Introduction to Particle Physics
- Chapter 2 -

Symmetries and Conservation Laws

Claudio Luci
SAPIENZA
UNIVERSITÀ DI ROMA

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Chapter summary:

- Conservation laws
- Discrete and continuous symmetries
- Baryonic and leptonic number
- parity
- Charge conjugation
- Time Reversal, CPT and Identical particles
- Isospin
- Gell-mann-Nishijima's formula

Conservation Laws

Whatever is not forbidden, it happens

$$n \nrightarrow p + e^- \quad ; \quad p \nrightarrow n + e^+ + \nu \quad ; \quad \mu \nrightarrow e + \gamma$$

- If something does not happen, it means that is violated some conservation law

1. "Sacred" conservation laws: they derive from a symmetry of the Lagrangian (Noether's theorem)

   - *Time translation → energy*
   - *Spatial translation→ momentum*
   - *Rotation invariance → angular momentum*
   - *gauge invariance (U(1)) → elettrical charge*
   - *etc… etc…*

1. "empirical" conservation laws: they are not "protected" by a symmetry of the Lagrangian and could be violated in some process

   - for instance: baryonic number or leptonic number

Two kind of symmetries

$$|\psi'\rangle = U|\psi\rangle \quad ; \quad \langle\psi'|\psi'\rangle = \langle\psi|\psi\rangle$$

U = unitary operator of the symmetry

- **Continuous symmetries:** for instance spatial translation
  - They can be derived from the identity by doing infinitesimal transformation;
  - it exists a generator of the transformation:

$$U(\alpha)=e^{i\alpha F}$$

α = real parameter;
F = generator of the transformation

  - **F** is a hermitian operator that commutes with the Hamiltonian → it is an observable
  - **Additive quantum numbers**

- **Discrete symmetries**: for instance parity
  - you can not get it by doing "little steps"
  - **Multiplicative quantum numbers**

Baryonic Number

- Experimentally it was observed in nuclear reactions that the number of nucleons was always conserved; this continued to be true also taking into account the "strange" fermions like Λ e Σ.

- Stuchelberg made the hypothesis that the baryonic number must be conserved

- Nowadays we say that in a reaction must be conserved the number of quarks minus the number of antiquarks:

$$B=\frac{1}{3}\left(N_q - N_{\bar{q}}\right)$$

- However we have not found (yet) a symmetry of the Hamiltonian that "protects" this conservation law, therefore there are theories (like GUT for instance) that violate the conservation law of baryonic number.

- The violation of baryonic number is one of the "base" ingredients to explain the "disappearance" of the antimatter.

Leptonic Number

- Experimentally it is observed that the leptonic number is conserved too
- Moreover we have (we had) three independent conservation laws for the three different leptons (electron, muon and tau) because it does not exist the decay:

$$\mu \to e + \gamma \quad \text{but} \quad \mu \to e + \bar{\nu}_e + \nu_\mu$$

$$\text{B.R.}(\mu \to e + \gamma) < 10^{-11} \quad [\text{experimental limit before MEG exp. (2008)}]$$

- However in 2002 we had the final experimental evidence of neutrino oscillations (that is mass eigenstates are not equal to flavor eigenstates), therefore **the leptonic number is not conserved separately any longer**

$$\text{B.R.}(\mu \to e + \gamma) \approx 10^{-55} \quad [\text{SM value}]$$

- New theories beyond SM (like SuSy for instance) foresee: $\text{B.R.}(\mu \to e + \gamma) \approx 10^{-11 \div -15}$

$$\text{B.R.}(\mu^+ \to e^+ + \gamma) < 4.2 \cdot 10^{-13} \quad [\text{MEG result (2016)}]$$

$$\text{B.R.} \approx 4 \cdot 10^{-14} \quad [\text{sensitivity expected by MEG-II}]$$

Parity

$$x,\ y,\ z \ \rightarrow\ -x,\ -y,\ -z$$

$$\vec{r} \Rightarrow -\vec{r}$$
$$t \Rightarrow t$$
$$\vec{p} \Rightarrow -\vec{p}$$
$$\vec{r} \times \vec{p} \Rightarrow \vec{r} \times \vec{p}$$
$$\vec{s} \Rightarrow \vec{s}$$

- The eigenvalue of P are: +1 e -1
- A wave function can or can not have a given parity.
  In case it has a parity, it can be even (eigenvalue +1)
  or odd (eigenvalue -1). For instance spherical harmonics.
- The parity of spherical harmonics is:

$$Y_l^m(\vartheta,\varphi) \rightarrow Y_l^m(\pi-\vartheta,\pi+\varphi) = (-1)^l\, Y_l^m(\vartheta,\varphi)$$

- The strong and e.m. interactions do conserve the parity

- Some reactions are observed while other ones are not observed, like for instance

$$\pi^- + d \rightarrow n + n \quad ; \quad \pi^- + d \not\rightarrow n + n + \pi^0$$

- This can be explained by assigning to the pion an **intrinsic parity**

N.B. the parity operator does not act on the spin wave function

Pion intrinsic parity: η_π

It has been measured in the absorption of slow pions in deuterium

$$\pi^- + d \rightarrow n + n$$

- The conservation of parity requires: $\eta_\pi \cdot \eta_d \cdot (-1)^{L_i} = \eta_n \cdot \eta_n \cdot (-1)^{L_f}$

deuteron: $S_d = 1 \Rightarrow \eta_d = 1 \quad \left[\eta = (-1)^{L+S+1}\right]$

$$\eta_d = 1,\ \eta_n \cdot \eta_n = 1,\ L_i = 0 \Rightarrow \eta_\pi = (-1)^{L_f}$$

•The two neutrons are identical fermion, therefore the total wave function must be antisymmetric. The initial state has J=1 because $S_\pi = 0$, $S_d=1$ and $L_i = 0$.

$|\psi_{nn}^{(1)}\rangle = |J=1, S=1, L_f = 0,2\rangle \Rightarrow \psi$ symmetric

$|\psi_{nn}^{(2)}\rangle = |J=1, S=1, L_f = 1\rangle \Rightarrow \psi$ antisymmetric

$|\psi_{nn}^{(3)}\rangle = |J=1, S=0, L_f = 0,2\rangle \Rightarrow \psi$ symmetric

$$\eta_\pi = (-1)^1 = -1$$

Charge conjugation

- The charge conjugation operator transform a particle into its antiparticle, who has all internal quantum number of opposite sign (electrical charge, strangeness, magnetic moment, baryonic/leptonic number, etc…).

- Only the "neutral" particles can be eigenstate of the charge conjugation operator **C**; for instance π^0 and photon are eigenstates of C … but neutron or neutrino are not!

$$C|\alpha,\psi\rangle = C_\alpha|\alpha,\psi\rangle \quad \left[\text{eigenstate of C}\right]$$
$$C|a,\psi\rangle = |\bar{a},\psi\rangle \quad \left[\bar{a}\text{=antiparticle of a}\right]$$

- The eigenvalue C_α are +1 and -1

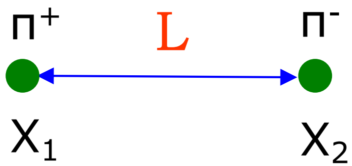
- Eigenstates of C can be constructed by using particle-antiparticle pair, where the operator C exchange the two particles;
- If the state is symmetric or antisymmetric due to exchange, we have:

$$C|a,\psi_1;\bar{a},\psi_2\rangle = |\bar{a},\psi_1;a,\psi_2\rangle = \pm|a,\psi_1;\bar{a},\psi_2\rangle$$

in this case $|a,\psi_1;\bar{a},\psi_2\rangle$ is an eigenstate of C

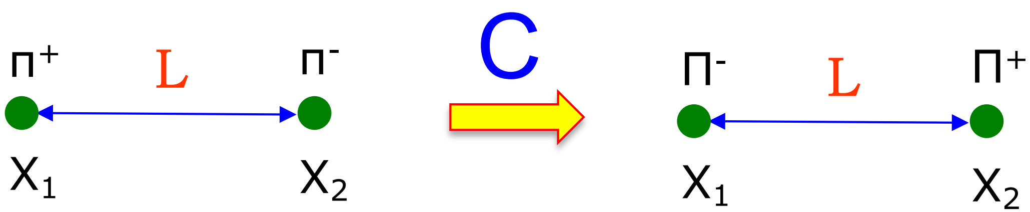
$\pi^+ \pi^-$ state

- Let's take a $\pi^+ \pi^-$ pair in a state of angular momentum L:



$$C|\pi^+\pi^-,L\rangle = (-1)^L|\pi^+\pi^-,L\rangle$$

because exchanging the two pions is equivalent to invert their spatial position



- N.B. let's recall that the spin of the pion is zero, so we do not have to take into account the spin in the overall state wave function

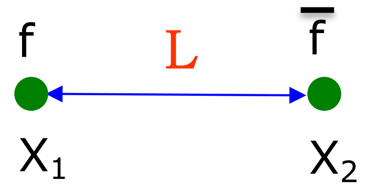
$f\bar{f}$ state

- **fermion pair of spin ½ .**
- To be kept in mind the following properties of the spin wave function:

$$|\uparrow\uparrow\rangle \quad ; \quad \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \quad ; \quad |\downarrow\downarrow\rangle \qquad [\text{Symmetric}]$$

S=1, S_Z=1 S=1, S_Z=0 S=1, S_Z=-1

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \quad [S=0,\ S_z=0] \qquad [\text{antisymmetric}]$$



- The two fermions exchange introduce, because of the spin, a factor $(-1)^{S+1}$
- Moreover, because of the opposed intrinsic parity of the fermion-antifermion pair (see Dirac eq.), we have to introduce another factor -1.
- Therefore the eigenvalue of the operator charge conjugation for this state is:

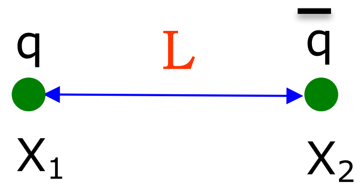
$$C|f\bar{f},J,L,S\rangle = (-1)^L \cdot (-1)^{S+1}(-1)|f\bar{f},J,L,S\rangle = (-1)^{L+S}|f\bar{f},J,L,S\rangle$$

- **This result imposes some constraints to the quark content of the mesons**

π^0 charge conjugation

$S_{\pi^0} = 0$

S = spin of $q\bar{q}$ pair forming the π^0
L = angular orbital momentum of $q\bar{q}$ pair



- The sum of S and L of the quark-antiquark pair gives the spin of the pion.

$$\Rightarrow \ L + S = 0 \quad \Rightarrow \ C_{\pi^0} = (-1)^{L+S} = (-1)^0 = 1$$

- Experimentally we find that the dominant decay channel is: $\pi^0 \to \gamma\gamma$

$$C\,|\pi^0\rangle = C_{\pi^0}\,|\pi^0\rangle \ ;$$

$$C\,|\gamma\gamma\rangle = C_\gamma \cdot C_\gamma\,|\gamma\gamma\rangle = 1 \cdot |\gamma\gamma\rangle \text{ since } C_\gamma^2 = 1$$

- Since the e.m. interactions are invariant per charge conjugation, we must have $C_{\pi^0} = 1$ according to the quark model.

N.B. : $C_\gamma = -1$ since ($q \to -q$; $E,B \to -E, -B$) $\Rightarrow$ C-parity of 3 photons is: $(C_\gamma)^3 = -1$

$$R = \frac{\Gamma(\pi^0 \to 3\gamma)}{\Gamma(\pi^0 \to 2\gamma)} < 3 \cdot 10^{-8}$$

Naively one would expect something of the order α=1/137

questions

$$R = \frac{\Gamma\left(\pi^0 \to 3\gamma\right)}{\Gamma\left(\pi^0 \to 2\gamma\right)} < 3 \cdot 10^{-8}$$

1) From an experimental point of view, why we can not simply say R=0?

1) How can we be sure that the π^0 decays in two photons? Maybe we could have just “lost” and additional photon.

2) Does the phase space play a role in considering the third photon? In other words, could be the suppression an effect of the phase space?

Time Reversal (t → -t)

- The operators P and C are hermitian and unitary operators; they have eigenstates and give rise to multiplicative quantum numbers;
- The time reversal operator T is antiunitary and does not have eigenstates;
- The correct definition of T in quantum mechanics was given by Wigner:

$$T\psi(t) = \psi^*(-t) \quad \Rightarrow \quad -i\frac{d\psi^*(t')}{dt'} = H\psi^*(t')$$

now let's take the complex conjugate of the equation and we go back to the original Schrödinger equation:

$$i\frac{d\psi(t')}{dt'} = H\psi(t')$$

PROVIDED THAT THE HAMILTONIAN H IS REAL !!!

- If H contains some imaginary terms then Time Reversal symmetry is violated !!

N.B. Time Reversal violation can not be searched in some particle decays since there are no quantum numbers associated to T

CPT theorem

- There is no fundamental reason in Nature to enforce the P, C and T symmetry separately;
- however a quantum field theory that is invariant for a Lorentz transformation, must be also invariant for a CPT symmetry transformation;
- this brings some consequences: for instance a particle and its antiparticle must have the same mass and the same life time;
- possible violations of CPT are looked for in the mass differences between particle and antiparticle.
- for instance the ASACUSA experiment at CERN established in 2003 the following limit for proton and antiproton mass difference:

$$\frac{\left| m_p - m_{\bar{p}} \right|}{m_p} \leq 10^{-8}$$

Identical particles

- The Quantum Mechanics affirms that in a system composed by identical particles (also called **indistinguishable** particles), we can not perform any measurement able to identify the swap of any two particles of the system:

$$\Rightarrow \quad \left|\zeta_2,\zeta_1\right\rangle = e^{i\alpha}\left|\zeta_1,\zeta_2\right\rangle$$

- if we do a second swap of the same particles:

$$\left|\zeta_1,\zeta_2\right\rangle = e^{i\alpha}\left|\zeta_2,\zeta_1\right\rangle = e^{i2\alpha}\left|\zeta_1,\zeta_2\right\rangle \quad \Rightarrow \quad e^{i2\alpha} = 1 \quad \Rightarrow \quad e^{i\alpha} = \pm 1$$

- BOSONS: $e^{i\alpha} = 1$ (symmetric particles)
- FERMIONS: $e^{i\alpha} = -1$ (antisymmetric particles)

N.B. the total wave function can be the product of various parts:

$$\zeta = \psi(spatial) \bullet \sigma(spin) \bullet \varphi(flavour) \bullet \tau(colour)$$

N.B. it is the total w.f. that must respect the symmetry and not the individual parts

A few conserved quantum numbers

- N.B. not all interactions do respect the various conservation laws. For instance:

Quantity	Strong	EM	Weak	Comments
Baryon number	Y	Y	Y	no $p \rightarrow e^+\pi^o$
Lepton number(s)	Y	Y	Y	no $\mu^- \rightarrow e^-\gamma$
top	Y	Y	***N***	discovered 1995
strangeness	Y	Y	***N***	discovered 1947
charm	Y	Y	***N***	discovered 1974,
bottom	Y	Y	***N***	discovered 1977
Isospin	Y	***N***	***N***	$p = n$ ($m_u \cong m_d$)
Charge con. (C)	Y	Y	***N***	part. → anti-part.
Parity (P)	Y	Y	***N***	1956
CP or Time (T)	Y	Y	y/n	small ***No***
CPT	Y	Y	Y	**sacred**

Interactions relative strength

Coupling Constants		
Strong	α_s	1
Electromagnetic	α	1/137
Weak	α_W	10^{-6}
Gravity	α_g	10^{-39}

- Due to interactions relative strength and to conservation laws, the interactions take place in the following order:
  1. Strong interaction (it acts only on quarks and gluons);
  2. e.m. interaction (for instance if there are photons or leptons as interacting particles; they do not interact strongly and e.m. comes first);
  3. weak interaction (for instance if there are neutrinos that interact only weakly or if there is a violation of a conservation law followed by strong and e.m. interactions, the weak interaction come first).
  4. (Gravity is not considered at all in particle physics)

A few example of reactions

- $\nu_\mu + p \rightarrow \mu^+ + n$

  weak interaction: forbidden
  (see lepton number)

- $\nu_e + p \rightarrow e^- + \pi^+ + p$

  weak interaction: allowed, but watch out the threshold energy.

- $\Lambda \rightarrow e^- + \pi^+ + \bar{\nu}_e$

  weak interaction: forbidden
  (see barion number)

- $K^+ \rightarrow \mu^- + \pi^0 + \bar{\nu}_\mu$

  weak interaction: allowed.
  (what about strangeness?)

- $K^+ \rightarrow \pi^+ + \pi^0$

  it happens (B.R.=20%). Which interaction? (mind the strangeness).

Why these reactions do not occur?

It is not always straightforward to find the reason

$K^{+}p \rightarrow \Lambda\pi^{+}\pi^{+}$ strangeness violation. It becomes evident by looking at the quark content.

$$B.R.(\phi(1020) \rightarrow KK) \approx 83\%$$
$$B.R.(\phi(1020) \rightarrow \pi^{+}\pi^{-}\pi^{o}) \approx 15\%$$

OZI rule

$$\frac{\sigma(\pi^{+}p \rightarrow \pi^{+}p)}{\sigma(\pi^{-}p \rightarrow \pi^{-}p)} = \frac{195\ \text{mb}}{22\ \text{mb}} = 8.86 \approx 9$$

isospin

$$\frac{\Gamma(\pi^{-} \rightarrow e^{-}\overline{\nu}_{e})}{\Gamma(\pi^{-} \rightarrow \mu^{-}\overline{\nu}_{\mu})} = 1.2x10^{-4}$$

helicity

Isospin

- Heisemberg proposed in 1932 that proton and neutron were two different states of the same particle: the nucleon.

- To implement this idea we can represent the nucleon as a two components column vector:

$$N=\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \quad ; \quad p=\begin{pmatrix}1\\ 0\end{pmatrix} \quad ; \quad n=\begin{pmatrix}0\\ 1\end{pmatrix}$$

- The mathematical formalism is identical to the one used by Pauli to describe the spin of the electron.

- The proton has $I_3 = ½$ while the neutron has $I_3 = -½$

- If the strong interactions are invariant for rotation in the isospin space, then the isospin must be conserved in all processes involving strong interaction.

Gell-Mann - Nishijima's formula

The third component of the isospin distinguish the electrical charge within an isospin multiplet

charge

$$Q = I_3 + \frac{1}{2}(B+S)$$

strangeness

Baryonic number

N.B. $B+S = Y$ (hypercharge)

$$\Rightarrow \quad Q = I_3 + \frac{1}{2}Y$$

The electromagnetic interaction breaks the isospin symmetry; as a consequence the masses within a multiplet are different (m_p different from m_n)

Isospin

Let's see a dynamical consequence of the isospin conservation

- Let's suppose to have two nucleons. From the rule of the addition of angular momentum we know that the total isospin can be 1 or 0.

Symmetric triplet; I = 1

a) $|1,1\rangle = pp$

b) $|1,0\rangle = \frac{1}{\sqrt{2}}(pn+np)$

c) $|1,-1\rangle = nn$

Antisymmetric isosinglet; I = 0

$|0,0\rangle = \frac{1}{\sqrt{2}}(pn - np)$

- It exists a bound state proton-neutron (deuteron), but do not exist bound states proton-proton or neutron-neutron, hence the deuteron must be an isospin singlet, otherwise they should exist also the other two states that differ by a rotation in the isospin space.

nucleon-nucleon scattering

Let's consider the following processes:

a) $p + p \rightarrow d + \pi^+$
b) $p + n \rightarrow d + \pi^0$
c) $n + n \rightarrow d + \pi^-$

the π has isospin 1 because it exists in three different states

- Since the deuteron has I=0, for the right hand processes we have:

$$d + \pi^+ = |1,1\rangle \;;\; d + \pi^0 = |1,0\rangle \;;\; d + \pi^- = |1,-1\rangle$$

while for the ones on the left we have:

$$p + p = |1,1\rangle \;;\; p + n = \frac{1}{\sqrt{2}}\left(|1,0\rangle + |0,0\rangle\right) \;;\; n + n = |1,-1\rangle$$

- Since the total isospin I must be conserved, only the state with I=1 will contribute. The scattering amplitudes have to be in the ratio:

$$1 : \frac{1}{\sqrt{2}} : 1$$

and the σ $2 : 1 : 2$

- The processes a) and b) have been measured, and once we take into account the e.m. interaction, they are in the predicted ratio.

pion-nucleon scattering

Let's consider the four reactions:

a) $\pi^+ + p \rightarrow \pi^+ + p$

b) $\pi^- + p \rightarrow \pi^0 + n$

c) $\pi^- + p \rightarrow \pi^- + p$

d) $\pi^- + n \rightarrow \pi^- + n$

The initial states are the composition of I=1 and I=1/2 that give I=1/2 and I=3/2

$$1 \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{3}{2}$$

- Let's express the various states in the base of the total isospin by using the Clebsch-Gordan coefficients

$$|\pi^+,p\rangle = \left|\tfrac{3}{2},+\tfrac{3}{2}\right\rangle \quad ; \quad |\pi^-,p\rangle = \sqrt{\frac{1}{3}}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$

$$|\pi^-,n\rangle = \left|\tfrac{3}{2},-\tfrac{3}{2}\right\rangle \quad ; \quad |\pi^0,n\rangle = \sqrt{\frac{2}{3}}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$

pion-nucleon scattering

- Let's write the four processes in the new base:

a) $|\frac{3}{2},+\frac{3}{2}\rangle \rightarrow |\frac{3}{2},+\frac{3}{2}\rangle$

b) $\sqrt{\frac{1}{3}}\,|\frac{3}{2},-\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}\,|\frac{1}{2},-\frac{1}{2}\rangle \rightarrow \sqrt{\frac{2}{3}}\,|\frac{3}{2},-\frac{1}{2}\rangle + \sqrt{\frac{1}{3}}\,|\frac{1}{2},-\frac{1}{2}\rangle$

c) $\sqrt{\frac{1}{3}}\,|\frac{3}{2},-\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}\,|\frac{1}{2},-\frac{1}{2}\rangle \rightarrow \sqrt{\frac{1}{3}}\,|\frac{3}{2},-\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}\,|\frac{1}{2},-\frac{1}{2}\rangle$

d) $|\frac{3}{2},-\frac{3}{2}\rangle \rightarrow |\frac{3}{2},-\frac{3}{2}\rangle$

- to compute the probability amplitude, we have to perform the scalar product $\langle f|S|i\rangle$

$$\langle \tfrac{3}{2}, I_3|S|\tfrac{3}{2}, I_3\rangle = A_{3/2} \;;\; \langle \tfrac{1}{2}, I_3|S|\tfrac{1}{2}, I_3\rangle = A_{1/2}$$

N.B. $A_{1/2} \neq A_{3/2}$

a) $A_{tot}=A_{3/2}$

b) $A_{tot}=\left(\frac{\sqrt{2}}{3}A_{3/2} - \frac{\sqrt{2}}{3}A_{1/2}\right)$

c) $A_{tot}=\left(\frac{1}{3}A_{3/2} + \frac{2}{3}A_{1/2}\right)$

d) $A_{tot}=A_{3/2}$

N.B. strong interactions do not mix states with different total isospin

pion-nucleon scattering

- The cross-sections of the four processes are proportional, by means of a factor K equal for all 4 processes (that takes into account the phase space, 2π factors, etc…), to:

$$\text{a)}\quad \sigma\left(\pi^{+} + p \rightarrow \pi^{+} + p\right) = K\left|A_{3/2}\right|^2$$

$$\text{b)}\quad \sigma\left(\pi^{-} + p \rightarrow \pi^{0} + n\right) = K\left|\frac{\sqrt{2}}{3}A_{3/2} - \frac{\sqrt{2}}{3}A_{1/2}\right|^2$$

$$\text{c)}\quad \sigma\left(\pi^{-} + p \rightarrow \pi^{-} + p\right) = K\left|\frac{1}{3}A_{3/2} + \frac{2}{3}A_{1/2}\right|^2$$

$$\text{d)}\quad \sigma\left(\pi^{-} + n \rightarrow \pi^{-} + n\right) = K\left|A_{3/2}\right|^2$$

- From these relations we infer that the processes a) and d) must have the same cross-section at the same energy. This has been verified experimentally.

- For the other processes we need to know $A_{1/2}$ and the relative phase between the amplitudes.

The Δ resonance

- The Δ resonance has isospin 3/2 (it exists in 4 states of different charge), therefore all processes in which it appears as formation resonance can proceed only through the channel with I=3/2. As a consequence:

$$\text{a)}\ \sigma\left(\pi^+ + p \rightarrow \pi^+ + p\right) = K\left|A_{3/2}\right|^2$$

$$\text{b)}\ \sigma\left(\pi^- + p \rightarrow \pi^0 + n\right) = K\left|\frac{\sqrt{2}}{3}A_{3/2}\right|^2 = \frac{2}{9}\left|A_{3/2}\right|^2$$

$$\text{c)}\ \sigma\left(\pi^- + p \rightarrow \pi^- + p\right) = K\left|\frac{1}{3}A_{3/2}\right|^2 = \frac{1}{9}\left|A_{3/2}\right|^2$$

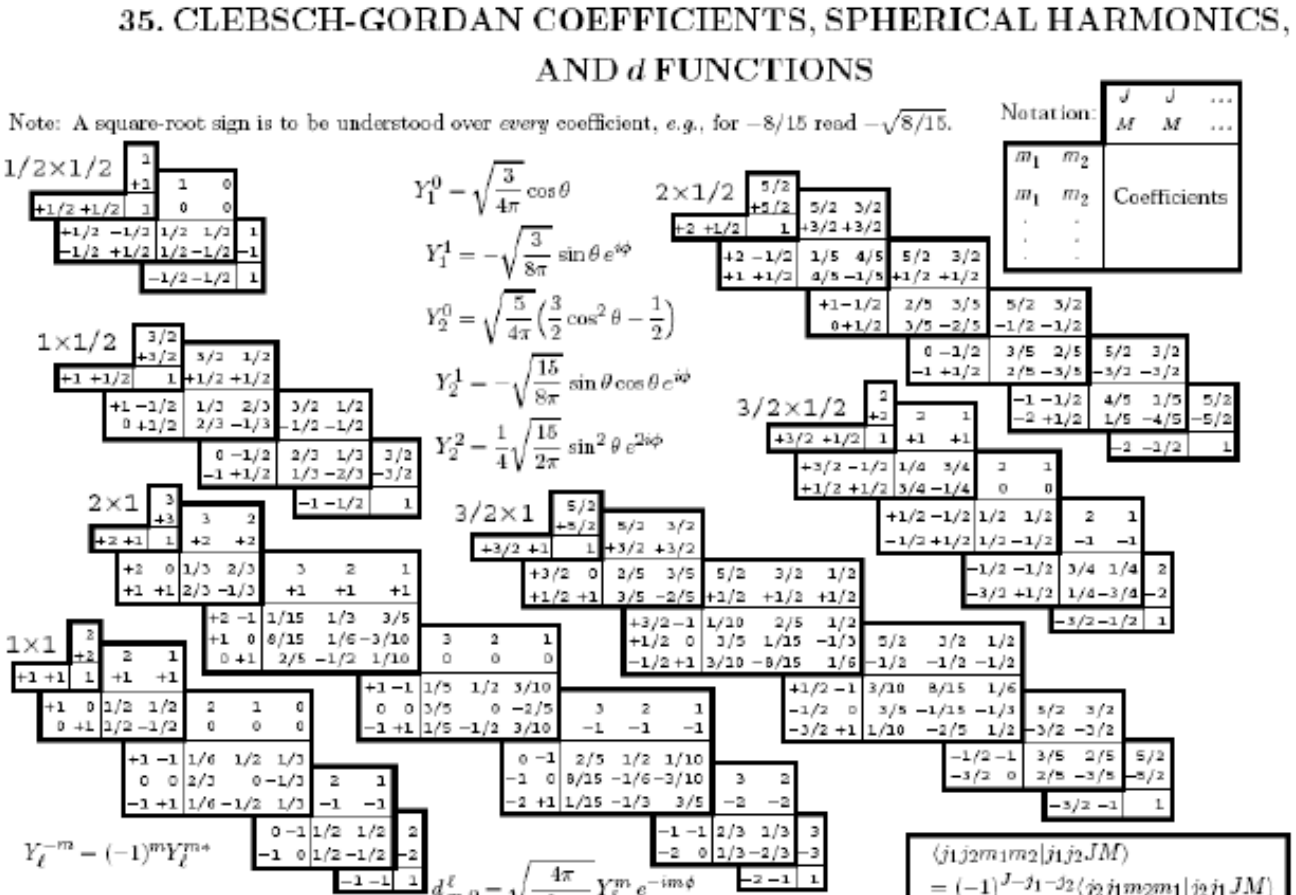
$$\text{d)}\ \sigma\left(\pi^- + n \rightarrow \pi^- + n\right) = K\left|A_{3/2}\right|^2$$

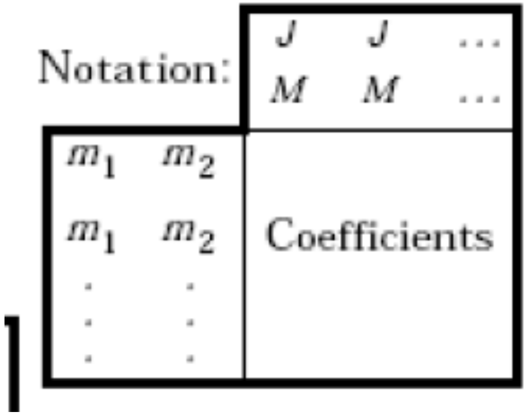
- from these relations we can infer now:

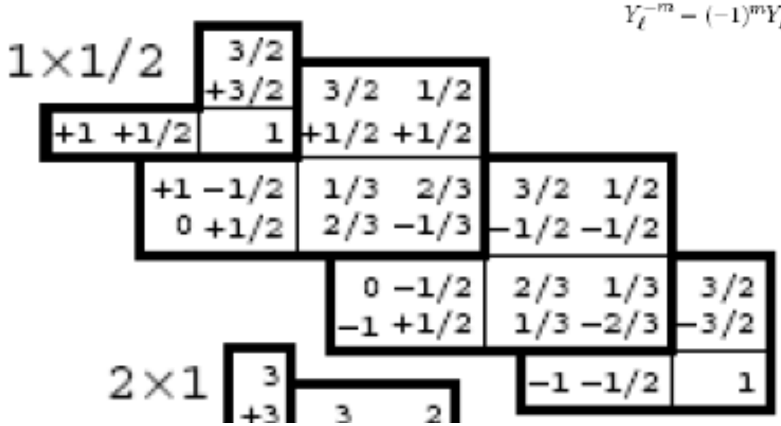
$$\frac{\sigma\left(\pi^+ + p \rightarrow \pi^+ + p\right)}{\sigma\left(\pi^- + p \rightarrow \pi^- + p\right)} = 9\ ;\quad \frac{\sigma\left(\pi^- + p \rightarrow \pi^0 + n\right)}{\sigma\left(\pi^- + p \rightarrow \pi^- + p\right)} = 2$$

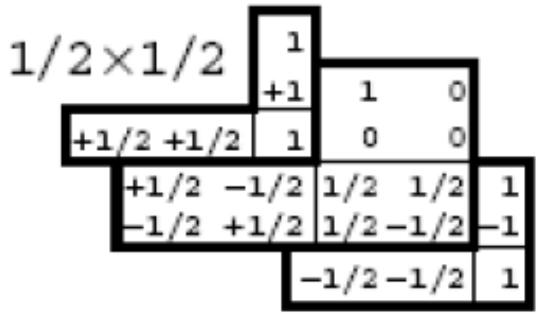
that we know they are true.

Clebsch-Gordan coefficients









exercise

- We have a system composed by a Σ^- and a proton. Write the total wave in the base of the total isospin of the system and compute the probability to find the system in a state of total isospin ½ .

- The Σ^- has I=1 e I_3=-1 (see Gell-mann-Nishijima formula) , while the proton has I=1/2 e I_3 = +1/2 . Combining together the two states we can have has total isospin ½ or 3/2 with the third component I_3 equal to -1/2.

$$|\Sigma^- p\rangle = \sqrt{\frac{1}{3}}\,|\frac{3}{2},-\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}\,|\frac{1}{2},-\frac{1}{2}\rangle$$

- The probability to find the system in a state of total isospin ½ is 2/3.

exercise

- The barion Λ decays in proton– π^- or neutron-π^0. In the decay a s-quark of the Λ transforms into a u-quark of the nucleon, therefore its strong isospin changes by ½. Assuming that in the Λ decays this selection rule is retained, and ignoring other small correction, deduce the ratio of the B.R. of the p- π^- decay with respect to the n- π^0 decay.

- The nucleone has isospin ½ while the pion has isospin 1, therefore a nucleon plus a pion can give total isospin equal to ½ or 3/2. The Λ has isospin zero, so in the total wave function of the system nucleon-pion we need to take into account only the component with isospin ½ , due to the selection rule ΔI = ½.

$$p+\pi^- = \left|\frac{1}{2};\frac{1}{2}\right\rangle + \left|1;-1\right\rangle = -\sqrt{\frac{2}{3}}\left|\frac{1}{2};-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{3}{2};-\frac{1}{2}\right\rangle \qquad n+\pi^0 = \left|\frac{1}{2};-\frac{1}{2}\right\rangle + \left|1;0\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{1}{2};-\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{3}{2};-\frac{1}{2}\right\rangle$$

- The transition probability is equal to the square of the w.f.:

$$\frac{B.R.\left(\Lambda\to p+\pi^-\right)}{B.R.\left(\Lambda\to n+\pi^0\right)} = \frac{\left|\langle p+\pi^- \mid \frac{1}{2};-\frac{1}{2}\rangle\right|^2}{\left|\langle n+\pi^0 \mid \frac{1}{2};-\frac{1}{2}\rangle\right|^2} = \frac{2/3}{1/3} = 2$$

- The experimental values are:

$$B.R.\left(\Lambda\to p+\pi^-\right) = 63.9\% \;\; ; \;\; B.R.\left(\Lambda\to n+\pi^0\right) = 35.8\%$$

- $$\frac{B.R.\left(\Lambda\to p+\pi^-\right)}{B.R.\left(\Lambda\to n+\pi^0\right)} = \frac{63.9}{35.8} = 1.78$$

  (most likely there is a higher order contribution with ΔI=3/2)

exercise

Il K^0_S può decadere in due pioni carichi oppure in due pioni neutri. Trovare il rapporto tra il B.R. del decadimento in pioni neutri rispetto a quello in pioni carichi. Si ricorda che per ragioni di simmetria lo stato finale deve avere isospin totale zero

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Nei decadimento deboli con ΔS=1 si ha ΔI=1/2, quindi dato che il K ha I=1/2, lo stato finale dei due pioni deve avere I=0 oppure I=1. La funzione d'onda dei due pioni deve essere simmetrica rispetto allo scambio delle due particelle, quindi dato che essi hanno spin zero e si trovano in uno stato di momento angolare l=0, anche la parte di isospin deve essere simmetrica, quindi I=0.

Utilizzando i coefficienti di Clebsh-Gordan si ha:

$$|0;0\rangle = +\sqrt{\frac{1}{3}}|1,+1;1-1\rangle - \sqrt{\frac{1}{3}}|1,0;1,0\rangle + \sqrt{\frac{1}{3}}|1,-1;1+1\rangle = \quad = +\sqrt{\frac{1}{3}}\pi^+\pi^- - \sqrt{\frac{1}{3}}\pi^0\pi^0 + \sqrt{\frac{1}{3}}\pi^-\pi^+$$

Di conseguenza abbiamo:

$$\frac{B.R.\left(K^0_S \to \pi^0 + \pi^0\right)}{B.R.\left(K^0_S \to \pi^+ + \pi^-\right)} = \frac{\left|\langle \pi^0\pi^0 \mid 0;0\rangle\right|^2}{\left|\langle \pi^+\pi^- \mid 0;0\rangle\right|^2} = \frac{1}{2}$$

I valori sperimentali sono:

$$B.R.\left(K^0_S \to \pi^0 + \pi^0\right) = 30.7\% \quad ; \; B.R.\left(K^0_S \to \pi^+ + \pi^-\right) = 69.2\%$$

$$\frac{B.R.\left(K^0_S \to \pi^0 + \pi^0\right)}{B.R.\left(K^0_S \to \pi^+ + \pi^-\right)} = \frac{30.7}{69.2} = 0.44$$

Probabilmente vi è un contributo di ordine superiore con ΔI=3/2

exercise

Dedurre attraverso quali canali di isospin possono avvenire le seguenti due reazioni:

a) $K^- + p \rightarrow \Sigma^0 + \pi^0$; b) $K^- + p \rightarrow \Sigma^+ + \pi^-$

Nel caso in cui il canale dominante sia quello con isospin 0 per entrambe le reazioni, trovare il rapporto tra le sezioni d'urto σ_a/σ_b

Ricordiamo l'isospin totale e la terza componente delle particelle coinvolte nella reazione e scriviamo lo stato iniziale ed i due stati finali in termini degli autostati di isospin utilizzando i coefficienti di Clebsh-Gordan.

$$K^- = \left|I = \tfrac{1}{2}; I_3 = -\tfrac{1}{2}\right\rangle \;;\; p = \left|I = \tfrac{1}{2}; I_3 = \tfrac{1}{2}\right\rangle \Rightarrow K^- + p = +\sqrt{\frac{1}{2}}|1;0\rangle - \sqrt{\frac{1}{2}}|0;0\rangle$$

$$\Sigma^0 = |I = 1; I_3 = 0\rangle \;;\; \pi^0 = |I = 1; I_3 = 0\rangle \Rightarrow \Sigma^0 + \pi^0 = +\sqrt{\frac{2}{3}}|2;0\rangle - \sqrt{\frac{1}{3}}|0;0\rangle$$

$$\Sigma^+ = |I = 1; I_3 = 1\rangle \;;\; \pi^- = |I = 1; I_3 = -1\rangle \Rightarrow \Sigma^+ + \pi^- = +\sqrt{\frac{1}{6}}|2;0\rangle + \sqrt{\frac{1}{2}}|1;0\rangle + \sqrt{\frac{1}{3}}|0;0\rangle$$

Di conseguenza la reazione a) può avvenire soltanto attraverso il canale di isospin totale 0, mentre la reazione b) può avvenire attraverso il canale con isospin 0 ed anche con isospin 1.

Nel caso in cui il canale dominante sia quello con isospin 0 per entrambe le reazioni, allora il rapporto tra le sezioni d'urto è pari al rapporto dei quadrati dei coefficienti di C.G. dell'autostato di isospin 0 nei due stati finali:

$$\frac{\sigma_a}{\sigma_b} = \left|\frac{\langle \Sigma^0 + \pi^0 | 0;0\rangle}{\langle \Sigma^+ + \pi^- | 0;0\rangle}\right|^2 = \left|\frac{-\sqrt{\frac{1}{3}}}{\sqrt{\frac{1}{3}}}\right|^2 = 1$$

SAPIENZA
UNIVERSITÀ DI ROMA

End of chapter 2