ELEMENTARY PARTICLE PHYSICS
Current Topics in Particle Physics
Laurea Magistrale in Fisica,
curriculum Fisica Nucleare e Subnucleare
Lecture 2

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Preliminaries

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Bibliography

♠ Bibliography

- K.A. Olive et al. (Particle Data Group), *The Review of Particle Physics*, Chin. Phys. C, 38, 090001 (2014)(PDG) update 2015, http://pdg.lbl.gov/
- F. Halzen and A. Martin, *Quarks and Leptons: An introductory course in Modern Particle Physics* , Wiley and Sons, USA(1984).

♠ Other basic bibliography:

- A.Das and T.Ferbel, *Introduction to Nuclear Particle Physics* World Scientific,Singapore, 2[nd] Edition(2009)(DF).
- D. Griffiths, *Introduction to Elementary Particles* Wiley-VCH,Weinheim, 2^{nd} Edition(2008),(DG)
- B.Povh *et al.*, *Particles and Nuclei* Springer Verlag, DE, 2[nd] Edition(2004).(BP)
- D.H. Perkins,*Introduction to High Energy Physics* Cambridge University Press, UK, 2[nd] Edition(2000).
- Y.Kirsh & Y. Ne'eman, *The Particle Hunters*

♠ Particle Detectors bibliography:

- William R. Leo *Techniques for Nuclear and Particle Physics Experiments*, Springer Verlag (1994)(LEO)
- C. Grupen, B. Shawartz *Particle Detectors*, Cambridge University Press (2008)(CS)
- *The Particle Detector Brief Book*,(BB) http://physics.web.cern.ch/Physics/ParticleDetector/Briefbook/

Specific bibliography is given in each lecture

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3. Hard interactions of quarks and gluons: Introduction to LHC Physics
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10. Dark matter
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    - Direct searches

Specific Bibliography

♠ Bibliography of this Lecture

- F. Halzen and A. Martin, *Quarks and Leptons: An introductory course in Modern Particle Physics* , Wiley and Sons, USA(1984)(HM).
- K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014) and 2015 update. (PDG15) Updated September 2015 by B. Foster, A.D. Martin, R.S. Thorne) and M.G. Vincter*Structure Functions* http://pdg.lbl.gov/2015/reviews/rpp2015-rev-structure-functions.pdf

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2 Electrodinamics of Spin-1/2 particles

3 The structure of hadrons

4 Partons
- Quarks
- Gluons

5 Structure function in QCD

The cross section in Rutherford scattering

Rutherford cross section

$$\frac{d\sigma_b(\theta)}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{4T}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

T= kinetic energy, θ scattering angle Z_1, Z_2 incoming, and taget charge

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Electrodinamics of spinless particles, Mandelstam variables

For a scattering process $\boldsymbol{AB \rightarrow CD}$ **(spinless)**in the center-of-mass frame:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big|_{\mathrm{cm}} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|\mathcal{M}|^2$$

where $s = (E_A + E_B)^2, |\boldsymbol{p}_A| = |\boldsymbol{p}_B| = p_i, |\boldsymbol{p}_C| = |\boldsymbol{p}_D| = p_f.$

p_i, p_f are initial and final momenta.We expect to have two independent kinematic variables *e.g.* the incident energy and the scattering angle. We like to express the invariant amplitude $\mathcal{M}$ as function of variables invariant under Lorentz transformation.

Generally are used Mandelstam variables:

$$s = (p_A + p_B)^2 \qquad t = (p_A - p_C)^2 \qquad u = (p_A - p_D)^2$$

but due to energy-momentum conservation and $p_i = m_i^2$ only two of the three are independent

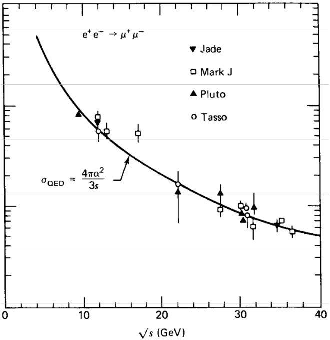
Electrodinamics of spin 1/2 particles

Now we **introduce the spin** the calculations for electromagnetic interactions of spin-$\frac{1}{2}$ leptons and quarks.
Using $\alpha = e^2/4\pi$

$$\frac{d\sigma}{d\Omega}\Big|_{cm} = \frac{\alpha^2}{4s}(1 + \cos^2\theta)$$

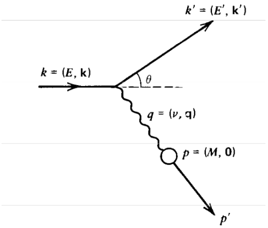
To have the reaction cross section integrating on θ and ϕ.

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$



Let's introduce $e^- \mu^- \to e^- \mu^-$ in laboratory frame studying kinematics relevant to the parton model.

q is the momentum transfer between electron and target. In the lab system where the μ (or proton) is at rest before the interaction.



$$\boldsymbol{q = k - k'}$$

1. Elastic scattering: $\nu = -\frac{q^2}{2M}$
2. In general :

$$\frac{E'}{E} < \frac{1}{1 + 2\frac{E}{M}\sin^2\frac{\theta}{2}}$$

[1] $q^2 = Q^2 = -2k \cdot k' \simeq -2EE'(1 - \cos\theta) \simeq -4EE'\sin^2\frac{\theta}{2}$

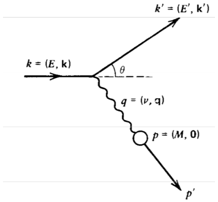
$$q^2 = -2p \cdot q = 2\nu M \quad \text{so } \boldsymbol{\nu \equiv E - E' = -\frac{q^2}{2M}}$$

[1] Derivation HM par. 6.8 pa 131. R. Paramatti, Appunti cinematica relativistica.

$e^-\mu^- \to e^-\mu^-$ in laboratory

Let's introduce the kinematics of $\boldsymbol{e^-\mu^- \to e^-\mu^-}$ in laboratory frame, where the initial μ^- is at rest, relevant to the parton model.

Later these results will be applied to electron-quark scattering probing the structure of hadrons.



The process $\boldsymbol{e^-\mu^- \to e^-\mu^-}$ in the laboratory frame

$e^-(k)\mu^-(p) \to e^-(k')\mu^-(p')$

q = k - k'

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left\{\cos^2\frac{\theta}{2} - \frac{q^2}{M^2}\sin^2\frac{\theta}{2}\right\}$$

- A powerful technique for exploring the internal structure of target is to bombard with a high energy beam of electrons and observe the angular distribution and energy of scattered electrons.

Size of Nuclei: Mott Scattering

The higher-energy electrons probe deeper into the nucleus.Electrons interact mainly through the electromagnetic force, and are not sensitive to the nuclear force.They probe the **distribution of charge** (*form factor*)in a nucleus, and the radius of the charge distribution can be defined as an effective size of the nucleus. At relativistic energies, the magnetic moment of the electron also contributes to the scattering cross section.

- The reference cross section for a **point-like center** on a **massive structureless target** calculated from electromagnetic scattering(Mott scattering)(c=1, $\hbar = 1$):

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{Z^2\alpha^2E^2}{4k^4\sin^4\frac{\theta}{2}}\left(1 - v^2\sin^2\frac{\theta}{2}\right)$$

k is the only momentum of incoming particle in the process $k = |\vec{k}_i| = |\vec{k}_f|$,$v^2$ the velocity($v = k/E$) and θ, scattering angle $\widehat{\vec{k}_i\vec{k}_f}$. In this the initial and final state spins are accounted.

Form factor

If the target is **not point-like**,with a charge distribution $\rho(\vec{x})$, the form factor $F(\vec{q})$, decreases the cross section for elastic scattering of electrons from that of a point-like center as: [2]:

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \boldsymbol{|F(\vec{q})|^2}$$

For any given spatial charge distribution $\rho(\vec{x})$ normalized to unity, the form factor of the target in terms of its Fourier transform $F(\vec{q})$ in momentum transfer is($\hbar = 1$):

$$F(\vec{q}) = \int e^{i\vec{q}\vec{x}} \rho(\vec{x}) d^3x$$

For little momentum transfers:

$$F(\vec{q}) = \int \left(1 + i\vec{q}\vec{x} - \frac{\vec{q}\vec{x}^2}{2}\right) \rho(\vec{x}) d^3x$$

[2]not taking in account spins $\sigma_{\text{Mott}} \rightarrow \sigma_{\text{Rutherford}}$

Form factor

If the charge distribution has a spheric distribution $\rho(\vec{x}) = \rho(x)$:

$$F(|\vec{q}|) = 1 - \frac{1}{6}|\vec{q}|^2 < \boldsymbol{r^2} > + ...$$

• Then the the scattering angle is a measure of $< r^2 >$ mean quadratic radius of target.

Thus, deviations from the distribution expected for point-scattering $\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}$ provide a measure of size (and structure) of the objects involved in the collision.

• For high-energy scattering of electrons on a massive nuclear target can be related to the Rutherford formula as:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = 4\cos^2\frac{\theta}{2}\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}}$$

Because electrons are thought to be point particles, the observed distribution, therefore, reflects the size of the nuclear target.

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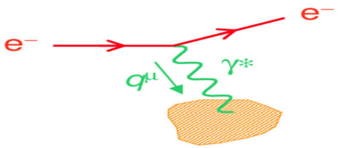
Proton structure

The electron proton scattering is extremely useful to study proton structure because the electron has not strong interactions then it is possible use virtual gamma to study the internal structure of proton:



photon wavelenght :

$$\lambda \sim \frac{h}{\boldsymbol{q}} \sim \frac{1\ \text{GeV} \cdot \text{fm}}{|\boldsymbol{q}|}$$

Electron-proton elastic scattering. Proton as muon

The above discussion **cannot** applied directly to **proton**, because is involved not only proton charge but also magnetic moment and proton recoil (nucleus is heavier). Dirac magnetic moment e/2M (M = proton mass). Then using electron muon scattering results the $\boldsymbol{e^- p \rightarrow e^- p}$ **elastic scattering** replacing mass of μ with M, proton mass:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \left(\frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}}\right) \frac{E'}{E} \left\{ \cos^2 \frac{\theta}{2} - \frac{q^2}{M^2} \sin^2 \frac{\theta}{2} \right\}$$

$$\frac{E'}{E} = \frac{1}{1 + \frac{2E}{M} \sin^2 \frac{\theta}{2}} \quad \text{is from recoil of target}$$

E and E' lepton energy loss.**BUT**

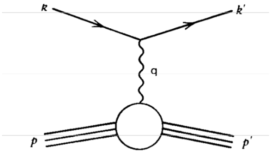
Since the proton has an extended structure the cross section for **electron-proton elastic scattering**(Rosenbluth formula) $\Longrightarrow$

Electron-proton elastic scattering. Proton Form factors

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left\{\left(F_1^2 - \frac{k^2q^2}{4M^2}F_2^2\right)\cos^2\frac{\theta}{2} - \right.$$

$$\left. - \frac{q^2}{M^2}(F_1 + kF_2)^2\sin^2\frac{\theta}{2}\right\}$$

- The two form factors $F_{1,2}(q^2)$ parametrize the ignorance of the detailed structure of proton as a unique blob.



• For $q^2 \to 0$ the probe are long-wavelength photons, and it does not make any difference that the proton has a structure $\sim$ 1fm. It is seen as a particle of charge e and magnetc moment $(1 + k)e/2M$. k the anomalous moment =1.79. Then $F_1(0) = 1$ $F_2(0) = 1$. Neutron $F_1(0) = 0$ $F_1(0) = 1$, $k_n = -1.91$.

Electron-proton elastic scattering. Proton Form factors

These **form factors** can me determined **experimentally** by **measuring $\frac{d\sigma}{d\Omega}$ as function of q^2 and θ** .If the proton was point like as the muon k = 0 and $F_1(q^2) = 1$ and we find again electron muon scattering formula. Just for completeness, sometime the previous formula is written as:

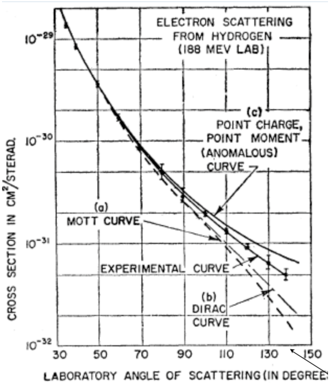
$$G_E \equiv F_1 + \frac{kq^2}{M^2}F_2 \qquad G_M \equiv F_1 + kF_2$$

$$\frac{d\sigma}{d\Omega}\bigg|_{\rm lab} = \Big(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\Big)\frac{E'}{E}\Big(\frac{G_E^2 + \tau G_M^2}{1 + \tau}\cos^2\frac{\theta}{2} +$$

$$+ 2\tau G_M^2\sin^2\frac{\theta}{2}\Big) \quad \text{with} \quad \tau \equiv \frac{-q^2}{4M^2}$$

The interference term disappeared it is possible show $G_E(q^2)$ and $G_M(q^2)$ are closely related to proton charge and magnetic moment distribution and generally are referred as electric and magnetic form factors. The data of angular dependence can be used to separate G_E and G_M.

Results



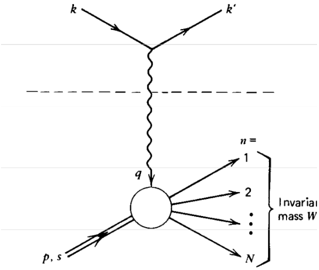
a) Point like proton, no magnetic effect (Mott $M \to \infty$)
b) Point like with spin $1/2$ and no anomalous moment
c) Point like with spin $1/2$ and anomalous moment

$1/\sqrt{Q^2} \sim 0.5\text{fm}.$

Inelastic proton scattering $ep \to eX$

Increasing the $-q^2$ of the photon an have a better spatial resolution it is possible to have a more detailed look at his structure. This can be done requiring a **large energy loss of the bombarding electron**.

Because of the large transfer of energy the **proton often will break up**.



- for modest $-q^2$ the proton is excited in a Δ state $ep \to e\Delta^+ \to ep\pi^0$. The invariant mass $W = (p + q)^2 \simeq M_\Delta$
- **$-q^2$ larger** the debris are so confused that the initial state **proton loses is identity** completely a new formalisms is necessary.

Inelastic proton scattering $ep \to eX$

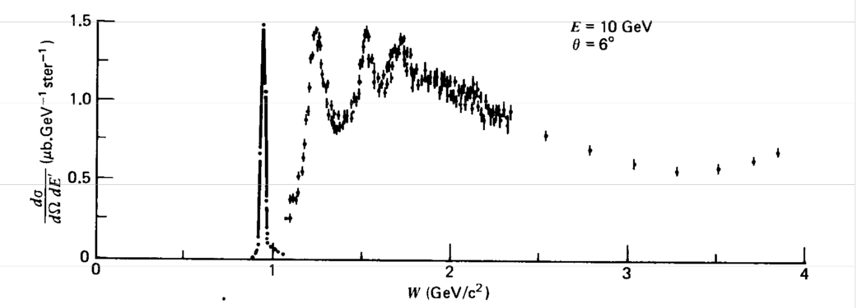


Figure : The $ep \to eX$ cross section as function of missing mass W. Data are from Stanford Linear Accelerator. The elastic peak W = M has been reduced by a factor 8.5.

- The peak correspond when the proton does not break up $(W \simeq M)$(elastic peak), broader peaks when target is excited to resonant baryon state; complicated multiparticle states with large invariant mas result in a smooth distribution in missing mass W.
- Elastic peak, $\Delta(1232)$, $N(1520)$

Variables

There are two independent variables:

$$q^2 \quad \text{and} \quad \nu \equiv \frac{p \cdot q}{M}$$

$$\text{the invariant mass } W^2 = M^2 + 2M\nu + q^2$$

Replacing with dimensioneless variables:

$$x = \frac{-q^2}{2p \cdot q} = \frac{-q^2}{2M\nu} \quad y = \frac{p \cdot q}{p \cdot k}$$

In the target proton rest frame:

$$x = E - E' \quad y = \frac{E - E'}{E'}$$

Inelastic scattering

It can be shown [3] the most general functional form for the description of hadronic current leads to a cross section with two functions W_1 and W_2. When the proton is broken up by the bombarding electron($ep \to eX$):

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{dE'd\Omega}}\right|_{\mathrm{lab}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\left\{W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right\}$$

That we shoud compare with elastic scattering ($ep \to ep$):

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d\Omega}}\right|_{\mathrm{lab}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\left(\frac{G_E^2 + \tau G_M^2}{1 + \tau}\cos^2\frac{\theta}{2} + \right.$$
$$\left. + 2\tau G_M^2\sin^2\frac{\theta}{2}\right) \quad \text{with} \quad \tau \equiv \frac{-q^2}{4M^2}$$

[3] Bibliography:HM

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Comparing muon and proton cross section

If point like spin-$\frac{1}{2}$ quarks reside inside the proton we should distinguish with small wavelength (large $-q^2$) virtual photon beam. If photon break up protons it should be an indication of structureless particles inside the proton and for small wavelengths the protons behave a a Dirac particle (quark). The p structure function became:

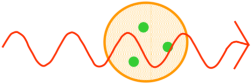
In laboratory frame:

muon

$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{lab} = \frac{(2\alpha E')^2}{q^4}\left\{\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right\}\delta\left(\nu + \frac{q^2}{2M}\right)$$

proton

$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{lab} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\left\{W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2}\right\}$$



$$2W_1^{point} = \frac{Q^2}{2m^2}\delta\left(\nu - \frac{Q^2}{2m^2}\right)$$

$$W_2^{point} = \delta\left(\nu - \frac{Q^2}{2m^2}\right)$$

$Q^2 \equiv -q^2$

m is the quark mass

Bjorken scaling

At large Q^2 the **inelastic** electron-proton scattering is behaving as **elastic** scattering of electron on a free quark inside the proton.Introducing dimensionless structure function:

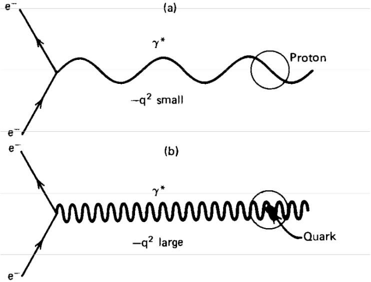
$$2mW_1^{\text{point}}(\nu, Q^2) = \frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$$\nu W_2^{\text{point}}\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$$\text{m is the quark mass}$$

The *point* notation indicates quark as structureless Dirac particle. ♠
These functions are only **function of the ratio** $\frac{Q^2}{2m\nu}$ and **not** of Q^2 and ν **independently**. The mass m serves as scale for the momenta Q^2, ν.

Bjorken scaling



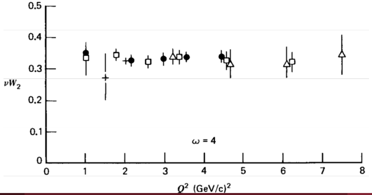
a) Elastic $ep \to ep$ scattering in which a large wave-length "photon beam" measures the size of proton through the elastic form factor analysis
b) In deep inelastic scattering (DIS) a short wavelength "photon beam" resolves the quarks within the proton provided $\lambda(\approx 1/\sqrt{-q^2}) \ll 1$ fm) (from HM)

Essentially what happens that at high Q^2 the photon is able to resolve pointlike quarks(as " muons")inside the proton and

$$MW_1(\nu, Q^2) \underset{\text{large } Q^2}{\rightarrow} F_1(\omega) \qquad \nu W_2(\nu, Q^2) \underset{\text{large } Q^2}{\rightarrow} F_2(\omega)$$

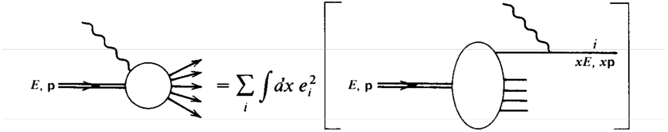
$$\omega = \frac{2q \cdot p}{Q^2} = \frac{2M\nu}{Q^2}$$

- we have replaced the mass m (quark as "muon") with the proton mass M to define the dimensionless variable ω.
- It is not so important it serves only to set the scale of the dimensional variable ω.
- Important: **the structure functions are function only of the ratio.** (Bjorken scaling).
- The presence of free quarks is signaled by the fact the inelastic structure functions are independent of Q^2 at a given value of ω.



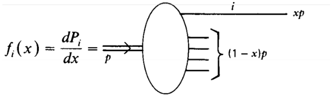
Are these particles (called partons) the quarks?

Try to make identification explicit:



This fig. means that the **various type of *point* partons make up a proton** (i = u, d ,.. quarks wirth a charge e_i and gluons that not interact with photons of course).

Each carrying a **fraction of x of parent proton's momentum and energy**, with a parton momentum distribution, $f_{i'}(x)$.



- Each parton i carries a fractionThe x of momentum p of proton:

$$\sum_{i'} \int \mathrm{d}x\; x f_{i'}(x) \;=\; 1$$

sum i' is extended to all partons, not only charged one i interacting with photons.

Parton kinematics

Table : Parton and Proton kinematics

	Proton	Parton
Energy	E	xE
Momentum	p_L	xp_L
	$p_T = 0$	$p_T = 0$
Mass	M	$m = (x^2E^2 - x^2p_L^2)^{\frac{1}{2}} = xM$

- both proton and children partons move along z axis ($i.e.$ $p_T = 0$) with longitudinal momenta p_L and xp_L.

It is conventional to redefine $F_{1,2}(\omega)$ as $F_{1,2}(x)$ at large Q^2 :

$$\nu W_2(\nu, Q^2) \underset{\text{large } Q^2}{\rightarrow} F_2(x) = \sum_i e_i^2 x f_i(x)$$

$$M W_1(\nu, Q^2) \underset{\text{large } Q^2}{\rightarrow} F_1(x) = \frac{1}{2x} F_2(x)$$

$$x = \frac{1}{\omega} = \frac{Q^2}{2M\nu}$$

$f_i(x) =$ probability distribution that the quark i carries a fraction x of proton momentum.

- the momentum fraction is found to be identical to the to x variable before introduced for x photon ($x = \frac{-q^2}{2p \cdot q} = \frac{-q^2}{2M\nu}$) $\longrightarrow$ The virtual photon must have just the right value of variable x to be asborbed by a parton with moment fraction x.
- The inelastic structure function $F_{1,2}(x)$ are function **only** of one variable, $\boldsymbol{x}$. They are independent of Q^2 at fixed x, satisfying Bjorken scaling .

- The quarks are **NOT** free in the proton, they are free on the time scale of the interaction, but are not free on a longer time scale. In particular their mass is not a well defined notion, since they are strongly interacting, they are virtual particles and they are not on the mass shell. In this sense their mass is a continuous function.
- During the short time in which the parton don't interact with other partons the virtual photon interact with quark, free particle not interacting with others
- $F_{1,2}(x)$ is derived (incoherence assumption) as addition of probability of scattering from single free parton, as in nuclear physics, but a nucleon can escape from nucleus as completely free particle, but colored parton has to recombine with non interacting spectator partons to form colorless hadrons into which proton breaks up (probability=1 , for color confinement)
- In hard collision the parton recoils as it were free to enable us to calculate $ep \to eX$ and the subsequent confining final state interactions don't affect the result.
- Picture valid as Q^2 virtual photon large and invarianat mass of final-state hadronic, W, large.

Parton model

Master formula of parton model

$$2xF_1(x) = F_2(x) = \sum_i e_i^2 x f_i(x)$$
$$x = \frac{1}{\omega} = \frac{Q^2}{2M\nu}$$

$F_{1,2}(x)$ structure function, M proton mass $\nu = \frac{q \cdot p}{M}$, x fraction of proton momentum carried by parton, $f_i(x)$ = probability distribution that the quark i carries a fraction x of proton momentum.

Structure Function

We have shown the most general functional form for the description of hadronic current leads to a cross section $ep \rightarrow ep$ with two functions W_1 and W_2. When the proton is broken up by the bombarding electron($ep \rightarrow eX$):

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{dE'd}\Omega}\right|_{\mathrm{lab}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\left\{W_2(\nu, q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu, q^2)\sin^2\frac{\theta}{2}\right\}$$

$$\mathrm{d}E'\mathrm{d}\Omega = \frac{\pi}{EE'}\mathrm{d}Q^2\mathrm{d}\nu = \frac{2ME}{E'}\pi y \mathrm{d}x\mathrm{d}y$$

$$\text{remember} \quad \nu = E - E' \qquad y = \frac{E - E'}{E}$$

$$x = \frac{-q^2}{2p\cdot q} = \frac{-q^2}{2M\nu} = \frac{Q^2}{2M\nu} \qquad y = \frac{p\cdot q}{p\cdot k} \underset{(\text{lab.})}{=} \frac{\nu}{E}$$

Structure Function

Invariant form $ep \to eX$ cross section

The invariant form of $ep \to eX$ cross section:

$$M\nu_{\text{max}} \frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y} = \frac{2\pi\alpha^2}{x^2y^2}\left\{xy^2F_1 + \left[(1-y) - \frac{Mxy}{2\nu_{\text{max}}}\right]F_2\right\}$$

$$x = \frac{Q^2}{2M\nu} \qquad y = \frac{\nu}{E}$$

$\nu_{\text{max}} = E$ in laboratory frame

Structure Function

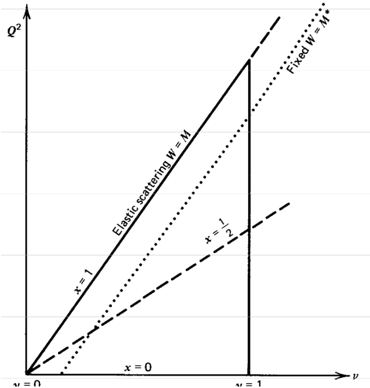


Figure : The triangle is the allowed kinematical region for $ep \to eX$ $\nu_{max} = E$ in laboratory frame. W is invariant mass of hadronic system.

a) M proton mass

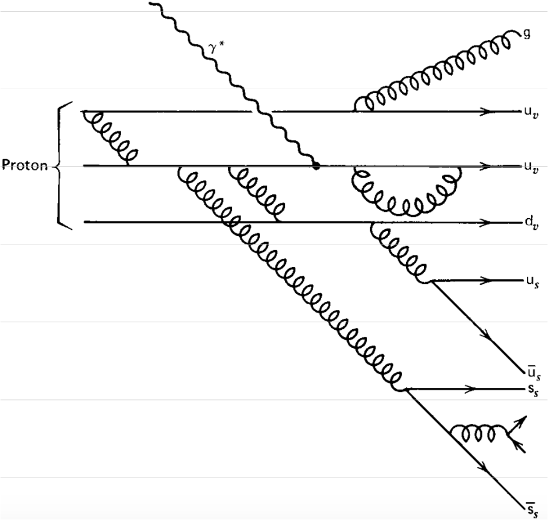
b) $x = \frac{Q^2}{2M\nu}$ In parton model, fraction of nucleon's momentum carried by struck quark.

c) $Q^2 = -q^2$ square of momentum transfer.

d) $W^2 = (p + q)^2 = M^2 + 2M\nu - Q^2$ mass squared of system X recoiling against scattered electron.

Quarks within proton

The photon see the proton structure as :



Quarks within proton

The measurement of **elastic form factors** provide us with information on **size of proton**, the measurement of **inelastic form factors** provide us at large Q^2 the **quark structure of proton**:

$$2xF_1(x) = F_2(x) = \sum_i e_i^2 x f_i(x)$$

The sum runs over the charged partons in the **proton**. For example:

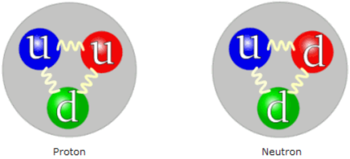
$$\frac{1}{x}F_2(x)^{\text{ep}} = \left(\frac{2}{3}\right)^2[u^p(x) + \bar{u}^p(x)] + \left(\frac{1}{3}\right)^2[d^p(x) + \bar{d}^p(x)] + \left(\frac{1}{3}\right)^2[s^p(x) + \bar{s}^p(x)]$$

$u^p(x), \bar{u}^p(x)$ probability distributions of u quarks and antiquarks in the proton. Neglected heavy quarks (c,b,t). 6 unknown quark structure functions. From inelastic structure function of **neutrons**.

$$\frac{1}{x}F_2(x)^{\text{en}} = \left(\frac{2}{3}\right)^2[u^n(x) + \bar{u}^n(x)] + \left(\frac{1}{3}\right)^2[d^n(x) + \bar{d}^n(x)] + \left(\frac{1}{3}\right)^2[s^n(x) + \bar{s}^n(x)]$$

• neutron and proton are in the same isospin doublet. There are many u quarks in a proton as d quarks in in a neutron.

$$u^p(x) \; = \; d^n(x) \equiv u(x) \quad d^p(x) \; = \; u^n(x) \equiv d(x) \quad s^p(x) \; = \; s^n(x) \equiv s(x)$$



Quark composition of a proton and a neutron (diagrams from *Wikipedia*

•The quantum number of the proton must be exactly *uud* of *valence* quarks. Proton has $u_v u_v d_v$ and many pair quarks -antiquarks of *sea*: $u_s\bar{u}_s$, $d_s\bar{d}_s$,$s_s\bar{s}_s$.....Let's immagine as radiated by valence quarks. Assume the 3 lightest flavor pair occur on the sea with roughly same frequency and momentum and neglect the others....

$$u_s(x) = \bar{u}_s(x) = d_s(x) = \bar{d}_s(x) = s_s(x) = \bar{s}_s(x) = S(x)$$

$$u(x) = u_v(x) + u_s(x) \quad d(x) = d_v(x) + d_s(x)$$

S(x) is the sea quark distribution common to all flavors.

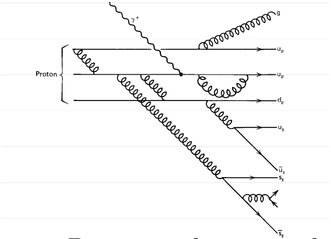


Figure : Proton: valence quarks, gluons slow debris of $q\bar{q}$ pair.

To get the quantum number of the proton: charge 1, baryon number 1, strangeness 0.

$$\int_0^1 [u(x) - \bar{u}(x)]\,\mathrm{d}x = 2$$

$$\int_0^1 [d(x) - \bar{d}(x)]\,\mathrm{d}x = 1$$

$$\int_0^1 [s(x) - \bar{s}(x)]\,\mathrm{d}x = 0$$

$u - \bar{u} = u - \bar{u}_s = u - u_s = u_v$ only distribution valence quarks
$d - \bar{d} = d - \bar{d}_s = d - d_s = d_v$ only distribution valence quarks
$s - \bar{s} = s - \bar{s}_s = 0.$

From already known:

$$2xF_1(x) = F_2(x) = \sum_i e_i^2 x f_i(x)$$

From, already known:

$$\frac{1}{x}F_2(x)^{\mathrm{ep}} = \left(\frac{2}{3}\right)^2[u^p(x) + \bar{u}^p(x)] + \left(\frac{1}{3}\right)^2[d^p(x) + \bar{d}^p(x)] + \left(\frac{1}{3}\right)^2[s^p(x) + \bar{s}^p(x)]$$

Proton

The proton is constituted by $u_v u_v d_v$ ($\bar{u}$ in pair with u(x) in the sea).

$$\frac{1}{x}F_2(x)^{\mathrm{ep}} = \frac{1}{9}[4u_v + d_v] + \frac{4}{3}S$$
$$\frac{1}{x}F_2(x)^{\mathrm{en}} = \frac{1}{9}[u_v + 4d_v] + \frac{4}{3}S$$

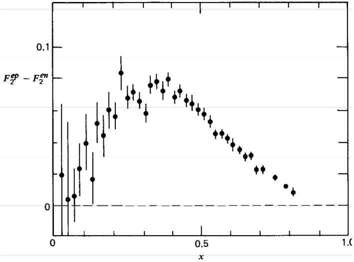
• where 4/3 is the sum of e_i^2 over six sea distribution.
• Gluons create $q\bar{q}$ pair in the sea and the S(x) distribution of the sea quarks common to all quark flavors is expected to have a bremmsstrahlung-like spectrum at small x and so the number of sea quarks grows as $x \to 0$.

u-v distribution

$$\frac{1}{x}F_2(x)^{\text{ep}} = \frac{1}{9}[4u_v + d_v] + \frac{4}{3}S$$
$$\frac{1}{x}F_2(x)^{\text{en}} = \frac{1}{9}[u_v + 4d_v] + \frac{4}{3}S$$

$$\frac{1}{x}(F_2(x)^{\text{ep}} - F_2(x)^{\text{en}}) = \frac{1}{3}(u_v - d_v)$$

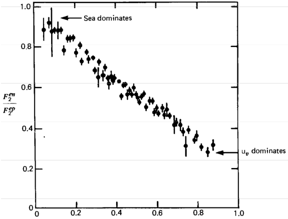
Subtracting the two equations it is possible to get the difference of the distributions of the u and d valence quarks in the proton: $\frac{x}{3(u_v - d_v)}$



Proton

- At small momentum ($x \approx 0$) the presence of valence quarks is overshadowed by these multiple, low-momentum $q\bar{q}$ pairs that make the sea S(X), (the u(x) structure function is very little or 0) this means.

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \underset{x \to 0}{\longrightarrow} 1$$



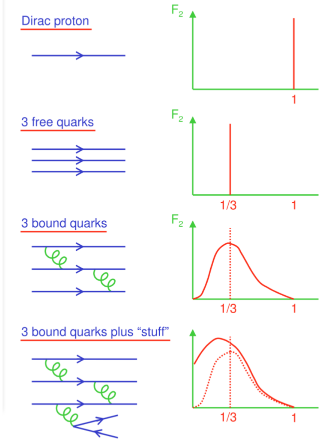
- Probing at large momentum part of proton structure ($x \approx 1$) the fast-valence quarks u_v, d_v dominate.

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \underset{x \to 1}{\longrightarrow} \frac{u_v \; + \; 4d_v}{4u_v \; + \; d_v}$$

For proton, there is an evidence that $u_v >> d_v$ at large x and the ratio $\rightarrow \frac{1}{4}$

F_2 in various proton model

How looks like F_2 in various proton model? How can interpreted the previous data?



1. The proton carries all momentum
2. Each quarks carries $\frac{1}{3}$ of proton momentum
3. From $2 \rightarrow 3$ once the quarks interact, they can redistribute the momenta among themselves, and the sharply defined momentum $x = \frac{1}{3}$ is washed out, becoming a distribution of momenta peaked around $x = \frac{1}{3}$.
4. Data on $F_2^{ep}(x)$ (see previous slide) at large Q^2 have the scenario 4. Sea quark solid line valence quark dotted line.

Quark structure function

Another approach is parametrize all large Q^2 data on $F_2^{en}(x), F_2^{ep}(x)$ in term of the valence and sea quark distribution and **extract the structure functions** taking in account the sum rules.

- $u(x) = u_v(x) + u_s(x) \longrightarrow \bar{u}(x)$ at small x as $xu_v(x) \rightarrow 0$.
- the **sea quarks** are **slow** respect their valence partner.

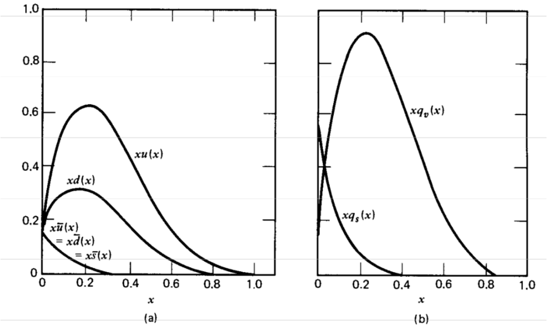
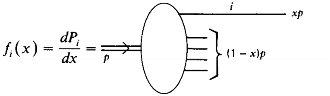


Figure : a)Quark structure function from deep inelastic scattering of data. b) Total valence a and sea quarks contribution to the structure of the proton.

Gluons within proton



- Each parton i carries a fraction x of momentum p of proton:

$$\sum_{i'} \int \mathrm{d}x \; x f_{i'}(x) \; = \; 1$$

The sum i' is extended to all partons, not only **charged** one i **interacting with photons.**

Summing on all partons we should get the total momentum p of the proton:

$$\int_0^1 \mathrm{d}x(xp)[u \; + \; \bar{u} \; + \; d \; + \; \bar{d} \; + \; s \; + \; \bar{s} \; = \; p \; - \; p_g$$

dividing by p

$$\int_0^1 \mathrm{d}x x[u \; + \; \bar{u} \; + \; d \; + \; \bar{d} \; + \; s \; + \; \bar{s} \; = \; 1 \; - \; \epsilon_g$$

$$\epsilon_g \; \equiv \frac{p_g}{p} \quad \text{momentum fraction carried by the gluons}$$

Gluons within proton

♣ **Gluons are not exposed by photon probe** (the gluons carry no electric charge), then are subtracted from the right side.

$$
\begin{aligned}
\frac{1}{x}F_2(x)^{ep} &= \frac{1}{9}[4u_v + d_v] + \cancel{\frac{4}{3}S} \\
\frac{1}{x}F_2(x)^{en} &= \frac{1}{9}[u_v + 4d_v] + \cancel{\frac{4}{3}S} \\
\int \mathrm{d}x F_2(x)^{ep} &= \frac{4}{9}\epsilon_u + \frac{1}{9}\epsilon_d = 0.18 \\
\int \mathrm{d}x F_2(x)^{en} &= \frac{1}{9}\epsilon_u + \frac{4}{9}\epsilon_d = 0.12
\end{aligned}
$$

$$
\text{where} \quad \epsilon_u \equiv \int_0^1 \mathrm{d}x(u + \bar{u})
$$

neglecting strange quarks carrying a small fraction of nucleon's momentum. ϵ_u, ϵ_d momentum carried by u, d quarks and anti-quarks.Solving the equation $\epsilon_u = 0.36, \epsilon_d = 0.18$.

Gluons within proton

The gluons carry the moment non carried by u, d quarks and antiquarks:

$$\epsilon_g \simeq 1 - \epsilon_u - \epsilon_d$$
$$\epsilon_u = 0.36, \epsilon_d = 0.18, \epsilon_g = 0.46$$

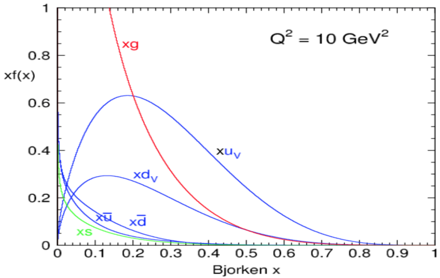
$\Longrightarrow$ the gluons accounts $\sim 50\%$ of momentum of charge quarks.

Partons

From Deep Inelastic Scattering(DIS) of leptons by nucleons reveals:

- Point-like Dirac particles inside hadrons through Bjorken scaling.
- A study of quantum numbers of these partons $\Longrightarrow$ identify with quarks
- From momentum distribution of quarks a large parte of proton's momentum is carried by neutral partons, not by quarks, gluons (QCD).

Structure functions



- u_v d_v similar shape
- $u_v \neq d_v$
- **Gluons are dominant constituent for $x < 0.2$**

u(x) d(x) cannot (yet be predicted from QCD[4])

[4]It is in a non perturbative regime

Contents

1 Reminders

2 Electrodinamics of Spin-1/2 particles

3 The structure of hadrons

4 Partons
- Quarks
- Gluons

5 Structure function in QCD

Gluons

- Quarks carry colors & electric charge: Red, Green, Blue
- Color is exchanged by 8 bicolored gluons
- Color interactions are assumed as a "copy of e.m. interactions" substituting $\sqrt{\alpha} \rightarrow \sqrt{\alpha}_s$. The 8 gluons are massless and have spin 1.
- Gluons themselves carry color charge and so they can interact with other gluons.
  There are ggg qqg vertex.
- at short distance α_s is sufficiently small $\rightarrow$ color interactions computed using perturbative QED.

The parton model ignores the role of gluons as the carriers of strong force associated to colored quarks.

It has been ignored that: The quark may irradiate a gluon before or after being struck by virtual photon γ^*.The inclusion of QCD diagrams:

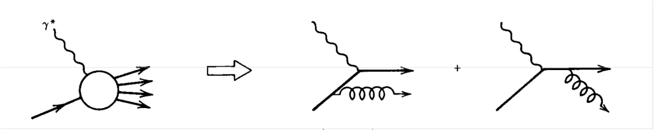


Figure : $\mathcal{O}(\alpha\alpha_s)$ contributions to $\gamma^* q \to qg$ to $ep \to eX$

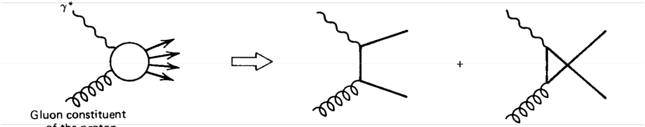


Figure : $\mathcal{O}(\alpha\alpha_s)$ gluon-initiated "hard "contributions to $\gamma^* q \to qg$ to $ep \to eX$

The pratic problem is to find the contribution of these γ^*-parton scattering diagram to the cross section for deep inelastic scattering.

- scaling property of structure functions no longer valid.
- the outgoing quark(and then direction of hadron jet) is not anymore collinear with virtual photon γ^*.

The gluon are emitted, the quark can recoil against a radiated gluon and **2 jets have a transverse p_T relative virtual photon**

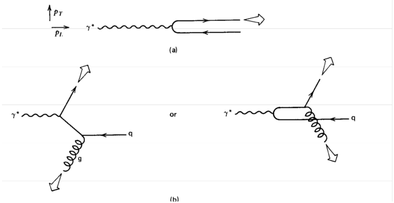


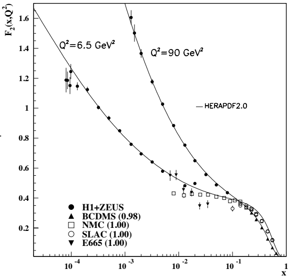
Figure : (a)Parton model diagram for $\gamma^* q \to q$, producing a jet with $p_T = 0$ (b) Gluon emission diagrams which produce jets with $p_T \neq 0$ (arrows).

Scaling violations of the structure interactions

One of the most striking **predictions** of the quark-parton model is that the structure functions $\boldsymbol{F_i}$ scale, *i.e.*, $F_i(x, Q^2) \rightarrow F_i(x)$ in the Bjorken limit that Q^2 and $\nu \rightarrow \infty$ with x fixed. They were **only function of $Q^2/2m\nu$ ratio** and not Q^2 and ν independently.

- This property is related to the **assumption that the transverse momentum of the partons** in the infinite-momentum frame of the proton is **small**.
- In QCD, however, the **radiation of hard gluons from the quarks violates this assumption**, leading to logarithmic scaling violations, which are particularly large at small x.

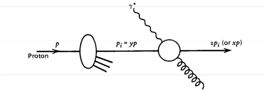
[a]



[a] Q = transfer moment, ν = lepton's energy loss in nucleon rest frame, x = fraction of the nucleon's momentum carried by struck quark.

Scaling violations -Reminders

- The **radiation of gluons** produces the **evolution of the structure functions**.
- As Q^2 increases, more and more gluons are radiated, which in turn split into $q\bar{q}$ pairs. .
- This process leads both to the softening of the initial quark momentum distributions and to the growth of the gluon density and the $q\bar{q}$ sea as x decreases.
- In QCD, the above process is described in terms of scale-dependent parton distributions $f_i(y)$.



γ^*- Parton Frame

$$p_i = \gamma p$$
$$z = \frac{Q^2}{2p_i \cdot q} = \frac{x}{y}$$

Figure : $\gamma^* q \rightarrow qg$, in $\gamma^* p \rightarrow X$.

Scaling violations -Reminders

The σ_T and σ_L are the γ^*p total cross section for transverse and longitudinal photons and $\sigma_0 \simeq \frac{4\pi^2\alpha}{s}$.

$f_i(y)$ parton structure function, probability i parton carries a fraction of the proton's momentum p;

$\hat{\sigma_T}$ the cross section for the absorption of transverse photon of momentum q by a parton of a momentum p_i; x is fixed and it has been integrated all over z, y subjected to the constrain $x = zy$. [5]

$$\left(\frac{\sigma_T}{\sigma_0}(x, Q^2)\right) = \sum_i \int_x^1 \frac{\mathrm{d}y}{y} f_i(y) \left(\frac{\hat{\sigma_T}}{\hat{\sigma}_0}\left(\frac{x}{y}, Q^2\right)\right)$$

[5] $\hat{\sigma}$ are γ^*-parton quantities, σ parent process.

Scaling violations of the structure interactions -Reminders

The $\gamma^* \to qg$ cross section:

$$\frac{\mathrm{d}\hat{\sigma}}{p_T^2} \simeq e_i^2 \hat{\sigma}_0 \frac{1}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z)$$

$$\hat{\sigma}_0 = \frac{4\pi^2\alpha}{\hat{s}} \qquad P_{qq}(z) = \frac{4}{3}\left(\frac{1 + z^2}{1 + z^2}\right)$$

$P_{qq}(z)$ = probability of a quark emitting a gluon and so becoming a quark with momentum reduced by a fraction z. [6]

[6] Singularity $z \to 1$ singularity is associated with an emission of a "soft" massless gluon.(example of infrared divergence)

Scaling violations of the structure interactions -Reminders

- The presence of gluon emission is signaled by a quark jet and gluon jet in final state, neither moving along direction of virtual photon $\Longrightarrow p_T^{\text{jet}} \neq 0$
- This p_T distribution of quarks has to be embeded into electron-proton (formula in previous page)

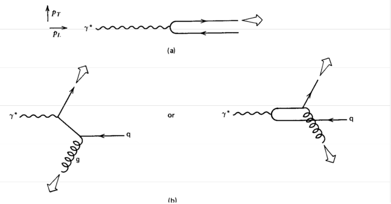


Figure : (a)Parton model diagram for $\gamma^* q \rightarrow q$, producing a jet with $p_T = 0$ (b) Gluon emission diagrams which produce jets with $p_T \neq 0$ (arrows).

p_T distribution

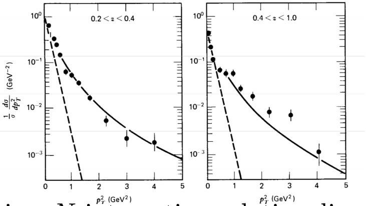


Figure : p_T^2 of hadrons in μN interaction relative direction virtual photon (from HM). The dashed line is without gluons emission

- The **hadron emerge with $p_T^{jet} \neq 0$ signaling the presence of gluon emission**.
- In a parton model without gluons, all final-state jets would be collinear with virtual photon. The hadron fragments will be therefore nearly collinear with photon (spread $\simeq$ 300 MeV uncertaintly principle for confined quarks).
- From data an excess of large p_T hadrons, which are fragments of the quark and gluon jets recoiling against one other.

Scaling violation

Let examinate how gluon bremmstrahlung how contribute to structure function. The γ^*-parton cross section is:

$$\begin{aligned}\hat{\sigma}(\gamma^* q \to qg) &= \int_{\mu^2}^{(p_T^2)_{\max}} \mathrm{d}p_T^2 \frac{\mathrm{d}\hat{\sigma}}{p_T^2} \\ &\simeq e_i^2 \hat{\sigma}_0 \int_{\mu^2}^{(p_T^2)_{\max}} \mathrm{d}p_T^2 \frac{1}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z) \\ &\simeq e_i^2 \hat{\sigma}_0 \left(\frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2} \right)\end{aligned}$$

It has been integrated to $(p_T^2)_{\max}$ of gluon. The lower limit μ on transverse momentum is for a cutoff to regularize the divergence at $(p_T^2) \to 0$.

The parton model structure function was:

$$\frac{F_2(x,Q^2)}{x} = \sum_i e_i^2 \int_x^1 \frac{\mathrm{d}y}{y}\delta\left(1 - \frac{x}{y}\right) = \sum_i e_i^2 f_i(x)$$

$$q(y) \equiv f_q(y) \text{ quark structure function}$$

$$\text{Now with gluon bremmstralung } \gamma^* q \to qg$$

$$\frac{F_2(x,Q^2)}{x} = \sum_i e_i^2 \int_x^1 \frac{\mathrm{d}y}{y} q(y)\left(\left(1 - \frac{x}{y}\right) + \frac{\alpha_s}{2\pi} P_{qq}\frac{x}{y}\log\frac{Q^2}{\mu^2}\right)$$

- **the presence of $\log Q^2$ indicate that the parton model scaling prediction for the structure functions should be violated.**
- In QCD F_2 is a function of Q^2 and x
- the variation of Q^2 is only logaritmic
- violation of Bjorken scaling is a signature of gluon emission

We can rewrite in another form as including gluon bremmstrahlung:

$$\frac{F_2(x, Q^2)}{x} = \sum_i e_i^2 \int_x^1 \frac{\mathrm{dy}}{y} (q(y) + \Delta(y, Q^2)) \delta\big(1 - \frac{x}{y}\big)$$

$$\text{where} \quad \Delta q(x, Q^2) = \frac{\alpha_s}{2\pi} \log\left(\frac{Q^2}{\mu^2}\right) \int_x^1 \frac{\mathrm{dy}}{y} q(y)\ P_{qq}\left(\frac{x}{y}\right)$$

- **The quark density $q(x, Q^2)$ now depends from Q^2** $\Longrightarrow$. Interpretation: It is arising from a photon with larger Q^2 probing a wider range of p_T^2 within the proton.
- Q^2 evolution is determined by QCD trough last equation.

- As Q^2 is increased the photon start to "see" evidence for point-like valence quarks within proton.
- If the quarks are not interacting, no further structure would be resolved as Q^2 was increased and the scaling $q(x)$ would be set in, and the and parton model would be satisfactory.
- QCD predicts that on increasing the resolution $Q^2 >> Q_0^2$ we should see that each quark is sourronded by a cloud of partons.
- The number of resolved which share proton's momentum increase with Q^2.
- There is an increased probability of finding quark at small x and decreased to find one at high x, because the high-momentum quarks lose momentum radiating gluons

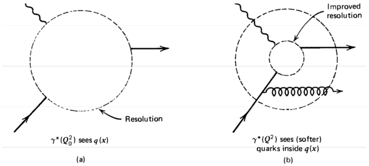


Figure : The quark structure of the proton of the proton as seen by virtual photon as Q^2 increases.

Altarelli-Parisi evolution equation

Considering the change of quark density $\Delta q(x, Q^2)$ when is probing a further interval in $\Delta \log Q^2$ is possible rewrite an integro differential **equation Altarelli-Parisi evolution equation** (DGLAP)

$$\frac{\mathrm{d}}{\mathrm{d}\log \mathrm{Q}^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{\mathrm{d}y}{y} q(y, Q^2) P_{qq}\left(\frac{x}{y}\right)$$

• The equation express the fact a quark with momentum fraction $xq(x, Q^2)$ could have come from a quark with higher momentum fraction $yq(y, Q^2)$, which has radiate a gluon.

The probability that this happens is $\propto \alpha_s P_{qq}\left(\frac{x}{y}\right)$. The integral is the sum over all possible momentum fractions $y(> x)$ of the parent

• QCD predicts the breakdown of scaling and allows to computr the dependence of structure function on Q^2

• Knowing the structure function at give x, Q_0^2, one can EVOLVE the function to x and Q^2.The evolution of $q(x, Q^2)$ is computable.

Scaling violation

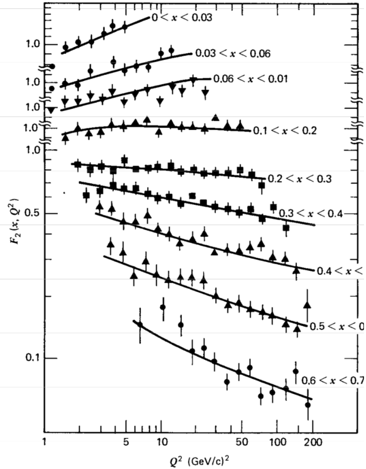


Figure : Deviation from scaling . With increasing Q^2, the structure function $F_2(x, Q^2$ increase at small x and decreases at large x. (from HM, CDHS data)

Summary

1. QCD predicts the breakdown of scaling and allows to compute the dependence of structure function from Q^2.
2. Given the quark structure function some reference point $q(x, Q_0^2)$ it is possible derive at any value of Q^2 using Altarelli-Parisi equation.

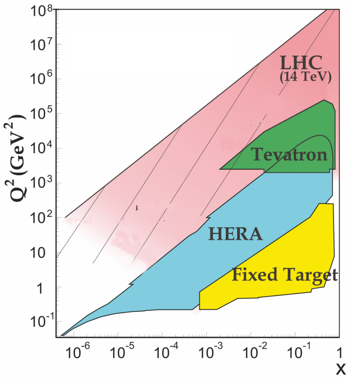
Altarelli Parisi equation

$$\frac{\mathrm{d}}{\mathrm{d}\log \mathrm{Q}^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{\mathrm{d}y}{y} q(y, Q^2) P_{qq}\left(\frac{x}{y}\right)$$

A quark with momentum fraction $xq(x, Q^2)$ could have come from a quark with higher momentum fraction $yq(y, Q^2)$, which has radiate a gluon, with a probability $\propto \alpha_s P_{qq}\left(\frac{x}{y}\right)$. The integral is the sum over all possible momentum fractions $y(> x)$ of the parent.

Parton Distribution Functions kinematic ranges

- The Parton Distribution Functions(PDFs) are determined from data for deep inelastic lepton-nucleon scattering and for related hard-scattering processes initiated by a nucleons.
- The kinematic range of fixed-target and colliders are complementary.



Parton distributions-main points

- There is one independent PDF **for each parton in the proton** $\boldsymbol{u(x,Q^2), d(x,Q^2), g(x,Q^2)....}$
- A total **13 PDF**
- At Leading Order PDF are interpreted as:**Probability of find a parton of a given flavor carrying a fraction x of total proton momentum**
- Once QCD corrections included, PDF became scheme independent and have no probabilistic interpretation

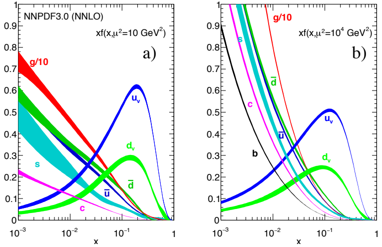
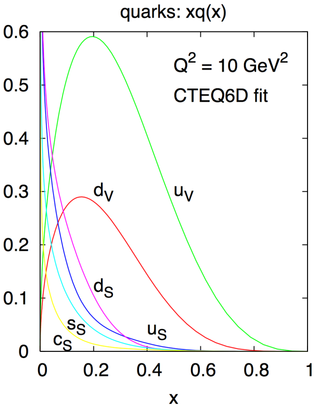


Figure : $xf(x)$, f(x) parton distribution $(f(x) = u_v, d_v, \bar{u}, \bar{d}, s, c, b, g)$ from PDG15

Parton distributions-main points



- **Valence quarks** u_v are **hard**
- **Sea quarks** u_s are **fairly soft**

Parton distributions-main points

- **Shape and normalization** of PDFs are very different for each flavor, reflecting the different underlying **dynamics** that determines each PDF flavor
- QCD imposes **momentum** and **valence sum rules** valid to all order in perturbation theory:

Momentum Sum Rules $$\sum_{i'} \int \mathrm{d}x \; x f_{i'}(x) \;=\; 1$$

Valence Sum Rules $$\int_0^1 [u(x) \;-\; \bar{u}(x)] \; \mathrm{dx} \;=\; 2$$

$$\int_0^1 [d(x) \;-\; \bar{d}(x)] \; \mathrm{dx} \;=\; 1$$

$$\int_0^1 [s(x) \;-\; \bar{s}(x)] \; \mathrm{dx} \;=\; 0$$