# Introduction to Particle Physics
## - Chapter 6 -

## Hadron structure and parton model

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# Chapter summary:

- Electron-proton elastic scattering.
- Cross-sections of Rutherford, Mott and Rosenbluth.
- Results of the Hofstadter's experiment.
- Dimension of the proton
- Scale invariance.
- Electron-proton inelastic scattering.
- Bjorken's scaling.
- Structure functions.
- Parton hypothesis.
- Relation of Callan-Gross.
- Valence quarks and sea quarks.
- Mention to scaling violation in QCD.

# Elastic electron-proton scattering

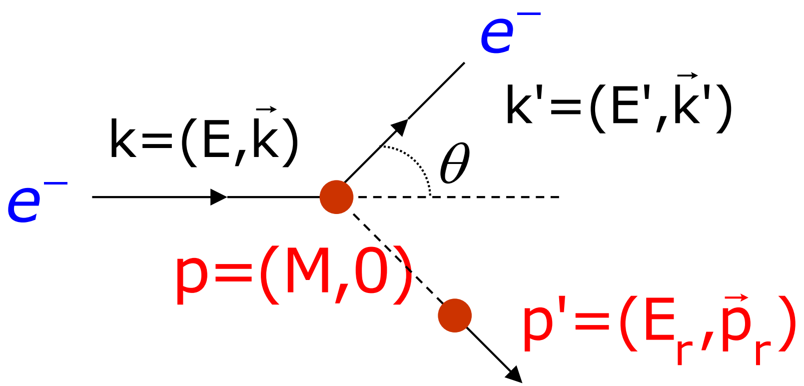
- **Elastic scattering:**

in the 1950' Hofstadter used electrons with energies ranging from 100 to 500 MeV to probe the charge distribution inside the nucleus. The results of the experiments showed that the proton is not a pointlike particle and permitted to measure its dimension.

- **Inelastic scattering:**

in 1967 Friedman, Kendall e Taylor started a series of experiments at SLAC with electrons up to 20 GeV to study the internal structure of the proton and to chase the quarks. The experiments showed that the proton is made of pointlike particles.

# Elastic scattering: kinematical variables




We measure only the quantities related to the electron, that is:

- initial energy E
- final energy E′
- scattering angle θ

- From the quadrimomentum conservation we have:

$$k_\mu + p_\mu = k'_\mu + p'_\mu \quad \Rightarrow \quad E_r = E + M - E' \quad ; \quad \vec{p}'_r = \vec{k} - \vec{k}'$$

$$\left(k_\mu + p_\mu\right)^2 = \left(k'_\mu + p'_\mu\right)^2$$

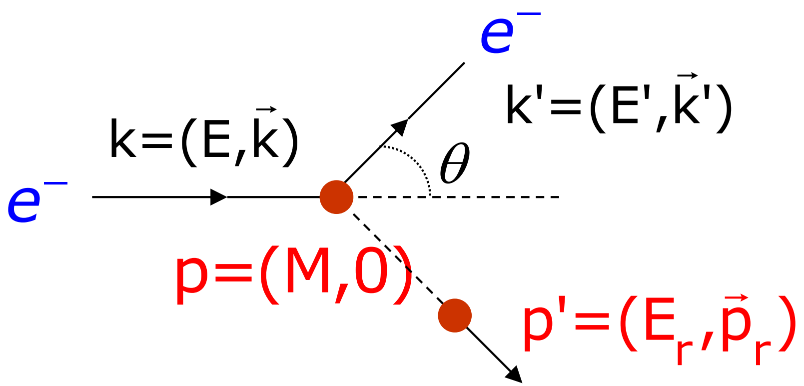
$$k_\mu k^\mu + p_\mu p^\mu + 2k_\mu p^\mu = k'_\mu k^{\mu\prime} + p'_\mu p^{\mu\prime} + 2k'_\mu p^{\mu\prime}$$

$$m_e^2 + m_p^2 + 2k_\mu p^\mu = m_e^2 + m_p^2 + 2k'_\mu p^{\mu\prime}$$

$$\Rightarrow \boxed{k_\mu p^\mu = k'_\mu p^{\mu\prime}}$$

This holds for elastic scattering only

# Elastic scattering: kinematical variables




$$\mathrm{E}_r = E + M - E'$$
$$\vec{\mathrm{p}}'_r = \vec{k} - \vec{k}'$$

$$k_\mu p^\mu = k'_\mu p^{\mu}{}'$$

$$\text{EM-0} = E'E_r - \vec{k}'\cdot\vec{p}_r = E'(E+M-E') - \vec{k}'\cdot(\vec{k}-\vec{k}') = E'E + E'M - (E')^2 - kk'\cos\theta + (k')^2$$

- If we neglect the electron mass, we have:

$$(E')^2 - (k')^2 = m_e^2 = 0 \quad ; \quad \mathrm{k{=}E} \;\; ; \;\; \mathrm{k'{=}E'}$$

$$\mathrm{E}' = \frac{E}{1 + \frac{E}{M}(1-\cos\theta)}$$

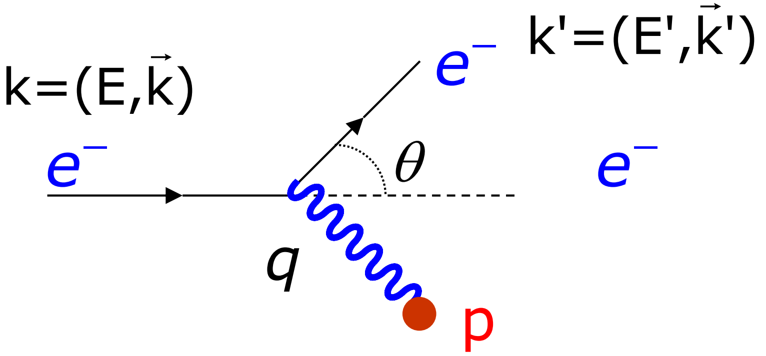
$$\boxed{\mathrm{E}' = \frac{E}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}}$$

N.B. E'=E per M→∞

N.B.: $\left[1-\cos\theta = 2\sin^2\frac{\theta}{2}\right]$

# Elastic scattering: quadrimomentum transfer

- We assume that in the interaction there is only one photon exchange:




- From the quadrimomentum conservation we have:

$$K_\mu = K'_\mu + q_\mu \quad \Rightarrow \quad q_\mu = K_\mu - K'_\mu$$

$$q_\mu^2 = \left(K_\mu - K'_\mu\right)^2 = K_\mu K^\mu + K'_\mu K'^\mu - 2K_\mu K'^\mu = 2m_e^2 - 2K_\mu K'^\mu \simeq -2K_\mu K'^\mu$$

$$q_\mu^2 = -2\left(EE' - KK'\cos\theta\right) = -2EE'\left(1-\cos\theta\right) = -4EE'\sin^2\frac{\theta}{2}$$

We prefer to use the positive variable $Q^2$

$$\boxed{Q^2 = -q^2 = 4EE'\sin^2\frac{\theta}{2}}$$

N.B. this relation is valid also for an inelastic scattering, where E and E' are independent variables. In case of elastic scattering we have: →

$$E' = \frac{E}{1+\frac{2E}{M}\sin^2\frac{\theta}{2}}$$

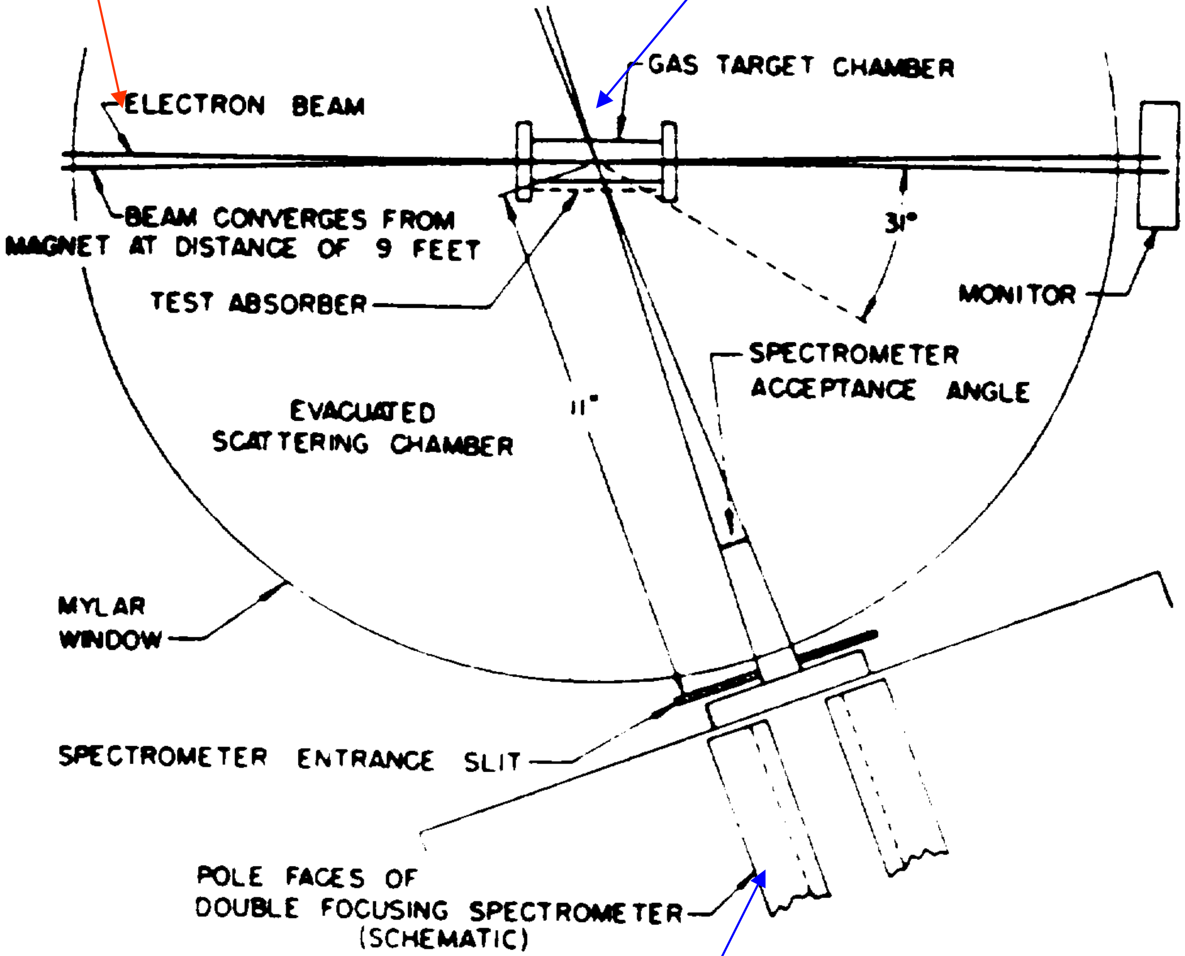
# Hofstadter's experiment (1956)

In 1953-55 Hofstadter and collaborators executed several experiments to measure the dimension and the structure of various nuclei.

In 1956 Hofstadter and McAllister used a gaseous target of hydrogen or helium to study the dimension of the proton and the neutron.

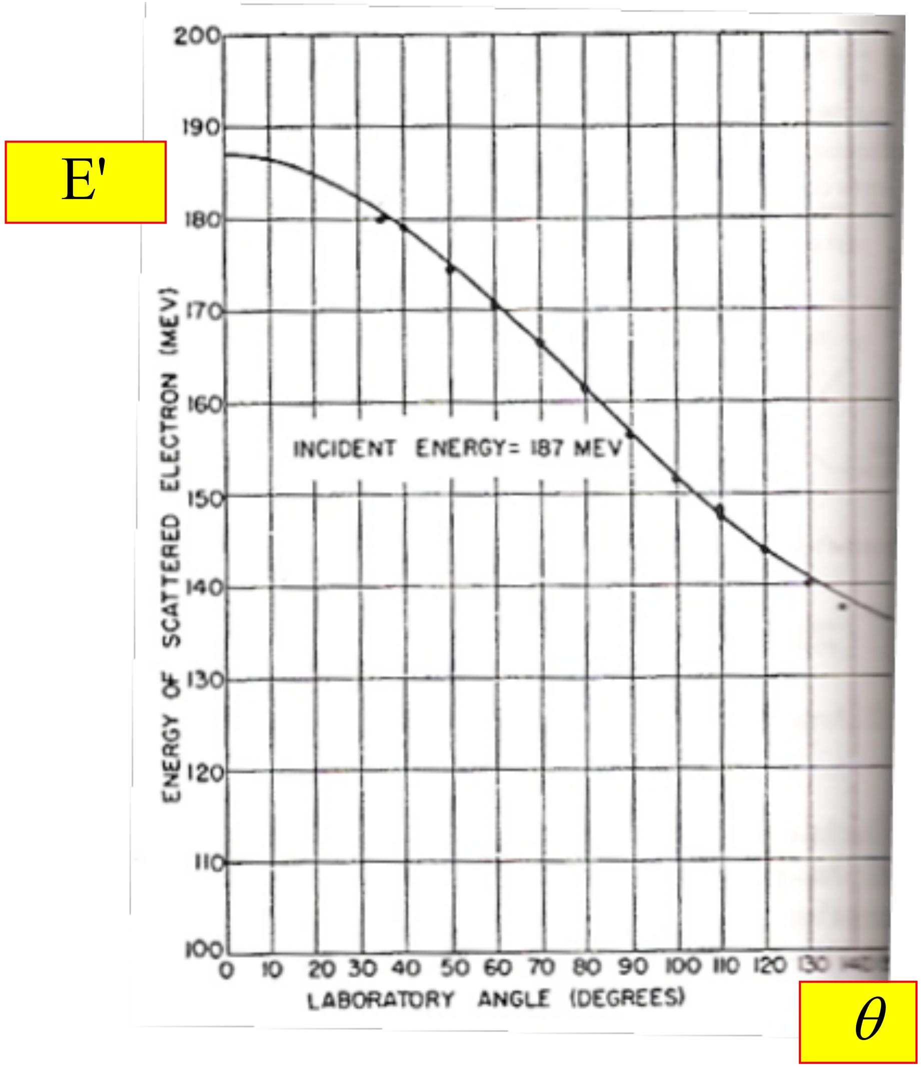
Target: proton o helium

Beam: 188 MeV electron




Spectrometer to measure the scattering angle. The angle θ range from 35° to 138°

# Hofstadter: energy of scattered electron




$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}$$

In the elastic scattering E' and θ are not independent variables

$$Q^2 = 4EE'\sin^2\frac{\theta}{2}$$

At every angle we have different $Q^2$

The graph shows the correlation between the energy of the scattered electron and the scattering angle.

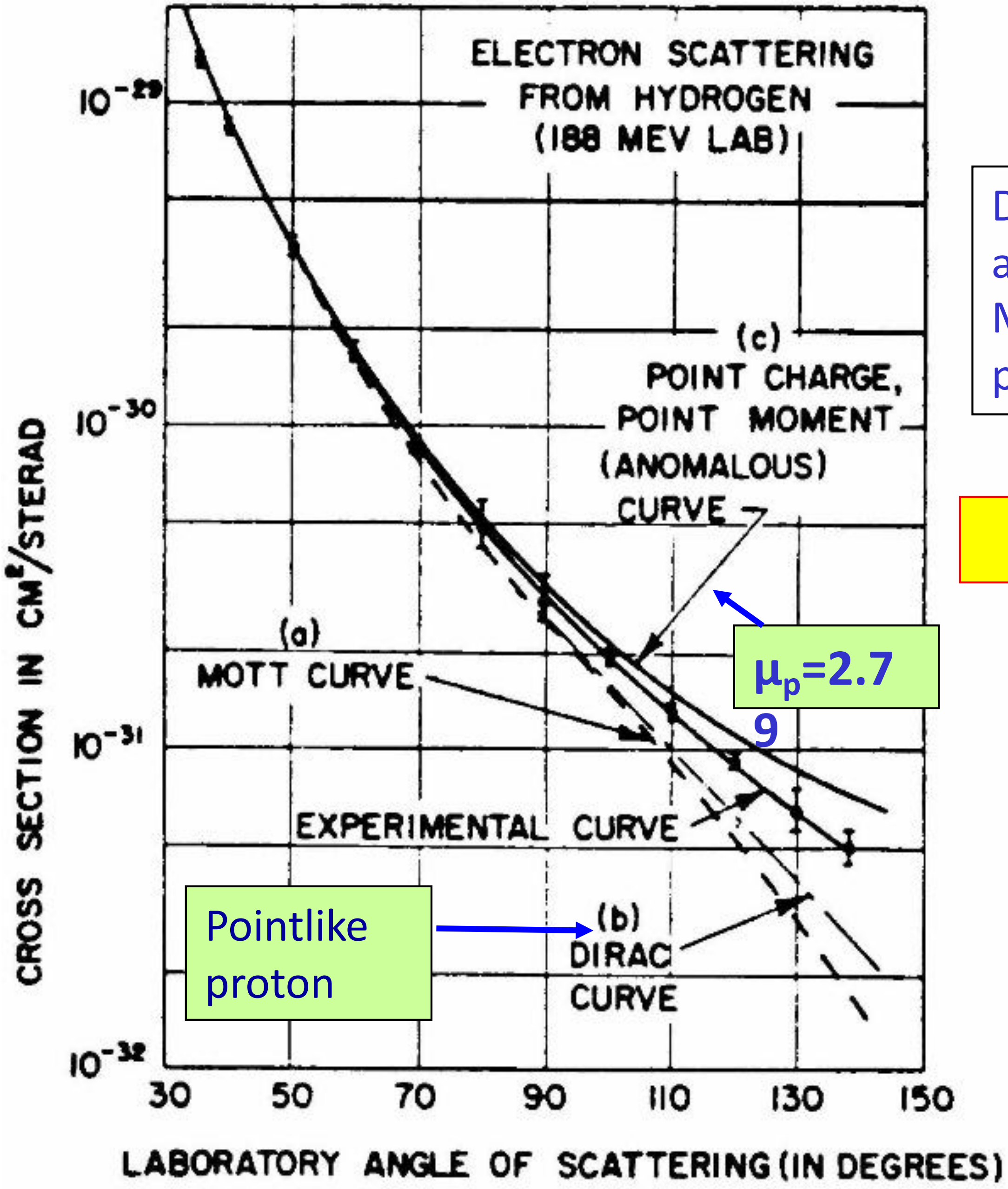
# A few values of E' and $Q^2$

| E(GeV) | θ (gradi) | E' (GeV) | $Q^2$ (GeV $^2$) |
|---|---|---|---|
| **0.188** | 60 | 0.171 | 0.03 |
| **0.188** | 120 | 0.145 | 0.08 |
| **0.5** | 60 | 0.395 | 0.20 |
| **0.5** | 120 | 0.278 | 0.59 |
| **2** | 60 | 0.97 | 1.94 |
| **2** | 120 | 0.48 | 2.86 |
| **5** | 60 | 1.36 | 6.82 |
| **5** | 120 | 0.55 | 8.34 |

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}$$

$$Q^2 = 4EE'\sin^2\frac{\theta}{2}$$

# Results of Hofstadter

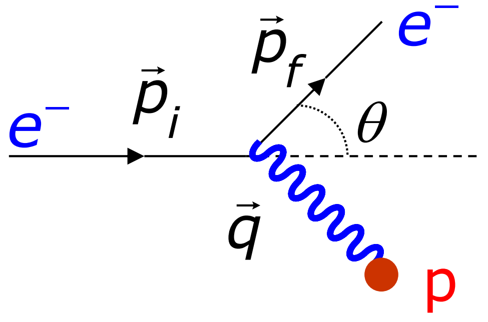



Data suggest that the proton has a structure. Hofstader e McAllister deduced that the proton has an average radius of:

$$\langle r \rangle = (0.74 \pm 0.24) \times 10^{-13} \text{cm}$$

# Rutherford's formula

Let's consider the Coulomb scattering of an electron from a proton at rest.




If the electron was without spin and the proton had an infinit mass, we would have the Rutherford's scattering:

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4p_i^2}\sin^4\frac{\theta}{2}}$$

To be noticed the $1/p^2$ dependence

- The momentum carried by the photon is: $\vec{q} = \vec{p}_i - \vec{p}_f \quad \Rightarrow \quad q^2 = p_i^2 + p_f^2 - 2\vec{p}_i \bullet \vec{p}_f$
- If the proton has an infinite mass, the electron conserves its kinematical energy:

$$\Rightarrow \left|\vec{p}_i\right| = \left|\vec{p}_f\right| \qquad \Rightarrow \qquad q^2 = 2p_i^2\left(1-\cos\theta\right) = 4p_i^2\sin^2\frac{\theta}{2}$$

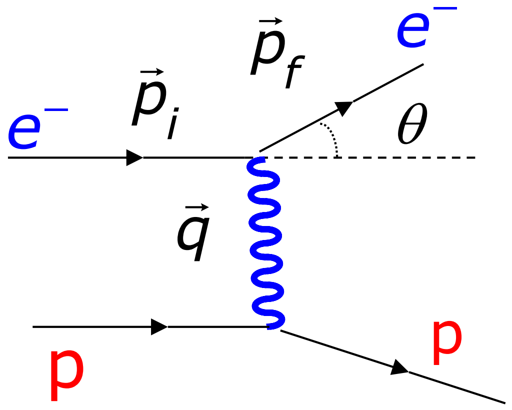
- moreover: $d\Omega = 2\pi\sin\theta d\theta = 2\pi d\left(-\cos\theta\right)$

$$dq^2 = 2p_i^2 d\left(-\cos\theta\right) \qquad \Rightarrow \qquad d\Omega = \frac{\pi}{p_i^2}dq^2$$

$$\boxed{\frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{q^4}}$$

Rutherford's formula

# Mott's cross-section

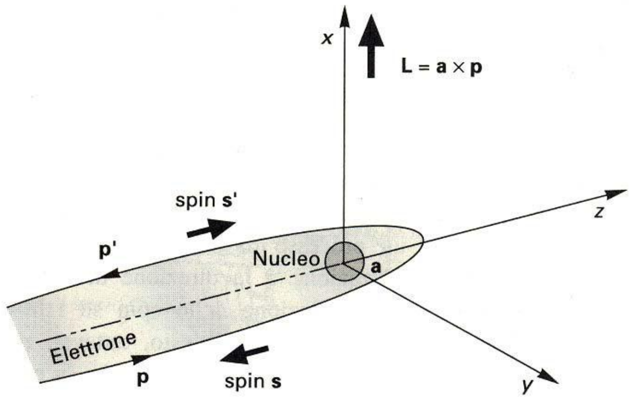



$\vartheta$ = scattering angle of the electron
M = proton mass
E = energy of the incoming electron

- Let's take into account the <u>electron spin</u> and the <u>proton recoil</u>. The proton is still considered as a scalar pointlike particle. What we get is the Mott's cross-section.

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \cdot \frac{\cos^2\left(\frac{\vartheta}{2}\right)}{1+\frac{2E}{M}\sin^2\left(\frac{\vartheta}{2}\right)}$$

From the formula we see that for θ=π the cross section is zero. This is related to the electron elicity and to have considered the proton without the spin




At 180° the electron must do a spin-flip, that can not be done if the proton has spin zero.

# "Dirac" cross-section

- The Mott's cross section takes into account the electrical interaction between the electron charge and the proton charge, but it does not consider the spin interaction.
- The cross-section can be improved by introducing in the calculation also the spin of the proton, assumed to be a Dirac pointlike particle of spin ½.
- The electron will interact both with the charge and the magnetic moment of the proton (that we know is different from the one predicted by Dirac for a pointlike particle).
- The magnetic interaction is associated to the spin-flip of the proton, therefore is preferred the scattering at an angle of 180°. The cross-section can be written as:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Dirac} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[1 + \frac{Q^2}{2M^2}\tan^2\left(\frac{\theta}{2}\right)\right]$$

Magnetic interaction: it is important at large angle et at large values of $Q^2$

# Magnetic moment

$$\vec{\mu} = \frac{q}{2m}\vec{L}$$

Magnetic moment of a particle with electric charge q, mass m and angular orbital momentum L

$$\vec{\mu} = g\frac{qh}{2m}\vec{S}$$

S=spin; g=gyromagnetic ratio. Classically g=1

- The Dirac's theory predicts that the electron has g=2 (and of course spin ½ ħ)

$$\mu_e = \frac{eh}{2m_e} \approx 5.79\times10^{-5}\ \frac{eV}{T}$$

Bohr magneton

- Replacing in the formula the electron mass with the proton mass, we have the nuclear magneton.

$$\mu_N = \frac{eh}{2m_N} \approx 3.1525\times10^{-8}\ \frac{eV}{T}$$

- The measured values of the proton and neutron magnetic moments are:

$$\mu_P = \frac{g_p}{2}\cdot\mu_N = +2.79\cdot\mu_N\ ;\ \mu_n = \frac{g_n}{2}\cdot\mu_N = -1.91\cdot\mu_N$$

- If they were two Dirac particles, we would have had: $\mu_P = \mu_N\ \ ;\ \ \mu_n = 0$

- Therefore the anomalous part of the magnetic moment of the proton and neutron, expressed in terms of nuclear magneton, is:

$$\boxed{\kappa_P = 1.79\ \ ;\ \ \kappa_n = -1.91}$$

# Dirac's cross-section with anomalous magn. mom.

- Rosenbluth carried out the computation of the cross-section by taking into account the anomalous magnetic moment of the proton and assuming that the proton WAS NOT a pointlike particle.
- This implies the introduction of two form factors that take into account the charge and the "electric current" distributions inside the proton.
- Here we report the expression of the Rosenbluth's cross-section in the limit of q→0 (no form factors needed). In this case the cross-section differs from the Dirac's one only for the presence of the anomalous magnetic moment κ.

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{Rosemb.\\ q\to 0}} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[\left(1+\frac{\kappa^2 Q^2}{4M^2}\right)+\frac{Q^2}{2M}(1+\kappa)\ \tan^2\left(\frac{\theta}{2}\right)\right]$$

At the same transferred $Q^2$ we have:

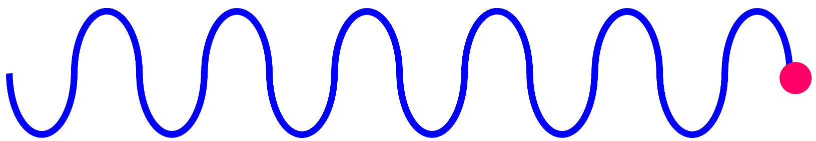
$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{Rosemb.\\ q\to 0}} > \left(\frac{d\sigma}{d\Omega}\right)_{Dirac} > \left(\frac{d\sigma}{d\Omega}\right)_{Mott}$$

But, as we have seen, the "truth" is in the middle, so we need to introduce two form factors in the cross-section.

# 1/2 Why do we need a form factor?

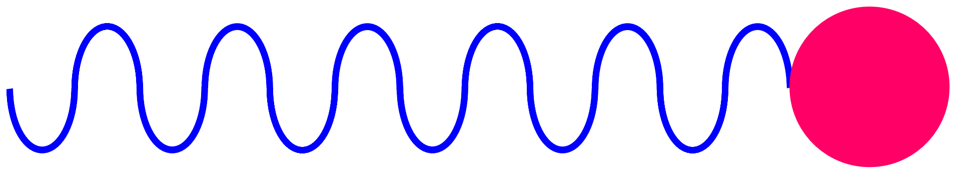
- A photon can "probe" dimensions similar to its wave length λ. Objects whose size is much smaller than λ are seen by the photon as pointlike objects.

$$\lambda = \frac{2\pi}{|\vec{q}|}$$



- If a charge is indeed a pointlike charge, the photon will always see the charge as pointlike whatever it is the photon wave length. Therefore in the interaction process between the photon and the charge DOES NOT enter a "scale" that distinguish the effect of "not being pointlike" of the charge.

- On the contrary, when the charge is NOT pointlike and it has a dimension similar to λ, the photon will not interact with the charge as a whole, but rather with the individual parts of the charge and we will have interference phenomena.



- This behaviour is described by the form factor that takes care of the charge distribution.

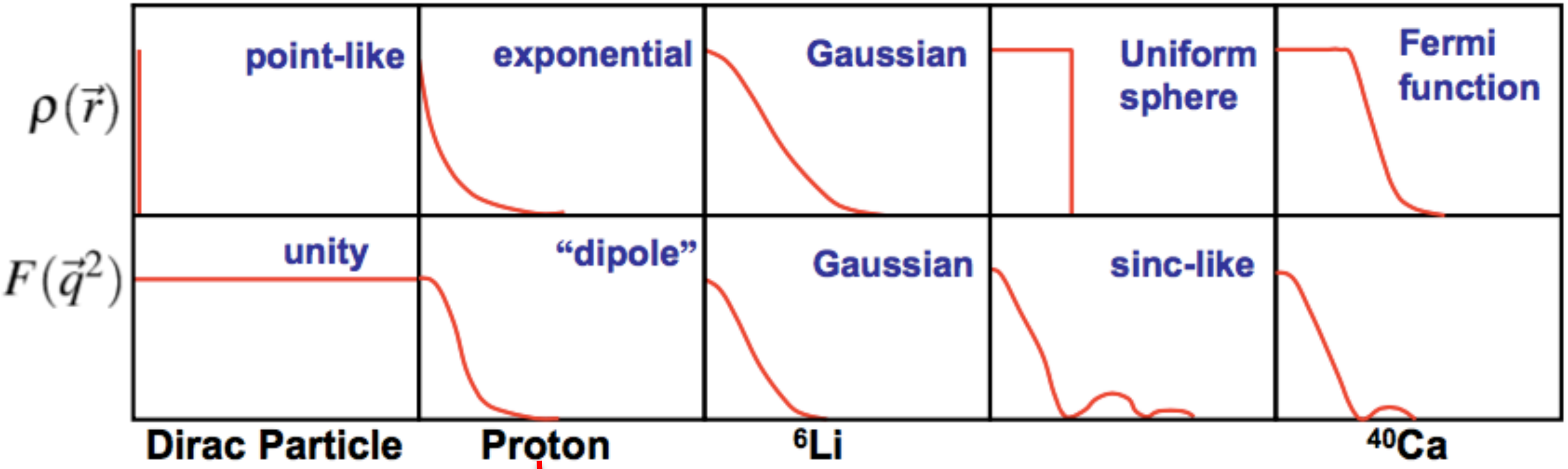
# Form factors

$$F(\vec{q}^{\,2}) = \int \rho(\vec{r}) \cdot e^{i\vec{q}\cdot\vec{r}}\, dV$$

$\rho(\vec{r})$: charge density

$$\Rightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{misurata} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[F\left(\vec{q}^{\,2}\right)\right]^2$$

From a measurement of the cross-section and by a comparison with the pointlike Mott's cross-section (for instance), we can deduce F($q^2$), and from the latter the charge distribution within a nucleus (or the proton).




a:scale factor

$$F(q^2) = \frac{1}{\left(1 + \dfrac{q^2}{a^2\hbar^2}\right)^2}$$

$$q \ll a \quad \Rightarrow \quad F(q^2) = 1$$

$$q \gg a \quad \Rightarrow \quad F(q^2) = \frac{1}{q^4} \ll 1$$

# Rosenbluth's cross-section

- The Rosenbluth's cross-section takes into account both the electric and magnetic interactions of the electron with the proton.
- We need two form factors, one electric ($G_E$) and another one magnetic ($G_M$).
- Later we will show the computation made by Rosenbluth.

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rosembluth} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[\frac{G_E^2 + \frac{Q^2}{4M^2}G_M^2}{1+\frac{Q^2}{4M^2}} + \frac{Q^2}{2M^2}G_M^2 \cdot \tan^2\left(\frac{\theta}{2}\right)\right]$$

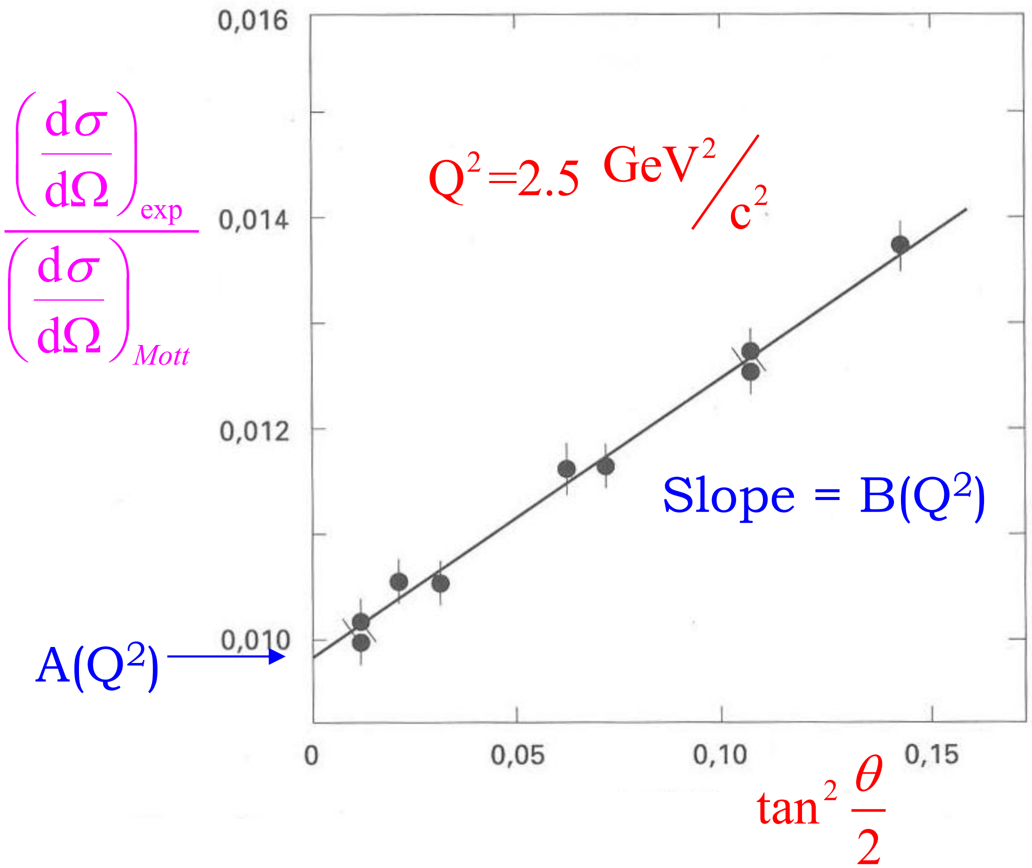
- In practice the formula can be re-written in the following way:

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)_{Ros.}}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = A(Q^2)+B(Q^2)\cdot\tan^2\left(\frac{\theta}{2}\right)$$

where: $A = \dfrac{G_E^2 + \frac{Q^2}{4M^2}G_M^2}{1+\frac{Q^2}{4M^2}}$ ; $B = \dfrac{Q^2}{2M^2}G_M^2$

# Measurement of the form factors

- The form factors $G_E$ and $G_M$ can be determined by doing a series of experiments at different values of $Q^2$ and measuring the differential cross-section $d\sigma/d\Omega$ as a function of $\theta$




$$A = \frac{G_E^2 + \dfrac{Q^2}{4M^2} G_M^2}{1 + \dfrac{Q^2}{4M^2}}$$

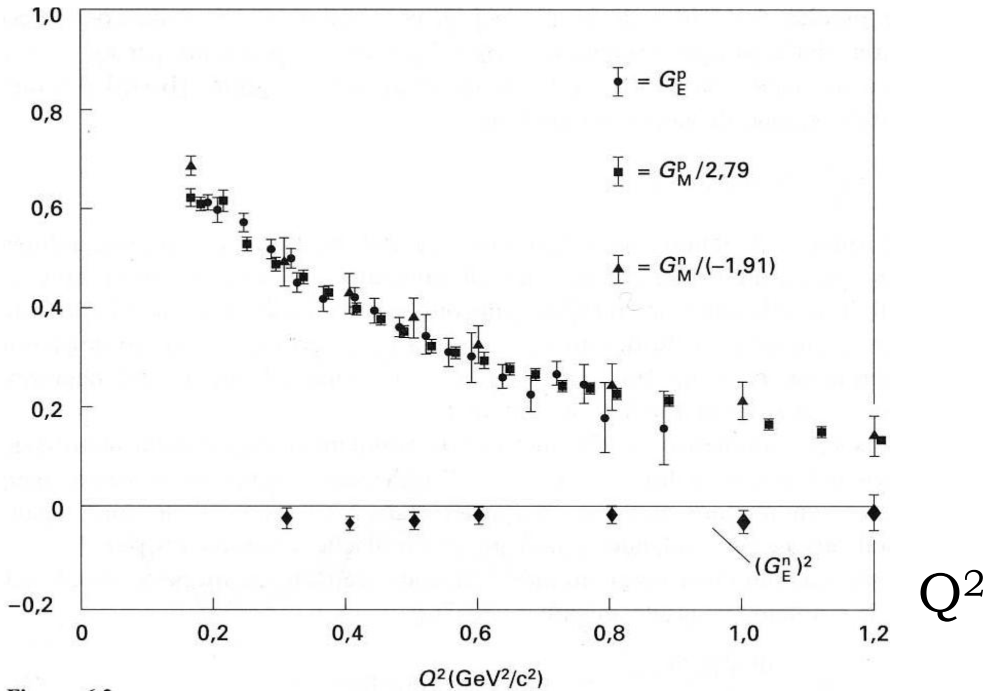
$$B = \frac{Q^2}{2M^2} G_M^2$$

- The magnetic form factor, at a given value of $Q^2$, can be determined through the slope, then from this value and from the intercept, we can determine the electric form factor

# Scale law of the form factors

- The experimental results show that the proton and neutron form factors are related by a very simple relationship:

$$G_E^p(Q^2) = \frac{G_M^p(Q^2)}{\mu_p} = \frac{G_M^n(Q^2)}{\mu_n} \quad ; \quad G_E^n(Q^2) = 0$$




This behaviour can be described by a dipole formula:

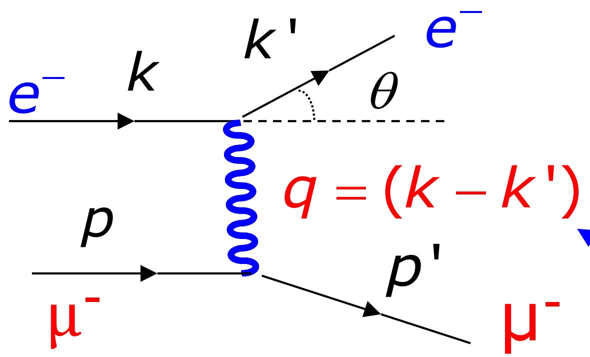
$$\boxed{G(Q^2) = \frac{1}{\left(1+\frac{Q^2}{m^2}\right)^2}}$$

where $\boxed{m^2 = 0.71\ \mathrm{GeV}^2}$

- The dipole formula is compatible with an exponential charge distribution:

$$\boxed{\rho \approx e^{-mr} \quad \Rightarrow \quad \sqrt{\overline{r^2}} \approx 0.8\ \mathrm{fm}}$$

# Elastic scattering electron-muon




(transferred quadrimomentum)

Let's consider a hypothetical scattering of an electron with a target made of muons. Both beam and target are pointlike particles, so no form factors here!

- The interaction is electromagnetic. The fundamental Feynman's diagram requires only one photon exchange.
- The matrix element of the process can be obtained by taking into account the interaction between the Dirac current of the electron with the one of the muon.

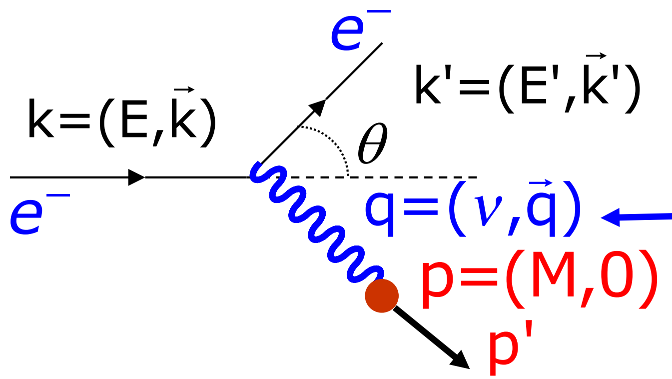
$$M_{\mathrm{fi}} = \left[\bar{u}(k')\gamma^{\mu}u(k)\right]\frac{e^2}{q^2}\left[\bar{u}(p')\gamma_{\mu}u(p)\right]$$

- To find the cross-section we need to square the matrix element $M_{fi}$. Since the beam (electron) and the target (muon) are not polarized, we need to take the average over the initial spin states and to sum up over the final spin states.

$$\overline{\left|M_{\mathrm{fi}}\right|^2} = \frac{8e^4}{q^4}\left[(k'\cdot p')(k\cdot p) + (k'\cdot p)(k\cdot p') - \mathrm{m}^2(p'\cdot p) - \mathrm{M}^2(k'\cdot k) + 2\mathrm{m}^2\mathrm{M}^2\right]$$

m= electron mass; M= muon massa

# Elastic scattering electron-muon




Kinematical variables in the reference system where the muon is at rest (that is the Laboratory frame)

(mnemonics: ν=photon energy = photon frequency!)

- **Let's compute $M_{fi}^2$ in the Lab frame and neglecting the electron mass.**

$$\boxed{q=k-k'=p'-p \quad \Rightarrow \quad p'=k-k'+p} \quad \Rightarrow \quad \overline{|M_{fi}|^2} = \frac{8e^4}{q^4}\left[-\frac{1}{2}q^2(k\cdot p - k'\cdot p) + 2(k'\cdot p)(k\cdot p) + \frac{1}{2}M^2q^2\right]$$

- In the experiment we measure only the quantities related to the electron (Energies (E,E') and scattering angle θ), that introduced in $M_{fi}^2$, give:

$$\overline{|M_{fi}|^2} = \frac{8e^4}{q^4}2EE'M^2\left[\cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2}\sin^2\left(\frac{\theta}{2}\right)\right]$$

- In order to get the differential cross-section of the electron scattering in the solid angle dΩ and energy in the range E' and E'+dE', we need to add to the matrix element $M_{fi}^2$ also the contribution of the phase space:

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2E'^2}{q^4}\left[\cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2}\sin^2\left(\frac{\theta}{2}\right)\right]\delta\left(\nu + \frac{q^2}{2M}\right)$$

# Elastic scattering electron-muon

$$E' = \frac{E}{1+\frac{E}{Mc^2}(1-\cos\vartheta)}$$

In the elastic scattering, the energy of the outgoing electron (scattered electron) and the scattering angle are NOT independent variables, but we have this relationship:

- If we do the integral of the cross-section with respect the energy E', we get:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4\left(\frac{\theta}{2}\right)} \frac{1}{1+\frac{2E}{M}\sin^2\left(\frac{\theta}{2}\right)} \left[\cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2}\sin^2\left(\frac{\theta}{2}\right)\right]$$

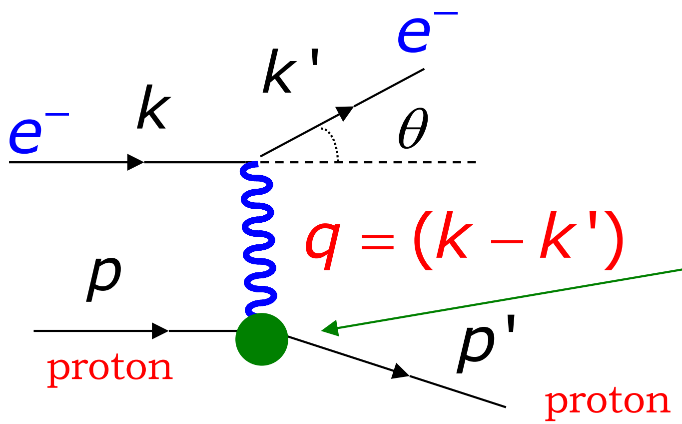
- which can be written as: $\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[1-\frac{q^2}{2M^2}\cdot\tan^2\left(\frac{\theta}{2}\right)\right]$

where $\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{\alpha^2\cos^2\left(\frac{\theta}{2}\right)}{4E^2\sin^4\left(\frac{\theta}{2}\right)\left[1+\frac{2E}{M}\sin^2\left(\frac{\theta}{2}\right)\right]}$

To be kept in mind that the term proportional to $\sin^2(\theta/2)$ in the Mott's cross-section is due to the interaction of the electron spin with the muon magnetic moment.

The formula of the differential cross-section plays a very important role in the understanding of the proton structure in the electron-proton scattering.

# Elastic scattering electron-proton




The proton-photon vertex is not a Dirac vertex because the proton is not a pointlike particle

- The matrix element is given also in this case by the interaction of the electron current with the proton current:

$$M_{fi} = J_\mu^{elec.} \frac{1}{q^2} J^\mu_{prot.}$$

- electron current: $J_\mu^{elec.} = -e \cdot \bar{u}(k') \cdot \gamma_\mu \cdot u(k)$

- The proton must follow the Dirac's equation, however the complex proton structure shows up in the vertex of the coupling proton-photon that is different from the one between electron and photon. The proton current can be written as:

$$J^\mu_{prot.} = e \cdot \bar{u}(p') \cdot \Gamma^\mu \cdot u(p)$$

$J^\mu$ must be a Lorentz quadrivector.

- The most general form to write $\Gamma^\mu$, using the Gordon decomposition, is:

$$\Gamma^\mu = \left[ \underbrace{F_1(q^2)}_{\text{Electric int.}} \gamma^\mu + \frac{\kappa}{2M} \underbrace{F_2(q^2)}_{\text{Magnetic int.}} i\sigma^{\mu\nu} q_\nu \right]$$

# Rosenbluth's cross-section

$$\Gamma^\mu = \left[ F_1(q^2)\gamma^\mu + \frac{\kappa}{2M} F_2(q^2) i\sigma^{\mu\nu} q_\nu \right] \quad ; \quad \sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]$$

- k=1.79 nuclear magneton
- M is the proton mass

• From the amplitude we can compute the cross-section, by summing up over the spin final states and averaging over the spin initial states. In this way we get the Rosenbluth cross-section:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Ros.} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left\{ \left[ F_1(q^2) - \frac{\kappa^2 q^2}{4M^2} F_2(q^2) \right] - \frac{q^2}{2M}\left[F_1(q^2) + \kappa F_2(q^2)\right] \tan^2\left(\frac{\theta}{2}\right) \right\}$$

- If $q^2 \to 0$, the virtual photon has a large wave lenght and it is not sensitive to the details of the proton structure, that it is seen as a pointlike particle. In this limit we must have:

$$\lim_{q^2 \to 0} F_1(q^2) = 1 \quad \text{and} \quad \lim_{q^2 \to 0} F_2(q^2) = 1$$

- If the proton were a pointlike particle like the muon, κ would be zero and $F_1(q^2)$ would be equal to 1 for whatever value of $q^2$, and we obtain the Dirac vertex. In this way we end up with the same formula obtained previously for the electron-muon scattering:

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[ 1 - \frac{q^2}{2M^2} \cdot \tan^2\left(\frac{\theta}{2}\right) \right]$$

# Proton form factors

$$\left(\frac{d\sigma}{d\Omega}\right)_{Ros.} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left\{\left[F_1(q^2) - \frac{\kappa^2 q^2}{4M^2} F_2(q^2)\right] - \frac{q^2}{2M}\left[F_1(q^2) + \kappa F_2(q^2)\right]\tan^2\left(\frac{\theta}{2}\right)\right\}$$

- The Rosenbluth's formula can be written in a different way by introducing the electric and magnetic form factors of the proton:

$$G_E = F_1 + \frac{\kappa q^2}{4M^2} \cdot F_2 \quad ; \quad G_M = F_1 + k \cdot F_2$$

$$\lim_{q^2 \to 0} G_E = 1 \quad ; \quad \lim_{q^2 \to 0} G_M = 1 + \kappa = \mu_p$$

($\mu_p$ = proton magnetic moment)

$$\Rightarrow \left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[\frac{G_E^2 - \frac{q^2}{4M^2} G_M^2}{1 - \frac{q^2}{4M^2}} - \frac{q^2}{2M^2} G_M^2 \cdot \tan^2\left(\frac{\theta}{2}\right)\right]$$

- we use to write the formula by using the variable: $\boxed{Q^2 = -q^2}$ $\quad$ N.B. $Q^2 > 0$

$$\Rightarrow \frac{\left(\frac{d\sigma}{d\Omega}\right)_{Ros.}}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = A(Q^2) + B(Q^2) \cdot \tan^2\left(\frac{\theta}{2}\right)$$

where: $A = \dfrac{G_E^2 + \frac{Q^2}{4M^2} G_M^2}{1 + \frac{Q^2}{4M^2}} \quad ; \quad B = \dfrac{Q^2}{2M^2} G_M^2$

# Inelastic scattering electron-proton

## The quest for quarks

## Rutherford's Legacy in Particle Physics: Exploring the Proton

Jerome I. Friedman
MIT

Talk by J.I. Friedman at CERN in November 2011 at the conference to celebrate the centenary of the Rutherford's atom.

**Prevailing model of the proton in the 1960's**

NUCLEAR DEMOCRACY

BOOTSTRAP MODEL

**Particles are composites of one another**

$$p = \pi^+ + n + \ldots..$$

$$n = \pi^- + p + \ldots\ldots$$

**Particles have diffuse substructures and no elementary building blocks**

# Still Friedman speaking ….

## Are Quarks Real?

**MANY UNSUCCESSFUL SEARCHES**

– Accelerators, Cosmic rays, Terrestrial environment
Sea water, Meteorites, Air, etc.

**FRACTIONAL CHARGES**

– Considered by many to be unreasonable

***GENERAL POINT OF VIEW IN 1966***

*Quarks most likely just mathematical representations*

*Useful but* NOT *real* !

**Particles have diffuse substructures and no elementary building blocks**

# Still Friedman speaking ….

## Implausibility of Quark Model

" …the idea that mesons and baryons are made primarily of quarks is hard to believe.."

M. Gell-Mann 1966

" Additional data are necessary and very welcome to destroy the picture of elementary constituents."

J. Bjorken 1967

" I think Professor Bjorken and I constructed the sum rules in the hope of destroying the quark model."

K. Gottfried 1967

" Of course the whole quark idea is ill founded."

J.J. Kokkedee 1969

# Linac e$^{\pm}$ at SLAC

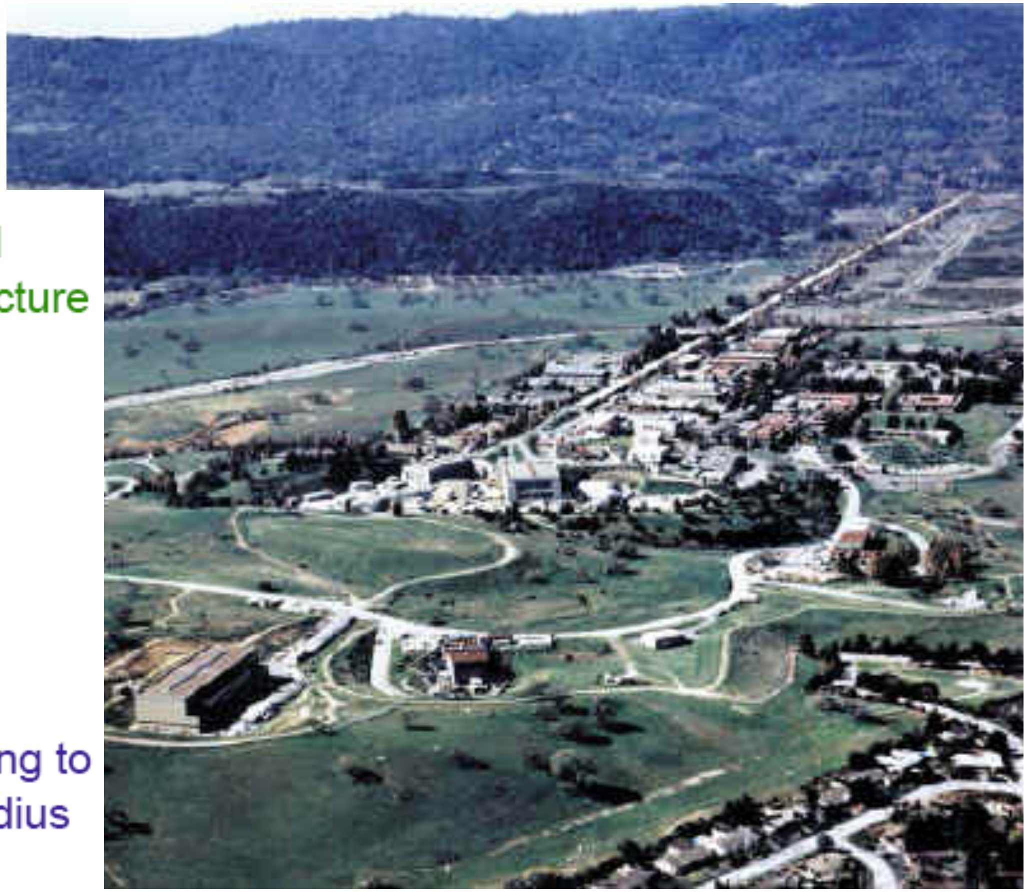
At SLAC, a laboratory near San Francisco, the "monster" begins to function in 1966. It is a linear accelerator of electrons up to 20 GeV, 2 miles long.

Friedman

- CIT-MIT-SLAC Collaboration designed and constructed spectrometer complex to study structure of proton, utilizing ELASTIC SCATTERING

- Electron ideal probe:
  - Structure known: "point particle"
  - Interaction understood: QED

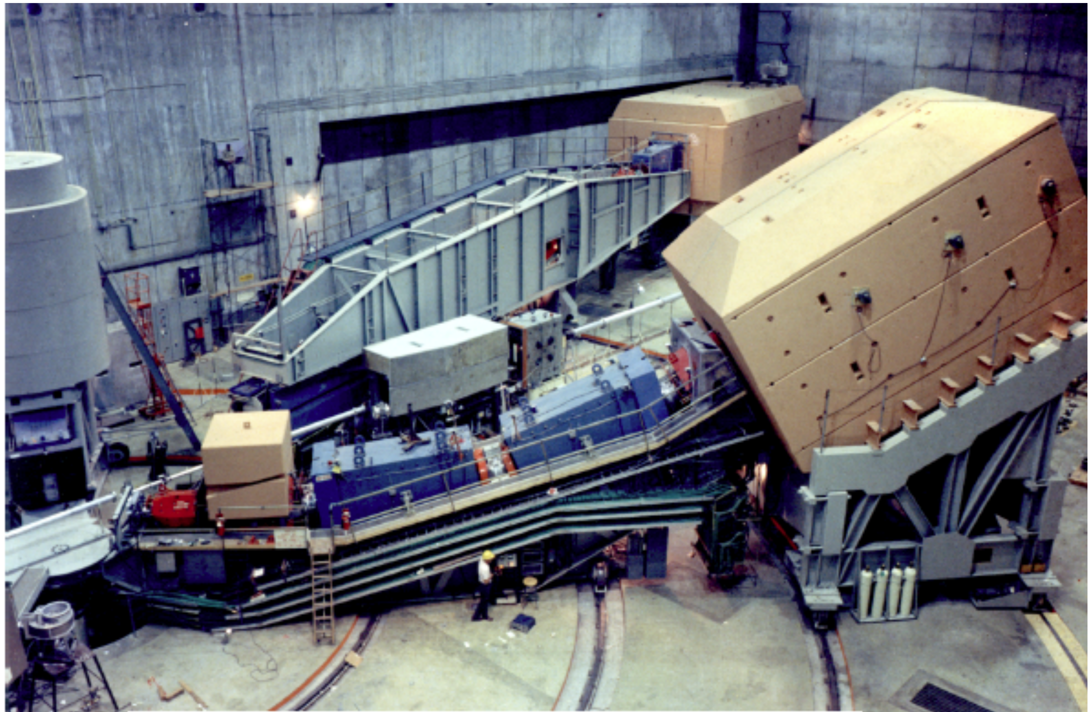
In 1950's, Hofstadter used Elastic e-p scattering to measure the proton's form factor & r.m.s. radius



2/2

# First results on the elastic scattering

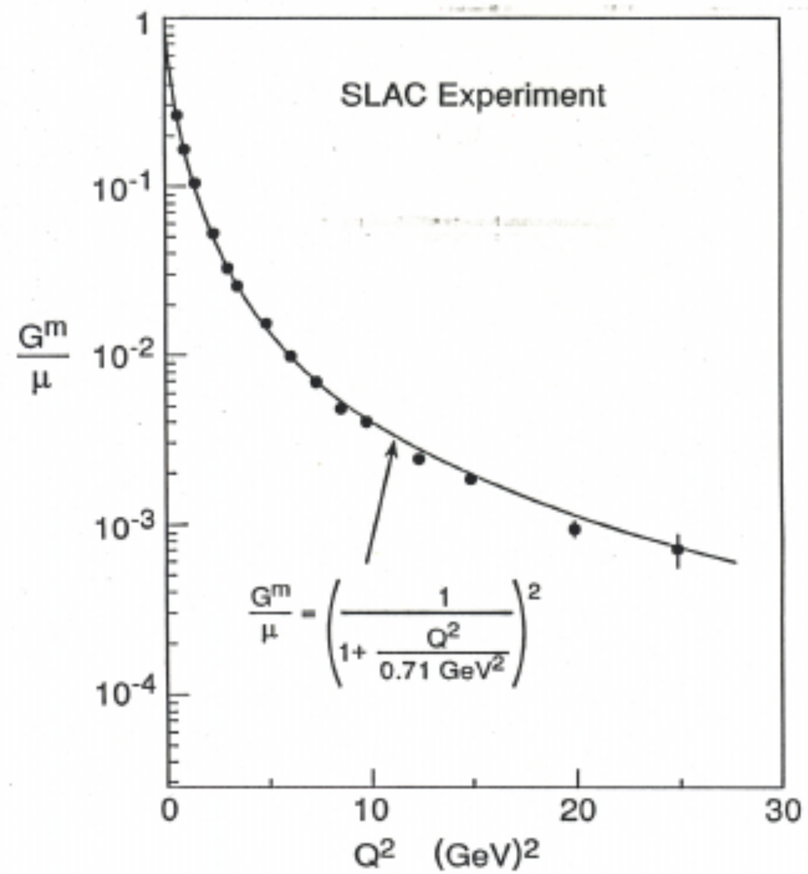
The detector is conceptually identical to the one used by Hofstadter but is has to be larger to measure the higher electron energies (up to 500 MeV for Hofstadter while here we arrive up to 20 GeV)



SLAC Magnetic Spectrometers

Friedman

Magnetic Form Factor of Proton




Extended earlier measurements at CEA & DESY

# Friedman: in 1967 we changed program

1967 MIT-SLAC begins Inelastic Program

$$e + p \Rightarrow e + \text{Anything}$$

## Inelastic vs. Elastic Scattering

- Elastic scattering provides information about the charge and magnetic moment distributions averaged over time
- Inelastic scattering can provide a "snapshot" of the structure

$$\Delta t = h/\Delta E$$

$\Delta E$ is energy lost by electron.

$\Delta E = 2$ GeV $\quad \Delta t = 3\times10^{-25}$ sec

for $v = c$

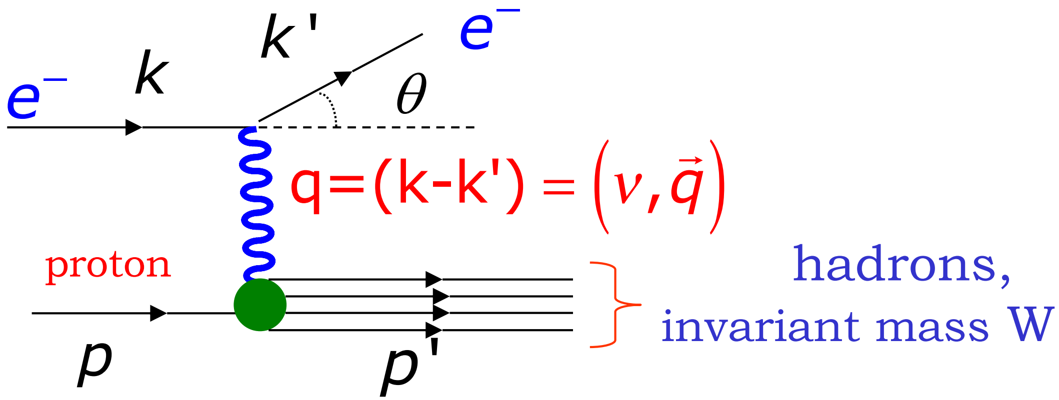
motion during "snapshot" is

$\approx 10^{-14}$ cm.

DEEP INELASTIC SCATTERING REQUIRED FOR LARGE $\Delta E$

The deep inelastic scattering is used to look for substructures inside the proton.

# Inelastic scattering electron-proton

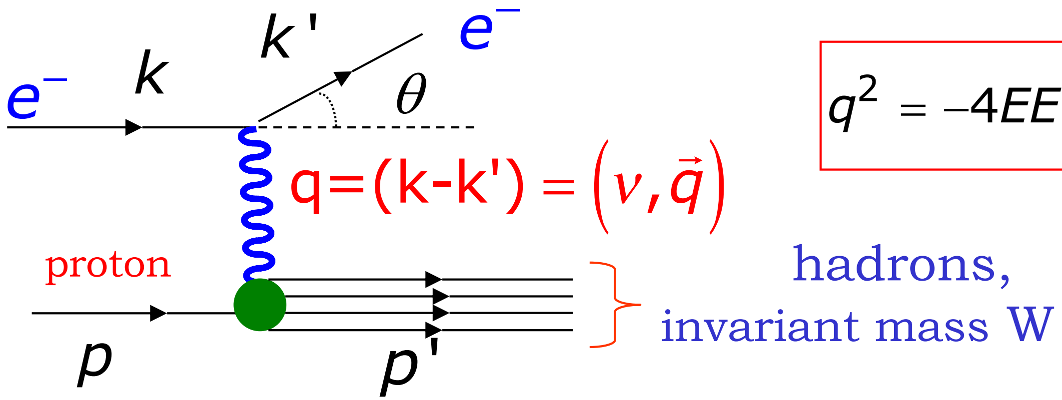



- In the elastic scattering the particle in the initial state, electron and proton, conserve their identity.
- If we increase the quadrimomentum transferred q (that is the energy transferred from the electron to the proton), the proton can be excited in one of its resonant states, for instance the $\Delta^+$, returning afterward to its fundamental state by emitting a pion.

$$e + p \rightarrow e + \Delta^+ \rightarrow e + p + \pi^0$$

- Increasing even more the $q^2$ transferred, the proton loose completely its identity and are produced many hadrons to replace it.
- In any case we assume that the interaction is dominated by one photon exchange only.
- Also in the case of inelastic scattering we measure only the quantities related to the electron, that is its energy and scattering angle.
- The cross-section measured with this method is called **inclusive cross-section**, because it does not discriminate between the several hadron states related to the proton part, but they are all summed-up together.

# Kinematical relationship




$$q^2 = -4EE'\sin^2\frac{\theta}{2}$$

$$p + k = p' + k'$$
$$q = k - k' = p' - p$$
$$q = (\nu, \vec{q}) \; ; \; p = (M, 0)$$
(the proton is at rest)

- We must get rid of p’ in the equations since we do not measure anything related to the hadron state:

$$p' = p + q \quad \Rightarrow \quad p'^2 = p^2 + q^2 + 2p\cdot q$$

- Let’s consider an elastic scattering. In this case the proton remains a proton, theredore $p'^2 = M^2$.

$$M^2 = p^2 + q^2 + 2p\cdot q \quad \Longrightarrow \quad \boxed{\nu = -\frac{q^2}{2M}}$$

- In the elastic scattering there is a relationship between the transferred energy ν (measured in the Laboratory) and the transferred quadrimomentum $q^2$.

- Let’s consider the inelastic scattering: in this case ν and $q^2$ are independent variables.

$p'^2 = W^2$ W is the invariant mass of the hadron produced in the reaction

$p\cdot q = M\nu$ N.B. p and q are evaluated in the laboratory

$$\boxed{Q^2 = -q^2} \quad \Longrightarrow \quad \boxed{Q^2 = 2M\upsilon + M^2 - W^2}$$

# Kinematical relationship: x and $Q^2$



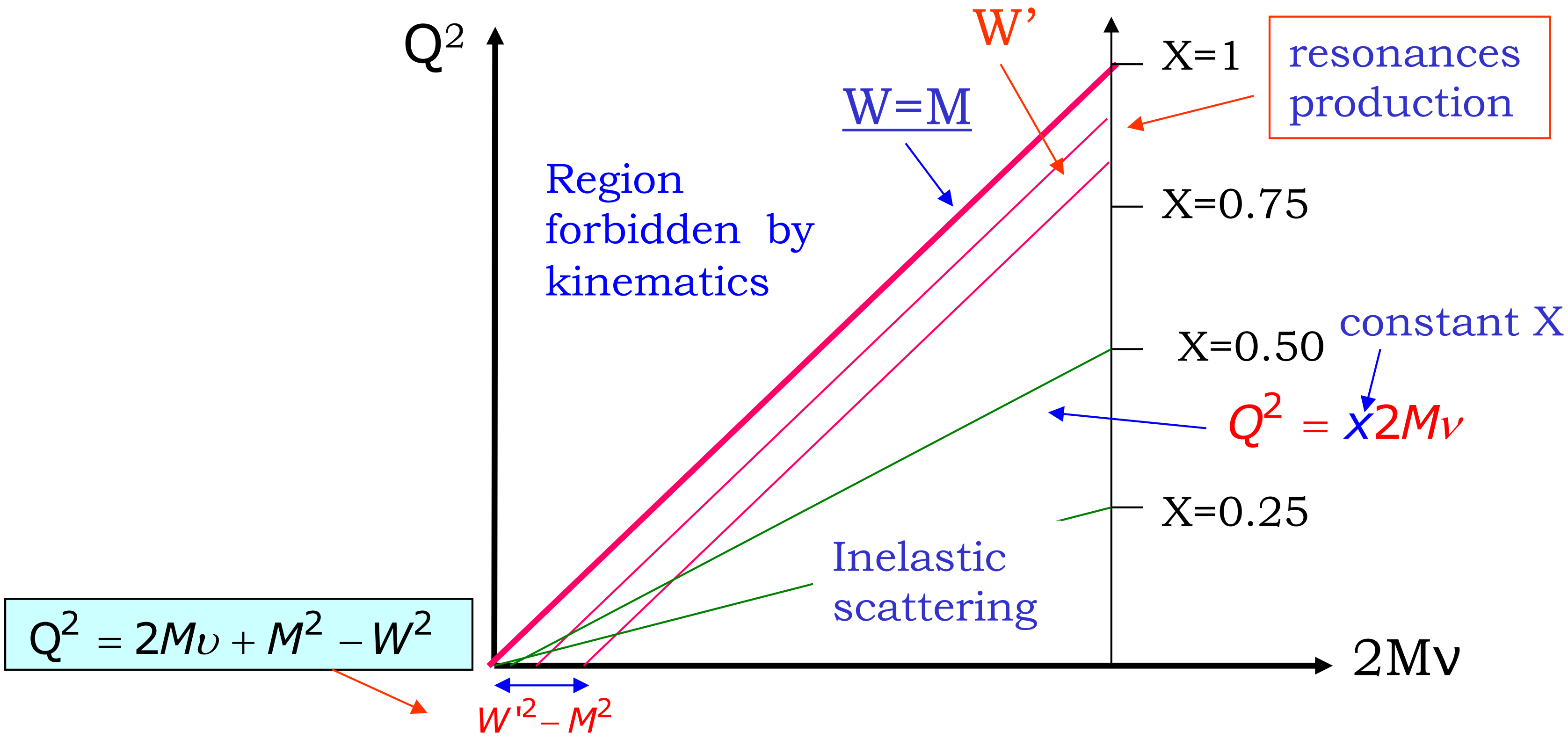
- Let's introduce a new kinematical variable, x (Bjorken variable), which will be very important in the deep inelastic scattering studies.

$$\boxed{x = \frac{Q^2}{2M\nu}}$$

(N.B. in the elastic scattering x = 1)

$$\Rightarrow \quad Q^2 = x2M\nu$$

- The regions where both $Q^2$ and ν are big are called of deep inelastic scattering. These regions are important because is there where we can investigate the internal proton structure.



# Experimental setup

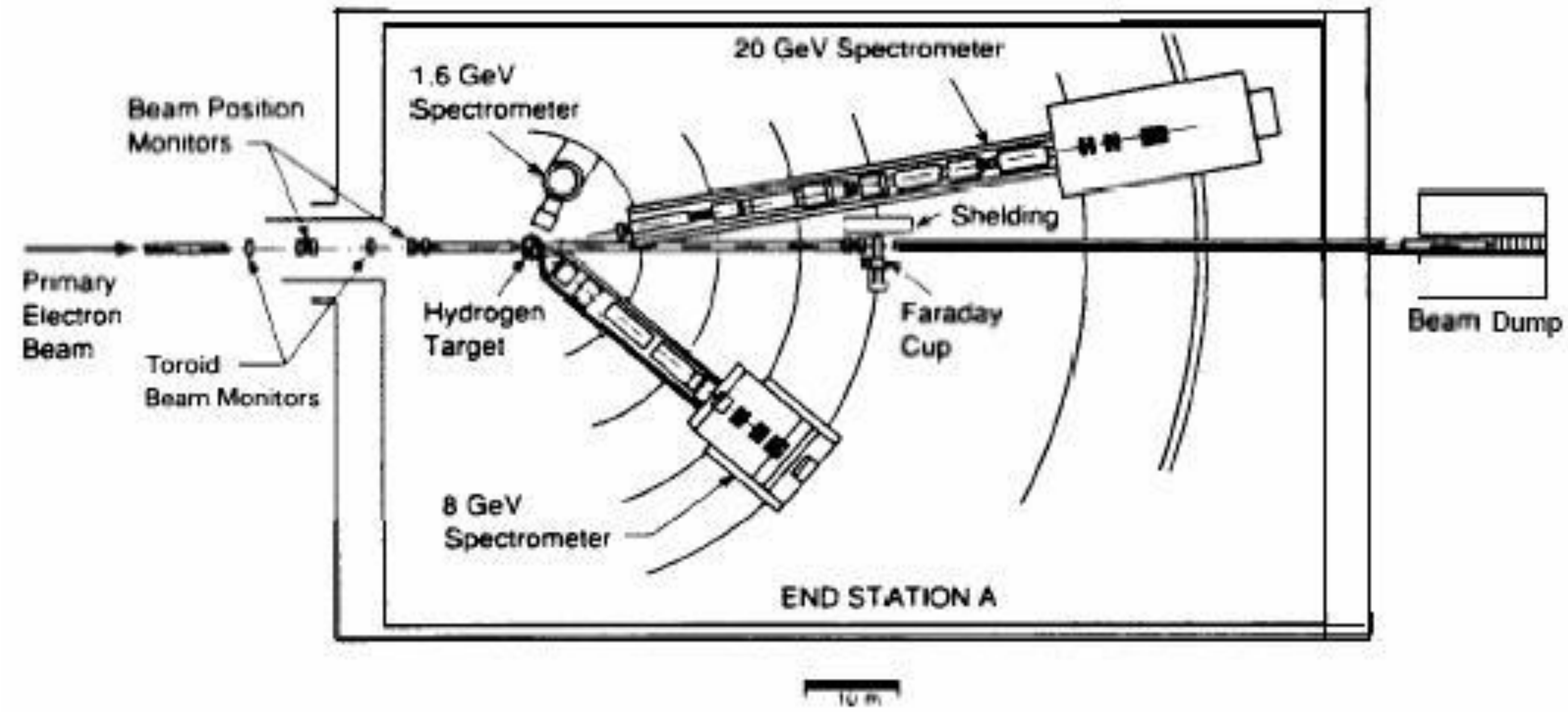



Fig. 14. Layout of spectrometers in End Station A. All three spectrometers can be rotated about the pivot. The 20 GeV spectrometer can be operated from about $1\frac{1}{2}°$ to 25°, the 8 GeV from about 12° to over 90°. The 1.6 GeV spectrometer coverage is from ~ 50° – 150°.

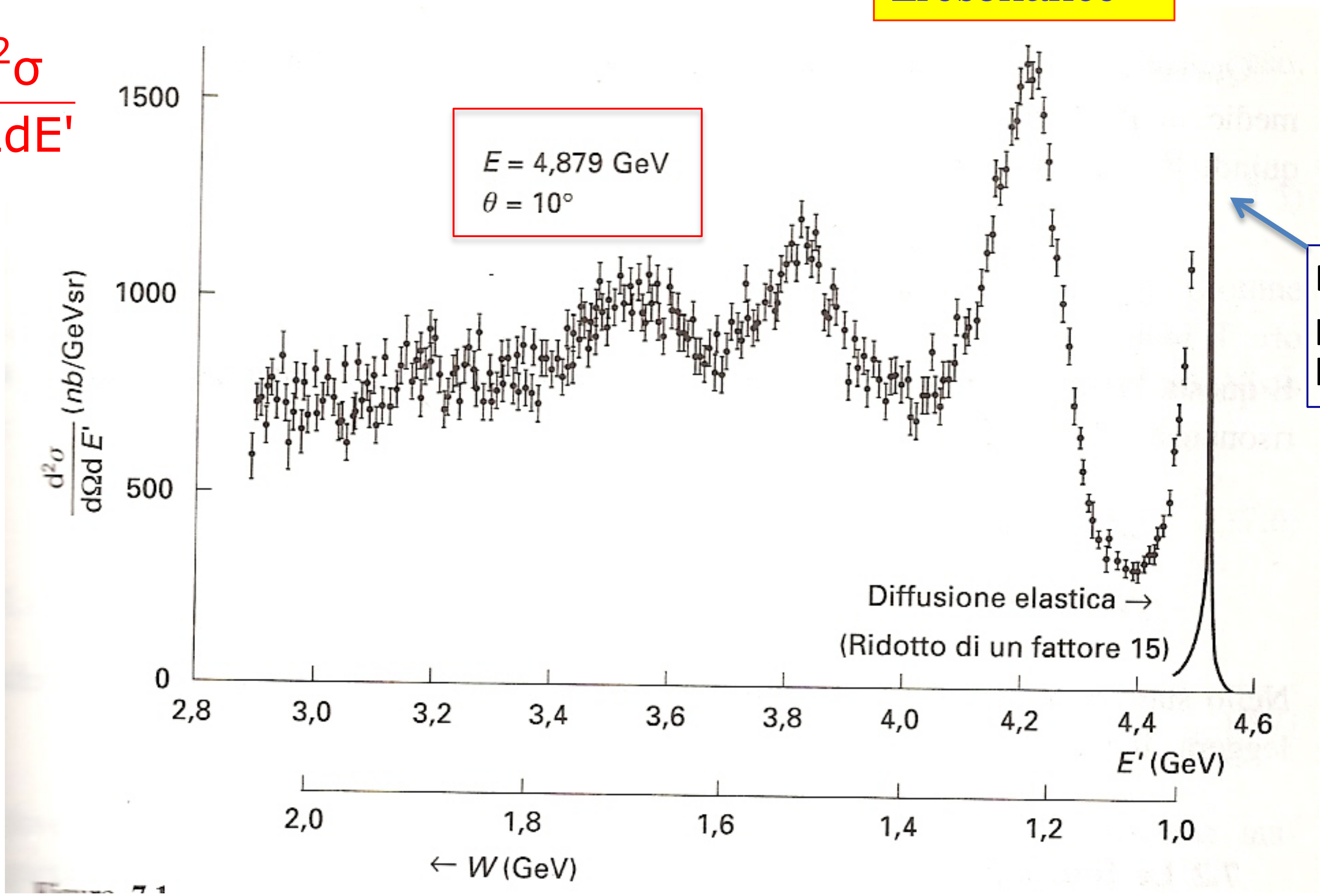
There were three spectrometers to measure the energy and the angle of the scattered electron.

In the paper by Friedman, Kendal, Taylor et al. in 1969, is reported the inelastic differential cross-section measured at θ=6° and 10°, with the energy of the incoming electron ranging between 7 and 17 GeV. The quadrimomentum transferred $Q^2$ went up to 7.4 $GeV^2$

# Example of inelastic scattering

$\Delta$resonance

$$\frac{d^2\sigma}{d\Omega dE'}$$

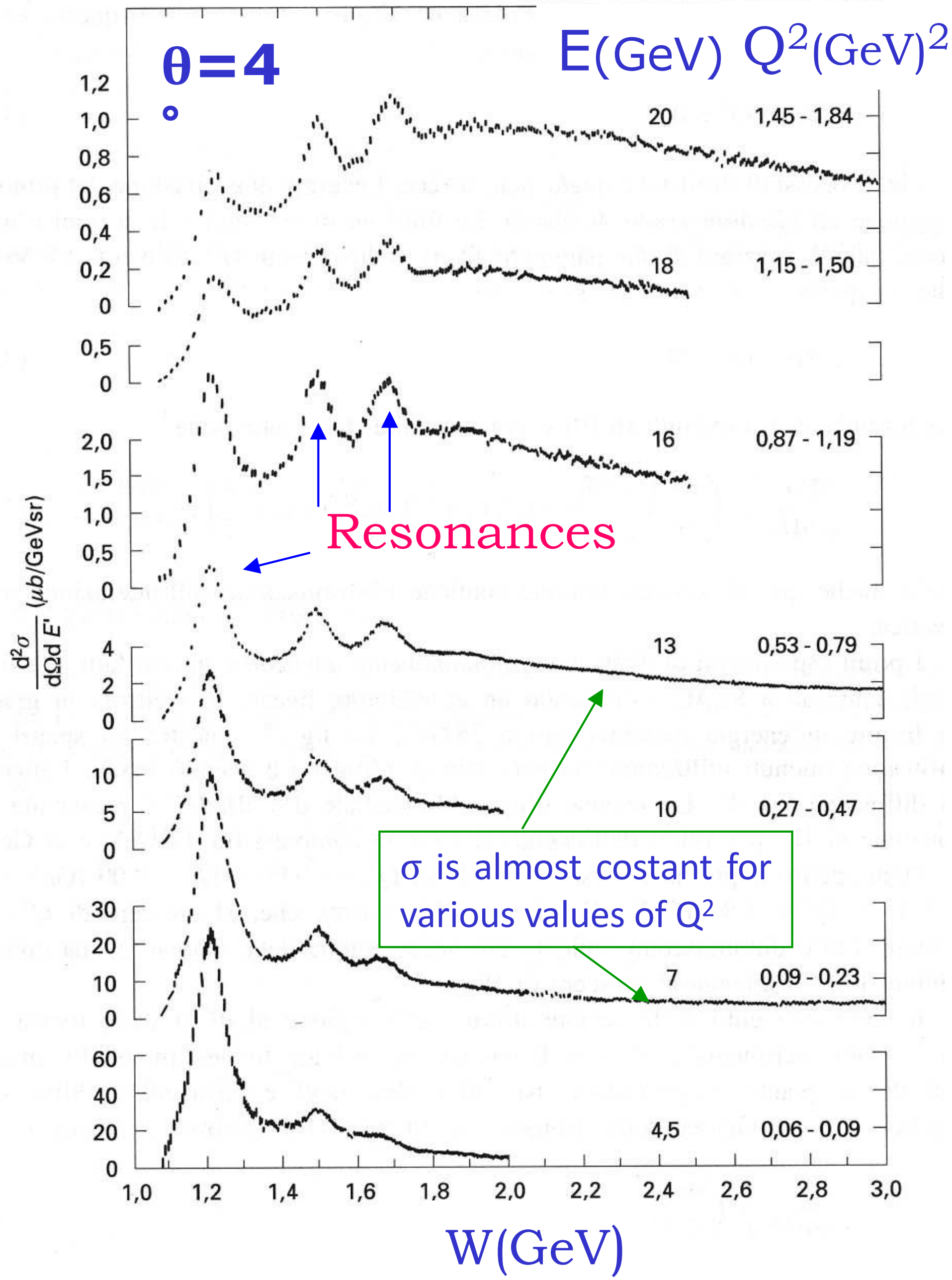



Elastic scattering peak reduced by a factor 15

We can see the production of several resonances with masses around 1.2 GeV, 1.5 GeV and 1.7 GeV

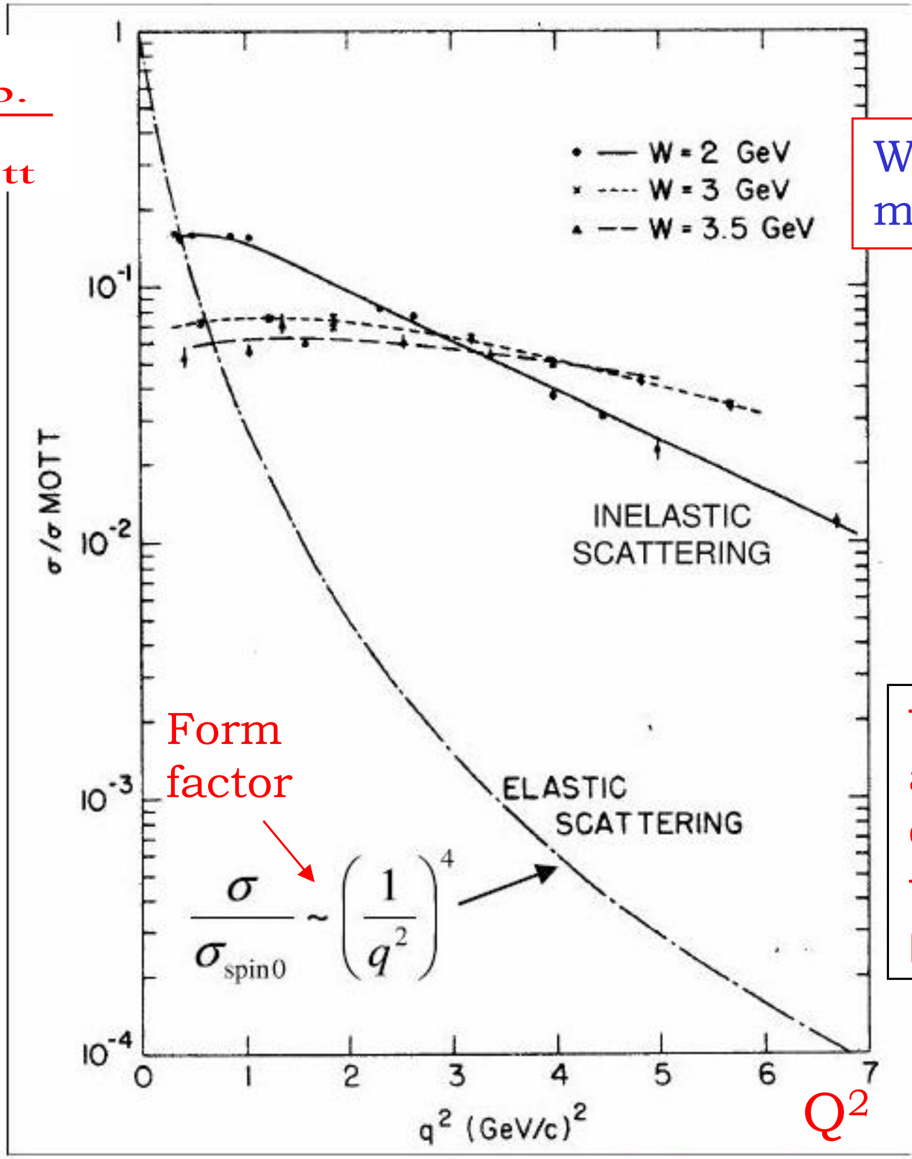
# Double differential cross-section

$$\frac{d^2\sigma}{d\Omega dE'}$$



# Comparison elastic and inelastic x-section

$$\frac{\sigma_{exp.}}{\sigma_{Mott}}$$




W=invariant mass

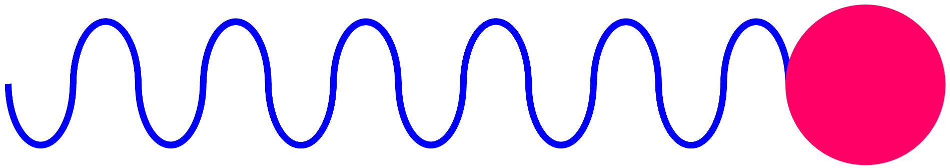
The energy of the incoming electron is in the range 7 – 17 GeV

The data clearly are not in agreement with the elastic cross-section foreseen, pointing to the existence of pointlike particles inside the proton.

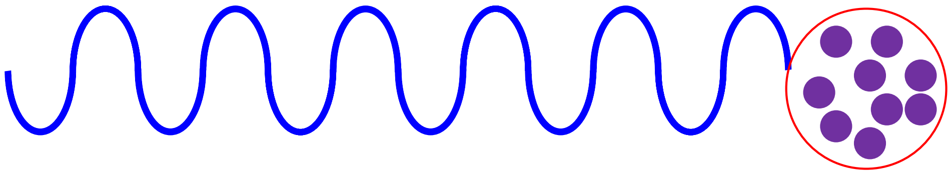
M.Breidenbach et al., Phys. Rev. Lett. **23** (1969) 935

# Scale invariance

- When the charge is NOT pointlike and has a dimension similar to $\lambda$, the photon will not interact with the charge as a whole, but with its single pieces and we will have the interference among the various scattering processes, giving rise to the form factor describing the charge distribution:



- Let's suppose now that the object is not made by a uniform charge distribution, but it is made by N pointlike charges:



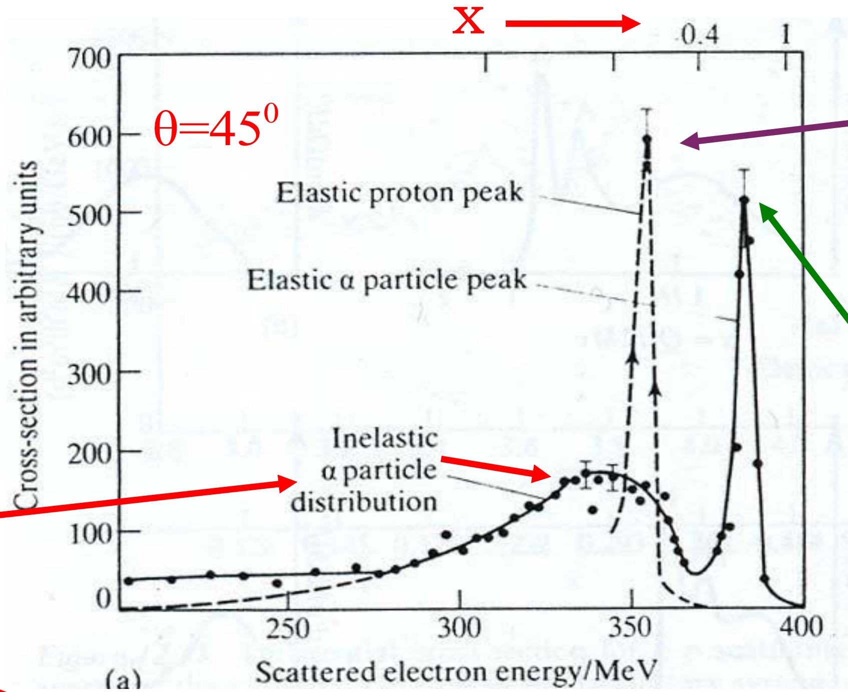
- When $\lambda$ is bigger than the distance between the various charges, we still need a form factor describing the charge distribution, as in the case of a continuous charge density.
- Buth when the photon wave length is much smaller than the distance between the charges, the photon will interact with the single poinlike charges. The interaction will occur only with the pointlike charge and we will not have the interference phenomenon as in the case of a continuous charge distribution (therefore we do no longer need a form factor).

- Moreover, even if we continue to decrease the photon wave length, the interaction always happens with a pointlike charge, so we will not have another scale related to the dimension of inner constituents (since they are pointilike). This is the scale invariance or scaling.

# 400 MeV electron scattering on α

Hofstadter 1956)

N.B.$Q^2$ increases with the increase of θ




$x'=Q^2/2m_p\nu=1$

$x'=Q^2 4/2(4m_p)\nu$

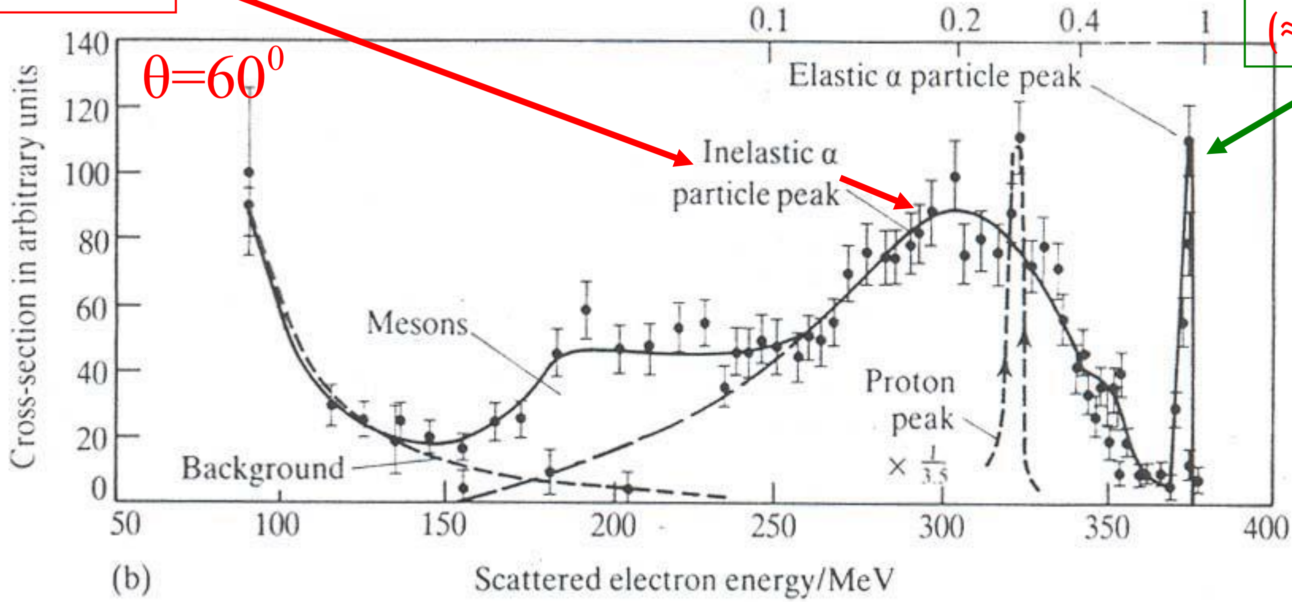
$x=Q^2/2m_\alpha\nu$

$\Rightarrow x'=4x=1$

$\Rightarrow x=0.25$

The inelastic peak has just a small decrease with the increasing of $Q^2$ (≈150 → ≈80)

The elastic peak decrease at the increasing of $Q^2$ (≈500 → ≈100)



# Scale invariance

- Let's consider a dipole form factor: $$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\Lambda^2_{nucleus}}\right)^2}$$
- In this expression $\Lambda_{\text{nucleus}}$ determines the scale of the phenomenon under study; the behaviour of the cross-sections, through the form factor, is a function of the relative values of $Q^2$ and $\Lambda_{\text{nucleus}}$

$$\text{If} \quad Q^2 \ll \Lambda^2_{\text{nucleo}} \Rightarrow F(Q^2) \rightarrow 1$$

- In this situation the photon has a wave length very large and it is not sensitive to the details of the target internal structure; the scattering happens as if the target were pointlike.
- Increasing $Q^2$, the elastic pointllike cross-section diminishes because of the factor $1/Q^2$
- However, if we imagine that there is another scale $\Lambda_{\text{nucleon}}$ but $Q^2 \ll \Lambda_{\text{nucleon}}$, it won't be detected any nucleon internal structure. Therefore we will have a quasi elastic scattering by the nucleons forming the nucleus. The scattering is quasi elastic because the nucleons are not free and are subject to the Fermi motion.
- The quasi elastic scattering happens for $x \approx 1/N$, where N is the number of the nucleus constituents.

- The form factor in the quasi elastic cross-section is independent from $Q^2$, that is does not depend from the scale (scale invariance; scaling)

# Inelastic scattering ep: cross-section

- Let's recall the cross-section of the elastic scattering e-μ, that we will write it in a different way:

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{q^4}\left[\cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2}\sin^2\left(\frac{\theta}{2}\right)\right]\delta\left(\nu + \frac{q^2}{2M}\right) \Rightarrow \frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4}\cdot S$$

where S represents the structure of the target.

- In case of the elastic scattering e-μ, where both e and μ are without structure, we have:

$$S_{e\mu\rightarrow e\mu} = \left[\cos^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{2M^2}\sin^2\left(\frac{\theta}{2}\right)\right]\delta\left(\nu - \frac{Q^2}{2M}\right)$$

where M is the muon mass. The δ express the fact that ν and $Q^2$ are not independent variables.

- In case of the inelastic scattering ep —> eX, ν and $Q^2$ are independent variable, therefore we have:

$$S_{ep\rightarrow eX} = W_2(Q^2,\nu)\cos^2\left(\frac{\theta}{2}\right) + 2W_1(Q^2,\nu)\sin^2\left(\frac{\theta}{2}\right)$$

- The complex structure of the proton is indicated by the presence of two structure functions $W_1$ and $W_2$, that for values of $Q^2$ less than 1 $(GeV/c)^2$ (as we will see), are function both of $Q^2$ and ν.

- Bjorken made the hypothesis that if the proton were composed by pointlike particles, the quasi elastic scattering with these particles should exhibit a scale invariance, that is the cross-section should be independent from $Q^2$ and being only function of the ratio $x=Q^2/2M\nu$

# Bjorken's scaling

- In other words, according to the Bjorken scaling, the structure functions $W_1$ e $W_2$ must reduce to the pointlike ones in the limit of deep inelastic scattering:

$$2W_1 = \frac{Q^2}{2m^2}\delta\left(\nu - \frac{Q^2}{2m}\right) \quad ; \quad W_2 = \delta\left(\nu - \frac{Q^2}{2m}\right)$$

m is the mass of the constituent particles

- If we recall that: $\delta(ax) = \frac{1}{a}\delta(x)$ we can re-write the relations as:

$$2mW_1(Q^2,\nu) = \frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right) = x \cdot \delta(1-x) \quad ; \quad \nu W_2(Q^2,\nu) = \delta\left(1 - \frac{Q^2}{2m\nu}\right) = \delta(1-x)$$

As we can see the two structures functions are only function of x (Bjorken's scaling).

- we could say that with the increase of $Q^2$, hence with the wave length that is getting shorter and shorter, the elastic scattering from the proton that can be considered as a coherent action of all quarks inside the proton, it can be replaced by the incoherent overposition of the elastic scattering of the single pointlike quarks.

- to summarize, in the limit $Q^2 \to \infty$, $\nu \to \infty$ where $x = \frac{Q^2}{2m\nu}$, we have:

$$\underset{\text{magnetic int.}}{mW_1(Q^2,\nu) \to F_1(x)} \quad ; \quad \underset{\text{Electric int.}}{\nu W_2(Q^2,\nu) \to F_2(x)}$$

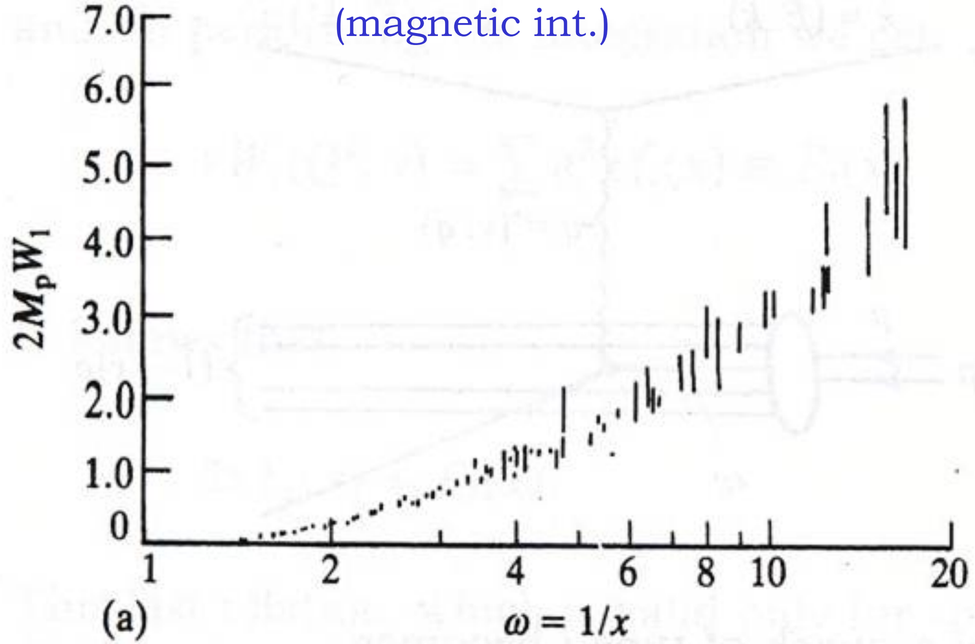
- The experimental data confirm this hypothesis for $Q^2 > (1\ \text{GeV/c})^2$

# Experimental evidence of the scaling

1/2

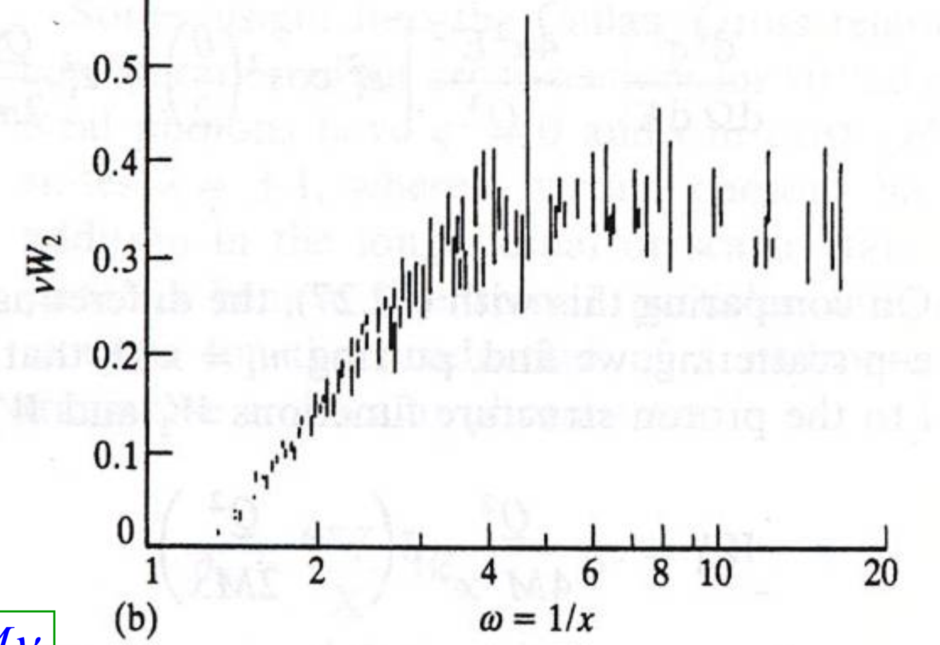
$$mW_1\left(Q^2,\nu\right) \rightarrow F_1(x)$$

(magnetic int.)




$$\nu W_2\left(Q^2,\nu\right) \rightarrow F_2(x)$$

(electric int.)



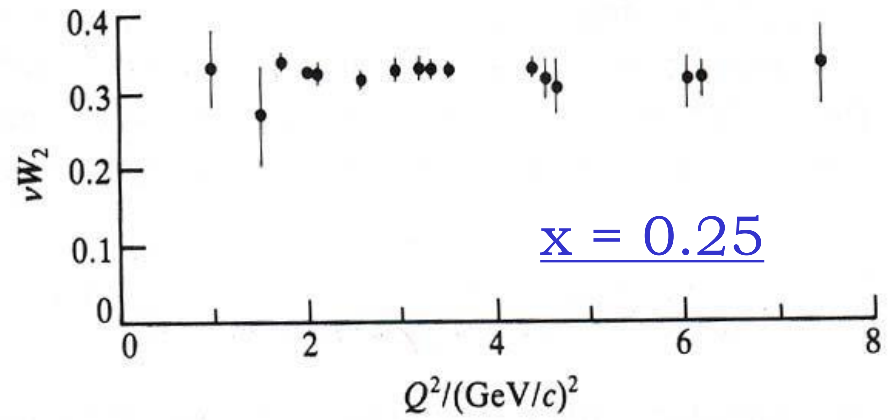

$$x = \frac{Q^2}{2M\nu} \quad \Rightarrow \quad \omega = \frac{2M\nu}{Q^2}$$

The data show that $F_1$ and $F_2$ are monotonic functions of only one variable.

# Experimental evidence of the scaling

Friedman and Kendal, 1972

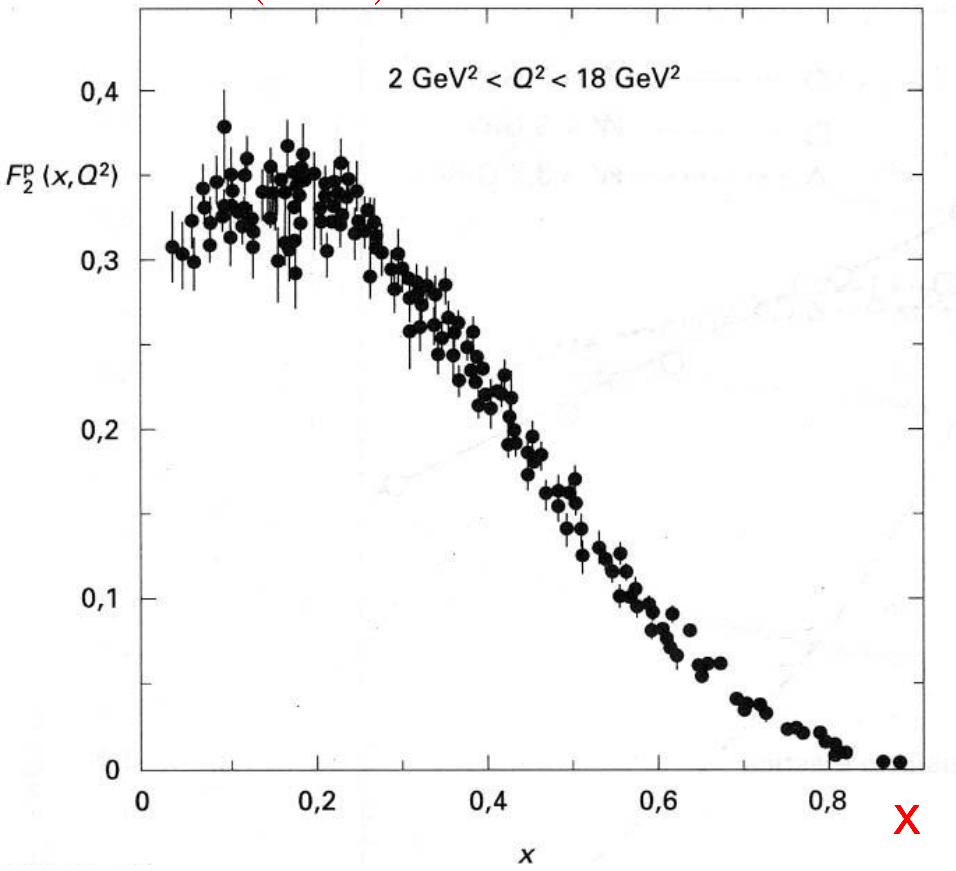
$\nu W_2(Q^2,\nu) \rightarrow F_2(x)$




x = 0.25

$F_2$ is not a function of $Q^2$

$\nu W_2(Q^2,\nu) \rightarrow F_2(x)$




x

# Again Friedman speaking

*Non–Constituent Models proposed to explain Scaling*

"OLD PHYSICS"

Vector Dominance

Resonance Models { Veneziano / N's and Δ's

Regge Poles

Diffraction Models

- Many attempts were made to use "Old Physics" models to explain results without success.
- But **Quark model** was not regarded as valid by most physicists.

?

- Theoretical contribution that helped resolve puzzle:
  – R. Feynman -- Parton Model

# Scaling interpretation a la Feynman

- The first “physical” interpretation of the scaling was given by Feynman in 1969. Feynman made the hypothesis that the proton is made by pontlike particles called **PARTONS.**

- Feynman postulated that each parton carries a fraction x of the energy and momentum of the proton.

- There are several types of partons. A parton of type i has the quadrimomentum:

$$\begin{cases} p_i = x \cdot p & \text{(P is the proton quadrimomentum )} \\ m_i = x \cdot M & \text{(M is the proton mass)} \end{cases}$$

- We can prove that the fraction “x” of the parton quadrimomentm is just equal to the Bjorken variable x:

$$x = \frac{Q^2}{2M\nu}$$

(we recall that ν is the energy transferred from the electron to the proton evaluated in the Laboratory frame)

- If we put m=xM in this formula, we get:

$$\nu = \frac{Q^2}{2xM} = \frac{Q^2}{2m}$$

This is the relationship between ν and $Q^2$ in an elastic scattering on a pointlike particle of mass m.

# Proof of x-Feynman = x-Bjorken

- Let's consider a reference system where the proton has a momentum so large that we can neglet all masses and all transverse momentum (infinite momentum frame).
- In this frame the proton quadrimomentum is:

$$p \equiv (E,\vec{p}) = (p,0,0,p)$$

- The proton is seen as a shower of partons, all of them with zero tranverse momentum with respect to the proton direction. Each of the parton carries a fraction x of the proton mass, momentum and energy.

$(xp+q)$ Parton quadrimomentum acquired after the scattering.

$(xp+q)^2 \approx m^2 \approx 0$ (Parton mass) $\Rightarrow x^2p^2 + 2xp\cdot q + q^2 = 0$

$p^2 = M^2 \approx 0$ Proton mass (that can be neglected)

$\Rightarrow x = -\dfrac{q^2}{2p\cdot q}$ p·q is a relativistic invariant and it can be computed in any frame, for instance in the Lab frame where the proton is at rest. $p = (M,0,0,0)$

While for the photon we have: $q = (\nu,\vec{q})$ where $\nu = E\text{-}E'$ is the energy transferred from the electron to the proton in the Laboratory reference system.

$$\Rightarrow p\cdot q = M\nu \quad \Rightarrow \quad \left[\nu = \frac{p\cdot q}{M}\right] \qquad \boxed{\Rightarrow x = -\frac{q^2}{2M\nu} = \frac{Q^2}{2M\nu}} \qquad \text{c.v.d.}$$

# Again Friedman

## If Partons are Quarks

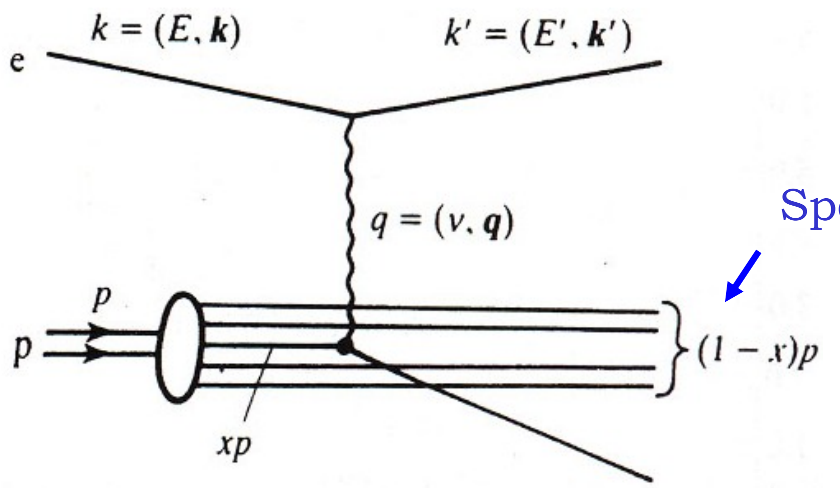
1) They must be spin 1/2 particles

2) They must have fractional charges consistent with the quark model

Do Partons have Fractional Charges (+2/3, -1/3)?

- Comparisons of Electron Scattering and Neutrino Scattering provided the answer.
- First neutrino results came from Large Heavy Liquid Bubble Chamber at CERN "Gargamelle" (1971-1974)

# Electron-parton scattering




Spectator partons

- Assuming that partons are pointlike particles of spin ½ , we can write down the differential cross-section of the eleastic scattering electron parton startining from the formula of the scattering e-μ:
- In the formula we must replace α with $\alpha e_i$, where $e_i$ is the fractional charge of the parton.

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4}\left[e_i^2 \cos^2\left(\frac{\theta}{2}\right) + e_i^2 \frac{Q^2}{2m_i^2}\sin^2\left(\frac{\theta}{2}\right)\right]\delta\left(\nu - \frac{Q^2}{2m_i}\right)$$

- If we compare this formula with the inelastic cross-section electron-proton (we recall: $m_i = xM$)

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4}\left[W_2(Q^2,\nu)\cos^2\left(\frac{\theta}{2}\right) + 2W_1(Q^2,\nu)\sin^2\left(\frac{\theta}{2}\right)\right]$$

$$\Rightarrow \boxed{W_1^i = e_i^2 \frac{Q^2}{4x^2M^2}\delta\left(\nu - \frac{Q^2}{2m_i}\right)} \quad ; \quad \boxed{W_2^i = e_i^2 \cdot \delta\left(\nu - \frac{Q^2}{2m_i}\right)}$$

# Callan-Gross relationship

- Partons that do not partecipate in the scattering behave as "spectators".
- The contributions of the single quarks to the inelastic differential cross-section are summed-up incoherently (there is no interference, we sum up the cross-sections and not the amplitudes).

- Every partons carries a fraction x of the proton quadrimomentum, where x can be different from parton to parton. Let's call $f_i(x)$ the probability that the parton of type i has the fraction x of the proton quadrimomentum (actually $f_i(x)$ is a density probability).

$$\Rightarrow W_1(Q_2,\nu) = \sum_i \int e_i^2 \frac{Q^2}{4M^2x^2} f_i(x)\delta\left(\nu - \frac{Q^2}{2Mx}\right)dx$$

$$\Rightarrow MW_1(Q_2,\nu) = \sum_i \frac{e_i^2}{2} f_i(x) \equiv F_1(x) \quad \left[x = Q^2/2M\nu\right]$$

- in the same way $$W_2(Q_2,\nu) = \sum_i \int e_i^2 f_i(x)\delta\left(\nu - \frac{Q^2}{2Mx}\right)dx$$

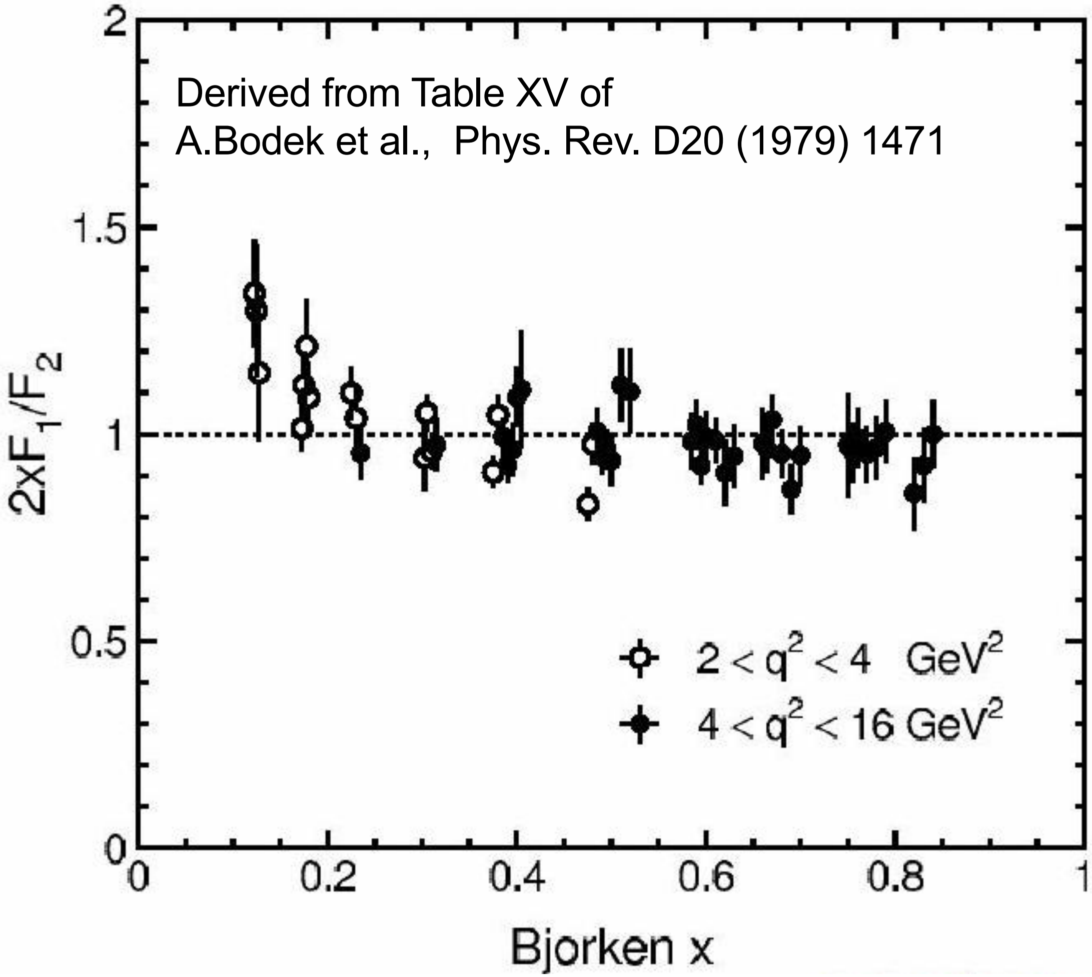
$$\Rightarrow \nu W_2(Q_2,\nu) = \sum_i e_i^2 x f_i(x) \equiv F_2(x)$$

- By comparing the two relations we have:

$$\boxed{2xF_1(x) = F_2(x)}$$

This is called Callan-Gross relationship and it is valid only for partons of spin ½ .
From its experimental evidence we deduce that the partons do have spin ½

# Experimental evidence of the C.G. relation




$$2xF_1(x) = F_2(x)$$

S=1/2

S=0

- Data confirm the existence inside the proton of poinlike particles of spin ½

- We remind you that the structure function $F_1$ is related to the magnetic interaction, so a particle of spin zero does not have any magnetic interaction.

# Quark structure of the nucleon

- The Bjorken's scaling can be summarized by these relations:

$$MW_1(Q_2,\nu) \to F_1(x) = \sum_i \frac{e_i^2}{2} f_i(x) \quad ; \quad \nu W_2(Q_2,\nu) \to F_2(x) = \sum_i e_i^2 x f_i(x)$$

- The hadron quantum numbers derive from their quark composition; however if we add quark-antiquark pair, the quantum numbers won't change. We distinguish valence quarks from sea quarks.
- By using the quantum numbers of u, d and s quarks, we can write down $F_2$, in the ep scattering, as:

$$F_2^{ep} = x\left\{\frac{4}{9}\left[u^p(x)+\bar{u}^p(x)\right]+\frac{1}{9}\left[d^p(x)+\bar{d}^p(x)\right]+\frac{1}{9}\left[s^p(x)+\bar{s}^p(x)\right]\right\}$$

$u^p(x)$ is the pdf (probability density function) of the quark u in the proton, that is the probability that the parton has a fraction of the proton momentum between x and x+dx

- For the electron-neutron scattering we can write down:

$$F_2^{en} = x\left\{\frac{4}{9}\left[u^n(x)+\bar{u}^n(x)\right]+\frac{1}{9}\left[d^n(x)+\bar{d}^n(x)\right]+\frac{1}{9}\left[s^n(x)+\bar{s}^n(x)\right]\right\}$$

- quarks u and d belong to an isospin doublet, therefore for the invariance of strong interaction for rotations in the isospin SU(2) space, we would expect that:

$$\begin{cases} u^p(x) = d^n(x) \equiv u(x) \\ d^p(x) = u^n(x) \equiv d(x) \\ s^p(x) = s^n(x) \equiv s(x) \end{cases}$$

Plus similar constraints for the other heavier $q\bar{q}$ pairs

# Valence quarks and sea quarks

- For every quark we can write down this expression: $q(x) = q_v(x) + q_s(x)$ → Sea; $q_v$ → valence

- Since the proton valence quarks are u and d, we must have: $q_v(x) = 0$ for $s, \bar{s}, \bar{u}, \bar{d}$
  therefore in the proton the quark s (and the other heavier quarks) and the antiquarks, must belong to the sea, while for the quark u and d we have:

$$u(x) = u_v(x) + u_s(x) \quad ; \quad d(x) = d_v(x) + d_s(x)$$

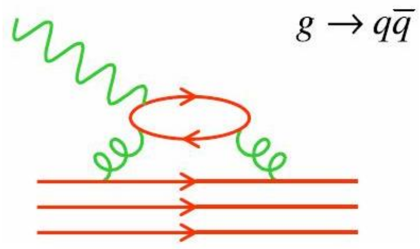
- If we do a simplified assumption that the three light quarks appear in the sea with the same probabiity and the same momentum distribution, we have:

$$u_s(x) = d_s(x) = s_s(x) = \bar{u}_s(x) = \bar{d}_s(x) \equiv s(x)$$

- With this parametrization the proton and neutron structure functions become:

$$F_2^{ep} = \frac{x}{9}\left[4 \cdot u_v(x) + d_v(x)\right] + \frac{4}{3} x \cdot s(x) \quad ; \quad F_2^{en} = \frac{x}{9}\left[u_v(x) + 4 \cdot d_v(x)\right] + \frac{4}{3} x \cdot s(x)$$

N.B. The sea quarks are originated from the quark-quark strong interactions mediated by gluons exchange. From time to time the virtual gluons produce virtual quark-antiquark pairs and the photon will interact with the virtual quarks.

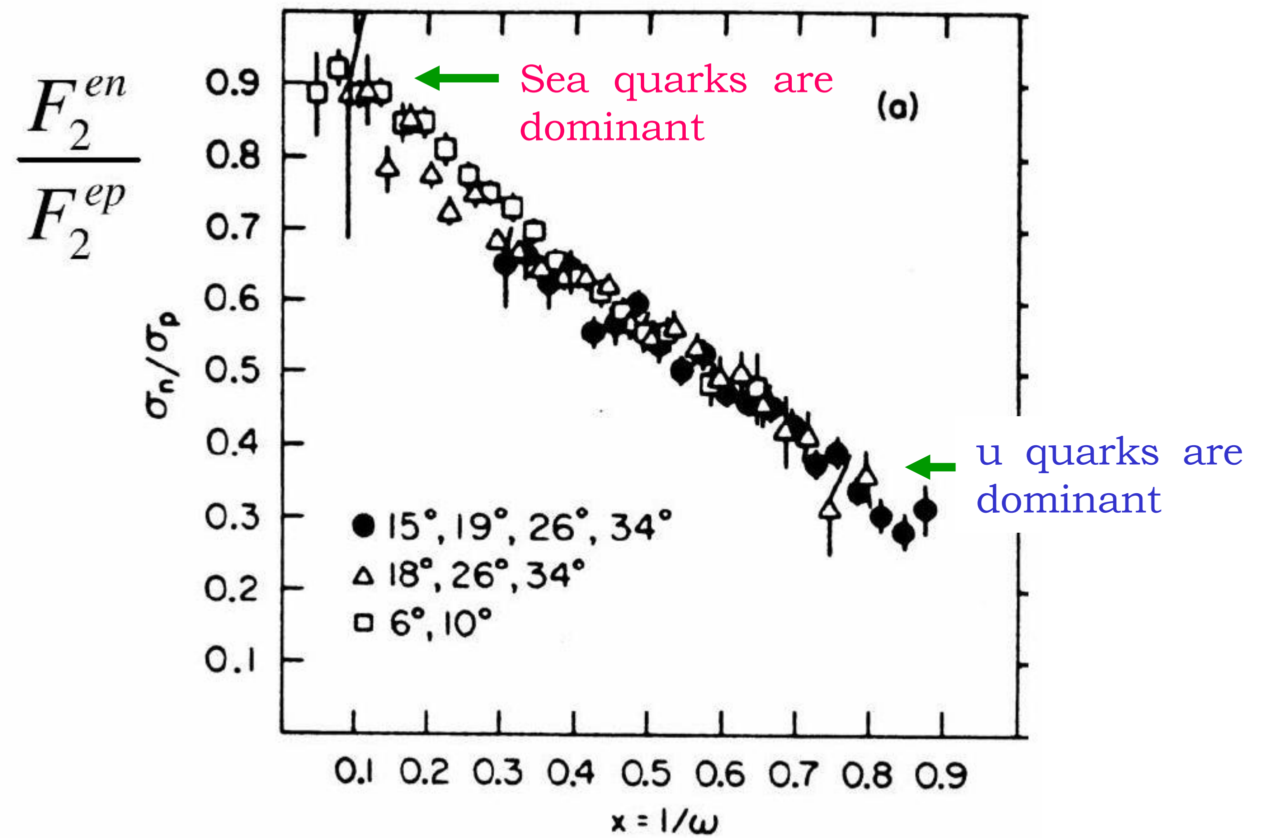


# Experimental evidence of the sea quarks

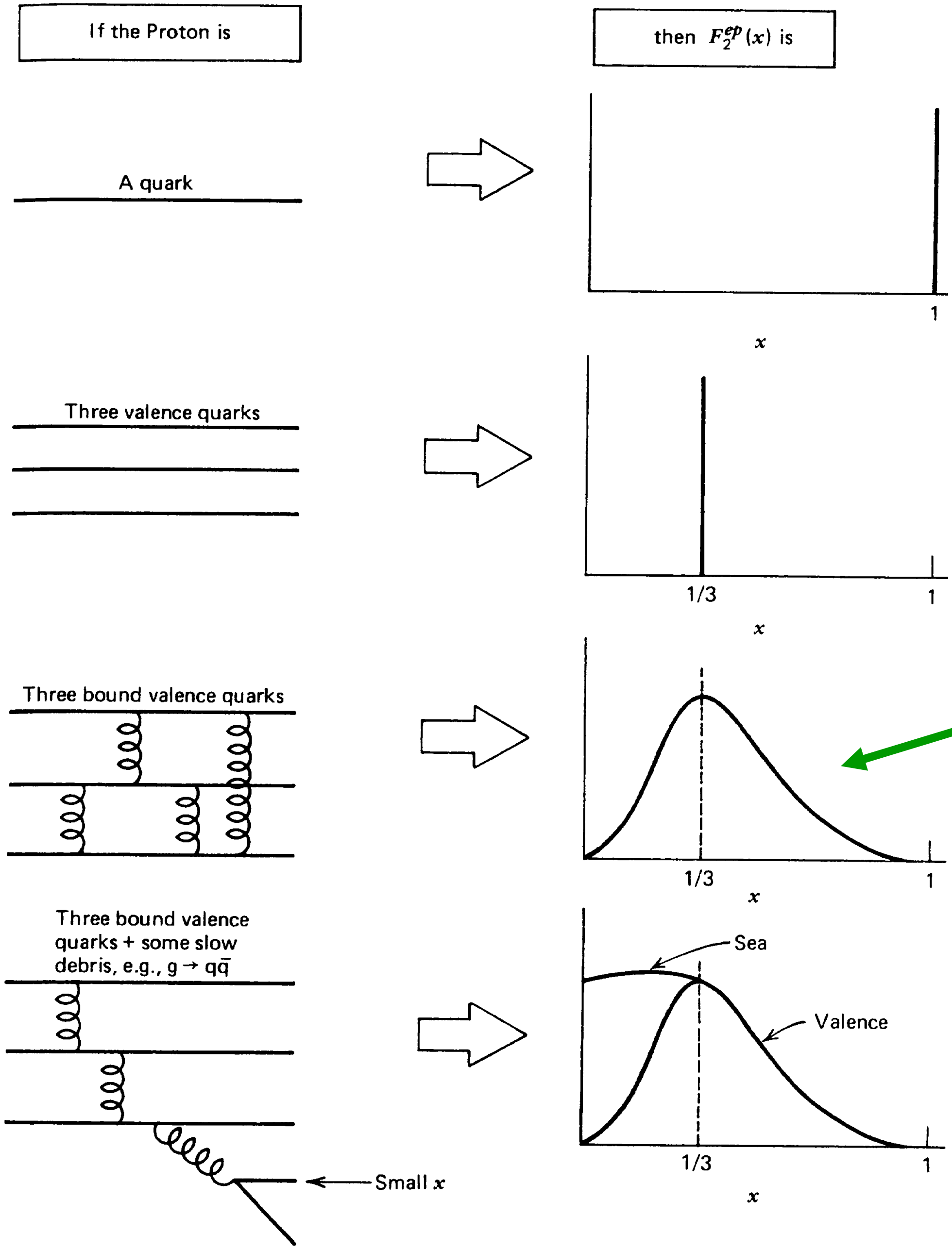
- Let's evaluate the ratio between the proton and neutron structure functions as a function of x.
- Let's consider two extreme cases:

a) Sea quarks are dominant; it should happen at small values of x: $\dfrac{F_2^{en}(x)}{F_2^{ep}(x)} \rightarrow 1$

b) Let's consider the case where $u_v$ is dominant (we recall that $u_v$ is the pdf in the proton and the d pdf in the neutron $\dfrac{F_2^{en}(x)}{F_2^{ep}(x)} \rightarrow \dfrac{u_v + 4d_v}{4u_v + d_v} \rightarrow \dfrac{1}{4}$



# $F_2$ for various proton compositions



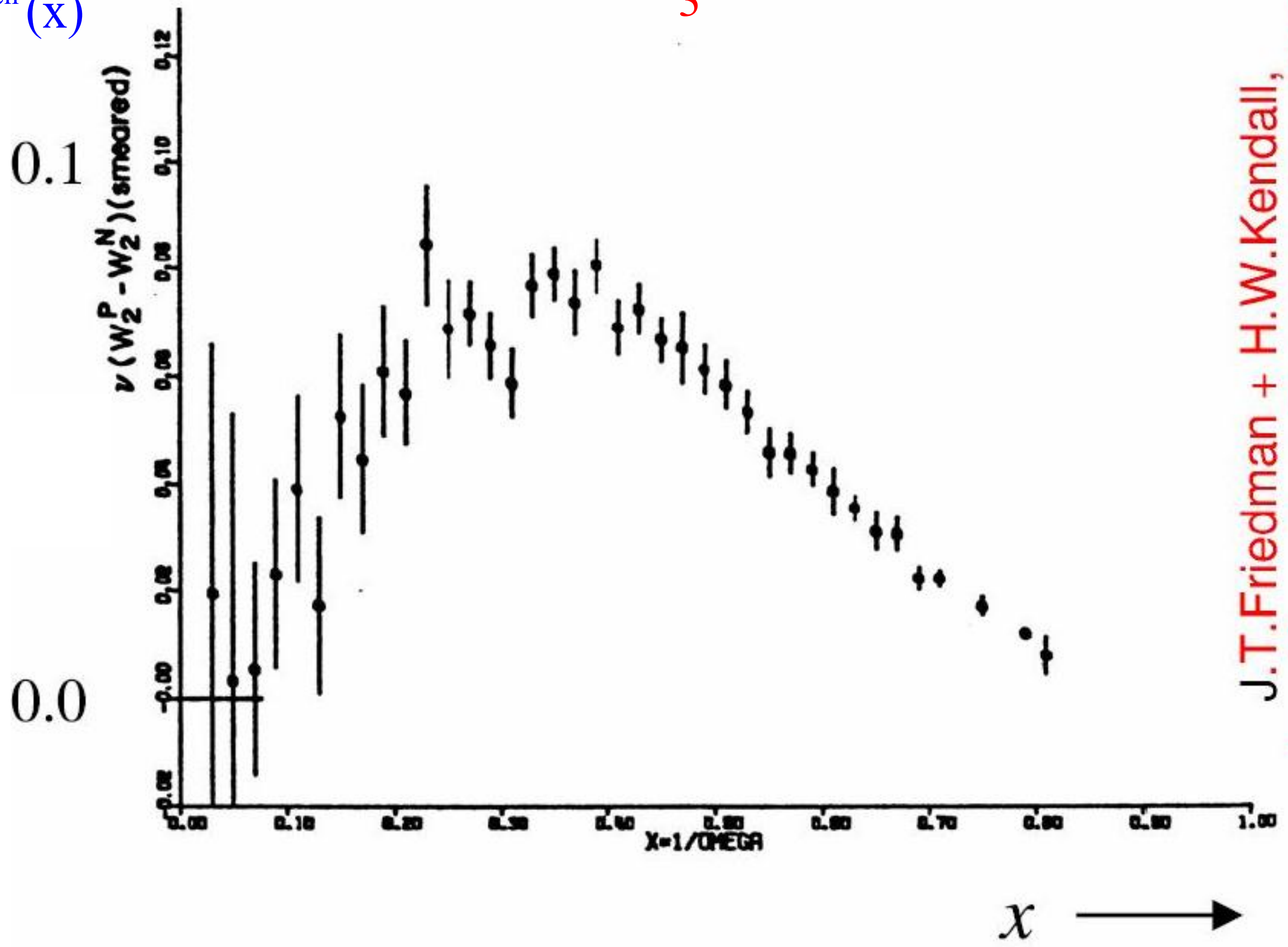

The interactions between quarks smear the momentum among them

# Experimental check of $F_2$ form

- Let's recall that: $F_2^{ep} = \frac{x}{9}\left[4 \cdot u_v(x) + d_v(x)\right] + \frac{4}{3} x \cdot s(x)$ ; $F_2^{en} = \frac{x}{9}\left[u_v(x) + 4 \cdot d_v(x)\right] + \frac{4}{3} x \cdot s(x)$
- Subtracting the two expressions we get rid of the sea quark contribution:

$$F_2^{ep}(x) - F_2^{en}(x) = \frac{x}{3}\left[u_v(x) - d_v(x)\right]$$

$F_2^{ep}(x) - F_2^{en}(x)$




J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. **22** (1972) 203

# Integrals of the structure functions

- Let's recall the form of $F_2$:

$$F_2^{ep} = x\left\{\frac{4}{9}\left[u^p(x)+\bar{u}^p(x)\right]+\frac{1}{9}\left[d^p(x)+\bar{d}^p(x)\right]+\frac{1}{9}\left[s^p(x)+\bar{s}^p(x)\right]\right\}$$

$$F_2^{en} = x\left\{\frac{4}{9}\left[u^n(x)+\bar{u}^n(x)\right]+\frac{1}{9}\left[d^n(x)+\bar{d}^n(x)\right]+\frac{1}{9}\left[s^n(x)+\bar{s}^n(x)\right]\right\}$$

- It is x·f(x), therefore from its integral we can deduce the fraction of proton momentum carried by the charged particles inside the proton. Let's neglet in the calculation the quark s.

$$\begin{cases} \int_0^1 F_2^{ep}dx = \frac{4}{9}\int_0^1 x(u+\bar{u})dx + \frac{1}{9}\int_0^1 x(d+\bar{d})dx \equiv \frac{4}{9}f_u + \frac{1}{9}f_d & \text{(proton)} \\ \int_0^1 F_2^{en}dx = \frac{4}{9}\int_0^1 x(d+\bar{d})dx + \frac{1}{9}\int_0^1 x(u+\bar{u})dx \equiv \frac{4}{9}f_d + \frac{1}{9}f_u & \text{(neutron)} \end{cases}$$

$$\begin{cases} f_u = \int_0^1 x(u+\bar{u})dx \\ f_d = \int_0^1 x(d+\bar{d})dx \end{cases}$$

$f_u$ = fraction of the proton momentum carried by all quarks u and $\bar{u}$ ; the same is true for $f_d$

- The experimental measurements give:

$$\left.\begin{aligned} \int_0^1 F_2^{ep}dx &= \frac{4}{9}f_u + \frac{1}{9}f_d \approx 0.18 \\ \int_0^1 F_2^{en}dx &= \frac{4}{9}f_d + \frac{1}{9}f_u \approx 0.12 \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} f_u &\approx 0.36 \\ f_d &\approx 0.18 \end{aligned}}$$

Only 50% of the proton momentum is carried by quarks and antiquarks; the rest is carried by the gluons.

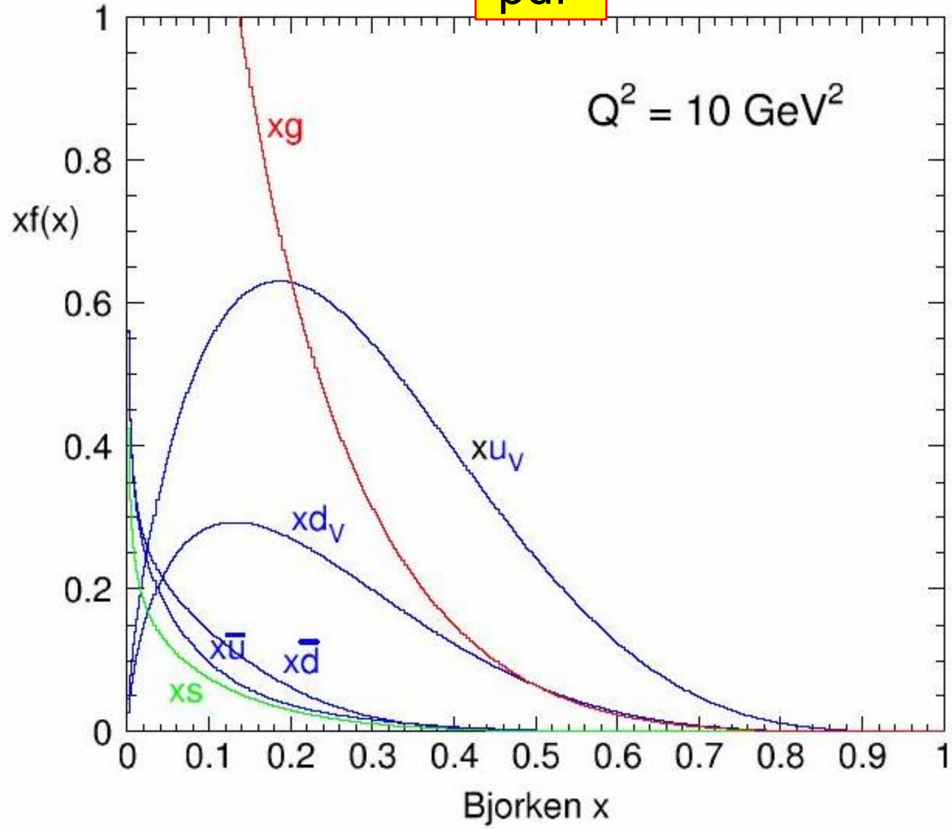
# Sum rules

- The quark probability density functions (pdf) must satisfy some sum rules. For instance in the proton we have:

$$\begin{cases} \int_0^1 \left[u(x)-\bar{u}(x)\right]dx = 2 & \longleftarrow \text{ 2 u valence quarks} \\ \int_0^1 \left[d(x)-\bar{d}(x)\right]dx = 1 & \longleftarrow \text{ 1 d valence quark} \\ \int_0^1 \left[s(x)-\bar{s}(x)\right]dx = 0 & \longleftarrow \text{ No strangeness} \end{cases}$$

- In order to confirm these sum rules , we need to analyse neutrinos and antineutrinos scattering, that happens through the weak interaction mediated by a W exchange. This allows to distinguish between quarks and antiquarks.

pdf



# Neutrino nucleon-interactions

- Now we just give a few formulas about the neutrino (antineutrino) nucleon scattering.
- We consider only the charged current interactions (W exchange); the charge and leptonic number conservation allow these reactions:

$$\begin{cases} \nu_\mu d \to \mu^- u \quad ; \quad \nu_\mu \bar{u} \to \mu^- \bar{\mathrm{d}} \\ \bar{\nu}_\mu u \to \mu^+ \mathrm{d} \quad ; \quad \bar{\nu}_\mu \bar{d} \to \mu^+ \bar{\mathrm{u}} \end{cases}$$

- Let's recall the naming conventions of the pdf inside the proton and neutron

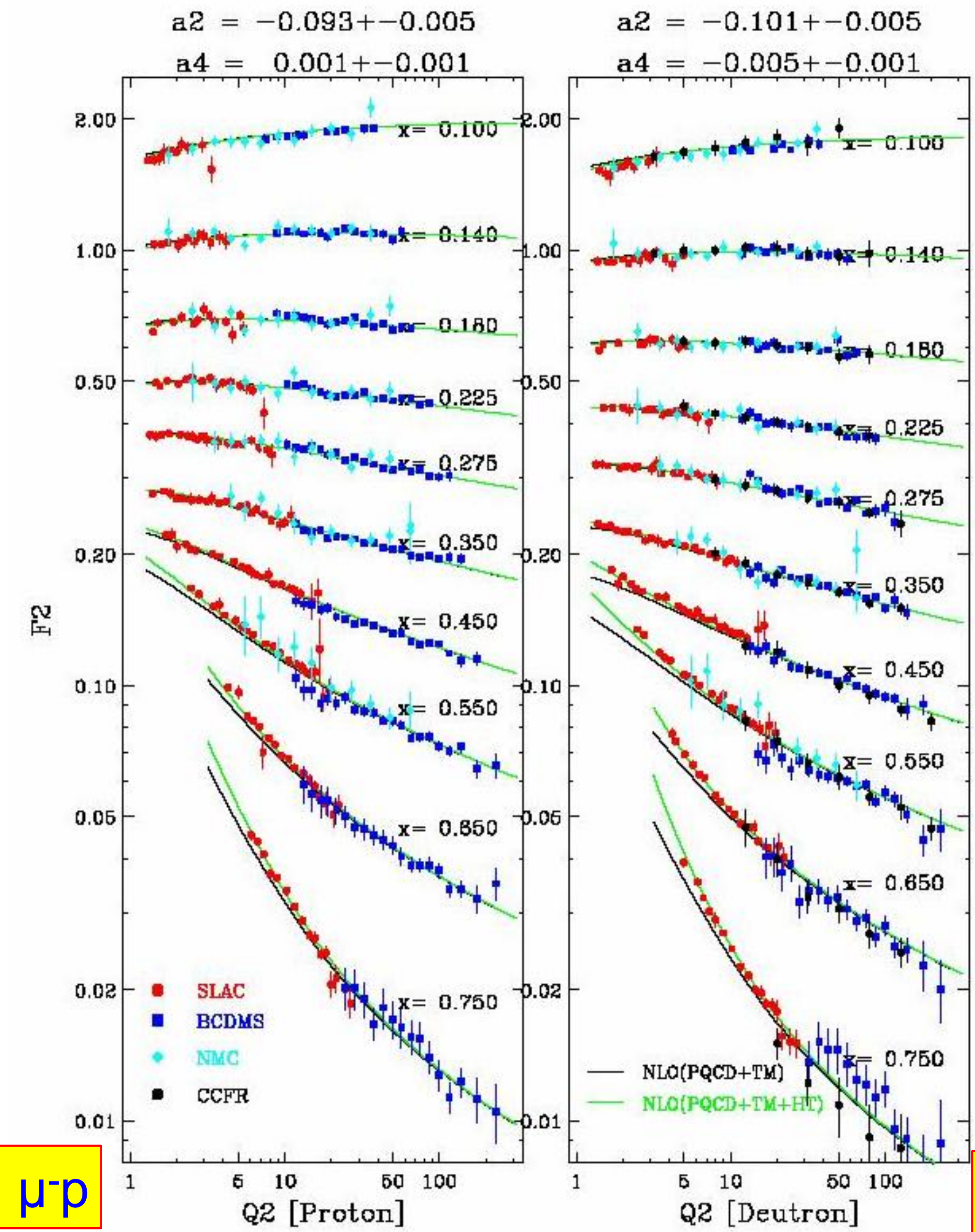
$$\begin{cases} \mathrm{u^p(x) = d^n(x) \equiv u(x)} \\ \mathrm{d^p(x) = u^n(x) \equiv d(x)} \\ \mathrm{s^p(x) = s^n(x) \equiv s(x)} \end{cases}$$

- For the sake of simplicity we use isoscalar target, that is with the same number of protons and neutrons. Keep in mind that neutrino interacts with d and $\bar{\mathrm{u}}$ while the antineutrino with u and $\bar{\mathrm{d}}$

$$\begin{cases} \mathrm{d^p(x) + d^n(x) = d(x) + u(x) \equiv Q(x)} \\ \mathrm{\bar{u}^p(x) + \bar{u}^n(x) = \bar{u}(x) + \bar{d}(x) \equiv \bar{Q}(x)} \end{cases}$$

- In the neutrino (antineutrino) interaction intervene a third structure function called F3, due to the interference between vectorial and axial currents.

# Scaling violation: $F_2(x, Q^2)$

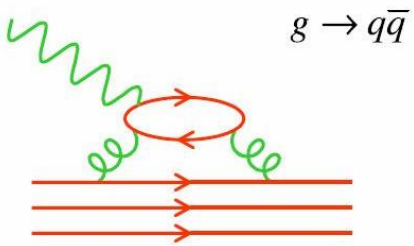



$e^- p$, $\mu^- p$

$e^- d$, $\mu^- d$, $(\nu d)$

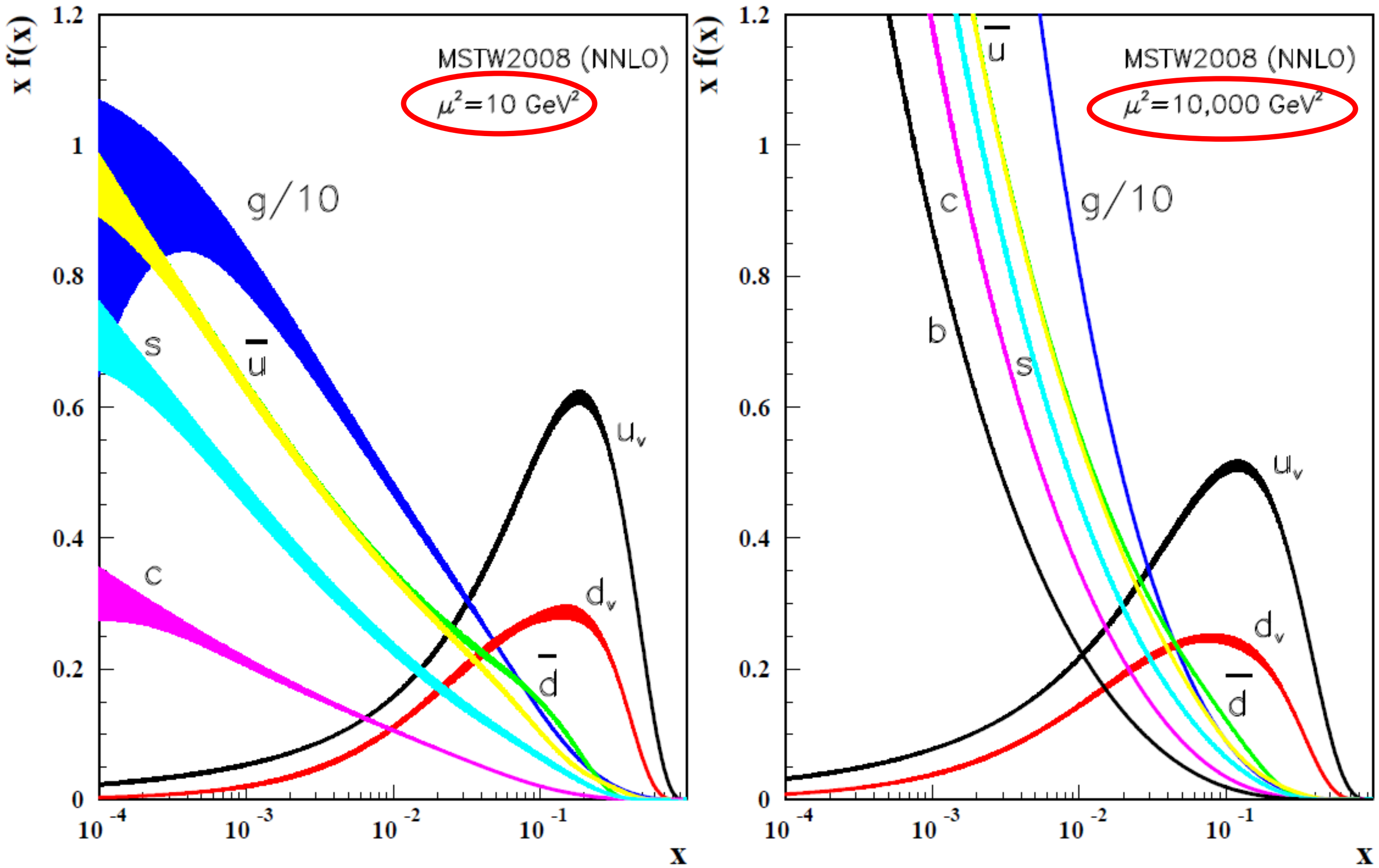
At high x $F_2$ is not flat any longer in $Q^2$, but it becomes a function of Q2 as well.

This is not due to a further scale inside the quarks, but it is a pure QCD effect due to the gluon splitting.



# DGLAP evolution equation

- DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) are the authors who first wrote the QCD evolution equation.
- QCD Evolution Equations for Parton Densities valid in the theory of the strong interactions , determine the rate of change of parton densities (probability densities to find a quark or a gluon in the proton) when the energy scale chosen for their definition is varied.



# End of chapter 6