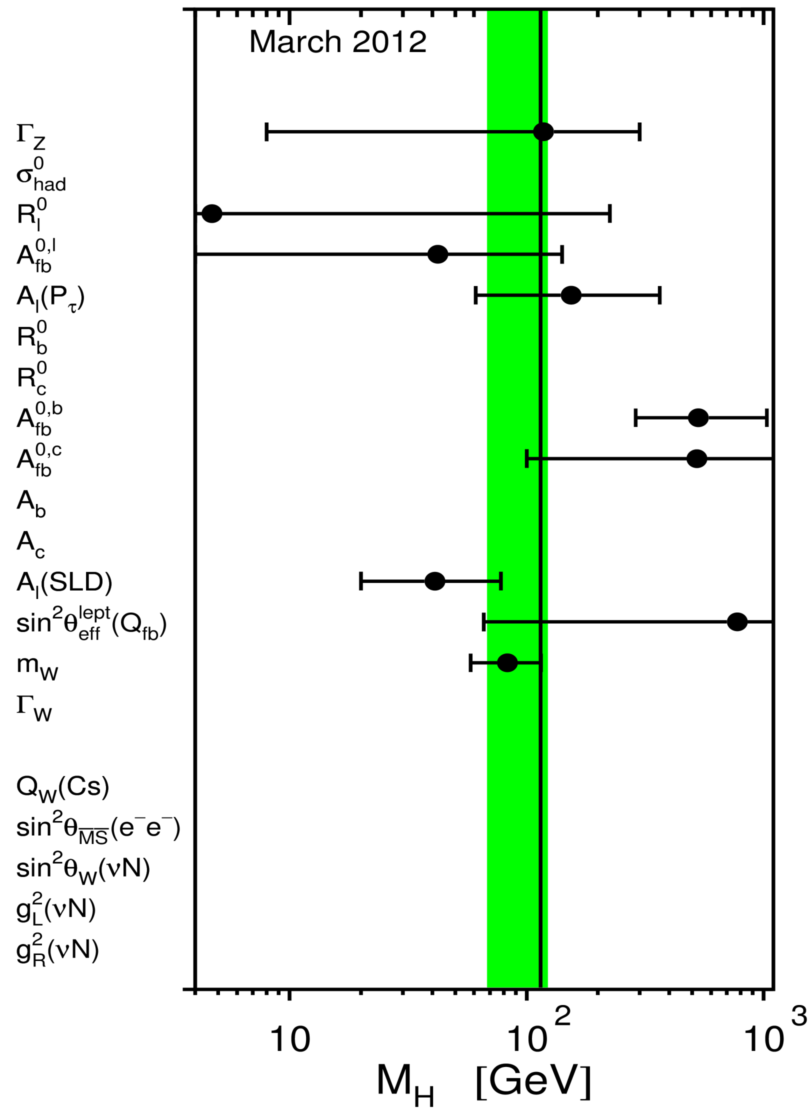
Alternative approach: I want to get a probability density function for M_H in the SM using all the available information, from precision physics and from direct searches (obviously excluding LHC results) to see if the particle that has been discovered at LHC has a mass compatible with the SM prediction (p.d.f $\neq$ 0)

Few observables are really sensitive to the Higgs

Simplified analysis using

$M_W, \sin^2\theta_{eff}^{lept.}$

asymmetries



- Parametrization:

$$\sin^2\theta_{eff}^{lept} = (\sin^2\theta_{eff}^{lept})^\circ + c_1\,A_1 + c_5\,A_1^2 + c_2\,A_2 - c_3\,A_3 + c_4\,A_4,$$

$$M_W = M_W^\circ - d_1\,A_1 - d_5\,A_1^2 - d_2\,A_2 + d_3\,A_3 - d_4\,A_4,$$

where

$$A_1 = \ln\frac{M_H}{100\ \mathrm{GeV}}, \qquad A_2 = \frac{\Delta\alpha_{had}^{(5)}}{0.02761} - 1,$$

$$A_3 = \left(\frac{m_t}{175\ \mathrm{GeV}}\right)^2 - 1, \quad A_4 = \frac{\alpha_s(M_Z)}{0.118} - 1,$$

c_i, d_i > 0 theoretical coefficients (depend upon the RS)

- Two quantities normally distributed

$$W = \sin^2\theta_{eff}^{lept} - (\sin^2\theta_{eff}^{lept})^\circ - c_2 A_2 + c_3 A_3 - c_4 A_4,$$

$$Y = M_W^\circ - M_W - d_2 A_2 + d_3 A_3 - d_4 A_4$$

- Likelihood of our indirect measurements Θ ={W, Y} is a two-dimensional correlated normal

$$f(\underline{\theta}\,|\,\ln(m_H)) \propto e^{-\chi^2/2}$$

$$\chi^2 = \underline{\Delta}^T \mathbf{V}^{-1} \underline{\Delta}, \quad V_{ij} = \sum_l \frac{\partial\Theta_i}{\partial X_l}\cdot\frac{\partial\Theta_j}{\partial X_l}\cdot\sigma^2(X_l), \quad \underline{\Delta} = \begin{pmatrix} w - c_1\ln(/100) - c_5\ln^2(/100) \\ y - d_1\ln(/100) - d_5\ln^2(/100)\end{pmatrix}$$

- Using Bayes' theorem the likelihood is turned into a p.d.f. of M_H via a uniform prior in ln (M_H)

$$f(M_H \mid ind.) = \frac{M_H^{-1}\, e^{-(\chi^2/2)}}{\int_0^{\infty} M_H^{-1}\, e^{-(\chi^2/2)}\, \mathrm{d}M_H}.$$

Likelihood

Bayes' theorem: $$f(\mu|x) = \frac{f(x|\mu) \cdot f(\mu)}{\int f(x|\mu) \cdot f(\mu)\, d\mu}$$

prior

How f(M_H | *ind*) is going to be modified by the results of the direct search experiments?

Ideal experiment (sharp kinematical limit, M_K) with outcome no candidate:

- f(M_H) must vanish below M_K (we did not observe)
- Above M_K the relative probabilities cannot change (experiment is not sensitive there)

$$f(M_H \mid dir.\,\&\, ind.) = \begin{cases} 0 & M_H < M_K \\ \dfrac{f(M_H \mid ind.)}{\int_{M_K}^{\infty} f(M_H \mid ind.)\, \mathrm{d}M_H} & M_H \geq M_K\,, \end{cases}$$

Just Bayes theorem:

$$f(M_H \mid dir. \,\&\, ind.) \propto f(dir. \mid M_H) \cdot f(M_H \mid ind.)$$

Likelihood for the ideal experiment:

$$f(dir. \mid M_H) = f(\text{“zero cand.”} \mid M_H) = \begin{cases} 0 & M_H < M_K \\ 1 & M_H \geq M_K \,. \end{cases}$$

Step function

Real life:
no sharp kinematical limit, step function should be replace by a smooth curve that goes to zero for low masses and to 1 for $M_H \to M_{Keff}$
Normalize the likelihood to the no signal case (pure background)
(Constant factors do not play any role in Bayes' theorem)

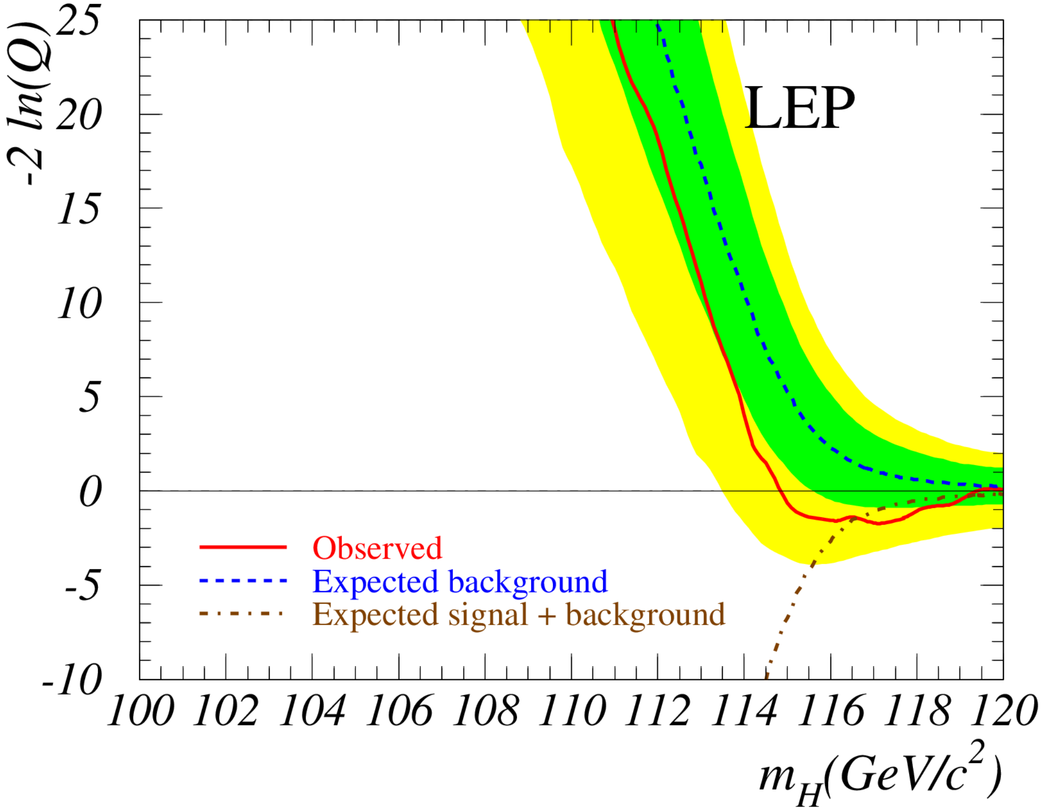
$$\mathcal{R}(M_H) = \frac{L(M_H)}{L(M_H \to \infty)}$$

Likelihood ratio
(should be providwed by the experiments)

$$f(M_H \mid dir. \,\&\, ind.) = \frac{\mathcal{R}(M_H)\, f(M_H \mid ind.)}{\int_0^\infty \mathcal{R}(M_H)\, f(M_H \mid ind.)\, \mathrm{d}M_H}$$

Role of $\mathcal{R}(M_H)$

$\mathcal{Q} = \mathcal{R}$



$\mathcal{R} = 1$

Region where the experiment is not sensitive;
shape of f(M_H | *ind*) does not change

$\mathcal{R} < 1$

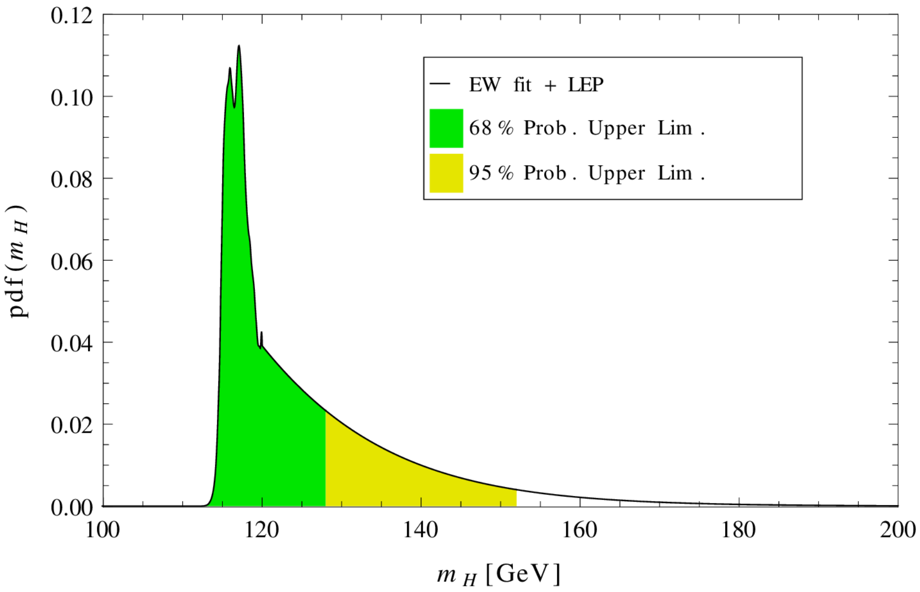
Probability is decreased,
p.d.f. is pushed above M_K
$\mathcal{R}(M_H) \to 0$ cuts the region

$\mathcal{R} > 1$

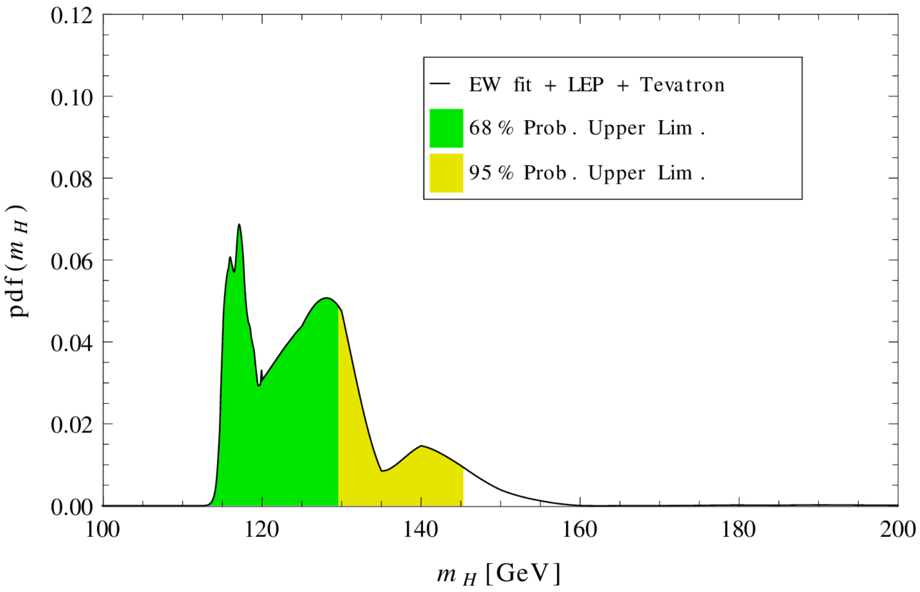
Probability is increased,
p.d.f is streched below M_K,
very large $\mathcal{R}(M_H)$ prompt a discovery

Combining direct and indirect information:

LEP

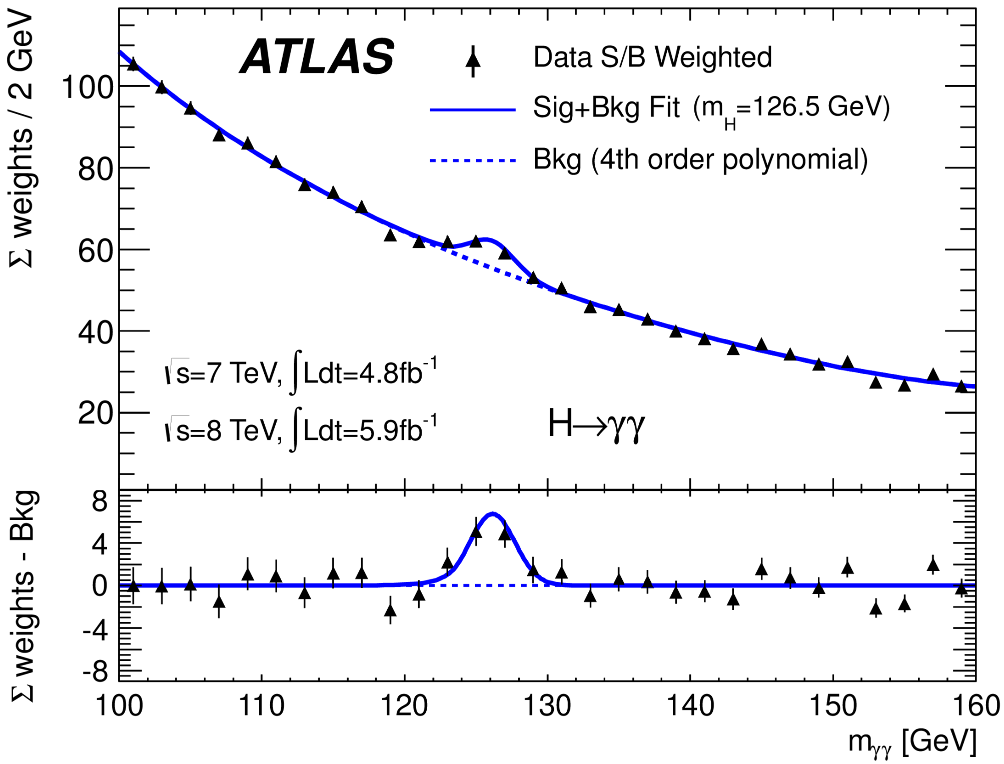


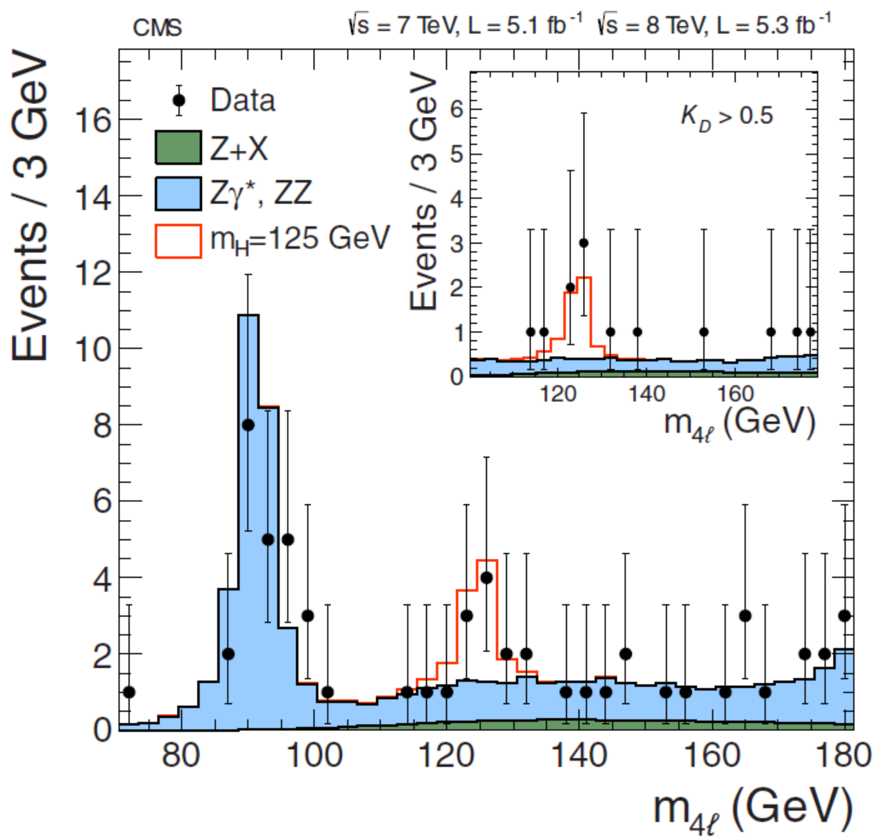
LEP+ TEVATRON



SM: M_H between 114 and 160 GeV with 95% probability below 145 GeV

The Higgs sector: LHC
4^{th} of July 2012





Clear evidence of a new particle
with properties compatible with those of the SM Higgs boson

It is where the SM predicts it should be

New Physics effects, where they could be?

New particles are going to contribute to the W,Z self-energies (process-independent contributions) and to vertices (for specific processes). With $M_{NP} >> M_Z$ where and what kind of "large" effects can we expect?

Self-energy: 3 types of NP contributions

★ $$\alpha(M_Z)T \equiv \frac{A_{WW}(0)}{M_W^2} - \frac{A_{ZZ}(0)}{M_Z^2} \propto \Pi_{11}(0) - \Pi_{33}(0)$$

isospin violation

Isosplitted particles: effects grow as the difference in the mass squared between partners of multiplet. Top contributes quadratically, Higgs logarithmically

★ $$\frac{\alpha(M_Z)}{4s^2c^2}S \equiv \frac{1}{M_Z^2}\left\{A_{ZZ}(M_Z^2) - A_{ZZ}(0) - \frac{c^2-s^2}{c\,s}\left[A_{\gamma Z}(M_Z^2) - A_{\gamma Z}(0)\right] - A_{\gamma\gamma}(M_Z^2)\right\}$$
$$\propto \Pi_{3Y}(M_Z^2) - \Pi_{3Y}(0)$$

No-effects that grow quadratically with the masses, but constant terms possible ($\neq 0$, $M_{NP} \to \infty$)

Top and Higgs logaritmically

$$\star \quad \frac{\alpha(M_Z)}{4s^2c^2}U \equiv \frac{A_{WW}(M_W^2)-A_{WW}(0)}{M_W^2} - c^2\frac{A_{ZZ}(M_Z^2)-A_{ZZ}(0)}{M_Z^2}$$

$$- \frac{1}{M_Z^2}\left[2cs\left(A_{\gamma Z}(M_Z^2)-A_{\gamma Z}(0)\right) - s^2 A_{\gamma\gamma}(M_Z^2)\right]$$

$$\propto \frac{1}{M_W^2}\left[\Pi_{11}(M_W^2)-\Pi_{11}(0)\right] - \frac{1}{M_Z^2}\left[\Pi_{33}(M_Z^2)-\Pi_{33}(0)\right]$$

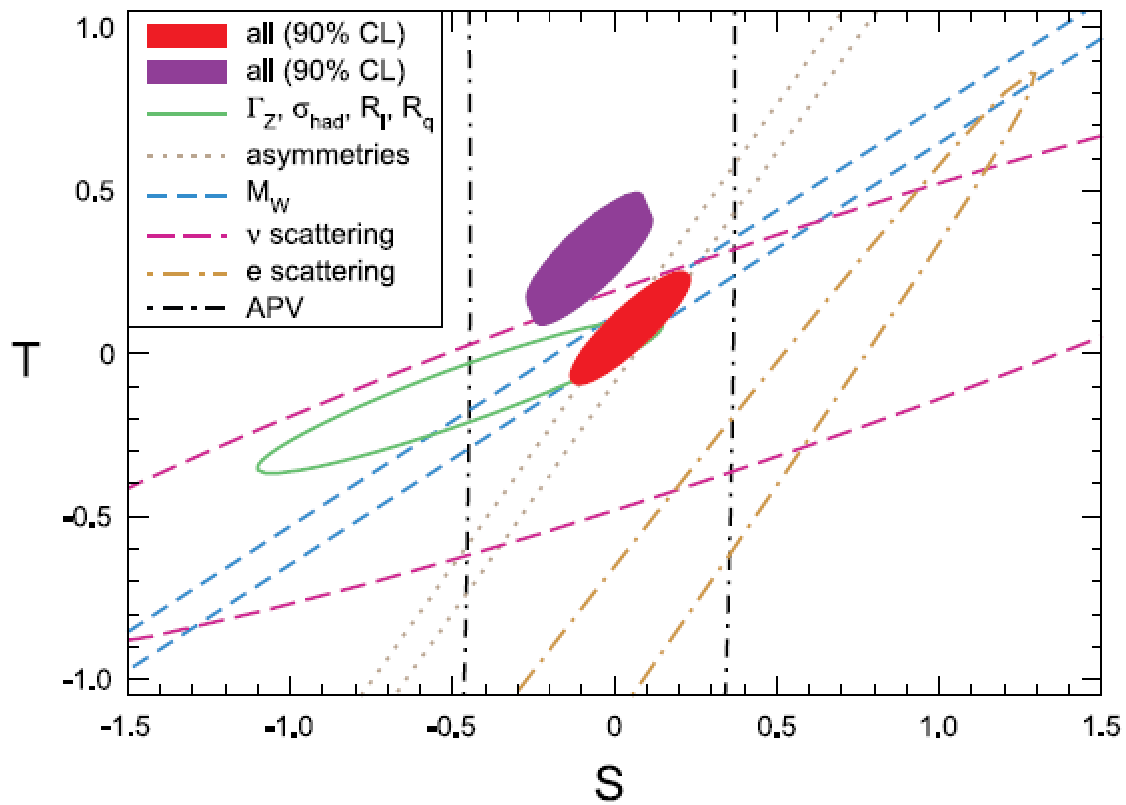
Isospin violation in the derivatives

U in many models is usually very small
U=0

Two parameters fit:

$115.5\,\mathrm{GeV} < M_H < 127\,\mathrm{GeV}$

$600\,\mathrm{GeV} < M_H < 1\,\mathrm{TeV}$



Before the discovery of the Higgs one could envisage a situation in which NP contributions were going to mask the effect of a heavy Higgs ("conspiracy").

Simple explanation:

$$\begin{aligned} \hat{\rho} &= \rho_0 + \delta\rho \, (\rho_0^{\rm SM} = 1, \delta\rho \leftrightarrow T) \\ \Delta\hat{r}_W &\leftrightarrow S \end{aligned}$$

$$\begin{aligned} \sin^2\theta_{eff}^{lept} &\sim \frac{1}{2}\left\{1 - \left[1 - \frac{4A^2}{M_Z^2\,\hat{\rho}\,(1-\Delta\hat{r}_W)}\right]^{1/2}\right\} \\ &\sim (\sin^2\theta_{eff}^{lept})^\circ + c_1 \ln\left(\frac{M_H}{M_H^\circ}\right) + c_2\left[\frac{(\Delta\alpha)_h}{(\Delta\alpha)_h^\circ} - 1\right] - c_3\left[\left(\frac{m_t}{m_t^\circ}\right)^2 - 1\right] + \ldots \end{aligned}$$

$c_i > 0$

To increase the fitted M_H:

$$\left\{ \begin{array}{l} \hat{\rho} > 1 \rightarrow \quad \left\{ \begin{array}{l} \rho_0 > 1 \\ \delta\rho > 0 \end{array} \right. \\ \Delta\hat{r}_W < 0 \end{array} \right.$$

NP better to be of the decoupling type

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