Introduction to Particle Physics
- Chapter 3 -

Strange particles and SU(3)

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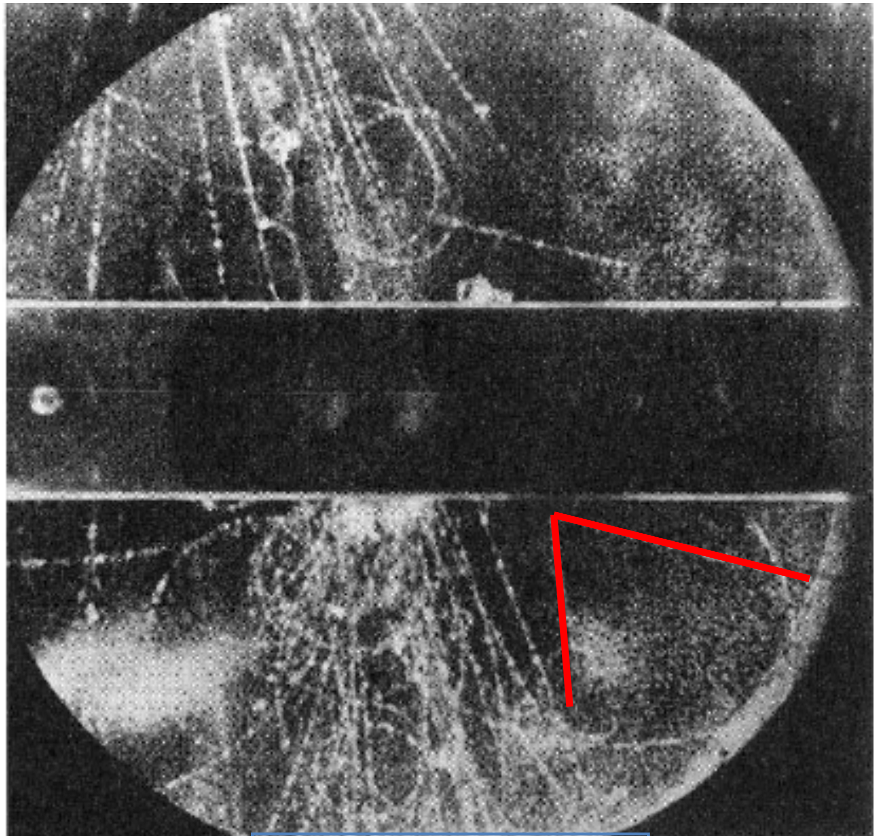
Chapter summary:

- The discovery of “strange” particles
- a new quantum number: the strangeness
- isospin and strangeness
- the particle zoo
- SU(3)

The discovery of strange particles

The pioneristic work on the “strange” particles were done by using cloud chambers at sea level and on top of the mountains and by using nuclear emulsion on aerostatic balloon.

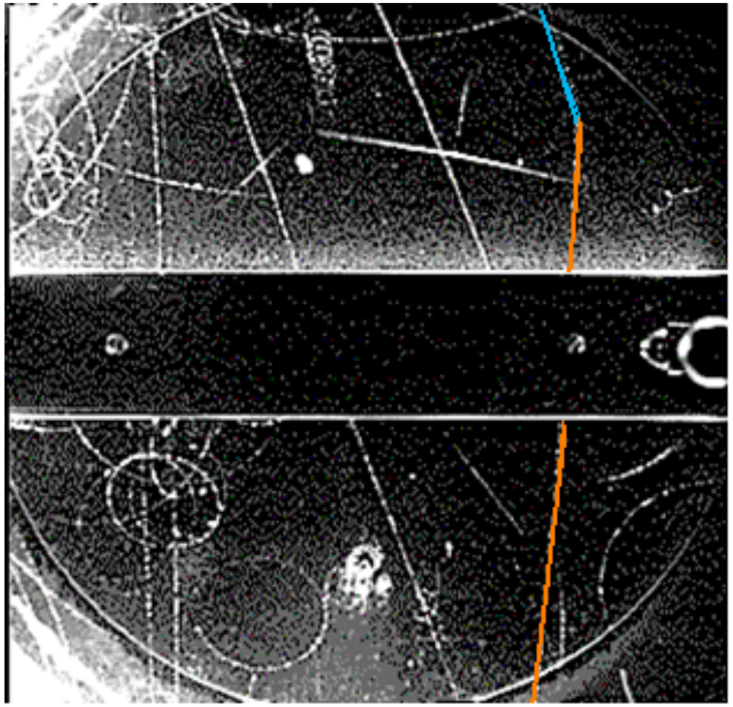


$$K^0 \to \pi^- \pi^+$$

- 1943 Leprince-Ringuet: identified a particle whose mass was 506±61 MeV.
- 1947 Rochester e Butler identified very clearly some neutral V particles in the data taken for one year with a cloud chamber at sea level.

The discovery of strange particles

Besides neutral V particles, they were also found other “strange” charged particles which decayed in one charged particle [plus neutral ones] (θ) or in three charged particles (τ).

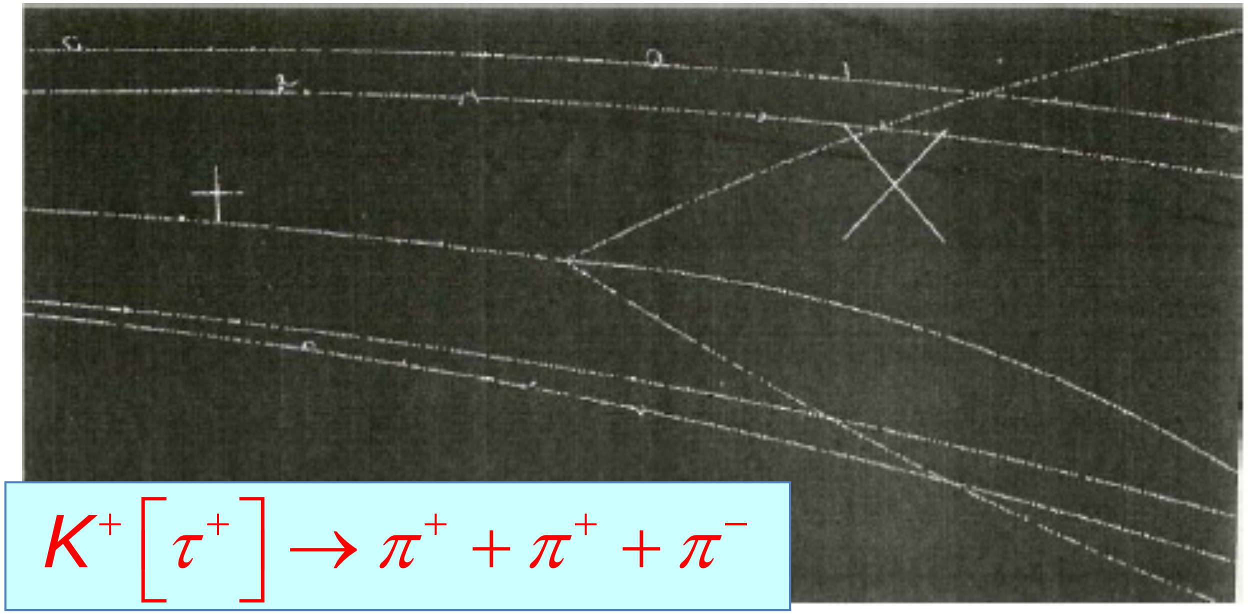


$$K^+ \rightarrow \mu^+ \nu$$

- associate production:

  in 1947 was evident that the new particles were always pair produced; one had mass around 500 MeV (K) and the other one has a higher mass, higher than the mass of the nucleon (it was called hiperon);
- The hyperon decayed into nucleon plus pion.

Decay of the K in three charged particles



$$K^+\left[\tau^+\right] \rightarrow \pi^+ + \pi^+ + \pi^-$$

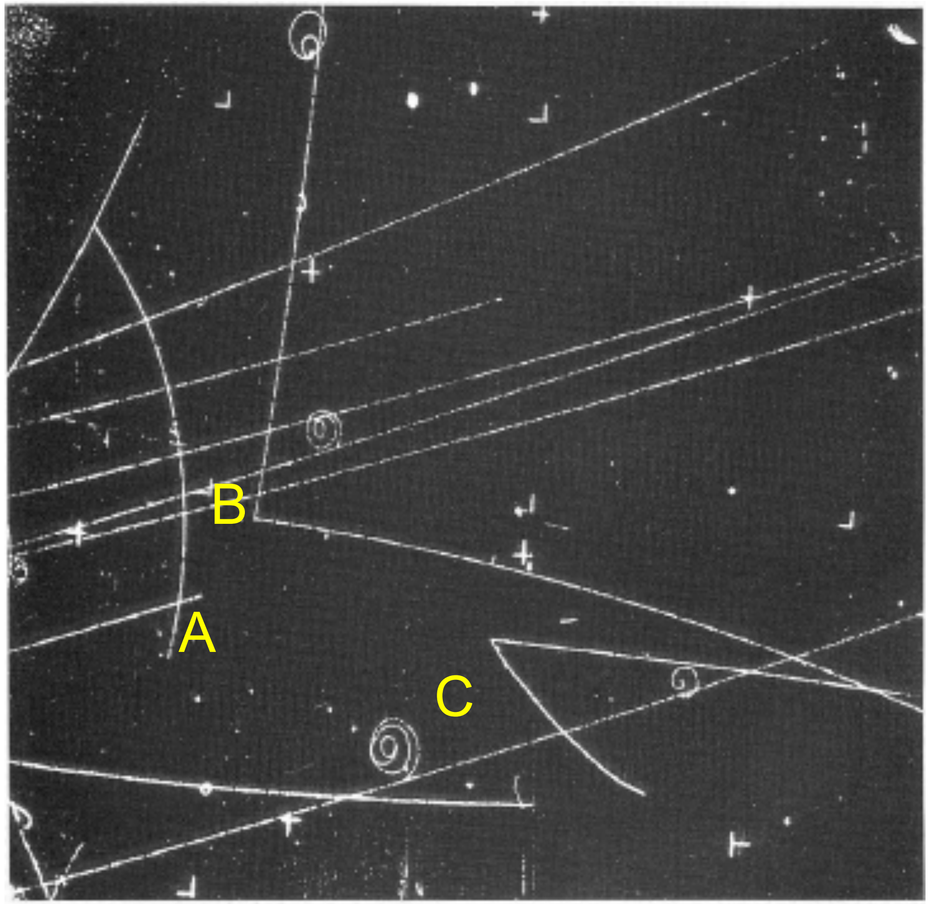
We will come back on this argument when we will talk about parity violation in the weak interactions

Moreover: $B.R.\left(K^+ \rightarrow \mu^+ + \nu\right) = 63.5\%$; $B.R.\left(K_s^0 \rightarrow \mu^+ + \mu^-\right) < 3.2 \cdot 10^{-7}$

→ GIM effect

Associate production: $\pi^- + p \rightarrow \Lambda + K$

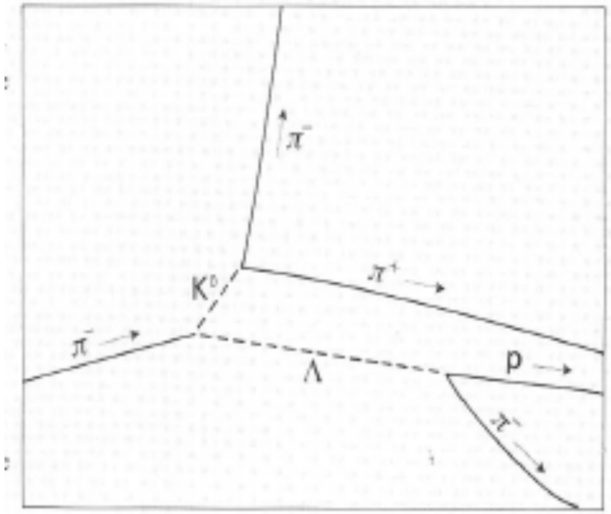
1 GeV/c π^- in a bubble chamber filled with liquid hidrogen



A) $\pi^- + p \rightarrow K^0 + \Lambda$

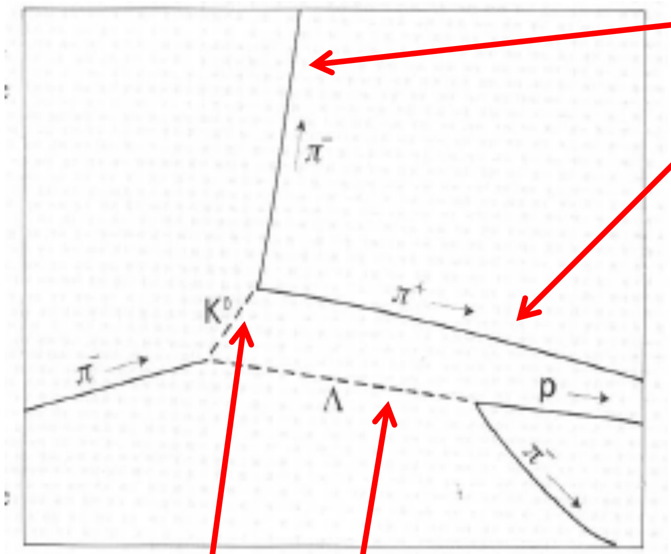
B) $K^0 \rightarrow \pi^- + \pi^+$

C) $\Lambda \rightarrow p + \pi^-$



N.B. : why K^0 and not anti-K^0?

Measurement of mass and life time



From the curvature radius we get the momentum of the charged particles and, by knowing the kind of particles, their energies.

Then we get the invariant mass of the mother.

$$m_K = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}$$

From the mass and the energy (E1+E2) we find the γ and then the β of the particle

$$\gamma = \frac{E}{m}$$

From the measure of the mean free path λ we get the life time τ of the particle

$$\lambda = \gamma\beta c\tau$$

Why are they strange particles?

- The production cross section of these particles is the order of mb, typical of the strong interactions;
- the life time are of the order of 10^{-10} s, typical of the weak interactions (e.m. int. $\sim 10^{-20}$ s , strong int. $\sim 10^{-23}$ s)

1. why the decay $\Lambda \rightarrow p + \pi^-$ does not happen through strong interactions?
2. Why the new particles are always produced in pairs?
3. (moreover τ-θ puzzle: same mass and same life time but opposed parity)

The strangeness

- An explanation of this anomaly was given in 1954 by Gell-Mann and Pais and, independently, by Nishijima.

  They introduced a new quantum number, the strangeness, that was conserved by the strong interaction but it is violated by the weak interaction.

- The strangeness is an additive quantum number. The "old" hadrons, the nucleon and the pion, have S=0, the hyperons S=-1 while the K mesons have S=± 1.

- In the production the strange particles must be produced in pairs (associate production) with opposed strangeness. The initial state has total strangeness equal to zero so, since the strong interaction conserves the strangeness, also the final state must have strangeness zero.

Example of associate production

$$\pi^- + p \rightarrow K^0 + \Lambda \quad ; \quad \pi^- + p \rightarrow K^0 + K^- + p$$

$$\pi^+ + n \rightarrow K^+ + \Lambda \quad ; \quad \pi^+ + n \rightarrow K^+ + K^- + p$$

$$\pi^- + p \rightarrow K^0 + \Sigma^0 \quad ; \quad \pi^- + p \rightarrow K^+ + \Sigma^-$$

$$\pi^+ + n \rightarrow K^+ + \Sigma^0 \quad ; \quad \pi^+ + n \rightarrow K^0 + \Sigma^+$$

$$\pi^+ + p \rightarrow K^+ + \Sigma^+$$

m($\pi^\pm$) = 139.6 MeV ; m(p) = 938.3 MeV ; m(n) = 939.6 MeV

m($K^\pm$) = 493.68 MeV ; m(K^0) = 497.67 MeV

m(Λ) = 1115.7 MeV

m($\Sigma^\pm$) = 1189.4 MeV ; m(Σ^0) = 1192.6 MeV

m(Ξ^0) = 1314.8 MeV ; m(Ξ^-) = 1321.3 MeV

(question: why are not produced the anti-hyperons)

Strangenes of K mesons

$$\pi^- + p \rightarrow K^0 + \Lambda \quad ; \quad \pi^- + p \rightarrow K^0 + K^- + p$$

$$\pi^+ + n \rightarrow K^+ + \Lambda \quad ; \quad \pi^+ + n \rightarrow K^+ + K^- + p$$

Moreover we do not observe the reaction: $\pi^- + n \rightarrow K^- + \Lambda$

K^0, Λ : opposed strangeness

K^0, K^- : opposed strangeness

Λ, K^- : same strangeness

K^+, Λ : opposed strangeness

K^+, K^- : opposed strangeness

$\bar{K}^0, K^-$: same strangeness

K^0, Σ : opposed strangeness

N.B. : by symmetry it must exist the anti-K^0

Isospin and strangenes of Σ and Ξ

$$Q = I_3 + \frac{1}{2}(B+S)$$

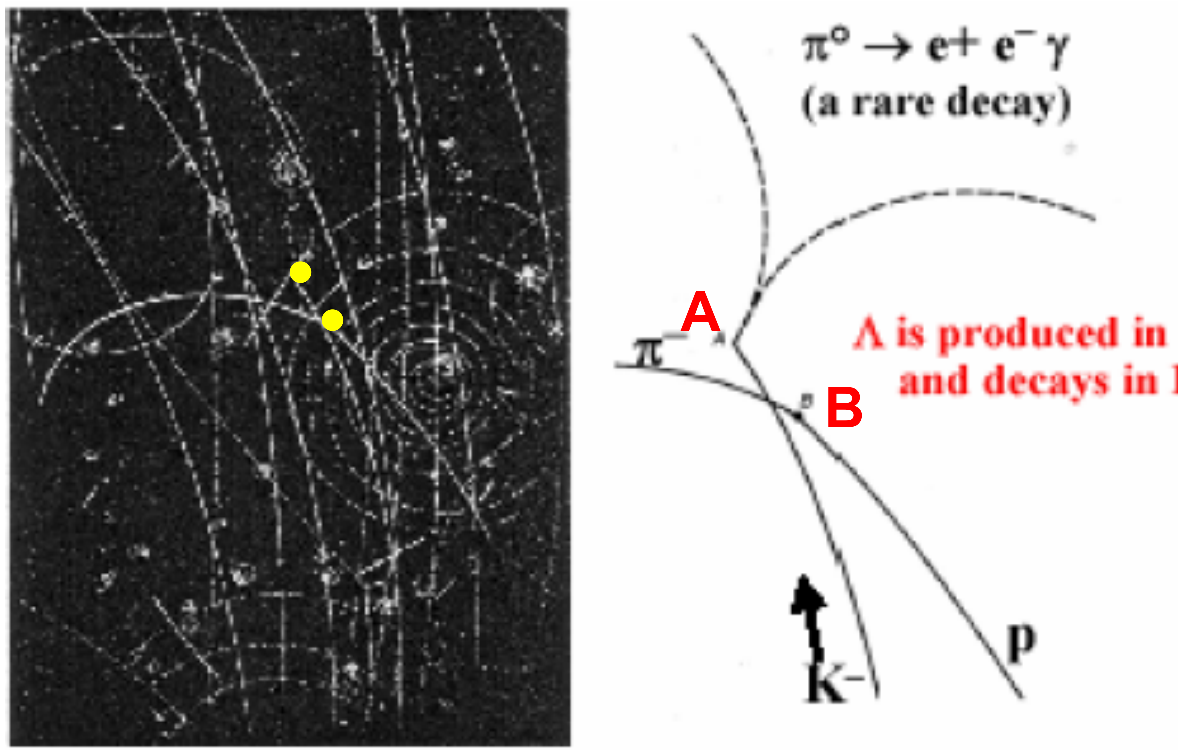
We infere the isospin

$$\left.\begin{array}{lll} Q(\Sigma^+)=1,\ B(\Sigma^+)=1,\ S(\Sigma^+)=-1 & \Rightarrow & I_3(\Sigma^+)=1 \\ Q(\Sigma^0)=0,\ B(\Sigma^0)=1,\ S(\Sigma^0)=-1 & \Rightarrow & I_3(\Sigma^0)=0 \\ Q(\Sigma^-)=-1,\ B(\Sigma^-)=1,\ S(\Sigma^-)=-1 & \Rightarrow & I_3(\Sigma^-)=-1 \end{array}\right\rangle \Rightarrow I=1$$

$$\left.\begin{array}{lll} Q(\Xi^0)=0,\ B(\Xi^0)=1,\ S(\Xi^0)=-2 & \Rightarrow & I_3(\Xi^0)=\frac{1}{2} \\ Q(\Xi^-)=-1,\ B(\Xi^-)=1,\ S(\Xi^-)=-2 & \Rightarrow & I_3(\Xi^-)=-\frac{1}{2} \end{array}\right\rangle \Rightarrow I=\frac{1}{2}$$

The production of strange particles

There have been used beams of charged K to produce new strange particles. We have an example of a K^- that it is stopped in a bubble chamber of liquid hidrogen.



A) $K^- + p \rightarrow \Lambda + \pi^0$

Strong Int.: strangeness is conserved.

B) $\Lambda \rightarrow p + \pi^-$

Weak Int.: strangeness is violated.

Interactions of the K mesons

We start from an initial state with strageness ± 1

$S = 1$, $B = 1$:

$K^+ p \rightarrow$	$K^+ p$					
$K^+ n \rightarrow$	$K^+ n$	$K^0 p$				

$S = -1$, $B = 1$:

$K^- p \rightarrow$	$K^- p$	$K^0 n$	$\pi^0 \Lambda^0$	$\pi^+ \Sigma^-$	$\pi^0 \Sigma^0$	$\pi^- \Sigma^+$
$K^- p \rightarrow$	$K^0 \Xi^0$	$K^+ \Xi^-$				
$K^- n \rightarrow$	$K^- n$	$\pi^- \Lambda^0$	$\pi^0 \Sigma^-$			
$K^- n \rightarrow$	$K^0 \Xi^-$					

With equal energy, the K^- produce more particles than K^+ because the hyperons (B=1) have strangeness = -1

For instance: $K^+ + n \rightarrow \bar{\Lambda} + p + n$ $\left[\text{S=1, B=1} \rightarrow \text{S=1, B=1} \right]$

(increase the threshold energy of the reaction)

Strange hyperons metastable

In the cosmic rays and in the accelerator experiments they have been found 6 strange hyperons metastable

	Q	S	m (MeV)	τ (ps)	$c\tau$ (mm)	Principal decays (BR in %)
Λ	0	−1	1116	263	79	$p\pi^-(64)$, $n\pi^0(36)$
Σ^+	+1	−1	1189	80	24	$p\pi^0(51.6)$, $n\pi^+(48.3)$
Σ^0	0	−1	1193	7.4×10^{-8}	2.2×10^{-8}	$\Lambda\gamma(100)$
Σ^-	−1	−1	1197	148	44.4	$n\pi^-(99.8)$
Ξ^0	0	−2	1315	290	87	$\Lambda\pi^0(99.5)$
Ξ^-	−1	−2	1321	164	49	$\Lambda\pi^-(99.9)$

To be noticed the life time typical of the e.m. interactions of the Σ^0. Why it is the only one that does not decays weakly?

Explain the B.R. of the Λ decays

The baryons (½)$^+$ and the mesons 0^-

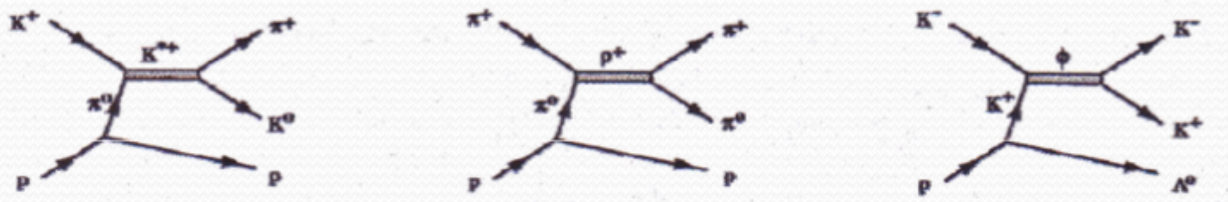
Let's classify the particles by using their spin and parity

barioni $\frac{1}{2}^+$	B	S	Y	I_3	Q	*mesoni* 0^-	B	S	Y	I_3	Q
p	+1	0	+1	+1/2	+1	K^+	0	+1	+1	+1/2	+1
n	+1	0	+1	−1/2	0	K^0	0	+1	+1	−1/2	0
Λ^0	+1	−1	0	0	0	η^0	0	0	0	0	0
Σ^+	+1	−1	0	+1	+1	π^+	0	0	0	+1	+1
Σ^0	+1	−1	0	0	0	π^0	0	0	0	0	0
Σ^-	+1	−1	0	−1	−1	π^-	0	0	0	−1	−1
Ξ^0	+1	−2	−1	+1/2	0	$\bar{K}^0$	0	−1	−1	+1/2	0
Ξ^-	+1	−2	−1	−1/2	−1	K^-	0	−1	−1	−1/2	−1

Mesonic resonances

Vector mesons that decay into pseudoscalar mesons

$J^P=1^-$ $J^P=0^-$



RISONANZA K* (K^{*+}; K^{*0})	→	M= 894 MeV/c^2; Γ= 51 MeV; I=1/2; S=+1
RISONANZA ρ (ρ^+; ρ^0; ρ^-)	→	M= 770 MeV/c^2; Γ= 150 MeV; I=1; S=0
RISONANZA ω	→	M= 783 MeV/c^2; Γ= 8.4 MeV; I=0; S=0
RISONANZA ϕ	→	M= 1019 MeV/c^2; Γ= 4.4 MeV; I=0; S=0

Mesonic resonances 1^-

	$m\ (MeV/c^2)$	$\Gamma\ (MeV)$	*decadimento*		
K^*	894	51		$K\pi$	
ρ	770	150		$\pi\pi$	
ω	783	8.4		$\pi^+\pi^0\pi^-$	
ϕ	1019	4.4	K^+K^-	$K^0\bar{K}^0$	$\pi^+\pi^0\pi^-$

mesoni 1^-	S	Y	I_3	Q
K^{*+}	+1	+1	+1/2	+1
K^{*o}	+1	+1	−1/2	0
ρ^+	0	0	+1	+1
ρ^0	0	0	0	0
ρ^-	0	0	−1	−1
ω	0	0	0	0
$\bar{K}^{*o}$	−1	−1	+1/2	0
K^{*-}	−1	−1	−1/2	−1
ϕ	0	0	0	0

Barionic resonances Σ^* and Ξ^*

There have been found barionic resonances with strangeness

RISONANZA Σ^*

Larghezza Γ= 37 MeV; $J^P=3/2^+$; I=1; S=-1

$k^- p \to \pi^- \Sigma^{*+} \to \pi^- \Lambda^o \pi^+$ → 1382.8 MeV/c²

$k^- p \to \pi^o \Sigma^{*o} \to \pi^o \Lambda^o \pi^o$ → 1383.7 MeV/c²

$k^- p \to \pi^+ \Sigma^{*-} \to \pi^+ \Lambda^o \pi^-$ → 1387.2 MeV/c²

RISONANZA Ξ^*

Larghezza Γ= 9 MeV; $J^P=3/2^+$; I=1/2; S=-2

$k^- p \to k^o \Xi^{*o}$ → 1531.8 MeV/c²

$\Xi^{*o} \to \Xi^- \pi^+$ o $\Xi^o \pi^o$

$k^- p \to k^+ \Xi^{*-}$ → 1531.8 MeV/c²

$\Xi^{*-} \to \Xi^- \pi^o$ o $\Xi^o \pi^-$

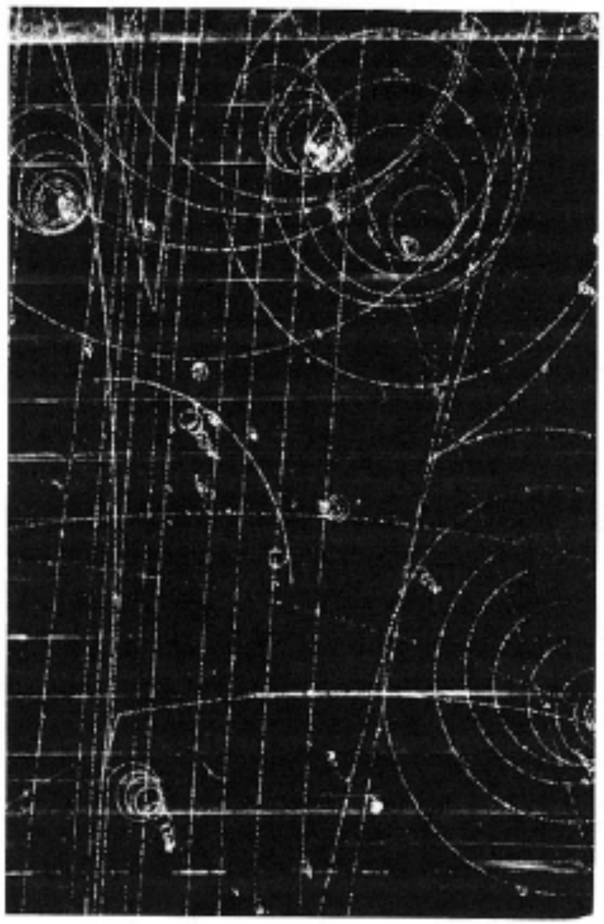
Ξ^- e Ξ^o sono barioni con $J^P=1/2^+$; S=-2; I=1/2

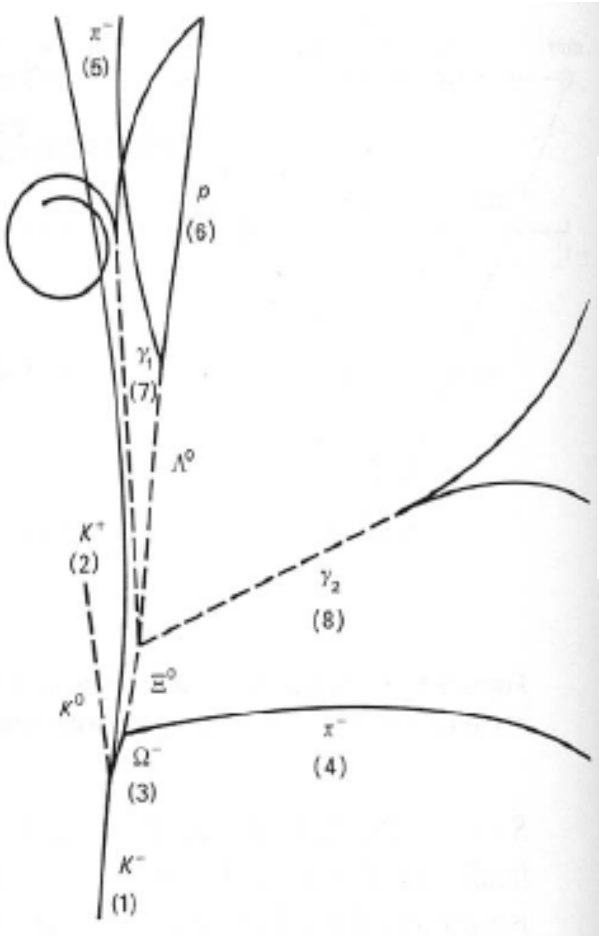
Baryonic resonances $(3/2)^+$

barioni $\frac{3}{2}^+$	S	Y	I_3	Q
Δ^{++}	0	+1	+3/2	+2
Δ^{+}	0	+1	+1/2	+1
Δ^{0}	0	+1	−1/2	0
Δ^{-}	0	+1	−3/2	−1
Σ^{*+}	−1	0	+1	+1
Σ^{*o}	−1	0	0	0
Σ^{*-}	−1	0	−1	−1
Ξ^{*o}	−2	−1	+1/2	0
Ξ^{*-}	−2	−1	−1/2	−1
Ω^{-}	−3	−2	0	−1

The discovery of the Ω^-

The Ω^- was predicted by Gell-Mann by using his new particle classification (the eightfold way)





$K^- + p \to \Omega^- + K^+ + K^0$

$\quad \hookrightarrow \Xi^0 + \pi^-$ ($\Delta S = 1$ weak decay)

$\quad\quad \hookrightarrow \pi^0 + \Lambda$ ($\Delta S = 1$ weak decay)

$\quad\quad\quad \hookrightarrow \pi^- + p$ ($\Delta S = 1$ weak decay)

$\quad\quad \hookrightarrow \gamma + \gamma$ (e.m. decay)

$\quad\quad\quad\quad \downarrow \quad\quad \downarrow$

$\quad\quad\quad\quad e^+e^- \quad e^+e^-.$

SU(3)

- SU(3) is the space of the unitary matrices 3x3 with trace equal to zero. There are 3^2-1=8 independent matrices.

- The fundamental of SU(2) is replaced by the triplet $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$

- The fundamental triplet transforms as

$$\psi \rightarrow \psi' = U\psi$$

The matrices U are unitary matrices 3x3.

- The canonical representation of the U matrices is:

$$U = e^{-\frac{1}{2} i \vartheta \hat{n} \cdot \vec{\lambda}}$$

- $\frac{1}{2}\vec{\lambda}$ are the 8 generators of the symmetry group.

Gell-mann matrices

- The λ matrices were introduced by Gell-Mann; they are equivalent to the Pauli σ matrices of per SU(2). Their standard representation is:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

There are 2 matrici diagonal matrices

The 8 generators satisfies the following commutation rules

$$\left[\frac{1}{2}\lambda_i, \frac{1}{2}\lambda_j\right] = i f_{ijk} \frac{1}{2}\lambda_k$$

$$f_{123} = 1; \; f_{147} = f_{246} = f_{257} = f_{345} = f_{516} = f_{637} = \frac{1}{2}; \; f_{458} = f_{678} = \frac{1}{2}\sqrt{3}$$

The f_{ijk} are antisymmetric for the exchange of two indexes

Hipercharge and isospin

- The diagonal generators identify two additive quantum numbers that can be used to identify the elements of a multiplet. The conventional choice is:

$$I_3 = \frac{1}{2}\lambda_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad Y = \frac{1}{\sqrt{3}}\lambda_8 = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

I_3 = isospin   Y = hipercharge

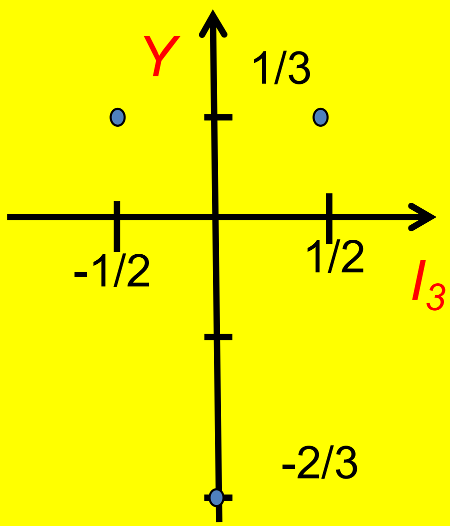
The 3 eigenstates of the fundamental triplet

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \; ; \; \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \; ; \; \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$I_3 = \frac{1}{2} \; ; \; I_3 = -\frac{1}{2} \; ; \; I_3 = 0$$

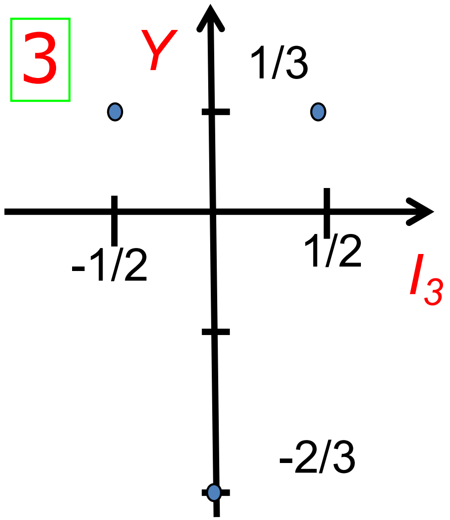
$$Y = \frac{1}{3} \; ; \; Y = \frac{1}{3} \; ; \; Y = -\frac{2}{3}$$

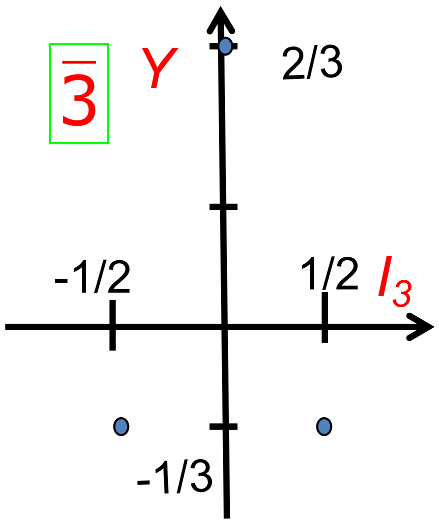
The states are represented in a bidimensional graph



Product of representations

N.B. The conjugate representation of SU(3), $\bar{3}$, transforms in a different way with respect to the 3 and has different quantum numbers.





What is a conjugate representation becomes more evident if we interpret them in terms of particles and anti-particles.

- $3 \otimes \bar{3} = 8 \oplus 1$
- $3 \otimes 3 = 6 \oplus \bar{3}$
- $3 \otimes 3 \otimes 3 = 3 \otimes \left(6 \oplus \bar{3}\right) =$

  $= 3 \otimes 6 \oplus 3 \otimes \bar{3} = 1 \oplus 8 \oplus 8 \oplus 10$

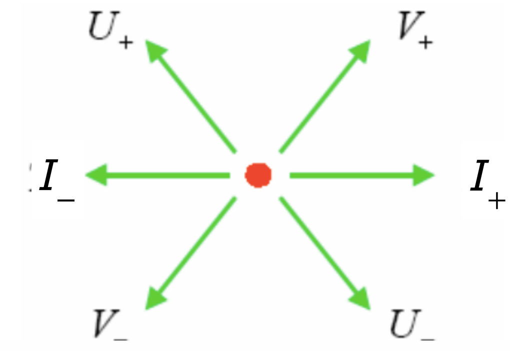
These are the irreducible representation of SU(3) that are characterised by the same quantum numbers identified by the Casimir Operators.

The states within a multiplet are identified by I_3 and Y; they are connected with each other by the ladder operators.

SU(3): ladder operators

We can use ladder operators to move from state to another with a multiplet. The ladders are constructed by using the symmetry generators.

$$I_i = \frac{1}{2}\lambda_i \quad \Rightarrow \quad \begin{aligned} I_\pm &= I_1 \pm I_2 \\ U_\pm &= I_6 \pm I_7 \\ V_\pm &= I_4 \pm I_5 \end{aligned}$$



- For example: $V_+ = I_4 + iI_5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \frac{i}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$V_+|u\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0 \quad ; \quad V_+|d\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0 \quad ; \quad V_+|s\rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = |u\rangle$$

- For the antiquark we have (watch out the minus sign):

$$V_+|\bar{u}\rangle = -|\bar{s}\rangle \quad ; \quad V_+|\bar{d}\rangle = 0 \quad ; \quad V_+|\bar{s}\rangle = 0$$

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End of chapter 3