Introduction to Particle Physics
- Chapter 4 -
Hadrons and the quark model

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last update : 070117

Chapter summary:

- Classification of the particles
- The flavour SU(3) symmetry
- Quarks
- Graphical construction of mesons and barions
- Mixing of the mesons with I_3 =0 and Y=0
- Quark composition of mesons and baryons
- OZI rule
- quark mass

Particle classification

- In the '50 were discovered new particles and resonances that were also considered as new particles.

- People tried to classify all these particles in a way to unveil their true nature (a similar work was done by Rydberg who found the formula describing the atomic spectra, or by Mendeleiev)

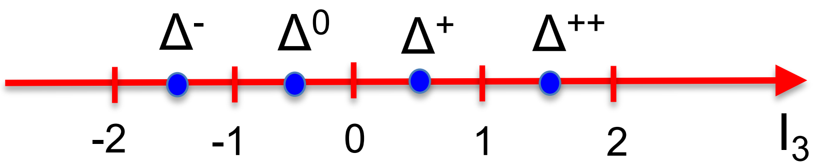
- A first symmetry encountered were associated to the isotopic spin (isospin); particles with the same isospin are exactly the same particle with respect to strong interactions, but the e.m. interactions break the symmetry and induce a mass difference of a few % among the particles of the same multiplet.

Particle classification

- To extend the symmetry it was tried to assemble several isospin multiplets in a larger group having the same spin and parity but with different strangeness (or hypercharge).

- There are a priori other possible choices, for instance same strangeness with different spin and parity, but all these ones did not work out well.

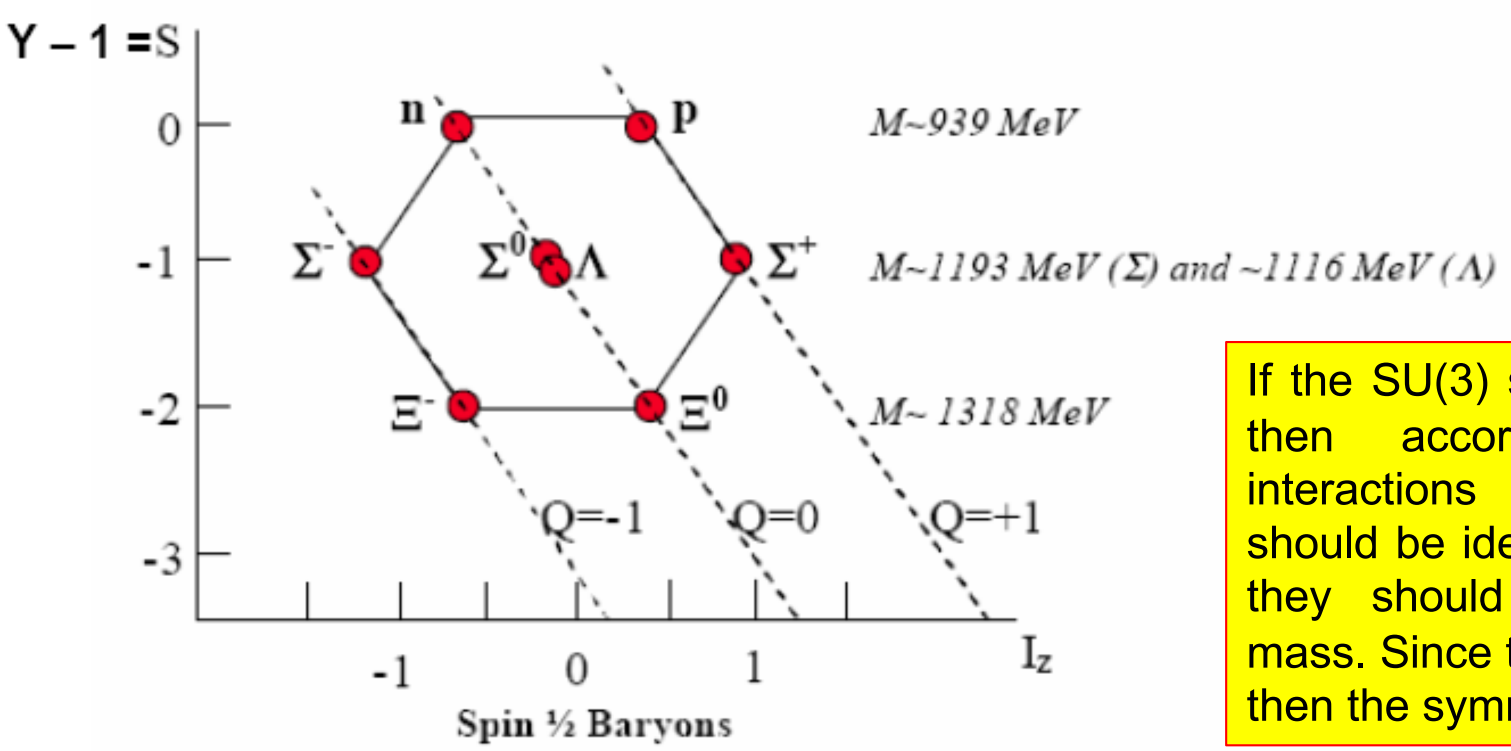
- The components of an isospin multiplet are represented as points on the horizontal axis I_3 whose position differ by one unit of isospin. For instance for the $\Delta(1232)$ we have:



$$Q = I_3 + \frac{1}{2}(B+S)$$

Baryons $(1/2)^+$

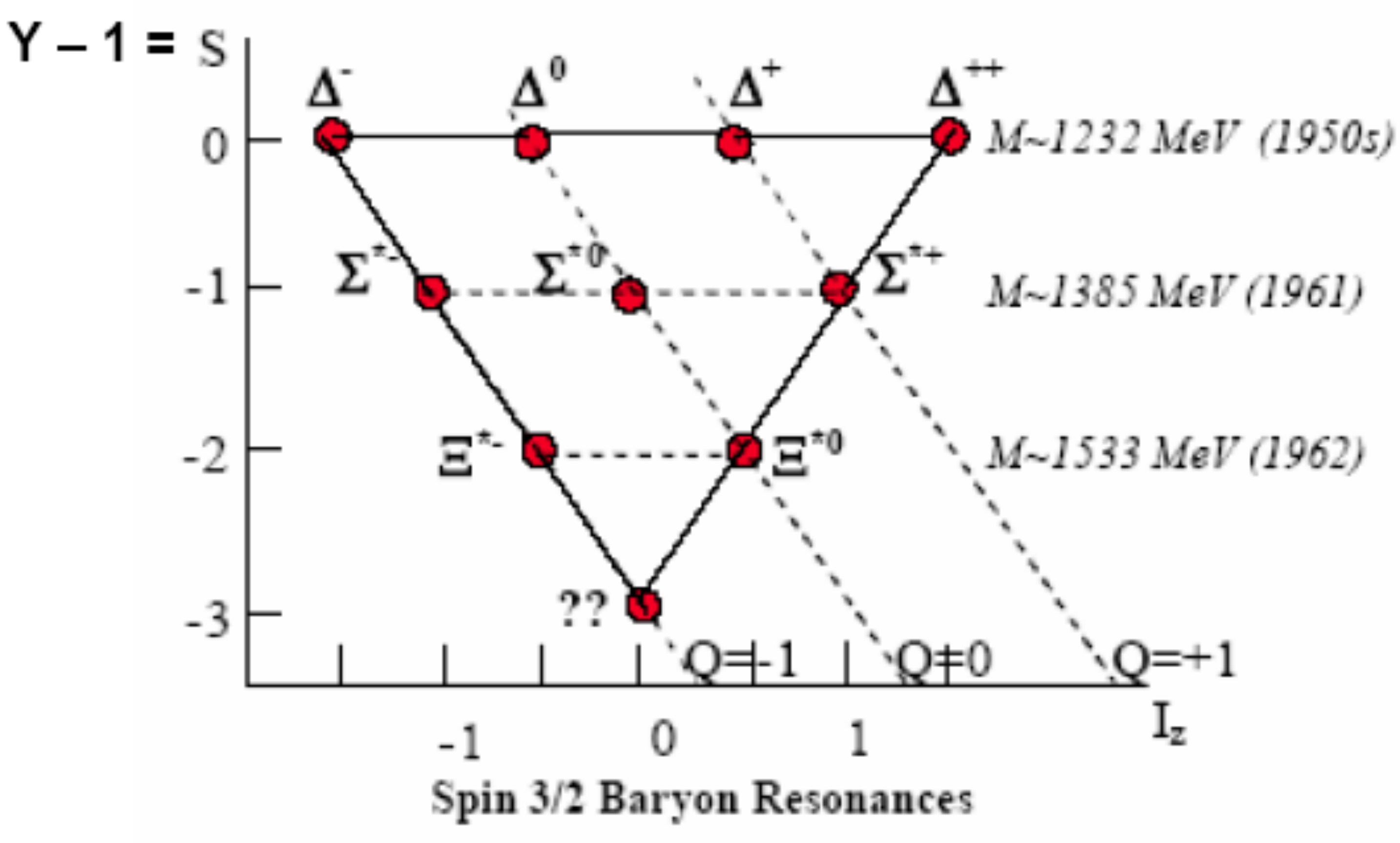
- There were 8 baryons of spin ½ and parity + already known when the eightfold way classification was proposed (1961: Gell-Mann and Ne'emann)



If the SU(3) symmetry is exact, then according to strong interactions the 8 particles should be identical, for instance they should have the same mass. Since this is not the case, then the symmetry is "broken".

N.B. the antibaryons fill up another SU(3) octet: $\overline{8}$

Baryons $(3/2)^+$



$\Delta m \approx 150$ MeV

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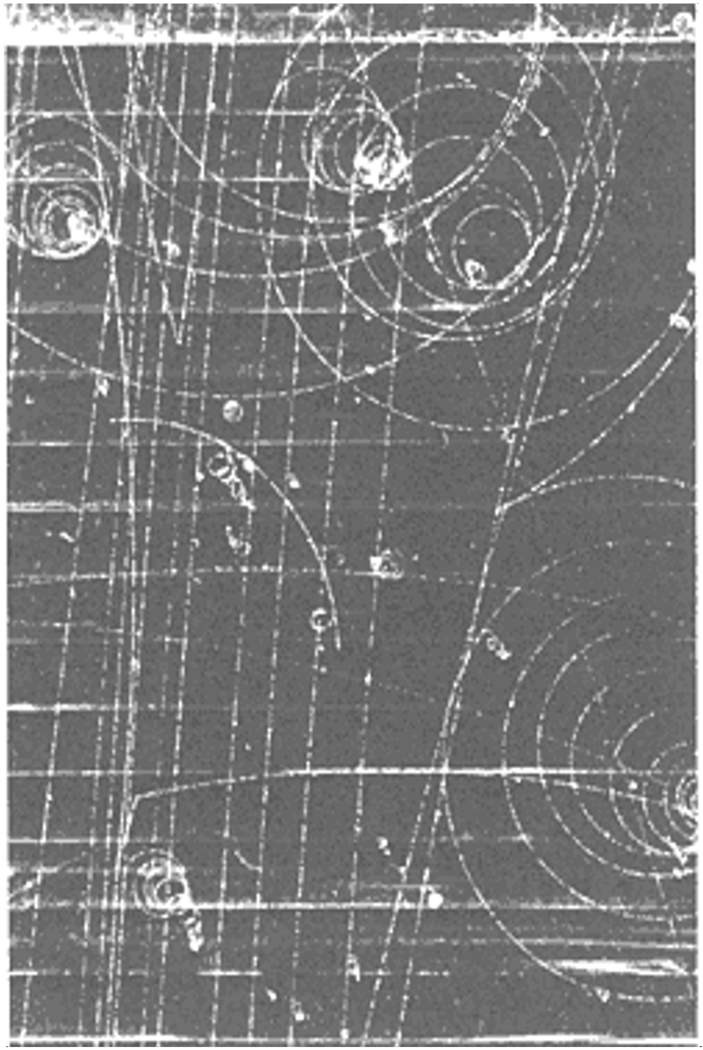
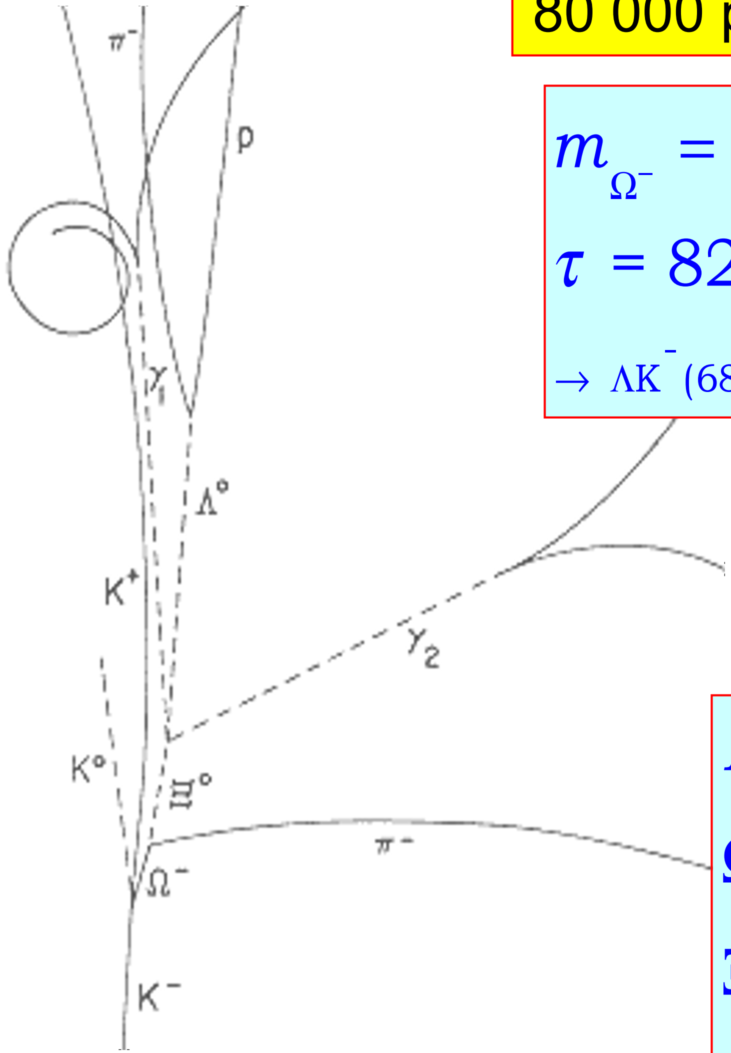
When the model was formulated the Ω^- was not discovered yet. Gell-Mann predicted the existence of a particle of strangeness equal to -3, electric charge -1, that decays weakly and with a mass about 1680 MeV. This particle was discovered in 1964 by Samios at the AGS.

February 1964: the discovery of the Ω^-

Bubble chamber at AGS; 80 000 pictures





$m_{\Omega^-} = 1672.45$ MeV

$\tau = 82$ ps

$\rightarrow \Lambda K^-$ (68%), $\Xi^0 \pi^-$ (24%), $\Xi^- \pi^0$ (9%)

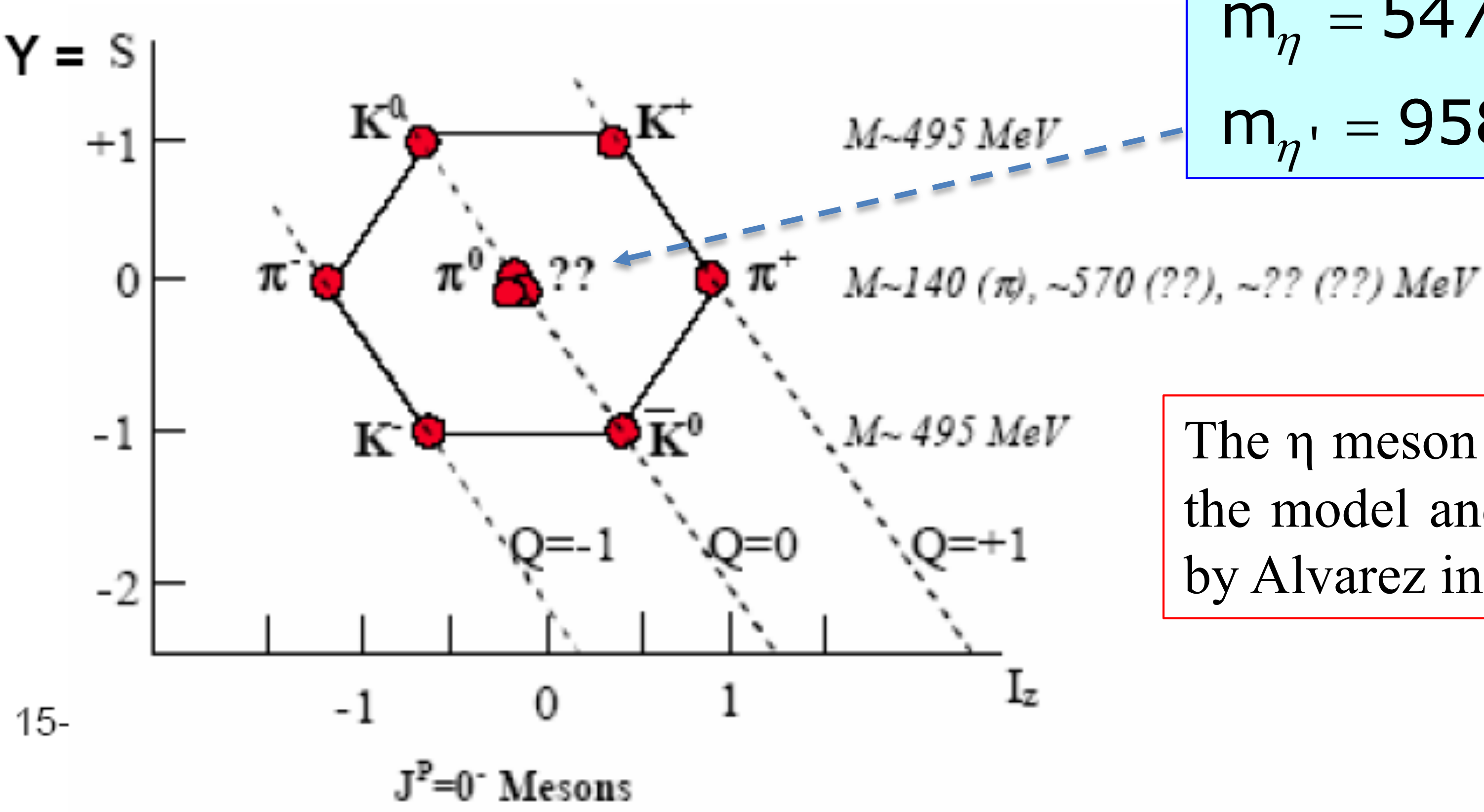
$K^- p \rightarrow \Omega^- K^+ K^0$

$\Omega^- \rightarrow \Xi^0 \pi^-$

$\Xi^0 \rightarrow \Lambda \pi^0$

$\Lambda \rightarrow p \pi^-$

Mesons 0^- (pseudoscalars)



$m_\eta = 547.7$ MeV

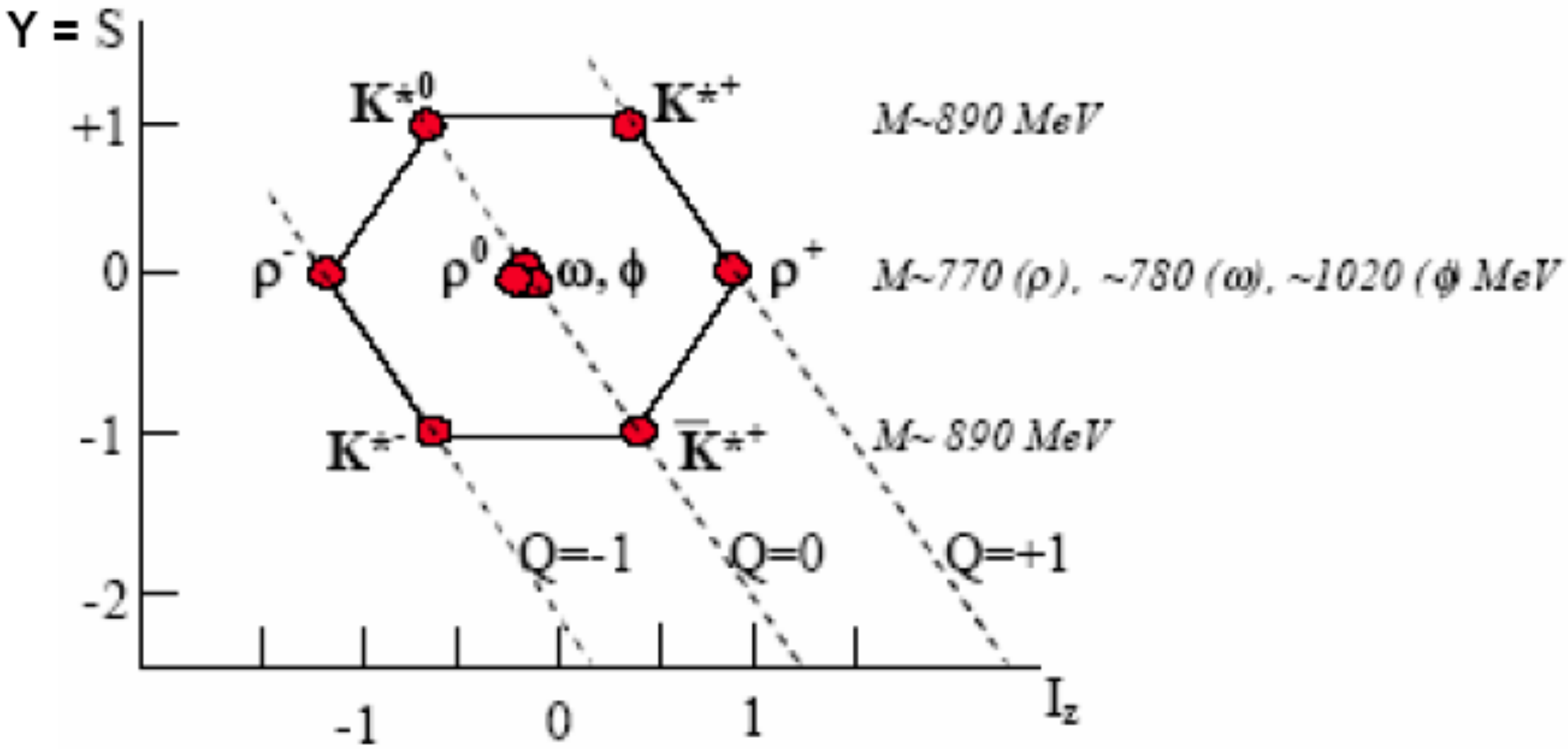
$m_{\eta'} = 958$ MeV

The η meson was predicted by the model and was discovered by Alvarez in 1961.

N.B. Particles and antiparticles appear in the same multiplet because mesons they have B=0.

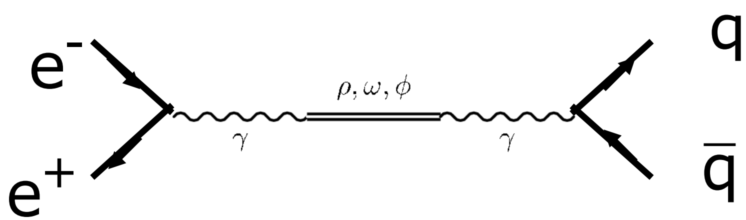
There are 9 particles in every multiplets, however the irreducible representations of SU(3) are 8+1, then one of the three particles having Y=0 and I_3=0 belongs to the SU(3) singlet. Actually there is a mixing between the singlet and the state of the octet with I=0, I_3=0 and Y=0.

Vector meson 1^-



N.B. ρ^0, ω e φ have the same quantum numbers of the photon.

→ Vector Dominance Model to explain hadronic interactions (1960)



Mass formula of Gell-mann - Okubo

- In order to explain the mass splitting between the states with different strangeness, Gell-Mann and Okubo proposed that the strong Hamiltonian could be divided in a symmetric part H_0 plus a part H' "semi strong" that broke the SU(3) symmetry.

- In this way they found some empirical mass formulae to explain the mass splitting.

- Nowadays these relations are seen as empirical formulae without any "physical" meaning.

Mass formula of Gell-mann - Okubo

- Baryons:

$$m = m_0 + m_1 Y + m_2 \left[I(I+1) - \frac{1}{4} Y^2 \right]$$

Example: in the decuplet we have Y = B+S = 2(I-1)

$$m = (m_0 + 2m_2) + Y \left(m_1 + \frac{3}{2} m_2 \right) \Rightarrow \Delta m = \text{costant} \approx 150 \text{ MeV (experimental)}$$

Example: in the octet ½ + we have:

$$2m_\Lambda + 2m_{\Xi^0} = m_{\Sigma^0} + 3m_\Lambda$$

$$\downarrow \qquad \qquad \downarrow$$

$$4515 \text{ MeV} \qquad 4539 \text{ MeV}$$

- Mesons:

$$m^2 = m_0^2 + m_1^2 Y + m_2^2 \left[I(I+1) - \frac{1}{4} Y^2 \right]$$

In the case of mesons we need to consider the square of the masses. The agreement with experimental data is worsened by the mixing between the SU(3) singlet and the singlet of the octet.

Example:

$$2m_{K^0}^2 + 2m_{K^0}^2 = 4m_{K^0}^2 = m_{\pi^0}^2 + 3m_\eta^2$$

$$\downarrow \qquad \qquad \downarrow$$

$$0.988 \text{ GeV}^2 \qquad 0.924 \text{ GeV}^2$$

Mass of the particles

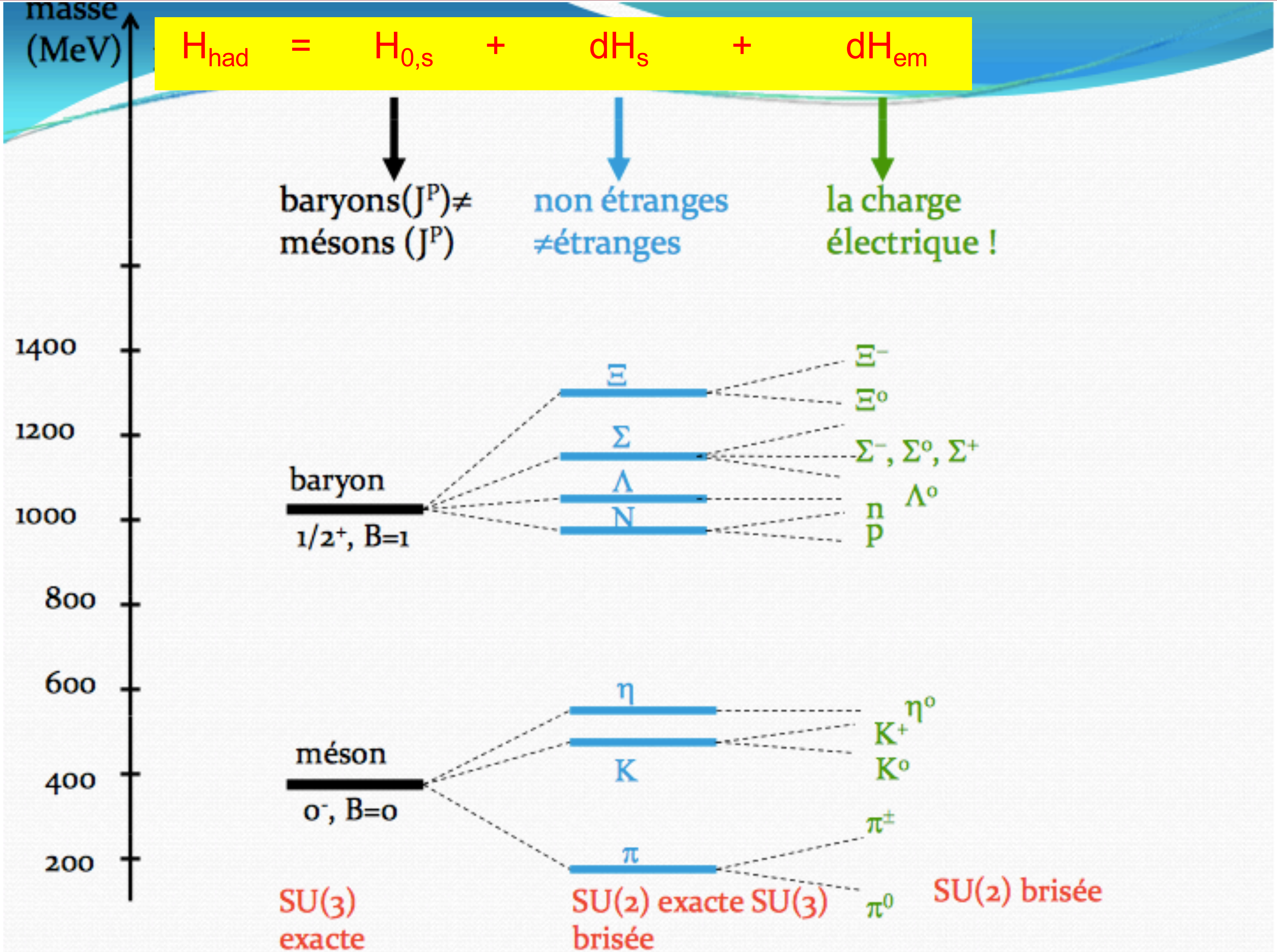
A SU(3) symmetry «exact» implies that all particles in a multplet must have the same mass…

π^-	0^-	140 MeV
π^0	0^-	135
$K^\pm$	0^-	494
K^0, $\bar{K}^0$	0^-	498
η	0^-	549
η'	0^-	958
$\rho^\pm$, ρ^0	1^-	770
ω	1^-	783
K^*	1^-	892
ϕ	1^-	1020

p	$½^+$	938 MeV
n	$½^+$	940
Λ	$½^+$	1160
Σ^+	$½^+$	1189
Σ^0	$½^+$	1192
Σ^-	$½^+$	1197
Ξ^0	$½^+$	1315
Ξ^-	$½^+$	1321
Ω	$3/2^+$	1672

… but this not the case!

Mass of the particles



quarks

Question: why $3 \otimes \bar{3}$ and $3 \otimes 3 \otimes 3$?

- In 1964 Gell-Mann, and independently Zweig, associated at every eigenstate of the fundamental triplet of SU(3) an elementary particle that he called quark (Zweig called these particles aces).

$$\underset{\text{up}}{\begin{pmatrix}1\\0\\0\end{pmatrix}} \quad ; \quad \underset{\text{down}}{\begin{pmatrix}0\\1\\0\end{pmatrix}} \quad ; \quad \underset{\text{strange}}{\begin{pmatrix}0\\0\\1\end{pmatrix}}$$

- Quarks are fermions of spin ½ .
- In this way we have:

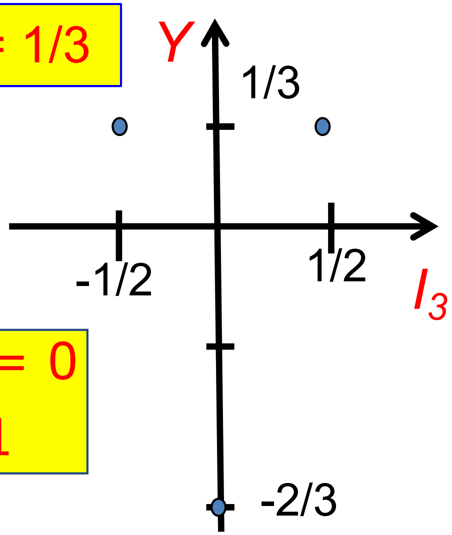
Baryons: $3 \otimes 3 \otimes 3 \Rightarrow qqq$ (they are composed by 3 quark)

Mesons: $3 \otimes \bar{3} \Rightarrow q\bar{q}$ (they are composed by a quark and an antiquark)

Quark quantum numbers

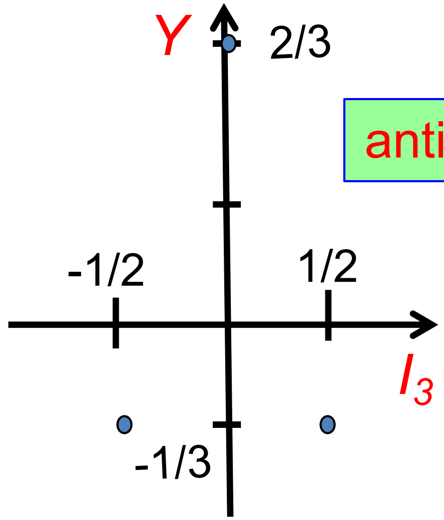
- We can find them by applying the operators I_3 and Y to the triplets:

quark: B= 1/3



S (u, d) = 0
S(s) = -1

anti-quark: B=-1/3



$S(\bar{u}, \bar{d}) = 0$
$S(\bar{s}) = 1$

- By using the formula $Q = I_3 + \frac{Y}{2}$ we can find the charge:

$$Q_u = \frac{2}{3} \;;\; Q_d = Q_s = -\frac{1}{3} \qquad ; \qquad Q_{\bar{u}} = -\frac{2}{3} \;;\; Q_{\bar{d}} = Q_{\bar{s}} = \frac{1}{3}$$

Quarks are just a mathematical “trick” or are they real particles?

Mesons 0^-

- Mesons are made of a quark-antiquark pair.

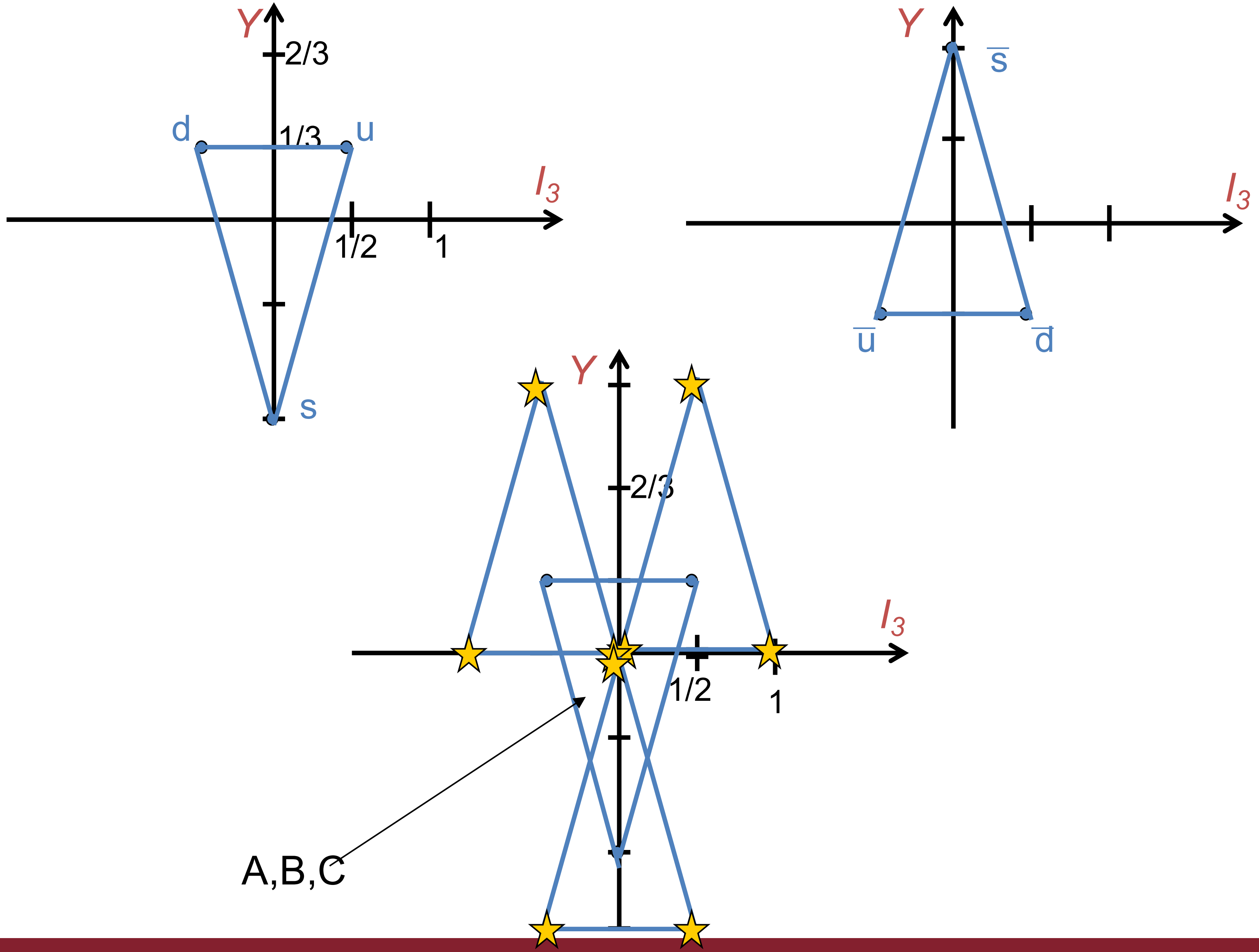
- Let's consider the ones in the S-state (L=0, lowest energy) and with opposite spin (S=0), then J=0 and parity P=-1.
  (reminder: the parity operator changes sign to the coordinates; the spatial part of the wave function goes like $(-1)^L$ while the spin part is not touched by the parity operation. Moreover fermions and antifermions have opposite intrinsic parity, hence the mesons have parity $(-1)^{L+1}$.

- Mesons can be constructed with a graphical method. For instance in the case of isospin we have:

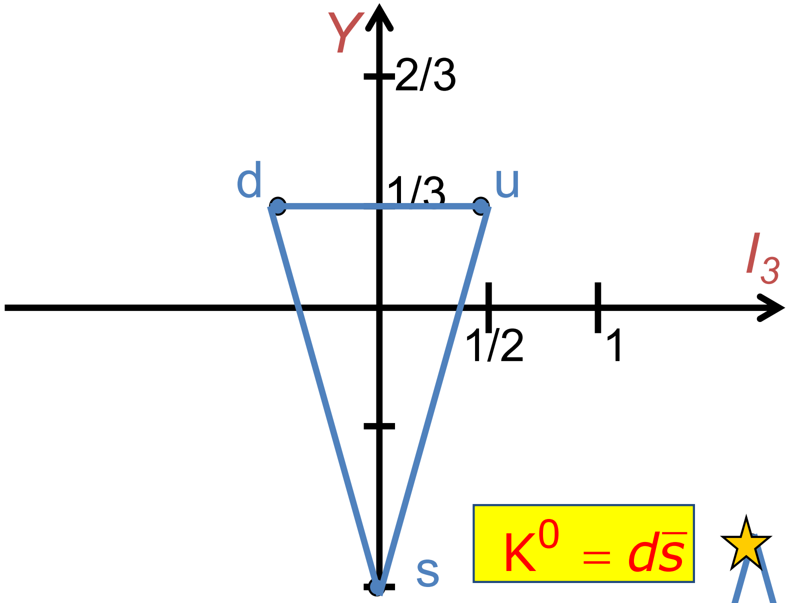
I_3 is an additive quantum number

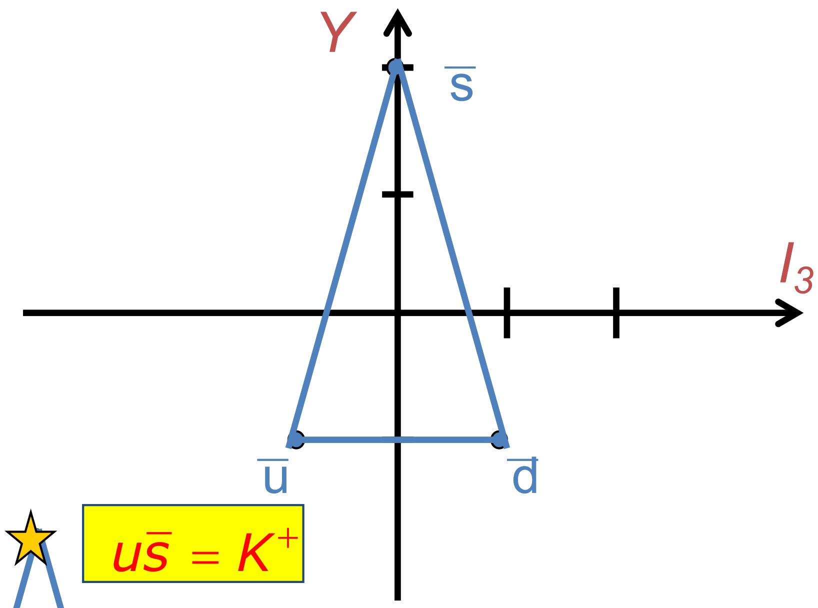
This diagram is obtained by superimposing the center of gravity of the antiquark multiplet in each place where is present a quark .

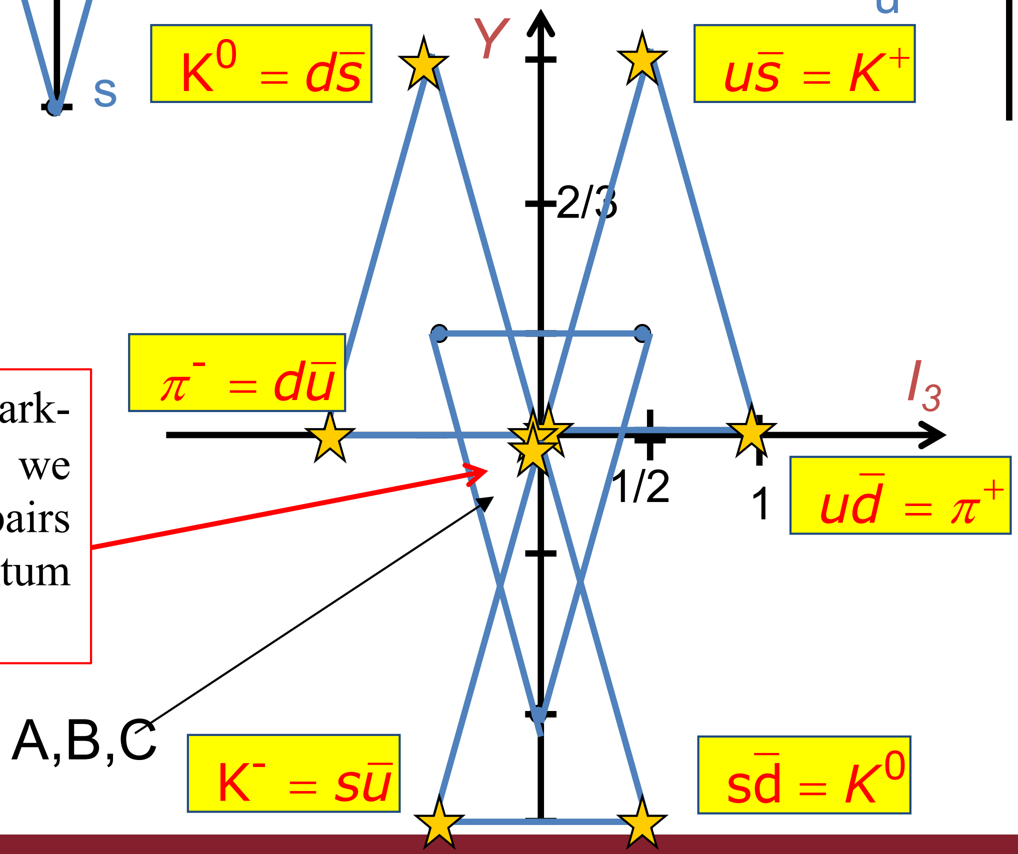
Graphical construction of the mesons 0^-



Graphical construction of the mesons 0^-







To which quark-antiquark pair do we identify the three pairs that have quantum numbers 0,0?

$$3 \otimes \bar{3} = 1 \oplus 8$$

Mesons 0^-

- The three states A, B, C with I_3=0 and Y=0 are orthogonal linear combinations of the states $u\bar{u} + d\bar{d} + s\bar{s}$

- Let's identify a state with $\{n, |I, I_3\rangle\}$ where n is the dimension of the representation.

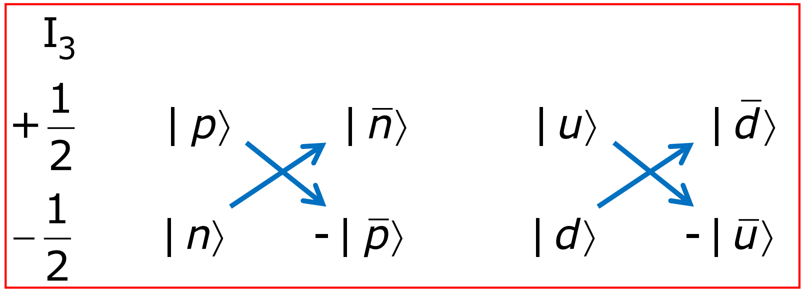
- The SU(3) singlet must contain, because of the symmetry, all three states with the same weight (a rotation in the SU(3) space must not change the state):

$$\eta_1 = \{1, |0,0\rangle\} = \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

- One of the other two states with I_3=0 must belong to the triplet with isospin=1, therefore it can be obtained with the ladder operators, in this case with the operators of lowering or raising the charge.

Charge conjugation of the nucleons

- Let's remind the behaviour of some nucleons through the charge conjugation operation. They appear a few "minus" signs in according to the Condon-Shortley convention.



$$\begin{pmatrix} u \\ d \end{pmatrix} \text{ e } \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix} \Rightarrow \begin{cases} I^- |\bar{d}\rangle = |-\bar{u}\rangle \\ I^+ |\bar{u}\rangle = |-\bar{d}\rangle \end{cases}$$

$I^{\pm}$: operator of isospin shift (raising and lowering of the charge)

- N.B. the quark s is an isospin singlet, therefore when we add it to an isospin doublet, it will not change the doublet properties:

$$\begin{pmatrix} u\bar{s} = K^+ \\ d\bar{s} = K^0 \end{pmatrix} \text{ e } \begin{pmatrix} s\bar{d} = \bar{K}^0 \\ -s\bar{u} = -K^- \end{pmatrix}$$

- Combining d with ū (or viceversa) we can have I=0 or I=1

Wave function of the π^0

- Let's apply the isospin shift operator that has the following property:

$$I^{\pm} | \Psi(I, I_3)\rangle = \sqrt{I(I+1) - I_3(I_3 \pm 1)}\, | \Psi(I, I_3 \pm 1)\rangle$$

- If we apply it to a quark we get: $\begin{cases} I^+ | d\rangle = | u\rangle \;;\; I^+ | \bar{u}\rangle = | -\bar{d}\rangle \;; \\ I^+ | u\rangle = I^+ | \bar{d}\rangle = 0 \end{cases}$

- Moreover: $\begin{cases} I^- | \Psi(1,1)\rangle = I^+ | \Psi(1,-1)\rangle = \sqrt{2}\, | \Psi(1,0)\rangle \\ I^+ | \Psi(1,0)\rangle = \sqrt{2}\, | \Psi(1,1)\rangle \;;\; I^- | \Psi(1,0)\rangle = \sqrt{2}\, | \Psi(1,-1)\rangle \\ I^+ | \Psi(1,1)\rangle = I^- | \Psi(1,-1)\rangle = 0 \end{cases}$

- By convention the wave function of the π^- is: $-d\bar{u}$

$$I^+ | \pi^-\rangle = I^+ | -d\bar{u}\rangle = | -\left[\left(I^+ d\right)\bar{u} + d\left(I^+\bar{u}\right)\right]\rangle = | -u\bar{u} + d\bar{d}\rangle = \sqrt{2}\left(\frac{1}{\sqrt{2}} | -u\bar{u} + d\bar{d}\rangle\right) = \sqrt{2}\, | \pi^0\rangle$$

- The π^0 is identified with the state: $\pi^0 = \frac{1}{\sqrt{2}}\left(d\bar{d} - u\bar{u}\right)$

- Indeed: $I^+ | \pi^0\rangle = I^+ \frac{| d\bar{d} - d\bar{u}\rangle}{\sqrt{2}} = \frac{| u\bar{d} + 0 - 0 - u\bar{d}\rangle}{\sqrt{2}} = \sqrt{2}\, | u\bar{d}\rangle = \sqrt{2}\, | \pi^+\rangle$

Relations between w.f. and physical states

- In order to find the singlet of the octet $\eta_8 = \{8, |0,0\rangle\}$ we need to find the quark composition that it is orthogonal to $\eta_1 = \{1, |0,0\rangle\}$ and to the π^0:

$$\eta_1 = \{1, |0,0\rangle\} = \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

$$\pi^0 = \frac{1}{\sqrt{2}}\left(d\bar{d} - u\bar{u}\right)$$

$$\eta_8 = \{8, |0,0\rangle\} = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

N.B. $I^{\pm} |\eta_8\rangle = 0$

- The physical states η and η' are a linear combination of η_1 e η_8, but since the mixing angle is small (~11°), we can do the identification:

$$\eta_8 \equiv \eta \quad ; \quad m_\eta = 548 \text{ MeV}$$

$$\eta_1 \equiv \eta' \quad ; \quad m_{\eta'} = 958 \text{ MeV}$$

Mesons 1^- (vector mesons)

- **The vector mesons 1^- have the same quark composition of the mesons 0^-, they are in the S-state (L=0) but the two quarks (actually quark-antiquark) have the spin that are parallel (S=1).**

- There are three mesons with I_3=0 and Y=0; one of them belongs to the isospin triplet ρ : ρ^+, ρ^-, ρ^0.
- ρ^0 has the same wave function of the π^0 (besides a factor "-1"):

$$\rho^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

- The SU(3) singlet ϕ_1 and the isospin singlet of the octet ϕ_8 are mixed to give the mass eigenstates ϕ and ω:

$$\omega = \phi_1 \cos\vartheta + \phi_8 \sin\vartheta$$
$$\phi = \phi_1 \sin\vartheta - \phi_8 \cos\vartheta$$

- N.B. in this case the mixing angle is $\theta \sim 35^o$

Exercise: computation of the ϑ angle

- Let'assume that the Hamiltonian matrix element between two states is equal to the "mass" squared:

$$M_\omega^2 = \langle \omega | H | \omega \rangle = M_1^2 \cos^2 \vartheta + M_8^2 \sin^2 \vartheta + 2M_{18}^2 \sin \vartheta \cos \vartheta$$
$$M_\phi^2 = \langle \phi | H | \phi \rangle = M_1^2 \sin^2 \vartheta + M_8^2 \cos^2 \vartheta - 2M_{18}^2 \sin \vartheta \cos \vartheta$$

- Since ω and ϕ are mass eigenstates, they are orthogonal:

$$M_{\omega\phi}^2 = \langle \phi | H | \omega \rangle = 0 = \left(M_1^2 - M_8^2\right) \sin \vartheta \cos \vartheta + M_{18}^2 \left(\sin^2 \vartheta - \cos^2 \vartheta\right)$$

- If we get rid of of M_{18} and M_1 from these three equations, we get:

$$\tan^2 \vartheta = \frac{M_\phi^2 - M_8^2}{M_8^2 - M_\omega^2}$$

- From the mass formula of Gell-Mann – Okubo we have:

$$M_8^2 = \frac{1}{3}\left(4M_{K^*}^2 - M_\rho^2\right)$$

$M_\rho = 776$ MeV
$M_{K^*} = 892$ MeV
$M_\omega = 783$ MeV
$M_\phi = 1020$ MeV

- If we put in the formula the measured values of the masses, we get:

$$\vartheta \approx 40^o$$

N.B. $\sin \vartheta = \frac{1}{\sqrt{3}}$ if $\vartheta \approx 35^o$

Mesons 1$^-$

- If we use $\sin\vartheta = \frac{1}{\sqrt{3}}$ we have:

$$\omega = \frac{1}{\sqrt{3}}\left(\phi_8 + \sqrt{2}\phi_1\right)$$

$$\phi = \frac{1}{\sqrt{3}}\left(\phi_1 - \sqrt{2}\phi_8\right)$$

- since:

$$\phi_1 = \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

$$\phi_8 = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

$M_\rho = 776$ MeV
$M_{K^*} = 892$ MeV
$M_\omega = 783$ MeV
$M_\phi = 1020$ MeV

- We have

$$\phi = s\bar{s} \quad ; \; \omega = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$$

- In this case of “ideal mixing”, that is almost true in practice, the φ is composed entirely from quark s and the ω from quarks u and d
- This implies that the mass of the ω should be similar to the one of the ρ^0 and the mass of φ should be higher, as it is observed experimentally.

Summary of the mixing

- We have the mixing in the states with I=0:
  - η, η' are linear combinations of η_1, η_8 that can mix between themselves because they have the same quantum numbers: ($I=I_3=S=0$)
  - The same thing happens to the physical states ω and ϕ that are the results of the mixing of ϕ_1 and ϕ_8

We need to introduce new parameters: the mixing angles between the states

$$|\omega\rangle = \sqrt{\frac{1}{2}}\left(u\bar{u} + d\bar{d}\right)$$

$$|\phi\rangle = s\bar{s}$$

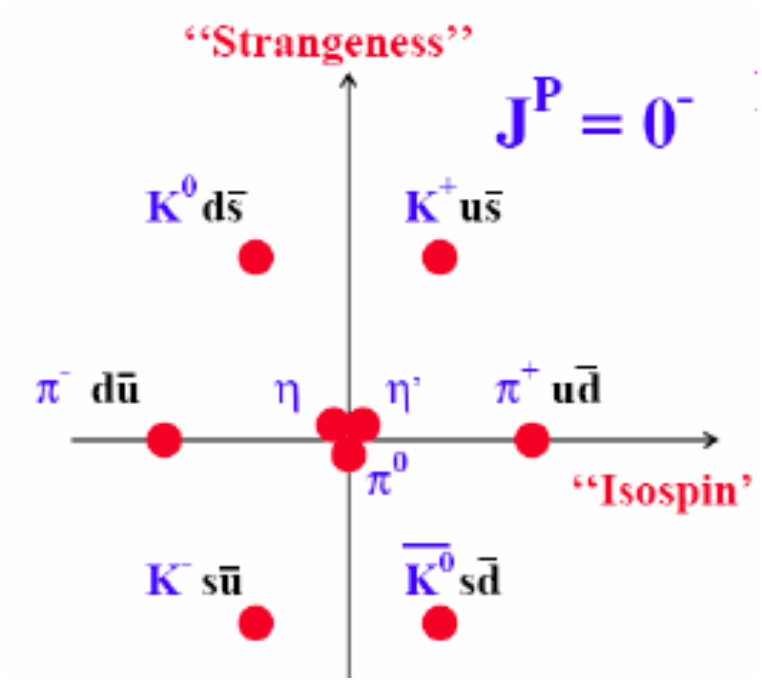
pure state $s\bar{s}$

$$|\eta\rangle = \sqrt{\frac{1}{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

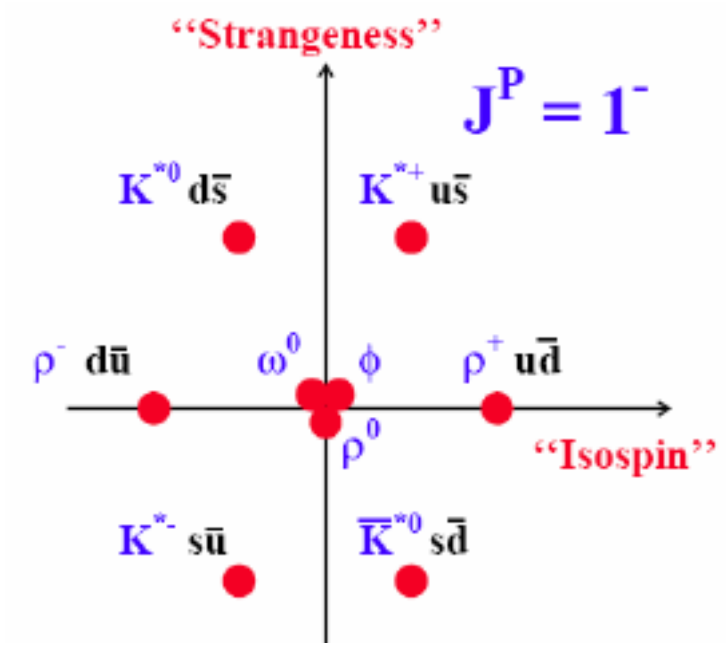
$$|\eta'\rangle = \sqrt{\frac{1}{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

Almost exact, there is just a small mixing

Quark content of the light mesons



Masses/MeV:
$\pi(140)$, $K(495)$
$\eta(550)$, $\eta'(960)$



Masses/MeV:
$\rho(770)$, $K^*(890)$
$\omega(780)$, $\phi(1020)$

Baryon graphical construction

- We use the same technique used to build the mesons. We just remind you that the baryons are made by 3 quarks..

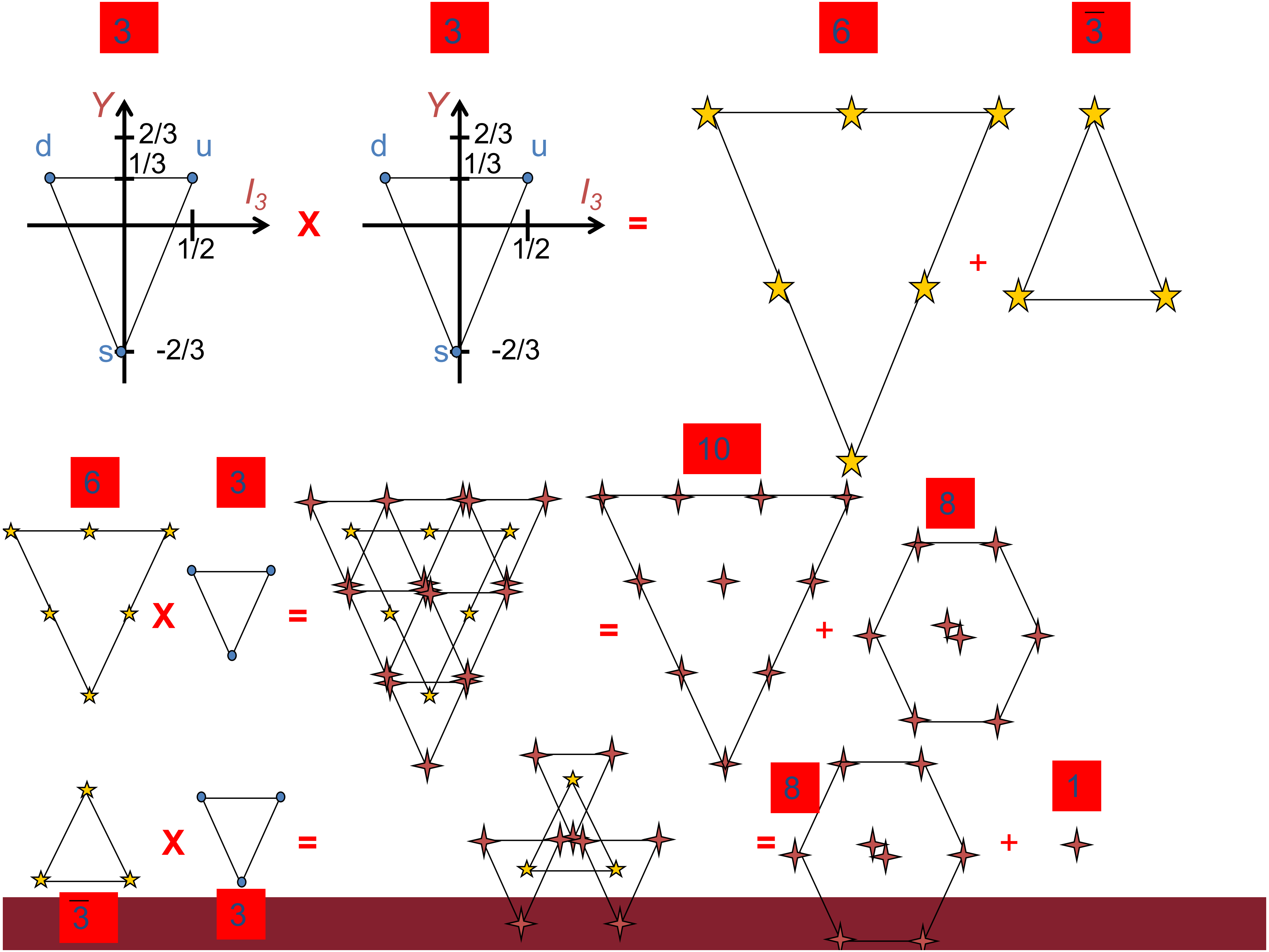
- Let's recall that:

$$3 \otimes 3 = 6 \oplus \bar{3}$$

$$6 \otimes 3 = 10 \oplus 8$$

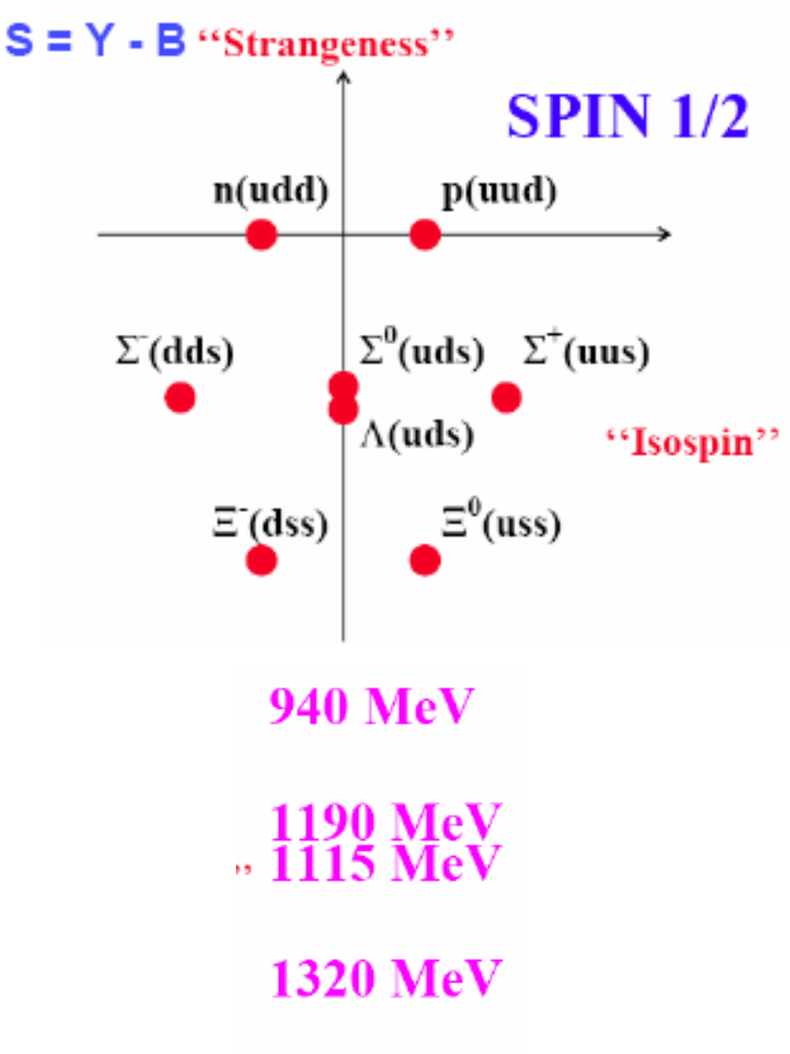
$$3 \otimes \bar{3} = 1 \oplus 8$$

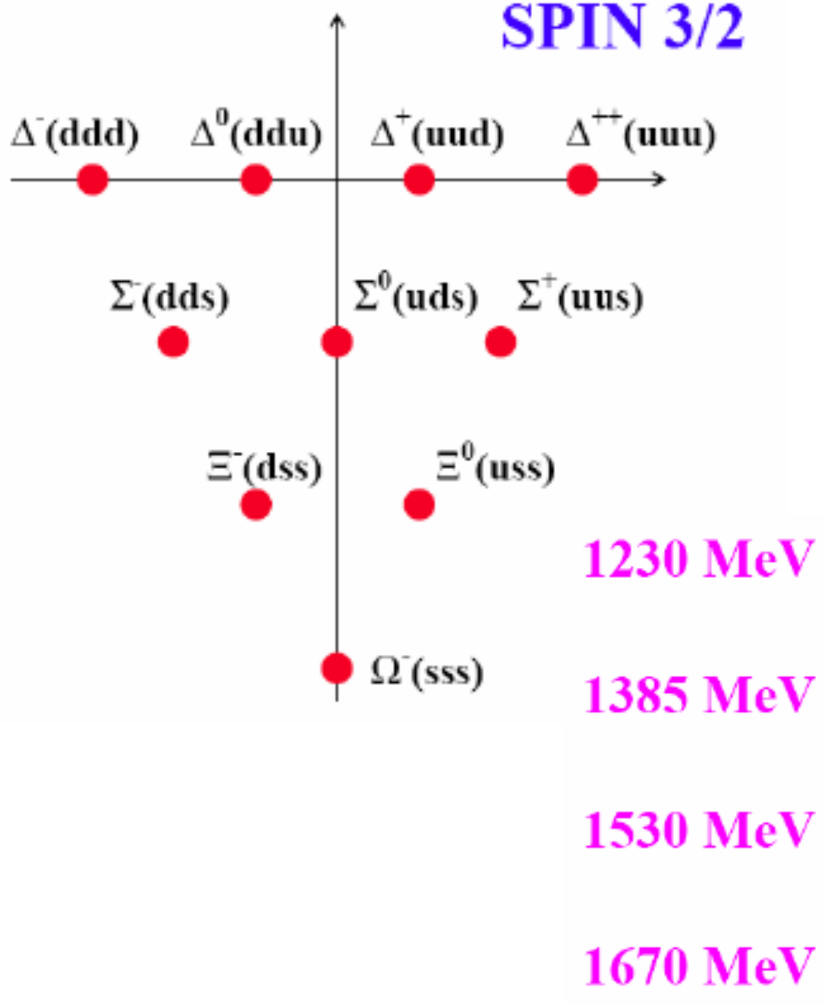
$$\Rightarrow \quad 3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$$



3
3
6
$\bar{3}$
Y
2/3
1/3
d
u
I_3
1/2
s
-2/3
X
Y
2/3
1/3
d
u
I_3
1/2
s
-2/3
=
+
6
3
X
=
10
=
8
+
$\bar{3}$
3
X
=
8
=
+
1

Quark content of the strange baryons





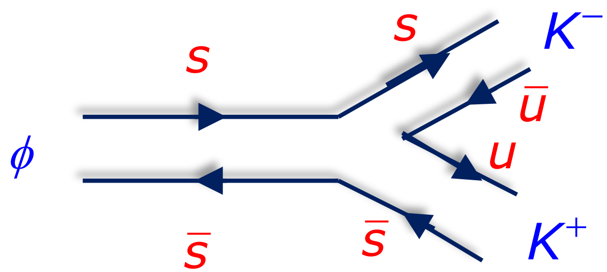
OZI rule

- The Φ quark composition, together with the OZI (Okubo, Zweig, Iizuka) rule, permits to better understand the Φ decays:

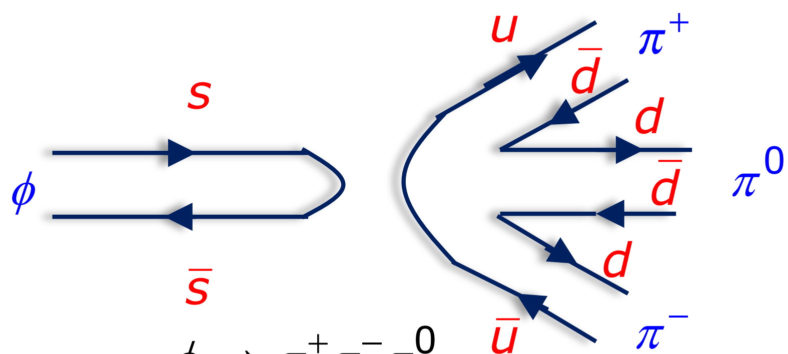
$$\phi(1020) \;\; \begin{array}{l} \rightarrow K^+K^- \\ \rightarrow K^0\bar{K}^0 \end{array} \Big\} \;\; 84\%$$

$$\rightarrow \pi^+\pi^-\pi^0 \quad 15\%$$

The phase space supports the decays of the Φ in 3π (Q ~ 600 MeV) with respect to Q ~ 24 MeV of the decay in KK



$\phi \rightarrow K^+K^-$



$\phi \rightarrow \pi^+\pi^-\pi^0$

OZI rule: whenever there are quark lines not connected, the diagrams are suppressed

The OZI rule can be explained in QCD by taking into account the gluons exchange

This is important to explain the long life time of the J/Ψ and of the Υ

Quark masses

- By using a "simple" model to derive the hadron mass from the quark masses and their interactions (spin dependent) , it is possible to derive the "effective" quark mass (you can see the details on the book by Burcham and Jobes):

quark	"free" mass (MeV)	Effective Mass Mesons (MeV)	Effective Mass Baryons (MeV)
u	5.6 ± 1.1	310	363
d	9.9 ± 1.1	310	363
s	199 ± 33	483	538

- Different bounding energy betweeb mesons and baryons

- The "free" mass or "current" mass is evaluated at the scale of 1 GeV/c^2

Quark masses

- The effective mass is different from the "free" (true!) quark mass.
- On the other hand, what is the mass of a particle?
  You can not weight it on a balance:

$$F = G\frac{mM}{r^2}$$

  - $E^2 - p^2 = m^2$? (but we don't have free quarks!)
  - propagator pole?
  - real part of the propagator?

- In any case in the Lagrangian that describes the quark interactions (Standard Model + QCD) we have to consider the quark "free" mass and not the effective mass:

$$L = m_u u\bar{u} + m_d d\bar{d} + m_s s\bar{s}$$

The typical QCD mass scale is Λ_{QCD}=200 MeV. The almost exact isospin symmetry derive from the fact that m_u~m_d «Λ_{QCD} ; while SU(3) is only an approximate symmetry because m_s~ Λ_{QCD} .

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End of chapter 4