Cabibbo-Kobayashi-Maskawa Matrix and CP Violation in Standard Model

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Lecture 5
Measurements of $\alpha$, $\beta$, $\gamma$
Constraints on Unitarity Triangle

Lezioni di Fisica delle Particelle Elementari
Outline of Today’s Lecture

- Measurements of Angle $\beta$ with rare B decays
  - Probing New Physics

- Measurements of Angle $\alpha$

- Measurements of Angle $\gamma$

- Constraints on Unitarity Triangle from CKM Measurements
Separating $B^0$ and $\bar{B}^0$ mesons

**Lepton Tag**

- $l^- \rightarrow \bar{B}^0$
- $l^+ \rightarrow B^0$

**Kaon Tag**

- $\sum Q < 0 \Rightarrow \bar{B}^0$
- $\sum Q > 0 \Rightarrow B^0$

**Kinematic Tag**

- $W^+$
- $\pi^+_h, \rho^+, \phi^+$
- $\bar{D}^0$
- $\pi_s$
Time Dependent B Oscillation (Or Mixing) at $\Upsilon(4S)$

\[ f_{\text{unmix}}(\Delta t) \propto e^{-\Gamma|\Delta t|} (1 + \cos \Delta m_d \Delta t) \]

\[ f_{\text{mix}}(\Delta t) \propto e^{-\Gamma|\Delta t|} (1 - \cos \Delta m_d \Delta t) \]

\[ A_{\text{mix}}(\Delta t) = \frac{f_{\text{unmix}} - f_{\text{mix}}}{f_{\text{unmix}} + f_{\text{mix}}} \]
Δt Spectrum of Mixed and Unmixed Events

perfect
flavor tagging & time resolution

- UnMixed
- Mixed

realistic
mis-tagging & finite time resolution

- UnMixed
- Mixed

\[ f_{\text{Unmix}}(\Delta t) = \begin{cases} f_{\text{Unmix}}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B_d}}}{4\tau_{B_d}} \times \left( 1 \pm \cos(\Delta m_d \Delta t) \right) \text{ionFunction} \\ f_{\text{Mix}}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B_d}}}{4\tau_{B_d}} \times \left( 1 \pm \cos(\Delta m_d \Delta t) \right) \text{ionFunction} \end{cases} \]

Unmixed: \( B^0_{\text{flav}} \bar{B}^0_{\text{tag}} \) or \( \bar{B}^0_{\text{flav}} B^0_{\text{tag}} \)

Mixed: \( B^0_{\text{flav}} B^0_{\text{tag}} \) or \( \bar{B}^0_{\text{flav}} \bar{B}^0_{\text{tag}} \)

w: the fraction of wrongly tagged events

\( \Delta m_d \): oscillation frequency

16 Nov 2006
Flavor Tagging Performance

The large sample of fully reconstructed events provides the precise determination of the tagging parameters required in the CP fit.

<table>
<thead>
<tr>
<th>Tagging category</th>
<th>Fraction of tagged events $\varepsilon$ (%)</th>
<th>Wrong tag fraction $w$ (%)</th>
<th>$Q = \varepsilon (1-2w)^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton</td>
<td>11.1 ± 0.2</td>
<td><strong>8.6 ± 0.9</strong></td>
<td>7.6 ± 0.4</td>
</tr>
<tr>
<td>Kaon</td>
<td><strong>34.7 ± 0.4</strong></td>
<td>18.1 ± 0.7</td>
<td>14.1 ± 0.6</td>
</tr>
<tr>
<td>NT1</td>
<td>7.7 ± 0.2</td>
<td>22.0 ± 1.5</td>
<td>2.4 ± 0.3</td>
</tr>
<tr>
<td>NT2</td>
<td>14.0 ± 0.3</td>
<td>37.3 ± 1.3</td>
<td>0.9 ± 0.2</td>
</tr>
<tr>
<td>ALL</td>
<td><strong>67.5 ± 0.5</strong></td>
<td></td>
<td><strong>25.1 ± 0.8</strong></td>
</tr>
</tbody>
</table>

**Highest “efficiency”**

**Smallest mistag fraction**
\[ R(\delta \Delta t) = (1 - f_{\text{tail}} - f_{\text{outl}}) \cdot G_{\text{core}}(\Delta \Delta t, S_{\text{core}}, \delta_{\text{core,i}}) + f_{\text{tail}} \cdot G_{\text{tail}}(\Delta \Delta t, S_{\text{tail}}, \delta_{\text{tail}}) + f_{\text{outl}} \cdot G_{\text{outl}}(\Delta \Delta t, \sigma_{\text{outl}} = 0 \text{ ps}, \delta_{\text{outl}} = 8 \text{ ps}) \]

\[ \sigma_{\text{core}} = S_{\text{core}} \cdot \sigma_{\text{evt}}^{\Delta t} \]

\[ \sigma_{\text{tail}} = S_{\text{tail}} \cdot \sigma_{\text{evt}}^{\Delta t} \]

Use the event-by-event uncertainty on $\Delta t$

Parameter

<table>
<thead>
<tr>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\text{Core}}$</td>
</tr>
<tr>
<td>$S_{\text{Tail}}$</td>
</tr>
<tr>
<td>$f_{\text{Tail}}$ (%)</td>
</tr>
<tr>
<td>$f_{\text{Outlier}}$ (%)</td>
</tr>
<tr>
<td>$\delta_{\text{Core,Lepton}}$</td>
</tr>
<tr>
<td>$\delta_{\text{Core,Kaon}}$</td>
</tr>
<tr>
<td>$\delta_{\text{Core,NT1}}$</td>
</tr>
<tr>
<td>$\delta_{\text{Core,NT2}}$</td>
</tr>
<tr>
<td>$\delta_{\text{Tail}}$</td>
</tr>
</tbody>
</table>

Different bias scale factor
For each tagging category
B$^0$B$^0$ Mixing Fit Result

Asymmetry($\Delta t$) = \[ \frac{N(\text{unmixed}) - N(\text{mixed})}{N(\text{unmixed}) + N(\text{mixed})} \approx (1 - 2\langle w \rangle) \times \cos(\Delta m_d \Delta t) \]

\[ \Delta m_d = 0.516 \pm 0.016 \text{ (stat)} \pm 0.010 \text{ (syst)} \text{ ps}^{-1} \]

\[ 29.7 \text{ fb}^{-1} \] published in PRL
Probing New Physics with $\beta$
Compare sin2β with “sin2β” from CPV in Penguin decays of B⁰

In SM, interference between B⁰ mixing and dominant b → ss (b → suu) [penguin amplitudes have no CKM phase] gives the same CPV (due to e⁻ⁱצעירים β) as in  b → ccs

Loop diagrams sensitive to high virtual mass scales ⇒ sensitive to new physics

NP coupling can bring in new phases that may cause deviations from expected "sin2β"

Both decays dominated by single weak phase

Tree:

\[ \bar{b} \rightarrow c \bar{s} \]

Penguin:

\[ \bar{b} \rightarrow s \bar{s} s \]

New Physics?

\[ \lambda_{J/\psi K^0_{S,L}} = \eta_{J/\psi K^0_{S,L}} \left( \frac{q}{p} \right)_B \left( \frac{V_{cb}^* V_{cs}}{V_{cb} V_{cs}^*} \right) \left( \frac{q}{p} \right)_K = \eta_{J/\psi K^0_{S,L}} e^{-2i\beta} \]

\[ \lambda_{\phi K^0_{S,L}} = \eta_{\phi K^0_{S,L}} \left( \frac{q}{p} \right)_B \left( \frac{V_{tb} V_{ts}^*}{V_{tb} V_{ts}} \right) \left( \frac{q}{p} \right)_K \sim \eta_{\phi K^0_{S,L}} e^{-2i\beta} \]

sin2β [charmonium] = sin2β [s-penguin]

Must be if one amplitude dominates
Standard Model Pollution in Penguin Mode

Decay amplitude of interest

$$\bar{B}^0 \to W^- b^+ t^- s^- g^- s^+ g^+$$

$$\propto V_{tb} V_{ts}^* \sim \lambda^2$$

SM Pollution

$$\bar{B}^0 \to W^- b^+ u^- s^- g^- s^+ g^+$$

$$\propto V_{ub} V_{us}^* \sim \lambda^2 R_u e^{-i\gamma}$$

Naive (dimensional) uncertainties on $\sin 2\beta$

$$O(\lambda^2) \sim 5\%$$

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The « Golden » Penguin mode $B^0 \to \phi K^0$

- Modes with $K_S$ and $K_L$ are both reconstructed

$B^0 \to \phi K_S \to K^+ K^- \pi^+ \pi^-$

$B^0 \to \phi K_L$ (Opposite CP)

Plots shown are 'signal enhanced' through a cut on the likelihood on the dimensions that are not shown, and have a lower signal event count

114 $\pm$ 12 signal events

98 $\pm$ 18 signal events

Opposite CP
CP analysis of ‘golden penguin mode’ $B^0 \rightarrow \phi K^0$

$B^0 \rightarrow \phi K_S^0 \rightarrow K^+K^-\pi^+\pi^-$

$B^0 \rightarrow \phi K_L^0$ (Opposite CP)

$S(\phi K_S) = +0.29 \pm 0.31\text{(stat)}$

$S(\phi K_L) = -1.05 \pm 0.51\text{(stat)}$

**BaBar**

Combined fit result
(assuming $\phi K_L$ and $\phi K_S$ have opposite CP)

$\eta_{\phi K^0} \times S_{\phi K^0} = +0.50 \pm 0.25 \pm 0.07 - 0.04$

$C_{\phi K^0} = +0.00 \pm 0.23 \pm 0.05$

Standard Model Prediction

$S(\phi K^0) = \sin2\beta = 0.72 \pm 0.05$

$C(\phi K^0) = 1-|\lambda| = 0$

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Time-Dependent CPV Results from Penguin Modes

\[ \sin(2\beta_{\text{eff}}) = \sin(2\phi_{1}^{\text{eff}}) \]

All s-Penguin modes exhibit “\(\sin2\beta\)” consistent with but slightly below \(\sin2\beta_{\text{Tree}}\) while Theory (Beneke et al) predicts same or higher.

Recall: Don’t get over excited for 2\(\sigma\) discrepancies!
What Are Penguins Telling Us?

Discrepancy?

2σ?
CKM Angle $\alpha$ From $(b \rightarrow u \ u \ d)$ Process: $B^0 \rightarrow \pi^+ \pi^-$

Neglecting Penguin diagram

$$\lambda(B \rightarrow \pi^+ \pi^-) = \left( \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left( \frac{V_{uc}^* V_{ub}}{V_{ud} V_{ub}^*} \right) \Rightarrow Im\lambda_{\pi\pi} = \sin(2\pi - 2\beta - 2\gamma) = \sin 2\alpha$$

Weak Phase in Penguin term is $\arg(V_{td}^* V_{tb})$ different from Tree, so it will modify $Im\lambda_{\pi\pi}$ and $|\lambda_{\pi\pi}|$ depending on its relative strength w.r.t Tree. (Penguins are large!)
Decay Amplitudes In $B^0 \rightarrow \pi^+\pi^-, \rho^+ \rho^-$

The decay can be described by the following processes:

- **Tree**:
  - $b \rightarrow u, d \rightarrow \bar{d}$
  - $V_{ub}$, $V_{ud}$
  - $\pi^+, \rho^-$

- **Penguin**:
  - $b \rightarrow u, c, t \rightarrow \bar{u}$
  - $\rho^+, \rho^-$

The ratio of amplitudes $|P/T|$ and strong phase difference $\delta$ cannot be reliably calculated!

\[ \lambda_{\pi^+\pi^-} = e^{i2\alpha} \frac{T + P e^{+i\gamma} e^{i\delta}}{T + P e^{-i\gamma} e^{i\delta}} \]

\[ A_{CP} = S \sin \Delta m t - C \cos \Delta m t \]

\[ C_{\pi^+\pi^-} \propto \sin \delta \]

\[ S_{\pi^+\pi^-} = \sqrt{1 - C_{\pi^+\pi^-}^2 \sin 2\alpha_{\text{eff}}} \]

One can measure $\delta\alpha_{\text{peng}}$ using isospin relation and bounds to get $\alpha$

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Time Dependent Asymmetry Measurement: \( B^0 \to \pi^+\pi^- \)

\[ B^0 \to \pi^+\pi^- \text{ (227M BB)} \]

\[ 467 \pm 33 B \to \pi^+\pi^- \]

68% of \( N_{\pi^+\pi^-} \)
5% of \( N_{K^+\pi^-} \)
13% of \( q\bar{q} \)

\[ m_{ES} \quad 5.2 \quad 5.25 \quad 5.3 \quad (\text{GeV/c}^2) \]

\[ A_{CP} = S \sin \Delta m t - C \cos \Delta m t \]

\[ S_{\pi\pi} = \sin 2\alpha_{\text{eff}} = -0.30 \pm 0.17 \pm 0.03 \]

\[ C_{\pi\pi} = -0.09 \pm 0.15 \pm 0.04 \]

\[ \sin 2\alpha = ?? \]
Estimating Penguin Pollution in $B^0 \rightarrow \pi^+\pi^-$, $\rho^+\rho^-$

$B \rightarrow \pi^+\pi^-, \pi^+\pi^0$ and $\pi^0\pi^0$ related by SU(2) $\Rightarrow$ Isospin relation between amplitudes $A^+, A^{+0}$ and $A^{00}$

$B \rightarrow \pi\pi$ states can have $I=0$ or 2; Gluonic Penguins contribute only to $I=0$ ($\Delta I=1/2$ rule)

$B \rightarrow \pi^+\pi^0$ has only tree amplitude $\Rightarrow$ $|A^{+0}| = |A^{-0}|$

$$
\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0} \\
\frac{1}{\sqrt{2}} \bar{A}^{+-} + \bar{A}^{00} = A^{-0}
$$

$A^{+-} = A(B^0 \rightarrow \pi^+\pi^-)$

$\bar{A}^{+-} = A(\bar{B}^0 \rightarrow \pi^+\pi^-)$

$A^{00} = A(B^0 \rightarrow \pi^0\pi^0)$

$\bar{A}^{00} = A(\bar{B}^0 \rightarrow \pi^0\pi^0)$

$A^{+0} = A(B^+ \rightarrow \pi^+\pi^0)$

$\bar{A}^{-0} = A(B^- \rightarrow \pi^-\pi^0)$

To constrain $\delta \alpha_{\text{penguin}}$ by isospin analysis requires $A^{00}$ and $\bar{A}^{00}$ to be very small or very large! $\Rightarrow$ Measure and constrain $C_{+-}, C_{00}, A^{+0}, A^{00}$ by rate and asymmetry measurements

Needs lots of statistics for $B \rightarrow \pi^0\pi^0$ and $B \rightarrow \pi^0\pi^0$ rate measurements
Constraining $\alpha$: $B^- \rightarrow \pi^- \pi^0$ Rate Measurement

- $B^- \rightarrow \pi^- \pi^0$ ($I=2$, $\Delta I=1/2$) has only tree amplitude, no penguin $\Rightarrow$ Base of Isospin triangle

\[ \begin{align*}
B^- & \quad \pi^0 \\
\text{u} & \quad \pi^- \\
\end{align*} \]

\[ \begin{align*}
B^- & \quad \pi^0 \\
\text{d} & \quad \pi^- \\
\end{align*} \]

- $B^+ \rightarrow \pi^+ \pi^0$ Events:
  - $N_{\pi^+\pi^0} = 379 \pm 41$
  - $N_{K^+\pi^0} = 682 \pm 39$

- $\mathcal{B}(B^+ \rightarrow \pi^+ \pi^0) = (5.8 \pm 0.6 \pm 0.4) \times 10^{-6}$
Constraining $\alpha : B \rightarrow \pi^0\pi^0$ Decay Diagrams

Difficult to calculate rates for such processes. Smaller the better for constraining $\alpha$.

**Grossman-Quinn bound:**

$$\sin^2(\alpha - \alpha_{\text{eff}}) \leq \frac{\mathcal{B}(B^0 \rightarrow \pi^0\pi^0) + \mathcal{B}(\bar{B}^0 \rightarrow \pi^0\pi^0)}{\mathcal{B}(B^+ \rightarrow \pi^+\pi^0) + \mathcal{B}(B^- \rightarrow \pi^-\pi^0)}$$
B → π^0π^0: Rate and Flavor Tagged Rate Asymmetry

First measurements

\[ BF_{\pi^0\pi^0} = (1.17 \pm 0.32 \pm 0.10) \times 10^{-6} \]
\[ C_{\pi^0\pi^0} = -0.12 \pm 0.56 \pm 0.06 \]

4.9σ BaBar

Measured by flavor of the other B → B_{\text{tag}}

\[ BF_{\pi^0\pi^0} = (2.32^{+0.41}_{-0.48} \pm 0.22) \times 10^{-6} \]
\[ C_{\pi^0\pi^0} = -0.43 \pm 0.51^{+0.16}_{-0.17} \]

6.0σ Belle

Average

\[ \frac{\Gamma(B \rightarrow \pi^0\pi^0) - \Gamma(B \rightarrow \pi^0\pi^0)}{\Gamma(B \rightarrow \pi^0\pi^0) + \Gamma(B \rightarrow \pi^0\pi^0)} = 0.28 \pm 0.39 \]

[BABAR, BELLE]

\[ B(B \rightarrow \pi^0\pi^0) = (1.51 \pm 0.28) \times 10^{-6} \]

Bad news is C_{\pi\pi} is not precise enough and B_{\pi^0\pi^0} is too large for useful G-Q bound.

274M BB
Angle $\alpha$ From $B \rightarrow \pi^+ \pi^-$: Bottomline

\[ \begin{align*}
\pi^+ \pi^- S_{\text{CP}} & \quad \text{HFAG} \\
\text{BaBar} & \quad \text{PRL 99 (2007) 021603} \\
& \quad -0.60 \pm 0.11 \pm 0.03 \\
\text{Belle} & \quad \text{HFAG} \\
& \quad \text{PRL 98 (2007) 211801} \\
& \quad -0.61 \pm 0.10 \pm 0.04 \\
\text{Average} & \quad \text{HFAG correlated average} \\
& \quad -0.61 \pm 0.08 \\
\text{standard model solution} & \quad \text{HFAG} \\
& \quad \text{PRL 2007} \\
\end{align*} \]

\[ \begin{align*}
\pi^+ \pi^- C_{\text{CP}} & \quad \text{HFAG} \\
\text{BaBar} & \quad \text{PRL 99 (2007) 021603} \\
& \quad -0.21 \pm 0.09 \pm 0.02 \\
\text{Belle} & \quad \text{HFAG} \\
& \quad \text{PRL 98 (2007) 211801} \\
& \quad -0.55 \pm 0.08 \pm 0.05 \\
\text{Average} & \quad \text{HFAG correlated average} \\
& \quad -0.38 \pm 0.07 \\
\text{standard model solution} & \quad \text{HFAG} \\
& \quad \text{PRL 2007} \\
\end{align*} \]

\[ \alpha = [80,107]^\circ \cup [156,171]^\circ \text{ @ 95\% Prob.} \]

Standard Model Solution:
\[ \alpha = (91 \pm 8)^\circ \text{ @ 68\% Prob.} \]
\[ \gamma = \phi_3 \equiv \arg \left[ \begin{vmatrix} \frac{V_{ud}V_{ub}^*}{|V_{cd}V_{cb}|} & -
\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}} 
\end{vmatrix} \right] \]
How To Measure $\gamma$?

- Interference is the key to $\gamma$

\[ A_{tot} = A_1 + A_2 e^{-i\gamma} e^{i\delta} \]

- Both charged and neutral B mesons can be used
  - Measure branching fractions for $B^+$
  - Time dependent studies for $B^0$

Both charged and neutral B mesons can be used:

- Measure branching fractions for $B^+$
- Time dependent studies for $B^0$
What to Watch Out For?

- Branching fractions of interesting B decays typically about $10^{-5}$ or smaller

- Every additional decay mode is important to increase statistics but...
  - Combining different decay modes not trivial

- Sensitivity to $\gamma$ strongly depends on $r_B = \frac{|A_2(b \rightarrow u)|}{|A_1(b \rightarrow c)|}$
  - Small values of $r_B$ make the measurement very difficult
  - Each decay mode has a different value of $r_B$

- Strong phase $\delta$ different for each final state
  - Combination of decay modes more complicated

- Experimentally need to determine: $r_B$, $\delta$, and $\gamma$
Experimental Techniques to Measure $\gamma$

- Many papers about $\gamma$ on the market

- Gronau-London-Wyler method with $B^- \rightarrow D^0 K^-$
  - CP eigenstates of $D^0$

- Atwood-Dunietz-Soni method with $B^- \rightarrow D^0 K^-$
  - Flavor eigenstates of $D^0$

- Dalitz Analysis of $B^- \rightarrow D^0 K^-, D^0 \rightarrow K_S \pi\pi$

- Time-dependent analysis $B^0 \rightarrow D^{(*)-} \pi/\rho$

- Search for decays $B^0 \rightarrow D^{(*)0} K^{(*)0}$

- Other methods
  - Charmless B decays ($K\pi$)
  - Variations of GWL and ADS method

$B \rightarrow D^{(*)} K^{(*)}$ decays
important ingredient for $\gamma$
Gronau-London-Wyler Method
with \( B^+ \rightarrow D^0 K^+ \)

\[
\begin{align*}
\bar{D}^0 / D^0 & \rightarrow K^+ K^-, \pi^+ \pi^- \\
K_S \pi^0 / \phi / \eta / \eta' 
\end{align*}
\]
Atwood-Dunietz-Soni Method in Pictures

Similar to GWL but replace CP eigenstates with flavor eigenstates of $D^0$

Combine dominant $b \rightarrow c$ transition with doubly Cabibbo-suppressed $D^0$ decays

Advantage: Both decay amplitudes are small but comparable  
- hopefully large $r_B$

Disadvantage: Effective BF($B^+ \rightarrow [K^-\pi^+]_D K^+$) $\sim 10^{-7}$
Time-Dependent Analysis and $\sin(2\beta + \gamma)$ with $B^0 \rightarrow D^{(*)}\pi$
CP violation from interference of decay and mixing with $B^0 \rightarrow D^{(*)}\pi/\rho$

- Advantage: Large branching fraction for favored decay ($\sim 3 \times 10^{-3}$)
- Disadvantage: Small BR for suppressed decay ($\sim 10^{-6}$) $\rightarrow$ Small CP violating amplitude!

\( V_{cb} V_{ud}^* = A \)

\( V_{ub} V_{cd}^* e^{i\delta} = r_B e^{-i\gamma} e^{i\delta} \)

Determines the sensitivity of the method

CKM angle

Strong phase difference
Time-dependent Decay Time Distributions at Asymmetric $e^+e^-$ Machines

\[ f \left( B^0 \rightarrow D^{(*)-}\pi^+, \Delta t \right) = N e^{-\Gamma|\Delta t|} \left\{ 1 + C^{(*)} \cos(\Delta m_d \Delta t) + S^{(*)} \sin(\Delta m_d \Delta t) \right\} \]
\[ f \left( \bar{B}^0 \rightarrow D^{(*)-}\pi^+, \Delta t \right) = N e^{-\Gamma|\Delta t|} \left\{ 1 - C^{(*)} \cos(\Delta m_d \Delta t) - S^{(*)} \sin(\Delta m_d \Delta t) \right\} \]
\[ f \left( \bar{B}^0 \rightarrow D^{(*)+}\pi^-, \Delta t \right) = N e^{-\Gamma|\Delta t|} \left\{ 1 + C^{(*)} \cos(\Delta m_d \Delta t) - \bar{S}^{(*)} \sin(\Delta m_d \Delta t) \right\} \]
\[ f \left( B^0 \rightarrow D^{(*)+}\pi^-, \Delta t \right) = N e^{-\Gamma|\Delta t|} \left\{ 1 - C^{(*)} \cos(\Delta m_d \Delta t) + \bar{S}^{(*)} \sin(\Delta m_d \Delta t) \right\} \]

Direct CP Violation

\[ C^{(*)} = \frac{1 - r^{(*)2}}{1 + r^{(*)2}} \approx 1 \]

Indirect CP Violation

\[ S^{(*)} = \frac{2r^{(*)}}{1 + r^{(*)2}} \sin(2\beta + \gamma - \delta^{(*)}) \]
\[ \bar{S}^{(*)} = \frac{2r^{(*)}}{1 + r^{(*)2}} \sin(2\beta + \gamma + \delta^{(*)}) \]

BaBar: $\beta\gamma = 0.56$

BaBar: $<|\Delta z|> \sim \beta\gamma c \tau$: 260 µm  
Average $\Delta z$ resolution: 190 µm
sin(2\beta+\gamma) Results in Pictures

- CP Asymmetry in Partially Reconstructed Sample
  - No significant CP asymmetry so far with either method
  - All measurements limited by statistics
Dalitz Analysis of $B^+ \rightarrow \bar{D}^0 \left( K_S \pi^+ \pi^- \right) K^+$
Possibly one of the cleanest methods to measure $\gamma$

$$A(B^+ \rightarrow [K_S \pi^+ \pi^-] K^+) = A(B^+ \rightarrow \bar{D}^0 K^+) + r_B A(B^+ \rightarrow D^0 K^+)$$

$$A(B^+ \rightarrow [K_S \pi^+ \pi^-] K^+) = A(B^+ \rightarrow \bar{D}^0 K^+) \left( f(m_+^2, m_-^2) + r_B e^{i(\gamma+\delta)} f(m_-^2, m_+^2) \right)$$

$$A(B^- \rightarrow [K_S \pi^+ \pi^-] K^-) = A(B^- \rightarrow D^0 K^-) \left( f(m_-^2, m_+^2) + r_B e^{i(-\gamma+\delta)} f(m_-^2, m_+^2) \right)$$

Measure $\gamma + \delta$ and $-\gamma + \delta$ from $B^+$ and $B^-$ decay rates
- Only 2-fold ambiguity in $\gamma$!
- Measure Dalitz structure $f(m_+^2, m_-^2) = A(\bar{D}^0 \rightarrow K_S \pi^+ \pi^-)$ with high statistics sample $D^{*-} \rightarrow D^0 [K_S \pi^+ \pi^-] \pi^-$.
**D* → K_S π⁺ π⁻ Dalitz Structure in D⁻ → D⁰π⁻**

\[ m_-^2 = M(K_S^0 π^-)^2 \]

\[ m_+^2 = M(K_S^0 π^+)^2 \]

81k events with 97% purity (92 fb⁻¹)

Isobar Model: sum of resonances and 1 non-resonant component

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Amplitude</th>
<th>Phase (deg)</th>
<th>Fit fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>K⁺(892)⁻</td>
<td>1.781 ± 0.018</td>
<td>131.0 ± 0.82</td>
<td>0.586</td>
</tr>
<tr>
<td>K⁺₀(1430)⁻</td>
<td>2.447 ± 0.076</td>
<td>-8.3 ± 2.5</td>
<td>0.083</td>
</tr>
<tr>
<td>K⁺₂(1430)⁻</td>
<td>1.054 ± 0.056</td>
<td>-54.3 ± 2.6</td>
<td>0.027</td>
</tr>
<tr>
<td>K⁺*(1410)⁻</td>
<td>0.515 ± 0.087</td>
<td>154 ± 20</td>
<td>0.004</td>
</tr>
<tr>
<td>K⁺*(1680)⁻</td>
<td>0.89 ± 0.30</td>
<td>-139 ± 14</td>
<td>0.003</td>
</tr>
<tr>
<td>K⁺*(892)+</td>
<td>0.1796 ± 0.0079</td>
<td>-44.1 ± 2.5</td>
<td>0.006</td>
</tr>
<tr>
<td>K⁺₀*(1430)+</td>
<td>0.368 ± 0.071</td>
<td>-342 ± 8.5</td>
<td>0.002</td>
</tr>
<tr>
<td>K⁺₂*(1430)+</td>
<td>0.075 ± 0.038</td>
<td>-104 ± 23</td>
<td>0.000</td>
</tr>
<tr>
<td>ρ(770)</td>
<td>1 (fixed)</td>
<td>0 (fixed)</td>
<td>0.224</td>
</tr>
<tr>
<td>ω(782)</td>
<td>0.0391 ± 0.0016</td>
<td>115.3 ± 2.5</td>
<td>0.006</td>
</tr>
<tr>
<td>f₀(980)</td>
<td>0.4817 ± 0.012</td>
<td>-141.8 ± 2.2</td>
<td>0.061</td>
</tr>
<tr>
<td>f₀(1370)</td>
<td>2.25 ± 0.30</td>
<td>113.2 ± 3.7</td>
<td>0.032</td>
</tr>
<tr>
<td>f₂(1270)</td>
<td>0.922 ± 0.041</td>
<td>-21.3 ± 3.1</td>
<td>0.030</td>
</tr>
<tr>
<td>ρ(1450)</td>
<td>0.516 ± 0.092</td>
<td>38 ± 13</td>
<td>0.002</td>
</tr>
<tr>
<td>σ</td>
<td>1.358 ± 0.050</td>
<td>-177.9 ± 2.7</td>
<td>0.093</td>
</tr>
<tr>
<td>σ’</td>
<td>0.340 ± 0.026</td>
<td>153.0 ± 3.8</td>
<td>0.013</td>
</tr>
<tr>
<td>Non Resonant</td>
<td>3.53 ± 0.44</td>
<td>127.6 ± 6.4</td>
<td>0.073</td>
</tr>
</tbody>
</table>

No D-mixing
No CP violation in D decays
Host Spots for $\gamma$ in the Dalitz Plot

- Sensitivity to $\gamma$ varies over the Dalitz plot
- Some resonances are better than others
- Perform analysis in each point of $(m_+ m_-)$

Example: $\gamma = 75$, $\delta = \pi$, $r_B = 0.125$

Babar preliminary

DCS $K^*(892)$

$\rho(770)$
Dalitz Structure in $B^\pm \rightarrow [K_S\pi^+\pi^-]K^\pm$ Data

~260 events

Dalitz Projection Plots
in signal region $m_{ES} > 5.27$ GeV/$c^2$
Constraints on $\gamma$ and $r_B$ with $B^- \to D^{(*)0} [K_S \pi^+ \pi^-] K^-$

211 million BB

\[
r_B = 0.118 \pm 0.079 (stat) \pm 0.034 (syst) +0.036_{-0.034} (dalitz)
\]

\[
r_B^* = 0.169 \pm 0.096 (stat) +0.030_{-0.028} (syst) +0.029_{-0.026} (dalitz)
\]

\[
\delta = 104^\circ \pm 45^\circ (stat) +17^\circ_{-21^\circ} (syst) +16^\circ_{-24^\circ} (dalitz)
\]

\[
\delta^* = 296^\circ \pm 41^\circ (stat) +14^\circ_{-12^\circ} (syst) \pm 15^\circ (dalitz)
\]

\[
\gamma = (70 \pm 31 (stat) +12_{-10} (syst) +14_{-11} (dalitz))^\circ
\]

Sensitivity to $\gamma$ decreases significantly for small $r_B$

Dalitz structure of background
PDF shapes
Dalitz amplitudes and phases

Modeling of Dalitz structure for signal

\[
BABAR
\]

$\gamma (\text{deg})$

Dalitz

\[
\sigma(\gamma) \text{ degree}
\]
Sensitivity of GWL, ADS, and Dalitz to $r_B$

Dalitz Analysis Dominates Current Knowledge of $\gamma$

Most promising method for determination of $\gamma$ in coming years

$\gamma = 88 \pm 16$ ([41,123] @ 95% prob)

$\gamma = -92 \pm 16$ ([−139,−57] @ 95% prob)
Constraints on Unitarity Triangle from Measurements of CKM Elements and CKM Angles
\[ V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

\[
V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \begin{pmatrix} 1 & 1 & e^{-i\gamma} \\ 1 & 1 & 1 \\ e^{-i\beta} & 1 & 1 \end{pmatrix}
\]
Determination of CKM Elements from Measurements

\[
\chi^2 = -2 \ln \mathcal{L}(y_{\text{model}}) = \mathcal{L}_{\text{exp}} \left[ x_{\text{exp}} - x_{\text{theo}}(y_{\text{model}}) \right] \times \mathcal{L}_{\text{theo}}(y_{\text{QCD}})
\]

Often Assumed to be Gaussian

Uniform likelihoods: "allowed ranges"

Frequentist

Bayesian

a-priori PDF

CKM Fitter

UTfit
http://www.utfit.org

Constraints on theoretical parameters

\[ y_{\text{theo}} = (A, \lambda, \rho, \eta, m_t \ldots) \]

Quarks confined in hadrons!

Measurement

Theoretical predictions

\[ x_{\text{theo}}(y_{\text{model}}) = y_{\text{theo}}, y_{\text{QCD}} \]
$B_d$ mixing + CPV in $K$ mixing ($\varepsilon_K$)

\[ \Delta m_d \propto f_{B_d}^2 B_{B_d} |V_{tb}|^2 |V_{td}|^2 \propto f_{B_d}^2 B_{B_d} |V_{cb}|^2 \lambda^2 \frac{((1-\bar{\rho})^2 + \eta^2)}{\eta(1-\bar{\rho})} \]

Allowed region

\[ \Delta m_d, \varepsilon_K \]

excluded area has CL $< 0.05$
$$\sqrt{\rho^2 + \eta^2} = \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|$$
$\text{BS Mixing constraint}$

\[ \frac{\Delta m_d}{\Delta m_s} \propto \frac{f_{B_d}^2 B_{B_d}}{f_{B_s}^2 B_{B_s}} \lambda^2 \left( (1 - \rho)^2 + \eta^2 \right) \]

Excluded area has CL < 0.05
$\alpha$ constraint
All measurements consistent, apex of (\(\rho, \eta\)) well defined.
Unitarity Triangle after All Constraints

Includes constraints from all CKM-related measurements
Impact of Angle Measurements on CKM Fit

Measurements of angles dominate the constraint on the UT apex!
Status of UT in 2007

All constraints

Only angles from B and $\varepsilon_K$ from Kaons
Future of CKM Physics

- BaBar & Belle are mature experiments and have a long term and a rich program for B physics (>2007)
  - Most CP asymmetry measurements are statistics limited
    - S-Penguins
    - $\alpha$ and $\gamma$ measurements (multiple to be sure)
    - CPV in B mixing remains to be discovered
  - Rare decays such as Radiative and Electroweak are clean probe of NP
    - e.g. F-B Asymmetry in $b \rightarrow s l^+ l^-$
    - CPV in $B \rightarrow s \gamma$ etc

- Tevatron is accumulating large B samples:
  - CDF has finally provided measurement of $B_s$ oscillation
  - They are the only current laboratory for studying $B_s$ and $\Lambda_b$ properties

- B Physics returns to Europe in with LHC-B !!
  - Will be an instrument for precision B physics
  - Precise exploration of CPV in $B_s$ and $B_d$ systems

- Super B Factory: studies started to evaluate possibility of very high luminosity B factory in near future