Cabibbo-Kobayashi-Maskawa Matrix and CP Violation in Standard Model

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Lecture 4 B⁰-B⁰ Oscillation CPV in Mixing and Decay Interference

Lezioni di Fisica delle Particelle Elementari

- Evolution of entangled 2-state quantum system
 - Example of B⁰-B⁰ oscillation
- Formalism of time-dependent CP violation
- Search for CP Violation in Mixing
- Observation of CP Violation in Interference between Decay and Mixing

CP Violating Processes



CP Violation in interference between Mixing and Decay





$B^0 - \overline{B}^0$ Oscillation

B⁰_d-B⁰_d Oscillation and CP Violation

Necessary ingredient for two types of CP Violation

in a two-state quantum system

- V_{tb} V_{td}^* đ B^0 $\overline{\mathbf{B}}^0$ Oscillation is an example of superposition principle đ $\overline{\mathbf{b}}$ V_{td} V_{tb}^*
 - Oscillation occurs because mass and flavor eigenstates are different
 - Flavor eigenstates $|B^0\rangle$ and $|\overline{B}^0\rangle$: physical states with definite quark structure and are produced as a consequence of the quark-level strong interactions.
 - CP eigenstates $|B_{CP=1}\rangle$ and $|B_{CP=-1}\rangle$: eigenstates of the CP operation

$$CP|B_{CP=1}\rangle = +|B_{CP=1}\rangle$$

 $CP|B_{CP=-1}\rangle = -|B_{CP=-1}\rangle$

• Mass eigenstates $|B_L\rangle$ and $|B_H\rangle$: eigenstates of the full Hamiltonian and, hence, with definite mass M and decay width $\Gamma \equiv 1/\tau$. These states evolve in time in a definite fashion according to

$$|B_L, t\rangle = e^{-\Gamma_L t} e^{-iM_L t} |B_L, t = 0\rangle$$
(2.28)

$$|B_H, t\rangle = e^{-\Gamma_H t} e^{-iM_H t} |B_H, t = 0\rangle.$$
(2.29)

Phenomenology of B⁰ Time Development

- An initially B⁰ or $\overline{B^0}$ system evolves with time as a mixture of flavor eigenstates $|\psi(t) = a|B^0\rangle + b|\overline{B}^0\rangle$
- Evolution regulated by time-dependent Schrödinger equation

• M and Γ computed to 2nd order of perturbation theory

$$M_{ij} = m_B \delta_{ij} + \langle i | H_W^{\Delta B=2} | j \rangle + P \sum_n \frac{1}{m_B - E_n} \langle i | H_W^{\Delta B=1} | n \rangle \langle n | H_W^{\Delta B=1} | j \rangle$$

$$\Gamma_{ij} = 2\pi \sum_n \delta(E_n - m_B) \langle i | H_W^{\Delta B=1} | n \rangle \langle n | H_W^{\Delta B=1} | j \rangle.$$

- Virtual intermediate states contribute to M
- Γ receives contributions from physical states to which B⁰ or B⁰ can decay

Mass Eigenstates of Effective Hamiltonian

Solving the Schroedinger equation

$$H|\psi\rangle=\lambda|\psi\rangle$$

Two complex eigenvalues

$$\lambda_{\pm} = M - \frac{i}{2}\Gamma - \sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12})}$$

• Mass eigenstates $|B_L, t\rangle = e^{-\Gamma_L t} e^{-iM_L t} |B_L, t = 0\rangle$ $|B_H, t\rangle = e^{-\Gamma_H t} e^{-iM_H t} |B_H, t = 0\rangle$

$$\begin{split} \Delta m_d &\equiv m_H - m_L \equiv \mathcal{R}e(\lambda_+ - \lambda_-) \\ \Delta \Gamma &\equiv \Gamma_H - \Gamma_L \equiv 2\mathcal{I}m(\lambda_+ - \lambda_-) \end{split} \qquad \begin{array}{l} \Gamma &= 1/\tau_{B^0} = \frac{1}{2}(\Gamma_H + \Gamma_L) \\ \Delta \Gamma &= \Gamma_H - \Gamma_L \end{array} \qquad \begin{array}{l} M = \frac{1}{2}(M_H + M_L) \\ \Delta \Gamma &= \Gamma_H - \Gamma_L \end{array} \qquad \begin{array}{l} \Delta m_d = M_H - M_L \end{aligned}$$

Interpretation of Effective Hamiltonian

 The effective Hamiltonian for the two-state system is not Hermitian since mesons decay



$$M_{12} = (V_{tb}V_{td}^*)^2 \frac{G_F^2}{8\pi^2} \frac{M_W^2}{m_B} S\left(\frac{m_t^2}{M_W^2}\right) \eta_B b_B(\mu) \langle B^0 | Q(\mu) | \overline{B}{}^0 \rangle$$
what we are after calculable perturbatively nonperturbative

Driving $B^0 \leftrightarrow B^0$ Oscillation



In B^o meson system, final states that both B^o and \overline{B}^{o} can decay into have very small rates Decays like b \rightarrow c \overline{c} d or b \rightarrow u \overline{u} d are suppressed due to associated CKM elements in W decay

$$\frac{\Gamma_{12}}{M_{12}} = O(\frac{m_b^2}{m_t^2}) \Box 1$$

B Oscillation is driven by M_{12} , which is dominated by Top quark in the loop

Differences between K and B Mesons

- Formalism for time evolution can be applied to both K and B mesons
- B mesons
 - Very few common states accessible by both B⁰ and B⁰
 - Comparable lifetime and oscillation frequency

 $\Delta\Gamma/\Gamma \lesssim \mathcal{O}(10^{-2}) \qquad \quad x_d \equiv \Delta m_d/\Gamma = 0.73 \pm 0.05$

Mass eigenstates have very similar lifetimes but different masses

 $\Delta \Gamma \ll \Delta m_d$

- Kaons
 - Mass eigenstates with similar masses
 - Very different lifetimes

$$\Delta \Gamma_{K} = \Gamma_{K} - \Gamma_{L} \cong \Gamma_{K} + \Gamma_{L} \cong \Gamma_{K}$$

Relation Between Mass and Flavor states



Time Development of Physical States

• Evolution of a pure B⁰ or B⁰ state at t=0

$$\left| B^{0}_{phys}(t) \right\rangle = \frac{1}{2p} \left(e^{-\Gamma_{L}t} e^{-iM_{L}t} \left(p \left| B^{0} \right\rangle + q \left| \overline{B}^{0} \right\rangle \right) + e^{-\Gamma_{H}t} e^{-iM_{H}t} \left(p \left| B^{0} \right\rangle - q \left| \overline{B}^{0} \right\rangle \right) \right)$$
$$\left| \overline{B}^{0}_{phys}(t) \right\rangle = \frac{1}{2q} \left(e^{-\Gamma_{L}t} e^{-iM_{L}t} \left(p \left| B^{0} \right\rangle + q \left| \overline{B}^{0} \right\rangle \right) - e^{-\Gamma_{H}t} e^{-iM_{H}t} \left(p \left| B^{0} \right\rangle - q \left| \overline{B}^{0} \right\rangle \right) \right)$$

After some math

$$\Gamma = 1/\tau_{B^0} = \frac{1}{2}(\Gamma_H + \Gamma_L)$$
$$\Delta \Gamma = \Gamma_H - \Gamma_L$$

$$M = \frac{1}{2}(M_{\rm H} + M_{\rm L})$$
$$\Delta m_{\rm d} = M_{\rm H} - M_{\rm L}$$

$$\left| B_{phys}^{0}(t) \right\rangle = g_{+}(t) \left| B^{0} \right\rangle + \left(q / p \right) g_{-}(t) \left| \overline{B}^{0} \right\rangle$$
$$\left| \overline{B}_{phys}^{0}(t) \right\rangle = \left(p / q \right) g_{-}(t) \left| B^{0} \right\rangle + g_{+}(t) \left| \overline{B}^{0} \right\rangle$$

$$g_{+}(t) = e^{-iMt} e^{-\Gamma t/2} \cos\left(\Delta m_d t/2\right)$$
$$g_{-}(t) = e^{-iMt} e^{-\Gamma t/2} i \sin\left(\Delta m_d t/2\right)$$

Prob of $B^0 \to \overline{B}^0$ oscillates as function of time !

Time evolution of B^0 and \overline{B}^0 mesons

$$|B^{0}(t)\rangle = e^{-iMt}e^{-\Gamma t} \left(\cos\frac{\Delta m t}{2} \quad |B^{0}\rangle + i \sin\frac{\Delta m t}{2} \cdot \frac{q}{p} |\overline{B^{0}}\rangle \right)$$
$$|\overline{B^{0}}(t)\rangle = e^{-iMt}e^{-\Gamma t} \left(i \sin\frac{\Delta m t}{2} \cdot \frac{p}{q} |B^{0}\rangle + \cos\frac{\Delta m t}{2} \quad |\overline{B^{0}}\rangle \right)$$

$$\frac{q}{p} = \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} = e^{-i2\beta}$$
$$P(B^0 \to \overline{B}^0) \propto$$
$$e^{-\Gamma t} \left(1 - \cos(\Delta m t)\right)$$

Slow oscillation compared to the lifetime



Quantum Entanglement in $\Upsilon(4S) \rightarrow B^0B^0$ Decays

O(4S) Spin =

- Strong interaction: CP is and flavor beauty number are conserved
 - Must have one **b** and one **anti-b** quarks in final state

$$|B_{\rm phys}^0 \overline{B}_{\rm phys}^0 \rangle = \frac{a}{\sqrt{2}} |B_L B_H \rangle + \frac{b}{\sqrt{2}} |B_H B_L \rangle$$

Time evolution given by mass eigenstates

$$|B_{\rm phys}^0\overline{B}_{\rm phys}^0;t_1,t_2\rangle = a \,\mathrm{e}^{i\lambda_+t_1}\mathrm{e}^{i\lambda_-t_2}|B_LB_H\rangle + b \,\mathrm{e}^{i\lambda_-t_1}\mathrm{e}^{i\lambda_+t_2}|B_HB_L\rangle$$

- Bose-Einstein Statistics requires wave function $|\Psi>$ to be symmetric at all times $|\Psi\rangle = |\Psi_{\text{flavor}}\rangle|\Psi_{\text{space}}\rangle$
- L=-1 implies asymmetric spatial wave function
- We need a=-b which means a B^0 and a B^0 meson at all times until one of them decays!
 - Example of Einstein-Podolsky-Rosen Paradox

With L = 1

Quantum Correlation at Y(4S)



- Decay of first B (B^0) at time t_{tag} ensures the other B is B^0
 - End of Quantum entanglement ! Defines a ref. time (clock)
- At t > t_{tag} , B⁰ has some probability to oscillate into B⁰ before it decays at time t_{flav} into a flavor specific state
- Two possibilities in the Y(4S) event depending on whether the 2nd B oscillated or not:

no oscillation/mixing $\Rightarrow B^0 \bar{B}^0$ in final state oscillation/mixing $\Rightarrow \bar{B}^0 \bar{B}^0$ in final state

Time Evolution of $\Upsilon(4S) \rightarrow B^0B^0$



Separating B^0 and \overline{B}^0 mesons





Time Dependent B Oscillation (Or Mixing) at Υ (4S)



CP Violation in Interference between Mixing and Decay



Time-Evolution of B Decays to CP Eigenstates

- Probability of $|B^0>|B^0> \rightarrow |f_{CP}>|f_{tag}>$ depends on
 - Difference Δt between decay time of the two B mesons
 - Decay amplitudes $A_{f_{CP}} = \langle f_{CP} | H | B^0, t \rangle$ $\overline{A}_{f_{CP}} = \langle f_{CP} | H | \overline{B}^0, t \rangle$
 - Oscillation parameter

$$\frac{q}{p} = -\frac{|M_{12}|}{M_{12}} = e^{-i2\beta}$$

• Flavor of tagging neutral B meson: B⁰ or B0

- Convenient parameter to describe time evolution
 - Takes into account combined effect of oscillation and decay

$$\lambda = \frac{q}{p} \frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}} = \eta_{f_{CP}} \frac{q}{p} \frac{\overline{A}_{\overline{f}_{CP}}}{A_{f_{CP}}}$$

Time-Dependent Decay Rates to CP Eigenstates

$$f_{B_{\text{tag}}=B^{0}}(t_{\text{tag}}, t_{f_{CP}}) \propto e^{-\Gamma(t_{f_{CP}} - t_{\text{tag}})} \left\{ 1 + \frac{1 - |\lambda_{f_{CP}}|^{2}}{1 + |\lambda_{f_{CP}}|^{2}} \cos[\Delta m_{d}(t_{f_{CP}} - t_{\text{tag}})] - \frac{2\mathcal{I}m\lambda_{f_{CP}}}{1 + |\lambda_{f_{CP}}|^{2}} \sin[\Delta m_{d}(t_{f_{CP}} - t_{\text{tag}})] \right\}$$

$$\begin{split} f_{B_{\text{tag}}=\overline{B}^{0}}(t_{\text{tag}},t_{f_{CP}}) &\propto e^{-\Gamma(t_{f_{CP}}-t_{\text{tag}})} \Big\{ 1 - \frac{1 - |\lambda_{f_{CP}}|^{2}}{1 + |\lambda_{f_{CP}}|^{2}} \cos[\Delta m_{d}(t_{f_{CP}}-t_{\text{tag}})] \\ &+ \frac{2\mathcal{I}m\lambda_{f_{CP}}}{1 + |\lambda_{f_{CP}}|^{2}} \sin[\Delta m_{d}(t_{f_{CP}}-t_{\text{tag}})] \Big\} \\ &\lambda = \frac{q}{p} \frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}} = \eta_{f_{CP}} \frac{q}{p} \frac{\overline{A}_{\overline{f}_{CP}}}{A_{f_{CP}}} \end{split}$$

- Expression and complexity of λ depends on specific final states
 - Decay amplitudes A and A can be more or less complicated depending on number of amplitudes contributing to total amplitude

Time-Dependent CP Asymmetry in Interference

$$a_{f_{CP}}(\Delta t) = \frac{f_{B_{\text{tag}}=B^{0}} - f_{B_{\text{tag}}=\overline{B}^{0}}}{f_{B_{\text{tag}}=B^{0}} + f_{B_{\text{tag}}=\overline{B}^{0}}} = \frac{1 - |\lambda_{f_{CP}}|^{2}}{1 + |\lambda_{f_{CP}}|^{2}} \cos \Delta m_{d} \Delta t$$
$$- \frac{2\mathcal{I}m\lambda_{f_{CP}}}{1 + |\lambda_{f_{CP}}|^{2}} \sin \Delta m_{d} \Delta t$$
$$\bullet \text{ CP Violation occurs if } |\lambda| = \left|\frac{q}{p}\right| \left|\frac{\overline{A}}{A}\right| \neq 1$$
$$\left|\frac{q}{p}\right| = 1 \quad \text{No CP Violation in Mixing}} \left|\frac{|\overline{A}|}{|A|}\right| = 1 \quad \text{No Direct } CP \text{ Violation}$$

• But even with $|\lambda|=1$ it is sufficient to have $\text{Im}\lambda \neq 0$

In Standard Model we expect $|\lambda| \cong 1$ in most of B decays

Simple Case with $|\lambda_{CP}| = 1$ $\Phi_M = \beta$ $A_f = A e^{i(\Phi_W + \delta)}$ $\bar{A}_f = \eta_{f_{CP}} A e^{i(-\Phi_W + \delta)}$ $\lambda_{f_{CP}} = \eta_{f_{CP}} e^{-2i(\Phi_W - \Phi_M)}$

 $a_{f_{CP}}(\Delta t) = -\mathcal{I}m\lambda_{f_{CP}}\sin\Delta m_d\Delta t = \eta_{f_{CP}}\sin2\Phi\sin\Delta m_d\Delta t$

- Very simple expression for CP violating asymmetry
- Amplitude of asymmetry defined by phase difference between mixing parameter q/p and ratio of decay amplitudes

- Complex phase Φ_M depends on specific final state
 - Can probe different angles of Unitarity triangle through different B decays

Why do We Need Time Dependence?



At Υ (4S): integrated asymmetry is zero \rightarrow must do a time-dependent analysis !

This is impossible to do in a conventional symmetric energy collider producing $\Upsilon(4S) \rightarrow BB$!!

CP Violation in Decay

CP Violation in Mixing

 CP Violation in interference between decay and mixing

CP Violation in Mixing





- Occurs when Mass eigenstates \neq CP eigenstates
- $(|q/p| \neq 1 \text{ and } < B_H|B_L > \neq 0)$
- The Box diagrams provide the required 2 phases Strong phases depend on quark masses and non-perturbative physics.
- Asymmetries are small and hard to calculate precisely (QCD

$$\begin{array}{c}
M_{12} \\
B^{0} \\
\hline
off-shell states f \\
ely (QCD) \\
\hline
-i \\ 2 \\
\Gamma_{12} \\
\hline
\end{array} \\
\begin{array}{c}
\overline{B}^{0} \\
\hline
on-shell \\
states f \\
\end{array}$$

$$a_{sl} = \frac{\Gamma\left(\overline{B}_{phys}^{0}(t) \to \ell^{+}\nu X\right) - \Gamma\left(B_{phys}^{0}(t) \to \ell^{-}\nu X\right)}{\Gamma\left(\overline{B}_{phys}^{0}(t) \to \ell^{+}\nu X\right) + \Gamma\left(B_{phys}^{0}(t) \to \ell^{-}\nu X\right)} = \frac{1 - \left|q / p\right|^{4}}{1 + \left|q / p\right|^{4}} \approx O(10^{-4})$$

Time-dependent *CP* Asymmetry:

$$A_{T}(t) = \frac{\Gamma(\overline{B}_{phys}^{0}(t) \rightarrow \ell^{+} \nu X) - \Gamma(B_{phys}^{0}(t) \rightarrow \ell^{-} \overline{\nu} X)}{\Gamma(\overline{B}_{phys}^{0}(t) \rightarrow \ell^{+} \nu X) + \Gamma(B_{phys}^{0}(t) \rightarrow \ell^{-} \overline{\nu} X)}$$

Search for asymmetry in same-sign dilepton sample same-sign $\ell^{\pm}\ell^{\pm}$ events occur in mixed events where one $\overline{B}^{0} \to B^{0} \to X\ell^{+}\upsilon$; other $B^{0} \to Y\ell^{+}\upsilon \implies \ell^{+}\ell^{+}$ one $B^{0} \to \overline{B}^{0} \to X\ell^{-}\upsilon$; other $\overline{B}^{0} \to Y\ell^{-}\upsilon \implies \ell^{-}\ell^{-}$

$$\mathcal{A}_{\tau}^{obs}(\Delta t) = \frac{\mathcal{N}(\ell^{+}\ell^{+},\Delta t) - \mathcal{N}(\ell^{-}\ell^{-},\Delta t)}{\mathcal{N}(\ell^{+}\ell^{+},\Delta t) + \mathcal{N}(\ell^{-}\ell^{-},\Delta t)} = \mathcal{A}_{\tau} \times \frac{\mathcal{S}(\Delta t)}{\mathcal{S}(\Delta t) + \mathcal{B}(\Delta t)}$$

 $S(\Delta t) = signal$ $B(\Delta t) = background$ from B decay and continuum

Time dependent measurement, time measured from ΔZ



Measurement region > 200μ m



 \Rightarrow To a good approximation:

$$|q/p| = 1 \text{ and } q/p = e^{-2i\varphi_M} = -|M_{12}|/M_{12}|$$

CP Violation in interference between decay and mixing

CPV In Interference Between Mixing and Decay



Neutral B Decays into CP final state f_{CP} accesible by both $B^0 \& \overline{B}^0$ decays

This is CPV when
$$\left|\frac{q}{p}\right| = 1$$
 and $\left|\frac{\overline{A}_{\overline{f}_{CP}}}{A_{f_{CP}}}\right| = 1$ and the CP parameter of interest is $\lambda_{f_{CP}} \equiv \eta_{f_{CP}} \frac{q}{p} \frac{\overline{A}_{\overline{f}_{CP}}}{A_{f_{CP}}}$

CPV Asymmetry is defined as :

$$a_{f_{CP}} = \frac{\Gamma\left(\overline{B}_{phys}^{0}(t) \to f_{CP}\right) - \Gamma\left(B_{phys}^{0}(t) \to f_{CP}\right)}{\Gamma\left(\overline{B}_{phys}^{0}(t) \to f_{CP}\right) + \Gamma\left(B_{phys}^{0}(t) \to f_{CP}\right)} = \frac{2Im\lambda_{f_{CP}}}{1 + \left|\lambda_{f_{CP}}\right|^{2}}\sin\left(\Delta m_{B}t\right) - \frac{\left(1 - \left|\lambda_{f_{CP}}\right|^{2}\right)}{1 + \left|\lambda_{f_{CP}}\right|^{2}}\cos\left(\Delta m_{B}t\right)}$$

When B decay is dominated by a single diagram, $\left|\lambda_{f_{CP}}\right| = 1 \Rightarrow a_{f_{CP}} = Im\lambda_{f_{CP}}\sin\left(\Delta m_{B}t\right)$

CP asymm. can be very large and can be cleanly related to CKM angles

Golden Decay Mode $B^0 \rightarrow J/\psi K^0$



Use Unitarity relation $V_{tb}V_{ts}^* + V_{cb}V_{cs}^* + V_{ub}V_{us}^*$ to rearrange terms $\overline{A} = \overline{A}_T + \overline{A}_P = V_{cb}V_{cs}^*(T_{c\overline{c}s} + P_c - P_t) + V_{ub}V_{us}^*(P_u - P_t)$ $= (V_{cb}V_{cs}^*) T + (V_{ub}V_{us}^*) P$

Since $\left| \frac{V_{ub} V_{us}^*}{V_{cb} V_{cs}^*} \right| \square \frac{1}{50} \implies (V_{cb} V_{cs}^*)$ T is the dominant amplitude expect $\left| \frac{\overline{A}}{A} \right| -1 = 10^{-2}$ Hence "Platinum" mode !

Golden Decay Mode $B^0 \rightarrow J/\psi K^0$



CPV In Interference Between Mixing and Decay: $B^0 \rightarrow J/\psi K^0$



Same is true for a variety of $B \rightarrow (cc)$ s final states

Time-Dependent CP Asymmetry with a Perfect Detector

Perfect measurement of time interval t=Δt
Perfect tagging of B⁰ and B⁰ meson flavors
For a B decay mode such as B⁰→ψKs with | λ_f|=1



Charmonium+K⁰ CP Sample for BABAR ('02)



Calibration with Flavor eigenstates



Observation of CP Violation (BaBar 2001)



14 Nov 2006

Events / 2.5 MeV/c²

 $\Delta t (ps)_9$

BABAR Result for sin2 β (July 2002)



Updated (ICHEP04) sin2β results from Charmonium Modes



Belle Results on sin2 β from Charmonium Modes





New Belle value lower than in '03 $\sin 2\beta = +0.728 \pm 0.056 \pm 0.023$ but still consistent with BaBar'04

CP Violation in B Decays Firmly Established



Lessons From sin2 β Measurement With B⁰ \rightarrow J/ ψ K⁰

- In 2001, large CP Violation in B system was observed in this mode by BaBar and Belle.
 - First instance of CPV outside the Kaon system.
- First instance of a CPV effect which was O(1) in contrast with the Kaon system
 - Confirms the 1972 conjecture of Kobayashi & Maskawa.
 - Excludes models with approximate CP symmetry (small CPV).
- In 2007 sin2β is a precision measurement (5%) and agrees well with the constraints in the ρ-η plane from measurements of the CKM magnitudes (will be discussed in tomorrow's lecture)
- Appears unlikely to find another **O(1)** source of CPV
 - enterprise now moves towards looking for corrections rather than alternatives to SM/CKM picture
- Focus now shifts to measurements of time-dependent asymmetries in rare B decays
- dominated by Penguin diagrams in the SM and where New Physics could 14 Nov 2006 contribute to the asymmetries

Tomorrow's Lecture

Measurements of α and γ

Constraints on Unitarity Triangle from measuments of CKM element magnitude and angles