Cabibbo-Kobayashi-Maskawa Matrix
and
CP Violation in Standard Model

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Lecture 1

Lezioni di Fisica delle Particelle Elementari

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for some of the material included in my slides

Outline of these lectures

- Origin of Cabibbo-Kobayashi-Maskawa (CKM) Matrix
  - What is it? Where does it come from?
  - Why do we care about it?
  - How is it related to the quark masses?
  - Why and how is it related to CP violation?
- Overview of experimental measurements of CKM elements

2 Lectures

- CP violation in the Standard Model
  - Little bit of History
  - Types of CP violation
  - B mesons and their importance
  - Formalism of CP Violation at B factories
- Overview of experimental measurements

3 Lectures

- Detailed discussion of one measurement

1 Lecture

Standard Model

$$SU(2)_L \times U(1)_Y$$

Weak Isospin (symbol L because only the LEFT states are involved)

Weak Hypercharge :

(LEFT and RIGHT states)

			I	I_3	Q	Y
	doublet L	ν_e	½	½	0	-1
		e_L^-	½	-½	-1	-1
Leptons	singlet R	e_R^-	0	0	-1	-2
	doublet L	u_L	½	½	2/3	1/3
		d_L	½	-½	-1/3	1/3
	singlet R	u_R	0	0	2/3	4/3
quarks	singlet R	d_R	0	0	-1/3	-2/3

Idem for the other families

Mass of Quarks in the Standard Model

- For each generation we have one left-handed SU(2) doublet, and two right-handed singlets

$$Q_L^I = \begin{pmatrix} U_L^I \\ D_L^I \end{pmatrix} = (3,2)_{+1/6}, \quad u_R^I = (3,1)_{+2/3}, \quad d_R^I = (3,1)_{-1/3}.$$

Eigenstates of weak interactions

- Quarks interact with Higgs field via Yukawa coupling

$$\mathcal{L}_Y = -\mathbf{G}_{ij}\overline{Q_{Li}^I}\phi d_{Rj}^I - \mathbf{F}_{ij}\overline{Q_{Li}^I}\tilde{\phi} u_{Rj}^I + \mathrm{H.c.}$$

Generic complex matrix of yukawa coupling constants

- Quarks acquire mass through because of spontaneous symmetry breaking

$$\mathcal{L}_M = -\sqrt{\frac{1}{2}}v\mathbf{G}_{ij}\overline{d_{Li}^I}d_{Rj}^I - \sqrt{\frac{1}{2}}v\mathbf{F}_{ij}\overline{u_{Li}^I}u_{Rj}^I + \mathrm{H.c.}$$

$$\mathbf{M}_d = \mathbf{G}v/\sqrt{2}, \quad \mathbf{M}_u = \mathbf{F}v/\sqrt{2}.$$

Mass matrices for up and down quarks. Elements are complex!

Weak Interactions and Mass Eignestates

- Diagonalize mass matrices to obtain mass eigenstates
  - Rotate quark fields by with unitary complex matrices V_{uL}, V_{uR}, V_{dL}, V_{dR}
  - Choose arbitrary phases so that M is diagonal

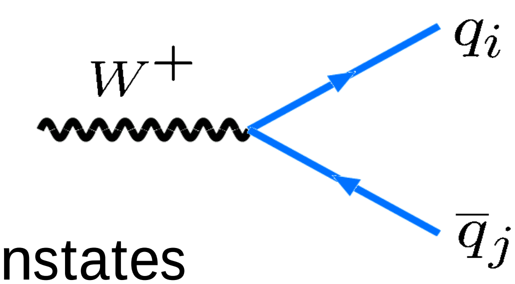
$$\mathbf{M}_d = \mathbf{G}v/\sqrt{2}, \quad \mathbf{M}_u = \mathbf{F}v/\sqrt{2}.$$

$$\mathbf{V}_{dL}\mathbf{M}_d\mathbf{V}^\dagger_{dR} = \mathbf{M}_d^{\rm diag}, \quad \mathbf{V}_{uL}\mathbf{M}_u\mathbf{V}^\dagger_{uR} = \mathbf{M}_u^{\rm diag}.$$

Universality of weak interactions: same constant g for all couplings

- Lagrangian for weak interactions of quarks

$$\mathcal{L}_W = -\sqrt{\frac{1}{2}} g \overline{u^I_{Li}} \gamma^\mu \mathbf{1}_{ij} d^I_{Lj} W^+_\mu + \text{h.c.}$$



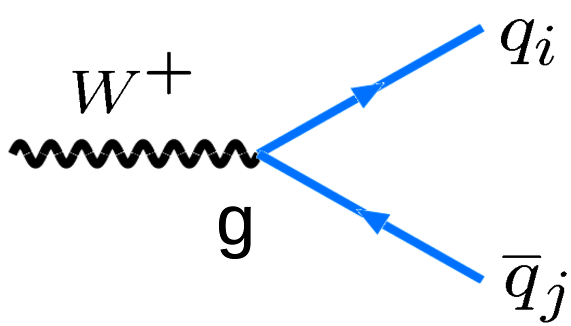
- Lagrangian after going from interaction to mass eigenstates

No more universal coupling constant!

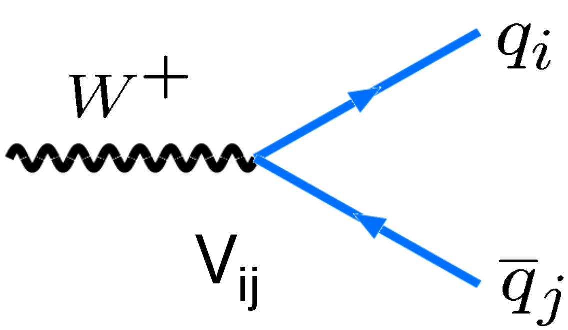
$$\mathcal{L}_W = -\sqrt{\frac{1}{2}} g \overline{u_{Li}} \gamma^\mu \boxed{\overline{\mathbf{V}}_{ij}} d_{Lj} W^+_\mu + \text{h.c.} \qquad \overline{\mathbf{V}} = \mathbf{V}_{uL}\mathbf{V}^\dagger_{dL}$$

No More Universality of Weak Interactions

- In absence of CKM matrix all weak interactions have same coupling
  - This is referred to as universality of weak interactions



- Because of CKM matrix coupling depends on quarks involved in the transition
  - Universality is broken!



Cabibbo-Kobayashi-Maskawa Matrix

$$V_{CKM} = V_{uL}^{\dagger} V_{dL} \qquad \mathbf{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Origin of CKM matrix is the difference between mass eigenstates and weak interaction eigenstates
- Lagrangian of Standard Model is diagonal in weak eigenstates with universal coupling constant
- Universality is broken when moving from interaction basis to mass basis necessary to obtain Lagrangian for mass terms after spontaneous symmetry breaking
- V_{CKM} is a unitary complex matrix

Properties of CKM Matrix

M(diag) is unchanged if $V_L'^f = P^f V_L^f$; $V_L'^f = P^f V_R^f$ $V(CKM) = P^u (CKM) P^{*d}$

P^f = phase matrix

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \to \begin{pmatrix} e^{-i\varphi_1} & 0 \\ 0 & e^{-i\varphi_2} \end{pmatrix} \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \begin{pmatrix} e^{-i\chi_1} & 0 \\ 0 & e^{-i\chi_2} \end{pmatrix} = \begin{pmatrix} V_{11} e^{-i(\varphi_1-\chi_1)} & V_{12} e^{-i(\varphi_1-\chi_2)} \\ V_{21} e^{-i(\varphi_2-\chi_1)} & V_{22} e^{-i(\varphi_2-\chi_2)} \end{pmatrix}$$

$$(\varphi_2-\chi_2) = (\varphi_2-\chi_1) + (\varphi_1-\chi_2) - (\varphi_1-\chi_1)$$

Among 4 phases, only 3 can be arbitrarly chosen and removed (so 2n-1)

Generally for a rotation matrix in complex plane

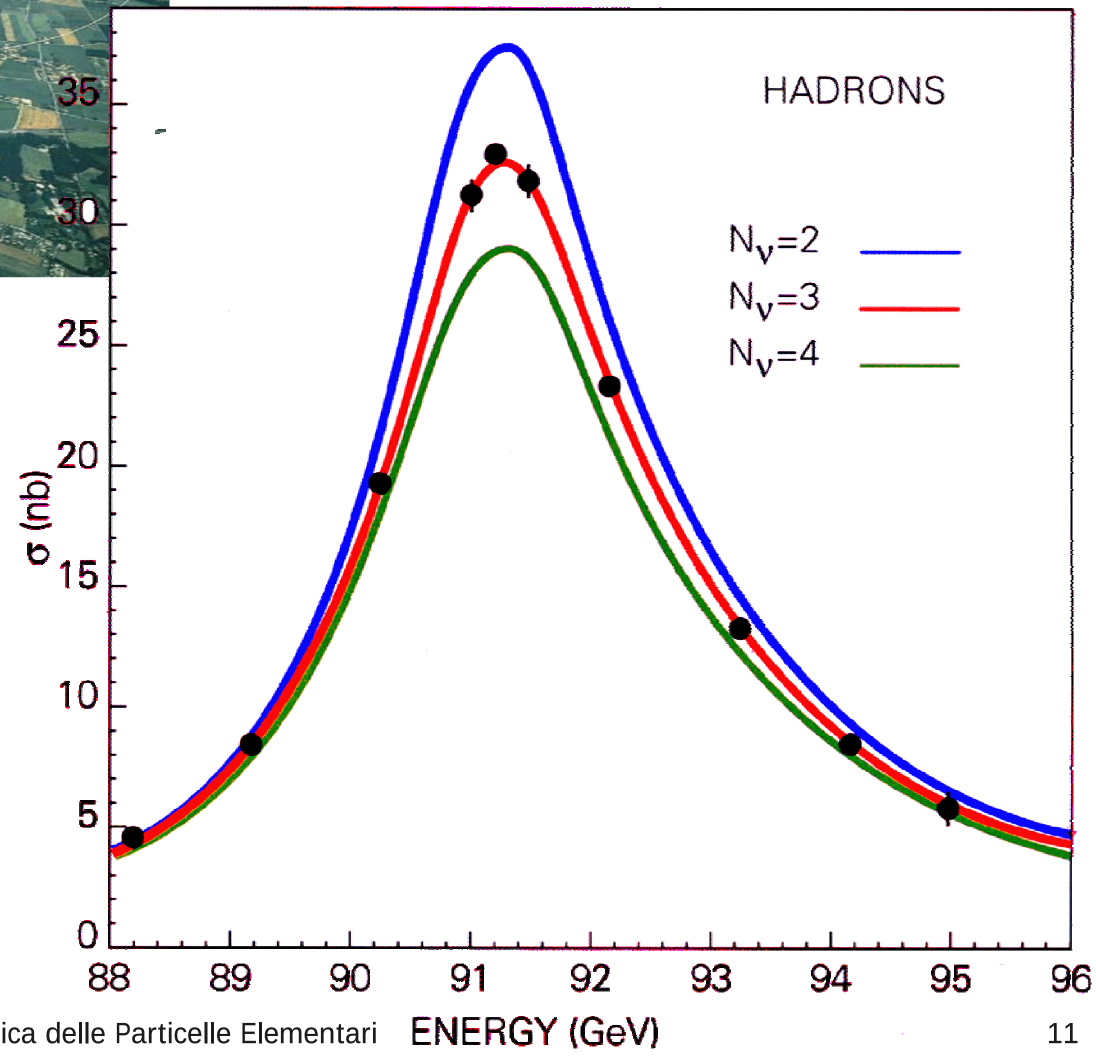
Quark families	# Angles	# Phases	# Irreducible Phases
n	n(n-1)/2	n(n+1)/2	n(n-1)/2-(2n-1)=(n-1)(n-2)/2
2	1	3	0
3	3	6	1
4	6	10	3

Necessary for CP Violation in SM

- Today we know there are three flavors, or generations of quarks
- But this was not the case when CKM matrix was first proposed in 1973!

How do we know there only 3 generations of matter?

Number of neutrino families from LEP @ CERN



Families of Matter known in 1972

Three Quarks for Muster Mark !…*Joyce*

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} ? \\ s \end{pmatrix}$$

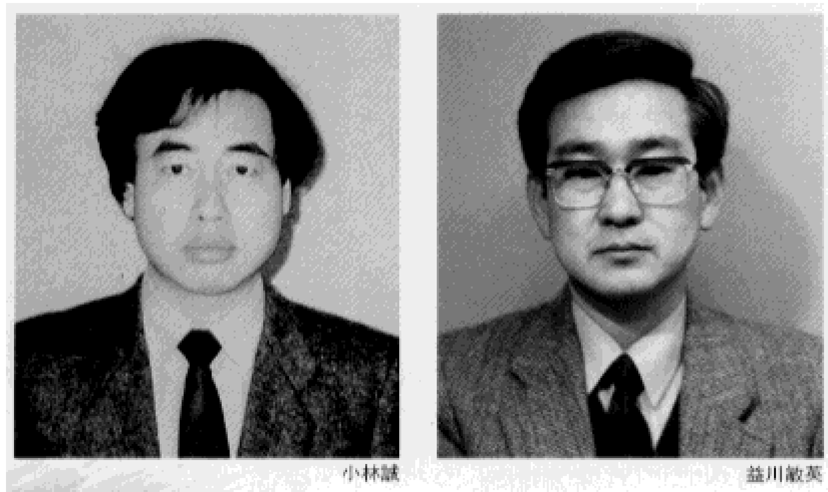
$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}$$

Only 2 families were known

Charm quark not even observed yet!

Kobayashi-Maskawa Mechanism of CP Violation

1972



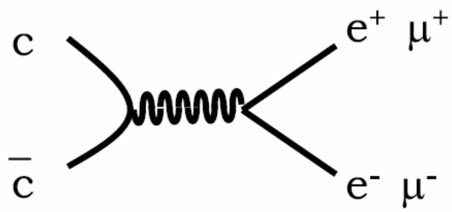
Two Young Postdocs at that time !

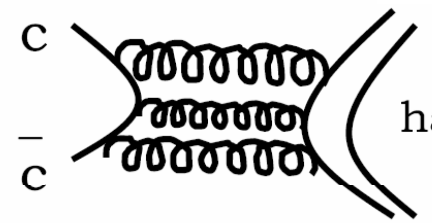
- Proposed a daring explanation for CP violation in K decays
- CP violation appears only in the charged current weak interaction of quarks
- There is a single source of CP Violation $\Rightarrow$ Complex Quantum Mechanical Phase δ_{KM} in inter-quark coupling matrix
- Need at least **3 Generation of Quarks** (then not known) to facilitate this
- **CP is NOT an approximate symmetry**, $\delta_{KM} \simeq 1$, it is MAXIMALLY violated !

1974: Discovery of charm in J/psi

Seen as a resonance $\quad$ ***m*~3.1 GeV** $\quad$ **Γ~10-100KeV**

•Brookhaven (p on Be target)

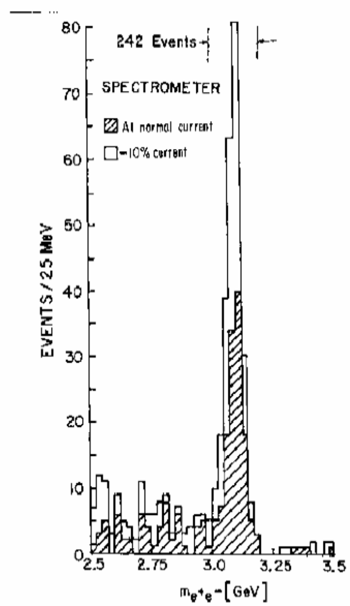




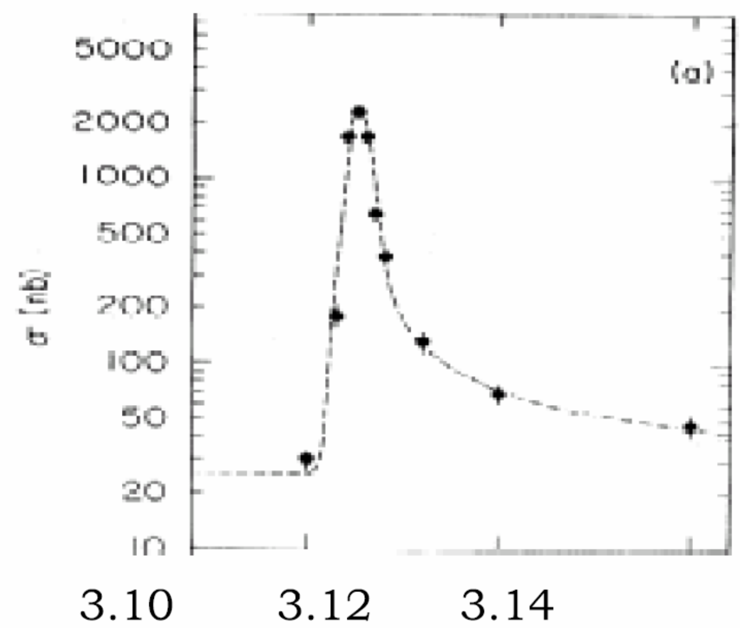
Γ~70 KeV

Γ(ee)~5 KeV
Γ(μμ)~5 KeV

e^+e^- final state



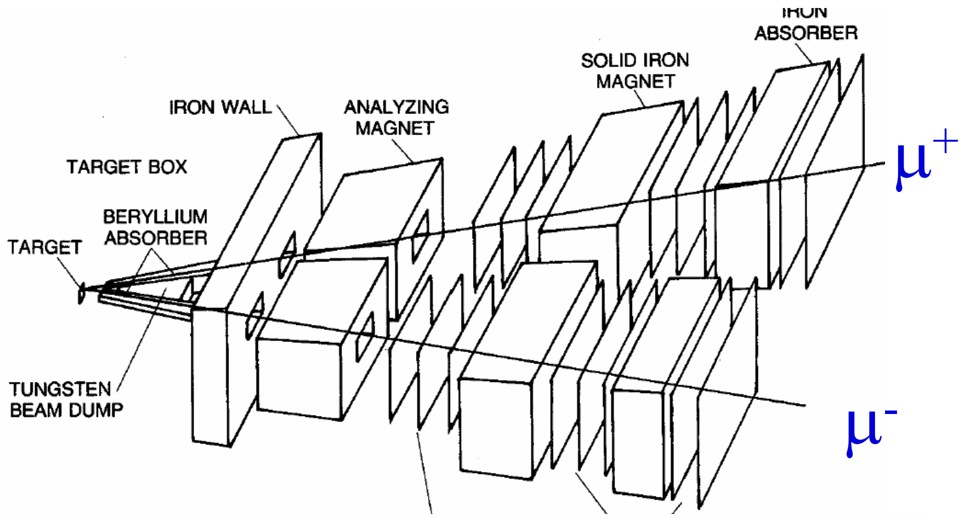
SLAC (e+e-)



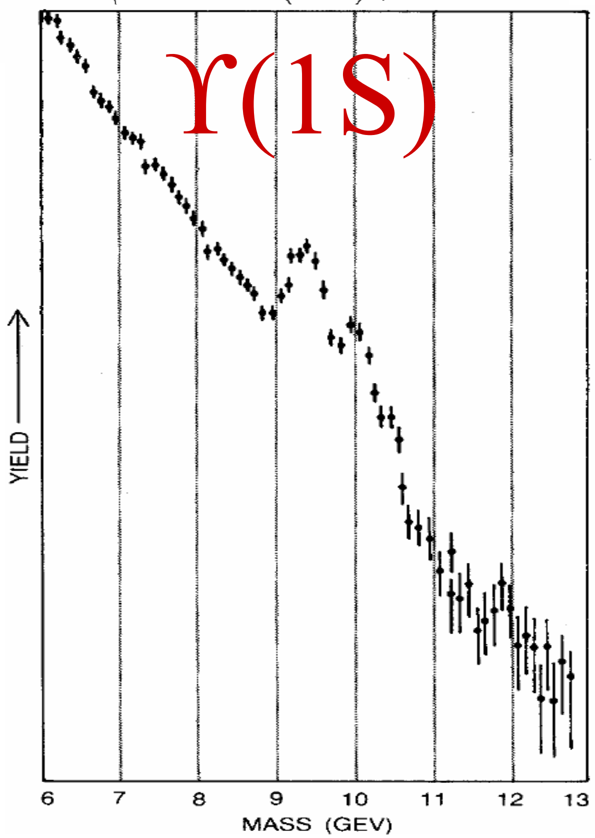
hadronic final state

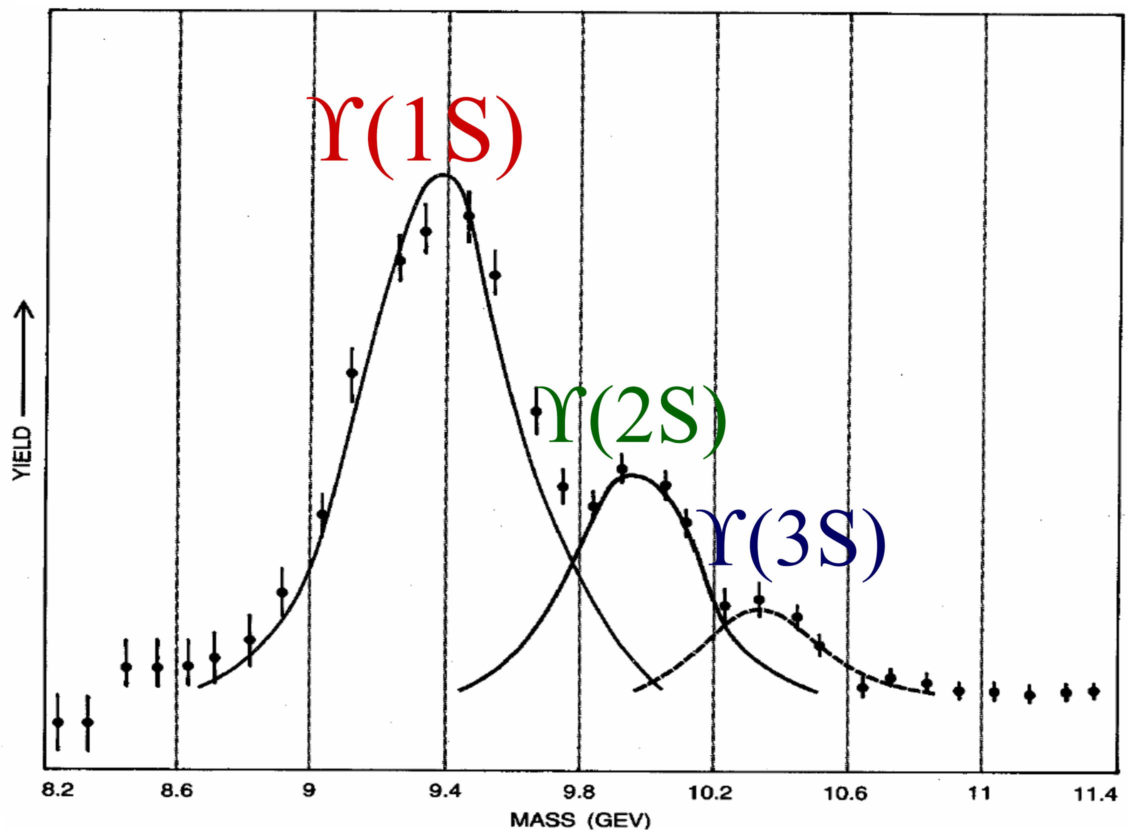
The decay through strong interaction is so suppressed that the electromagnetic interaction becomes important

1977: Discovery of bottom in Upsion(1S) @ FNAL

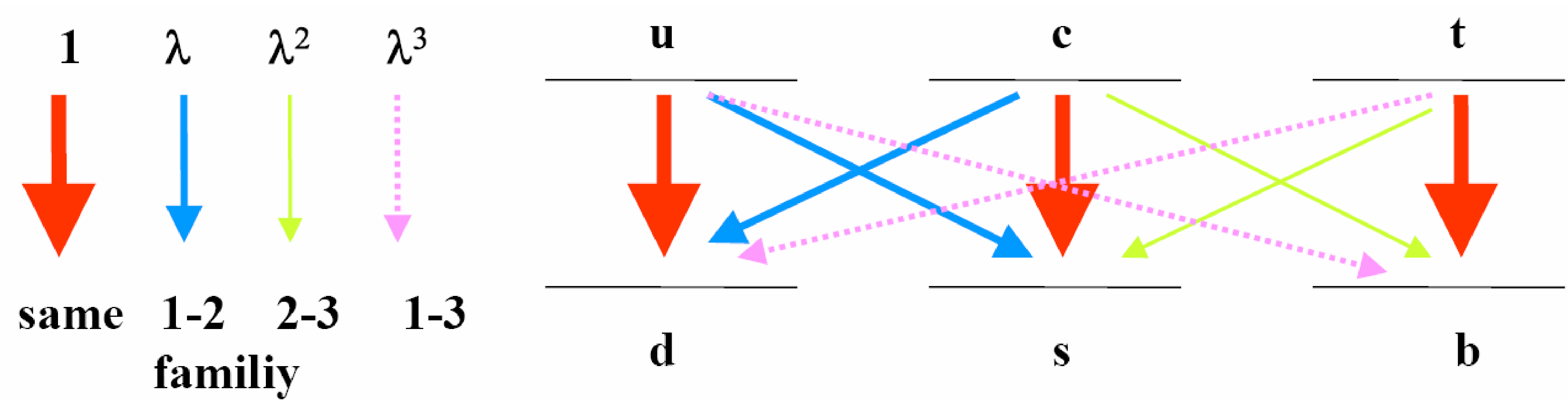


PRL **39** 252(1977)





Features of CKM Matrix



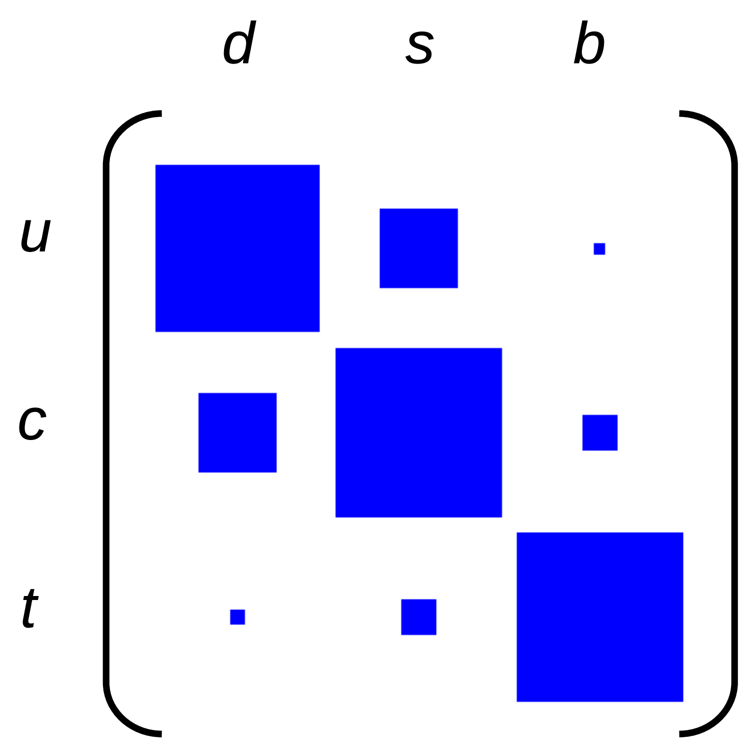
Diagonal elements ~ 1

V_{cb} , V_{ts} $\sim 4\times 10^{-2}$

V_{us} , V_{cd} ~ 0.2

V_{ub} , V_{td} $\sim 4\times 10^{-3}$

Relative magnitudes



Wolfenstein Parameterization of CKM Matrix

- Wolfenstein first saw a pattern with 4 parameters

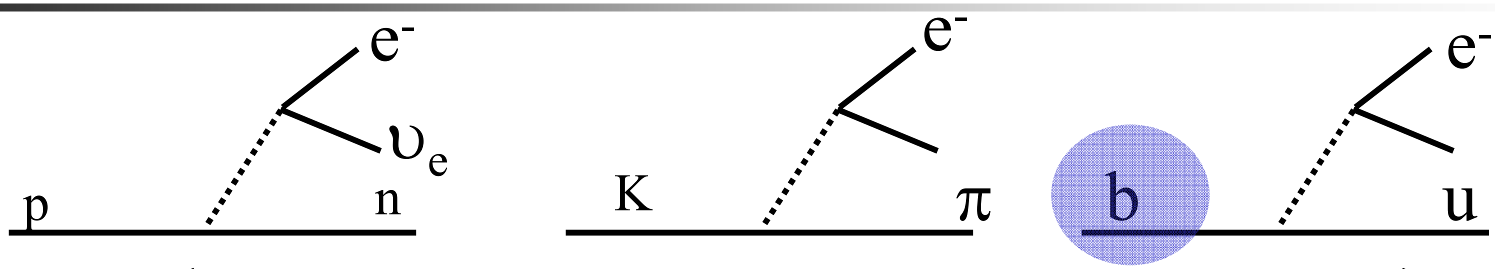
Cabibbo angle with 2 generations

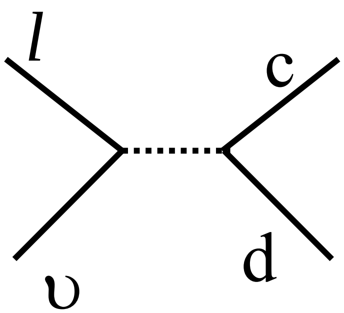
$$\mathbf{V}_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$$\begin{aligned} \lambda &= |V_{us}| &\approx 0.22 \\ A &= |V_{cb}|/\lambda^2 &\approx 0.80 \\ \sqrt{\rho^2+\eta^2} &= |V_{ub}|/\left(\lambda|V_{cb}|\right) &\approx 0.35 \\ \eta/\rho &= \tan\left[\arg\left(V_{ub}\right)\right] &\approx\ ? \end{aligned}$$

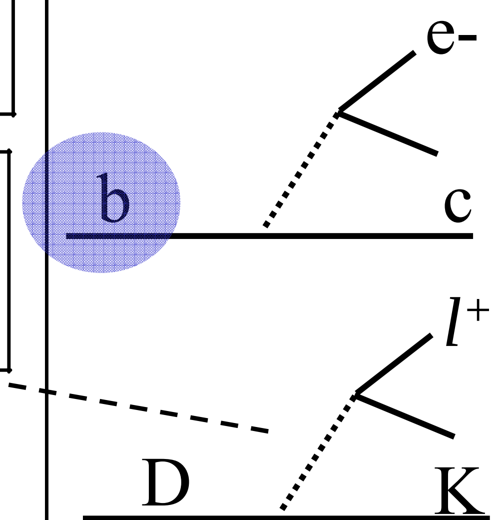
$$\begin{pmatrix} 1 & 1 & e^{-i\gamma} \\ 1 & 1 & 1 \\ e^{-i\beta} & 1 & 1 \end{pmatrix}$$

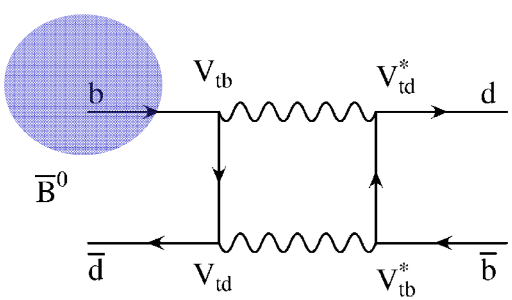
Measurements of CKM Element **Magnitudes**

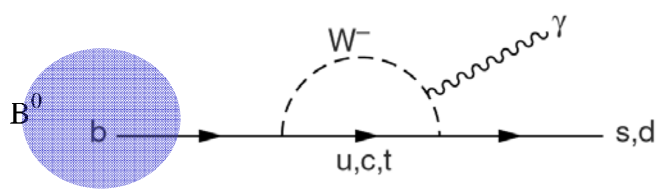


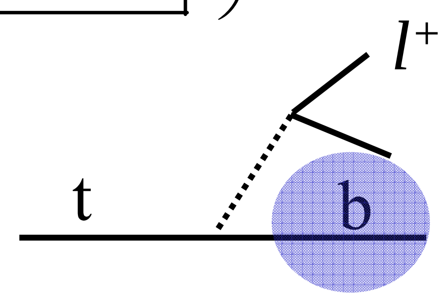


$V_{ud}=0.9738$ (±0.05%)	$V_{us}=0.2196$ (±0.4%)	$V_{ub}=0.004$ (±13%)
$V_{cd}=.224$ (±5.4%)	$V_{cs}=0.97$ (±11.0%)	$V_{cb}=0.041$ (±3.6%)
$V_{td}\approx 0.01$ (±100%)	$V_{ts}\approx 0.047$ (±17.0%)	$V_{tb}=0.94$ (±29%)









b quark plays a special role in determination of CKM elements!

Measuring CKM Elements

- Measurements related to first 2 generations briefly discussed here
  - Most measurements established since a while

- Mostly focus on decays of B mesons and related measurements because
  - B factories at SLAC and KEK since 1999 have allowed a detailed study of many B decays that were not available previously

  - B mesons are an excellent laboratory to study CP Violation
    - observations of 2 different types of CP violation in B mesons since 2001!
    - First observation in 1964 with neutral Kaons

- Redundant measurements of same observables in different processes allow to verify CKM paradigm
  - Discrepancies could be a sign of New Physics beyond Standard Model
  - For example: use measurements to verify unitarity of CKM matrix

Word of Caution

- CKM matrix elements describe processes at quark level but processes observed experimentally involve hadrons

- Theory is used to relate measurements with hadrons to quantities defined for quarks
  - HQET, OPE, Lattice QCD

- Ultimately must verify theories with measurements

- When models are used to interpret data this should be described clearly and some kind of error assigned to the model-dependency

CKM Elements in First Two Generations

$$\mathbf{V}_{CKM} = \begin{pmatrix} \boxed{\begin{matrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{matrix}} & \begin{matrix} V_{ub} \\ V_{cb} \end{matrix} \\ \begin{matrix} V_{td} & V_{ts} \end{matrix} & V_{tb} \end{pmatrix}$$

$$\mathbf{V}_{CKM} = \begin{pmatrix} \boxed{\begin{matrix} 1-\lambda^2/2 & \lambda \\ -\lambda & 1-\lambda^2/2 \end{matrix}} & \begin{matrix} A\lambda^3(\rho - i\eta) \\ A\lambda^2 \end{matrix} \\ \begin{matrix} A\lambda^3(1-\rho-i\eta) & -A\lambda^2 \end{matrix} & 1 \end{pmatrix} + O(\lambda^4)$$

Measuring $|V_{ux}|$ and $|V_{cx}|$

$|V_{ud}|$: 1) Super-allowed nuclear β-decays

2) Neutron β-decay

3) Pionic β-decay

$|V_{us}|$: 1) Semileptonic Kaon decays

2) Leptonic Kaon & Pion decay

$|V_{cd}|$, $|V_{cs}|$: 1) Dimuon production from neutrinos on nuclei

2) Semileptonic D-meson decays

$|V_{ud}|$: β Decays

Fermi-transitions: $0^+ \to 0^+$ within same isospin multiplet
pure vector-current (take advantage of CVC)

$$|V_{ud}|^2 = \frac{2\,\pi^3 \ln 2}{m_e^5} \cdot \frac{1}{2\,G_F^2\,(1+\Delta_R)\,Ft}\,, \qquad Ft = f \cdot t_{1/2} \cdot (1+\delta_R) \cdot (1-\delta_C)$$

Radiative Correction (nucleus-independent)
$\Delta_R = (2.40 \pm 0.08)\%$

1) PS Integral ($\sim E_0^5$)
2) Radiative Correction (nucleus-dependent)
3) Isospin-symmetry breaking

Neutron β-decays: $n \to p\, e^- \,\nu_e$

Vector transition: $G_V = g_V\, G_F\, |V_{ud}|$ (CVC <=> Isospin Cons.: $g_V=1$)

Axial-V. transition: $G_A = g_A\, G_F\, |V_{ud}|$ (PCAC: $g_A/g_V \equiv \lambda \neq 1$)

$$\frac{1}{\tau_n} = \frac{(m_e c^2)^5 \cdot G_F^2 \cdot |V_{ud}|^2}{2\,\pi^3\,\hbar\,(\hbar c)^6} \cdot (1+3\lambda^2) \cdot f\,(1+\delta_R) \cdot (1+\Delta_R)$$

=> **Measure τ_n and λ**

Gamov-Teller-transition => g_A

Fermi-transition => g_V

$|V_{us}|$: Semileptonic K Decays

K_{l3} decays: $K^+ \to \pi^0 l^+ \nu_l$ and $K_L \to \pi^- l^+ \nu_l$, $0^- \to 0^-$ (pure Vector transitions)

$$\Gamma_{K_{l3}} = \frac{\left(m_K c^2\right)^5 \cdot G_F^2 \cdot |V_{us}|^2}{192\ \pi^3\ \hbar\ \left(\hbar c\right)^6} \cdot C^2 \cdot |f_+(0)|^2 \cdot I \cdot \left(1+\Delta_R\right)\left(1+\delta_R\right)$$

Normalisation:

K^+_l: $C = 1/\sqrt{2}$

K^0_l: $C = 1$

Phase Space Integral: $I = I(f_+, (m_l/m_K)^2 f_0)$

=> K_{e3} preferred

$$< K(p_K)|\bar{u}\gamma^\mu s|\pi(p_\pi)> = C\left((p_K^\mu + p_\pi^\mu) f_+(q^2) + (p_K^\mu - p_\pi^\mu) f_-(q^2)\right),\ q^\mu = (p_K^\mu - p_\pi^\mu)$$

What Did We Learn?

- Semileptonic decays are main approach to measurement of these first 4 CKM elements
  - Measure branching fractions and lifetimes
  - One vertex is leptonic → No CKM element
  - One vertex is hadronic → Only 1 CKM element in decay amplitude
  - Extract CKM element for experimental measurement

- Where do we need theory and why
  - Hadronic part of semileptonic decay amplitudes parameterized via form factors
  - Hadronic vertex in leptonic decays parameterized with decay constants
  - Estimate form factors with lattice QCD

b Quark is Special!

- Processes involving b quark can be used to measure several CKM element magnitudes

- Large mass of b quark allows use of Heavy Quark Effective Theory (HQET) for reliable theoretical calculations
  - Important for interpretation of experimental measurements with B mesons

- B mesons are of particular interest for study of CP violation
  - We will discuss this in detail next week

- Highlights of b quark
  - Heavy mass: big phase space an hence variety of final states to decay to
  - Long lifetime: important for experimental techniques to identify B mesons
  - B0-B0bar oscillation: a fine example of quantum entanglement, important ingredient for CP violation
  - b$\rightarrow$u transitions: necessary ingredient for CP violation

Overview of B mesons

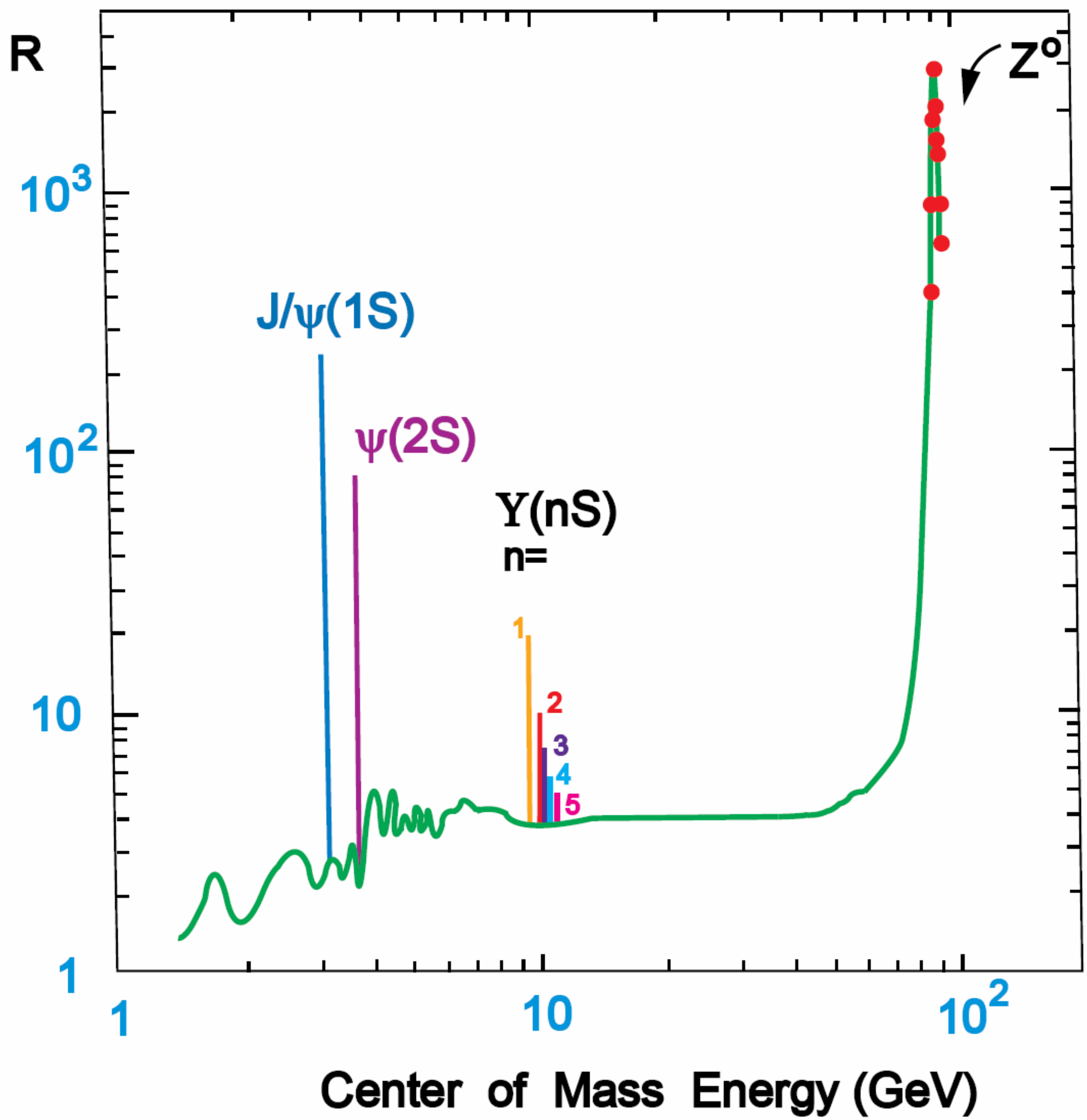
- Properties of B mesons
- B meson Production
- B decays

Summary of B properties

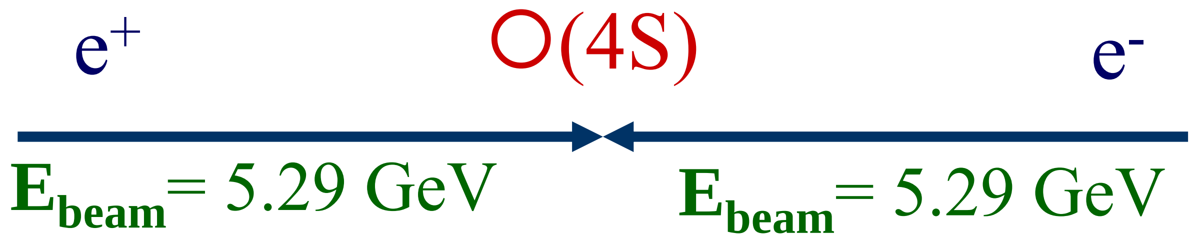
Particle, $I(J^P)$	Mass (in MeV/c^2)	Lifetime τ =1/Γ (in10^{-12} s)
B^0_d =(bd) , $I(J^P)$=½ (0^-)	5279.4 ± 0.5	1.536 ±0.014 & (cτ =460μm)
B^- = (bu), $I(J^P)$=½ (0^-)	5279.0 ± 0.5	1.671 ±0.018 & (cτ =501μm)
B^0_s =(bs), $I(J^P)$=0(0^-)	5369.6 ± 2.4	1.461 ±0.057 & (cτ =438μm)
Λ_b = (bud), $I(J^P)$=0($1/2^+$)	5624.0 ± 9.0	1.229 ±0.080 & (cτ =368μm)

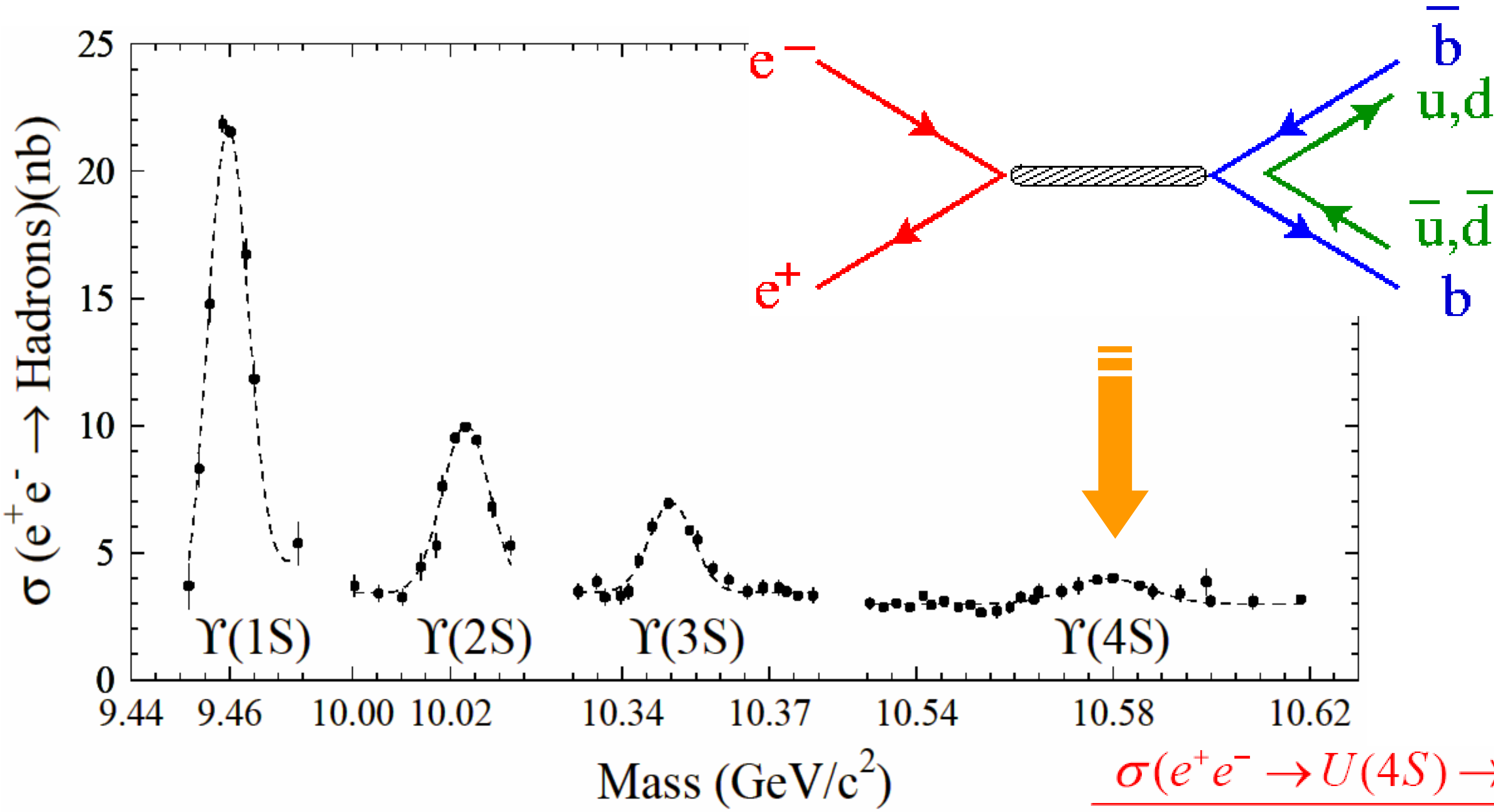
B Production in e^+e^- Collisions

$$R = \frac{e^+e^- \to \text{all}}{e^+e^- \to \gamma^* \to \mu^+\mu^-}$$



B Production at Upsilon resonance: B Factory

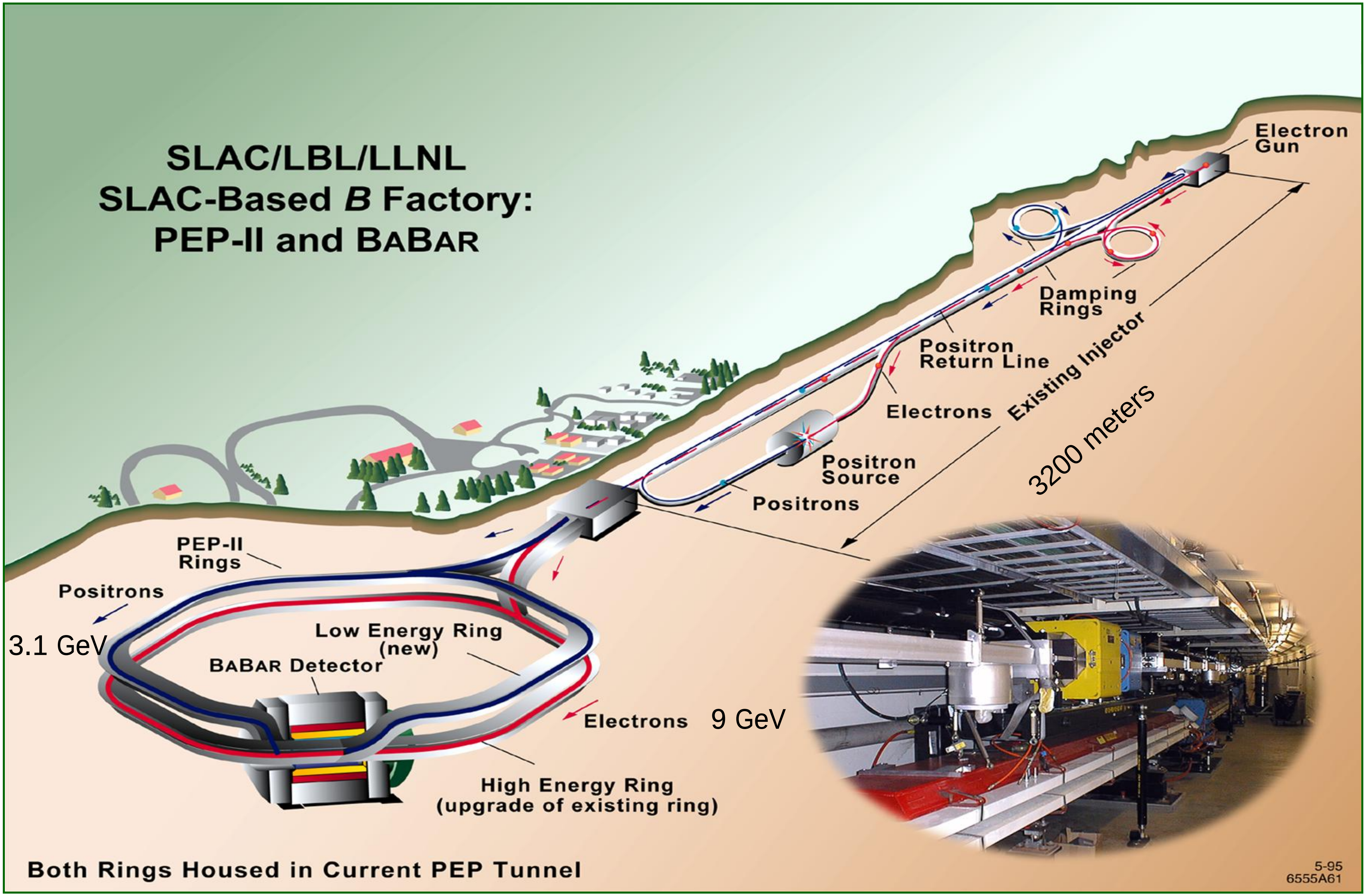




$$\frac{\sigma(e^+e^- \to U(4S) \to bb)}{\sigma(e^+e^- \to U(4S) \to TOT)} = \frac{1.2nb}{3.5nb} = 25\%$$

Enough energy to barely produce 2 B mesons, nothing else!
B Mesons produced with ~ 300 MeV momentum
Moving very slowly, don’t travel much before decay

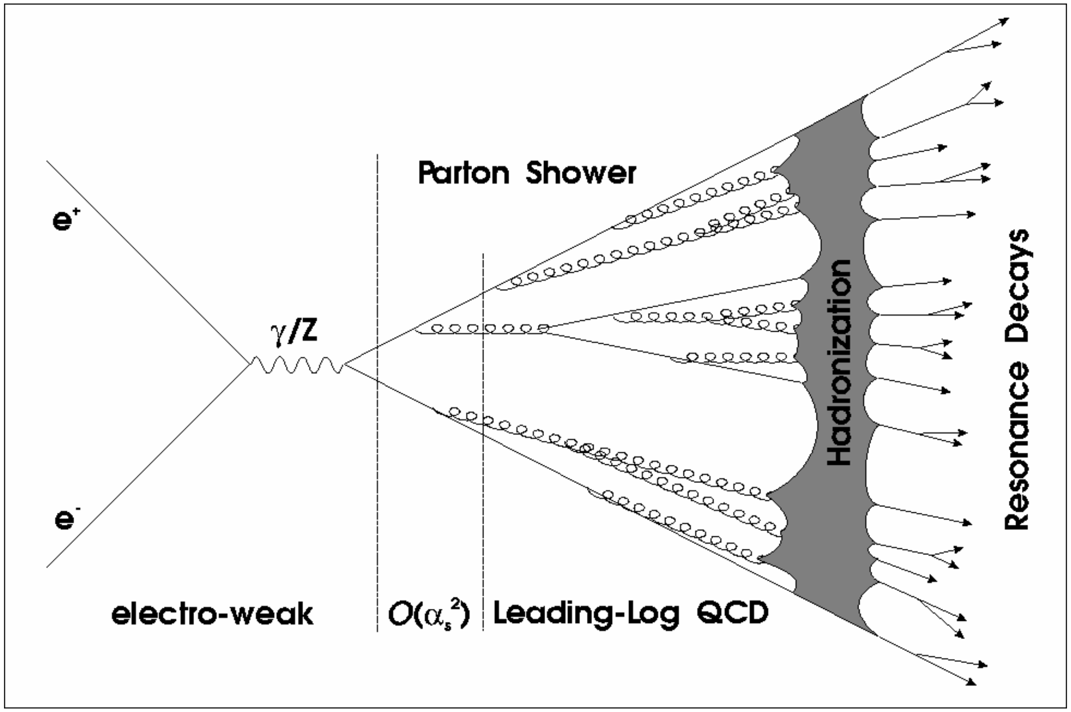
PEP-II Collider at SLAC (Stanford, CA)



PEP-II accelerator schematic and tunnel view

B Production at Z^0 Resonance

All types of B hadrons produced in Z → bb hadronization



b hadron	Fraction [%]
B^+, B^0	39.7 ± 1.0
B_s^0	10.7 ± 1.1
b baryons	9.9 ± 1.7

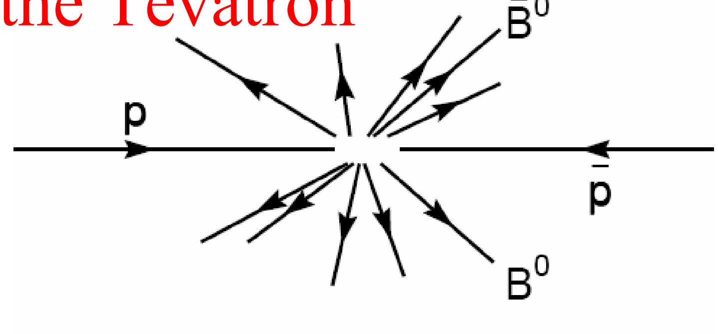
$$\frac{\Gamma(b\bar{b})}{\Gamma(TOT)} \sim 17\%$$

Average B momentum ~ 35 GeV
$\Rightarrow (\beta\gamma)_B \approx 7$ (highly relativistic)

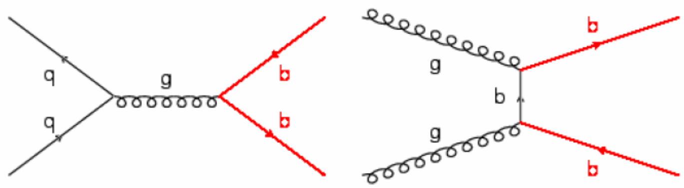
LEP/SLD Program ended in ‘95, made important contributions to b physics

B Production in pp Collisions

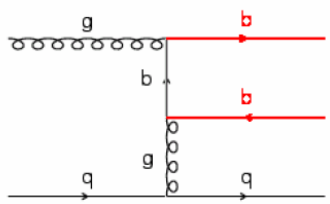
At the Tevatron



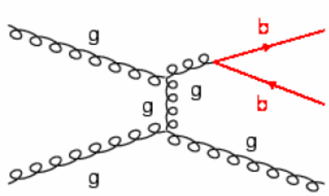
Lowest order

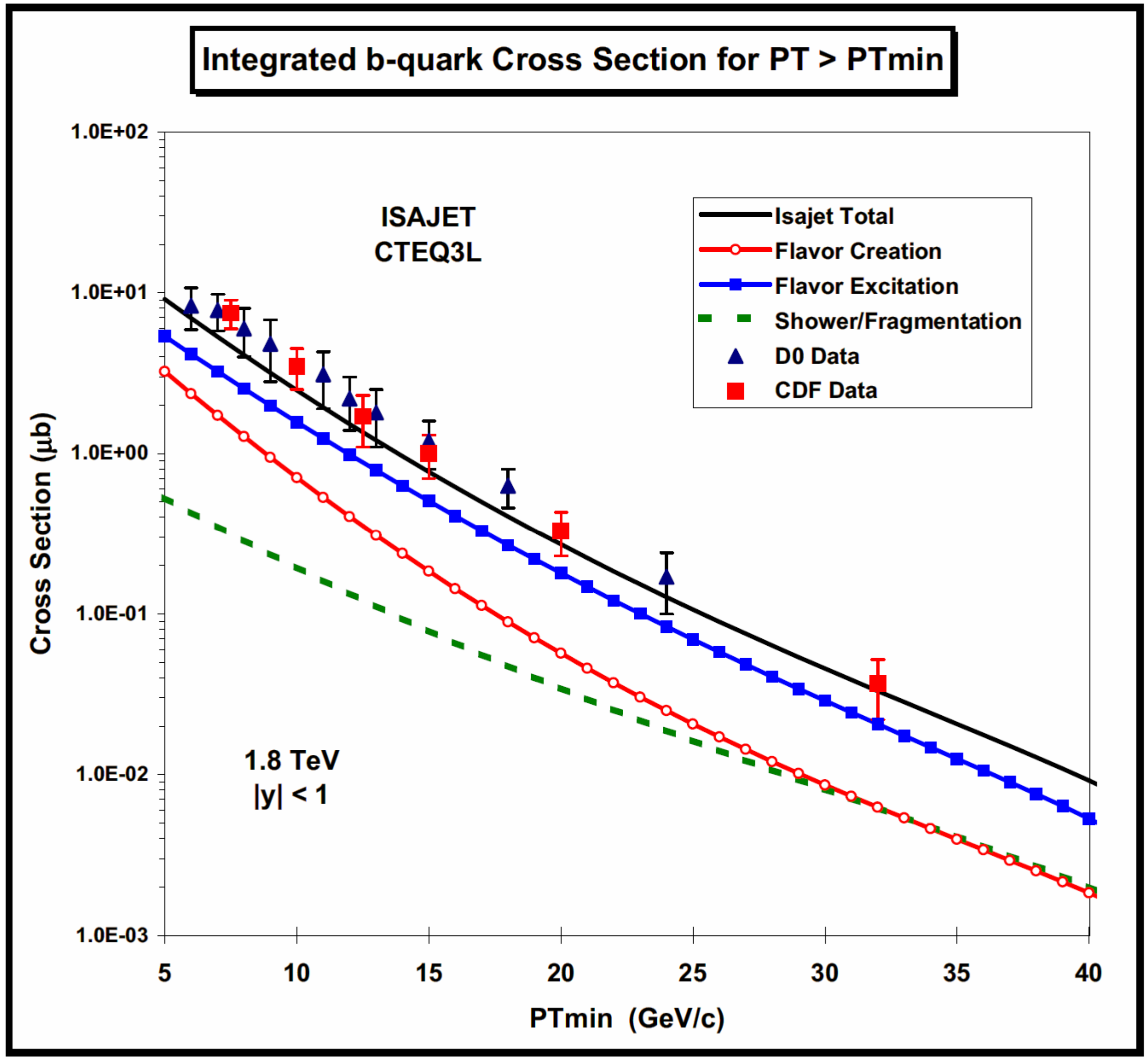


Flavor excitation



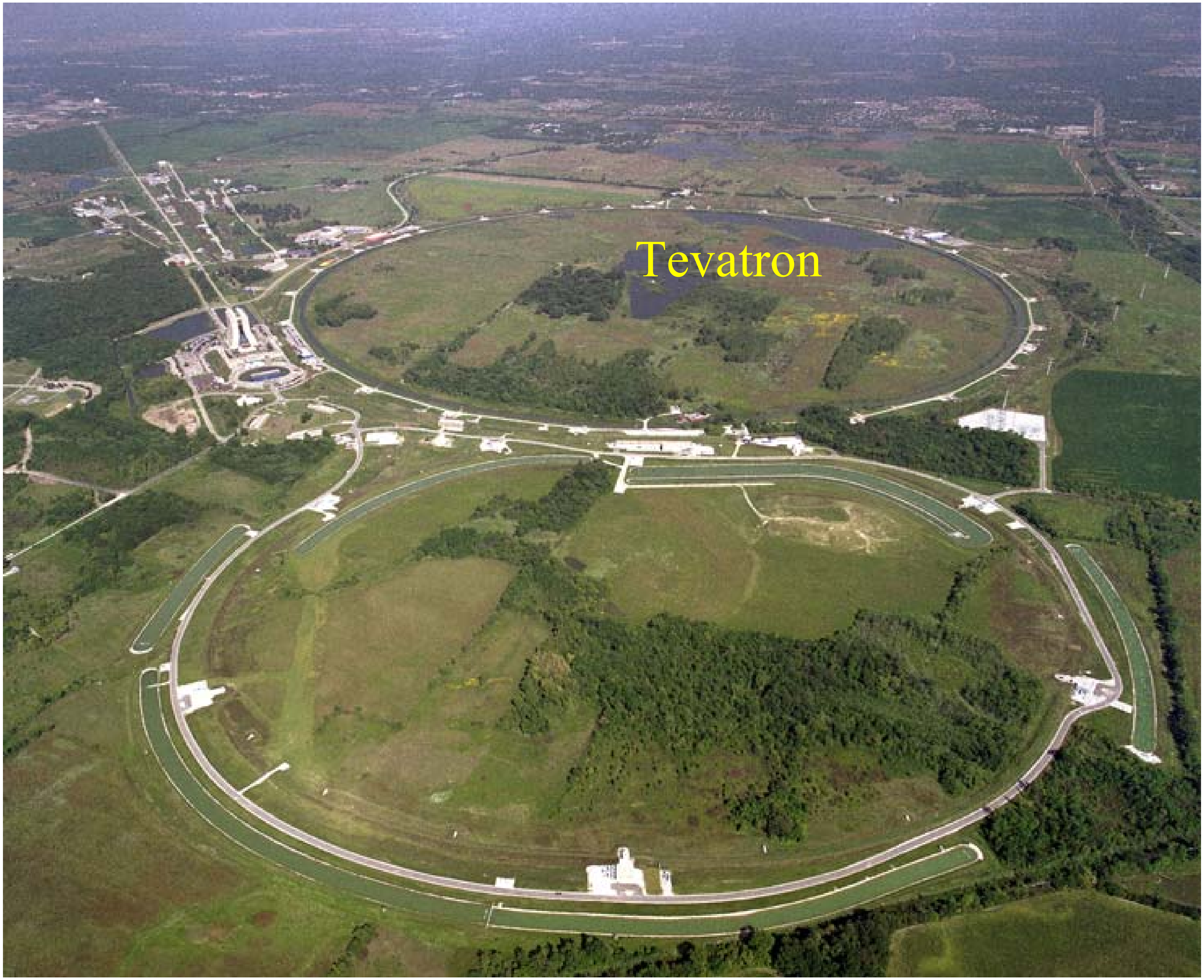
Gluon splitting





$$\sigma(b\bar{b}) \sim > 10\mu b \qquad \frac{\sigma(b\bar{b})}{\sigma(TOT)} = \frac{1}{1000}$$

Tevatron at Fermilab (Chicago, IL)



Summary of Past and Present Experiments

Experiments	*# of b events*	*Environment*	*Characteristics*
LEP Coll. ALEPH/DELPHI/ L3/OPAL	~1M (each expt.)	Z^0 decays (σ~6nb)	Back-to-back 45GeV b-jets All B hadrons produced **Stopped**
SLD	~0.1M	Z^0 decays (σ~6nb)	Back-to-back 45GeV b-jets All B hadrons produced Beam polarized **Stopped**
ARGUS	~0.2M	$\Upsilon(4S)$ decays (σ~1.2nb)	B mesons produced at rest B^0 and B^+ produced **Stopped**
CLEO	~9M	$\Upsilon(4S)$ decays (σ~1.2nb)	B mesons produced at rest B^0 and B^+ produced **Running at charm threshold**
Belle Babar	~130M (each expt.)	$\Upsilon(4S)$ decays (σ~1.2nb)	B mesons produced at rest B^0 and B^+ produced **Running**
TeVatron Coll. CDF/D0	~several	$p\bar{p}$ collider E(c.d.m)=1.8 TeV	Triggered events All B hadrons produced **Running**