Introduction to Particle Physics

- Chapter 8 -

# The neutral K system and CP violation

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# Chapter summary:

- The neutral K system
- Eigenstates of CP: $K^0_1$ and $K^0_2$
- Strangeness oscillations
- $K_1$ regeneration
- Fitch and Cronin experiment about CP violation
- Direct and indirect CP violation
- Introduction of the states $K^0_S$ and $K^0_L$
- Semileptonic decays of the $K^0_L$
- Operative definition of the charge positive sign
- Direct CP violation
- The parameter ε'

# The neutral K mesons

- The K mesons are the lightest mesons with strangeness: $M_{K^\pm} = 493.677 \pm 0.016$ MeV; $M_{K^0} = 497.648 \pm 0.027$ MeV

- The K belong to an isospin doublet as far as strong interactions are concerned:

$$\begin{pmatrix} K^+ = u\bar{s} \\ K^0 = d\bar{s} \end{pmatrix} \qquad \begin{pmatrix} \bar{K}^0 = \bar{d}s \\ K^- = \bar{u}s \end{pmatrix}$$

- These are the mass eigenstates that are also eigenstates of the strong interactions; these are the states that are produced in all processes where strong interactions take place.

- For instance: $\pi^- + p \to \Lambda^0 + K^0$ (associate production)

- The $\bar{K}^0$ production needs a more exotic process: $\pi^- + p \to \bar{\Sigma}^- + \bar{K}^0 + p + n$ (or $\pi^+ + p \to + \bar{K}^0 + K^+ + p$ )

- Since the threshold of the first reaction is lower than the second ones, we can have a pure beam of $K^0$ without any contamination from $\bar{K}^0$

- The K are not stable, they decay into particles with lower mass, but since they are the lightest strange particles, their decayd must be mediated by weak interactions (strangeness violation decays).
- From the study of the K decays (charged and neutral) it has been found the first hint of parity violation of the w.i.

- The weak interactions violate separately both C and P, but they seems to conserve the combined symmetry CP.
- Then it seems reasonable to assume that the K eigenstates that participate in the weak interactions are eigenstates of CP and not eigenstates of strangeness, that intervene in the strong interactions.

# CP eigenstates

- $K^0$ and $\overline{K^0}$ are eigenstates of strangeness but not of CP symmetry.

$$P|K^0\rangle = -|K^0\rangle \quad ; \quad P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$ (negative intrinsic parity)

$$CP|K^0\rangle = -C|K^0\rangle = -|\bar{K}^0\rangle \quad ; \quad CP|\bar{K}^0\rangle = -C|\bar{K}^0\rangle = -|K^0\rangle$$

- However the following linear combinatios are CP eigenstates with eigenvalues +1 and -1:

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad (CP=+1)$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad (CP=-1)$$

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- Pay attention that this definition is not unique because it depends on the arbitrary phase that intervenes in the application of the charge conjugation operator. If we use another definition with respect to the one adopted in the book by Burcham and Jobes, we get:

$$C|K^0\rangle = -|\bar{K}^0\rangle \;\Rightarrow\; CP|K^0\rangle = -C|K^0\rangle = |\bar{K}^0\rangle$$

$$C|\bar{K}^0\rangle = -|K^0\rangle \;\Rightarrow\; CP|\bar{K}^0\rangle = -C|\bar{K}^0\rangle = |K^0\rangle$$

⇒

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad (CP=+1)$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad (CP=-1)$$

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**In any case the CP eigenvalue of $K_1$ is +1 while the one of $K_2$ is -1**

- We must point out that $K_1$ is not the antiparticle of $K_2$, as we can see:

$$C|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(C|K^0\rangle - C|\bar{K}^0\rangle\right) = \frac{1}{\sqrt{2}}\left(|\bar{K}^0\rangle - |K^0\rangle\right) \neq |K_2^0\rangle$$

**This implies that $K_2$ and $K_1$ can have different masses and lifetimes**

# $K_1$ and $K_2$ decays

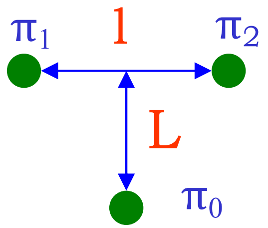
- We saw in the τ-θ puzzle that the K can decay into states with two pions or with three pions. We have to determine the CP eigenvalue of these two states and associate them to $K_1$ and $K_2$.

Two pions state: $\pi^0\pi^0$ and $\pi^+\pi^-$

- Let's call l the relative orbital angular momentum, so the parity of the state is $(-1)^l$
- since the $\pi^0$ is eigenstates of C and that $\pi^+$ and $\pi^-$ are antiparticles, the charge conjugation is equivalent to a parity operation, therefore: $C(\pi_1\pi_2)= (-1)^l$

$$\Rightarrow \; CP(\pi_1\pi_2) = +1$$

Three pions state: $\pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0$




- Since the K has spin zero, then l=L
- The Q of the reaction is small, about 90 MeV, so most likely l=L=0
- The Bose statistics for the system $\pi^0\pi^0\pi^0$ wants an even l, then l=2 is highly suppressed because angular momentum effect. So the system is in a S-wave state.

- From the above argument, $\pi_1\pi_2$ has CP=+1. The $\pi^0$ has C=+1 and P=-1, therefore the combination of the $\pi^0$ with the system $\pi_1\pi_2$ gives a state with overall CP eigenvalue equal to -1

$$|K_1^0\rangle \rightarrow \pi\pi \qquad (CP = +1)$$
$$|K_2^0\rangle \rightarrow \pi\pi\pi \qquad (CP = -1)$$

- The Q of the first reaction is much bigger than the second one (there is one pion less), therefore the decay rate ( Γ ) of $K_1$ is much bigger than the one of $K_2$

$$\tau_{K_2} >> \tau_{K_1}$$

$$\tau_{K_2} = 0.5 \cdot 10^{-7}\ s$$
$$\tau_{K_1} = 0.9 \cdot 10^{-10}\ s$$

# Strangeness oscillation

- Since $K^0$ and $\overline{K^0}$ are superposition of two states with different mass, they give rise to a phenomenon very important and very interesting, known as strangeness oscillation in the time evolution of the two eigenstates of the strong interactions ($K^0$ and $\overline{K^0}$).

- Let's suppose that at t=0 we produce a pure beam of $K^0$, for instance through the process $\pi^- p \rightarrow \Lambda^0 K^0$ :

The two states can be written as :

$$|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0\rangle + |K_2^0\rangle\right)$$

$$|\bar{K}^0\rangle = \frac{1}{\sqrt{2}}\left(|K_2^0\rangle - |K_1^0\rangle\right)$$

- At t=0 the $K^0$ wave function is: $|\Psi(t)\rangle = |K^0(t)\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0(t)\rangle + |K_2^0(t)\rangle\right)$

- For an unstable particle of mass m and lifetime $\tau = 1/\Gamma$, the time dependent wave function, in the center of mass of the particle where E=m, can be written as:

$$|\Psi(t)\rangle = |\Psi(0)\rangle\ e^{-imt} \cdot e^{-\frac{\Gamma}{2}t}$$

- this is consistent with the exponential decay law for unstable particles: $N(t) = |\Psi(t)|^2 = |\Psi(0)|^2\ e^{-\Gamma t} = N_0 e^{-\frac{t}{\tau}}$

- Since the states $K_1$ and $K_2$ are two different states for the weak interactions, they can have different masses and lifetimes, as they do, that we will call: $m_1, \Gamma_1$ and $m_2, \Gamma_2$ (let's recall that $\Gamma = 1/\tau$)

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0(0)\rangle \cdot e^{-im_1 t} e^{-\frac{1}{2}\Gamma_1 t} + |K_2^0(0)\rangle \cdot e^{-im_2 t} e^{-\frac{1}{2}\Gamma_2 t}\right)$$

# Strangeness oscillation

- At time t the intensity of the $K^0$ state in the beam is: $I(K^0) = \left|\langle K^0 \mid \Psi(t)\rangle\right|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle K^0 \mid K_1^0(t)\rangle + \langle K^0 \mid K_2^0(t)\rangle\right)\right|^2$

- $\langle K^0 \mid K_1^0(0)e^{-(im_1+\Gamma_1/2)t}\rangle = \frac{1}{\sqrt{2}}\langle K^0 \mid K^0\rangle e^{-(im_1+\Gamma_1/2)t}$
- $\langle K^0 \mid K_2^0(0)e^{-(im_2+\Gamma_2/2)t}\rangle = \frac{1}{\sqrt{2}}\langle K^0 \mid K^0\rangle e^{-(im_2+\Gamma_2/2)t}$

$$\Rightarrow \quad \langle K^0 \mid \Psi(t)\rangle = \frac{1}{2}\langle K^0 \mid K^0\rangle\left[e^{-im_1t}\cdot e^{-\frac{\Gamma_1}{2}t} + e^{-im_2t}\cdot e^{-\frac{\Gamma_2}{2}t}\right]$$

N.B. $\langle K^0 \mid \bar{K}^0\rangle = 0 \quad ; \quad \langle K^0 \mid K^0\rangle = 1$

$$\Rightarrow \quad \left|\langle K^0 \mid \Psi(t)\rangle\right|^2 = \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + e^{-\frac{\Gamma_1+\Gamma_2}{2}t}\cdot e^{i(m_2-m_1)t} + e^{-\frac{\Gamma_1+\Gamma_2}{2}t}\cdot e^{i(m_1-m_2)t}\right] = \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2e^{-\frac{\Gamma_1+\Gamma_2}{2}t}\cos(\Delta m t)\right]$$

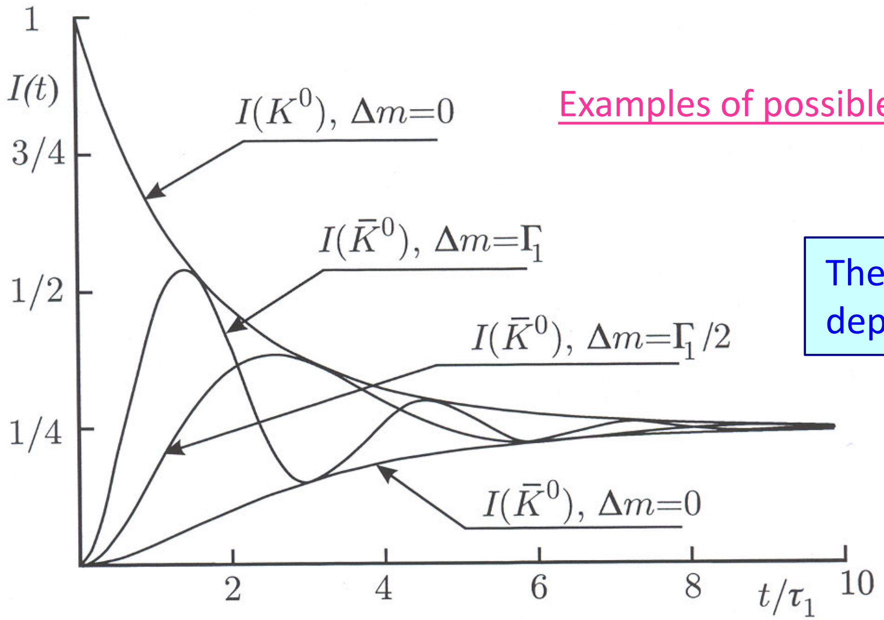
- For the $\overline{K}^0$ we have: $I(\bar{K}^0) = \left|\langle \bar{K}^0 \mid \Psi(t)\rangle\right|^2 = \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-\frac{\Gamma_1+\Gamma_2}{2}t}\cos(\Delta m t)\right]$

$\Delta m = m_2 - m_1$

- The intensities of the $K^0$ and $\overline{K}^0$ oscillate with the frequency $\Delta m/2\pi$. From the frequency measurement we can get the mass difference $\Delta m$.

N.B. $\Gamma_1 >> \Gamma_2$

$$\Gamma_1 \gg \Gamma_2$$
$$\Rightarrow e^{-\frac{t}{\tau_2}} \gg e^{-\frac{t}{\tau_1}}$$
$$\Rightarrow e^{-\frac{10\tau_1}{\tau_2}} \approx 1$$




Examples of possible oscillations

The oscillation form depends on $\Delta m$ and $\Gamma_1$

# Strangeness oscillation

- In order to measure the oscillation frequency we need to measure the intensity of the $K^0$ or $\bar{K}^0$ as a function of the time, namely as a function of the distance from the production point of the $K^0$ beam.
- It is convenient to measure the $K^0$ intensity since we start with a pure $K^0$ beam.
- To identify the presence of the $\bar{K}^0$ in the beam we exploit the different behaviour of the $K^0$ and $\bar{K}^0$ in the matter. Let's recall that the $K^0$ has strangeness +1 while the $\bar{K}^0$ has strangeness -1 therefore , since in the target there are no baryons with strangeness +1, the $K^0$ can only do elastic scattering or charge exchange, like for instance $K^0+p \to K^+ + n$.
- Instead the $\bar{K}^0$ has strangeness -1, therefore it can produce baryons with strangeness -1, like for instance:

$$\bar{K}^0+p \to \Lambda^0\pi^+ \ ; \ \bar{K}^0+p \to \Sigma^0\pi^+ \ ; \ \bar{K}^0+p \to \Sigma^+\pi^+\pi^- \ ; \ \text{etc..}$$

- By measuring the production of strange hyperons as a function of the distance from the production point, it is possible to determine the $\bar{K}^0$ intensity and then $\Delta m$

$$I(\bar{K}^0) = \left|\langle \bar{K}^0 \mid \Psi(t)\rangle\right|^2 = \frac{1}{4}\left[ e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-\frac{\Gamma_1+\Gamma_2}{2}t}\cos(\Delta m t)\right]$$

- Since $\Gamma_1 >> \Gamma_2$ the $K_1$ decays immediately, therefore we have:

$$I(\bar{K}^0) \approx \frac{1}{4}\left[ e^{-\Gamma_2 t} - 2e^{-\frac{\Gamma_1}{2}t}\cos(\Delta m t)\right] = \frac{1}{4} \quad \text{for} \quad \tau_1 << t << \tau_2$$

- Experimentally we measure: $\left|\Delta m \cdot \tau_1\right| = 0.477 \pm 0.002$
- From this result we get $\Delta m$, since we know $\tau_1$. The sign can be deduced from other experiment about $K_S$ regeneration and it is such that $m_2 > m_1$.

$$\Delta m = \left(0.535 \pm 0.002\right)\cdot 10^{-10}\hbar = \left(3.52 \pm 0.01\right)\cdot 10^{-6} eV$$

# Transitions with ΔS=2

- The strangeness oscillations happens because the $K^0$ and $\overline{K^0}$ can decay in the same final states, like for instance:

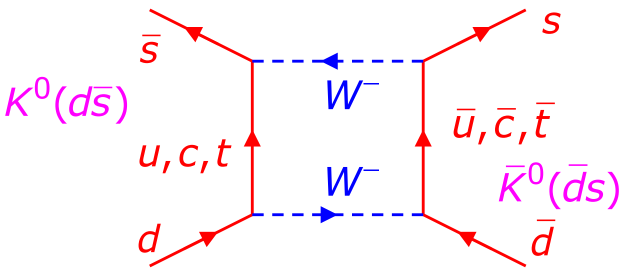
$$K^0 \to \pi^+\pi^- \leftarrow \bar{K}^0 \quad \text{or} \quad K^0 \to \pi^+\pi^-\pi^0 \leftarrow \bar{K}^0$$

- then we can have transitions from $K^0$ to $\overline{K^0}$ through an intermediate state of two or three pions:


- this is possible because $K^0$ and $\overline{K^0}$ are two neutral particles, one the antiparticle of the other, but they are distinct states (contrary to the $\pi^0$ that it is own antiparticle) because they have quantum numbers, in this case strangeness, that distinguish the two particles.
- The weak interactions do not distinguish the strangeness, therefore we can have transitions from a state to the other one mediated by weak interactions. These are second order transitions characterized by ΔS=2
- As far as the strong interactions are concerned the two states $K^0$ and $\overline{K^0}$ are orthogonal, while the weak interactions connect the two states.

$$\langle \bar{K}^0 \mid K^0 \rangle = 0 \quad ; \quad \langle \bar{K}^0 \mid H_{st} \mid K^0 \rangle = 0 \quad ; \quad \langle \bar{K}^0 \mid H_{weak} \mid K^0 \rangle \neq 0$$

- At quark level, the transition ΔS=2 happens through a box diagram like this:




From this diagramm we can compute Δm

$$\Delta m \approx \frac{G^2}{4\pi^2} f_k^2 m_c^2 \cos^2\theta_c \sin^2\theta_c$$

A computation made by Gaillard, Lee and Rosner, before 1974, using the measured values, predicted $m_c \approx 1.5$ GeV.

# $K_1$ regeneration

- In 1955 Pais and Piccioni suggested that the existence of the states $K_1$ and $K_2$ should give rise to a phenomenon known as $K_1$ regeneration.
- Let's suppose to produce a pure beam of $K^0$ and to let it advance in vacuum. Initially the beam consist in an equal mixture of the states $K_1$ and $K_2$

$$|\Psi(0)\rangle = |K^0(0)\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0(0)\rangle + |K_2^0(0)\rangle\right)$$

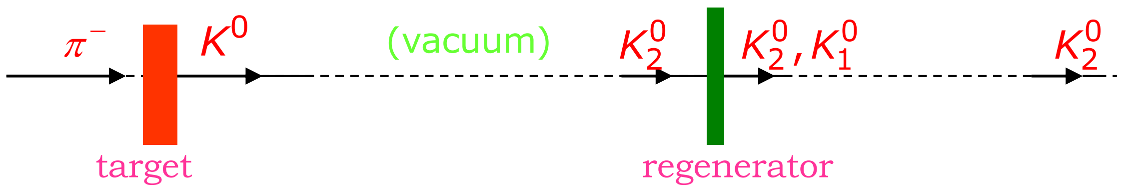
- Let's now choose $t >> t_1$; the $K_1$ component with short lifetime (that decades in two pions) will be completely decayed and the wave function will be:

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}|K_2^0(0)\rangle \cdot e^{-im_2 t} e^{-\frac{1}{2}\Gamma_2 t} \approx \frac{1}{\sqrt{2}}|K_2^0(0)\rangle \cdot e^{-\Gamma_2 t}$$

now the beam contains only the component $K_2$ with long lifetime that is composed by:

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

- Let's suppose now that we place a block of material in the beam:




- The $K^0$ and $\overline{K}^0$ have different strong interactions with matter, in particular the $\overline{K}_0$ has a bigger cross-section therefore it will be more strongly absorbed in the block.

# $K_1$ regeneration

- Let's call f and $\overline{f}$ the fraction $K^0$ e $\overline{K^0}$ that are left in the beam after the passage in the block:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(f\,|K^0\rangle + \bar{f}\,|\bar{K}^0\rangle\right)$$

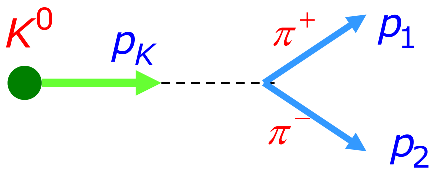
- In terms of the states $K_1$ and $K_2$ we have:

$$|\Psi\rangle = \frac{1}{2}\left[f\left(|K_1^0\rangle + |K_2^0\rangle\right) + \bar{f}\left(|K_2^0\rangle - |K_1^0\rangle\right)\right] = \frac{1}{2}\left[\left(f-\bar{f}\right)|K_1^0\rangle + \left(f+\bar{f}\right)|K_2^0\rangle\right]$$

- Since $f \neq \overline{f}$ the state with short lifetime has been regenerated by the presence of the material in the beam line.

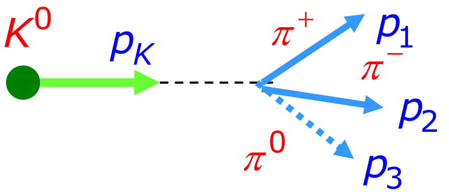
- This phenomenon can be verified experimentally by looking for K decays in two pions along the beam line before and after the regenerator.

N.B. to be sure that the two pions are coming from the K decay, we have to verify that their invariant mass is equal to the K mass and the momentum sum is equal to K initial momentum.




$$(p_1 + p_2)^2 = p_K^2 = m_K^2$$
$$\vec{p}_1 + \vec{p}_2 = \vec{p}_K$$

If in the decay is present a third pion that goes undetected, these relations are no longer valid:

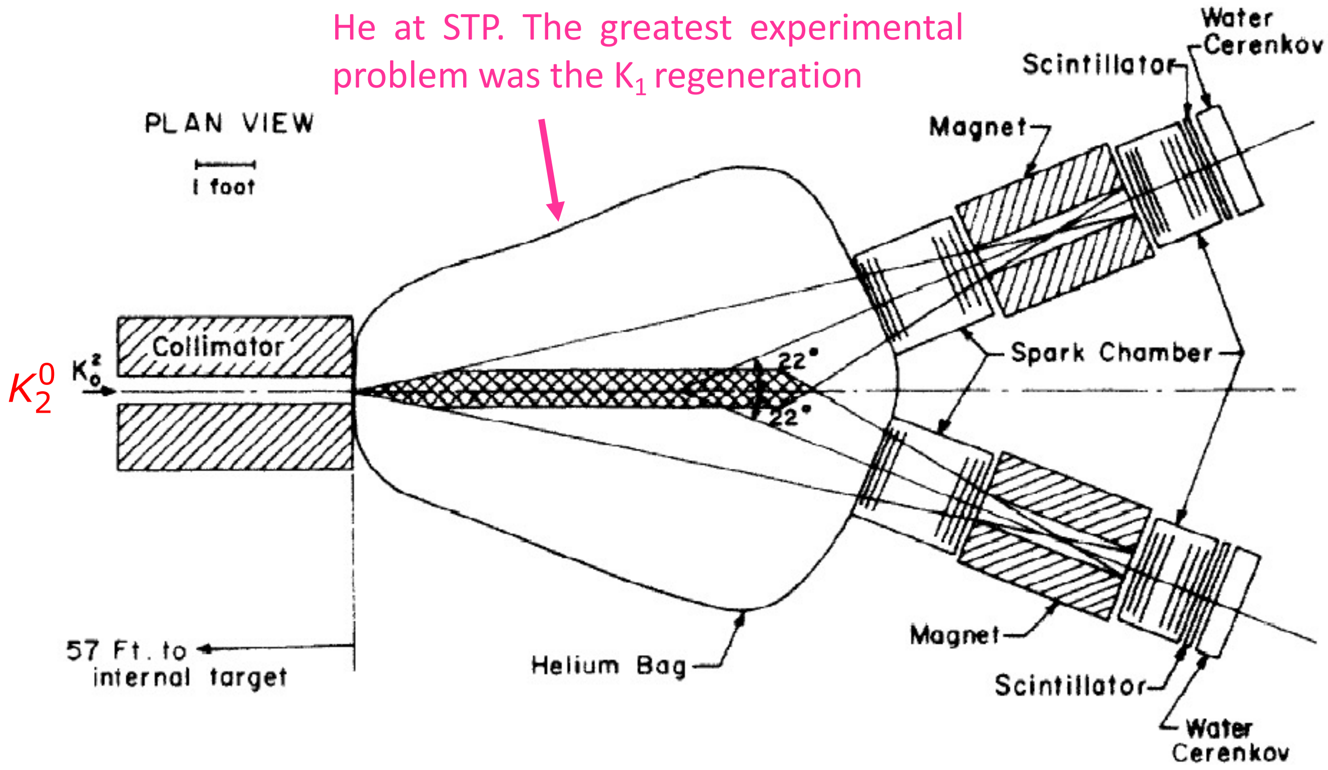



$$(p_1 + p_2)^2 \neq p_K^2 \Rightarrow \quad \neq m_K^2$$
$$\vec{p}_1 + \vec{p}_2 \neq \vec{p}_K$$

# Christenson, Cronin, Fitch, Turlay Exper.

- In 1963 Cronin, Fitch et al., made an experiment at the AGS accelerator at Brookhaven that was looking for two pions decays in a $K_2$ beam.
- The $K^0$ were produced by bombarding a Berillium target with a primary proton beam of 30 GeV, obtaining $K^0$ with momentum of ≈ 1 GeV/c
- The component with short lifetime had a decay lenght ($\gamma\beta c t_1$) of about 6 cm.
- The $K^0$ were made decay along a vacuum tube 15 m long, before to reach the experiment.
- The goal of the experiment was to put an upper limit the the B.R. of the $K_2$ in two pions.
- Instead the experiment observed the $K_2$ decay in two pions that was the first clear evidence of the CP violation in the weak interaction.

He at STP. The greatest experimental problem was the $K_1$ regeneration

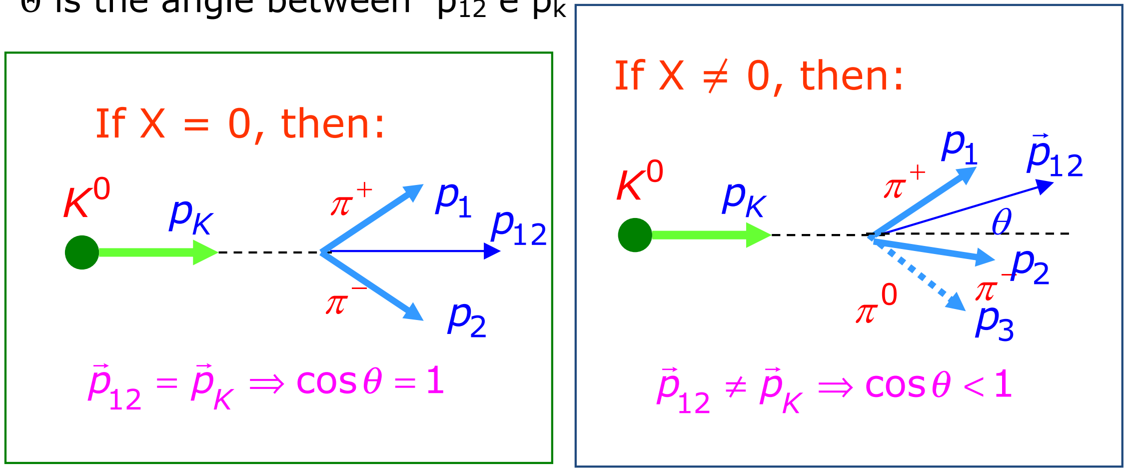


# Results of the experiment

$K_L^0 \to \pi^+\pi^- + X$

$\vec{p}_{12} = \vec{p}_1 + \vec{p}_2$

$\Theta$ is the angle between $p_{12}$ e $p_K$

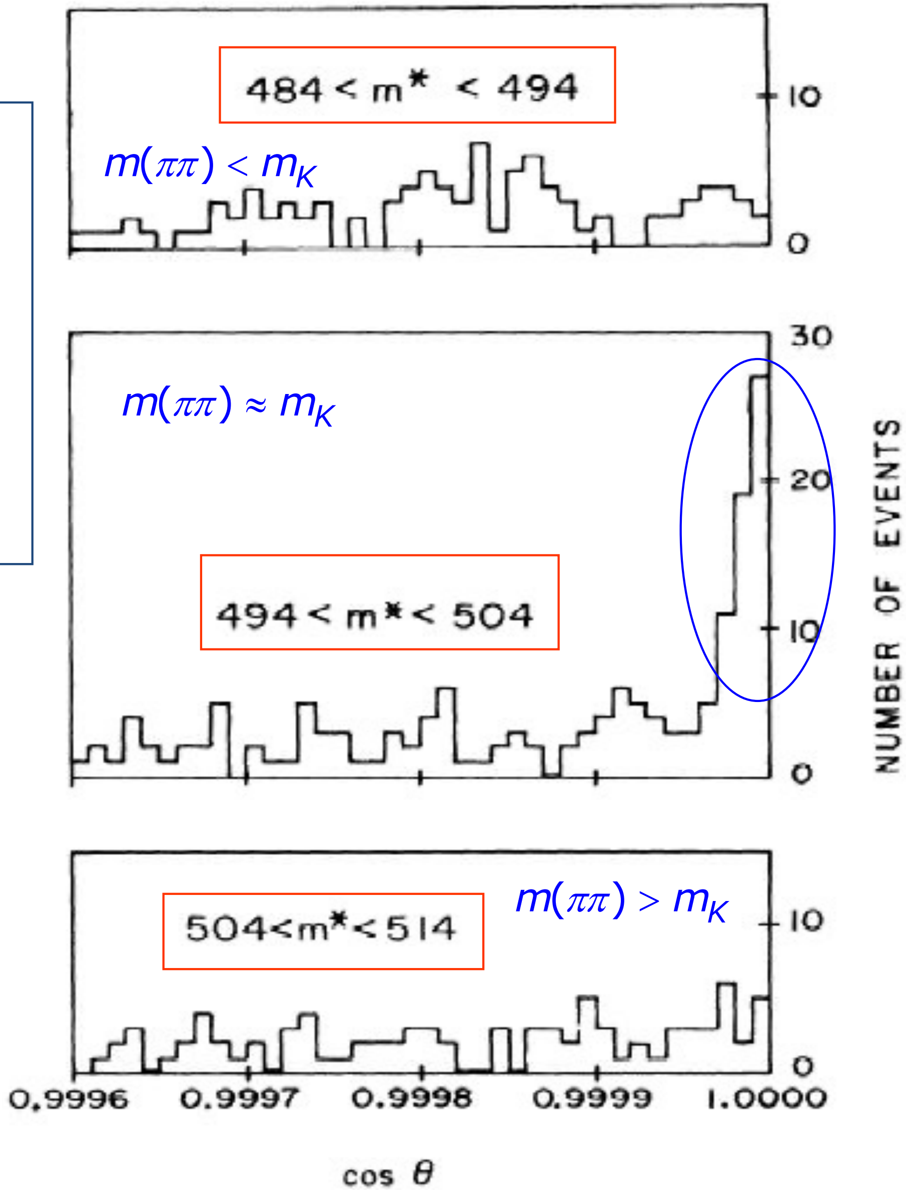



- The calibration of the apparatus was checked by placing a tungsten regenerator just before the experiment.
- The events in figure with $\cos\theta > 0.99999$ have an invariant of $499.1 \pm 0.8$ MeV
- The events in the peak, after background subtraction, are $45 \pm 9$ over a total of 22700 $K_2$ decays.

$$R = \frac{\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_L^0 \to \text{all charged})} = (2.0 \pm 0.4) \cdot 10^{-3}$$

The normalization is done with respect to all charged $K_L$ decays:

$\left[\text{B.R. } K_L^0 \to \pi^0\pi^0\pi^0 = 21\%\right]$



# CP violation

- The state $K_2$, that has CP = -1, can not decay in two pions if CP is conserved in the weak interactions.
- The 1963 experiment by Christenson, Cronin, Fitch and Turlay showed instead that the $K_2$ decays in two pions (n.b. the results of the experiment were published 1964)
- As a first step this lead to a change in the name of the neutral K states: the state with a short lifetime (where it is predominat the CP = +1 component) was called $K_S^0$ (K Short) and the state with a long lifetime (where it is predominant CP = -1) was called $K_L^0$ (K Long).

- The result found by Fitch and Cronin was:

$$R = \frac{\Gamma(k_L^0 \to \pi^+\pi^-)}{\Gamma(k_L^0 \to \text{all charged})} = (2.0 \pm 0.4) \cdot 10^{-3} \qquad \textit{Today}: \ \text{B.R.}\ \Gamma(k_L^0 \to \pi^+\pi^-) = (2.090 \pm 0.025) \cdot 10^{-3}$$

**We can no longer identify $K_S$ with $K_1$ and $K_L$ with $K_2$**

- Contrary to the parity violation, the CP violation gave a lot of theoretical problems to be incorporated in the various models/theories of the week interactions existing at that time.
- The transition $K_L \to \pi\pi$ can be explained in two ways: indirect CP violation and direct CP violation:

<u>Indirect violation</u>:

We suppose that the weak interactions do not violate CP, but the state $K_L$ is a linear superposition of the states $K_1$ and $K_2$, ; the observed decay in 2 pions is due to the $K_1$ component present in $K_L$

<u>Direct violation</u>:

In this case we suppose that the weak interactions violate directly the CP symmetry by connecting two states with different eigenvalues of CP. In this case we should observe the CP violation also in other weak processes.

# Indirect CP violation

- To take into account the CP violation in the $K_L$ decay we make the hypothesis that the eigenstates of the weak Hamiltonian , $K_S$ e $K_L$, are not eigenstates of CP but are a linear superpositions of the latter ($K_1$ and $K_2$). This mechanism is called indirect CP violation because the violation happens in the mixing of the states and NOT in the weak interactions matrix element.

$$|K_S^0\rangle = \frac{|K_1^0\rangle + \varepsilon |K_2^0\rangle}{\sqrt{1+|\varepsilon|^2}} \quad ; \quad |K_L^0\rangle = \frac{|K_2^0\rangle + \varepsilon |K_1^0\rangle}{\sqrt{1+|\varepsilon|^2}}$$

- ε is a small complex number that measure the amount of CP violation induced by the mixing of the $K^0$ states.

- The two states $K_S$ and $K_L$ are not CP eigenstates:

$$CP|K_S^0\rangle = \frac{CP|K_1^0\rangle + \varepsilon CP|K_2^0\rangle}{\sqrt{1+|\varepsilon|^2}} = \frac{|K_1^0\rangle - \varepsilon |K_2^0\rangle}{\sqrt{1+|\varepsilon|^2}} \neq |K_S^0\rangle$$

(the same is true for $K_L$)

- $K_S$ and $K_L$ are not even orthogonal states. The lack of "orthogonality" was expected since both states have the same decay channels, for instance the one in two pions.

$$\langle K_L^0 | K_S^0\rangle = \frac{1}{1+|\varepsilon|^2}\left(\langle K_2^0| + \varepsilon^* \langle K_1^0|\right)\left(|K_1^0\rangle + \varepsilon |K_2^0\rangle\right) = \frac{1}{1+|\varepsilon|^2}\left(\varepsilon\langle K_2^0|K_2^0\rangle + \varepsilon^*\langle K_1^0|K_1^0\rangle\right) = \frac{\varepsilon+\varepsilon^*}{1+|\varepsilon|^2} = \frac{2\mathcal{Re}(\varepsilon)}{1+|\varepsilon|^2}$$

the amount of non orthogonality is a measure of the amount of CP violation

- The observed decay of the $K_L$ in two pions is due to the decay in two pions of its component $K_1$. In principle it is also possible the $K_S$ decay in three pions due to its $K_2$ component, but its B.R. is very small ($3.2 \cdot 10^{-7}$) because it prevails the much faster decay in two pions.

# CP violation in the mixing

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad (CP=+1)$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad (CP=-1)$$

$$|K_L^0\rangle = \frac{|K_2^0\rangle + \varepsilon |K_1^0\rangle}{\sqrt{1+|\varepsilon|^2}}$$

We can express $K_L$ also in the base $K^0$ – anti $K^0$ (this is the formalism used in the B0 mixing)

$$|K_L^0\rangle = \frac{|K_2^0\rangle + \varepsilon |K_1^0\rangle}{\sqrt{1+|\varepsilon|^2}} = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[\frac{1}{\sqrt{2}}\left(K^0+\bar{K}^0\right) + \varepsilon\frac{1}{\sqrt{2}}\left(K^0-\bar{K}^0\right)\right] = \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}}\left[(1+\varepsilon)K^0 + (1-\varepsilon)\bar{K}^0\right]$$

$$|K_L^0\rangle = \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}}\left[q|K^0\rangle + p|\bar{K}^0\rangle\right] \qquad q = 1+\varepsilon \ ; \ p = 1-\varepsilon$$

In the direct CP violation we have (for instance):

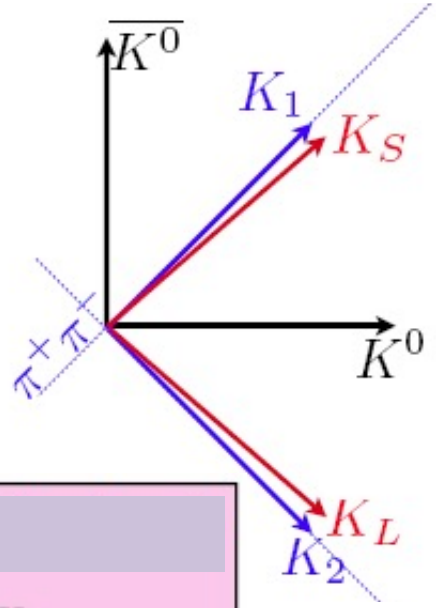
$$\Gamma\left(K^0 \to \pi^- + e^+ + \nu_e\right) \neq \Gamma\left(\bar{K}^0 \to \pi^+ + e^- + \bar{\nu}_e\right)$$

But even if the two Γ were equal, we would have an indirect CP violation in the mixing because because ε is not equal to zero, therefore the content of $K^0$ in $K_L$ is not equal to the anti-$K^0$ one.

Actually it is also present the direct CP violation, but at the level of per mille with respect to the indirect one, so it is very difficult to detect its effects.

# $K^0$ eigenstates and CP violation

Summary of the various $K^0$ system eigenstates and the two CP violation mechanisms




**Strangeness eigenstates**

$$K^0 = d\bar{s},\quad S=+1 \qquad CP(K^0) = \overline{K}^0$$
$$\overline{K}^0 = \bar{d}s,\quad S=-1 \qquad CP(\overline{K}^0) = -K^0$$

**CP eigenstates**

$$K_1 = \left(K^0 + \overline{K}^0\right)/\sqrt{2},\quad CP=+1 \quad \Rightarrow \pi\pi$$
$$K_2 = \left(K^0 - \overline{K}^0\right)/\sqrt{2},\quad CP=-1 \quad \Rightarrow \pi\pi\pi$$

**Mass eigenstates**

$$K_S = pK^0 + q\overline{K}^0 \cong K_1 + \bar{\varepsilon}K_2$$
$$K_L = qK^0 + p\overline{K}^0 \cong \bar{\varepsilon}K_1 + K_2$$

**"indirect" CP violation**

$\mathrm{Re}(\varepsilon) = 2.3 \times 10^{-3}$

$$|p|^2 + |q|^2 = 1$$

N.B. $\quad \langle K_S | K_L \rangle = 2\mathrm{Re}(\varepsilon) \neq 0$

$K_2 \to \pi\pi$

**"direct" CP violation**

$|\varepsilon'| << |\varepsilon|$

$$P\left(K^0 \to F\right) \neq P\left(\overline{K}^0 \to \overline{F}\right)$$

Have a *choice* when 'parameterizing' $K_S$ and $K_L$:

1. in terms of $K^0$ and $\overline{K}^0$
2. in terms of $K_1$ and $K_2$

In the $K^0$ system we use option 2) while in the $B^0$ system we use option 1)

# Summary about K decays

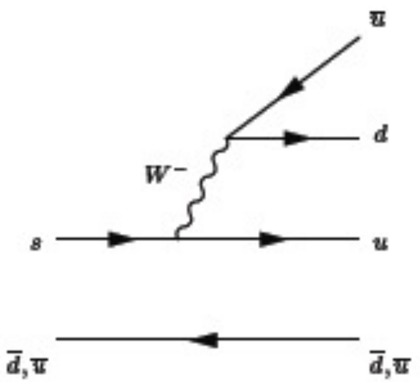
## Kaons...

Isospin / "Strangeness"

| Isospin | Strangeness −1 | Strangeness +1 |
|---|---|---|
| +1 | $\overline{K^0}$ $(s\bar{d})$ | $K^+$ $(\bar{s}u)$ |
| −1 | $K^-$ $(s\bar{u})$ | $K^0$ $(\bar{s}d)$ |

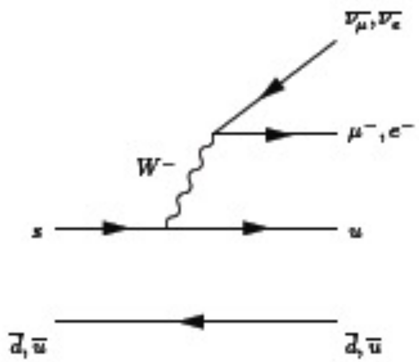
$m_K \sim 494$ MeV/$c^2$
No strange particles lighter than kaons exist
⇒Decay must violate "strangeness"

Strong force conserves "strangeness"
⇒Decay is a pure weak interaction



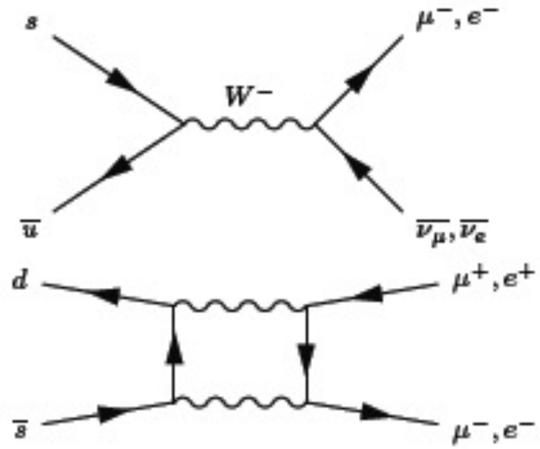

hadronic decays:

$$K^+ \to \pi^+\pi^0,\ \pi^+\pi^-\pi^+,\ \pi^+\pi^0\pi^0$$
$$K^- \to \pi^-\pi^0,\ \pi^-\pi^+\pi^-,\ \pi^-\pi^0\pi^0$$
$$K^0 \to \pi^0\pi^0,\ \pi^0\pi^0\pi^0,\ \pi^+\pi^-,\ \pi^+\pi^-\pi^0$$
$$\overline{K^0} \to \pi^0\pi^0,\ \pi^0\pi^0\pi^0,\ \pi^+\pi^-,\ \pi^+\pi^-\pi^0$$




semi-leptonic decays:

$$K^+ \to \pi^0\mu^+\nu_\mu,\ \pi^0 e^+\nu_e$$
$$K^- \to \pi^0\mu^-\overline{\nu_\mu},\ \pi^0 e^-\overline{\nu_e}$$
$$K^0 \to \pi^-\mu^+\nu_\mu,\ \pi^- e^+\nu_e$$
$$\overline{K^0} \to \pi^+\mu^-\overline{\nu_\mu},\ \pi^+ e^-\overline{\nu_e}$$




leptonic decays:

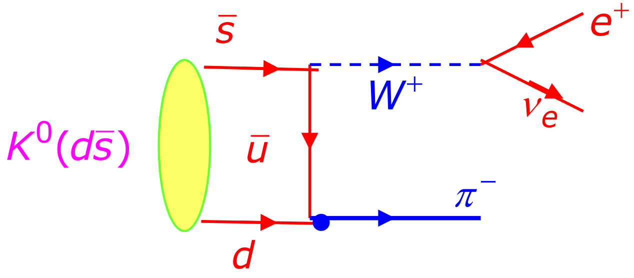
$$K^+ \to \mu^+\nu_\mu,\ e^+\nu_e$$
$$K^- \to \mu^-\overline{\nu_\mu},\ e^-\overline{\nu_e}$$
$$K^0 \to \mu^-\mu^+,\ e^-e^+$$
$$\overline{K^0} \to \mu^+\mu^-,\ e^+e^-$$

Hadronic and leptonic decays:
particle and anti-particle behave the same

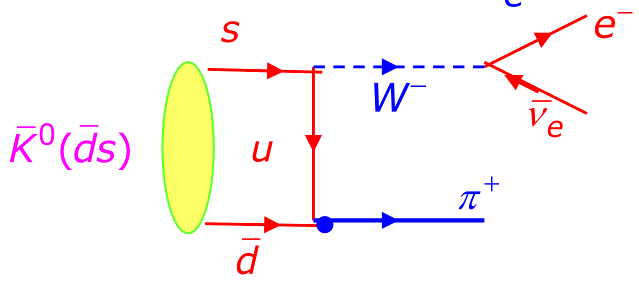
Semi-leptonic decays:
particle and anti-particle are distinct!
"ΔQ=ΔS rule"

# Semileptonic $K^0$ decays

$K^0 \rightarrow \pi^- + e^+ + \nu_e$




$\bar{K}^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e$




$\text{B.R.}(K_L^0 \rightarrow \pi^\pm e^\mp \nu_\mu) = (38.81\pm0.27)\%$ ← The two final states ($K^0$ and $\bar{K}^0$) are summed up

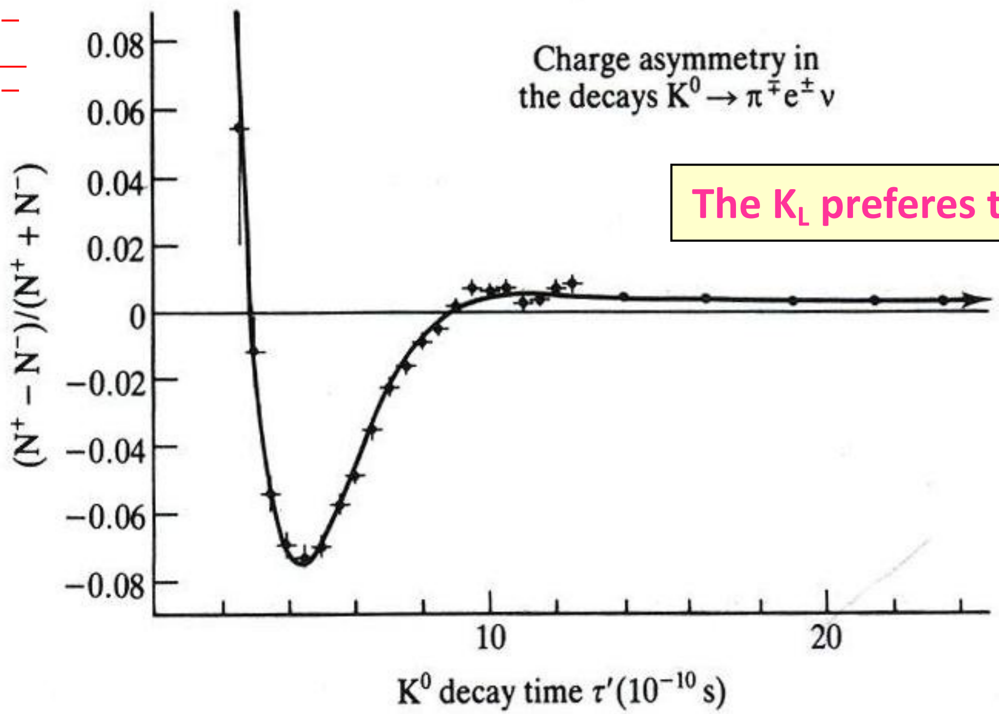
$\text{B.R.}(K_L^0 \rightarrow \pi^\pm \mu^\mp \nu_\mu) = (27.19\pm0.25)\%$ ← Phase space effect

- The two final states are CP conjugate (you can go from one to the other one with a CP transformation).
- **Let's recall that from the sign of the the charge of the lepton we know if we are dealing with a $K^0$ or $\bar{K}^0$ decay.**
- If CP was conserved, the $K_L$ would have the same decay rates in both final states because the $K_L$ would be an equiprobable mixture of $K^0$ and $\bar{K}^0$.
- From an experimental point of view, we start from a pure $K^0$ beam and we measure as a function of time the difference between the decays with a positron ($N^+$) and the decays with an electron ($N^-$) [strangeness oscillation].
- We wait long enough (that is we are far enough from the $K^0$ beam production point) in a such a way that the Ks component decays and we are left only with the $K_L$ component. Without the CP violation we would have an equal number of positron decays and electron decays.
- Therefore we measure as a function of time the charge asymmetry defined as follows:

$$\delta_l = \frac{N(K_L^0 \rightarrow \pi^- e^+ \nu_e) - N(K_L^0 \rightarrow \pi^+ e^- \bar{\nu}_e)}{N^+ + N^-} = \frac{2\mathcal{Re}(\varepsilon)}{1+|\varepsilon|^2}$$

# operative definition of the sign of the charge

$$\frac{N^+ - N^-}{N^+ + N^-}$$




The $K_L$ preferes the decay in positrons

$\tau'(10^{-10} s)$

$$\delta_l = \frac{N^+ - N^-}{N^+ + N^-} = (0.327 \pm 0.012)\%$$

- For the first time we have a process that is able to distinguish between matter and antimatter and can provide an operative definition of the sign of the electric charge.

The positive charge is the one carried by the lepton that it is preferentially produced in the $K_L$ decay.

- The CP violation treats in a different way matter and antimatter; it could explain why in the Universe now we have only matter and no antimatter anywhere (at least as far as we know at the moment).

# Direct CP violation

- Usually the CP violation is parametrized through the ratio of $K_S$ and $K_L$ decay amplitudes into a pair of charged or neutral pions:

$$\eta_{+-} = \frac{\langle \pi^+\pi^- \mid H \mid K_L^0 \rangle}{\langle \pi^+\pi^- \mid H \mid K_S^0 \rangle} = |\eta_{+-}|\ e^{i\phi_{+-}}$$

$$\eta_{00} = \frac{\langle \pi^0\pi^0 \mid H \mid K_L^0 \rangle}{\langle \pi^0\pi^0 \mid H \mid K_S^0 \rangle} = |\eta_{00}|\ e^{i\phi_{00}}$$

H is the Hamiltonian responsible of the transition between the initial and the final states

- If the CP violation in the $K_L$ decay is due only to the $K_1$ and $K_2$ mixing , then the $K_L$ decay in two charged pions or in two neutral pions is due to the $K_1$ component, therefore we should have:

$$\eta_{+-} = \frac{\langle \pi^+\pi^- \mid H \mid \varepsilon K_1^0 \rangle}{\langle \pi^+\pi^- \mid H \mid K_1^0 \rangle} = \varepsilon \quad \text{and} \quad \eta_{00} = \frac{\langle \pi^0\pi^0 \mid H \mid \varepsilon K_1^0 \rangle}{\langle \pi^0\pi^0 \mid H \mid K_1^0 \rangle} = \varepsilon$$

- The measured value of these parameters are (PDG 2016):

$$|\eta_{+-}| = (2.232 \pm 0.011)\cdot 10^{-3} \quad ; \quad \phi_{+-} = 43.4^\circ \pm 0.5^\circ$$

$$|\eta_{00}| = (2.220 \pm 0.011)\cdot 10^{-3} \quad ; \quad \phi_{00} = 43.7^\circ \pm 0.6^\circ$$

$$\left|\frac{\eta_{00}}{\eta_{+-}}\right| = 0.9950 \pm 0.0007 \quad ; \quad \phi_{00} - \phi_{+-} = 0.34^\circ \pm 0.32^\circ$$

- Data are consistent with the hypothesis of the CP violation in the $K_1$ and $K_2$ mixing.
- However the agreement is at the level of per cent therefore it is not excluded the direct CP violation, but if this one exists, it should be at the per mille level with respect to the indirect CP violation.
- It means looking for effects at $10^{-6}$ level in the neutral K decays. This the reason why the direct CP violation in the K decays has been observed only in 2002, almost 40 years later than the Fitch-Cronin experiment.

# Direct CP violation

- We can have $\eta_{+-}$ and $\eta_{00}$ different from zero even without the mixing of the eigenstates $K_S$ and $K_L$ ($\varepsilon$=0), if the weak Hamiltonian is able to connect states with different CP eigenvalues. This mechanism is know as direct CP violation.

- We have to evaluate the following matrix element: $\langle \pi\pi | H_w | K_L^0 \rangle$ or $\langle \pi\pi | H_w | K_s^0 \rangle$

- it is useful to decompose the two pions state in term of total isospin components. The pion has isospin 1, therefore the two pions system can have total isospin 0, 1, or 2.
- If we consider the total wave function of the two pions system, we have:

$$\psi = \varphi(spatial) \cdot \chi(spin) \cdot \xi(flavour)$$

- Pions are bosons, therefore the total wave function must be symmetric. We saw that spatial part is symmetric and the spin part is not present, therefore the flavour wave function must be symmetric as well; as a consequence the total isospin must be even, hence we have I=0 or I=2.

- By using the Clebsch-Gordan coefficients we have:

$$\langle \pi^+\pi^- | = \sqrt{\tfrac{1}{3}}\,\langle 2| + \sqrt{\tfrac{2}{3}}\,\langle 0| \quad ; \quad \langle \pi^0\pi^0 | = \sqrt{\tfrac{2}{3}}\,\langle 2| - \sqrt{\tfrac{1}{3}}\,\langle 0|$$

where: $\langle \pi^+\pi^- | = \frac{1}{\sqrt{2}}\left(\langle \pi_1^+\pi_2^- | + \langle \pi_1^-\pi_2^+ |\right)$ (symmetrized state)

- We have four amplitudes that describe the $K_S$ and $K_L$ decays in two pions:

$$\langle 0 | H_w | K_S^0 \rangle \quad ; \quad \langle 2 | H_w | K_s^0 \rangle$$
$$\langle 0 | H_w | K_L^0 \rangle \quad ; \quad \langle 2 | H_w | K_L^0 \rangle$$

- $H_w$ is the weak Hamiltonian responsible of the decays.

# Direct CP violation

- The K has isospin ½ therefore in one case we have a ΔI = ½ transition and in the other one we have ΔI = 3/2. The two transitions can have a different phase factor, therefore we have a phase shift in the final state composition that depends on the total isospin
- Let's call $\delta_0$ the phase shift of the I=0 component and $\delta_2$ the one of I=2, therefore we have:

$$\langle \pi^+\pi^-| = \sqrt{\frac{1}{3}}\, e^{i\delta_2}\langle 2| + \sqrt{\frac{2}{3}}\, e^{i\delta_0}\langle 0|$$

$$\langle \pi^0\pi^0| = \sqrt{\frac{2}{3}}\, e^{i\delta_2}\langle 2| - \sqrt{\frac{1}{3}}\, e^{i\delta_0}\langle 0|$$

- Let's define the amplitudes of the $K^0$ decays as follows: $A_0=\langle 0|H_w|K^0\rangle$ and $A_2=\langle 2|H_w|K^0\rangle$
- Assuming the CPT invariance, we can deduce also the amplitudes of the $\bar{K}^0$ decays. Let's recall that:

$$CP|K^0\rangle = -|\bar{K}^0\rangle \quad \Rightarrow \quad CPT|K^0\rangle = -\langle \bar{K}^0| \quad ; \quad CPT\langle 0| = |0\rangle \quad ; \quad CPT\langle 2| = |2\rangle$$

The two pions have CP=1

**(The Time Reversal changes the inital state in a final state and viceversa)**

- If we assume that the weak interactions are invariant under CPT, we have:

$$A_0=\langle 0|H_w|\bar{K}^0\rangle \xrightarrow{CPT} -\langle K^0|H_w|0\rangle = -A_0^*$$

$$A_2=\langle 2|H_w|K^0\rangle \xrightarrow{CPT} -\langle K^0|H_w|2\rangle = -A_2^*$$

- We get rid of one phase by choosing $A_0$ real. Let's recall the expression of $K_S$ and $K_L$ in terms of $K^0$ and $\bar{K}^0$:

$$|K_S^0\rangle = \frac{(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\bar{K}^0\rangle}{\sqrt{2(1+|\varepsilon|^2)}} \quad ; \quad |K_L^0\rangle = \frac{(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle}{\sqrt{2(1+|\varepsilon|^2)}}$$

- We can express the $K_S$ and $K_L$ transitions in two pions through the amplitudes $A_0$, $A_2$ and the term that express the CP violation.

# Direct CP violation

- Let's recall the direct CP violation parametrization:

$$\eta_{+-} = \frac{\langle \pi^+\pi^- \,|\, H \,|\, K_L^0 \rangle}{\langle \pi^+\pi^- \,|\, H \,|\, K_S^0 \rangle} = |\eta_{+-}|\ e^{i\phi_{+-}} \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 \,|\, H \,|\, K_L^0 \rangle}{\langle \pi^0\pi^0 \,|\, H \,|\, K_S^0 \rangle} = |\eta_{00}|\ e^{i\phi_{00}}$$

- We have to compute the four amplitudes. With the previous definitions we have:

$$\langle \pi^+\pi^- \,|\, H_w \,|\, K_L^0 \rangle = \text{costant} \cdot \left( \varepsilon(\mathcal{Re}A_2 e^{i\delta_2} + \sqrt{2}A_0 e^{i\delta_0}) + \mathcal{Im}A_2 e^{i\delta_2} \right)$$

$$\langle \pi^+\pi^- \,|\, H_w \,|\, K_S^0 \rangle = \text{costant} \cdot \left( \mathcal{Re}A_2 e^{i\delta_2} + \sqrt{2}A_0 e^{i\delta_0} + \varepsilon \mathcal{Im}A_2 e^{i\delta_2} \right)$$

$$\langle \pi^0\pi^0 \,|\, H_w \,|\, K_L^0 \rangle = \text{costant} \cdot \left( \varepsilon(\sqrt{2}\mathcal{Re}A_2 e^{i\delta_2} - A_0 e^{i\delta_0}) + \sqrt{2}\mathcal{Im}A_2 e^{i\delta_2} \right)$$

$$\langle \pi^0\pi^0 \,|\, H_w \,|\, K_S^0 \rangle = \text{costant} \cdot \left( \sqrt{2}\mathcal{Re}A_2 e^{i\delta_2} - A_0 e^{i\delta_0} + \varepsilon\sqrt{2}\mathcal{Im}A_2 e^{i\delta_2} \right)$$

$$\text{costant} = \frac{2}{\sqrt{3}} \frac{1}{\sqrt{1+|\varepsilon|^2}}$$

$$\eta_{+-} = \frac{\langle \pi^+\pi^- \,|\, H \,|\, K_L^0 \rangle}{\langle \pi^+\pi^- \,|\, H \,|\, K_S^0 \rangle} = \frac{\varepsilon \mathcal{Re}A_2 e^{i\delta_2} + \varepsilon\sqrt{2}A_0 e^{i\delta_0} + \mathcal{Im}A_2 e^{i\delta_2}}{\mathcal{Re}A_2 e^{i\delta_2} + \sqrt{2}A_0 e^{i\delta_0} + \varepsilon \mathcal{Im}A_2 e^{i\delta_2}} \approx \frac{\varepsilon + \frac{1}{\sqrt{2}} \mathcal{Im}\left(\frac{A_2}{A_0}\right) e^{i(\delta_2-\delta_0)}}{1 + \frac{1}{\sqrt{2}} \mathcal{Re}\left(\frac{A_2}{A_0}\right) e^{i(\delta_2-\delta_0)}} \approx$$

$$\approx \left\{ \varepsilon + \frac{1}{\sqrt{2}} \frac{\mathcal{Im}(A_2)}{A_0} e^{i(\delta_2-\delta_0)} \right\} \left\{ 1 - \frac{1}{\sqrt{2}} \frac{\mathcal{Re}(A_2)}{A_0} e^{i(\delta_2-\delta_0)} \right\} \approx \varepsilon + \frac{1}{\sqrt{2}} \frac{\mathcal{Im}(A_2)}{A_0} e^{i(\delta_2-\delta_0)} = \varepsilon + \varepsilon'$$

- With the same procedure we get: $\eta_{00} \approx \varepsilon - \sqrt{2} \dfrac{\mathcal{Im}(A_2)}{A_0} e^{i(\delta_2-\delta_0)} = \varepsilon - 2\varepsilon'$

$$\varepsilon' = \frac{1}{\sqrt{2}} \frac{\mathcal{Im}(A_2)}{A_0} e^{i(\delta_2-\delta_0)}$$

# Direct CP violation

- We can define the parameters that describe the direct CP violation in the neutral K system as follows:

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | H | K_L^0 \rangle}{\langle \pi^+\pi^- | H | K_S^0 \rangle} = \varepsilon + \varepsilon' \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 | H | K_L^0 \rangle}{\langle \pi^0\pi^0 | H | K_S^0 \rangle} = \varepsilon - 2\varepsilon'$$

- Let's recall that if the CP violation is only present in the $K_1$ e $K_2$ mixing (indirect violation), then $\eta_{+-}$ and $\eta_{00}$ must be equal, therefore ε' = 0.
- **The direct CP violation implies the existence of the parameter ε' different from zero.**

- Since the discovery of the CP violation in 1964 were realized several experiments to measure ε', however this measurement is very challenging from an experimental point of view because we have to measure a parameter of the order $10^{-6}$.

- The experimental procedure consist to measure a double ratio between partial widths, in a such a way that many systematic errors cancell out:

$$R = \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \frac{\Gamma(K_L^0 \to \pi^0\pi^0)/\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_S^0 \to \pi^0\pi^0)/\Gamma(K_S^0 \to \pi^+\pi^-)}$$

$$R^{-1} = \left|\frac{\eta_{+-}}{\eta_{00}}\right|^2 = \, = \frac{|\varepsilon+\varepsilon'|^2}{|\varepsilon - 2\varepsilon'|^2} \approx 1 + 6\,\mathcal{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)$$

- The existence of the direct CP violation implies that the violation can be observed in other decays besides the neutral K system, for instance in the charged K decays

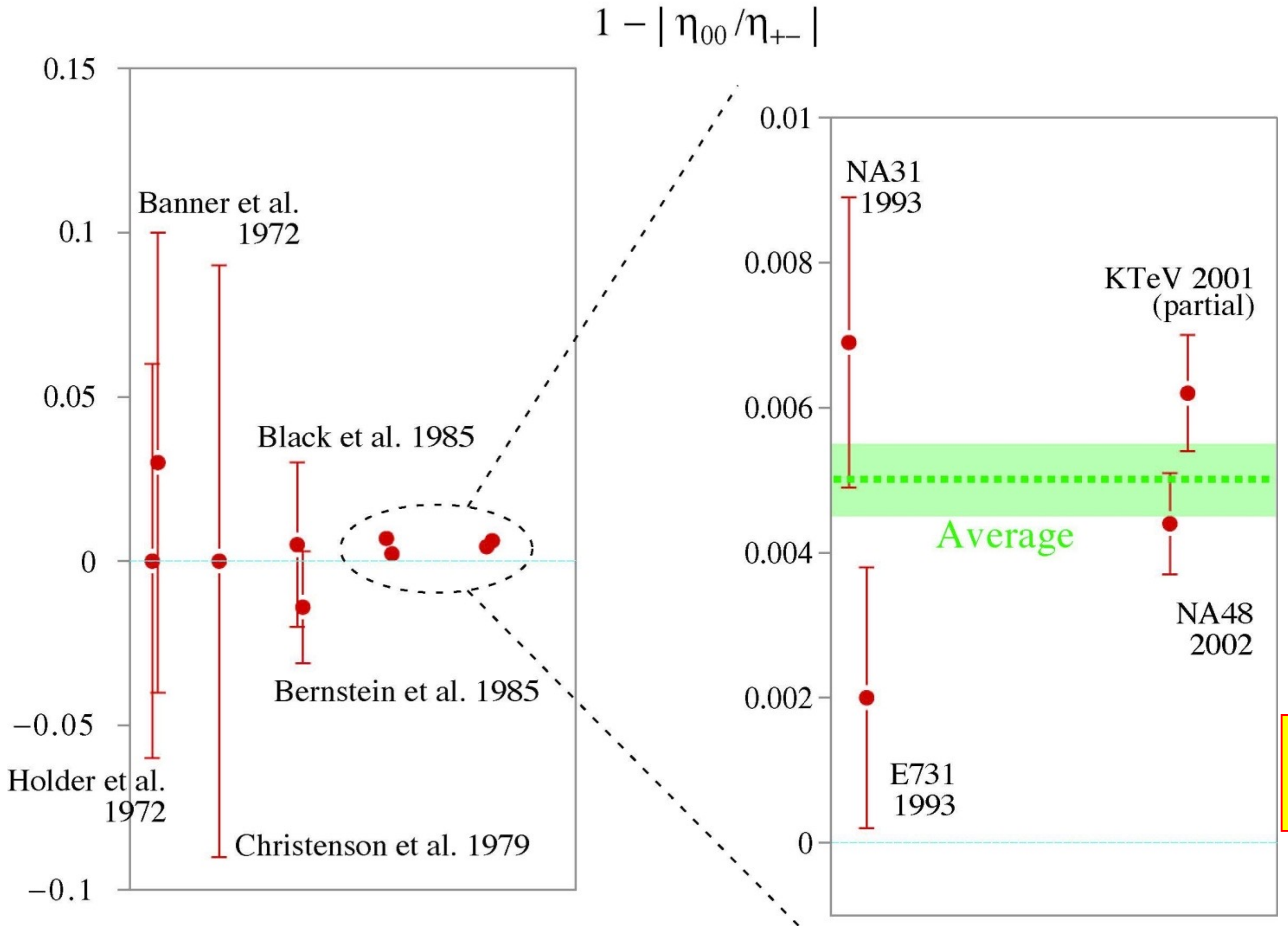
# Measurements of the direct CP violation

$$R = \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \frac{\Gamma(K_L^0 \to \pi^0\pi^0)/\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_S^0 \to \pi^0\pi^0)/\Gamma(K_S^0 \to \pi^+\pi^-)}$$

$$R^{-1} = \left|\frac{\eta_{+-}}{\eta_{00}}\right|^2 = = \frac{|\varepsilon+\varepsilon'|^2}{|\varepsilon-2\varepsilon'|^2} \approx 1+6\,\mathcal{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)$$

- A value of R different from one is the proof of the existence of the direct CP violation.

This measurement took almost 30 years of experiments before being achieved




$$\left|\frac{\eta_{00}}{\eta_{+-}}\right| = 0.9950 \pm 0.0008$$

$$\mathcal{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (1.67 \pm 0.26)\cdot 10^{-3}$$

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# End of chapter 8