Cabibbo-Kobayashi-Maskawa Matrix and CP Violation in Standard Model

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Lecture 3
Introduction to CP Violation

Lezioni di Fisica delle Particelle Elementari
Outline of Today’s Lecture

- What is CP Violation and why do we care?

- CKM matrix revisited
  - CP Violation in the Standard Model

- Experimental method to measure CP Violating effects
  - Quantum interference
Asymmetric Universe of Matter

- Universe is very empty but in a biased way
  
  \[
  \frac{n_{\text{baryon}}}{n_{\text{photons}}} \approx 10^{-18}, \quad \frac{N(\text{anti-baryon})}{N(\text{baryon})} \leq 10^{-4} - 10^{-6}
  \]

- Absence of anti-nuclei amongst cosmic rays in our galaxy
- Absence of intense $\gamma$-ray emission due to annihilation of distant galaxies in collision with antimatter galaxies

- The early universe believed to have equal amount of matter and anti-matter
  - What happened to the anti-matter?

- **CP Violation** is one of the three ingredients required to generate such an asymmetry after the Big Bang (A. Sakharov, 1967)
  - Baryon-number violating processes
  - Non-equilibrium state during expansion
  - C and CP Violation
C and P Symmetries and Fundamental Interactions

- **Parity, \( P \)**
  - Parity reflects a system through the origin. Converts right-handed coordinate systems to left-handed ones.
  - Vectors change sign but axial vectors remain unchanged
    - \( x \rightarrow -x, L \rightarrow L \)

- **Charge Conjugation, \( C \)**
  - Charge conjugation turns a particle into its anti-particle
    - \( e^+ \rightarrow e^-, K^- \rightarrow K^+, \gamma \rightarrow \gamma \)
CP Symmetry, particles and anti-particles

- CP symmetry transforms a particle in its anti-particle

- CP is violated if particles and anti-particles behave differently!
Weak Interactions and Symmetry Violation

- P and C are good symmetries of the strong and electromagnetic interactions

- Parity violation observed in 1957
  - Asymmetry in $\beta$ decays of $^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- + \nu$
  - Electrons produced mostly in one hemisphere

- Charge-conjugation violation 1958
  - Only left-handed neutrinos and right-handed anti-neutrinos

- CP believed to be a good symmetry, but …
A Shocker: Weak Interaction Violates Parity!

Observation of a spatial asymmetry in the $\beta$-decay electrons from $^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- + \nu$

- Cold $^{60}\text{Co}$ inside a Solenoidal B Field
- $^{60}\text{Co}$ nuclei spin aligned with B field direction
- $^{60}\text{Co}$ undergoes $\beta$ decay .......electron emitted
- Measure electron intensity w.r.t B field dir.
- Result: Electrons preferentially emitted opposite spin dir.

\[ I(\theta) = 1 - \frac{V_e}{c} \cos \theta \]

asymmetry of intensity $\rightarrow$ Weak interaction violated Parity
CP Violation in Kaons

- CP conservation implies
  - \( \text{CP} = +1 \)
  - \( \text{CP} = -1 \)

- CP violation in kaons observed in 1964

  0.2% of the time!

- No theoretical explanation!
Observation of CP Violation in Kaons

\[ A \left( \left| K_L^0 \right> \rightarrow 2\pi \right) \over A \left( \left| K_s^0 \right> \rightarrow 2\pi \right) = (2.27 \pm 0.02) \times 10^{-3} \]

2-body decay: the two \( \pi \) are back-to-back: \( |\cos \theta| = 1 \)
Complex Coupling Constants and CP Violation

<table>
<thead>
<tr>
<th>Fermion bilinear</th>
<th>Boson field $F$</th>
<th>$PF,P^\dagger$</th>
<th>$CF,C^\dagger$</th>
<th>$CPF,CP^\dagger$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\psi}\psi$</td>
<td>Scalar $S^+(t, \bar{x})$</td>
<td>$S^+(t, -\bar{x})$</td>
<td>$S^-(t, \bar{x})$</td>
<td>$S^-(t, -\bar{x})$</td>
</tr>
<tr>
<td>$\bar{\psi}\gamma^5\psi$</td>
<td>Pseudoscalar $P^+(t, \bar{x})$</td>
<td>$-P^+(t, -\bar{x})$</td>
<td>$P^-(t, \bar{x})$</td>
<td>$-P^-(t, -\bar{x})$</td>
</tr>
<tr>
<td>$\bar{\psi}\gamma_\mu\psi$</td>
<td>Vector $V^+_{\mu}(t, \bar{x})$</td>
<td>$V^+_{\mu}(t, -\bar{x})$</td>
<td>$-V^-_{\mu}(t, \bar{x})$</td>
<td>$-V^-_{\mu}(t, -\bar{x})$</td>
</tr>
<tr>
<td>$\bar{\psi}\gamma_\mu\gamma^5\psi$</td>
<td>Axial $A^+_{\mu}(t, \bar{x})$</td>
<td>$-A^+_{\mu}(t, -\bar{x})$</td>
<td>$A^-_{\mu}(t, \bar{x})$</td>
<td>$-A^-_{\mu}(t, -\bar{x})$</td>
</tr>
</tbody>
</table>

Table 2.1: Properties of charged boson fields and corresponding fermion bilinear terms under $P$, $C$, and $CP$. $\gamma^5$ and $\gamma^\mu$ are the Dirac matrices.

Generic interaction lagrangian with vector and axial fields

$$L = a \, V^+_{\mu}(t, \bar{x})V^{\mu-}(t, \bar{x}) + b \, A^+_{\mu}(t, \bar{x})A^{\mu-}(t, \bar{x}) + c \, V^+_{\mu}(t, \bar{x})A^{\mu-}(t, \bar{x}) + c^* \, A^+_{\mu}(t, \bar{x})V^{\mu-}(t, \bar{x})$$

$a, b$: real constants
$c$: complex constant

Lagrangian after CP transformation

$$CP\,LCP^\dagger = a \, V^-_{\mu}(t, -\bar{x})V^{\mu+}(t, -\bar{x}) + b \, A^-_{\mu}(t, -\bar{x})A^{\mu+}(t, -\bar{x}) + c \, V^-_{\mu}(t, -\bar{x})A^{\mu+}(t, -\bar{x}) + c^* \, A^-_{\mu}(t, -\bar{x})V^{\mu+}(t, -\bar{x}) .$$

Lagrangian invariant under CP IF AND ONLY IF $c = c^*$! $c$ must be real
Reminder Kobayashi-Maskawa Mechanism of CP Violation

1972

- Proposed a daring explanation for CP violation in K decay:

- CP violation appears only in the charged current weak interaction of quarks

- There is a single source of CP Violation \( \Rightarrow \) Complex Quantum Mechanical Phase \( \delta_{KM} \) in inter-quark coupling matrix

- Need at least 3 Generation of Quarks (then not known) to facilitate this

- CP is NOT an approximate symmetry, \( \delta_{KM} \approx 1 \), it is MAXIMALLY violated!
CKM Matrix Revisited

\[ \mathbf{V}_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

\[ \mathbf{V}_{\text{CKM}} = \begin{pmatrix} 1 & 1 & e^{-i\gamma} \\ 1 & 1 & 1 \\ e^{-i\beta} & 1 & 1 \end{pmatrix} \]

\[ \mathbf{V}_{\text{CKM}} = \begin{pmatrix} 1 - \lambda^2 / 2 & \lambda & A\lambda^3 (\rho - i\eta) \\ -\lambda & 1 - \lambda^2 / 2 & A\lambda^2 \\ A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \]

<table>
<thead>
<tr>
<th>Quark families</th>
<th># Angles</th>
<th># Phases</th>
<th># Irreducible Phases</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>n(n-1)/2</td>
<td>n(n+1)/2</td>
<td>n(n-1)/2 - (2n-1) = (n-1)(n-2)/2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>10</td>
<td>3</td>
</tr>
</tbody>
</table>

Only Source of CP Violation in SM

CP Violation built in the Standard Model through Kobayashi-Maskawa Mechanism!

Only one complex phase! All CP violating effects in SM related to each other B and K decays CP Violating phenomena are cause by the same complex phase.
Unitarity of CKM Matrix

\[ V^\dagger V = VV^\dagger = 1 \]

- All rows and columns must be orthonormal
  - 3 conditions for diagonal elements
  - 6 conditions for off-diagonal elements

\[ \begin{align*}
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 &= 1 \\
|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 &= 1 \\
|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 &= 1
\end{align*} \]

Magnitude of each term

Only condition with comparable size of all pieces and involving \( b \) decays
Unitarity Triangles

Unitarity condition of CKM Matrix $\rightarrow$ orthonormality of rows & columns

$\sum_{(i=u,c,t)} V_{ij}V_{ik}^* = \delta_{jk} ; \sum_{(i=d,s,b)} V_{ij}V_{kj}^* = \delta_{ik}$

$\Rightarrow$ three conditions are interesting for understanding SM predictions for CP violation

$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0,$

$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0,$

$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$

Each relation requires sum of three complex quantities to vanish

$\Rightarrow$ can be represented in the complex plane as a triangle

$\Rightarrow$ known as Unitarity Triangles

With the knowledge of $|V_{ij}|$ magnitudes, its instructive to draw the triangles
Three Unitarity Triangles Drawn to Common Scale

\[ V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \]
\[ V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \]
\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \]

(a) ds

(b) sb

(c) db

\[ \Rightarrow \] All sides of comparable length \( \lambda^3 \)

\[ \Rightarrow \] All angles are large

Experimentally $\Rightarrow$ hard to measure small numbers
easier to measure larger numbers as in (c)

One side is much shorter than the other two $\Rightarrow$ triangle collapses on a line
CKM Unitarity Triangle in B Decays

\[ V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \]

Angles of Unitarity Triangle

\[ \alpha = \phi_2 \equiv \arg \left( -\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right), \]

\[ \beta = \phi_1 \equiv \arg \left( -\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right), \]

\[ \gamma = \phi_3 \equiv \arg \left( -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right), \]

All lengths involve b decays
Large CP Asymmetries predicted, \( \propto \) UT angles
Measuring Complex Phase of CKM Matrix

- Branching fractions and lifetimes sensitive to magnitude of CKM elements
  - Decay probabilities usually include $|V_{ij}|^2$
  - We looked for decays involving only one CKM element to make interpretation of experimental result possible

- Complex phase of CKM is a relative phase between matrix elements

- We need processes with interference of two different CKM elements

$$A_1 = Ae^{i\alpha}$$
$$A_2 = Be^{i\beta}$$
$$A_{tot} = A_1 + A_2$$

$$|A_{tot}|^2 = |A|^2 + |B|^2 + ABe^{i(\alpha-\beta)} + ABe^{-i(\alpha-\beta)}$$

Sensitive to phase difference!
CP Violation

- CP violation can be observed by comparing decay rates of particles and antiparticles

\[ \Gamma(a \rightarrow f) \neq \Gamma(\bar{a} \rightarrow \bar{f}) \Rightarrow \text{CP Violation} \]

- The difference in decay rates arises from a different interference term for the matter vs. antimatter process. Analogy to double-slit experiment:

\[
\begin{align*}
\text{source} & \quad A_1 & A_2 \\
B^0 & \quad A_1 & f & \quad B^0 \\
\bar{B}^0 & \quad \bar{A}_1 & \bar{f} & \quad \bar{B}^0
\end{align*}
\]

Classical double-slit experiment:
Relative phase variation due to different path lengths: interference pattern in space
Identify B final states which are arrived at by two paths

In B\(^0\) system, B\(^0\) \(\rightarrow\) \(\bar{B}\)\(^0\) oscillation provides one path with the other path(s) come from weak decay of B hadron.

In B\(^\pm\) system \(\Rightarrow\) no oscillation possible, 2 (or more) amplitudes must come from different weak decay of B.

B Meson is heavy \(\Rightarrow\) many final states, multiple “paths.”

2 classes of B decays come into play: “Tree” \(\Rightarrow\) spectator decay like “Penguin” \(\Rightarrow\) FCNC loop diagrams with u,c,t.
Overview of CP Violating Processes

CP Violation in Decay
a.k.a. Direct CPV

\[ A(B \to f) \neq \bar{A}(\bar{B} \to \bar{f}) \]

CP Violation in Mixing

\[ A(B^0 \to \bar{B}^0) \neq A(\bar{B}^0 \to B^0) \]

CP Violation in interference between Mixing and Decay

\[ A(B^0 \to f_{cp}) \neq A(\bar{B}^0 \to f_{cp}) \]
CP Violation Is a Quantum Phenomenon

- CPV is due to Quantum interference between two or more amplitudes

- Phase of QM amplitudes is the key

- Need to consider two types of phases
  - **CP-conserving phases**: don’t change sign under CP
    - Sometimes called *strong phases* since they can arise from strong, final-state interactions
  - **CP-violating phases**: these do change sign under CP transformation
    - originate in the Weak interaction sector

\[
A = Ae^{i\phi}e^{i\delta}
\]

\[
\bar{A} = Ae^{-i\phi}e^{i\delta}
\]
How can CP asymmetries arise?

- Suppose a decay can occur through two different processes, with amplitudes $A_1$ and $A_2$

- First, consider the case in which there is a (relative) CP-violating phase between $A_1$ and $A_2$ only

\[
A = A_1 + a_2 e^{i\varphi_2}
\]

\[
\bar{A} = A_1 + a_2 e^{-i\varphi_2}
\]

→ No Direct CP asymmetry!

(Decay rate is different from what it would be without the phase)
How can CP asymmetries arise?

- Next, introduce a relative *CP-conserving* phase in addition to the relative *CP-violating* phase

\[
A = A_1 + a_2 e^{i(\varphi_2 + \delta_2)}
\]

\[
\bar{A} = A_1 + a_2 e^{i(-\varphi_2 + \delta_2)}
\]

- Now have a Direct CP Violation

\[
|A| \neq |\bar{A}|
\]
**Definition of CP Asymmetry**

\[
Asymmetry = \frac{\left| \overline{A} \right|^2 - \left| A \right|^2} {\left| \overline{A} \right|^2 + \left| A \right|^2} = \frac{2 |A_1||A_2| \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2)} {\left| A_1 \right|^2 + \left| A_2 \right|^2 + |A_1||A_2| \cos(\delta_1 - \delta_2) \cos(\phi_1 - \phi_2)}
\]

To extract the CP-violating phase from an observed CP asymmetry, we need to know the value of the CP-conserving phase difference.

**B system:** extraordinary laboratory for quantum interference experiments: many final states, multiple “paths” → Lots of channels for CP Violation
Direct CP Violation

\[ \left| B_{A(B \rightarrow f)} \right|^2 \neq \left| B_{\bar{A}(\bar{B} \rightarrow \bar{f})} \right|^2 \]
CPV in Decay a.k.a. Direct CP Violation

\[ A(B \to f) \neq \bar{A}(\bar{B} \to \bar{f}) \]

\[ \Gamma(B \to f) = \left| A_1 + A_2 e^{i\varphi_w k} e^{i\delta_{st}} \right|^2, \quad \Gamma(\bar{B} \to \bar{f}) = \left| A_1 + A_2 e^{-i\varphi_w k} e^{i\delta_{st}} \right|^2 \]

\[ A_{CP} = \frac{Br(\bar{B} \to \bar{f}) - Br(B \to f)}{Br(\bar{B} \to \bar{f}) + Br(B \to f)} \equiv \frac{|\bar{A}_f|^2 - |A_f|^2}{|\bar{A}_f|^2 + |A_f|^2} \neq 0 \to \text{Direct CPV} \]
Direct CP Violation in $B^0 \to K^- \pi^+$

**SM amplitude** = $\lambda^2 e^{i\gamma} T + P$

- Loop diagrams from New Physics (e.g. SUSY) can modify SM asymmetry via $P$
- Clean mode with “large” rate: $BF(B^0 \to K^+\pi^-) = (18.2 \pm 0.8) \times 10^{-6}$
  - Measure *charge* asymmetry, reject large $B \to \pi\pi$ background with Particle ID

**Čerenkov angle**

- $\pi$
- $K$

**$K\pi$ separation**

**$\Delta E$ (GeV)**

- Signal
- $B$ background

10 Nov 2006
Observation of Direct CPV in $B^0 \rightarrow K^-\pi^+$

$$A_{K^-\pi^+} \equiv \frac{\Gamma(B \rightarrow K^-\pi^+) - \Gamma(B \rightarrow K^+\pi^-)}{\Gamma(B \rightarrow K^-\pi^+) + \Gamma(B \rightarrow K^+\pi^-)}$$

$n_{K\pi} = 1606 \pm 51$

$A_{K\pi} = -0.133 \pm 0.030 \pm 0.009$

\begin{align*}
n(B^0 \rightarrow K^+\pi^-) &= 910 \\
n(B^0 \rightarrow K^-\pi^+) &= 696
\end{align*}
Confirmation of Direct CPV by Belle at ICHEP04

\[ A_{CP} = -0.101 \pm 0.025 \pm 0.005 \]

3.9\(\sigma\) significance

\[ 274M \bar{B} \bar{B} \]

Signal=2139 ±53

Non-Perturbative QCD uncertainties large,
Standard Model CP Violation not precisely predictable

⇒ insufficient to prove or rule out contribution from New Physics