

ELEMENTARY PARTICLE PHYSICS
Current Topics in Particle Physics
Laurea Magistrale in Fisica,
curriculum Fisica Nucleare e Subnucleare
Lecture 11

Simonetta Gentile*

* Università Sapienza, Roma, Italia.

December 10, 2017

Simonetta Gentile

terzo piano Dipartimento di Fisica *Guglielmo Marconi*

Tel. 0649914405

e-mail: simonetta.gentile@roma1.infn.it


pagina web: <http://www.roma1.infn.it/people/gentile/simo.html>

Bibliography

♠ Bibliography

- K.A. Olive et al. (Particle Data Group), *The Review of Particle Physics*, Chin. Phys. C, 38, 090001 (2014)(PDG) update 2015, <http://pdg.lbl.gov/>
- F. Halzen and A. Martin, *Quarks and Leptons: An introductory course in Modern Particle Physics*, Wiley and Sons, USA(1984).

♠ Other basic bibliography:

- A.Das and T.Ferbel, *Introduction to Nuclear Particle Physics* World Scientific, Singapore, 2nd Edition(2009)(DF).
- D. Griffiths, *Introduction to Elementary Particles* Wiley-VCH, Weinheim, 2nd Edition(2008),(DG)
- B.Povh et al., *Particles and Nuclei* Springer Verlag, DE, 2nd Edition(2004).(BP)
- D.H. Perkins, *Introduction to High Energy Physics* Cambridge University Press, UK, 2nd Edition(2000).
- Y.Kirsh & Y. Ne'eman, *The Particle Hunters* 

♠ Particle Detectors bibliography:

- William R. Leo *Techniques for Nuclear and Particle Physics Experiments*,
Springer Verlag (1994)(LEO)
- C. Grupen, B. Shawartz *Particle Detectors*,
Cambridge University Press (2008)(CS)
- *The Particle Detector Brief Book*,(BB)
<http://physics.web.cern.ch/Physics/ParticleDetector/Briefbook/>

Specific bibliography is given in each lecture

Lecture Contents - 1 part

1. Introduction. Lep Legacy
2. Proton Structure
3. Hard interactions of quarks and gluons: Introduction to LHC Physics
4. Collider phenomenology
5. The machine LHC
6. Inelastic cross section pp
7. W and Z Physics at LHC
8. Top Physics: Inclusive and Differential cross section $t\bar{t}$ W, $t\bar{t}$ Z
9. Top Physics: quark top mass, single top production
10. Dark matter
 - Indirect searches
 - Direct searches

Specific Bibliography

♠ Bibliography of this Lecture

- O. Lahav and A.R. Liddle *The Cosmological parameters*
The Review of Particle Physics (2017) C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update. (PDG-Rev-Cosmo) [rpp2016-rev-cosmological](#)
- M. Drees and G. Gerbier *Dark Matter*
The Review of Particle Physics (2017) C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update. (PDG-Rev-dark) [rpp2016-rev-dark-matter](#)
Katherine Freese *Status of dark matter in the universe*, Proceedings of 14th Marcel Grossman Meeting, MG14, University of Rome "La Sapienza", Rome, July 2015, [arXiv:1701.01840](#)

Specific Bibliography

♠ Bibliography of this Lecture

- Bertone, Particle Dark Matter: *Observations, Models and Searches*, Cambridge
- Freese, Lisanti and Savage, Rev. Mod. Phys. 85 (2013) 1561
- Kurylov and Kamionkowski, Phys. Rev. D 69 (2004) 063503
- Classic papers:
 - Lewin and Smith, Astrop. Phys. 6 (1996) 87
 - Jungman, Kamionkowski and Griest, Phys. Rep. 267 (1996) 195
Gondolo, arXiv: hep-ph/9605290
 - Spergel, Phys. Rev. D 37 (1988) 1353
 - Drukier, Freese and Spergel, Phys. Rev. D 33 (1986) 12
Goodman and Witten, Phys. Rev. D 31 (1985) 12.

1 Dark matter. Direct Detection

Our dark halo



The Sun is moving through the Milky Way's dark matter halo. So we expect a “**WIMP wind**” coming towards us on the Earth!

Direct detection of WIMPs

Billions of WIMPs may be passing through the Earth each second, but they very rarely interact.

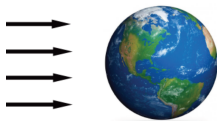


Figure : Schematic

Direct detection experiments operate underground and search for WIMPs via their scattering with atomic nuclei in the detector.

- WIMP **velocity** $\sim 10^{-3}c$
 \Rightarrow **non-relativistic**
- Expected **recoil energies**
 ~ 10 keV
- Expected < 1
event/kg/year

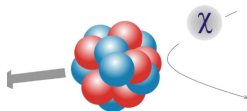
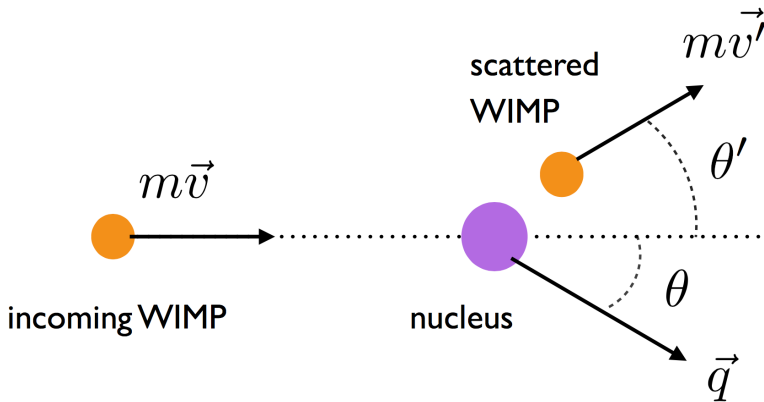


Figure : Schematic

WIMP-nucleus interaction

- WIMP-nucleus **elastic collision**

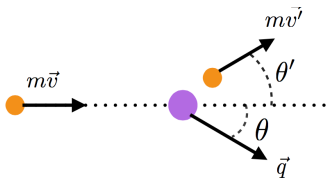


m : mass of WIMP

M : mass of nucleus

WIMP-nucleus interaction

- Energy-momentum conservation



$$\begin{aligned}\frac{1}{2}mv^2 &= \frac{1}{2}mv'^2 + \frac{q^2}{2M} \\ mv' \cos \theta' &= mv - q \cos \theta \\ mv' \sin \theta' &= q \sin \theta\end{aligned}$$

- Eliminate θ' and v'

$$q = 2\mu v \cos \theta \quad \mu = \frac{mM}{m + M} \text{ Reduced mass}$$

$$q = 2\mu v \cos \theta$$

- Magnitude of recoil momentum varies in the range:

$$0 \leq q \leq q_{\max} \equiv 2\mu v \quad E_{\max} \equiv \frac{2\mu^2 v^2}{M}$$

- Minimum WIMP speed required to produce a recoil energy E :

$$v_m = \frac{q}{2\mu} = \sqrt{\frac{ME}{2\mu^2}}$$

The expected event rate

- The **strongly simplified expected event rate** :

$$R \propto N\Phi_{\chi}\sigma$$

- N = number of target nuclei in the detector,
- Φ_{χ} = flux of WIMPs,
- σ = WIMP-nucleus cross section



$$\Phi_{\chi} = n \langle v \rangle$$

- n : WIMP number density $n = \frac{\rho}{m}$, ρ local DM mass density
- $\langle v \rangle$: average WIMP velocity with respect to the detector

The expected event rate

- Let's estimate the expected flux of WIMPs on Earth assuming:

$$\rho = 0.3 \text{ GeV}/\text{cm}^3, \quad \langle v \rangle = 220 \text{ km}/\text{s}, \quad m = 100 \text{ GeV}$$

$$\Phi_\chi = \frac{\rho}{m} \times \langle v \rangle = 6.6 \cdot 10^4 \text{ cm}^{-2} \text{ s}^{-1}$$

- The **flux is large enough** that even though WIMPs are weakly interacting, we would have a small but blue **potentially measurable rate** in direct detection experiments.

The expected event rate

- Strongly simplified **event rate per unit detector mass**, assuming¹ :

$$\sigma = 10^{-38} \text{cm}^2, M = 100 \text{GeV}$$
$$R = \frac{N}{NM} \Phi_{\chi} \sigma \sim 0.12 \text{ events/kg/yr}$$

- M = number of nuclei in the detector
- NM = detector mass
- More realistic and proper calculation of the event rate are necessary.

¹ $1 \text{kg} = 15.6 \cdot 10^{26} \text{ GeV}$, Xenon atom weight $2.1810 \cdot 10^{-22} \text{g}$.

The differential event rate

- To find the differential event rate, we need the following ingredients:
 - WIMP-nucleus scattering cross section which describes the interaction of a WIMP with the nucleus. **(particle physics input)**
 - Local DM density and velocity distribution in the detector reference frame **(astrophysics input)**.

The differential event rate

- The differential event rate per unit detector mass is determined from differential cross section²

$$\frac{d\sigma}{dE}$$

- Multiply the number N of the nuclei in the detector. Divide by the detector mass MN .
- Multiply the flux of WIMPs with velocity \vec{v} in the velocity space element d^3v :

$$nvf(\vec{v})d^3v, \quad n = \frac{\rho}{m}$$

- $f(\vec{v})$ WIMP velocity distribution in the *detector reference frame*.

² $R = \frac{N}{NM} \Phi_\chi \sigma$ and $\Phi_\chi = \frac{\rho}{m} \times \langle v \rangle$

The differential event rate

The differential event rate per unit detector mass ³:

$$\frac{dR}{dE} = \frac{\rho}{m} \frac{1}{M} \int_{v>v_m} d^3v \frac{d\sigma}{dE} v f(\vec{v})$$

Recap: Many **unknowns** enter in event rate:

- R , expected event rate
- E , recoil energy of detector nucleus
- $\frac{\rho}{m}$ DM number (ρ =DM density, m = DM mass)
- v , DM speed
- v_m , minimum DM speed required to produce a recoil energy E
- $f(\vec{v})$, DM velocity distribution in the *detector reference frame*.
- σ , DM-nucleus scattering cross section

³The divided by M

Particle physics input: cross section

The differential cross section

The WIMP-nucleus differential cross section encodes how DM interacts with ordinary matter:

- **WIMP-quark interaction:** strongly depends on the DM model, and is calculated in terms of an effective Lagrangian which describes the interaction of the WIMP candidate with quarks and gluons.
- **WIMP-nucleon cross section:** calculated using hadronic matrix elements which describe the nucleon content in quarks and gluons \implies *subject to large uncertainties*.
- **Total WIMP-nucleus cross section:** calculated by adding the spin and scalar components of nucleons.

The differential cross section

- The standard theoretical framework assumed for **direct detection experiments** is inspired by models of **supersymmetric DM**.
- It assumes interactions mediated by heavy particles (i.e. *contact interactions*), and includes only the **leading-order interactions in the non-relativistic limit**.

The differential cross section

- Differential **WIMP-nucleus scattering cross section** for standard contact interactions:

$$\frac{d\sigma}{dE} = \frac{\sigma_0}{E_{\max}} F^2(E) = \frac{M}{2\mu^2 v^2} \sigma_0 F^2(E)$$

- σ_0 : total scattering section with point-like nucleus.
- $F^2(E)$: **nuclear form factor** (normalized to 1); takes into account the finite size of the nucleus and encode the dependence on momentum transfer ($q = \sqrt{2ME}$).

The differential cross section

- When momentum transfer is small, the DM doesn't probe the size of the nucleus and coherently scatters off the entire nucleus:

$$F^2(E) \rightarrow 1$$

- As momentum transfer increases, the DM becomes sensitive to the spatial structure of the nucleus:

$$F^2(E) < 1$$

- The effect is strong for heavy target nuclei. \implies **Event rate suppression.**

The differential cross section

Two relevant contributions to the cross section:

- **Spin-independent (SI)**: coherent interaction of the WIMP with all nucleons; no dependence on nuclear spin.
- **Spin-dependent (SD)**: the WIMP interacts with the spin of the nucleus.

Considering both spin-independent and spin-dependent WIMP-nucleus interactions in the non-relativistic limit:

$$\frac{d\sigma}{dE} = \frac{M}{2\mu^2 v^2} [\sigma_0^{SI} F_{SI}^2 + \sigma_0^{SD} F_{SD}^2]$$

Spin-independent interaction

- The SI contribution to the cross section can arise from scalar couplings of DM to quarks, which occurs through the operator $(\bar{\chi}\chi)(\bar{q}q)$
- For a WIMP with scalar interactions, the SI **WIMP-nucleus** cross section is:

$$\sigma_0^{SI} = \frac{4\mu^2}{\pi} [Z f_p + (A - Z) f_n]^2$$

- where f_p, f_n are **couplings of the WIMP with point-like protons and neutrons**, respectively. Z and $A-Z$ are the number of protons and neutrons in the nucleus.

Spin-independent interaction

- To compare data from different direct detection experiments which have different target nuclei, it is convenient to consider the **WIMP-proton cross section**:

$$\sigma_{SI} = \frac{4\mu_p^2}{\pi} (f_p)^2$$

μ_p : WIMP proton reduced mass, $\frac{mM}{m+M}$ Reduced mass; M: mass of nucleus.

- Then we have:

$$\sigma_0^{SI} = \sigma_{SI} \left[Z + (A - Z) \left(\frac{f_n}{f_p} \right) \right]^2 \left(\frac{\mu}{\mu_p} \right)^2$$

Spin-independent interaction

- For most WIMP candidates one can assume $f_n \simeq f_p$ and the **SI scattering cross section of WIMPs with protons and neutrons** are roughly **comparable**. For identical couplings, $f_n = f_p$, and

$$\sigma_0^{SI} = \sigma_{SI} A^2 \left(\frac{\mu}{\mu_p} \right)^2$$

- The SI **cross section increases** rapidly with nuclear mass **A^2** . \implies **heavy target are favored**.
- The A^2 dependence comes from the fact that the contributions to the total SI cross section of a nucleus is a coherent sum over the individual protons and neutrons.

Spin-independent interaction

- The **SI form factor** is essentially a Fourier transform of the mass distribution of nucleus: the mass distribution of the nucleus:

$$F(\mathbf{q}) = \int d^3x \rho(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}}$$

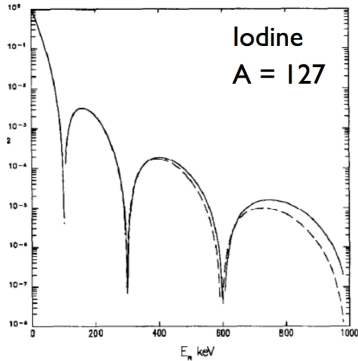
- An accurate approximation⁴

$$F(q) = 3e^{-q^2 s^2/2} \frac{\sin(qr) - qr \cos(qr)}{(qr)^3}$$

$s = 1$ fm skin thickness, a solid sphere, approximating spin-independent interaction with the whole nucleus (single outer shell nucleon for spin dependent) effective nuclear radius $r \approx A^{1/3}$

⁴J.D.Lewin, P.F. Smith *For a complete review: Review of mathematics, numerical factors, and corrections for dark matter experiments based on elastic nuclear recoil* *Astroparticle Physics* 6 (1996), 87 [Form factor, protected](#)

Spin-independent interaction



- Uncertainties in the determination of the nuclear form factor will affect the theoretical prediction for event rate.

Figure : Nuclear form factor

Spin-dependent interaction

- SD scattering is due to the interaction of a WIMP with the spin of the nucleus. It can arise from axial vector couplings of DM to quarks⁵ It happens only in detector nuclei with an **odd number of protons and/or neutrons**.
- Unlike the SI case, the two SD couplings may be quite different, and we cannot simplify the cross section as we did for the SI case.
- **No A^2 enhancement** of SD cross section as SI case.
- The **SD** is **not as significant as SI scattering** in direct detection experiments.
- The SD form factor depends on the spin structure of a nucleus



I will consider **only SI interactions** in my lecture.

⁵which occurs through the operator $(\bar{\chi}\gamma_{\mu}\gamma_5\chi)(\bar{q}\gamma_{\mu}\gamma_5q)$.

The differential event rate

$$\begin{aligned}\frac{dR}{dE} &= \frac{\rho}{m} \frac{1}{M} \int_{v>v_m} d^3v \frac{d\sigma}{dE} v f(\vec{v}) \\ \frac{d\sigma}{dE} &= \frac{M}{2\mu^2 v^2} \sigma_0 F^2(E) \\ \frac{dR}{dE} &= \rho \underbrace{\frac{\sigma_0 F^2(E)}{2m\mu^2}}_{\text{particle physics}} \underbrace{\int_{v>v_m} d^3v \frac{\vec{v}}{v}}_{\text{astrophysics}}\end{aligned}$$

- astrophysics
- particle physics

Halo integral

- Let's define **halo integral** as:

$$\eta(v_m) \equiv \int_{v > v_m} d^3v \frac{\vec{v}}{v}$$

- The event rate can be written as ⁶

$$\frac{dR}{dE} = \frac{\rho \sigma_0 F^2(E)}{2m\mu^2} \underbrace{\eta(v_m)}_{\text{astrophysics}}$$

$$\text{For SI case : } \frac{dR}{dE} = \frac{\rho A^2 \sigma_{SI} F^2(E)}{2m\mu_p^2} \underbrace{\eta(v_m)}_{\text{astrophysics}}$$

⁶ $\sigma_0^{SI} = \sigma_{SI} A^2 \left(\frac{\mu}{\mu_p}\right)^2$

Astrophysics input input: DM distribution

- The dark matter halo in the local neighborhood is most likely dominated by a smooth component with an **average density**:

$$\rho_\chi \approx 0.3 \text{ GeV cm}^{-3}$$

- The simplest model for this smooth component is often taken to be the SHM, Standard Halo Model an isothermal sphere with an **isotropic, Maxwellian velocity distribution** and rms velocity dispersion σ_v .⁷

⁷Freese, Lisanti and Savage, *Colloquium: Annual modulation of dark matter* Rev. Mod. Phys. 85 (2013) 1561 ??

Standard Halo Model

$$f_{\text{gal}}(\vec{v}) = \begin{cases} \frac{1}{N_{\text{esc}}} \left(\frac{3}{2\pi\sigma_v^2} \right)^{3/2}, [e^{-3\vec{v}^2/2\sigma_v^2} - e^{-3v_{\text{esc}}^2/2\sigma_v^2}] & \text{for } |\vec{v}| < v_{\text{esc}} \\ 0, & \text{otherwise} \end{cases}$$

$$N_{\text{esc}} = \text{erf}(z) - \frac{2}{\sqrt{\pi}} z e^{-z^2}$$

$z \equiv v_{\text{esc}}/v_0$ is a normalization factor

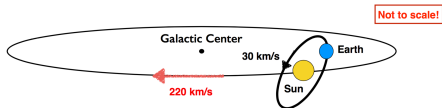
$$v_0 = \sqrt{2/3}\sigma_v$$

- $v_0 \approx 235$ km/s is the most probable speed.
- The Maxwellian distribution is **truncated at the escape velocity** v_{esc} to account for the fact that **WIMPs with sufficiently high velocities escape** the Galaxy's potential well and, thus, the high-velocity tail of the distribution is depleted.

**The Dark Matter velocity distribution
depends on the halo model.**

DM velocity distribution

To compute the event rate, we need the WIMP velocity, \vec{v} , distribution in the **detector reference frame**. Need to transform from the galactic frame to the detector frame.

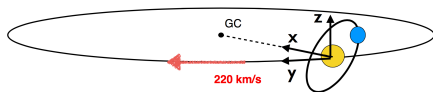


$$f_{\text{det}}(\vec{v}, t) = f_{\text{det}}(\vec{v} + \vec{v}_e(t)) = f_{\text{gal}}(\vec{v} + \vec{v}_s(t) + \vec{v}_e(t))$$

- $\vec{v}_e(t)$, Earth's velocity wrt the Sun
- $\vec{v}_s(t)$, Sun's velocity wrt the Galaxy

Sun's orbit around the Galaxy

Galactic coordinate system:



Origin at the position of the Sun.

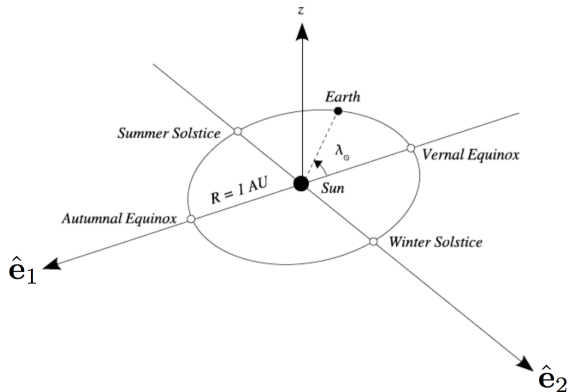
x-axis points towards the Galactic Center.

y-axis points towards the direction of the Galactic rotation.

z-axis points to the North Galactic pole

$$\vec{v}_s(t) = (0, 220, 0) + (10, 13, 7)\text{km/s}$$

Earth's orbit around the Sun



Sun's ecliptic longitude $\lambda(t)$ changes from 0 to 360 degrees as the Earth orbits the Sun.

Earth's orbit around the Sun

- In the approximation of circular orbit, we have

$$\lambda(t) = \frac{2\pi}{1yr}(t - 0.218)$$

- t is the time during the year running from 0 to 1 year, with $t=0$ on January first
- 0.218 is the fraction of year before the spring equinox (March 21).
- The position vector of the Earth is:

$$\vec{r}_e(t) = -[\cos \lambda(t)\hat{e}_1 + \sin \lambda(t)\hat{e}_2] \text{AU}$$

- 1 AU is the average distance between the Earth and the Sun.

Earth's orbit around the Sun

- **Earth's velocity** around the Sun is **time dependent**:

$$\vec{v}_e(t) = v_e [\sin \lambda(t) \hat{e}_1 - \cos \lambda(t) \hat{e}_2]$$

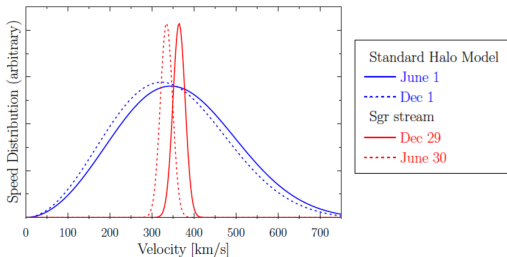
- $v_e = 29.8$ km/s
- Orthogonal vectors spanning the plane of the plane of the Earth's orbit:

$$\hat{e}_1 = (-0.0670, 0.4927, -0.8676)$$

$$\hat{e}_2 = (-0.9931, 0.1170, -0.01032)$$

DM velocity distribution

- Comparison of **two different model in the lab frame(on the earth)** after accounting the motion of Solar System relative to Galactic center:
 - SHM, *Standard Halo Model* an isothermal sphere with an isotropic, Maxwellian velocity distribution and rms velocity dispersion σ_v .
 - *Tidal stream*, the material in the stream has not had the time to spatially mix, the stream has a small velocity dispersion in comparison: $f_{\text{str}}(\vec{v}) = \delta^3(\vec{v})$



Freese et al. 1209.3339

The differential event rate

The **recoil spectrum falls off exponentially in the galactic rest frame** for the SHM (neglecting form factors), due to the exponential drop-off with velocity ⁸ Even when form factors and the motion of the Earth through the halo are accounted for, the **spectrum is still approximately exponential in the laboratory frame**:

$$\frac{dR}{dE} \sim e^{-E/E_0} \quad E_0 \sim \mathcal{O}(10\text{keV}) \text{ For typical WIMP and target mass}$$

• **Large contribution to the rate is at low energies.**

8

$$f_{\text{gal}}(\vec{v}) = \begin{cases} \frac{1}{N_{\text{esc}}} \left(\frac{3}{2\pi\sigma_v^2}\right)^{3/2}, [e^{-3\vec{v}^2/2\sigma_v^2} - e^{-3v_{\text{esc}}^2/2\sigma_v^2}] & \text{for } |\vec{v}| < v_{\text{esc}} \\ 0, & \text{otherwise} \end{cases}$$

$$N_{\text{esc}} = \text{erf}(z) - \frac{2}{\sqrt{\pi}} z e^{-z^2}$$

$z \equiv v_{\text{esc}}/v_0$ is a normalization factor

$$v_0 = \sqrt{2/3}\sigma_v$$

The differential event rate

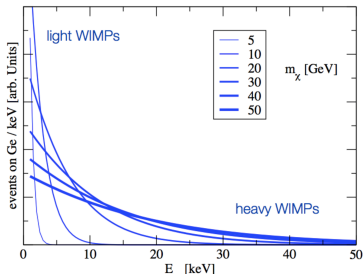
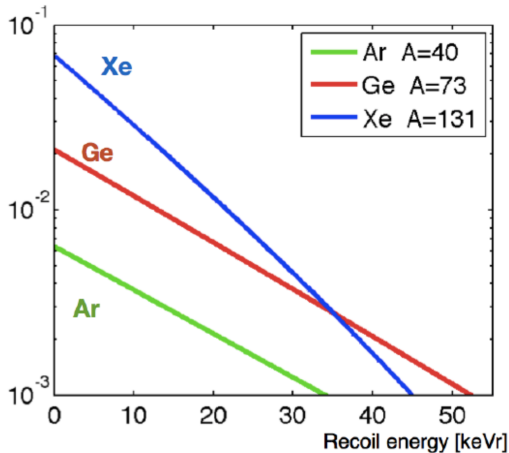


Figure : Event rate

- **Spectrum** is featureless: **exponentially falling off**.
- Spectrum is shifted to **low energies for low WIMP masses**. To detect light WIMPs, need low energy threshold (and light target nuclei).
- Expect **different rates for different targets**.
- Rate **depends on A^2** \implies heavier targets are favored in direct detection experiments.

The differential event rate

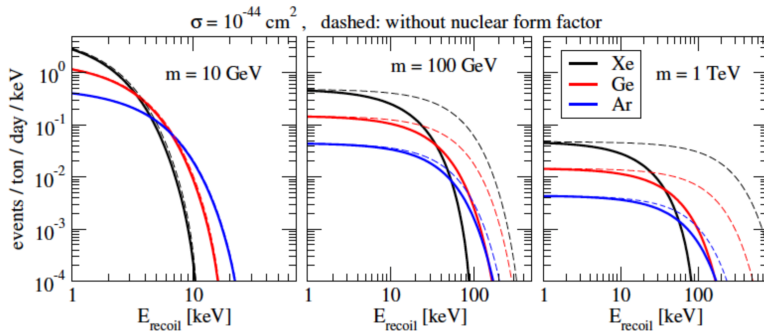


$$m = 100 \text{ GeV}$$

$$\sigma_{\text{SI}} = 10^{-44} \text{ cm}^2$$

$$\rho = 0.3 \text{ GeV/cm}^3$$

The differential event rate



Nuclear form factor is less important for light WIMPs

- We discussed the different ingredients which enter in the expected event rate in direct detection experiments.
 - **particle physics input**: cross section. SI cross section scales as A^2
 - **astrophysics input**: local DM density and velocity distribution. Maxwell distribution in the Standard Halo Model.
- **Recoil spectrum exponentially falling off** and featureless.
- **Direct detection experiment need to achieve low energy threshold.**