

Cabibbo-Kobayashi-Maskawa Matrix and CP Violation in Standard Model

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SAPIENZA
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Lecture 2
Measurement of CKM Elements

Lezioni di Fisica delle Particelle Elementari

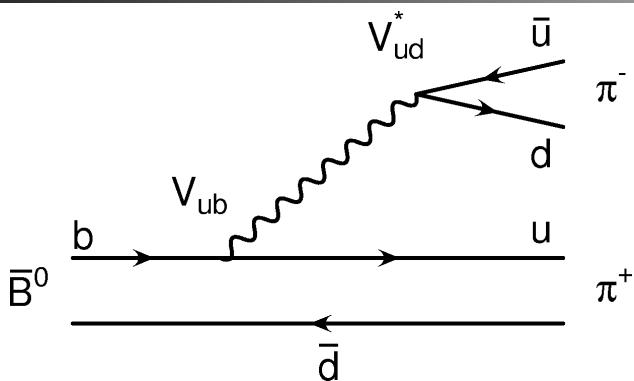
Outline of this Lecture

- Introduction to diagrams
 - Catalog of B decays
- Primer of B meson reconstruction at B Factories
 - Kinematic variables
 - Background suppression
- Measurement of V_{ub} and V_{cb} and V_{tb} in details
- Challenges of measuring V_{td} and V_{ts}

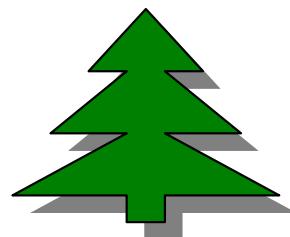
Catalog of B decays

Introduction to Diagram Jargon!

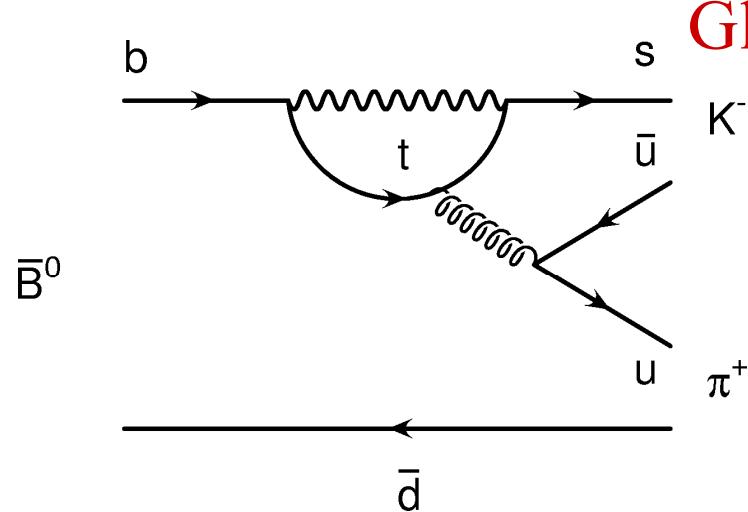
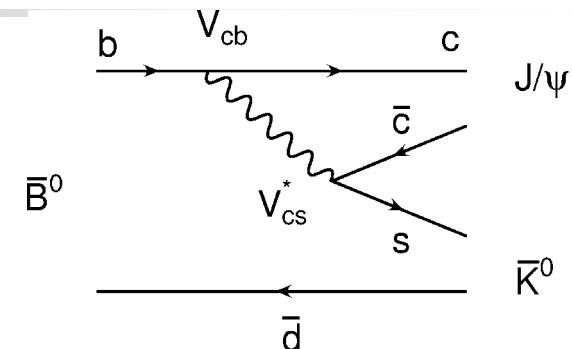
Spectator



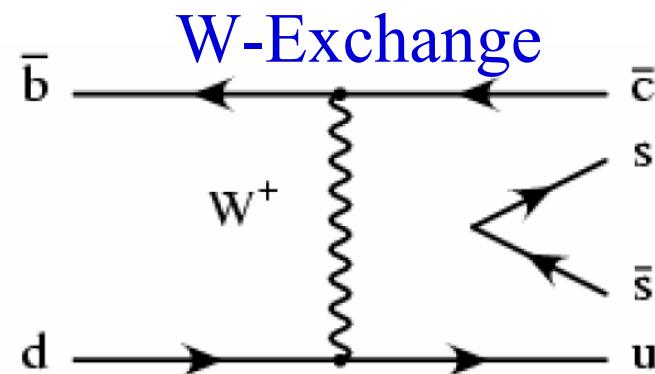
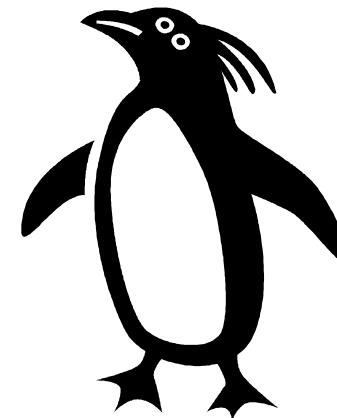
Tree Diagrams

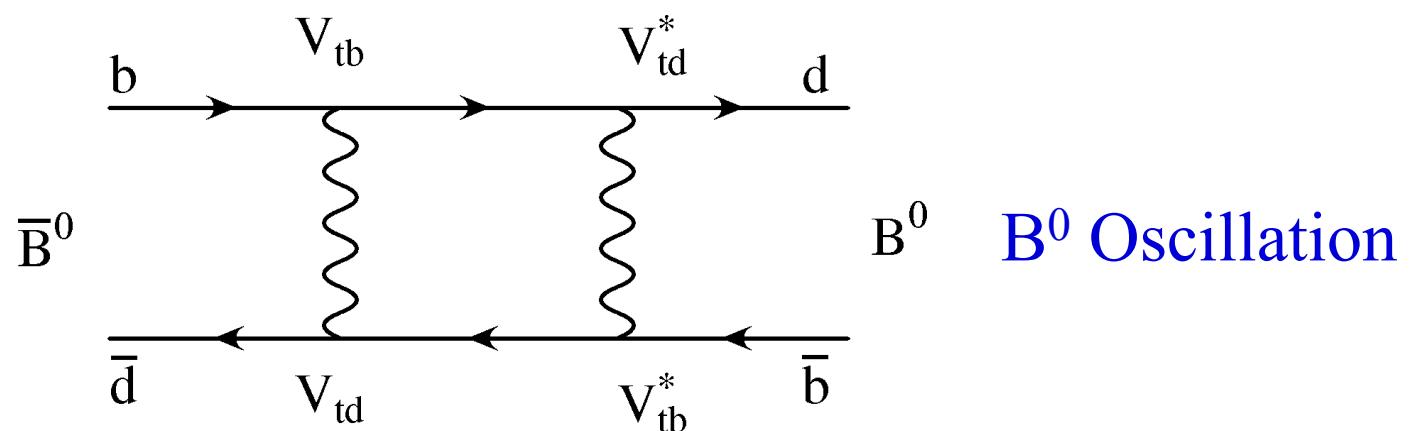
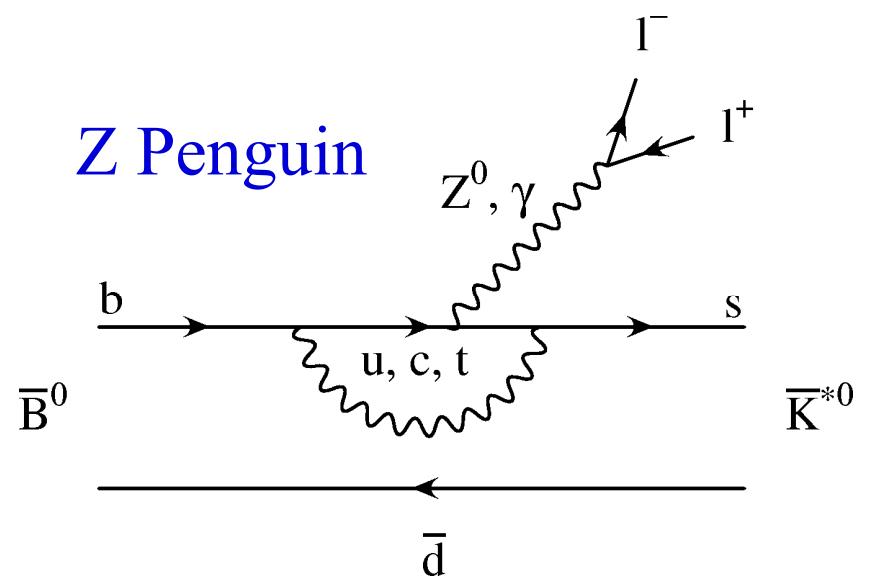
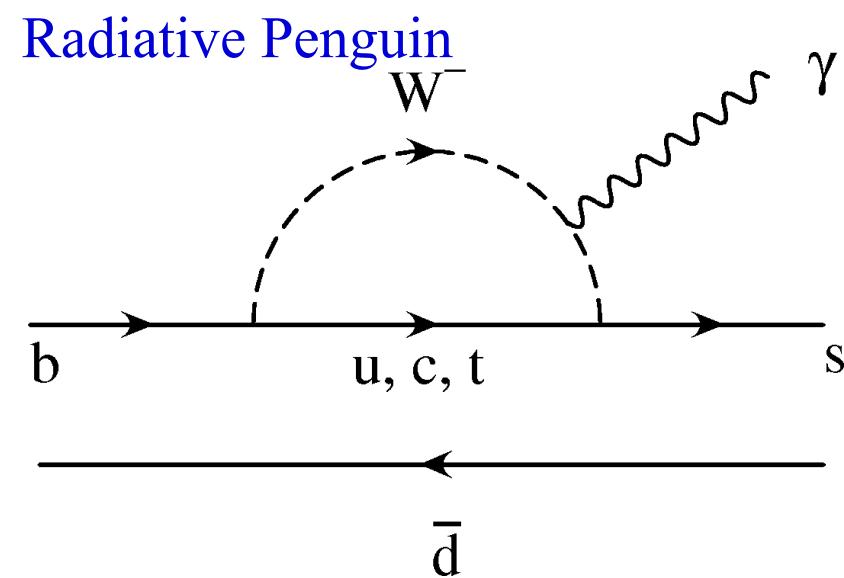
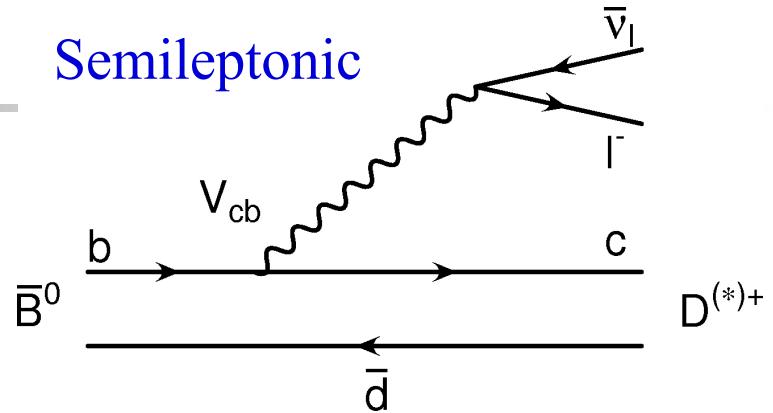
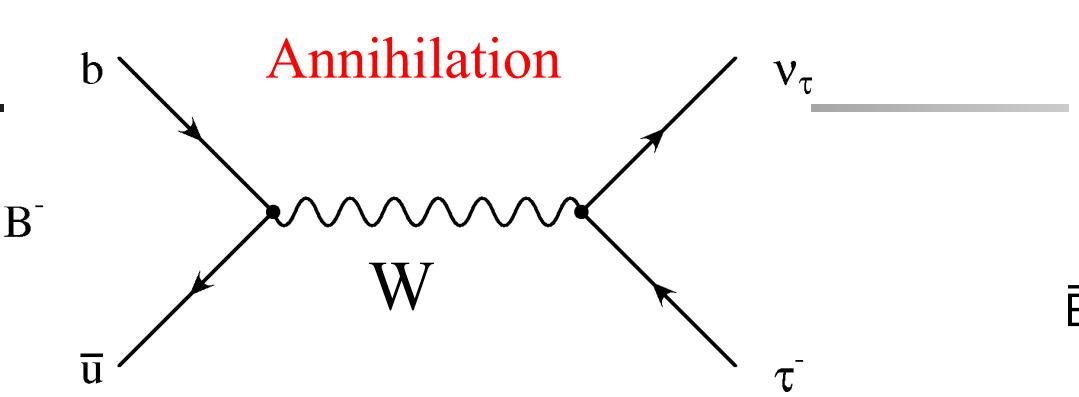


Color Suppressed



Gluonic Penguin



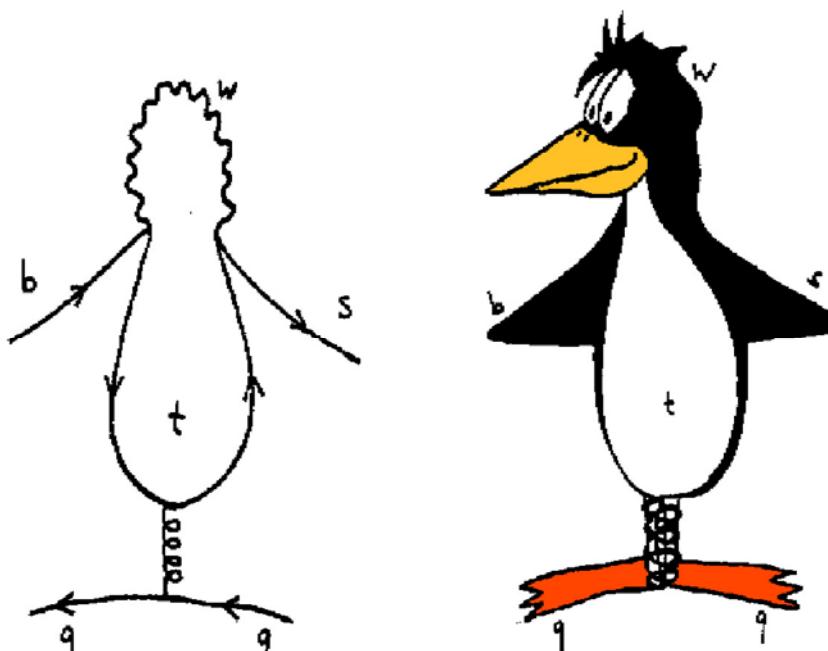


Penguins in B Decays!?

Ref: Preface to Shifman's 1999 book, ITEP Lectures on Particle Physics and Field Theory, John Ellis recalls how the gluon interference diagram came to be called a penguin diagram.

One night in spring 1977, Ellis lost a bet during a game of darts. His penalty required that he use the word "penguin" in a journal article. "For some time, it was not clear to me how to get the word into this b quark paper that we were writing at the time," Ellis wrote.

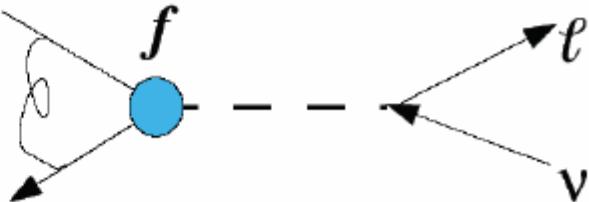
"Then, one evening I stopped on my way back to my apartment to visit some friends living in Meyrin, where I smoked some illegal substance. *Later, when I got back to my apartment and continued working on our paper, I had a sudden flash that the famous diagrams looked like penguins.* So we put the name into our paper, and the rest, as they say, is history."



Topology of Tree Decay Amplitudes

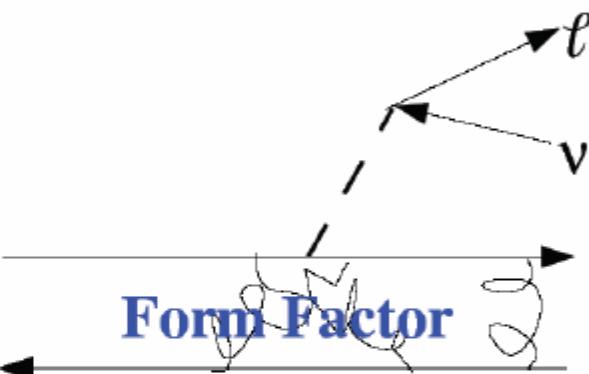
Leptonic

Hadronic & Leptonic
current do factorize



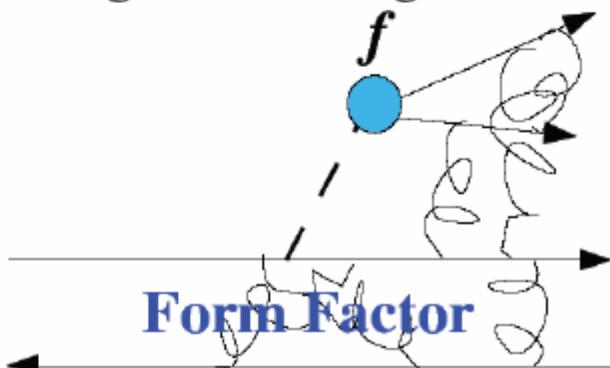
Semileptonic

(In most cases best
way to extract $|V_{ij}|$)



Hadronic

No factorization in naïve sense
due to gluon exchange



- * Low energy QCD: decay constant f
- * Lattice QCD starts to get precise

Exclusive Decays:

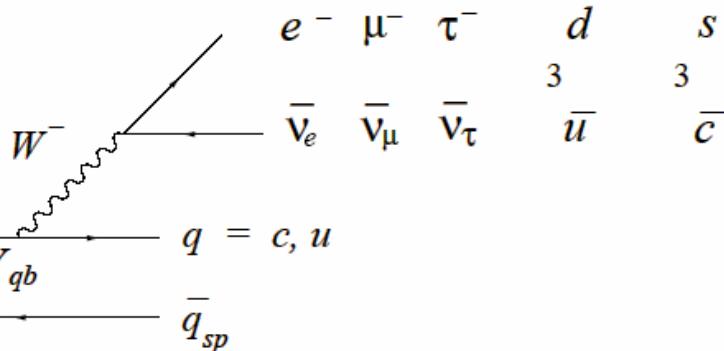
- * FF: Symmetries (χ & HQS)
- * FF: Lattice QCD, Sum Rules; ...

Inclusive Decays:

- * Operator Product Expansion

Theoretical developments:
e.g. QCD Factorisation approach
Not used for $|V_{ij}|$ extraction (yet)

Summary of b-quark Decay

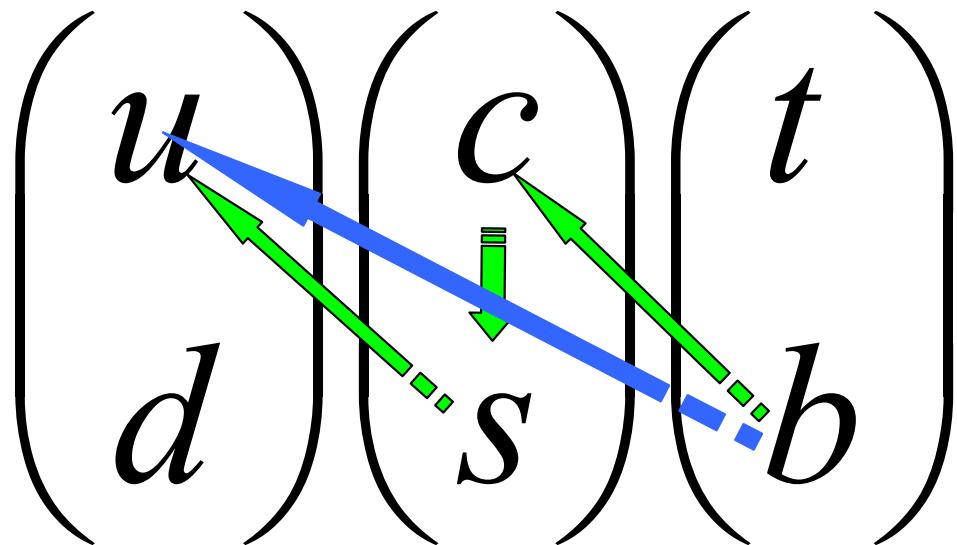


Mode	N_c	f_{QCD}	f_{ps}	V_{xb}	\mathcal{B}
$c\bar{u}d'$	3	1.3	0.52	0.038	48%
$c\bar{c}s'$	3	1.3	0.25	0.038	23%
$c e \bar{\nu}_e$	1	1.0	0.52	0.038	12%
$c \mu \bar{\nu}_\mu$	1	1.0	0.52	0.038	12%
$c \tau \bar{\nu}_\tau$	1	1.0	0.13	0.038	3%

$u\bar{u}d'$	3	1.3	1.00	0.003	0.6%
$u\bar{c}s'$	3	1.3	0.52	0.003	0.3%
$u e \bar{\nu}_e$	1	1.0	1.00	0.003	0.1%
$u \mu \bar{\nu}_\mu$	1	1.0	1.00	0.003	0.1%
$u \tau \bar{\nu}_\tau$	1	1.0	0.25	0.003	<0.1%

$s(d)g, \ s(d)\gamma, \ s(d)Z^0\dots$

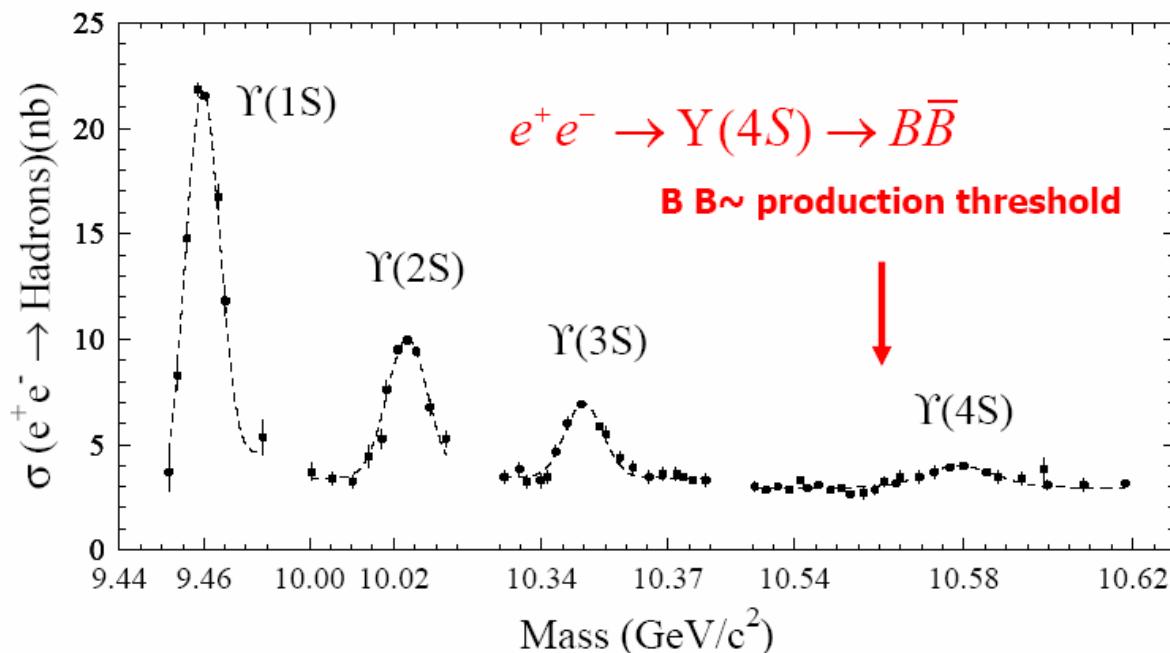
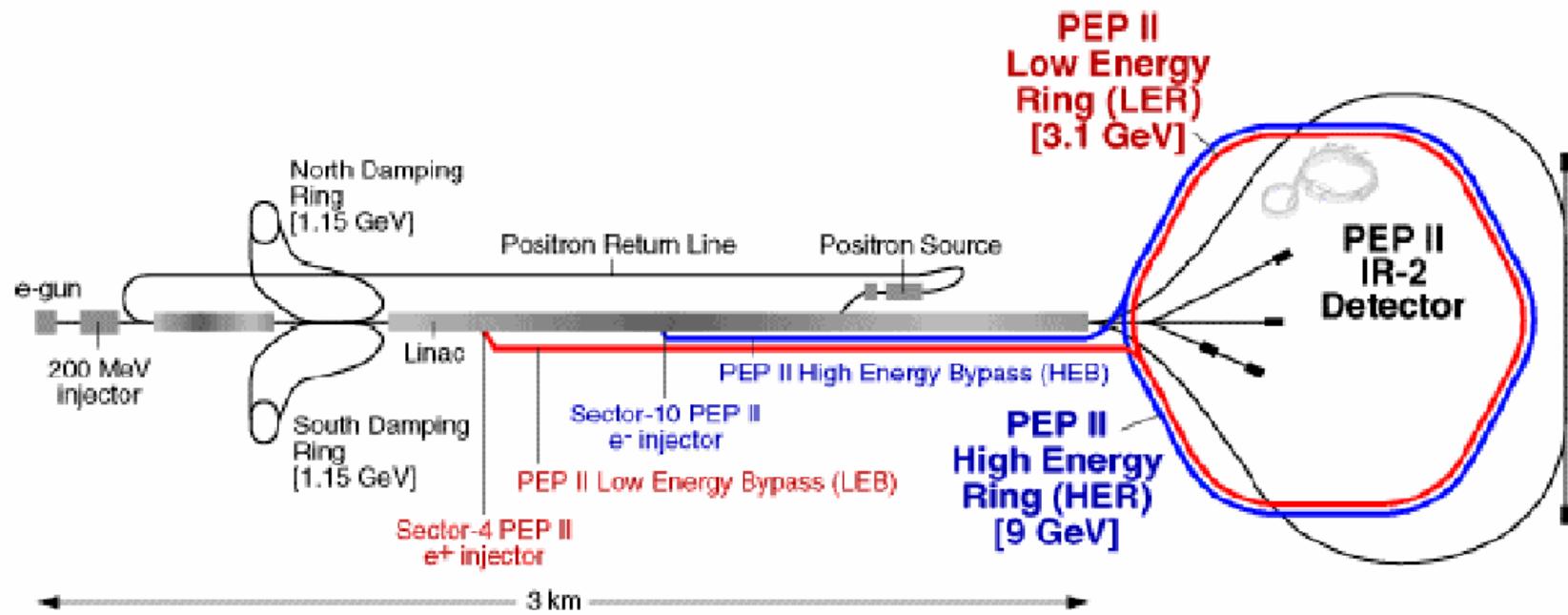
$\mathcal{O}(10^{-3})$



- 99% of B's \rightarrow D's
- 66(13)% of B's $\rightarrow K^+(K^-)$: flavor tagging
- 10% semi-leptonic BR: flavor tagging
- 7×10^{-4} of B's $\rightarrow J/\psi \rightarrow \mu^+ \mu^-$
- mean track multiplicity for single B ~ 5.5

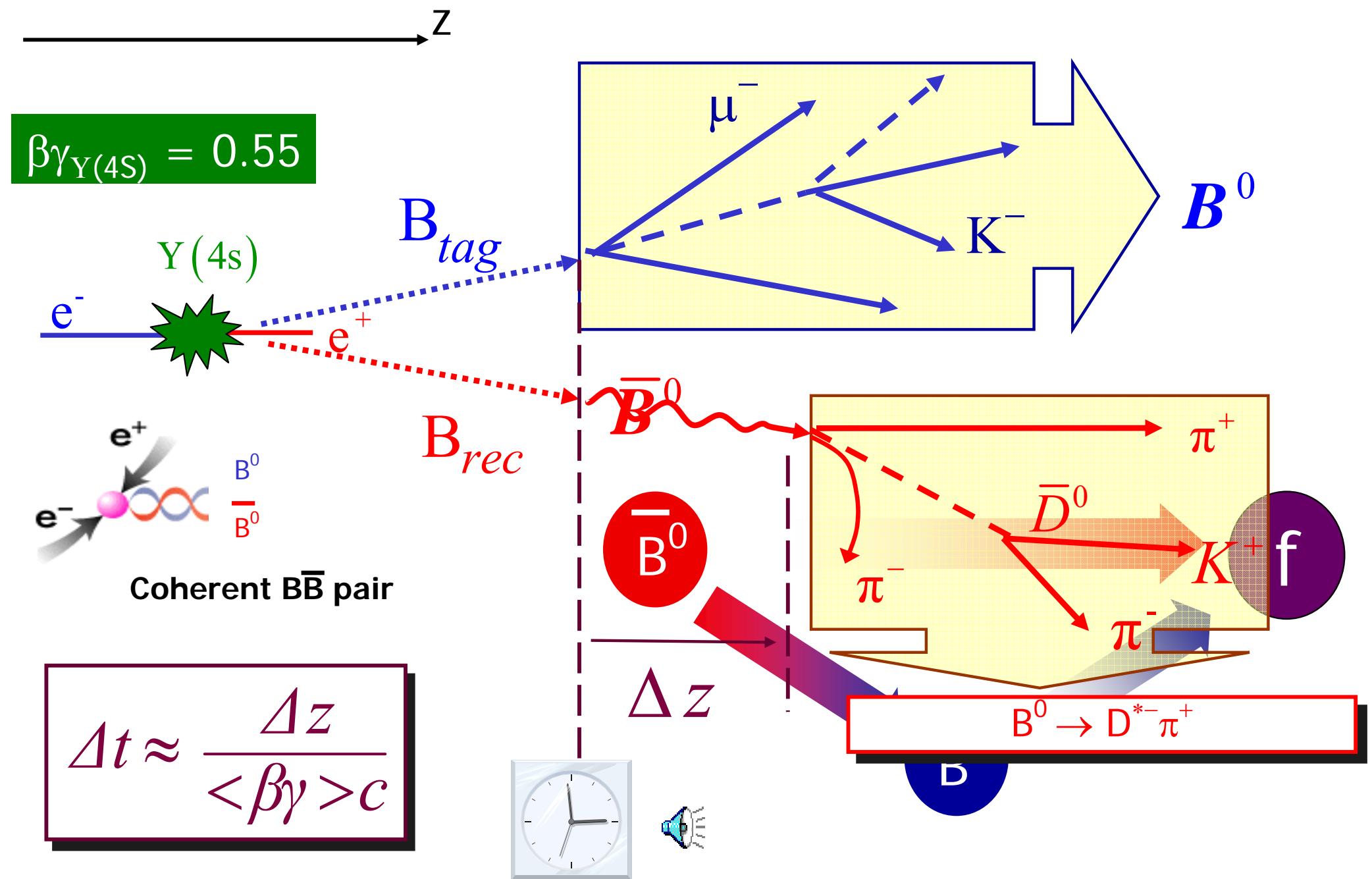
Brief Primer of B Reconstruction at e^+e^- B Factory

PEP-II Asymmetric B-Factory at SLAC



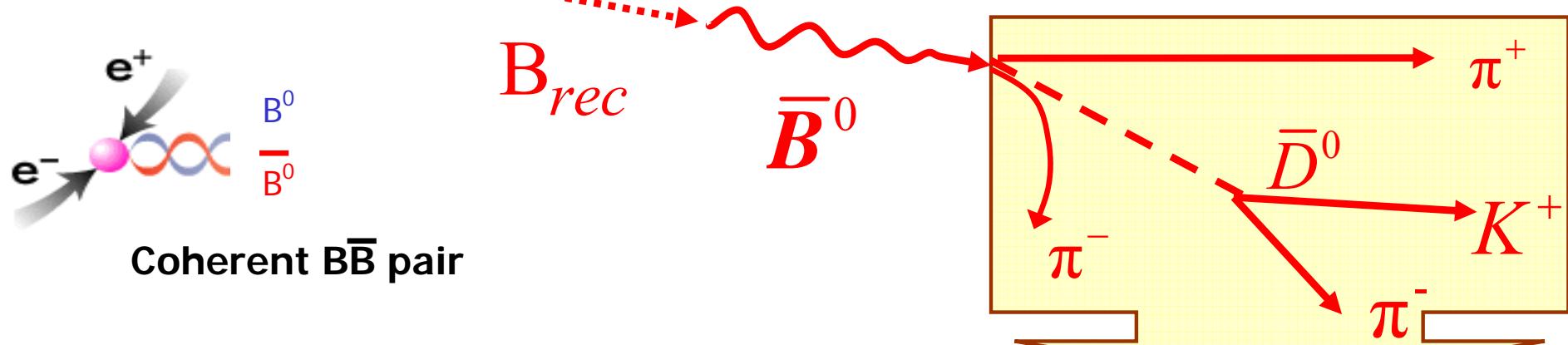
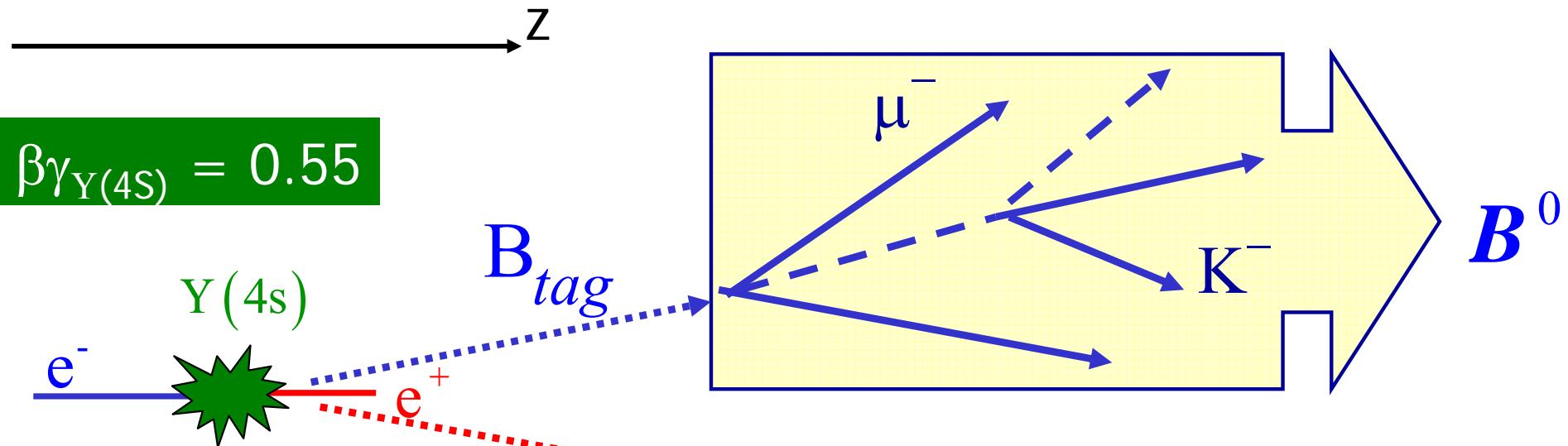
- 9 GeV e^- on 3.1 GeV e^+
- $Y(4S)$ boost in lab frame
 - $\beta\gamma = 0.55$

Time Evolution of $\Upsilon(4S) \rightarrow B^0 \bar{B}^0$



Snapshot of $B\bar{B}$ Event at BaBar

Inclusive Reconstruction: Look at some of decay products carrying information about their mother



Exclusive Reconstruction: All particles in final state are identified

$$B^0 \rightarrow D^{*-} \pi^+$$

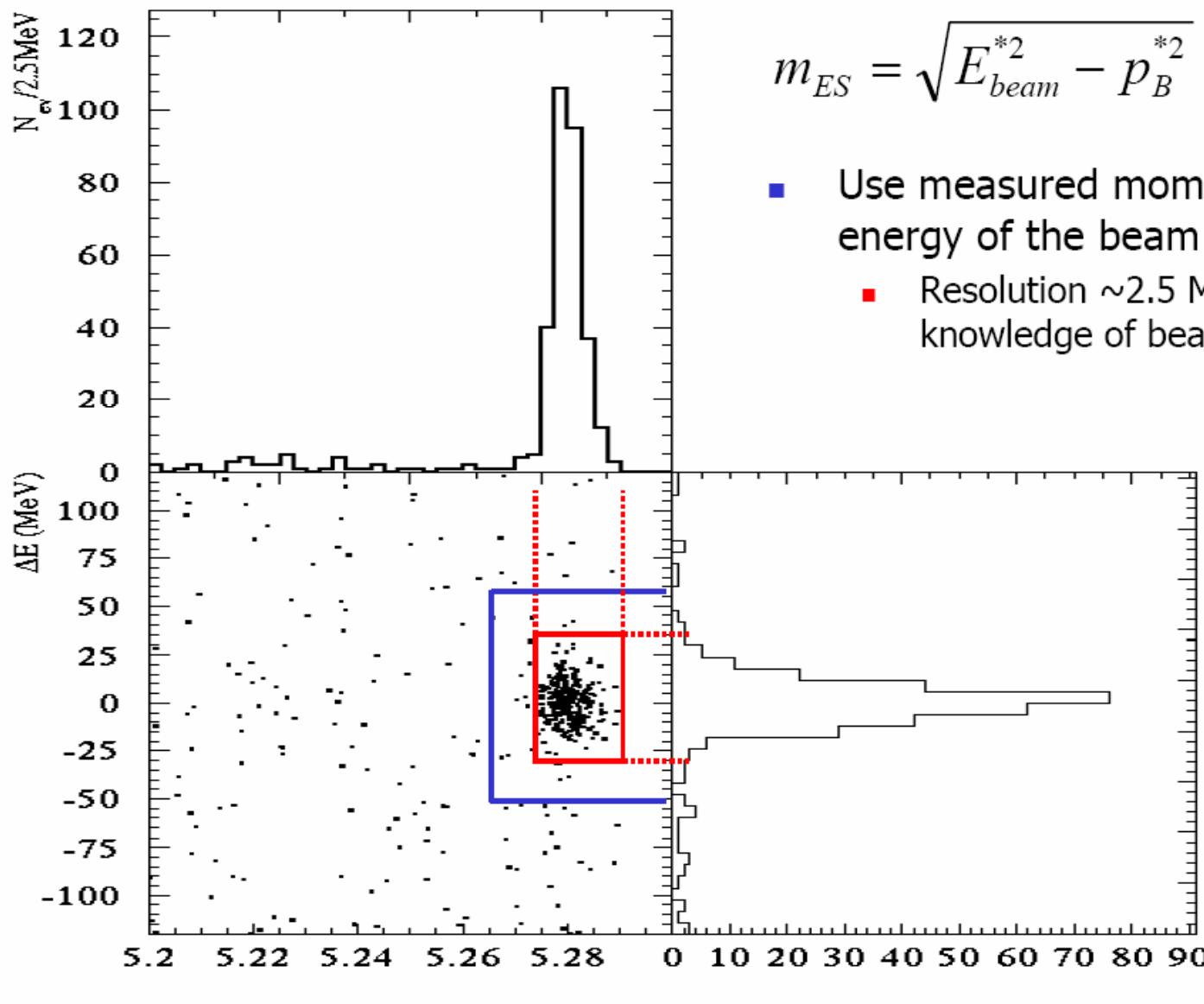
Exclusive or Inclusive?

- Exclusive Selection
 - All products in selected final states are found in the detector
 - Conservation laws connect measured quantities between initial and final states
 - Advantages:
 - Typically better signal to noise ratio
 - Kinematic constraints remove most of combinatorial background
 - Disadvantages:
 - Usually requires more reliance on theoretical models and theory for interpretation of results

- Inclusive Selection
 - Not all particles in final state selected
 - No kinematic relation between initial and final state
 - Advantage:
 - Closer to transition diagram at quark level, hence typically less dependent on theory models
 - Disadvantage:
 - More background because of reduced constraints

Ingredients of B Reconstruction

- Take advantage of clean environment in $e^+e^- \rightarrow \gamma(4S) \rightarrow B\bar{B}$
 - Energy of each B meson is known in the center of mass



$$m_{ES} = \sqrt{E_{beam}^{*2} - p_B^{*2}}$$

Energy-substituted mass

- Use measured momenta of B daughters and energy of the beam
 - Resolution $\sim 2.5 \text{ MeV}/c^2$ dominated by knowledge of beam energy

Energy difference

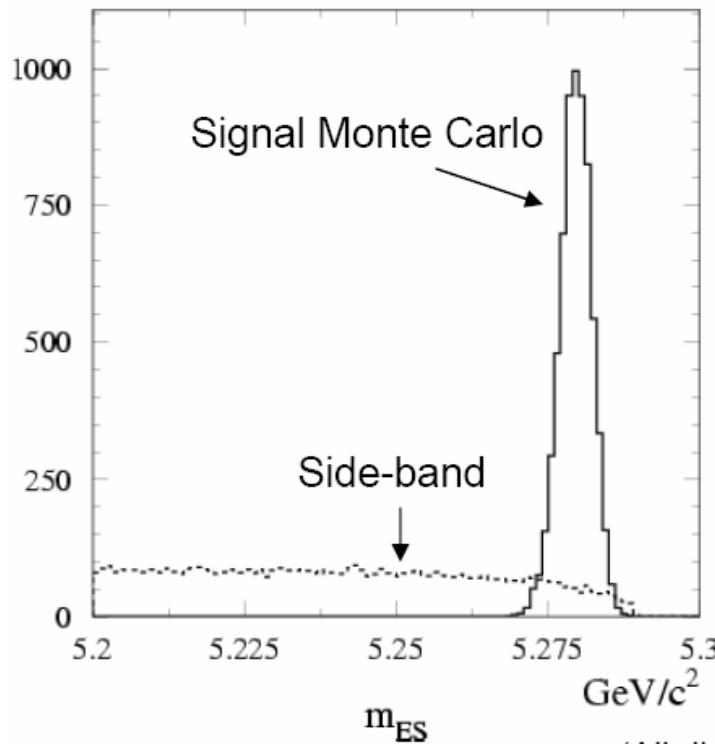
$$\Delta E = E_B^* - E_{beam}^*$$

- Difference between total reconstructed and expected energy
 - Dominated by detector energy resolution
 - Resolution depends on particles in final states

Kinematic Variables

$$m_{\text{ES}} = \sqrt{E_{\text{beam}}^{*2} - p_B^{*2}}$$

$\sigma(m_{\text{ES}}) \approx 2.6 \text{ MeV}/c^2$

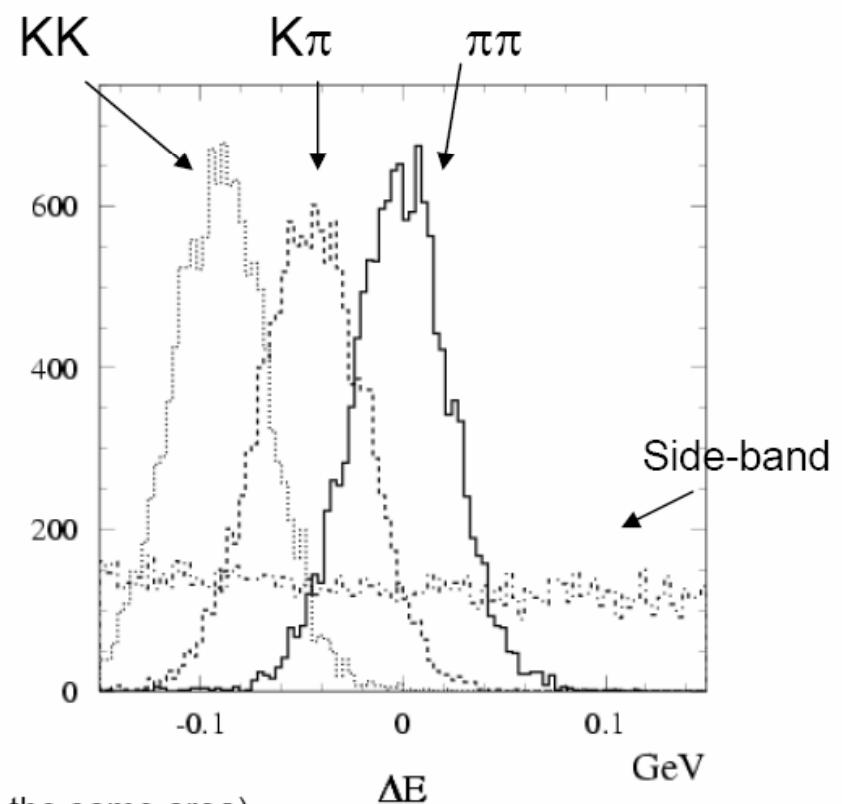


(All distributions are normalized to the same area)

$$\Delta E = E_B^* - E_{\text{beam}}$$

- Dominated by tracking resolution
- Assume π mass for tracks
- Momentum dependent shift for $K\pi$ and KK

$\sigma(\Delta E) \approx 26 \text{ MeV}$



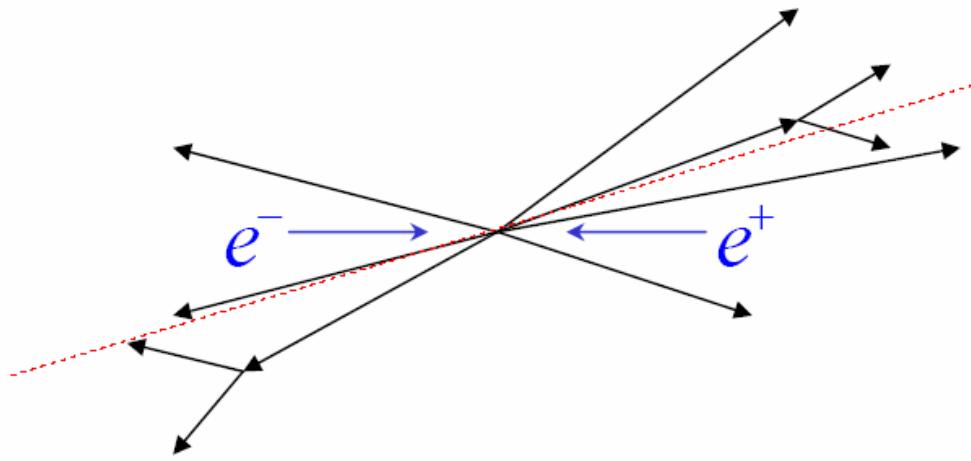
Continuum Background Rejection

- Main source of background: continuum $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$)

- Branching fraction of interesting B decays $\leq 10^{-4}$
- Branching fraction of D decays: $\simeq 10^{-2}$

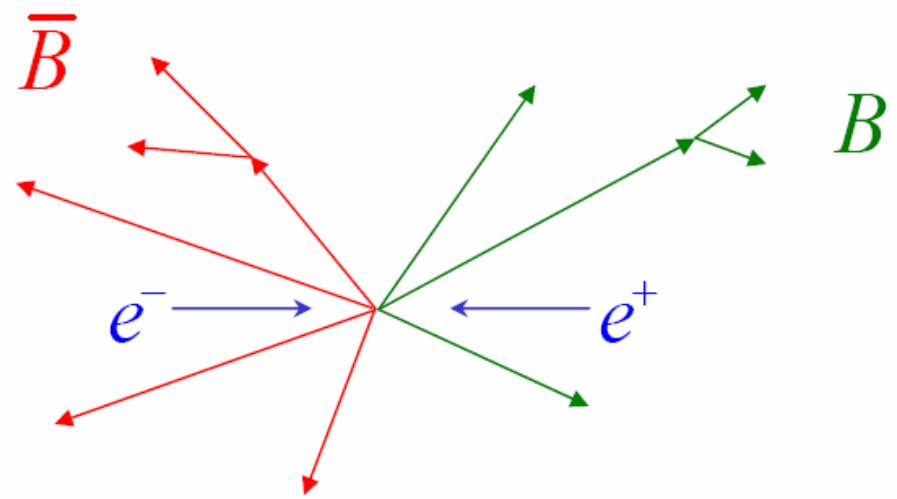
Overall branching
fraction $< 10^{-6}$

- Distinguish signal and background based on event topology
 - Neutral networks
 - Fisher discriminant



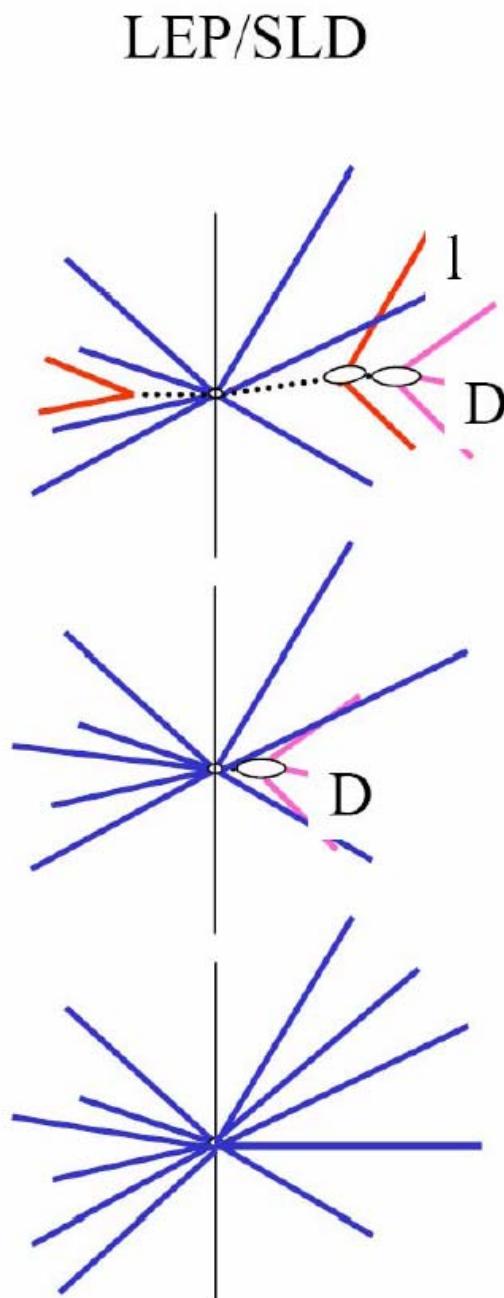
Thrust Axis

$$e^+e^- \rightarrow q\bar{q}$$



$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$$

Event Topology at LEP/SLD vs. B Factories

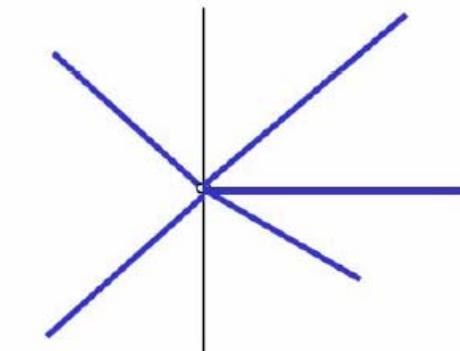


b ⇔ non-b

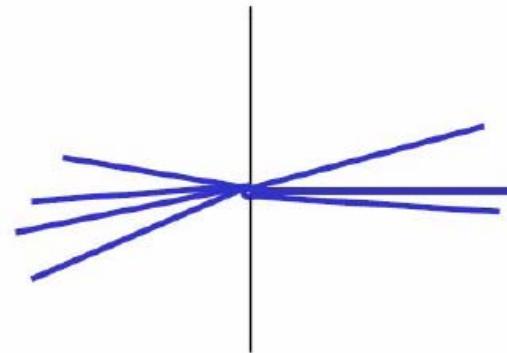
- displaced vertices
- leptons
- topology

B-factories

B-events

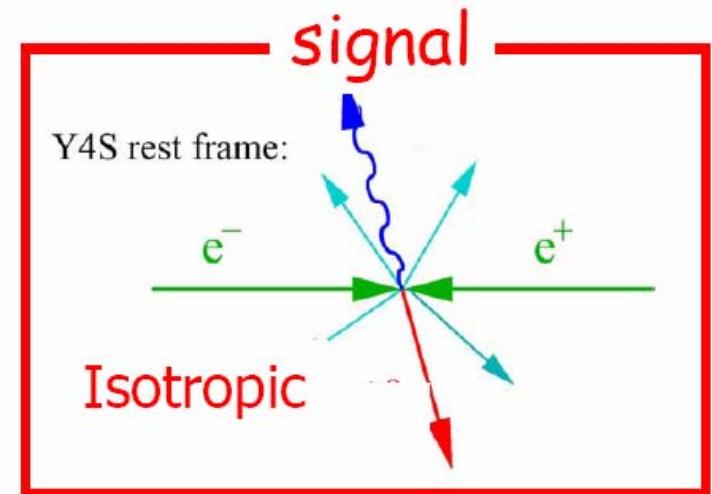
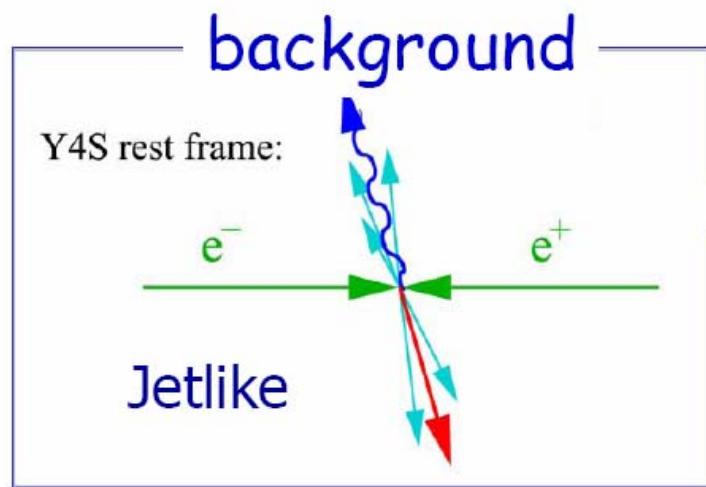


D-events

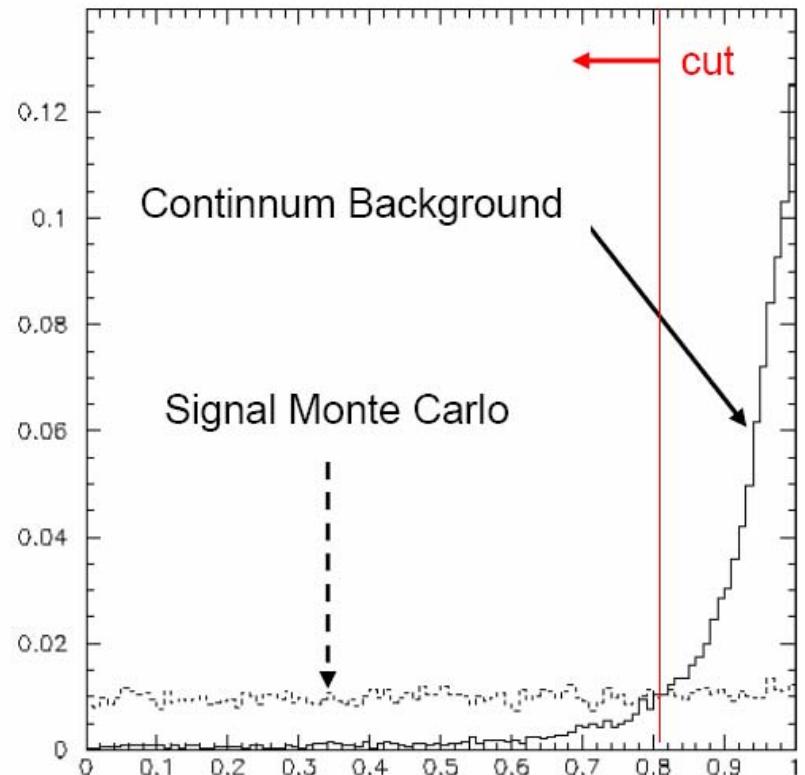


Light quarks-events

Background Fighting: Sphericity Angle

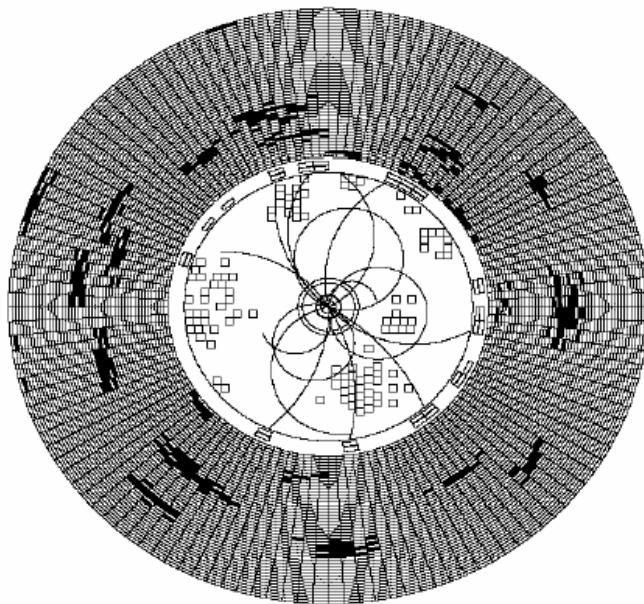


- $\cos\theta_s$: Angle between the B candidate and the rest of event

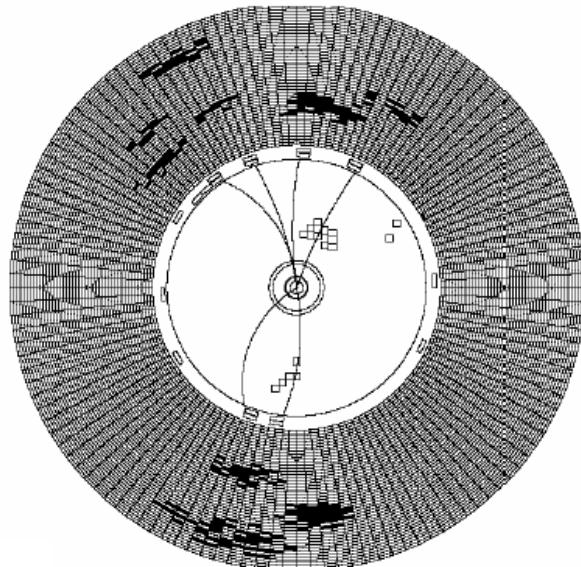


2nd Fox-Wolfram Moment

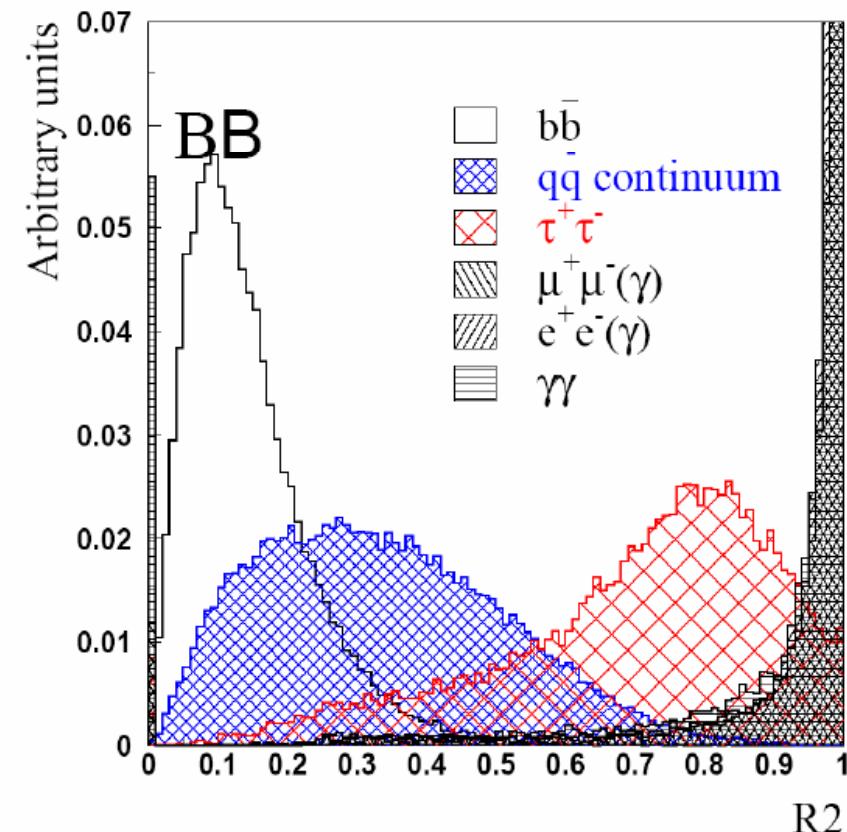
$\Upsilon(4S) \rightarrow B\bar{B}$ Decay



$e^+e^- \rightarrow u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}$ decays

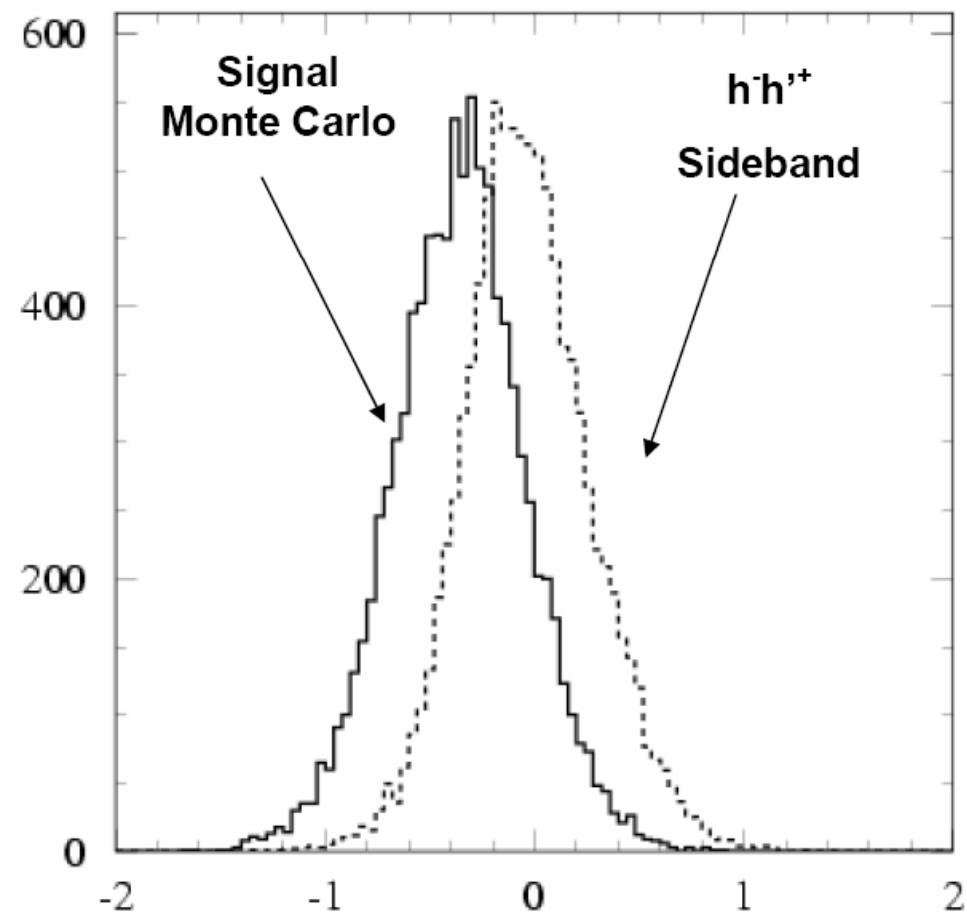
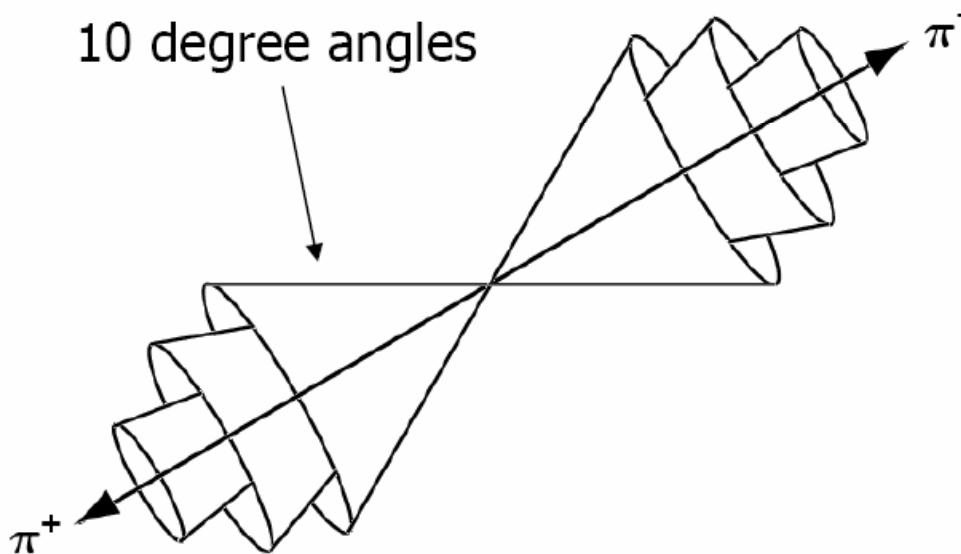


Differences in the event topology (Isotropic B Vs jet-like Continuum) and Energy flow structure in these events used to construct continuum background suppression tools.



Background Fighting: Fisher Discriminant

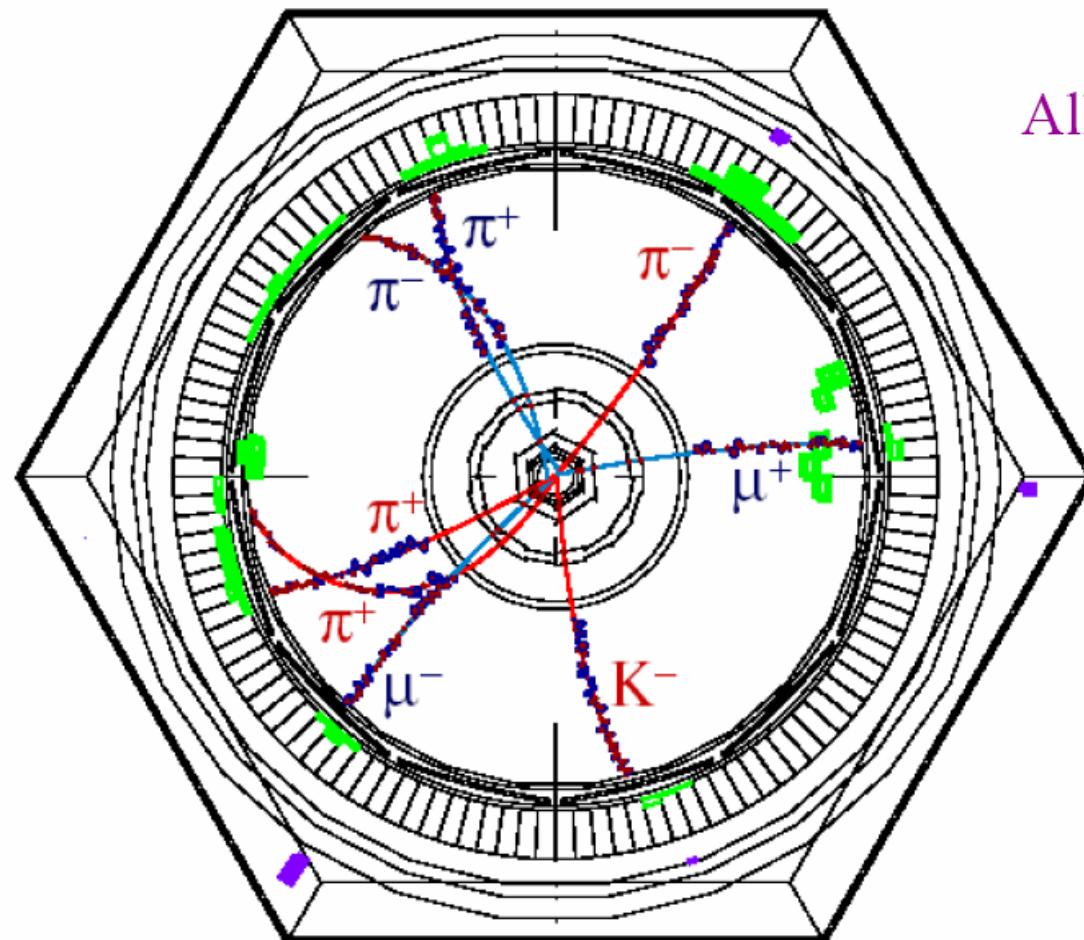
- Optimized linear combination of energy flow into cones about candidates
- Sensitive only to the rest of event
- Studied and calibrated on data: $B \rightarrow D^0\pi^-$, h^-h^{+} sideband
- Validated with Monte Carlo



A Completely Reconstructed $\Upsilon(4S)$ event at BaBar

$$\overline{B}^0 \rightarrow D^{*+} \pi^-$$

$$B^0 \rightarrow \psi(2S) K_s^0$$



All particles accounted for
Nothing Missing !

$$D^{*+} \rightarrow D^0 \pi^+$$

$$\psi(2S) \rightarrow \mu^+ \mu^-$$

$$D^0 \rightarrow K^- \pi^+$$

$$K_s^0 \rightarrow \pi^+ \pi^-$$

Measurement of $|V_{ub}|$ and $|V_{cb}|$

$$\mathbf{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & \boxed{V_{ub}} \\ V_{cd} & V_{cs} & \boxed{V_{cb}} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\mathbf{V}_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \boxed{A\lambda^3(\rho-i\eta)} \\ -\lambda & 1-\lambda^2/2 & \boxed{A\lambda^2} \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$|V_{cb}|$ Exclusive

$|V_{cb}|$ from $B^0 \rightarrow D^{(*)} l \bar{\nu}$ Decays

Differential measurement of $B(B^0 \rightarrow D^* l \bar{\nu})$ allows for the extraction of $|V_{cb}|$ through the expression:

$$\frac{d\Gamma}{dw} \propto |V_{cb}|^2 F^2(w) G(w)$$

Form factor of $B \rightarrow D^*$ transition

$$w = v_B \cdot v_D = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}$$

Known kinematic factor

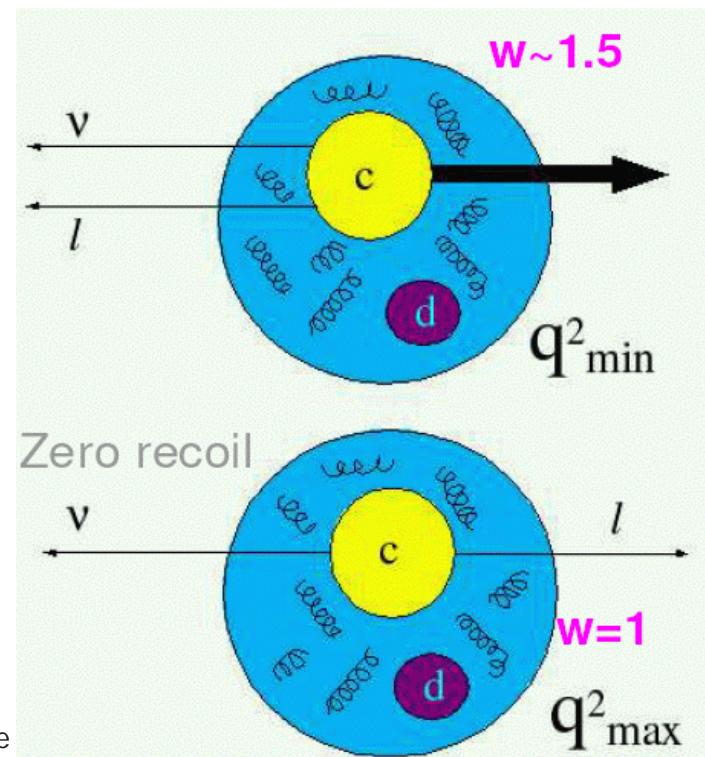
HQET and LQCD provide calculation at zero recoil

In reality the formula is slightly more complicated:

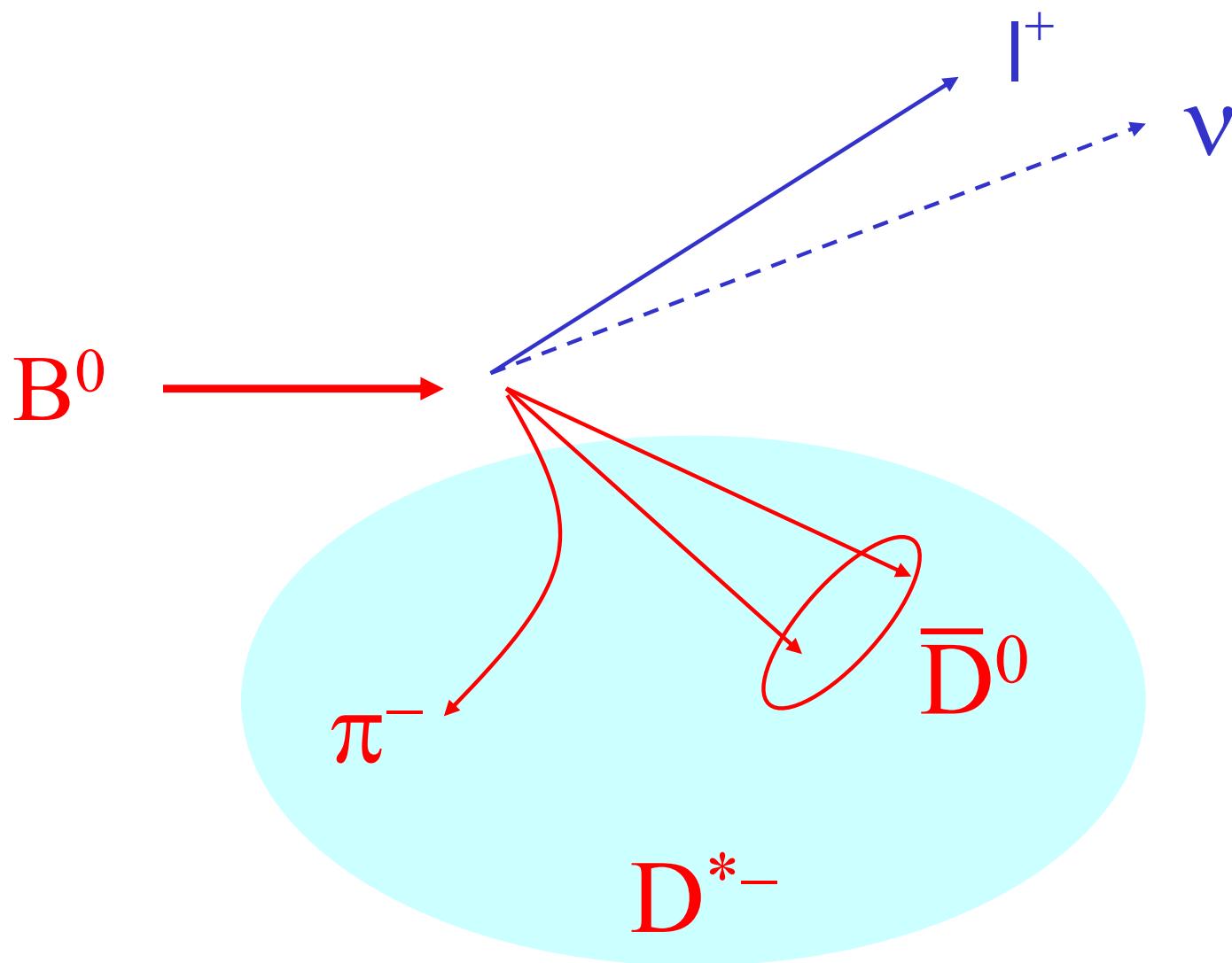
$$F^2(w)G(w) = h_{A_1}^2(w)\sqrt{w-1}(w+1)^2 \left\{ 2 \left[\frac{1-2wr+r^2}{(1-r)^2} \right] \times \left(1+R_1(w)^2 \frac{w-1}{w+1} \right) + \left[1+(1-R_2(w)) \frac{w-1}{1-r} \right]^2 \right\}$$

where $r = \frac{M_{D^*}}{M_{B^0}}$

$F \rightarrow 1$ for $w \rightarrow 1$

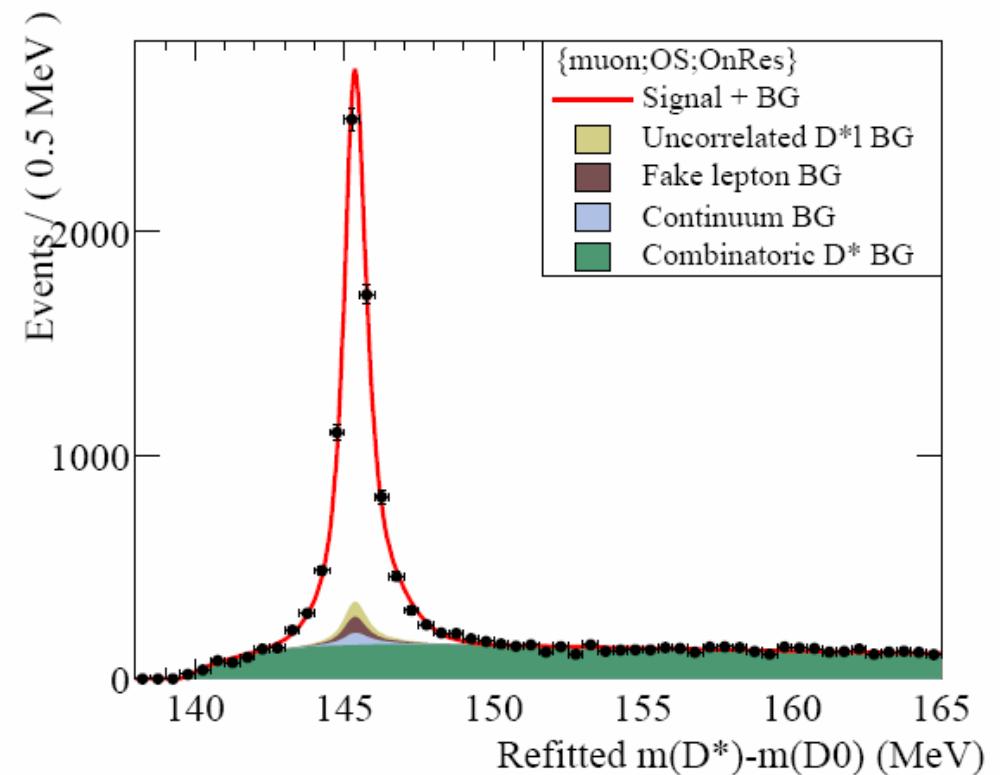
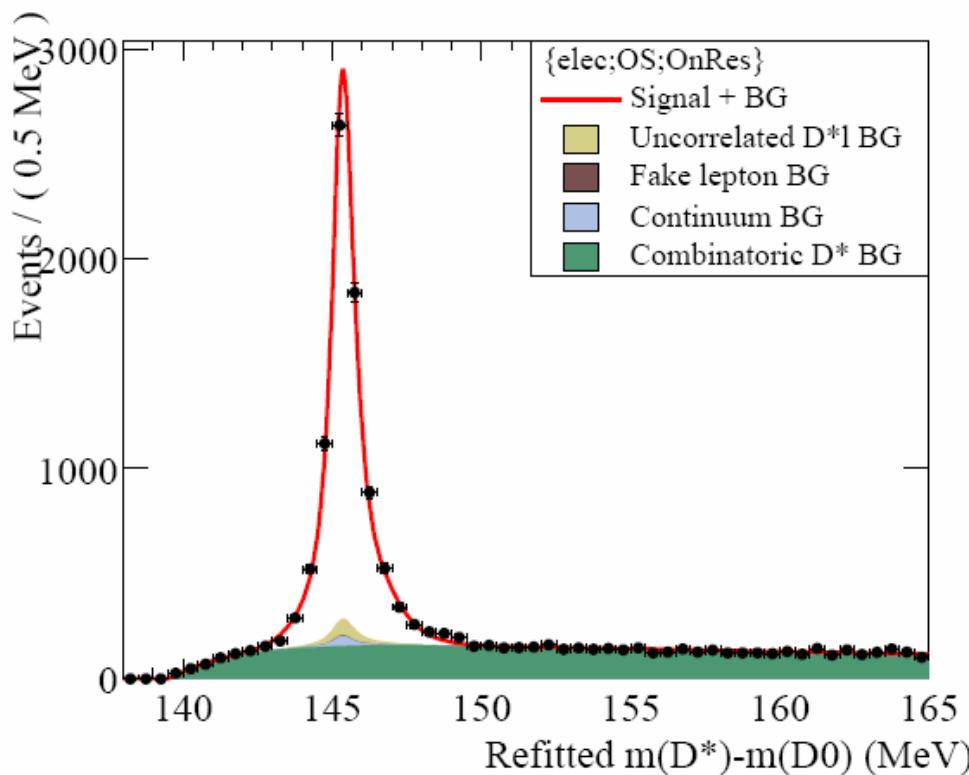
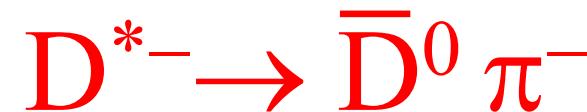


$B^0 \rightarrow D^{(*)} l \nu$ Decays



$D^{*+} - D^0$ mass difference

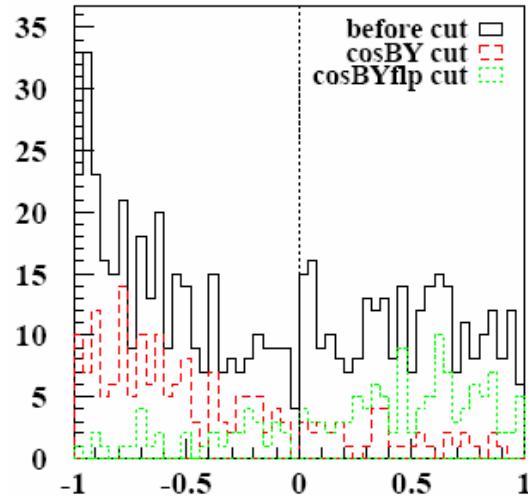
- D^0 mass: 1864 MeV
- D^* mass: 2010 MeV
- Fixed momentum for soft pion
 - Only experimental resolution



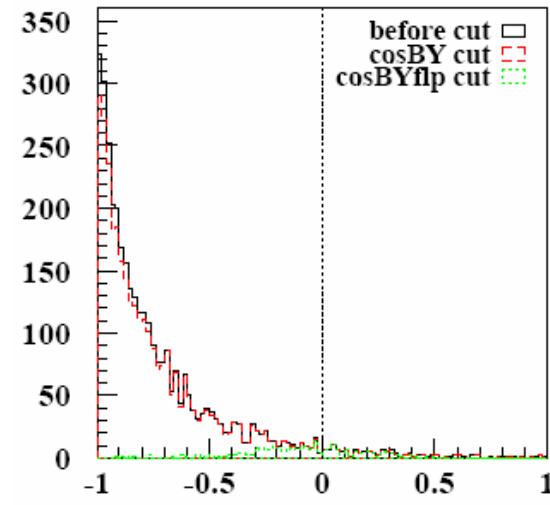
Angular Variables for $B^0 \rightarrow D^{(*)} l \nu$ Selection

- Angle between D^* and lepton

Background

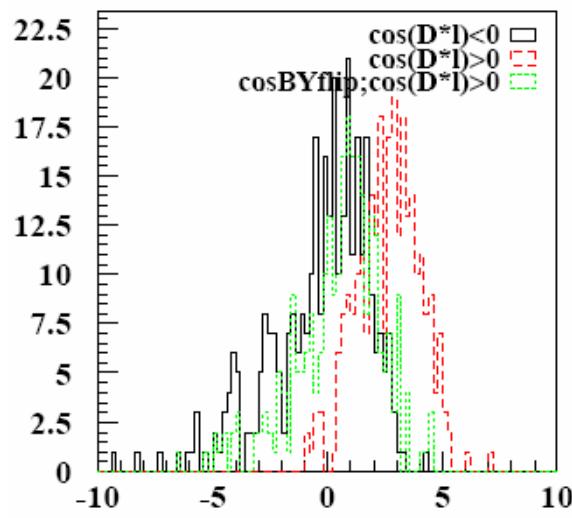


Signal

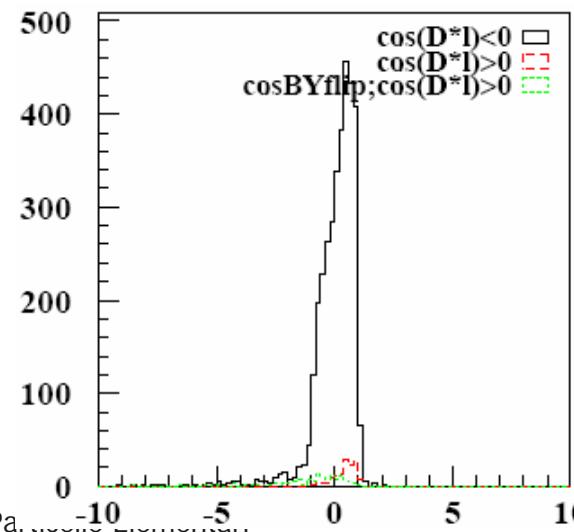


- Angle between B^0 and the $D^* l$ system
 - From kinematic quantities since B direction not measured

Background



Signal



Extraction of $|V_{cb}|$

Measurement:

- Determine number of $B^0 \rightarrow D^{*-} l^+ \nu$ candidates as function of w
- Obtain $h_{A1}(w)|V_{cb}|$ distribution
- Fit differential spectrum and extrapolate to $w=1$

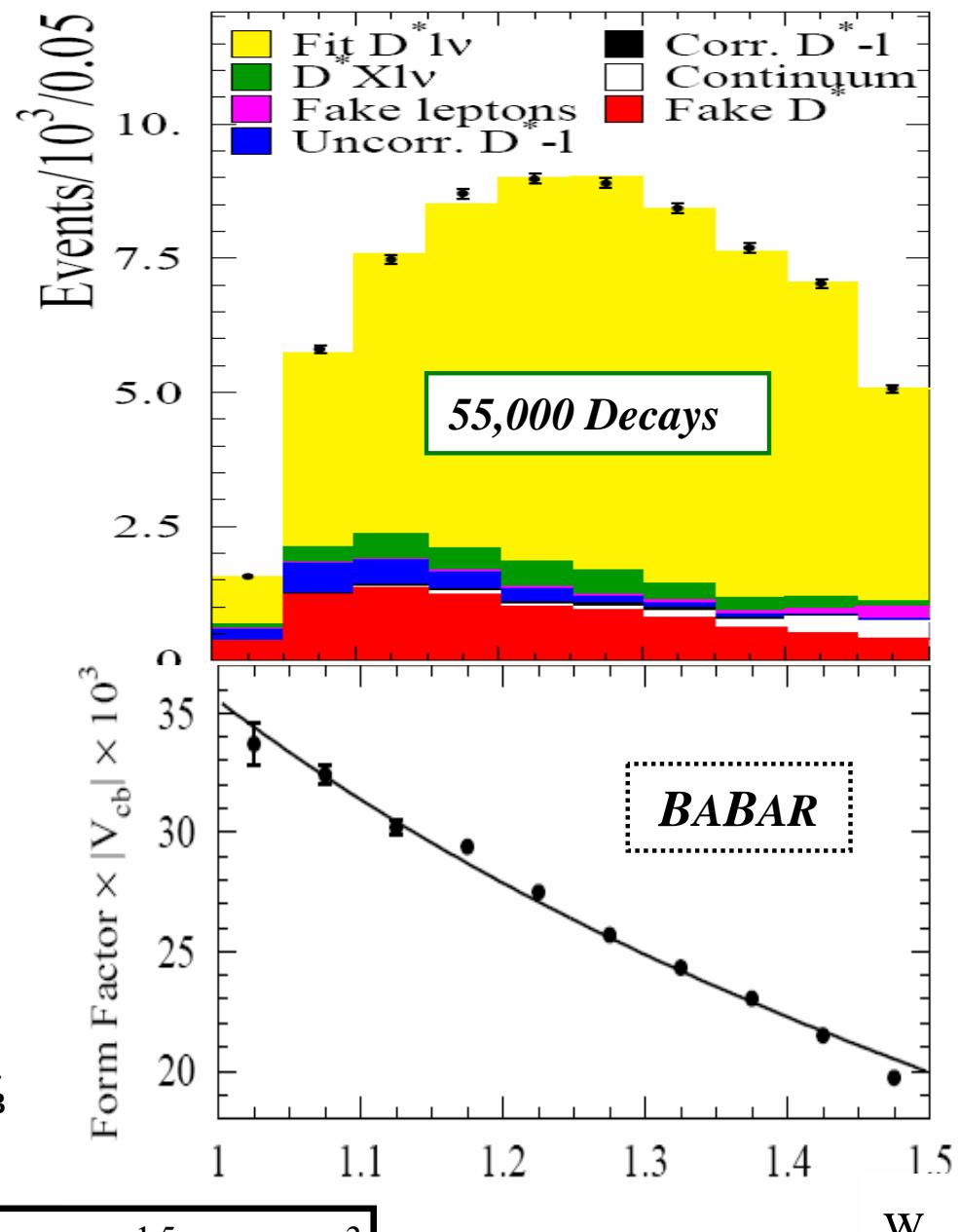
In BaBar:

$$h_{A_1}(1)|V_{cb}| = (35.5 \pm 0.3_{\text{stat}} \pm 1.6_{\text{syst}})$$

and using

$$h_{A1}(w=1) = 0.919^{+0.030}_{-0.035}$$

Hashimoto et al.
PRD 66, 014503
(LQCD)



$$|V_{cb}| = (38.7 \pm 0.3_{\text{stat}} \pm 1.7_{\text{syst}} \pm 1.3_{A_1}) \times 10^{-3}$$

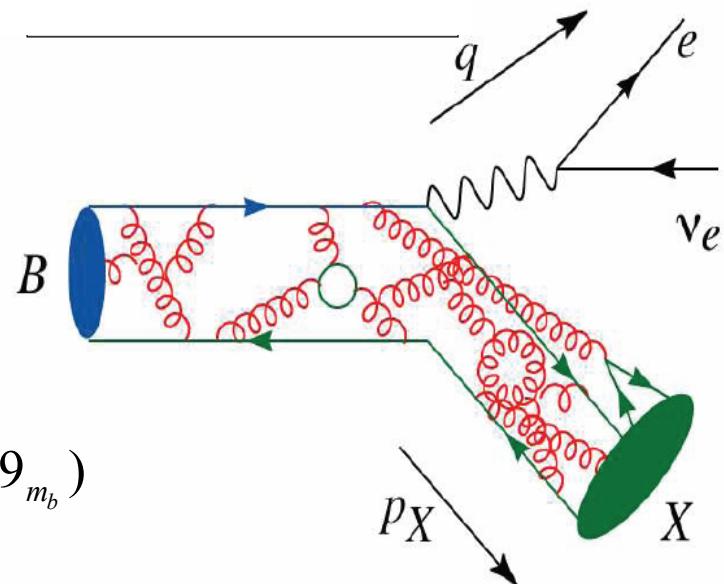
$|V_{ub}|$ Inclusive

$|V_{ub}|$ from Inclusive Semileptonic B Decays

Inclusive charmless semileptonic B decays,

$B \rightarrow X_u l \bar{\nu}_l$, allow for the measurement of $|V_{ub}|$

$$|V_{ub}| = 0.00424 \sqrt{\frac{B(B \rightarrow X_u l \bar{\nu}_l)}{0.002} \frac{1.61 \text{ ps}}{\tau_B}} \times (1 \pm 0.028_{\text{OPE}} \pm 0.039_{m_b})$$



Theoretically relatively simple, but

- hadronization effects and Fermi motion of b quark
- non-perturbative parametrizations (Shape Function, SF) affected by large uncertainties

Experimentally challenging, because

- $B \rightarrow X_c l \bar{\nu}_l$ background (60 times higher) separated in limited region of phase space
- extrapolation to full phase space introduces uncertainties

Lots of Theory Needed as Input for Interpretation

Traditional method:

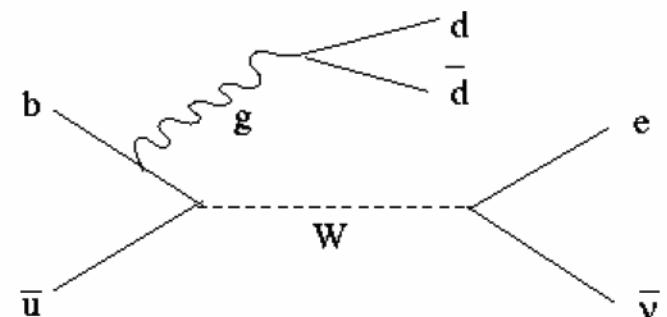
- * Measure event yield in restricted phase space
- * Extrapolate to full BF (usually DeFazio & Neubert)
- * Full BF $\Rightarrow |V_{ub}|$

State-of-the-art:

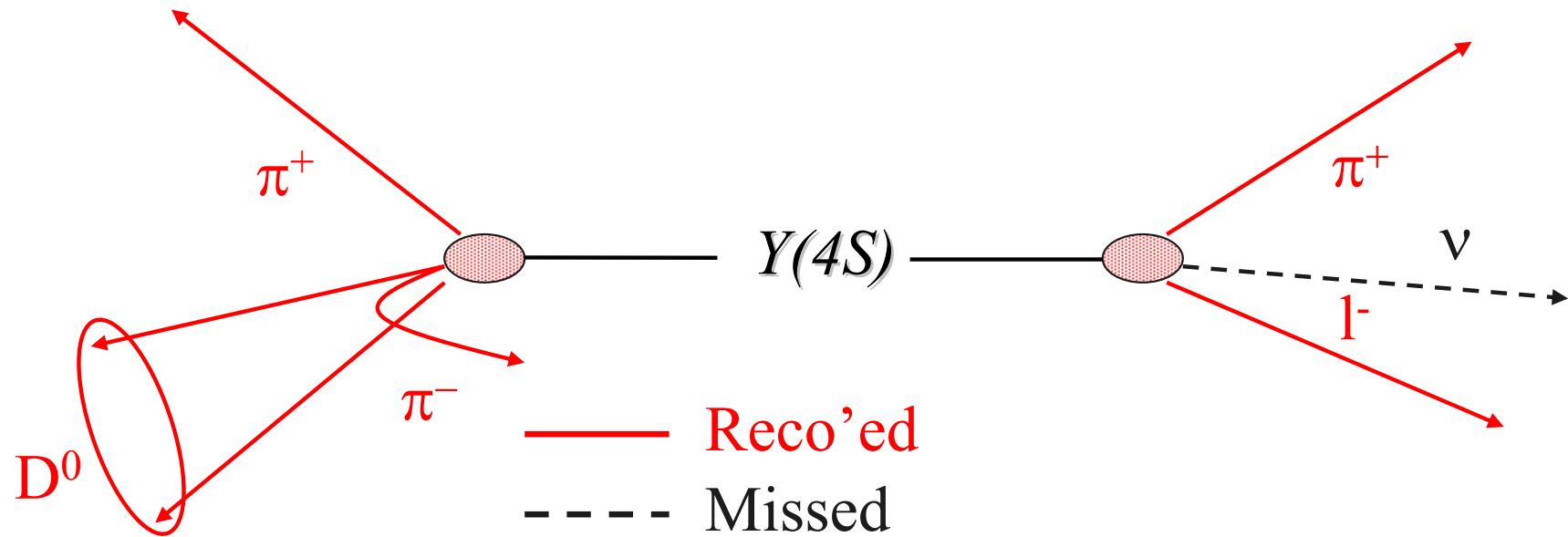
- * Partial BF in restricted phase space ('unfolded')
Not depending on theoretical calculation on the market
- * Direct translation: Partial BF $\Rightarrow |V_{ub}|$ using existing calculations
- * Today: Bosch, Lange, Neubert, Paz (Used for recent results)
Bauer, Ligeti, Luke (Used for M_x - q^2 extraction)
Aglietti, Ricciardi, Ferrera (new; not used yet)
....

Additional Uncertainties ('estimated'):

- * Subleading Shape Functions
(contributing differently in $B \rightarrow X_s \gamma$)
- * Weak Annihilation (important @ high q^2)



$|V_{ub}|$ inclusive from m_X and recoil

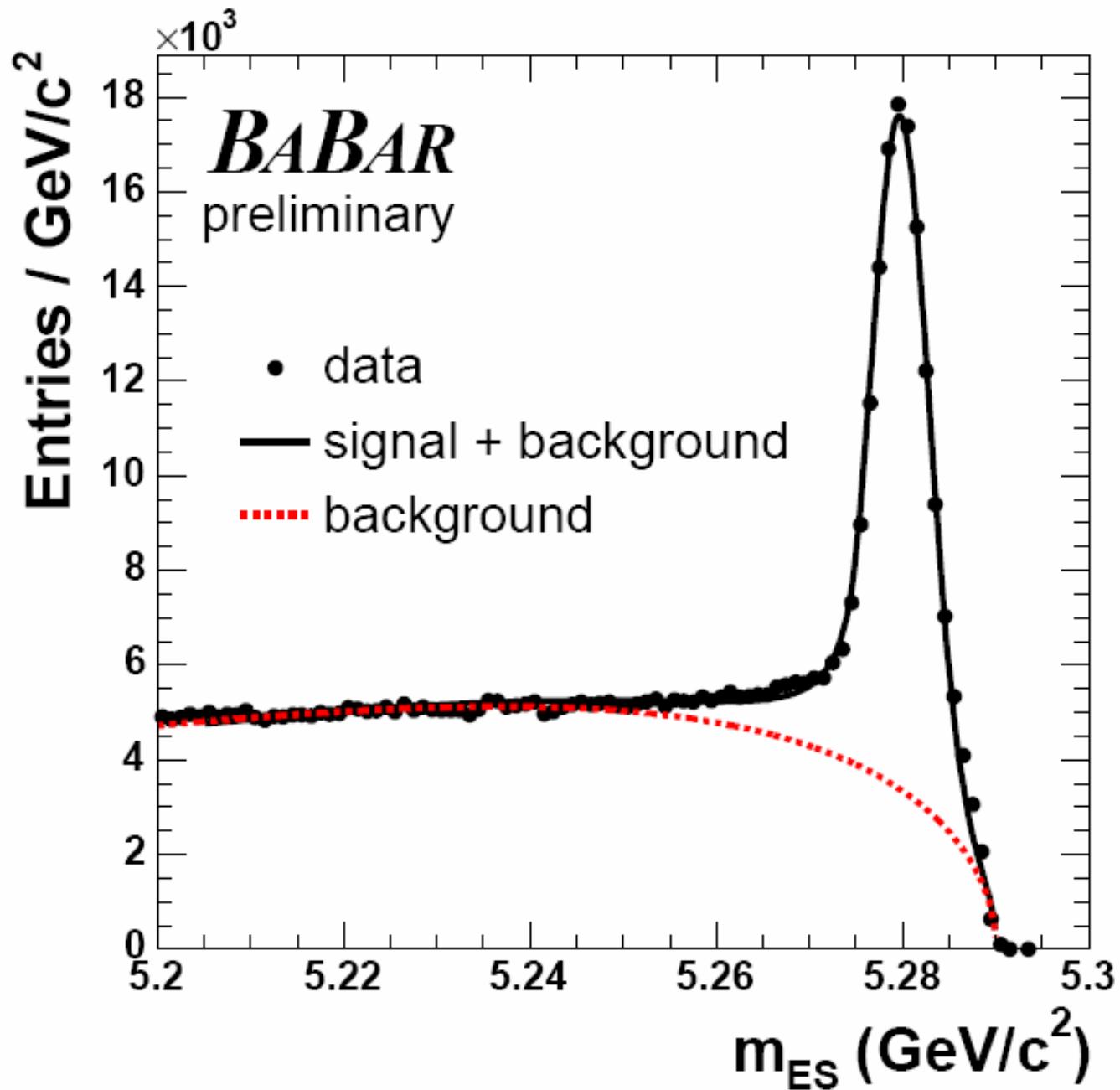


- Background $B \rightarrow X_c l \bar{\nu}$ separated using invariant mass of the X system (M_X)
- Full reconstruction of one B meson to improve signal/background ratio
- Look for semileptonic decay on the recoil side

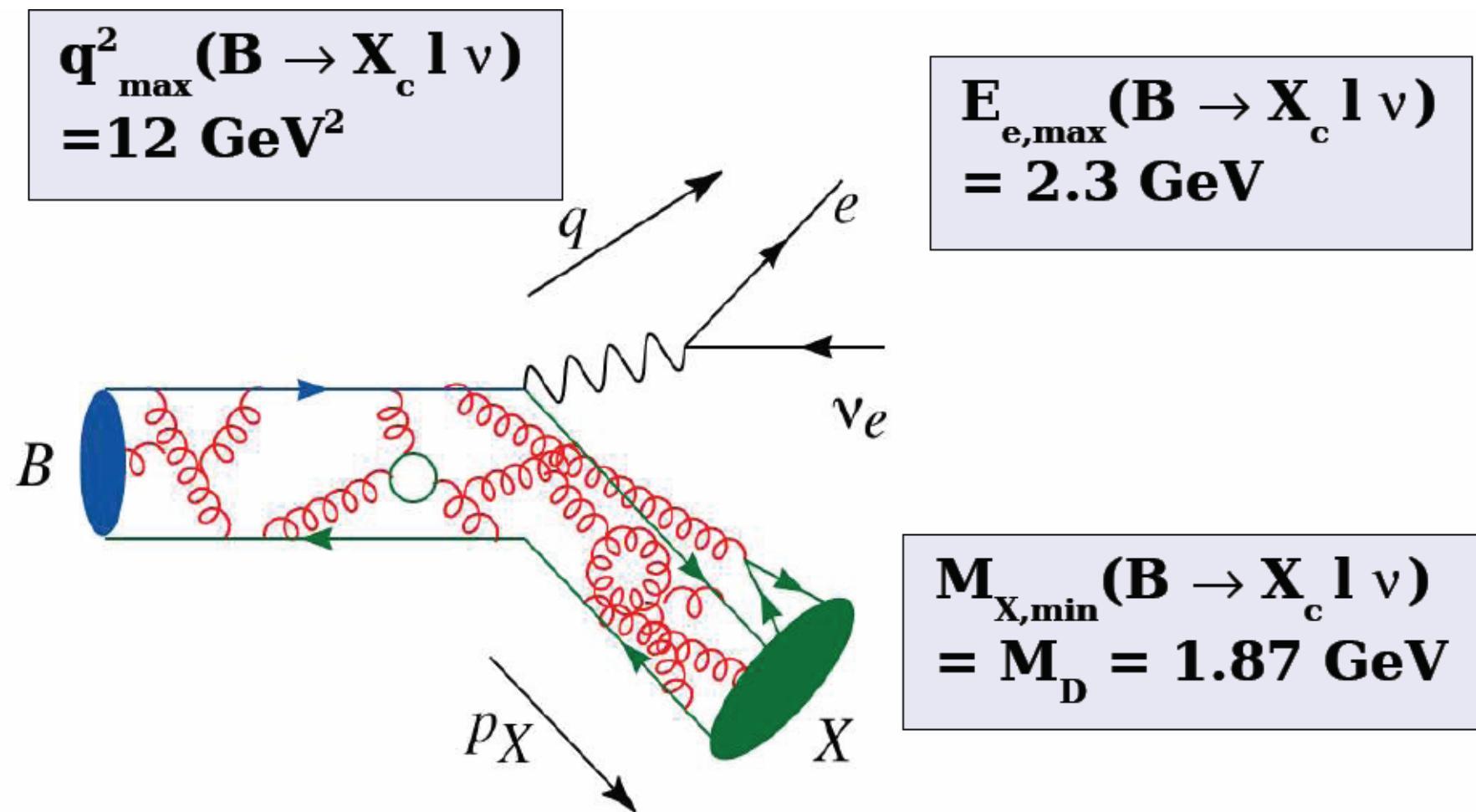
Fully Reconstructed B Mesons with Leptons in Recoil

Fully reconstruct as many decay modes as possible on one side

On the other side use lepton identification to clean up the sample:
 $p^* > 0.5 \text{ GeV}$ for electrons
 $p^* > 0.8 \text{ GeV}$ for muons



Kinematic Variables Useful for $|V_{ub}|$

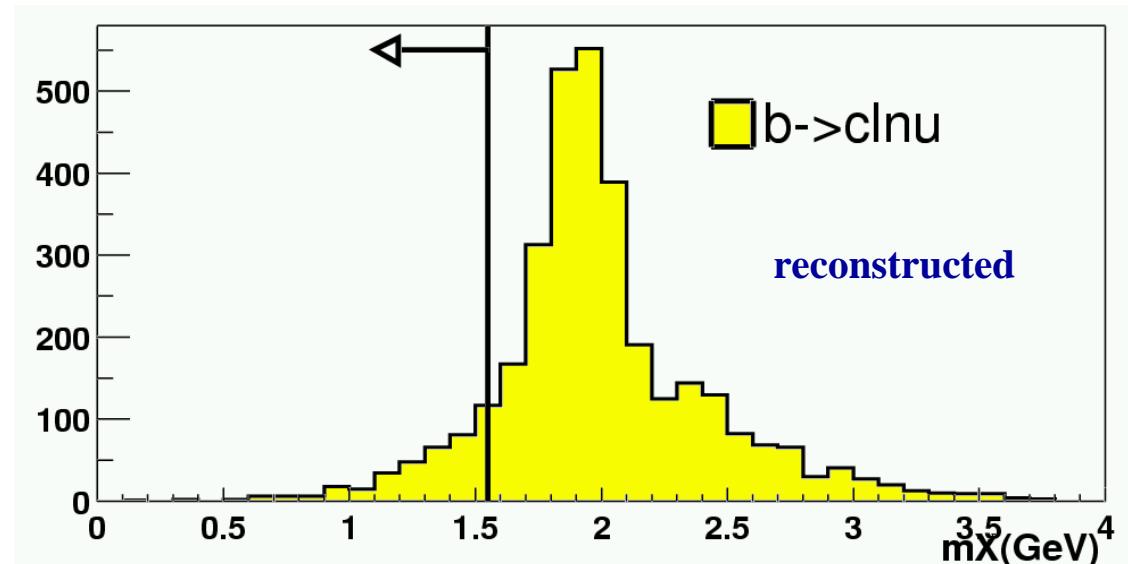


M_x to separate $b \rightarrow c l \bar{\nu}$ from $b \rightarrow u l \bar{\nu}$

$B \rightarrow D^* l \bar{\nu}$

$B \rightarrow D^* l \bar{\nu}$

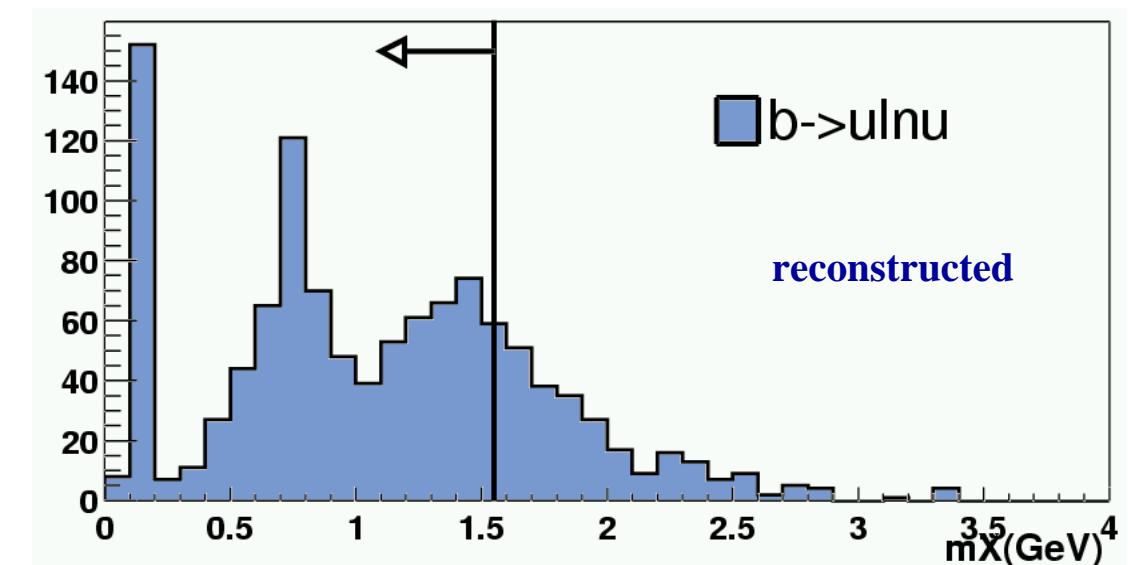
$B \rightarrow D^{**} l \bar{\nu}$



$B \rightarrow \pi l \bar{\nu}$

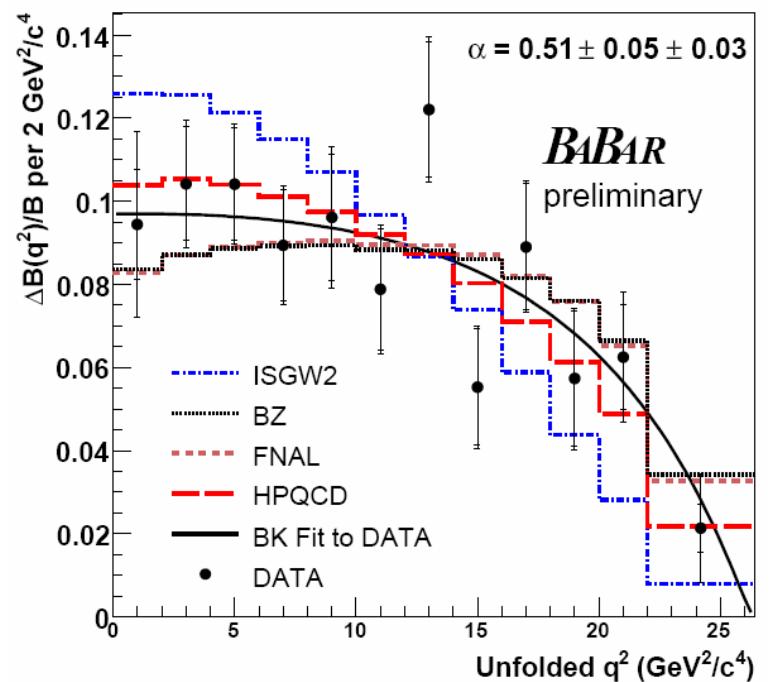
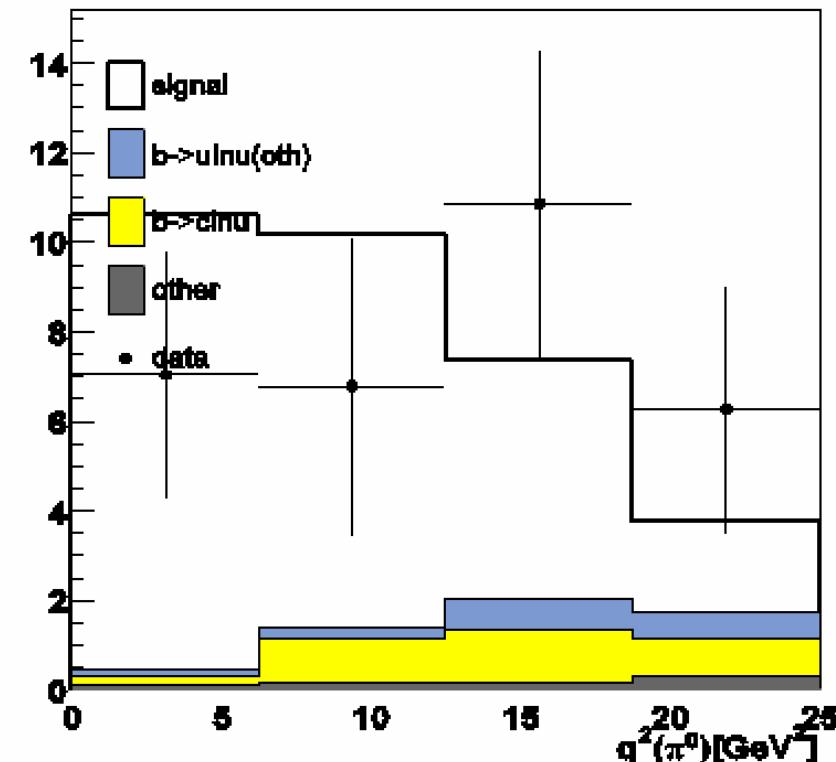
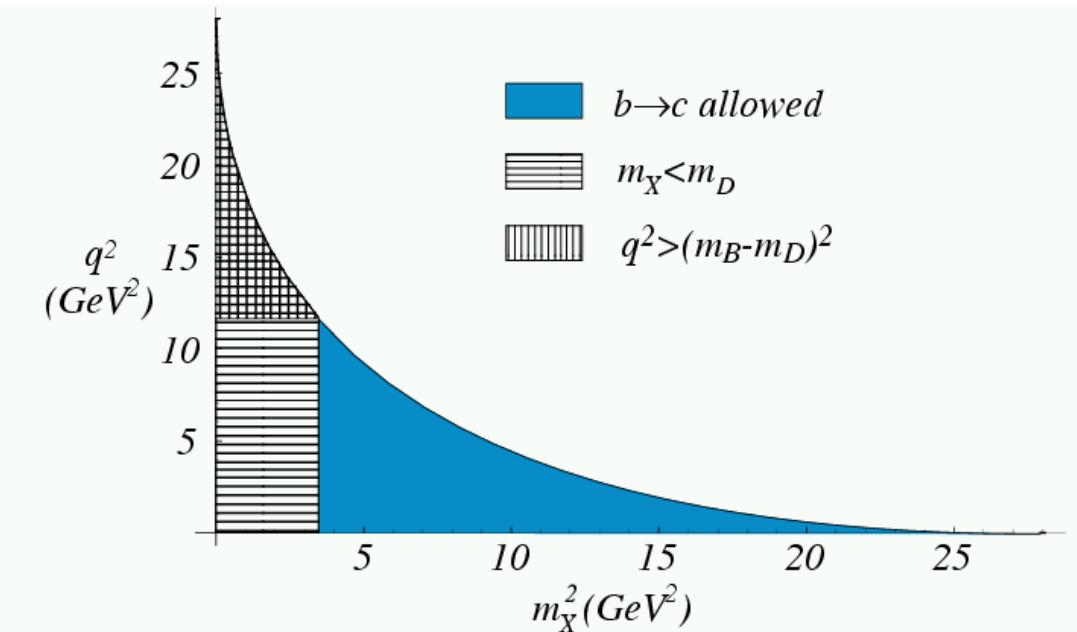
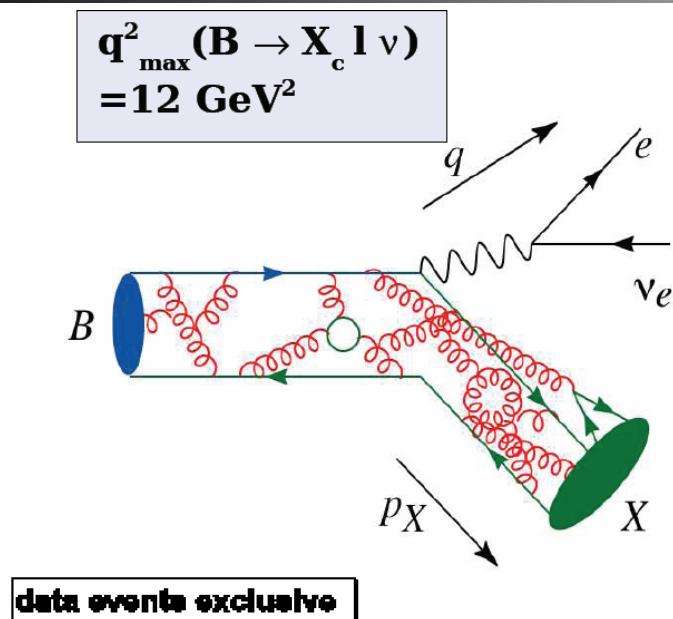
$B \rightarrow \rho l \bar{\nu}$

$B \rightarrow \eta l \bar{\nu}$



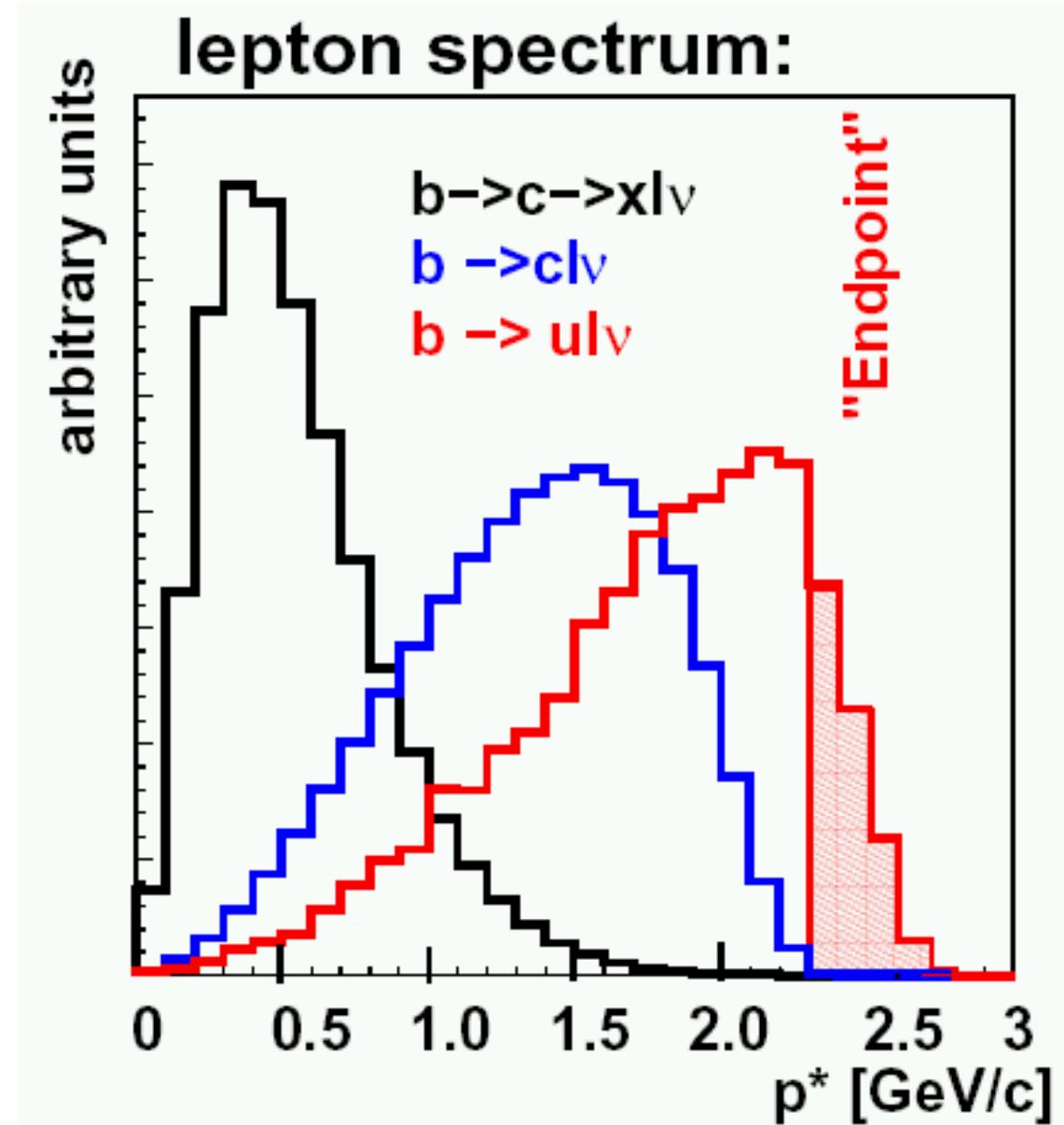
About 60-80% of signal for $M_x < M_D$

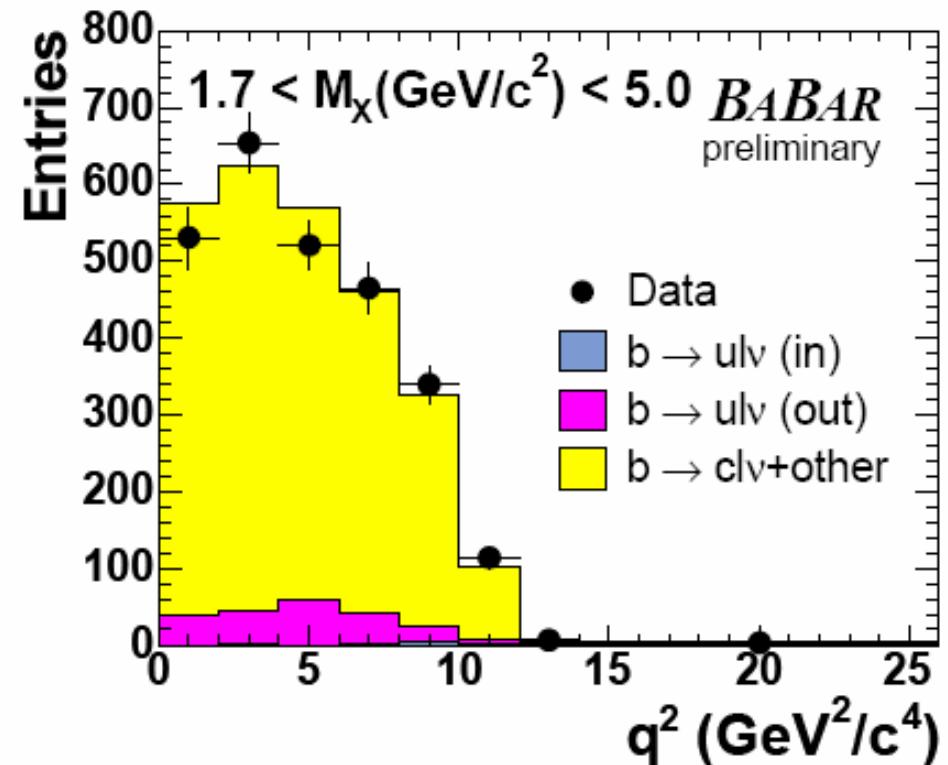
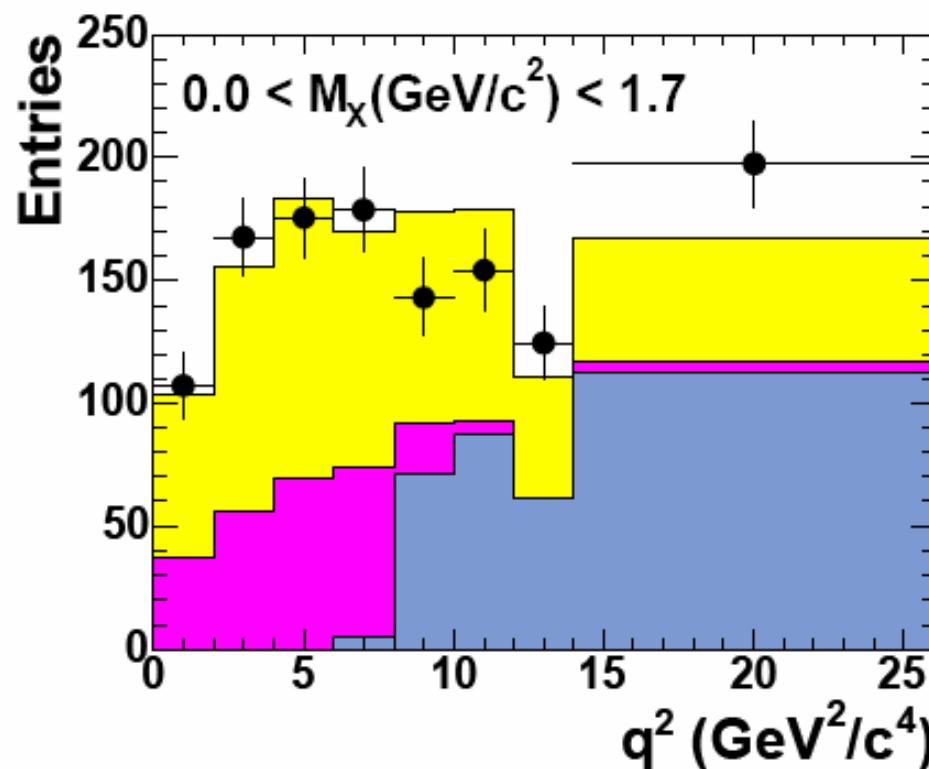
q^2 Spectrum



Lepton spectrum in Semileptonic Decays

- Cascade (secondary) leptons have lower momentum
- Requirement on momentum of leptons provides good separation between signal and cascade leptons
- Higher end of the spectrum mostly from decays mediated by V_{ub}
 - Lighter hadrons allows more momentum for leptons



$|V_{ub}|$ inclusive from m_x vs q^2 

$$|V_{ub}|^{\text{BLL}} = (4.82 \pm 0.26_{\text{stat}} \pm 0.25_{\text{syst}} \pm 0.46_{\text{th+SF}}) \times 10^{-3}$$

$$|V_{ub}|^{\text{BLNP}}_{\text{Belle } B \rightarrow X_s \gamma} = (5.00 \pm 0.27_{\text{stat}} \pm 0.26_{\text{syst}} \pm 0.46_{\text{SF}} \pm 0.28_{\text{th}}) \times 10^{-3}$$

$$|V_{ub}|^{\text{BLNP}}_{\text{BABAR } b \rightarrow c \ell \bar{\nu}} = (4.65 \pm 0.24_{\text{stat}} \pm 0.24_{\text{syst}}^{+0.46}_{-0.38\text{SF}} \pm 0.23_{\text{th}}) \times 10^{-3}$$

- Results vary slightly depending on particular calculation or experimental input to shape functions
- Uncertainty dominated by theory!

Measurement of $|V_{tb}|$

$$\mathbf{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & \boxed{V_{tb}} \end{pmatrix} \quad \mathbf{V}_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & \boxed{1} \end{pmatrix}$$

$|V_{tb}|$ measured with CDF at Tevatron

- $|V_{tb}|$ expected to be unity to order $\mathcal{O}(\lambda^4)$
- Measurements of all other CKM elements and unitarity of the matrix provide strong constraints on $|V_{tb}|$
- Top quark was discovered in 1995 by both CDF and D0
- Only measurement from CDF in (surprise!) semileptonic decays
 - Distinguish $t \rightarrow b$ transitions from t decays to lighter d and s quarks
 - Count number of events with multiple b tags

$$\frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = 0.94^{+0.31}_{-0.24}$$

Statistical \oplus Systematic

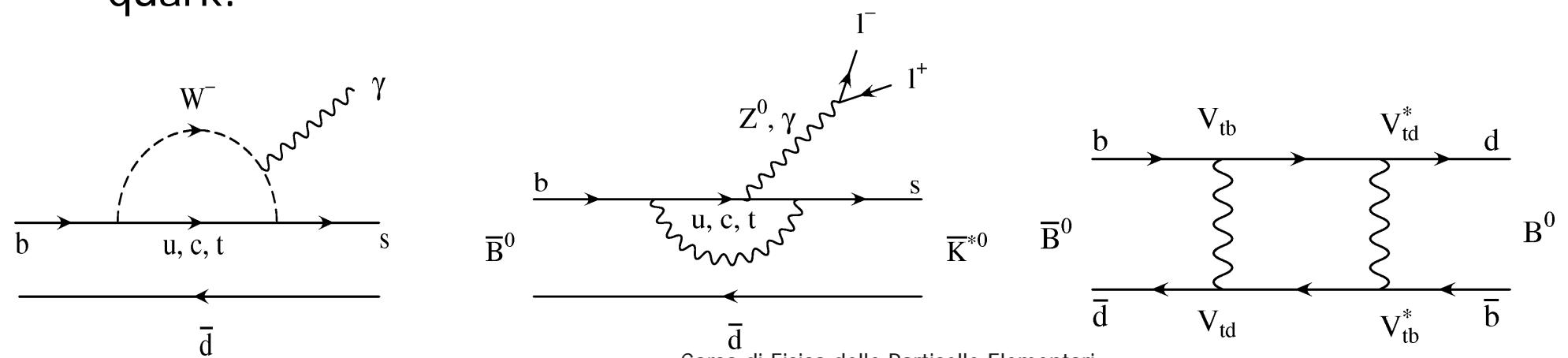
Measurement of $|V_{td}|$ and $|V_{ts}|$

$$\mathbf{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ \boxed{V_{td}} & \boxed{V_{ts}} & V_{tb} \end{pmatrix}$$

$$\mathbf{V}_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ \boxed{A\lambda^3(1-\rho-i\eta)} & \boxed{-A\lambda^2} & 1 \end{pmatrix}$$

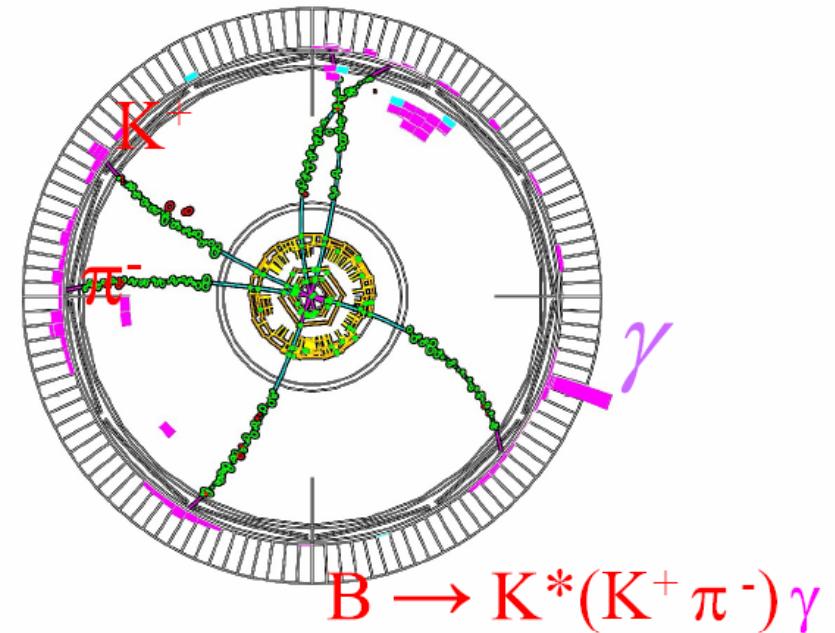
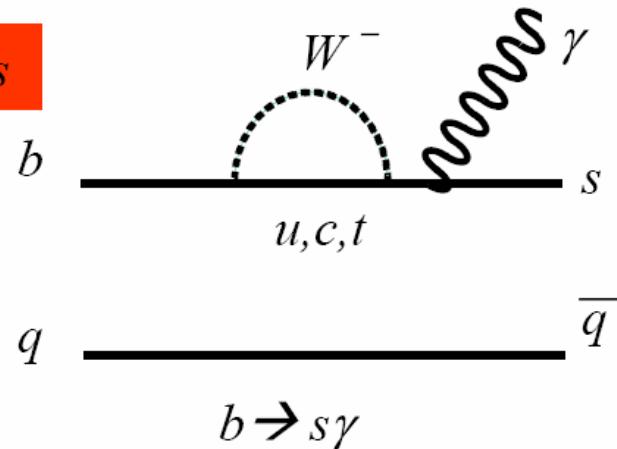
No Trees for Top! Bottom Help Us!

- Top quark is very heavy with very short lifetime
- Top decays too quickly before forming top-flavored hadrons or toponium ($t\bar{t}$ quarkonium)
 - None of tree diagrams used for measurement of other CKM elements
- Use higher order processes involving top quark. But which ones?
 - Loop diagram → Rare processes in Standard Model
 - Sensitive to new particles and New Physics
- Since $|V_{tb}| \sim 1$, use loop diagrams with b quark will certainly have a top quark!



Penguin Decays of B Mesons

Radiative B decays



Inclusive decays are cleaner
(excl. depends upon not very well known form factors)

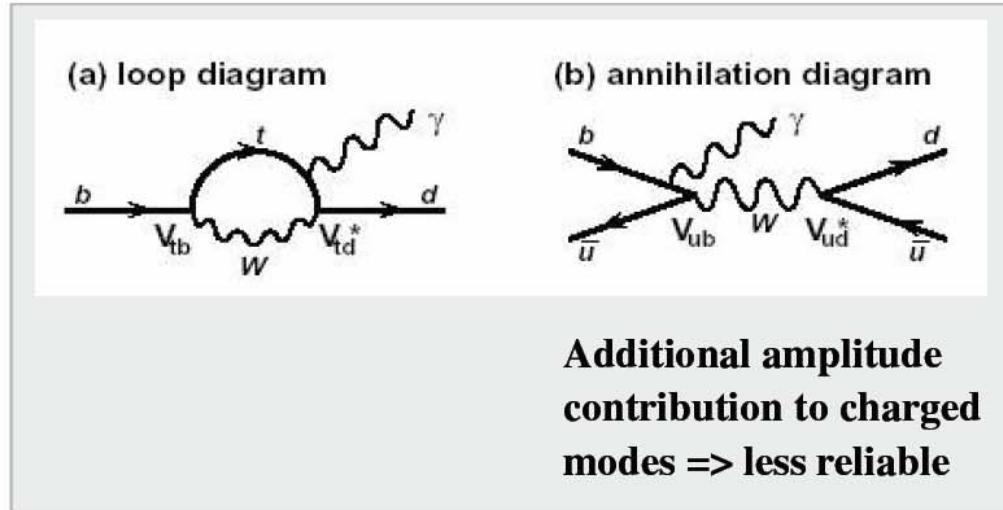
Loops sensitive to New Physics (heavy “objects” in the loop)

Photon energy spectrum depends on the quark mass and Fermi movement
→ important for addressing theoretical error for V_{cb} (see later)

if $b \rightarrow d\gamma$ is also measured : $Br(b \rightarrow d\gamma)/Br(b \rightarrow s\gamma) \propto |V_{td}/V_{ts}|^2$

Radiative Penguin Decays

$B \rightarrow \rho \gamma (\propto |V_{td}|^2)$ & $B \rightarrow K^* \gamma (\propto |V_{ts}|^2)$ sensitive to New Physics



Ratio of BFs predicted more cleanly than individual rates: SU(3) breaking correction

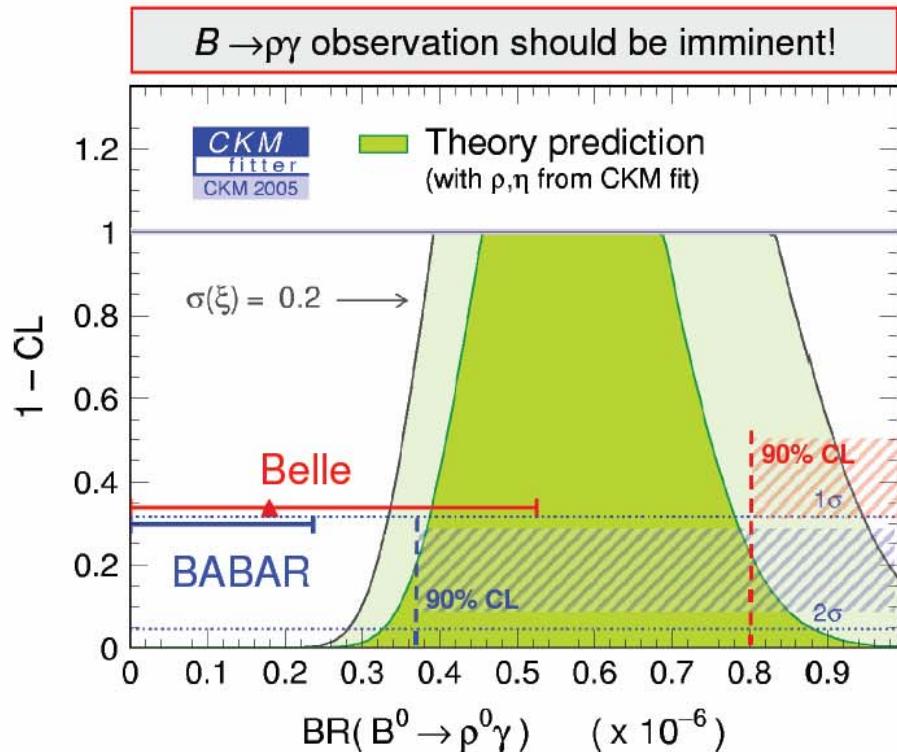
$$\frac{BF(B^0 \rightarrow \rho^0 \gamma)}{BF(B^0 \rightarrow K^{*0} \gamma)} = 1.023 \frac{1}{2} \left| \frac{V_{td}}{V_{ts}} \right|^2 \xi^{-2} (1 + \Delta)$$

Kinematical Factor

SU(3)-breaking:
 $\xi = 1.2 \pm 0.1 (?)$ WA, c-u-peng.:
 $\Delta < 0.04$

Ali, Parkhomenko, EPJ C23 (2002) 89
 Bosch, Buchalla, NP B621 (2002) 459
 (and later papers); errors from CKM 05

The situation until June 2005:



$$BF(B^0 \rightarrow K^{*0} \gamma) = (40.1 \pm 2.0) 10^{-6}$$

Limits on radiative decays

Heavy Flavor Averaging Group
July 2005

Compilation of B^0 Semi-leptonic and Radiative Branching Fractions
All branching fractions are in units of 10^{-6}

In PDG2004 New since PDG2004 (preliminary) New since PDG2004 (published)

RPP#	Mode	PDG2004 Avg.	BABAR	Belle	CLEO	New Avg.
162	$K^*(892)^0\gamma$	43 ± 4	$39.2 \pm 2.0 \pm 2.4$	$40.1 \pm 2.1 \pm 1.7$	$45.5^{+7.2}_{-6.8} \pm 3.4$	40.1 ± 2.0
163	$K^0\phi\gamma$	< 8.3		< 8.3		< 8.3
164	$K^+\pi^-\gamma$ †	4.6 ± 1.4		$4.6^{+1.3+0.5}_{-1.2-0.7}$		4.6 ± 1.4
–	$K^0\pi^+\pi^-\gamma$	New	$18.5 \pm 2.1 \pm 1.2$	$24 \pm 4 \pm 3$		19.5 ± 2.2
–	$K^+\pi^-\pi^0\gamma$	New	$40.7 \pm 2.2 \pm 3.1$			40.7 ± 3.8
165	$K^*(1410)^0\gamma$	< 130		< 130		< 130
166	$K^+\pi^-\gamma$ (N.R.) †	< 2.6		< 2.6		< 2.6
167	$K_1(1270)^0\gamma$	< 7000		< 58		< 58
168	$K_1(1400)^0\gamma$	< 4300		< 12		< 12
169	$K_2^*(1430)^0\gamma$	13 ± 5	$12.2 \pm 2.5 \pm 1.0$	$13 \pm 5 \pm 1$		12.4 ± 2.4
–	$K^0\eta\gamma$	New		$8.7^{+3.1+1.9}_{-2.7-1.6}$		$8.7^{+3.6}_{-3.1}$
173	$\rho^0\gamma$	< 1.2	$0.0 \pm 0.2 \pm 0.1$	$1.17^{+0.35+0.09}_{-0.31-0.08}$	< 17	0.38 ± 0.18
174	$\omega\gamma$	< 1.0	$0.5 \pm 0.3 \pm 0.1$	$0.58^{+0.35+0.07}_{-0.27-0.08}$	< 9.2	$0.54^{+0.23}_{-0.21}$
175	$\phi\gamma$	< 3.3	< 0.94		< 3.3	< 0.94
237	$K^0e^+e^-$	< 0.54	$0.14^{+0.16}_{-0.11} \pm 0.02$ ‡	$-0.070^{+0.129+0.014}_{-0.082-0.028}$ ‡	< 8.45	$0.045^{+0.090}_{-0.080}$
238	$K^0\mu^+\mu^-$	$0.56^{+0.29}_{-0.24}$	$0.60^{+0.34}_{-0.27} \pm 0.05$	$0.626^{+0.217+0.038}_{-0.181-0.041}$	< 6.64	$0.618^{+0.185}_{-0.155}$
239	$K^0l^+l^-$	< 0.68		$0.328^{+0.134+0.022}_{-0.113-0.026}$		$0.328^{+0.136}_{-0.116}$
240	$K^*(892)^0e^+e^-$	< 2.4	$1.03^{+0.33}_{-0.29} \pm 0.12$	$1.85^{+0.55}_{-0.49} \pm 0.19$		$1.28^{+0.30}_{-0.29}$
241	$K^*(892)^0\mu^+\mu^-$	1.3 ± 0.4	$0.89^{+0.39}_{-0.33} \pm 0.14$	$1.85^{+0.35}_{-0.31} \pm 0.10$		1.48 ± 0.26
243	$K^*(892)^0l^+l^-$	1.17 ± 0.30		$1.69^{+0.26}_{-0.24} \pm 0.11$		$1.69^{+0.28}_{-0.26}$

† $1.25 \text{ GeV}/c^2 < M_{K\pi} < 1.6 \text{ GeV}/c^2$ ‡ Central values are not significant.

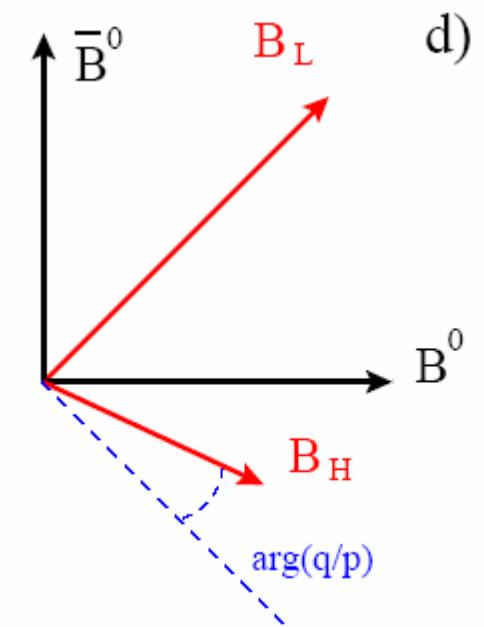
B_d^0 - \bar{B}_d^0 Oscillation

- We will discuss the oscillation phenomenon in great detail in the next 2 lectures
 - A necessary ingredient for 2 types of CP Violation
- Oscillation occurs because flavor eigenstates are not eigenstates of weak interactions
 - Nature produces mesons in flavor eigenstates (i.e. quark content): $B_0 = bd$

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

$$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

- A physical state B^0 at $t=0$ can become B^0 at $t' > t$
- Frequency of oscillation given by mass difference
 $\Delta m := m_H - m_L = 0.502 \pm 0.007 \text{ ps}^{-1}$



$|V_{td}|$ from Δm_d

SM prediction (top-loop dominates):

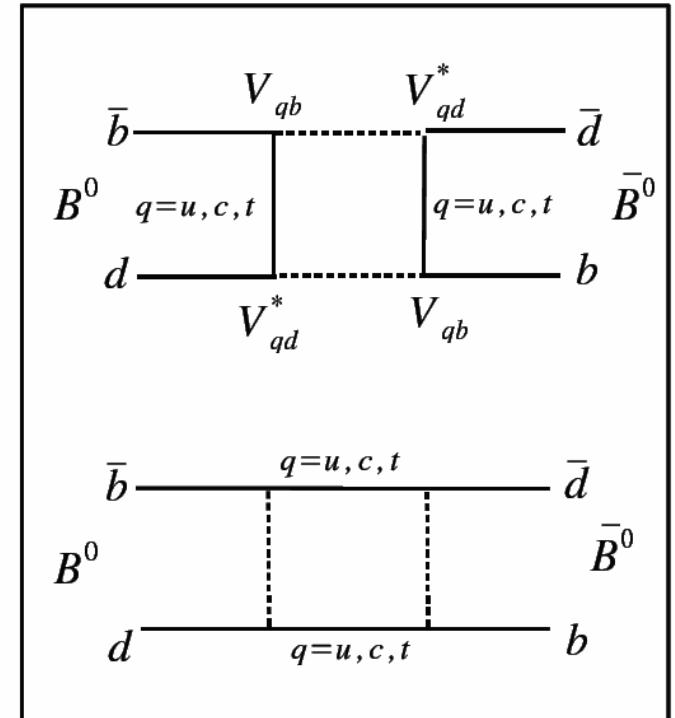
$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_{B_d} m_W^2 \eta_B S_0(x_t) f_{B_d}^2 B_d |V_{td} V_{tb}^*|^2$$

Hadronic matrix element

$$\langle \bar{B}^0 | (\bar{b} \gamma^\mu (1 - \gamma^5) d)^2 | B^0 \rangle$$

$$= \frac{8}{3} m_{B_d}^2 f_{B_d}^2 \hat{B}_d$$

Loop integral
("Inami-Lim"-function)
including radiative corr.



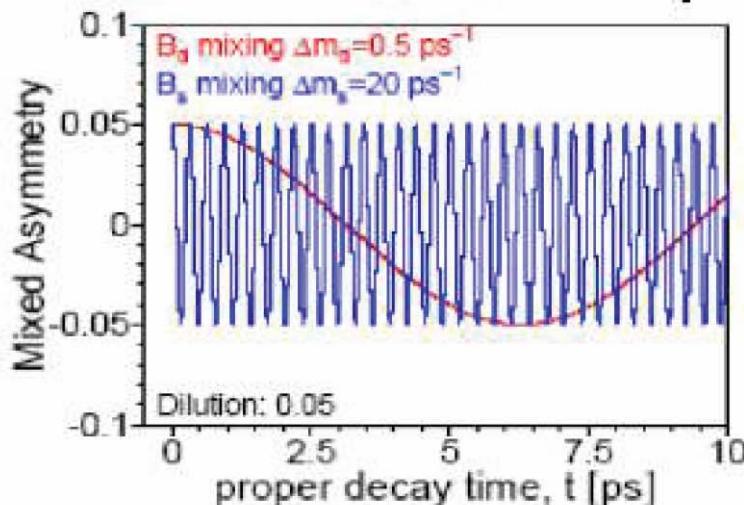
$$S_0(x_i) = x_i \left(\frac{1}{4} + \frac{9}{4(1-x_i)} - \frac{3}{2(1-x_i)^2} \right) - \frac{3}{2} \left(\frac{x_i}{1-x_i} \right)^3 \ln x_i \quad x_i = \left(\frac{m_i}{M_W} \right)^2$$

$$S_0(x_i, x_j) = x_i x_j \left[\left(\frac{1}{4} + \frac{3}{2(1-x_i)} - \frac{3}{4(1-x_i)^2} \right) \frac{\ln x_i}{x_i - x_j} + (x_j \leftrightarrow x_i) - \frac{3}{4} \frac{1}{(1-x_i)(1-x_j)} \right]$$

$|V_{ts}|$ from Δm_s

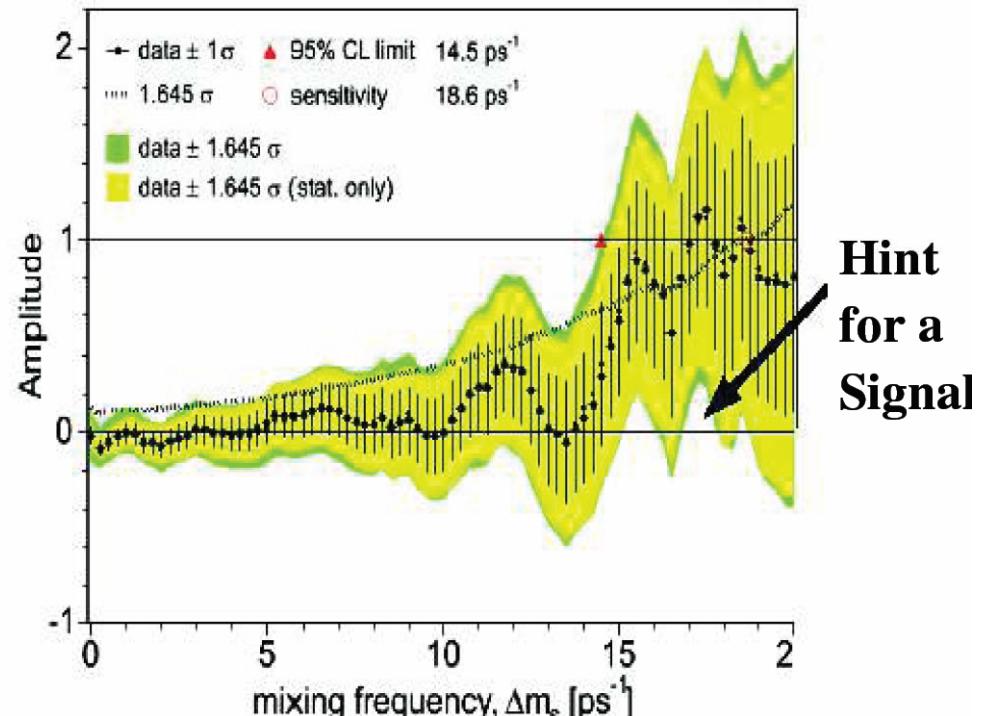
$$\Delta m_s = \frac{G_F^2}{6\pi^2} m_{B_s} m_W^2 \eta_B S_0(x_t) f_{B_s}^2 B_s |V_{ts} V_{tb}^*|^2$$

=> B_s -Oscillations much more rapid than B_d -Oscillations ($\sim O(30)$)



So far, only lower limits from LEP, SLD, Tevatron experiments.

Combined amplitude spectrum (World average + recent CDF II):



Amplitude Method

$$P_{s,\pm}(t) = \frac{1}{2\tau} e^{-t/\tau} (1 \pm A \cos(\Delta m_s t))$$

=> Scan m_s and fit parameter A

$$m_s = m_{s,\text{true}} \Rightarrow A=1; A=0 \text{ else}$$

Combining Δm_d and Δm_s

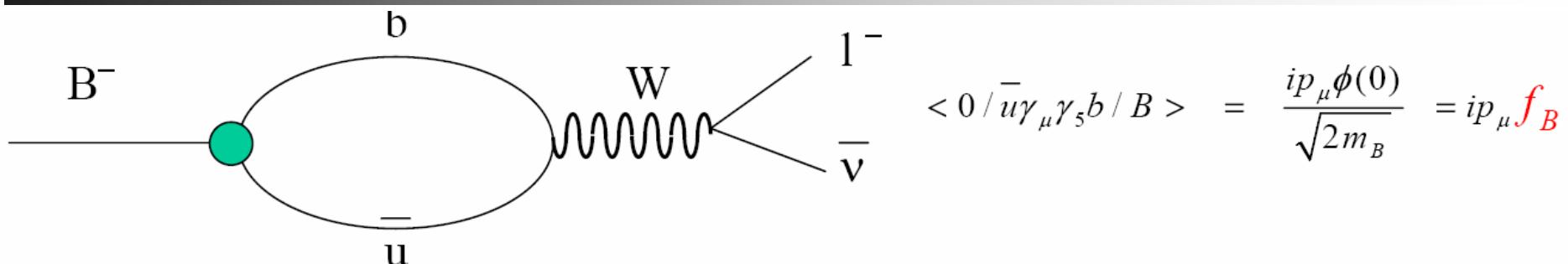
$$f_{B_s}^2 B_s = \frac{f_{B_s}^2 B_s}{f_{B_d}^2 B_d} f_{B_d}^2 B_d = \xi^2 f_{B_d}^2 B_d$$

=> Measurement constrains $f_{B_d}^2 B_d$
since ξ is better known from
Lattice QCD:

$$\xi = 1.21 \pm 0.04 \pm 0.05_{\text{chiral logs}}$$

- By measuring both oscillation frequencies can obtain constraints on $|V_{td}|$ and $|V_{ts}|$
- But what about the B decay constant?
- Can be measured from data!

B Decay Constant f_B



$$\langle 0 | \bar{u} \gamma_\mu \gamma_5 b / B \rangle = \frac{i p_\mu \phi(0)}{\sqrt{2m_B}} = i p_\mu f_B$$

Vacuum saturation approx : the matrix element of V-A current is calculated between the vacuum and the pseudoscalar meson, only the axial current contributes (pseudovector*pseudoscalar \rightarrow vector p_μ)

f_B translates the probability that the quark and the antiquark meet to decay
(or the size of the B meson wave function at the origin)

NON PERTURBATIVE
QCD parameter

Let's consider the oscillation diagram

Soft gluon exchange between quark
and antiquark in the meson and gluon
loops around each quark

$$\langle \bar{B}_q^0 / O(\Delta B = 2) / B_q^0 \rangle \propto \langle \bar{B}_q^0 / \bar{b} \gamma_\mu (1 - \gamma_5) q / 0 \rangle \langle 0 / \bar{b} \gamma_\mu (1 - \gamma_5) q / B_q^0 \rangle \propto f_{B_d}^2 m_B^2$$

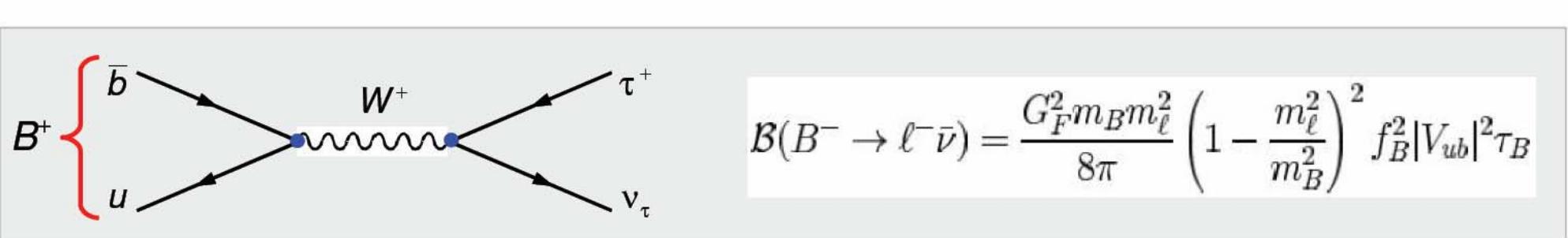
A new constant B_B is inserted to take all possible deviations from vacuum saturation approximation into account

$$\infty \rightarrow f_{B_d}^2 B_{B_d} m_B^2 \rightarrow \text{Gluon exchange between } B^0 \text{ and } \bar{B}^0 \quad B_B \sim 1$$

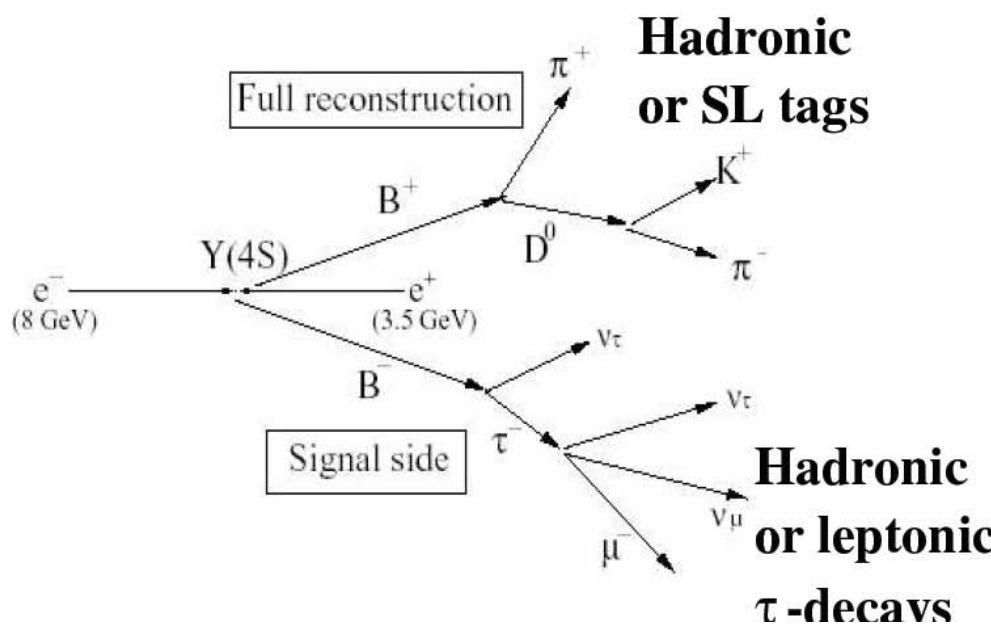
There is a factor $8/3$

f_B from Rare Decay $B \rightarrow \tau\nu$

- Helicity-suppressed annihilation decay sensitive to $(f_B * |V_{ub}|)^2$
- Powerful together with Δm_d : removes f_B (Lattice QCD) dependence
- Sensitive, e.g., to charged Higgs replacing the W -propagator



Experimental techniques:



BABAR: $BF(B^+ \rightarrow \tau^+ \nu_\tau) < 2.6 \cdot 10^{-4}$ @ 90% C.L.

Datta, SLAC seminar 2005

Belle: $BF(B^+ \rightarrow \tau^+ \nu_\tau) < 1.8 \cdot 10^{-4}$ @ 90% C.L.

EPS 2005

Prediction from global CKM fit:

$$BF(B^+ \rightarrow \tau^+ \nu_\tau) = (8.2^{+1.7}_{-1.3}) \cdot 10^{-5}$$

$$(+5.0 \text{ @ } 95\% \text{ C.L.})$$

CKMfitter Group, CKM'05