Collider Particle Physics
- Chapter 1 -

Accelerators

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Chapter Summary

- Electrostatic accelerators
- LINAC
- Circular accelerators
- MEG experiment
- Bending and focusing in circular accelerators
- Particle dynamics in the transvers plane
- Beam injection and extraction
- Acceleration and phase stability
- Luminosity in a collider

Accelerators in the world

where accelerators are used

Industry
- Material studies and processing
- Food sterilization
- Ion implantation

Security
- Airports & boarders
- Nuclear security
- Imaging

Energy
- Destroying radioactive waste
- Energy production
- Nuclear fusion
- Thorium fuel amplifier

World wide about >30’000 particle accelerators are in operation with a large variety of applications.

Health
- Diagnostic and imaging
- X-rays
- Cancer therapy
- Radioisotope production

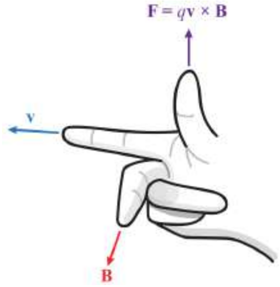
Research (<1%)
- Particle Physics
- Storage rings & Colliders
- Material science
- Light sources
- R&D

How can we accelerate particles?

How can we increase the energy of a particle?

A *charged* particles that travels through an electro-magnetic field feels the **Lorentz force**:

$$\boxed{\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})}$$



Magnetic field *B*:

Force acts perpendicular to path.

→ **Can change direction of particle**

→ cannot accelerate

Electric field E:

Force acts parallel to path.

→ **Can accelerate**

→ not optimal for deflection

Numeric Example:

$v = c,\ B = 1\mathrm{T}$

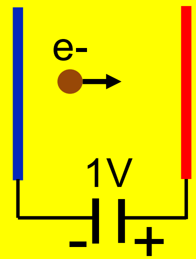
$E = vB = 3\mathrm{x}10^{8}\ \mathrm{m/s\ x\ 1T}$

$\boldsymbol{E = 300\ \mathrm{MV/m}}$

Technical limit for el. field:
$\mathrm{E} \propto 1\mathrm{MV/m}$

$$\begin{aligned} \Delta E = & \quad q \int_{r_1}^{r_2} (\vec{v} \times \vec{B} + \vec{E}) d\vec{r} \\ = & \quad q \int_{r_1}^{r_2} \vec{E} d\vec{r} = qU. \end{aligned}$$

$(\vec{v} \times \vec{B}) d\vec{r}$ = 0



$\boldsymbol{\Delta E = 1\ eV}$

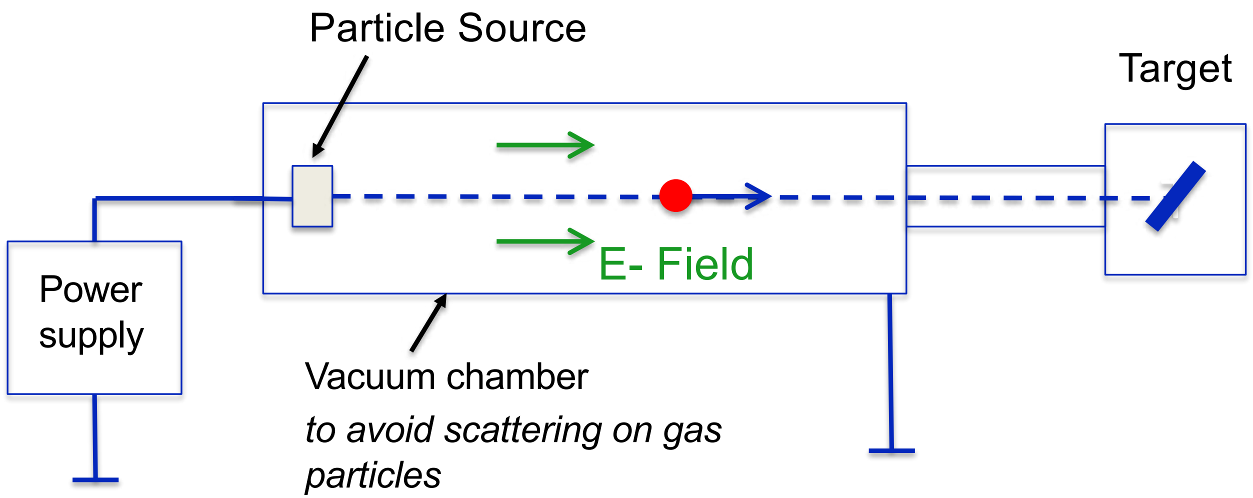
Which types of accelerators exists?
And
how do they work?

Basic accelerator

Electro-static accelerator (most basic accelerator)

→ Charged particle travels through a fixed high voltage *U*

Energy gain: $\Delta E = eU$



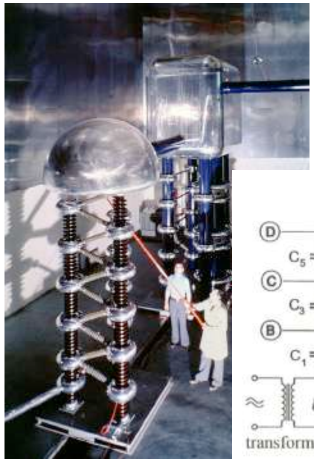
Final particle energy is limited by a maximum reachable voltage.

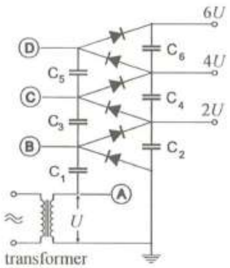
Max. voltage limited by corona formation and discharge to **~10MV**.

Electrostatic accelerators: ~ 1930

Cockcroft-Walton cascade generator

1928





Concept:

rectifier circuit, built of capacitors and diodes (Greinacker circuit)

Limitation:

Electrical discharge in air (Paschen Law)

Max. Voltage ~ 1 MV

Van de Graaff accelerator

1930

Concept:

mechanical transport of charges via rotating belt

Electrode in high pressure gas to suppress discharge (SF_6)

Max. Voltage ~ 1- 10 MV

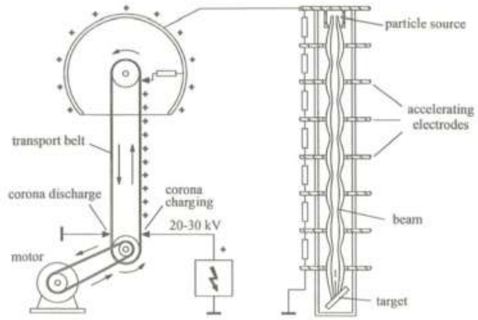


Image Credit: K. Wille

Tandem Van de Graaff accelerator

1936

at MPI Heidelberg

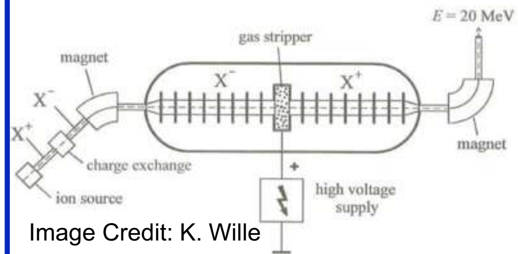


Image Credit: K. Wille

Concept:

Generate negative ions, strip off electrons in the center, use voltage a 2nd time with now positive ions

Max. Voltage ~ 25 MV

Historically largely used as 1st stage accelerators for proton and ion beams.

Electrostatic Accelerator Limitation

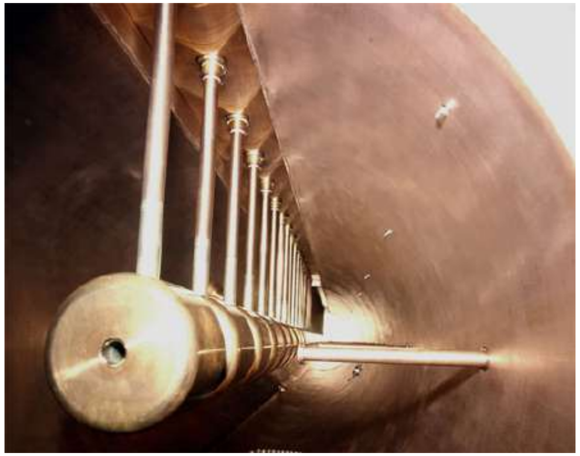
Electrostatic

Limitation:

Generation of max. (direct) voltage before sparking.

Acceleration over one stage or gap.

Radio Frequency



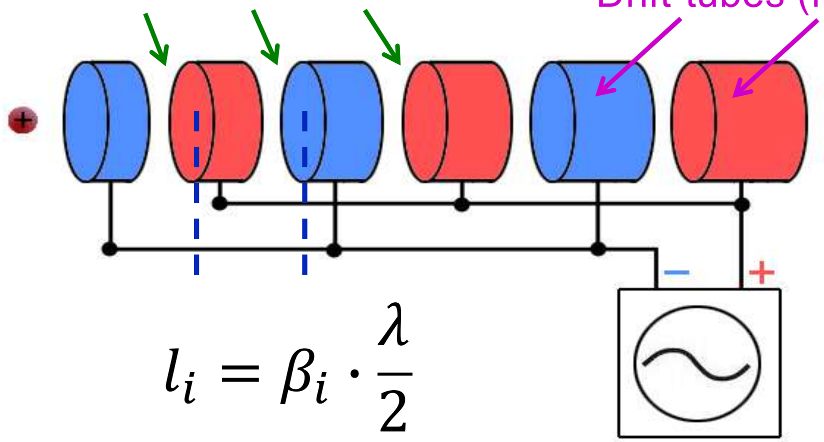
Solution:

Use alternating (RF) voltages and pass the particles through many acceleration gaps of the same voltage.

1925 idea by Ising

1928 first working RF accelerator by Wideroe

LINear ACcelerator (LINAC): functionalities



$$l_i = \beta_i \cdot \frac{\lambda}{2}$$

Energy gain after *n* gaps:

$$E = n\, q\, V_{RF} \sin \phi_s$$

n *No. of acceleration gaps*
q *Charge of the particle*
V_{RF} *Peak voltage of RF System*
ϕ_s *synchronous phase w.r.t. RF field*

Question
Once build, can we use the LINAC to accelerate any particle we like?

- High-frequency RF field (turn-over frequency MHz): $\lambda = c/f_{RF}$
- Particle should only feel the field when the field direction is synchronized.
- Drift-tubes screen the field as long as the field has the reversed polarity.
  - The more energy the particle gains, the faster it becomes (non-relativistic regime)
  - → Drifts have to increase in length.

→ Particles have to be clustered into packages (bunches).

Excercise: LINAC

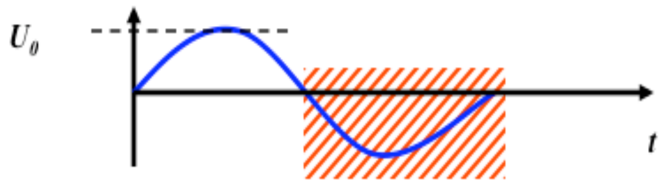
Question

Once build, can we use the LINAC to accelerate any particle we like?

This question could be rephrased to:

How does the drift tube length l_i depend on the particle type?

Drift tubes provide shielding of the particles during the negative half wave of the RF.



Time span of the negative half wave: $\tau_{RF}/2$

Length of the Drift Tube: $l_i = v_i * \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_i = \frac{1}{2} m v^2$

$$v_i = \sqrt{2E_i/m} \qquad E_i = iqV_{RF} \sin \phi_S$$

$$\boxed{l_i = \frac{1}{f_{RF}} \sqrt{\frac{i\, q\, V_{RF}\, \sin \phi_s}{2m}}}$$

valid for non-relativistic particles ...

So the answer is **no**. The drift tube length depends on the charge-to-mass-ratio (*q/m*) of the particle and the RF system. For a given RF system bandwidth only a certain range of *q/m* leads to a synchronized acceleration. One knob to play could be the charge state for ions, which may allow to get closer to the design *q/m*.

LINAC limitation

Linear

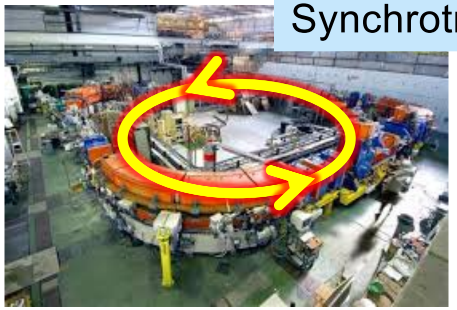
LINAC

Consists of a chain of many accelerating gaps placed on a straight line.

Particles pass the accelerator only ONCE.

The final energy is limited by length.

Circular



Synchrotron

Cyclotron

PSI cyclotron, currently doing first class physics

Use magnets that bend particles on a circular orbit.

Particles circulate over MANY turns and can gain more energy at each passage through the acceleration gap.

Cyclotron – "spiral version of a LINAC"

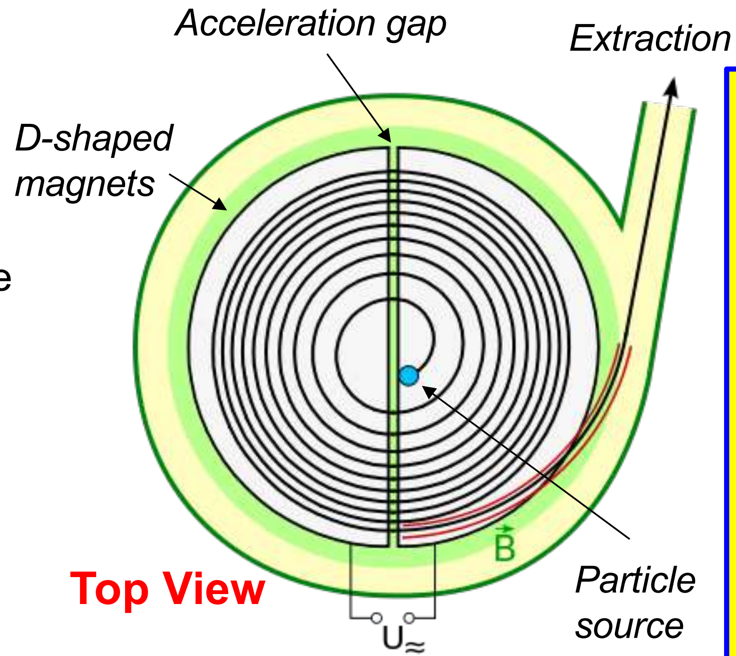
1929 proposed E.O. Lawrence
1931 built by Livingston

- **Particle Source in the middle**
- Acceleration gap connected to RF source between the two D-shaped magnets.
- **Constant vertical magnetic field** to guide the particles in the horizontal plane. **The radius of particle trajectory becomes larger and larger with larger energy.**
- Particles extracted with a deflector magnet or an electrode.

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \longrightarrow F_L = q\, v\, B \longrightarrow$$ Vertical B, No E

$$F_c = m\frac{v^2}{r} \longrightarrow$$ centrifugal force

$$F_L = F_c \longrightarrow \omega = \frac{v}{r} = \frac{qB}{m} \longrightarrow$$ revolution period

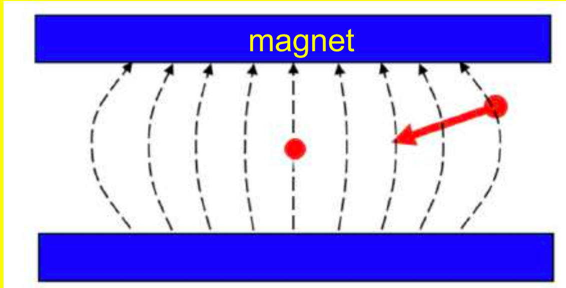


$\omega_{RF} = \omega_{cyclotron}$

f_{RF} = const.
B = const.

Weak focusing

Side View



B field is decreasing moving outward from the center.

A component of the Lorentz force prevents the particles to hit the magnet walls

Same principle of weak focusing is working in the dipole magnets

Cyclotron limitation

Constant revolution frequency for constant mass:

$$\omega = \frac{v}{r} = \frac{Bq}{m} = \frac{Bq}{m(E)}$$

f_{RF} = const.

B = const.

[*Well … it is the relation between p and v, or p and E that is different*]

But, for relativistic particles the mass is not constant!

The classical cyclotron only valid for particles up to few % of speed of light.

→ **Not useful for electrons** … already relativistic at ~500 keV.

Modifications:

Synchro-cyclotron

f_{RF} (E)

B (E) or B = const.

Isochronous cyclotron

f_{RF} = const.

B (r)

Common accelerator for medium energy protons and ions up to ~60MeV/n, used for nuclear physics, radio isotope production, hadron therapy.

Modern"cyclotrons"can reach > 500 MeV (PSI, TRIUMF, RIKEN)

Let's open a parenthesis

(it is not part of the exam program)

Fatti non foste a viver come bruti
ma per seguir virtute e canoscenza

Paul Scheerer Institut (PSI) cyclotron [near Zurich]

1974

- Diameter ~15m
- Injection energy 72 MeV
- Accelerates protons to E = 590 MeV (i.e. 0.8c) in 186 revolutions

It produce a proton beam of 2.4 mA, a world record.

$$N_p = \frac{2.4 \cdot 10^{-3}}{1.6 \cdot 10^{-19}} \approx$$

$$1.5 \cdot 10^{16} \ \ \text{prot/s}$$

They are used to produce high intensity muon beam, ~ 10^8 muon/s.

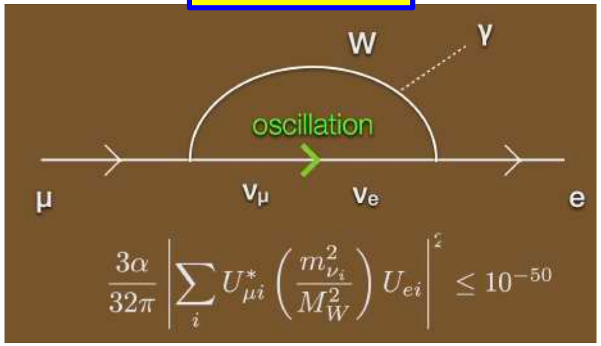
8 sector magnets

4 acceleration cavities

First stage accelerator feeding a smaller cyclotron before the large PSI ring cyclotron is a Cockraft-Walton accelerator.

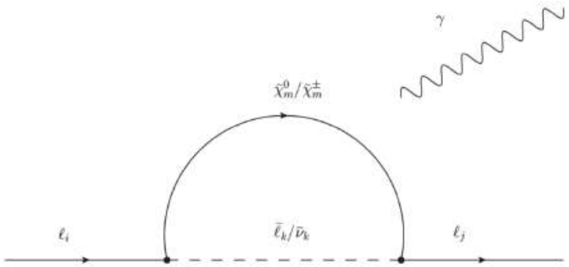
MEG experiment at PSI

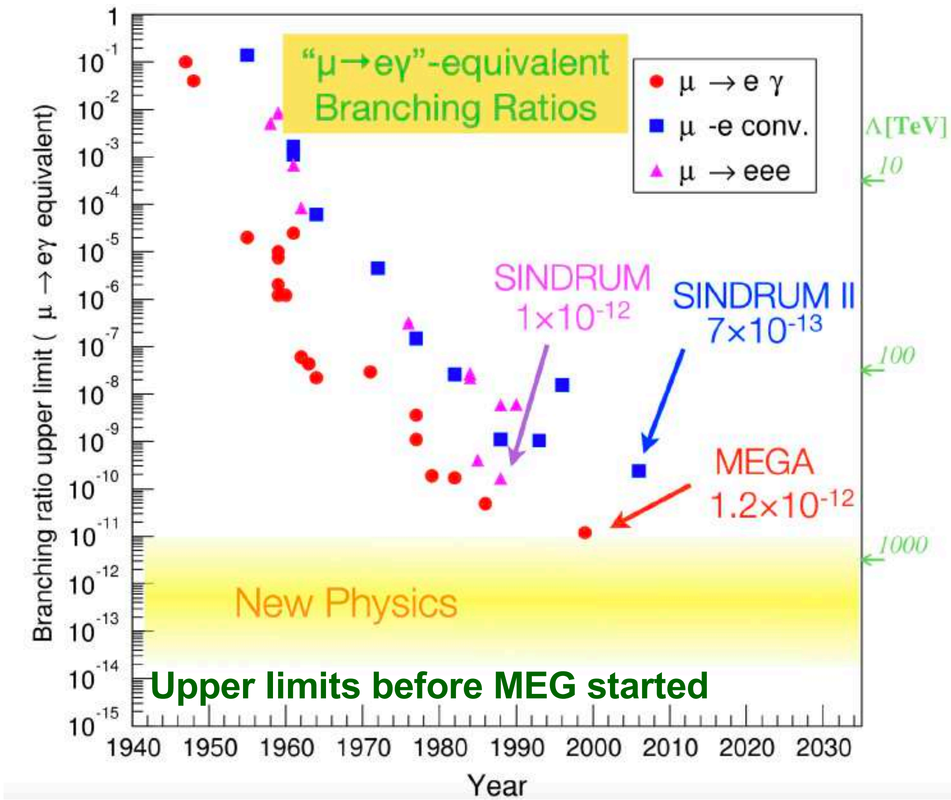
$\mu \rightarrow e + \gamma$



- In the SM, even with massive neutrinos, the B.R. is pratically zero

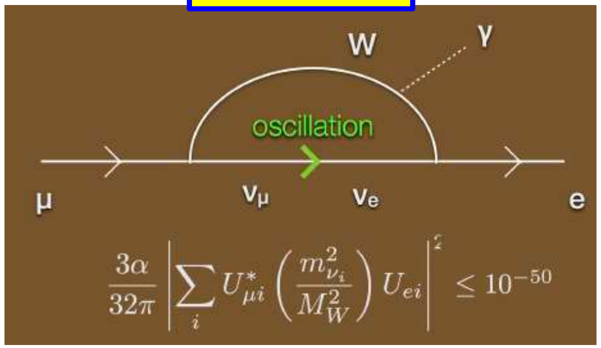
- However, if we have new particles in the loop, the B.R. is enhanced.

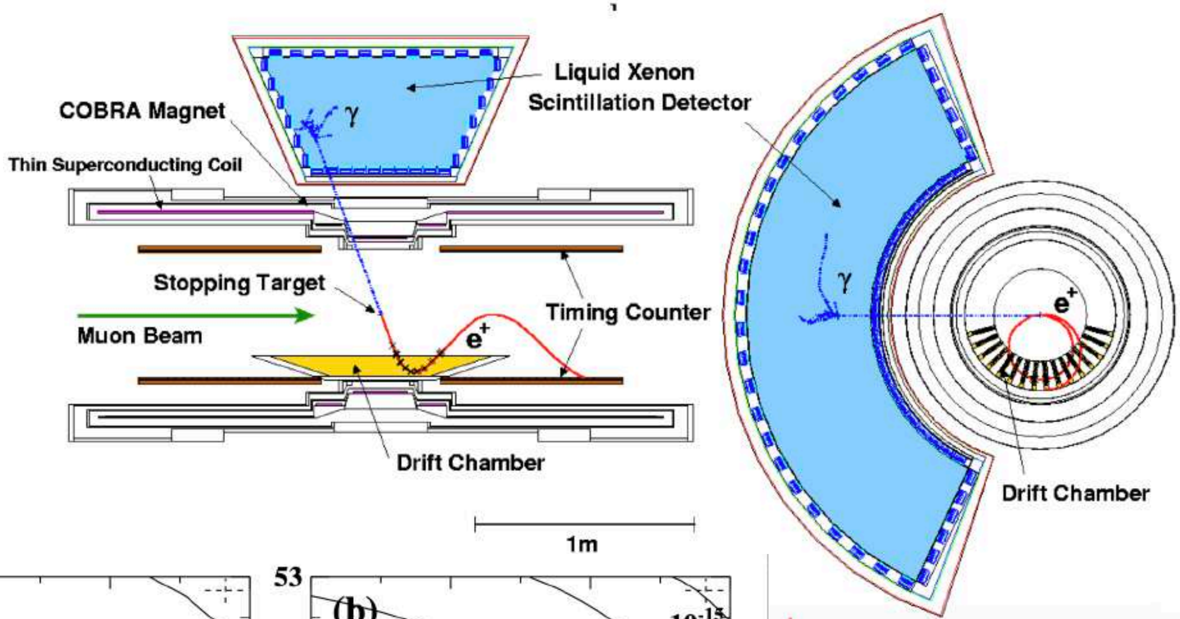


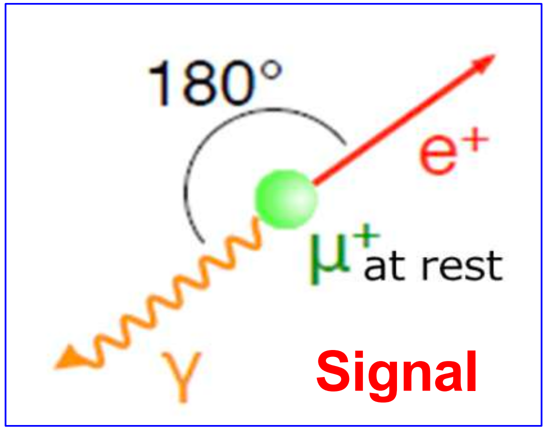


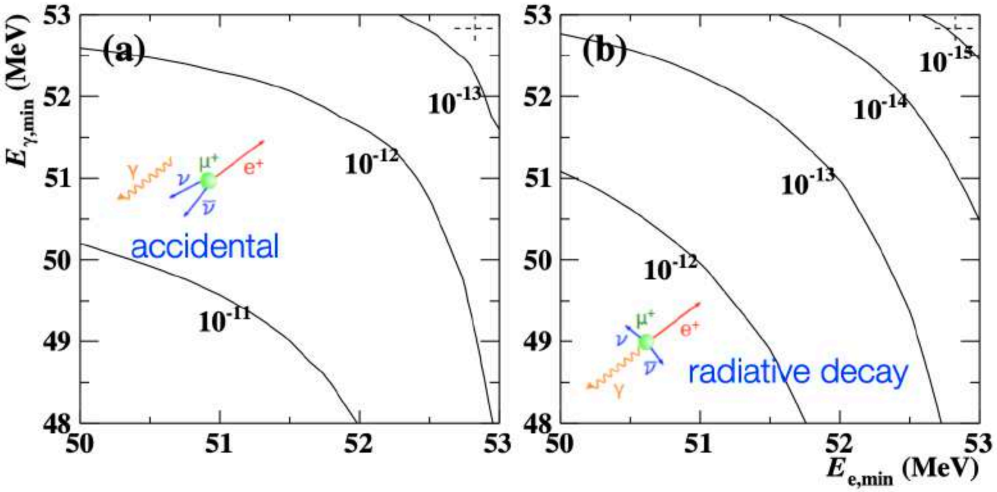
MEG experiment at PSI

$\mu \to e + \gamma$





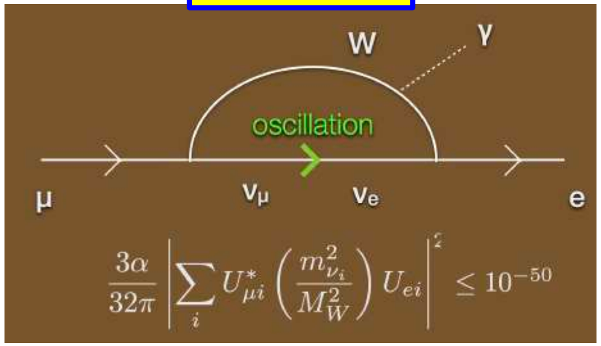


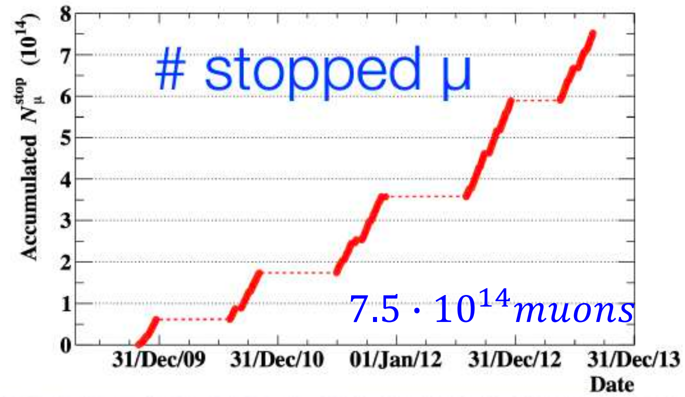


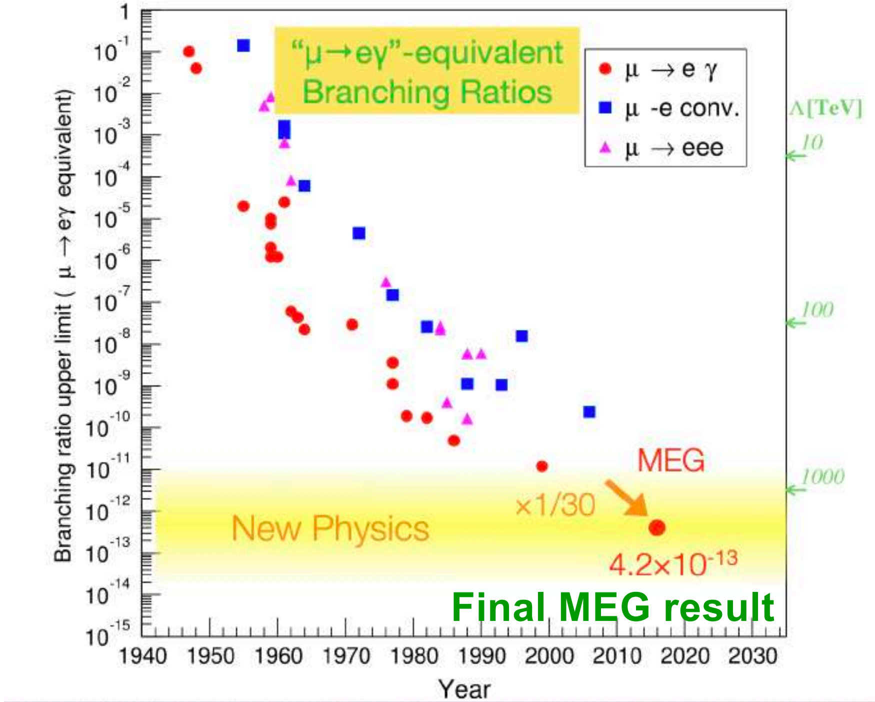
Background rejection is essential for this measurement.

MEG experiment at PSI

$\mu \rightarrow e + \gamma$





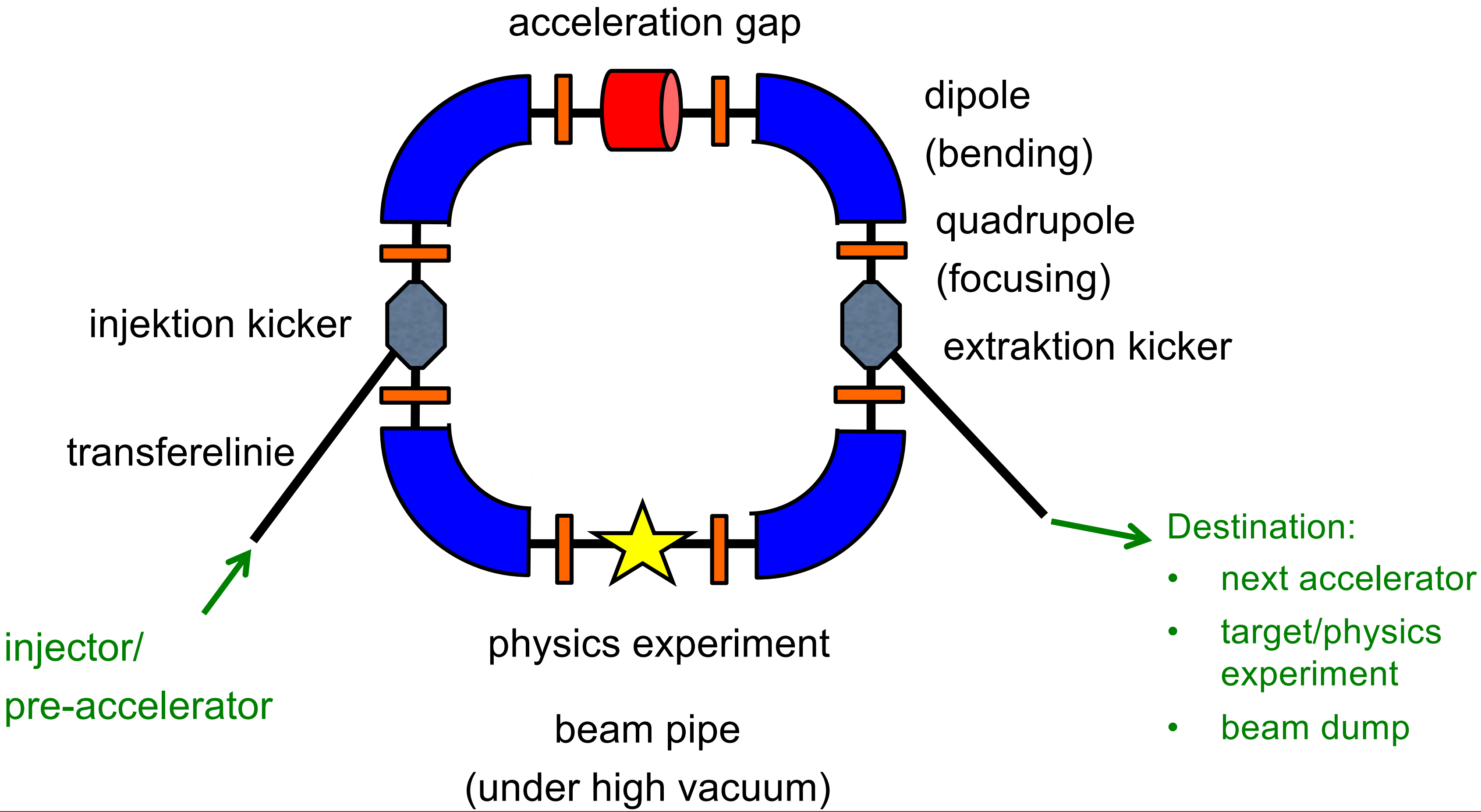


They are excluding part of the new physics band

Let’s close the parenthesis

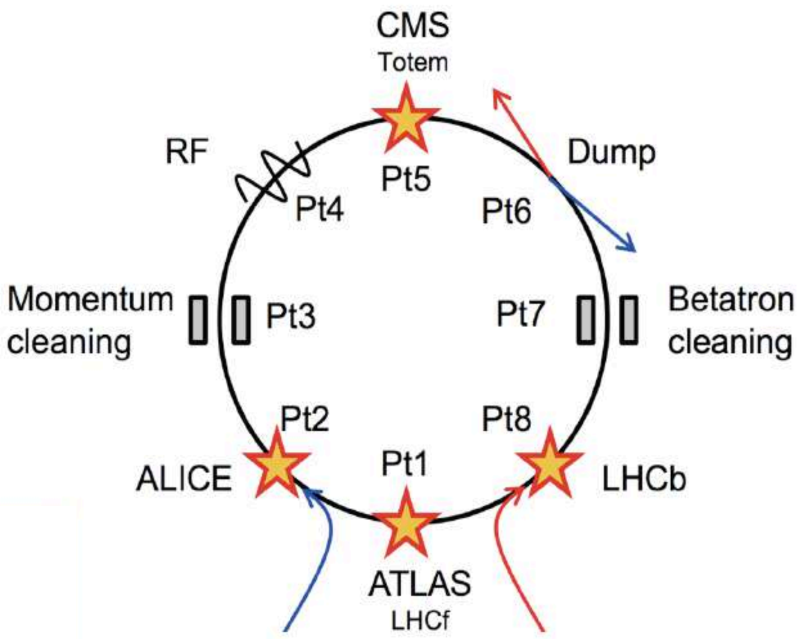
Basic Synchrotron

Synchrotrons are THE accelerators to reach highest particle energies and are able to store the beam over many hours.



Most famous example

The largest machine in the world The Large Hadron Collider (LHC)



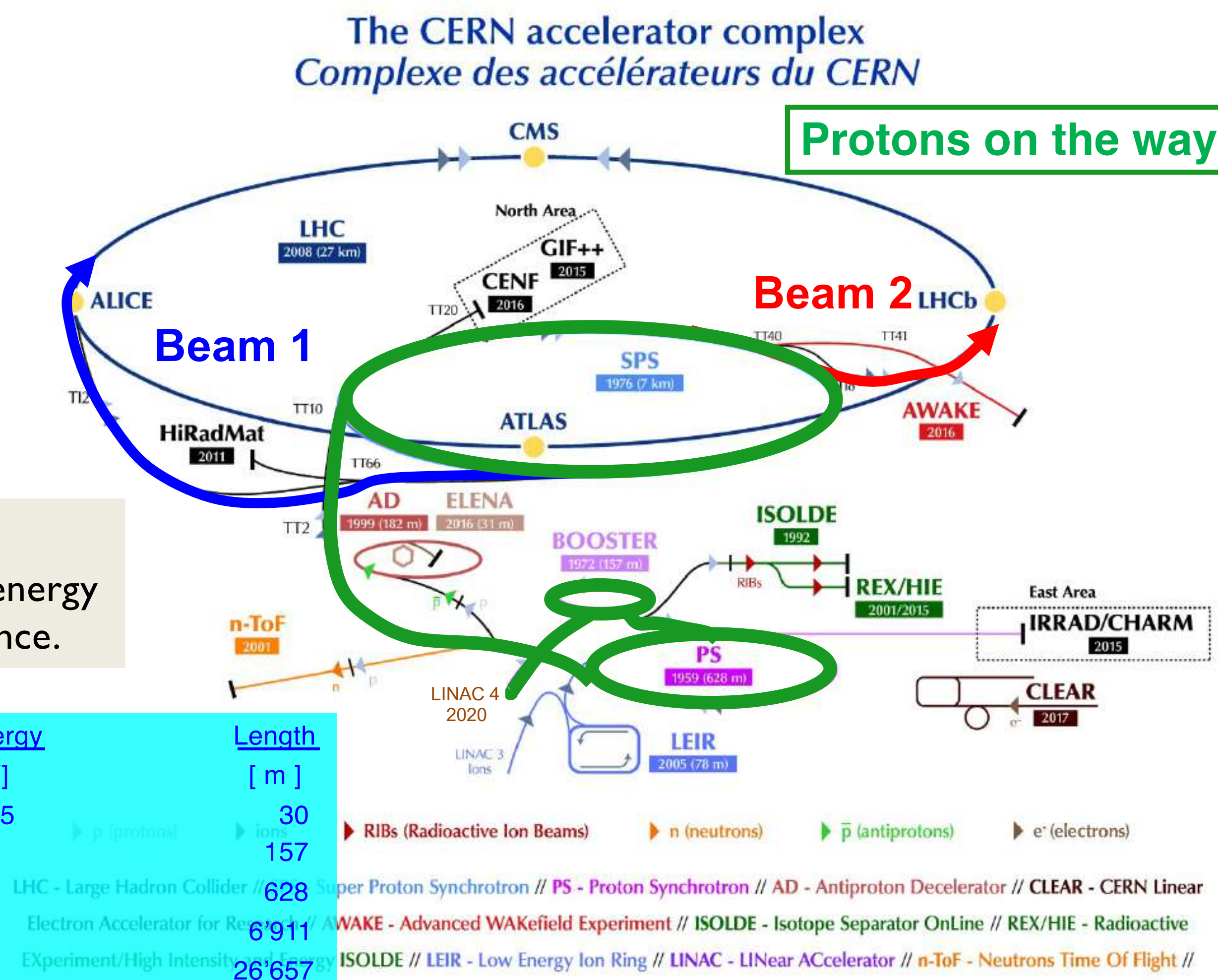
27 km circumference

100m underground

Accelerates protons and heavy-ions to E = 6.8 TeV (2022).

Collides 2 counter-rotating beams in 4 physics experiments.

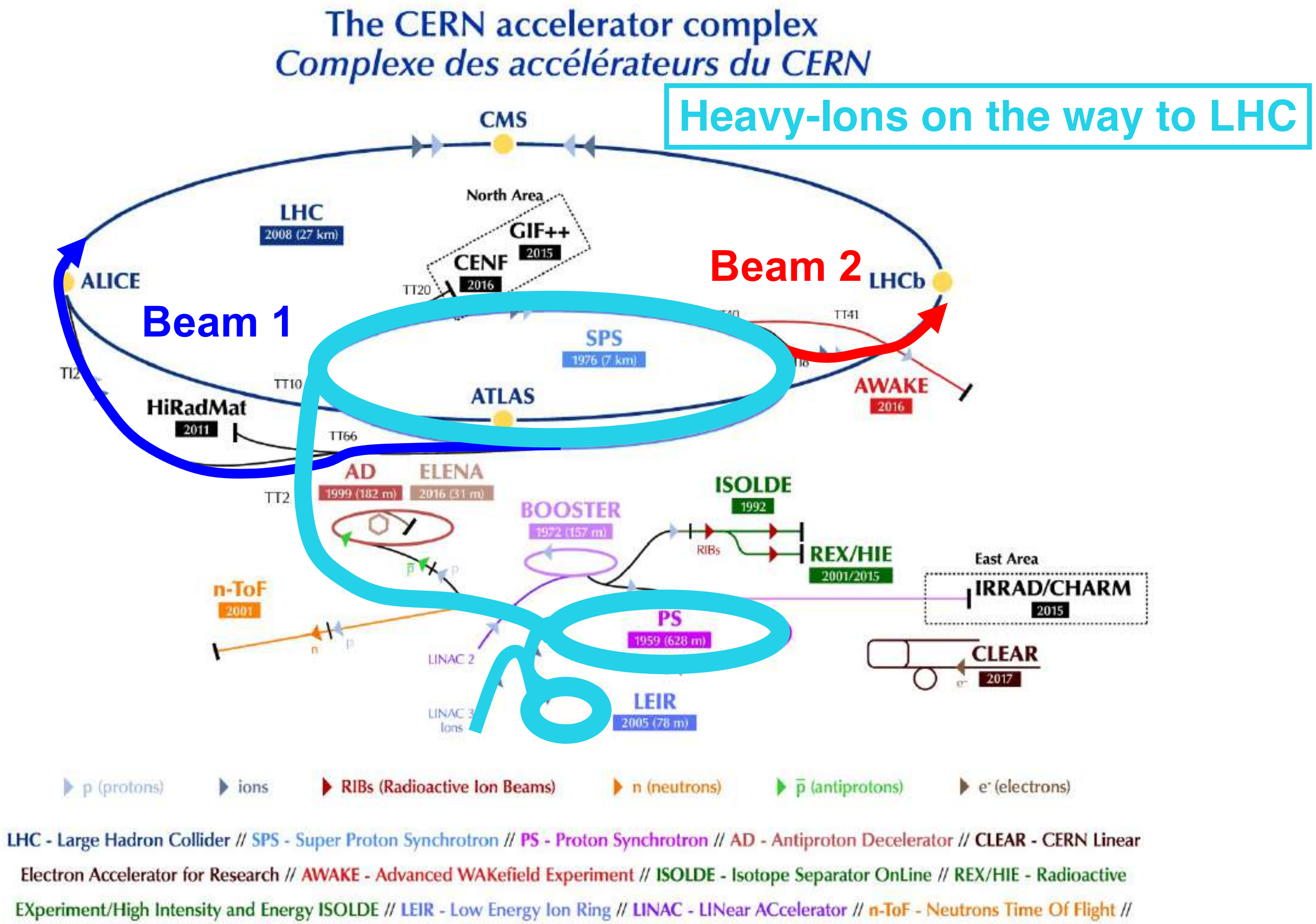
Getting particles into the LHC



1) Linear → Circular
2) The higher the particle's energy the larger the circumference.

	Year	Top energy [GeV]	Length [m]
Linac 4	2020	0.05	30
PSB	1972	1.4	157
PS	1959	26.0	628
SPS	1976	450.0	6'911
LHC	2008	7000.0	26'657

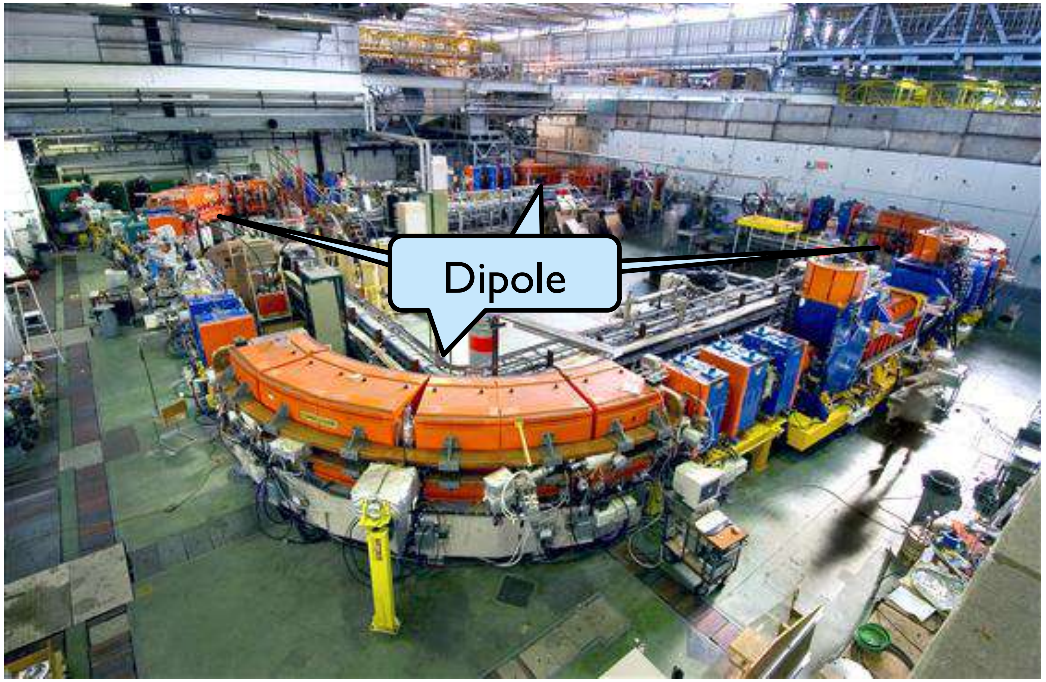
Getting particles into the LHC

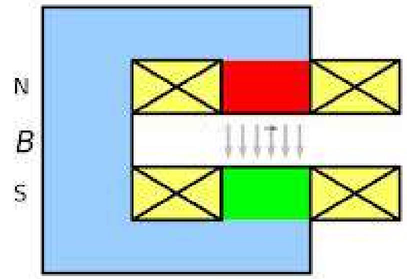


Synchrotron:
bending and focusing

Bending

Vertical magnetic field to bend in horizontal plane.





LEIR has 4 dipoles, each with 90º bending angle, to keep particles on a circular orbit

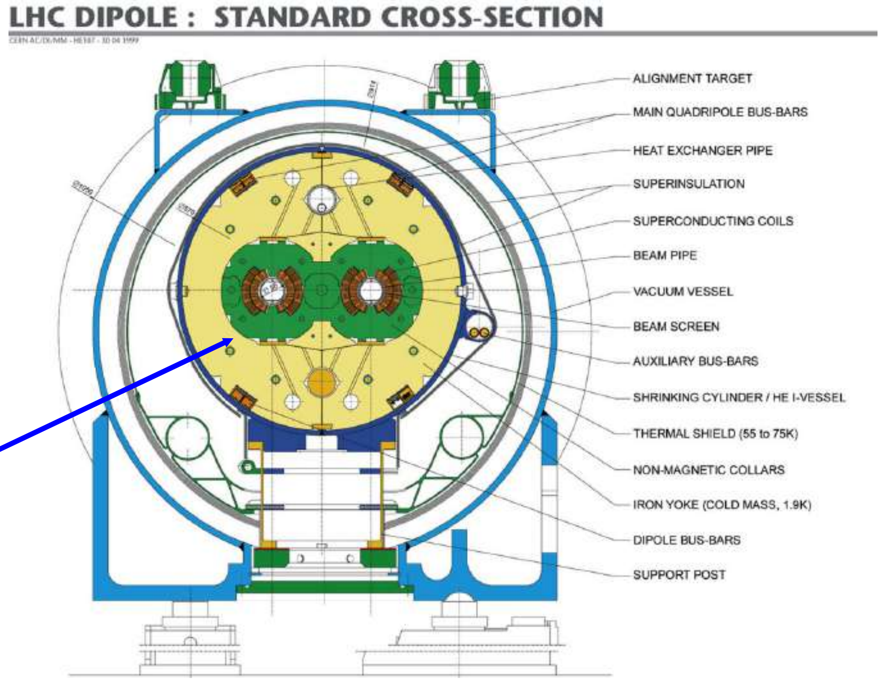
LEIR (Low Energy Ion Ring)

- 78m circumference
- first circular accelerator for CERN's heavy-ions on the way to LHC
- 2.5 sec to accelerate ion bunches from 4.2 MeV/n to 72 MeV/n

Bending at LHC

LHC has 1232 superconducting dipole magnets, each 15 m long and able to deflect the beam by 0.29°.

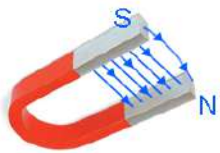
8.33 Tesla (max 2 T in iron)
11.7 kA (superconducting coil)

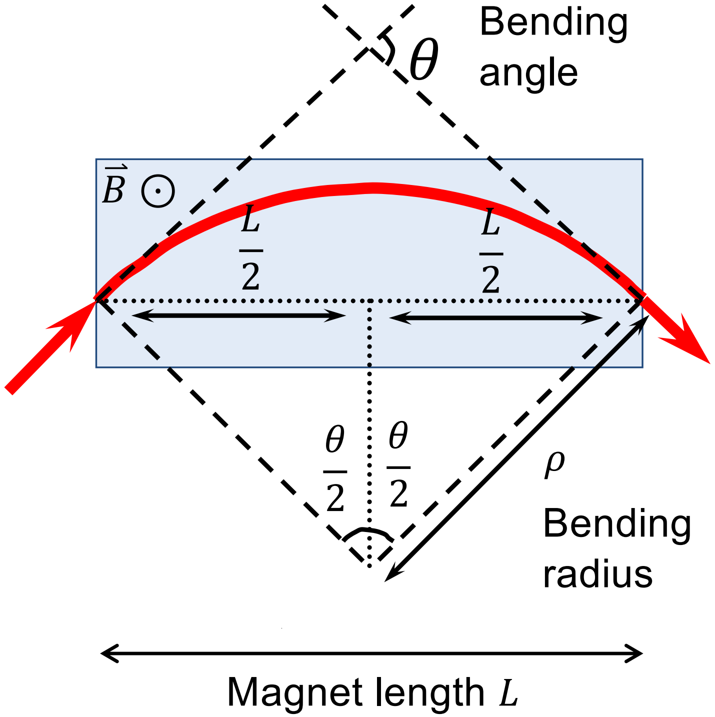


The superconducting coils are cooled to 1.9 K (the cosmic background radiation is at 2.7 K). LHC is the coldest point in the Universe (on a large scale).

Deflection of a charged particle

Charged particles are deflected in a magnetic field





The ideal circular orbit

Lorentz Force $\quad F_L = q\, v\, B$

Centrifugal Force $\quad F_{centr} = \dfrac{\gamma\, m_0\, v^2}{\rho}$

$$\underbrace{F_L = F_{centr}}$$

$$\boxed{\frac{p}{q} = B\,\rho}$$

$B\,\rho$ = Beam rigidity

q = charge

$p = \gamma\, m_0 v$ momentum

B = mag. Field strength

ρ = bending radius

Required Magnetic Field Strength

Full circle

$$\alpha = \int \frac{dl}{\rho} = \int \frac{Bdl}{B\rho} = 2\pi \xrightarrow[\frac{p}{e} = B\rho]{\int B\,dl \approx N\,l\,B} \boxed{B = 2\pi p / (qNl)}$$

N: number of magnets
l: length of a magnet

Example SPS:

- Particle:
  - p = 450 GeV/c
  - q = +1e (proton)
- Dipole magnets:
  - l = 6.2m
  - ρ = 735m
  - N = 744

$$B \approx \frac{2\pi \times 450\ \ GeV}{744 \times\ 6.2\ m \times 3 \times 10^8 \frac{m}{s} \times\ \ e} = 2.0\ T$$

normal conducting magnet

Example LHC:

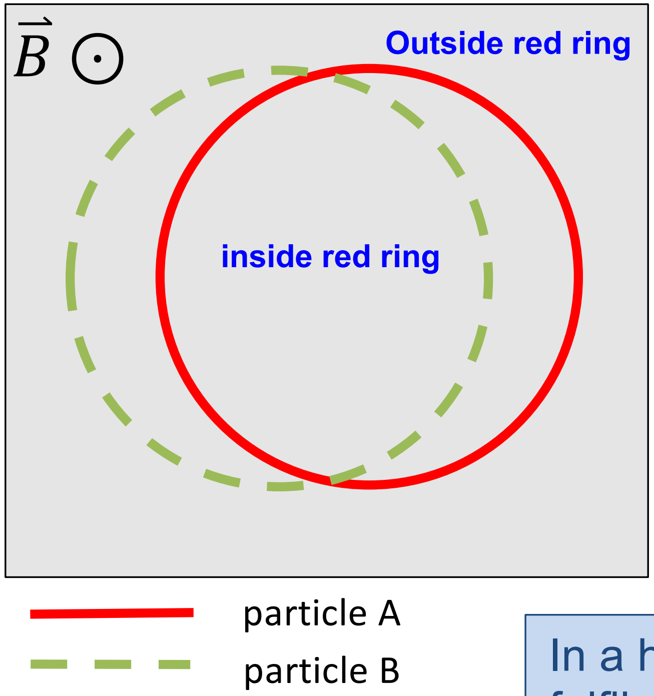
- Particle:
  - p = 7000 GeV/c
  - q = +1e (proton)
- Dipole magnets:
  - l = 15m
  - ρ = 2803m
  - N = 1232

$$B \approx \frac{2\pi \times 7000\ \ GeV}{1232 \times\ 15\ m \times 3 \times 10^8 \frac{m}{s} \times\ \ e} = 8.3\ T$$

superconducting magnet

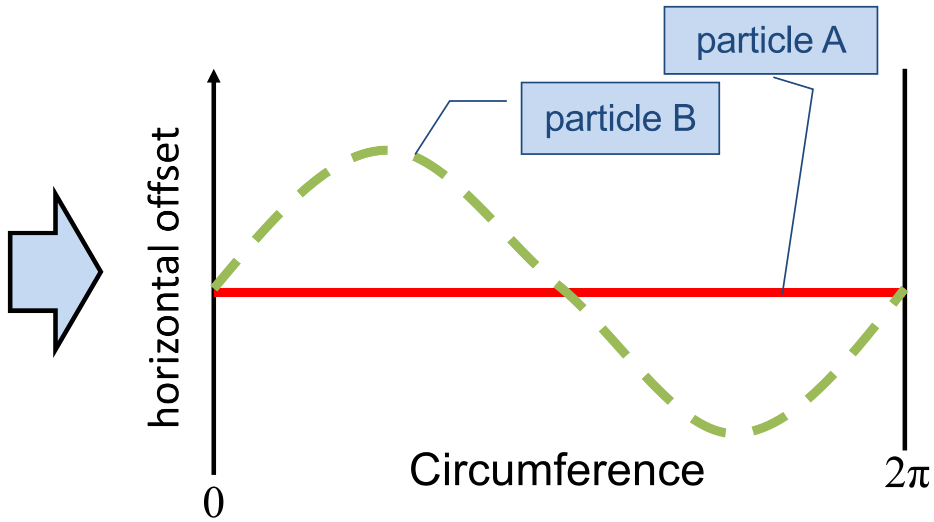
Particles oscillation

Example: two charged particles, with the same momentum, in a homogeneous magnetic field



"horizontal" movement (distance between the two orbits)

Particle B, while it is turning, go outside and inside the trajectory of the particle A

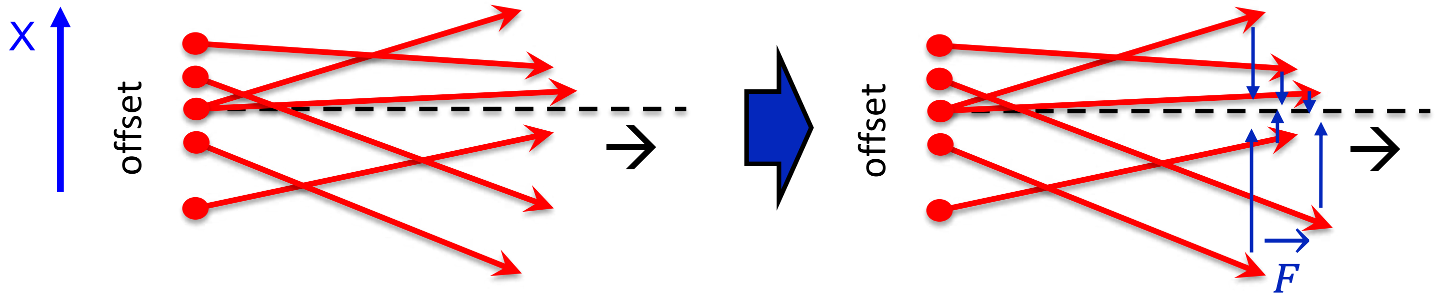


In a homogeneous magnetic field, particles with varying initial conditions fulfil oscillations around the design orbit → **Betatron-Oscillation**

design orbit = trajectory of ideal particle → defined by dipole magnets

Beam focusing

A bunch contains many particles with different initial conditions.



Many different positions, angles and energy offsets

We need a focusing force that keeps the particles close to the design orbit.

Focusing force should rise as a function of the distance to the design orbit.

Beam focusing

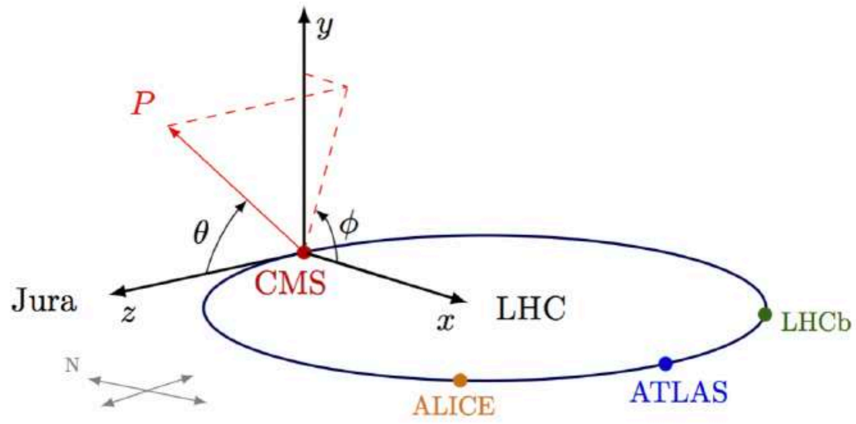
Requirement:
Lorentz force linearly increasing as a function of distance from design orbit.
→ Linearly increasing magnetic field.

$$F(x) = q \cdot v \cdot B(x)$$

Taylor series as a function of distance from magne

$$B_y(x) = B_{y0} + \frac{\partial B_y}{\partial x} x + \frac{1}{2}\frac{\partial^2 B_y}{\partial x^2} x^2 + \frac{1}{3!}\frac{\partial^3}{\partial x}$$

dipole *quadrupole* *sextupole* *octupo*



Normalize to p/q: $\frac{p}{q} = B\,\rho$

$$\frac{B_y(x)}{p/q} = \frac{1}{\rho} + \boxed{kx} + \frac{1}{2} m x^2 + \frac{1}{3!} n x^3 + \ldots$$

quadrupole

N.B. if you need a force along x, B component has to be along y since particle velocity is along z

Beam focusing

Focusing of particles with quadrupoles: strong focusing

$$F(x) = q \cdot v \cdot B(x)$$

with the vertical (y) and horizontal (x) quadrupole fields

$$B_y = g \cdot x$$
$$B_x = g \cdot y$$

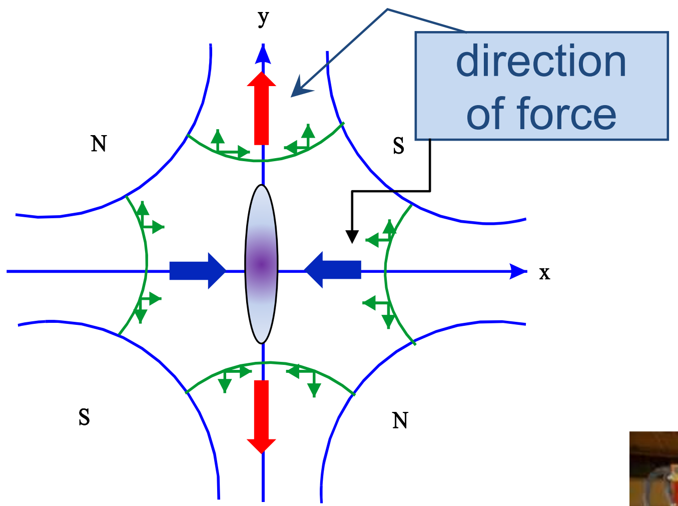
where g is the gradient

$$g = \frac{2\mu_0 n I}{r^2} \left[\frac{T}{m}\right]$$

Normalized gradient = focusing strength

$$k = \frac{g}{p/q} [m^{-2}]$$

I coil current
n number of windings
r distance magnet center to pole
μ_0 permeability of free space



Do you see the problem with this?

Quadrupoles focus in one plane, but defocus in the other!

quadrupole magnet

Focusing analogous to geometrical optics

Focusing of particles with quadrupoles is similar to focusing of light with lenses.

A series of alternating focusing and defocusing lenses will focus:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Consider:

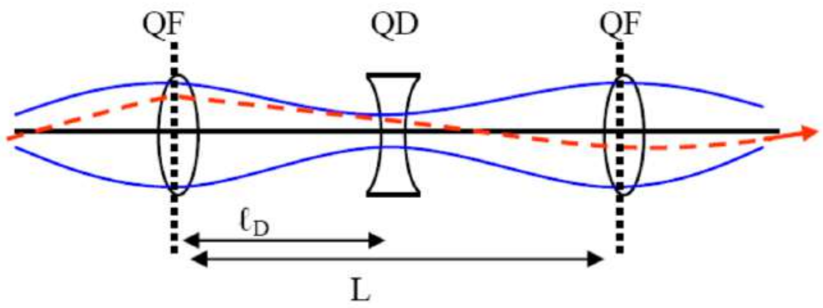
$$f_1 = f$$
$$f_2 = -f$$

Then:

$$F = \frac{f^2}{d} > 0$$

In a synchrotron quadrupoles are lenses with the focal length:

$$f = \frac{1}{k \cdot l_Q}$$



Typical alternating

F = focusing

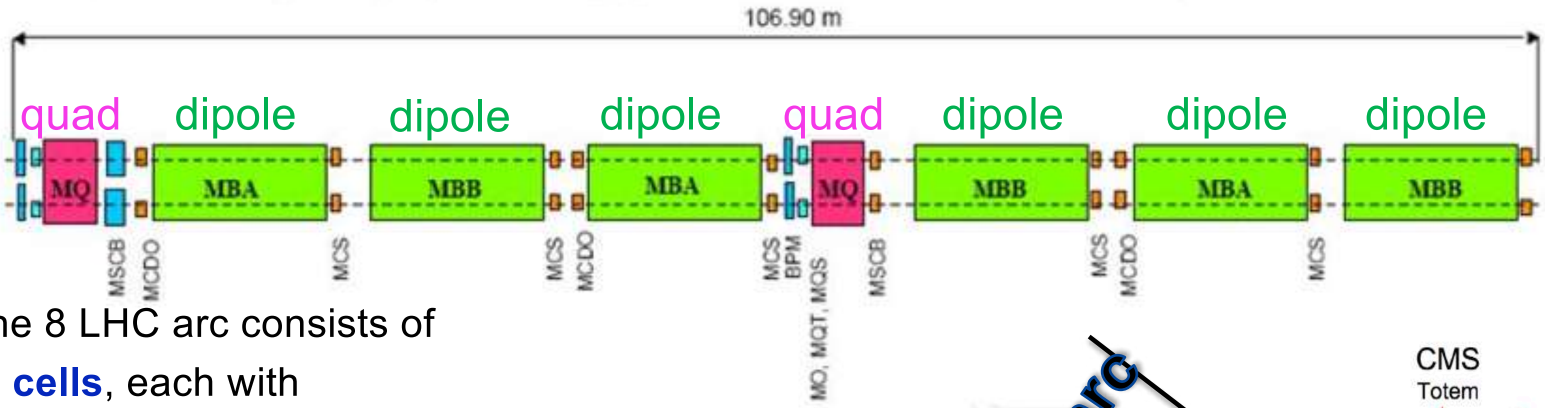
0 = nothing (dipole, RF, …)

D = defocusing

0

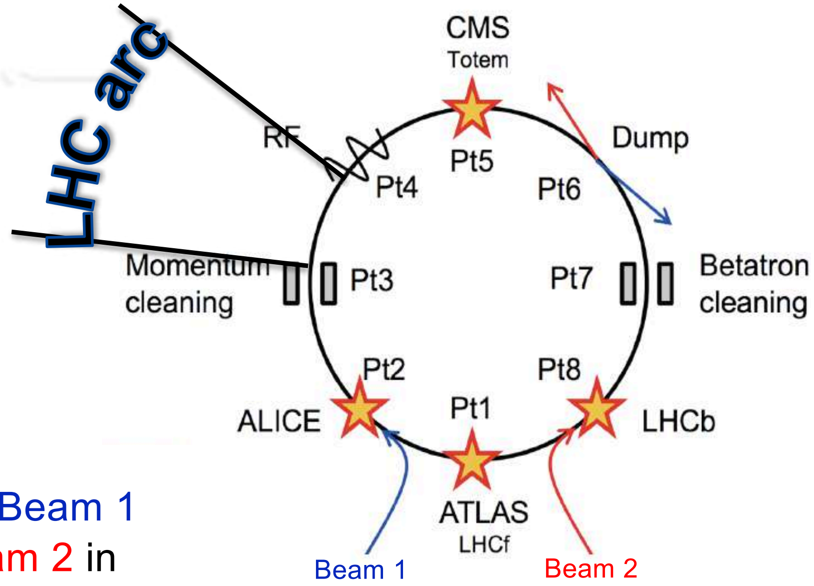
lattice of quadrupoles in an accelerator

The LHC FODO cells



Each of the 8 LHC arc consists of **23 FODO cells**, each with

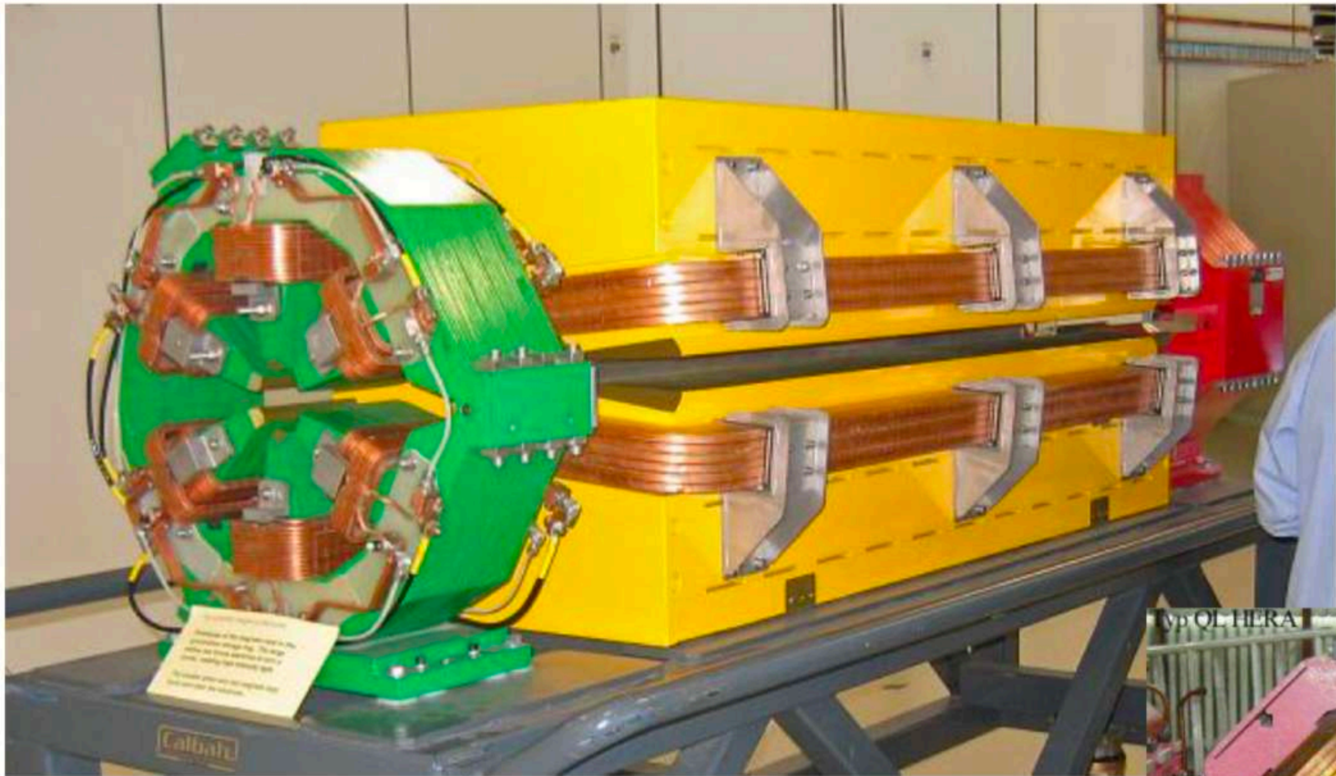
- **2 Quadrupoles**
- **6 Dipoles**
- Additional instrumentation and corrector magnets are installed in between for beam control.



LHC quadrupole

A focusing magnet for Beam 1 is a defocusing for Beam 2 in the same plane.

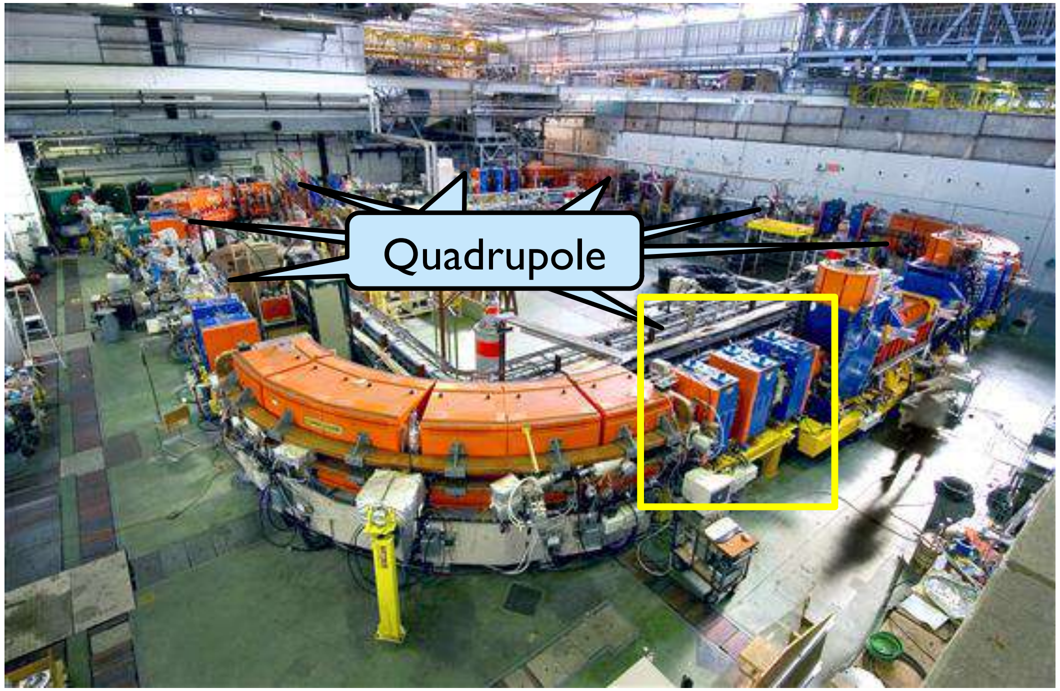
Example of magnets



a sextupole in front of
a "C" dipole

a quadrupole of the
HERA accelerator

Beam focusing



LEIR – first circular accelerator for CERN‘s heavy-ions on the way to LHC

How does a particle move in an accelerator

(No need to remember all equations. This is only meant to give you the big picture and the “namings”)

Particle motion

Focusing force that keeps the particles close to the design orbit, which rises as a function of the distance.

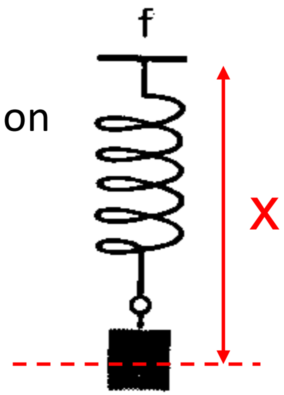
$$F(x) = q \cdot v \cdot B(x)$$

Classical free harmonic oscillator

→ experiences restoring force proportional to the displacement x when displaced from equilibrium position

Second law of motion: $\vec{F} = m\vec{a}$

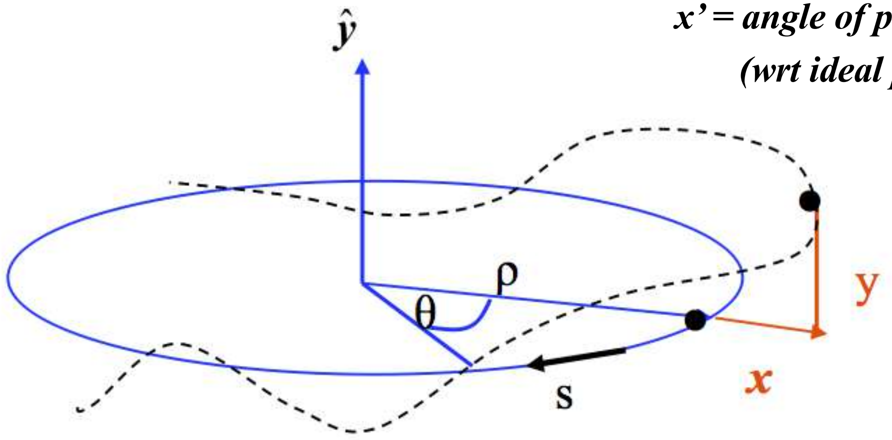
$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = m\ddot{x} = -kx$$



Solution of equation of motion is sinusoidal oscillation: $x(t) = A\cos(\omega t + \varphi)$

Coordinate system

Use different coordinate system: **Frenet-Serret** rotating frame



- The ideal particle defines "design" trajectory: *x=0, y=0*
  → *travels through the center of all magnets.*
- *x, y* << ρ

Look at the particle motion along the path length *s*.

Toward the equation of motion

$$F_x = m \cdot \ddot{x}$$

Describes motion as a function of time.

But what we need is something like $F_x = Mx''$

$$\dot{x} = \frac{dx}{dt}$$

$$x' = \frac{dx}{ds}$$

→ Replace free parameter time ***t*** by path length ***s.***

→ Compare to Lorentz force $F(x) = q \cdot v \cdot B(x)$

Taylor expansion of normalize magnetic field:

$$\frac{B_y(x)}{p/q} = \frac{1}{\rho} + kx + \frac{1}{2}\cancel{m}x^2 + \frac{1}{3!}n\cancel{x}^3 + \ldots\cancel{\textit{higher orders}}$$

dipole *quadrupole* *sextupole* *octupole*

Only consider **linear** terms: **dipole & quadrupole** fields!

$$\frac{B_y(x)}{p/q} \approx \frac{1}{\rho} + kx$$

Equation of motion

Horizontal motion:

$$\boxed{\begin{aligned} x'' + Kx &= 0 \\ y'' - ky &= 0 \end{aligned}}$$

Vertical motion:

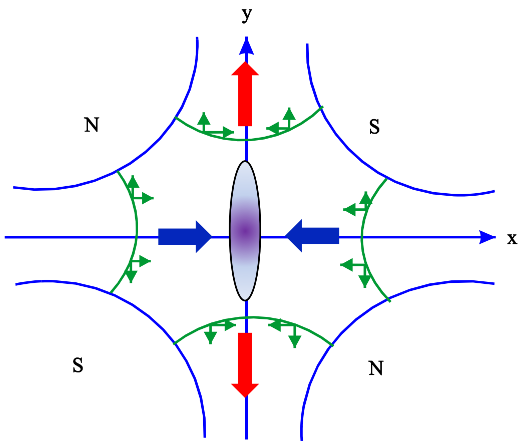
Where $K = \dfrac{1}{\rho^2} + k$

with k as the quadrupole focusing strength and ρ the bending radius.

In vertical:

→ In general, no dipoles: $\dfrac{1}{\rho^2} = 0$

→ Sign change of force direction: $k \Longleftrightarrow -k$



Assuming the motion in the horizontal and vertical plane are independent → **Particle motion in x & y is uncoupled**

Solving the equation of motion – focusing quadrupole

Equation of motion in horizontal plane

$$x'' + Kx = 0$$

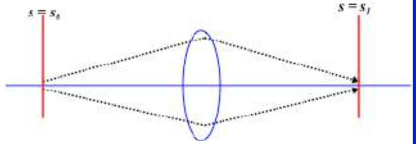
Equation of the **harmonic oscillator** with spring constant K.

Ansatz

For **K > 0 (focusing)** the solution can be found with this ansatz and boundary conditions:

$$x(s) = a_1 \cos(\omega s) + a_2 \sin(\omega s)$$

$$s = 0 \rightarrow \begin{cases} x(0) = x_0, \\ x'(0) = x'_0 \end{cases}$$



Solution

Inserting these into the equation of motion yields:

$$x(s) = x_0 \cos(\sqrt{K}s) + x'_0 \frac{1}{\sqrt{K}} \sin(\sqrt{K}s)$$

$$x'(s) = -x_0\sqrt{K}\sin(\sqrt{K}s) + x'_0 \cos(\sqrt{K}s)$$

Transfer Matrix

Use matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = M_{foc} \cdot \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

Focusing Quadrupole

$$M_{foc} = \begin{pmatrix} \cos(\sqrt{K}s) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}s) \\ -\sqrt{K}\sin(\sqrt{K}s) & \cos(\sqrt{K}s) \end{pmatrix}$$

Solving the equation of motion – defocusing quadrupole

Equation of motion in horizontal plane

$$x'' + Kx = 0$$

Equation of the **harmonic oscillator** with spring constant K.

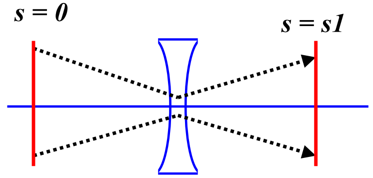
Defocusing Quadrupole

For **K < 0 (defocusing)** the new ansatz is:

$$x(s) = a_1 \cosh(\omega s) + a_2 \sinh(\omega s)$$

Defocusing Quadrupole

$$M_{defoc} = \begin{pmatrix} \cosh(\sqrt{|K|}s) & \frac{1}{\sqrt{|K|}}\sinh(\sqrt{|K|}s) \\ \sqrt{|K|}\sinh(\sqrt{|K|}s) & \cosh(\sqrt{|K|}s) \end{pmatrix}$$

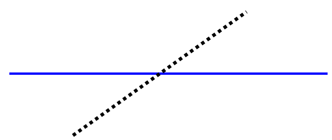


Drift Space

For **K = 0 (drift)** the ansatz is: $x(s) = x_0' s$

Drift Space

$$M_{drift} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$



Dipole

For **K = 1/ρ² (dipole)** use the result for a focusing dipole and insert K.

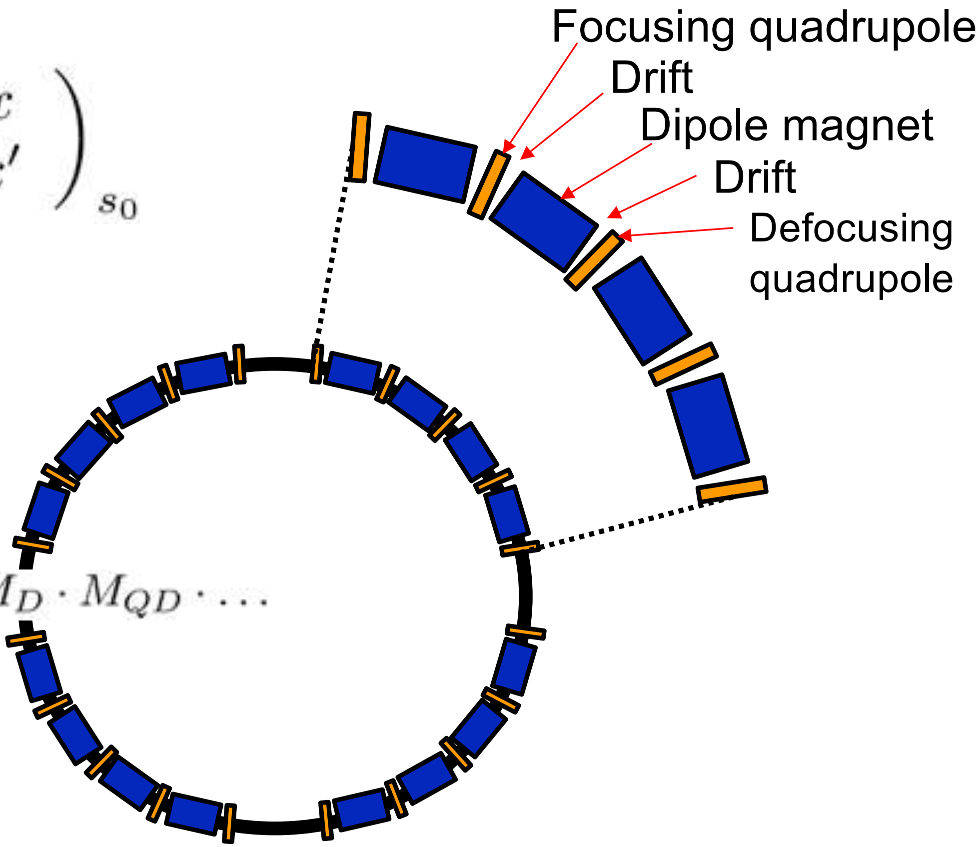
Dipole

$$M_{dipole} = \begin{pmatrix} \cos(\frac{s}{\rho}) & \rho\sin(\frac{s}{\rho}) \\ -\frac{1}{\rho}\sin(\frac{s}{\rho}) & \cos(\frac{s}{\rho}) \end{pmatrix}$$

Particle tracking

Knowing the initial coordinates at $s=s_0$, we can use the transfer matrix to calculate the effect of an element to the particle's trajectory and get its new coordinates at $s=s_1$.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$



For a sequence of elements:

$$M_{total} = M_{QF} \cdot M_D \cdot M_{Bend} \cdot M_D \cdot M_{QD} \cdot \ldots$$

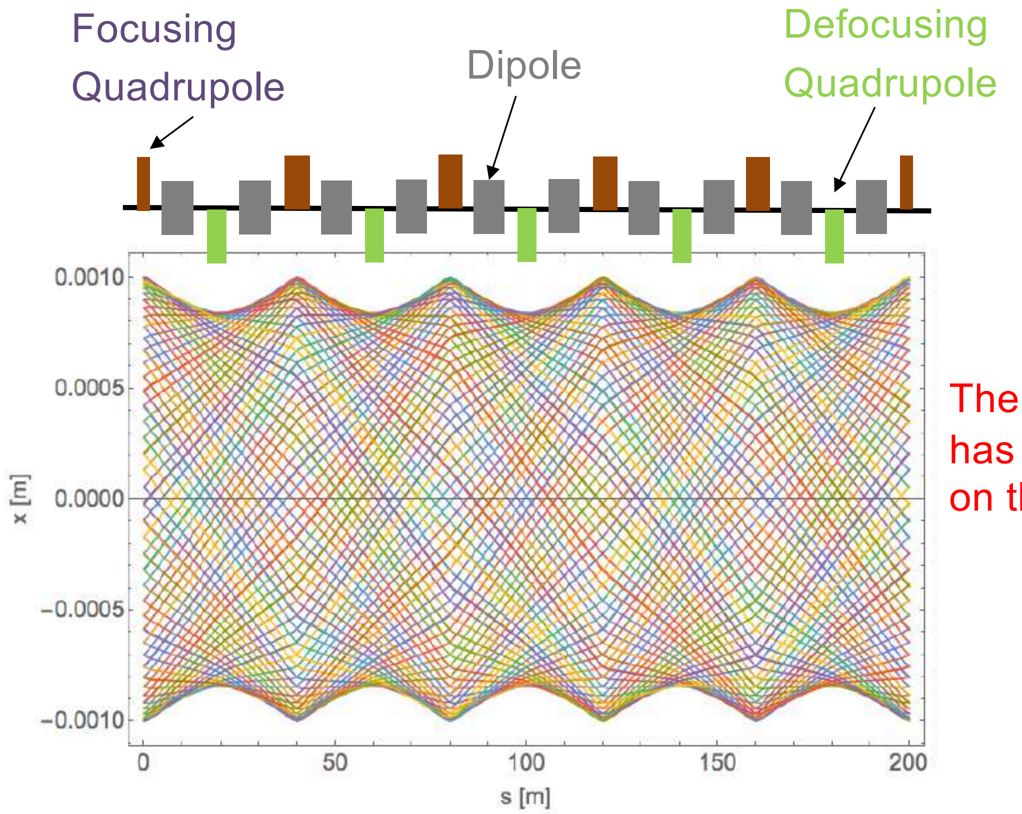
Building up the particles path through the accelerator …

How does a particle trajectory look like?

Initial coordinates

x_0 = 0.001m (1 mm)

x_0' = 0



The envelope of all trajectories has a periodicity that depends on the lattice

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

Hill's equation

We had ...

$$x'' + Kx = 0$$

But, around the accelerator *K* is not constant and does depend on *s*!

$$x''(s) + K(s)x(s) = 0$$

Hill's equation

- $K(s+L) = K(s)$ → periodic function, where L is the "lattice period"

- General solution of Hill's equation:

$$x(s) = \sqrt{2J_x\beta_x(s)}\cos(\psi(s) + \phi)$$

It is a **quasi harmonic oscillation**, where **amplitude and phase depend on the position s** in the ring.

The Beta function

General solution of Hill's equation

$$x(s) = \sqrt{2J_x \beta_x(s)} \cos(\psi(s) + \phi)$$

Integration constants: determined by initial conditions

The **beta function** is a periodic function determined by the focusing properties of the lattice, i.e. quadrupoles

$$\beta(s+L) = \beta(s)$$

The **"phase advance"** of the oscillation between the point s_0 and point s in the lattice.

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

The Tune

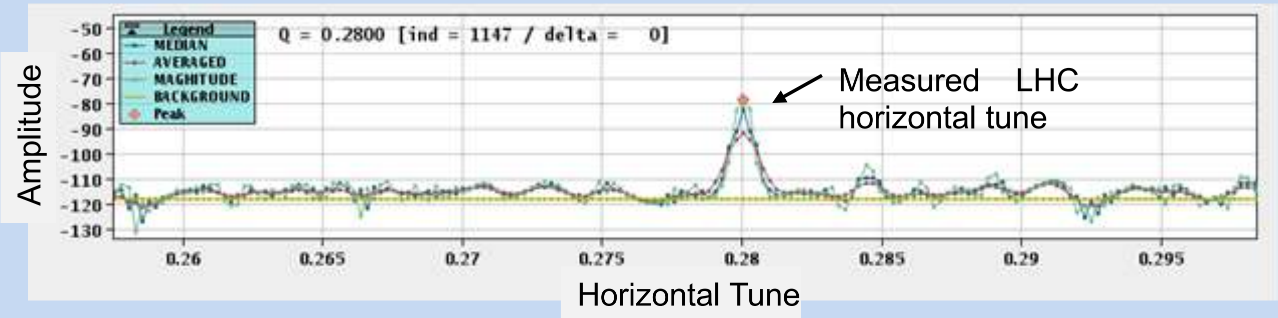
The number of oscillations per turn is called **"tune"**

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)} \xrightarrow{\text{full turn}} Q = \frac{1}{2\pi} \int \frac{ds}{\beta(s)}$$

The tune is an important parameter for the **stability of motion** over many turns. It has to be **chosen appropriately, measured and corrected**.

Tune Measurement

1) Measure beam position at one location turn by turn.
2) Beam position will change $\propto \cos(2\pi Q i)$.
3) Perform FFT to get frequency of oscillation → tune



Courant-Snyder Parameters: $\alpha(s), \beta(s), \gamma(s)$

General solution of Hill's equation: $x(s) = \sqrt{2J_x\beta_x(s)}\cos(\psi(s) + \phi)$

Define: $\alpha(s) = -\frac{1}{2}\beta'(s) \qquad \gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$

$\alpha(s), \beta(s), \gamma(s)$ are called **Courant-Snyder parameters or Optics parameters**

Let's assume for $s(0) = s_0$, $\psi(0) = 0$, $\beta(0) = \beta_0$ and $\alpha(0) = \alpha_0$

Defines ϕ from initial conditions: x_0 and x'_0, β_0 and α_0.

Re-write transfer matrix with optics parameters:

$$M = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}(\cos\psi + \alpha_0 \sin\psi) & \sqrt{\beta\beta_0}\sin\psi \\ \frac{(\alpha_0 - \alpha)\cos\psi - (1+\alpha\alpha_0)\sin\psi}{\sqrt{\beta\beta_0}} & \sqrt{\frac{\beta_0}{\beta}}(\cos\psi - \alpha\sin\psi) \end{pmatrix}$$

Once we know α and β, we can compute the single particle trajectories between two locations without remembering the exact lattice structure and strength of each element!

Phase Space

General solution of Hill's equation: $x(s) = \sqrt{2J_x\beta_x(s)}\cos(\psi(s) + \phi)$

J_x is called **action** and can be written as:

$$J_x = \frac{1}{2}\left(\gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2\right)$$

which is the equation of an **ellipse** in the **phase-space x, x'.**

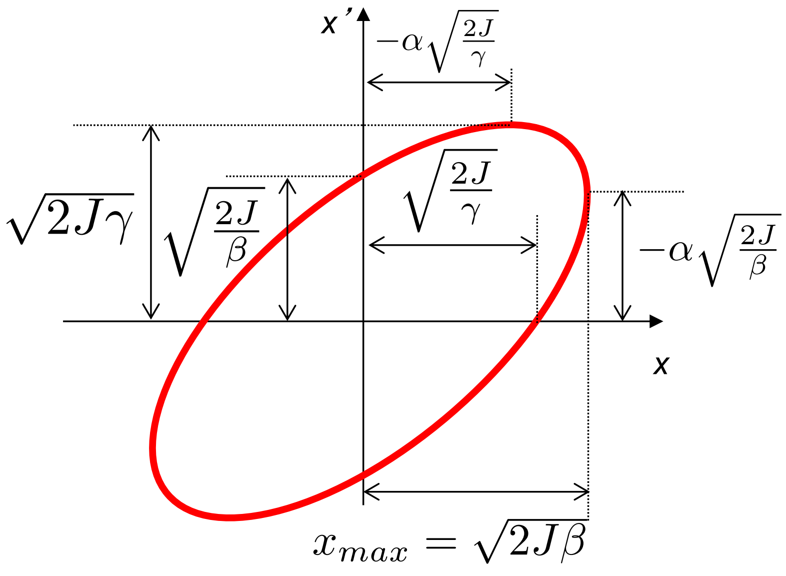
The shape and orientation of ellipse are defined by the Courant-Snyder parameters.

The area of the ellipse is:

$$A = 2\cdot\pi\cdot J_x$$

x-x' phase space

(trajectory offset vs. angle)



Emittance and beam size

At a given location: $x = \sqrt{2\beta_x J_x}\cos\psi_x$

The mean square value of this is:

$$\langle x^2 \rangle = 2\beta_x \langle J_x \cos^2\psi_x \rangle = \beta_x \langle J_x \rangle = \beta_x \epsilon_x$$

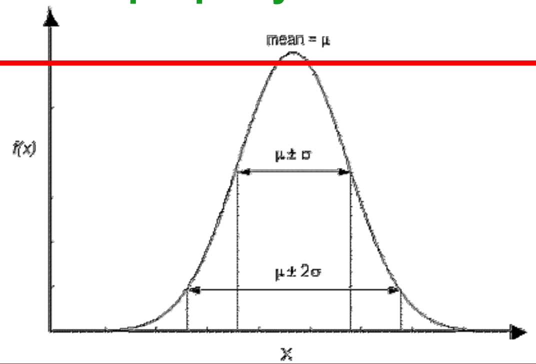
assumes action and phase uncorrelated, and uniform distribution in phase from 0 to 2π.

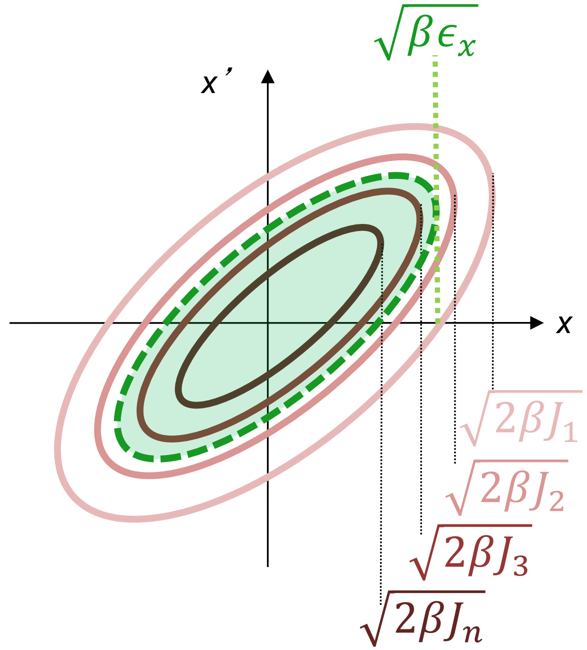
Defines ***emittance*** of particle distribution:

$$\langle J_x \rangle = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2} := \epsilon_x$$

ϵ_x is an **intrinsic beam property** that is defined at it's creation.

In LHC it is defined by the injector chain properties





Typically the distribution of particles in a bunch follows a Gaussian shape:

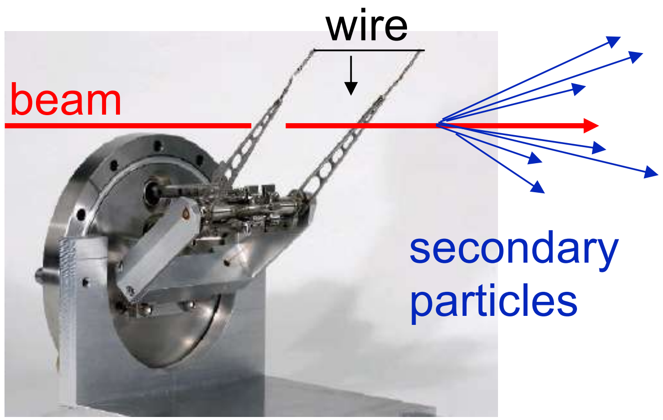
$$\rho(x) = \frac{N}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{x^2}{2\sigma_x^2}}$$

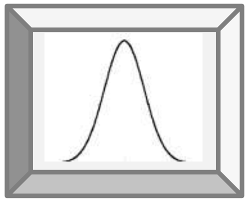
Therefore, $\sigma_x = \sqrt{\langle x^2 \rangle} = \sqrt{\epsilon_x \beta_x}$

describes the one sigma **beam size**.

Beam size and emittance measurement

Principle of a wire-scanner beam size measurement



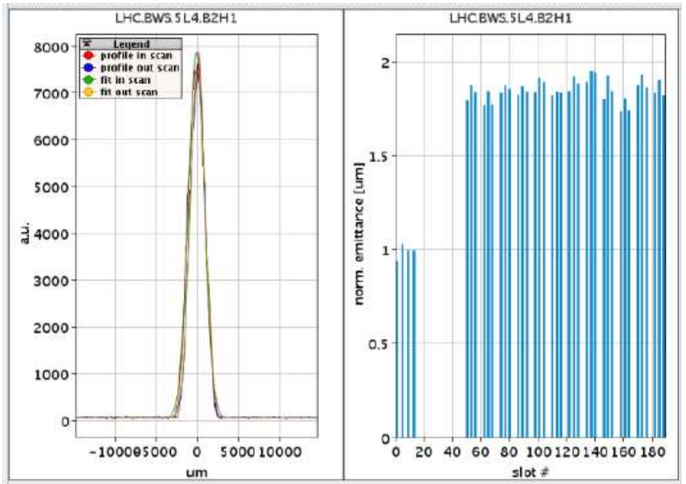


$$\sigma_x = \sqrt{\varepsilon \beta_x}$$

Gaussian fit to profile → beam size σ

Knowledge of β-function → emittance ε

Single horizontal bunch profile



LHC measurement

Emittance calculated from profile measurement.

All circulating bunches.

Beam size around the accelerator

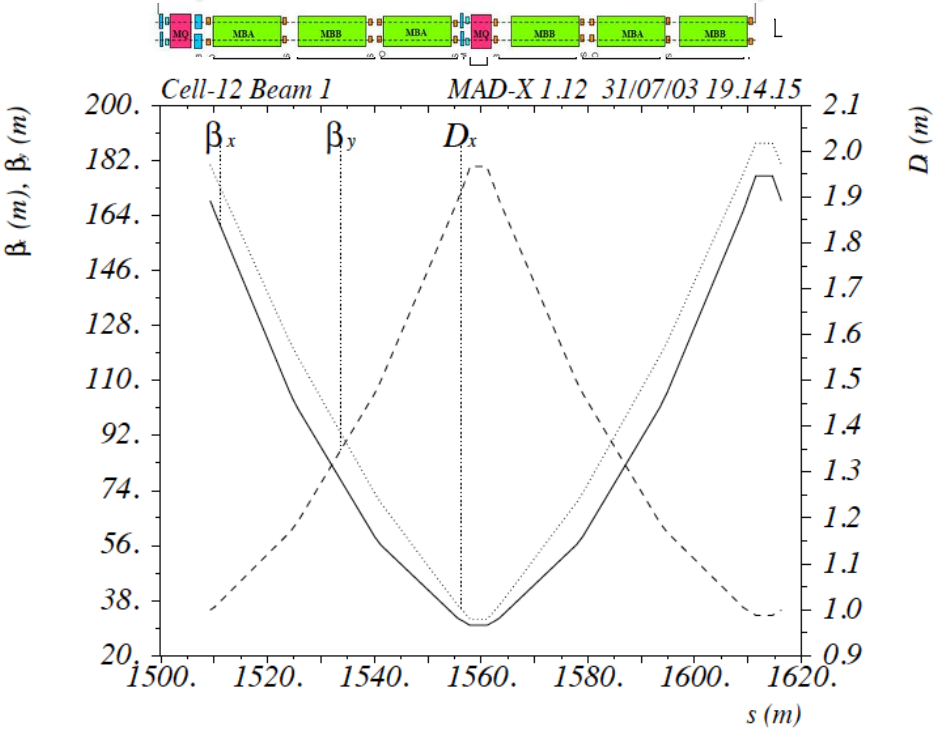
The β-function is periodic

→ It changes along the cell.

→ The beam size changes along the cell!

$$\sigma = \sqrt{\varepsilon\beta}$$

FODO



Max. horizontal beam size in the **focusing quadrupoles**

Max. vertical beam size in the **defocusing quadrupoles**

The regular LHC FODO cell:

- *Phase advance: 90°*
- *Maximum beta: 180 m*

Things to remember

Phase space

A space that represents all possible states of a system.

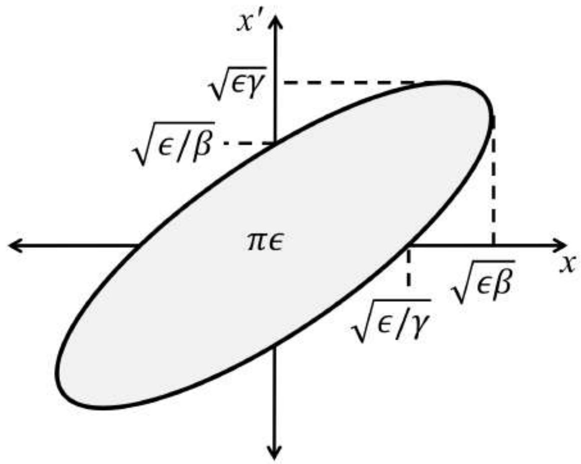
A particle's trajectory points or coordinates at a given element draw an ***ellipse in phase space***.

The orientation and shape of that ellipse is described by the optical (Courant-Snyder) parameters. → β-function

The area of that ellipse is $\propto$ ***emittance***.

Emittance is a beam property that cannot be changed by focusing.

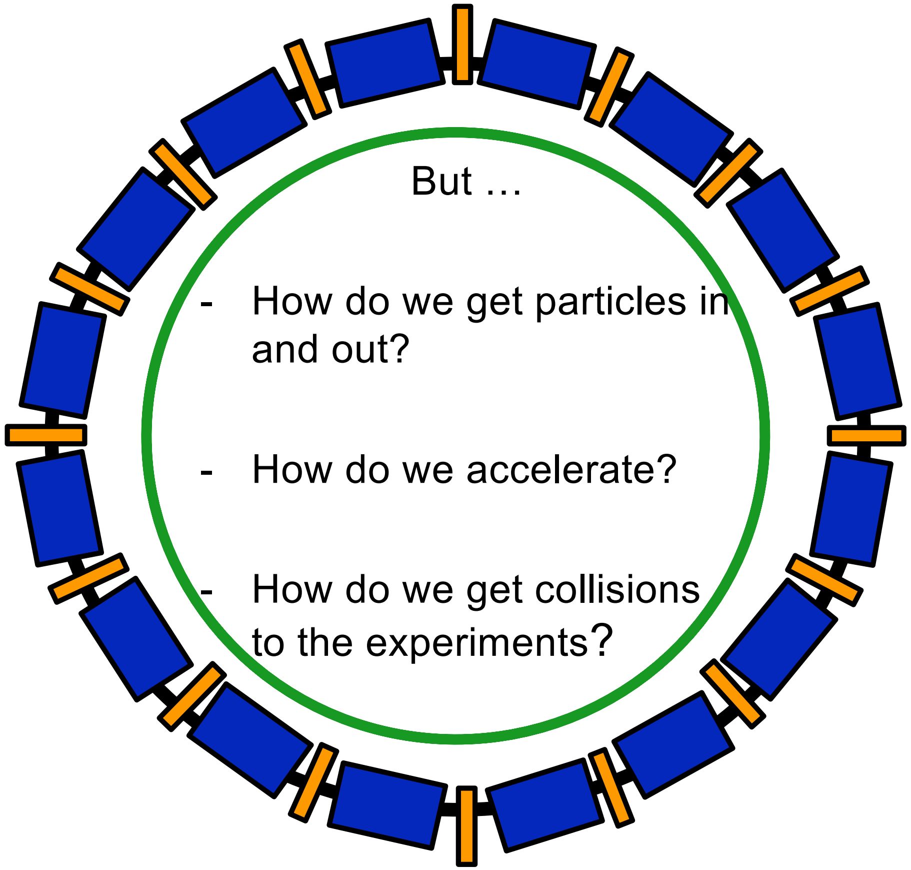
The ***beam size*** of a particle ensemble is defined by $\sigma = \sqrt{\epsilon\beta}$.



Beam Injection/extraction

What we learned so far?

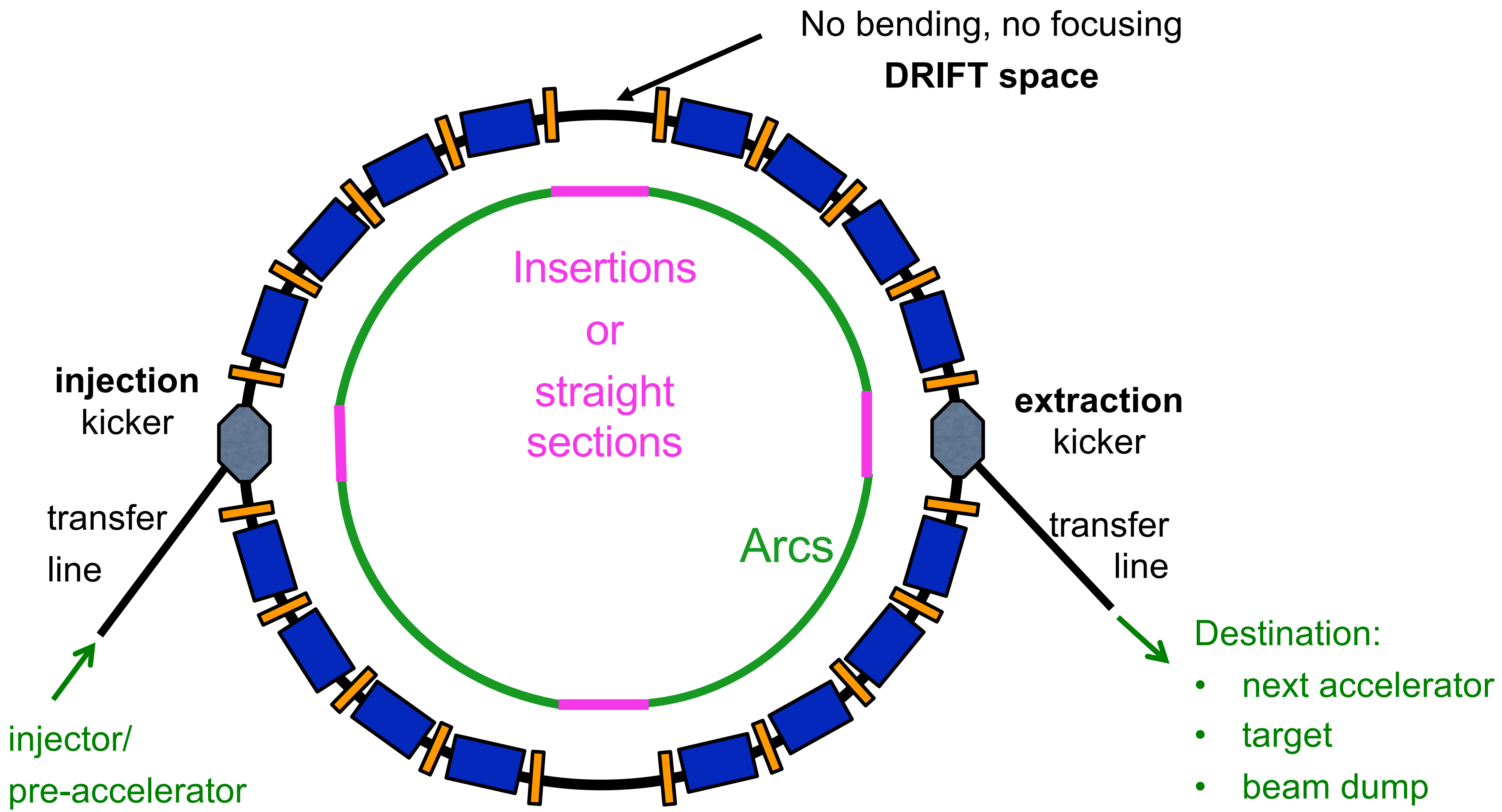
We know, how particles behave along the magnetic lattice of an accelerator.



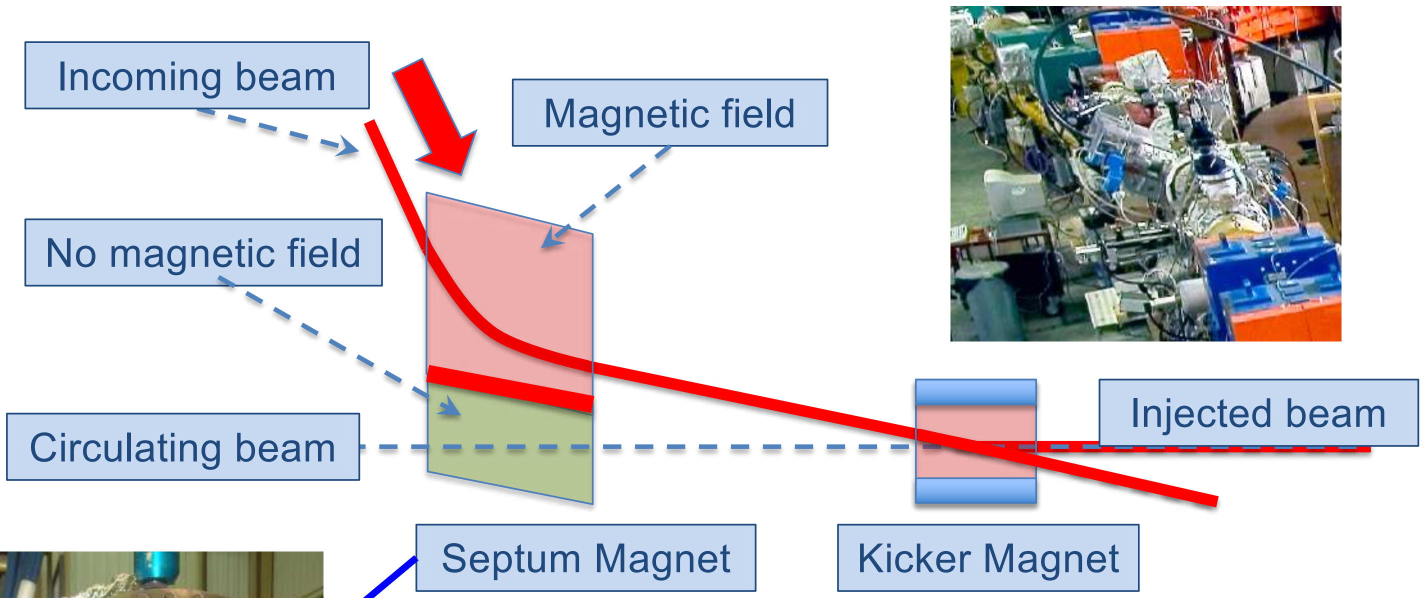
But …

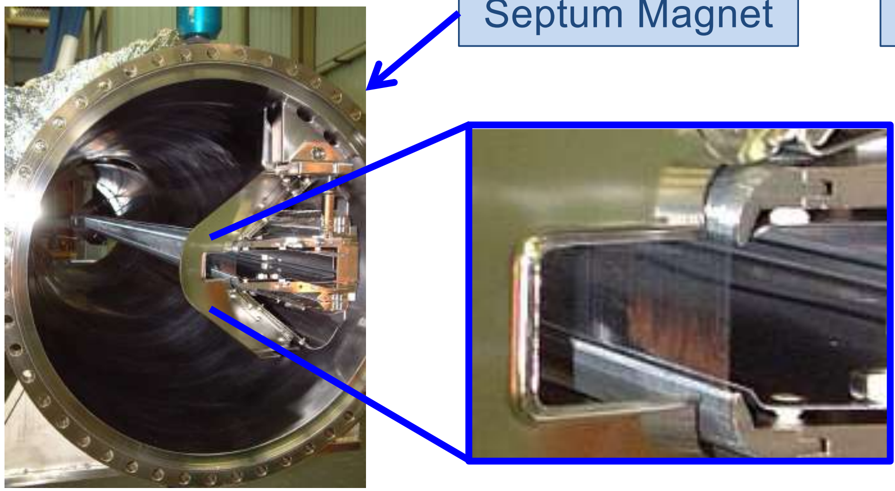
- How do we get particles in and out?
- How do we accelerate?
- How do we get collisions to the experiments?

Straight Sections and Insertions



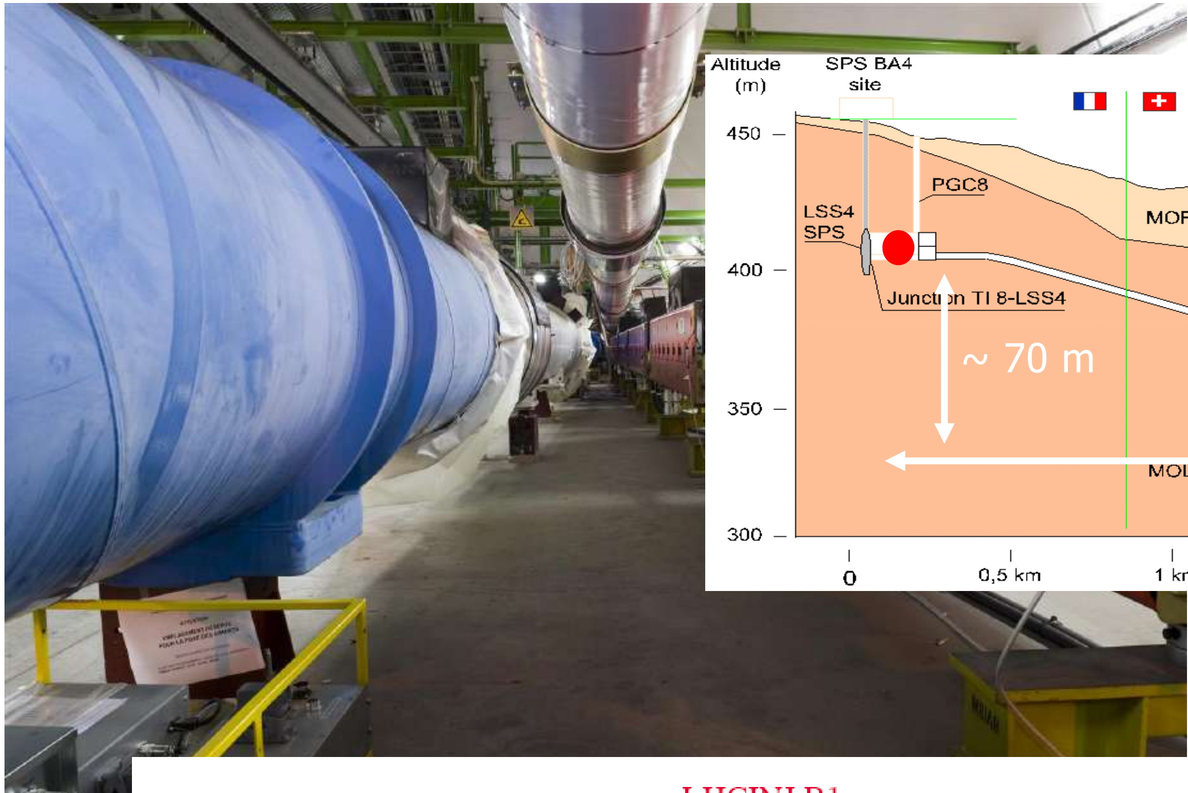
Injection and extraction

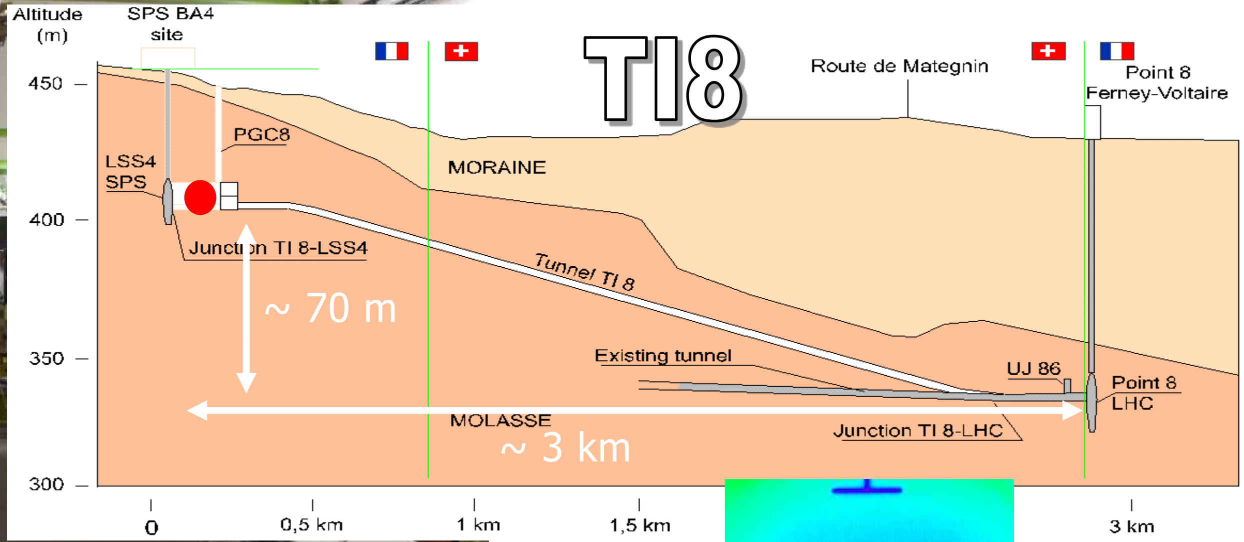


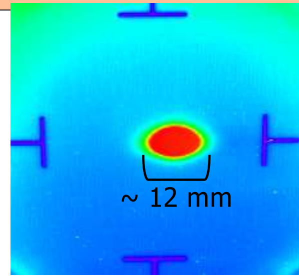


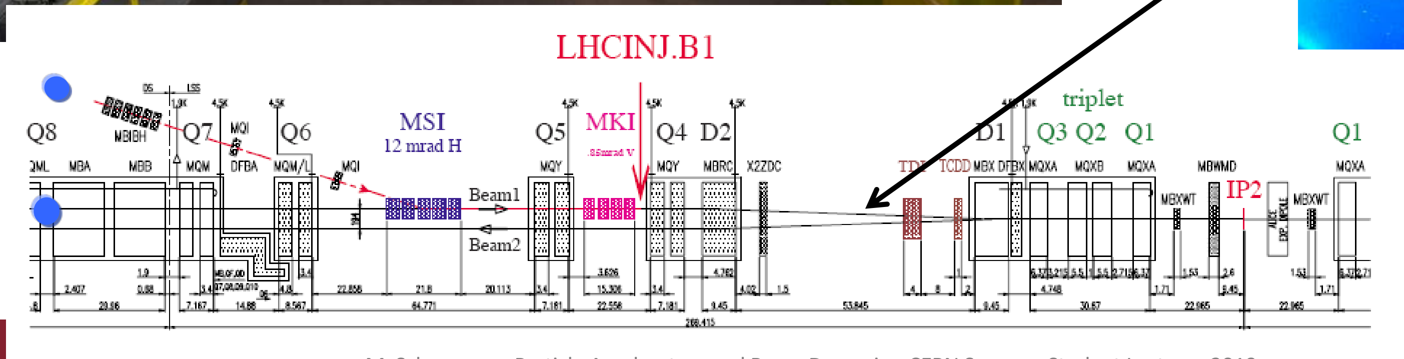
Extraction follows the same principle, but the beam travels in the **opposite direction**.

Injection of Beam 2 into LHC



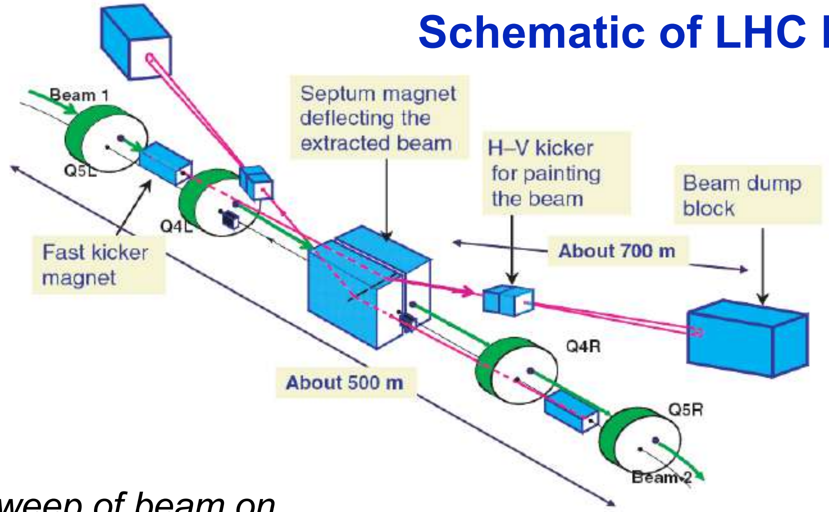






Beam dump – How to safely kill the LHC beam

Schematic of LHC beam dump system



LHC beam stores ~360MJ energy.

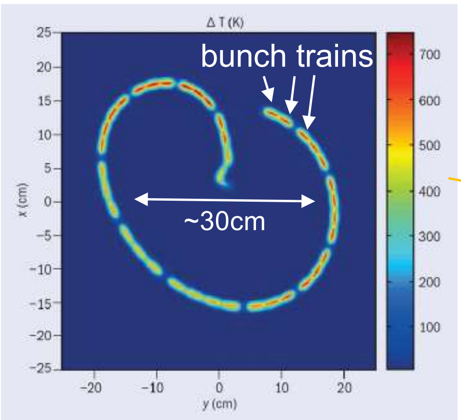
Let's compare it to the kinetic energy of a frecciarossa train whose mass is **500 ton**

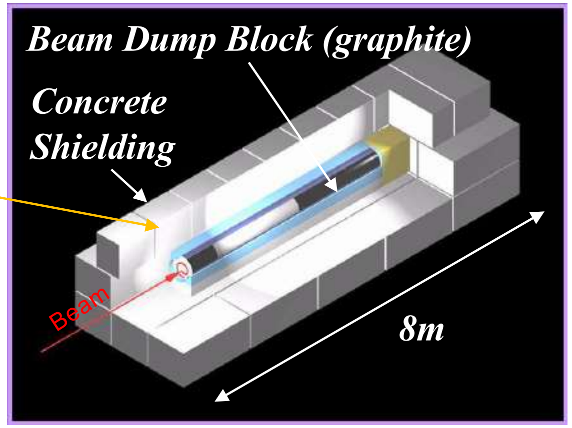
$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 3.6 \cdot 10^8}{5 \cdot 10^5}} =$$

$$= 37.9\ \frac{m}{s} \cong \mathbf{140}\ \frac{\mathbf{km}}{\mathbf{h}}$$

Better be careful

Sweep of beam on beam dump window





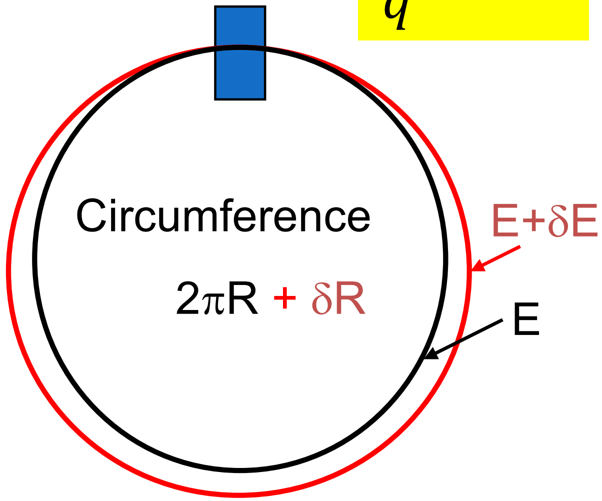
acceleration

RF Acceleration and magnet field increase

What about the magnetic field during acceleration?

cavity

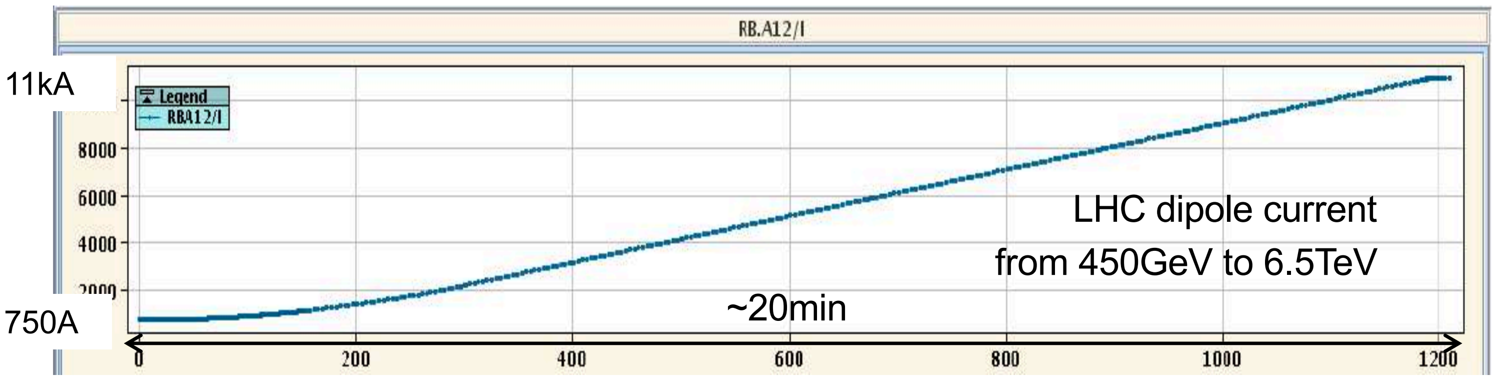
$$\frac{p}{q} = B\,\rho$$



Beam rigidity needs to be increased proportionally to increasing energy.

→ Machine radius is constant.

→ Need to increase dipole field accordingly!



Acceleration without magnetic field increase

LHC magnetic dipole field at 450 GeV:

$$B = \frac{p}{q\rho} = \frac{450\,\mathrm{GeV}/c}{e \times 2803\,\mathrm{m}} = 0.535\,\mathrm{T}$$

Required bending radius at 7 TeV with B_{inj}=0.5T:

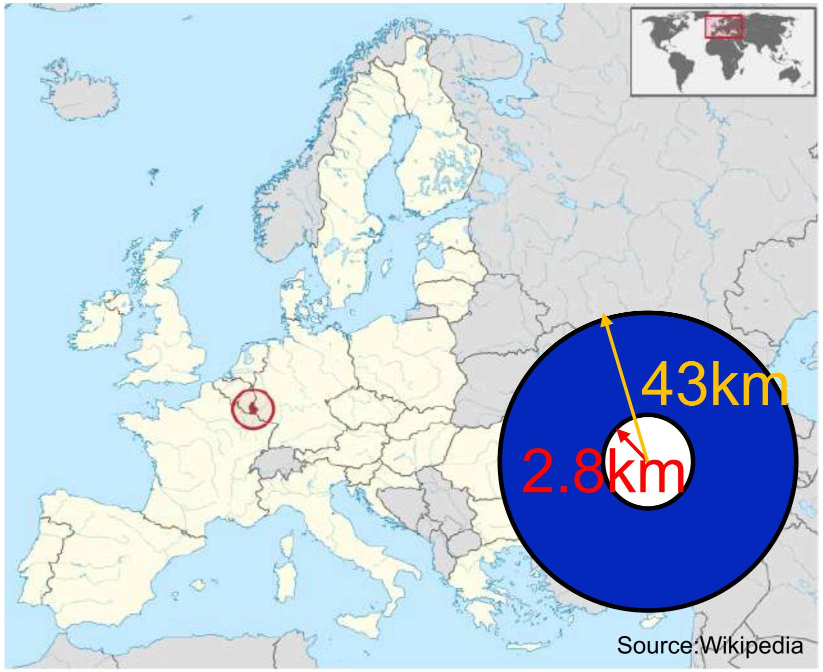
$$\rho = \frac{p}{qB} = \frac{7\,\mathrm{TeV}/c}{e \times 0.535\,\mathrm{T}} = 43.6\,\mathrm{km}$$

How does the bending radius changes, when accelerating without adjusting the magnetic field?

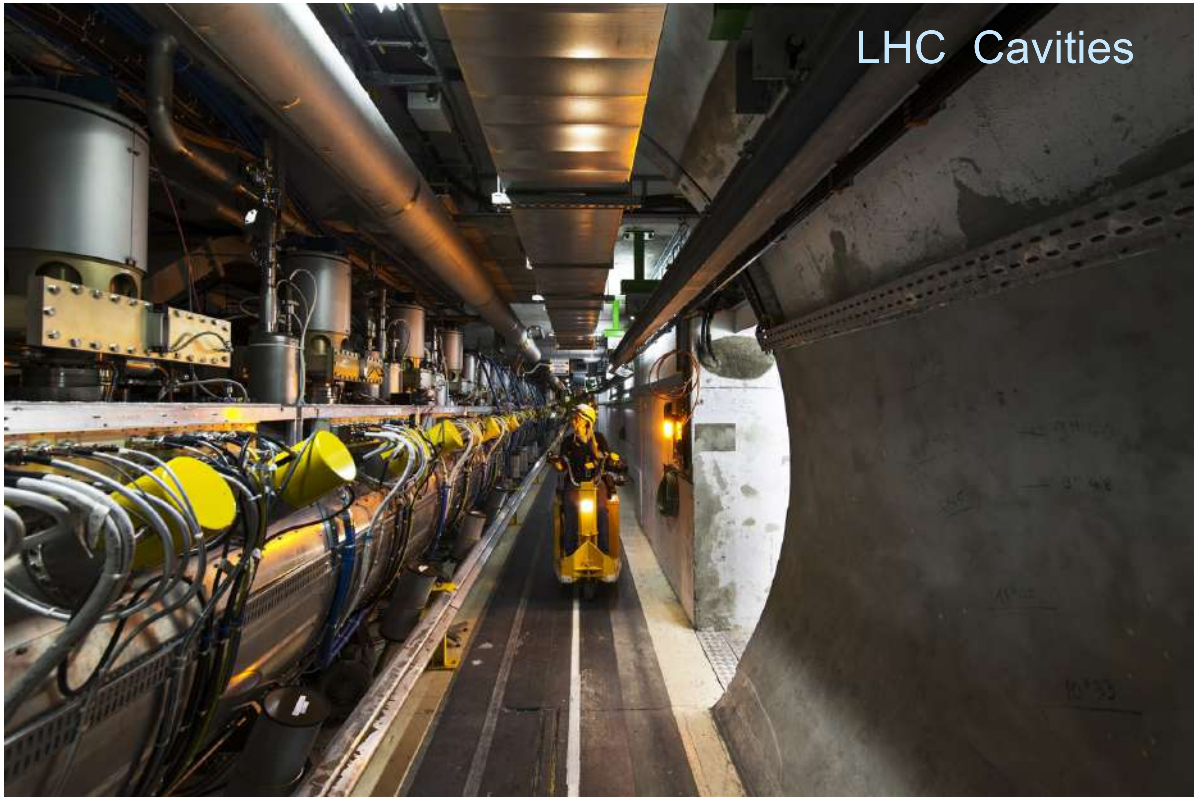
$$\frac{p}{q} = B\,\rho$$

Equivalent to 270km circumference (pure dipole field! without any insertions or quadrupoles)

Magnet surface = 5800km^2
→Area of Brunei (South-Eastern Asia)
→ Area of 2x Luxemburg



Example: LHC accelerating system



LHC has

- 8 superconducting cavities per beam
- Accelerating field 5 MV/m
- Can deliver 2 MV/cavity (peak voltage)
- Operating at 400 MHz
- Beam aperture (radius) ~30cm
- Energy gain/turn during ramp 485 keV (11245 turns/s)

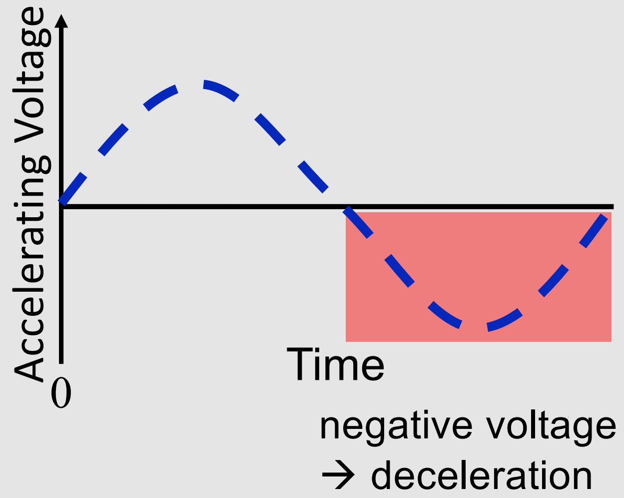
Going from 450 GeV (injection energy) up to 6.8 TeV (collision energy) takes about 20 minutes.

RF acceleration

Accelerating voltage is changing with time. That has two consequences:

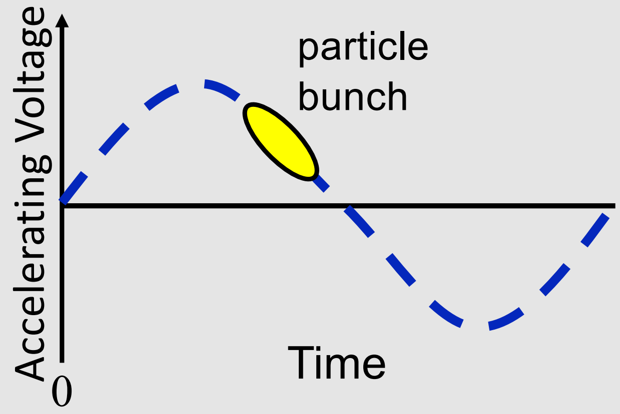
Need ***synchronization*** between beam and RF phase to gain energy.

There is a ***synchronous RF phase*** for which the energy gain fits the increase of the magnetic field.



Not all particles see the same voltage, because they arrive at different times.

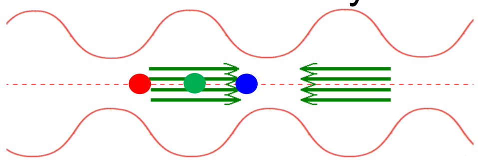
Not all particles gain the same energy.



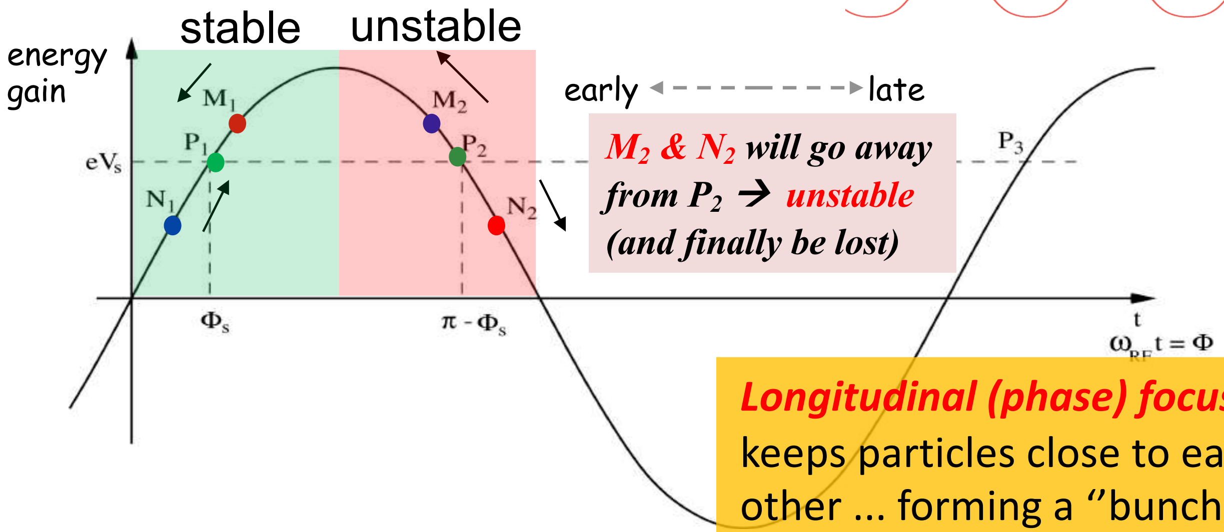
Phase stability (non-relativistic regime)

Assume the situation where ***energy increase is transferred into a velocity increase*** (non-relativistic regime).

field in a cavity



Particles P_1, P_2 have the synchronous phase.



M_2 & N_2 will go away from P_2 → unstable (and finally be lost)

Longitudinal (phase) focusing keeps particles close to each other ... forming a ''bunch"

- Ideal particle
- Particle with $\Delta t < 0$ (early) →lower energy gain → gets slower
- Particle with $\Delta t > 0$ (late) →higher energy gain → gets faster

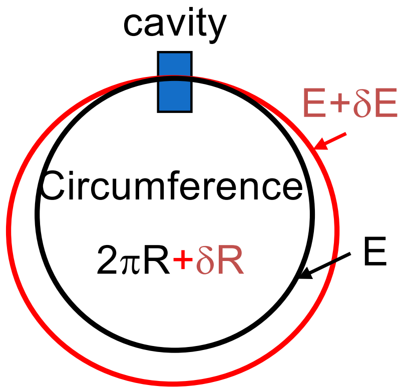
→ ***M_1 & N_1 will move towards P_1 → stable***

Phase stability (relativistic regime)

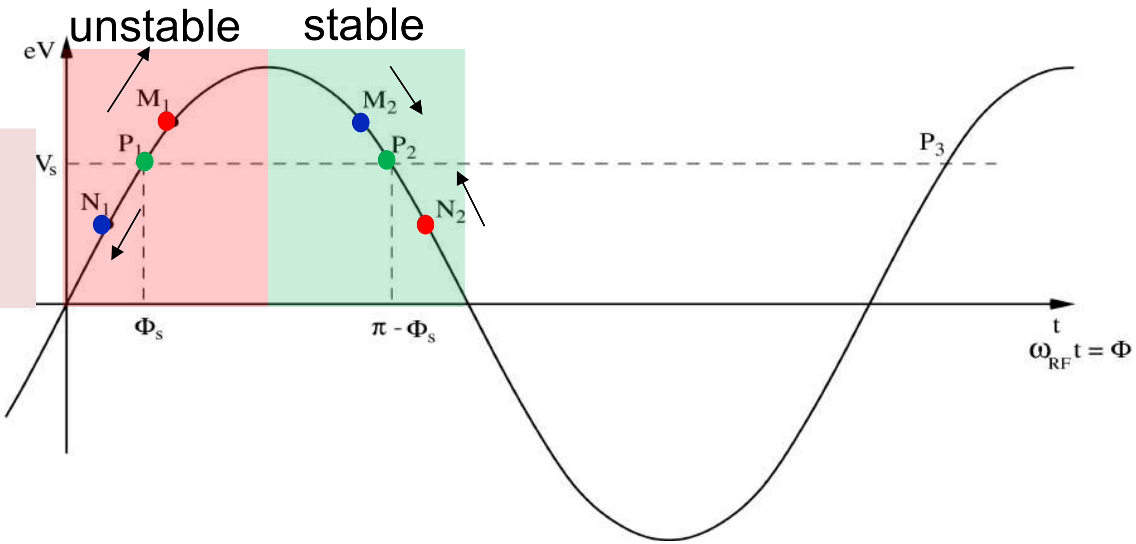
Now assume relativistic energies ($v \approx c$):

An ***increase in momentum*** transforms into a ***longer orbit and thus a longer revolution time.***

$$\frac{p}{q} = B\,\rho$$

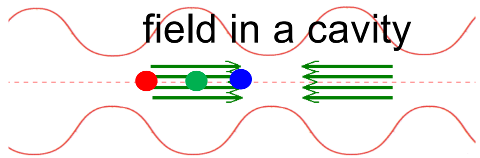


M_1 & N_1 will go away from P_1
→ unstable (and finally be lost)



- Ideal particle
- Particle with Δt < 0 → higher energy gain → gets longer orbit
- Particle with Δt > 0 → lower energy gain → gets shorter orbit

M_2 & N_2 will move towards P_2 → stable

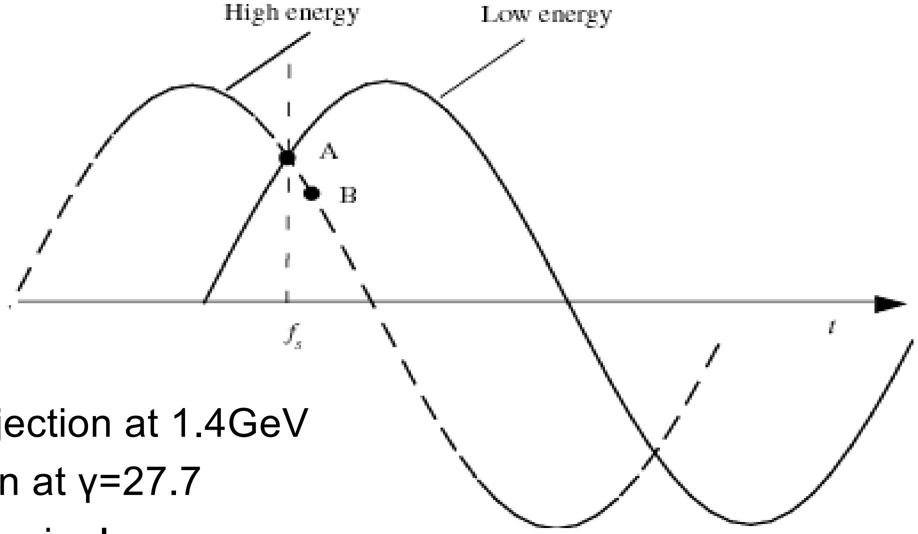


Crossing transition

The previously stable synchronous phase becomes unstable when v => c and the gain in path length overtakes the gain in velocity → ***Transition***

Transition from one slope to the other during acceleration → ***Crossing Transition.***

The RF system needs to make a rapid change of the RF phase, a 'phase jump'.



In the PS: γ_t is at ~6 GeV, injection at 1.4GeV

In the SPS: γ_t= 22.8, injection at γ=27.7

=> no transition crossing!

In the LHC: γ_t is at ~55 GeV, also far below injection energy

Transition crossing not needed in leptons machines, why?

Synchrotron Oscillation

Like in the transverse plane the particles are oscillating in longitudinal space.

Particles keep *oscillating around the stable synchronous particle* varying phase and dp/p.

Typically one synchrotron oscillation takes many turns (much slower than betatron oscillation)

Phase-space ellipse defines ***longitudinal emittance***.

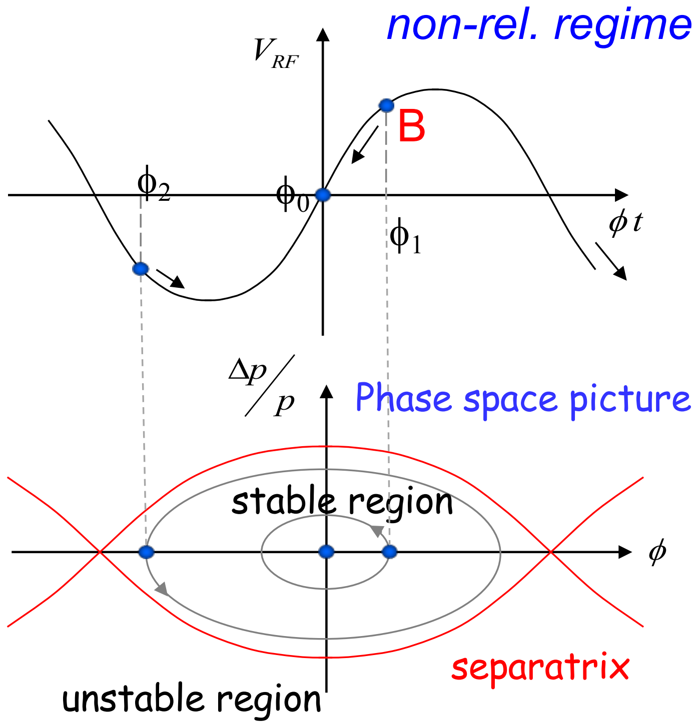
Separatrix is the trajectory separating stable and unstable motion.

Stable region is also called ***bucket***.
→ Harmonic number h = number of buckets:

$$f_{RF} = h\, f_{rev}$$

Simple case (no accel.): B = const.

- Stable phase: $\phi_0 = 0$
- Particle B oscillates around ϕ_0.



Emittance during Acceleration

What happens to the emittance if the reference momentum P_0 changes?

Can write down transfer matrix for reference momentum change:

$$M_x = \begin{pmatrix} 1 & 0 \\ 0 & P_0/P_1 \end{pmatrix} \Rightarrow \epsilon_{x1} = \frac{P_0}{P_1}\epsilon_{x0}$$

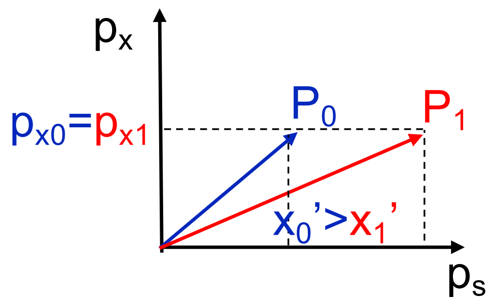
The emittance shrinks with acceleration!

With $P = \beta\gamma mc$ where γ, β are the relativistic parameters.

The conserved quantity is

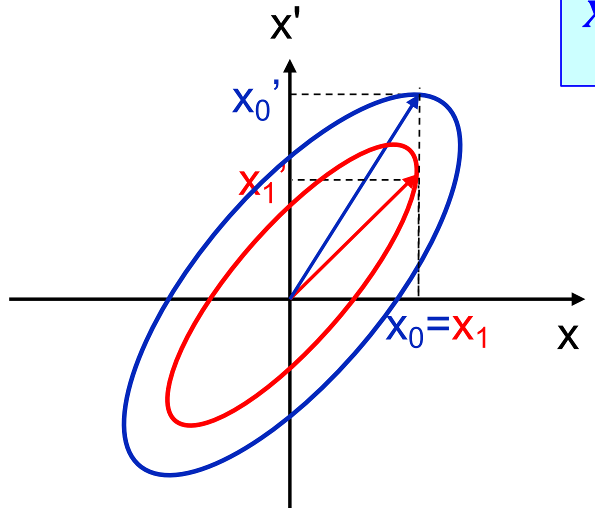
$$\beta_1\gamma_1\epsilon_{x1} = \beta_0\gamma_0\epsilon_{x0}$$

It is called ***normalized emittance***.



Only longitudinal momentum changes during the acceleration

$$X_0' \approx \sin\theta = \frac{P_{x0}}{P_0}$$



How big are the beams in the LHC?

Normalized emittance at LHC : $\varepsilon_n = 3.5\ \mu m$

→ ε_n preserved during acceleration.

$$\boxed{\varepsilon_{7TeV} = \varepsilon_{450GeV} \frac{\gamma_{450GeV}}{\gamma_{7TeV}}}$$

The **geometric emittance:**

- Injection energy of 450 GeV: $\varepsilon = 7.3$ nm
- Top energy of 7 TeV: $\varepsilon = 0.5$ nm

$$\boxed{\sigma = \sqrt{\varepsilon \beta}}$$

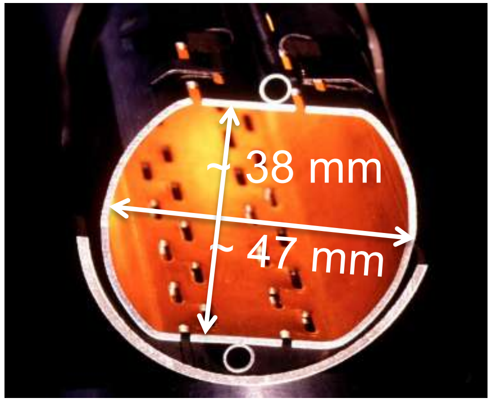
The corresponding max. **beam sizes** in the arc,
at the location with the maximum beta function ($\beta_{max} = 180$ m):

- $\sigma_{450GeV} = 1.1$ mm
- $\sigma_{7TeV} = 300\ \mu m$

Aperture requirement: $a > 10\ \sigma$

LHC beam pipe radius:

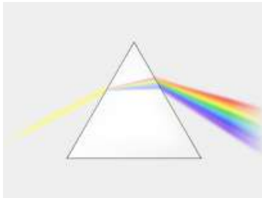
- Vertical plane: 19 mm ~ 17 σ @ 450 GeV
- Horizontal plane: 23 mm ~ 20 σ @ 450 GeV



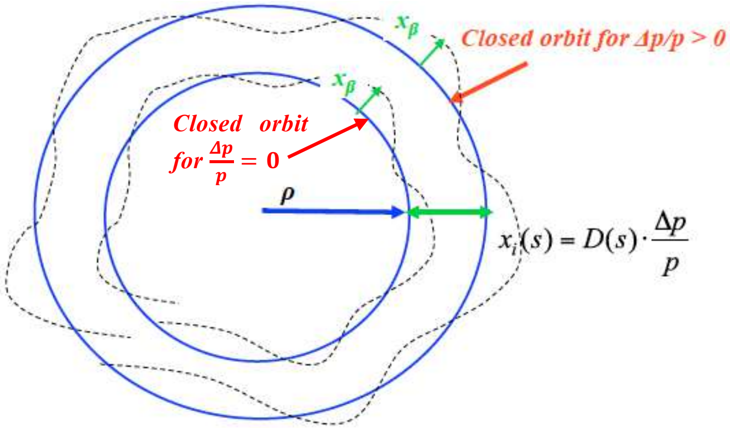
Transverse-Longitudinal Coupling: Dispersion

Dipole magnets generate dispersion:

→ Particles with different momentum are bent differently.



Due to the momentum spread in the beam $\frac{\Delta p}{p}$, this has to be taken into account for the particle trajectory.



$$\boxed{x(s) = x_\beta(s) + D(s)\frac{\Delta p}{p}}$$

Dispersion function *D(s)* corresponds to the trajectory of a particle with momentum offset $\frac{\Delta p}{p} = 1$.

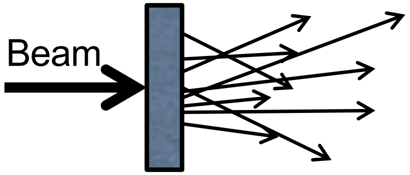
This also has an effect on the beam size:

$$\sigma = \sqrt{\beta\varepsilon} \quad \rightarrow \quad \sigma = \sqrt{\beta\varepsilon + D^2\left(\frac{\Delta p}{p}\right)^2}$$

Experiments and Luminosity

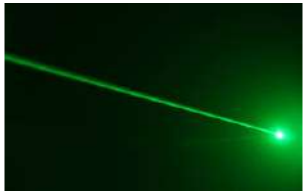
Each accelerator and experiment requires specific beam properties.
Fundamentally different are:

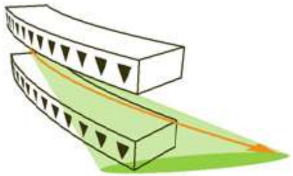
Fixed Target:



Secondary particles

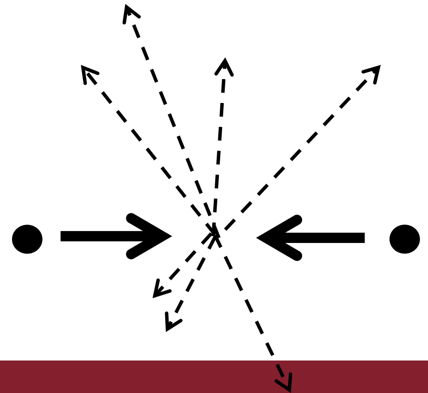
Light Sources:





Particles that are bent to a circular orbit emit energy/light.

Collider:



“Smashing” Modes and Center-of-Mass Energy

The ***center-of-mass energy*** defines
the upper limit of the newly created particle’s mass.

Fixed Target	**Collider**

$$E \propto \sqrt{E_{beam}}$$

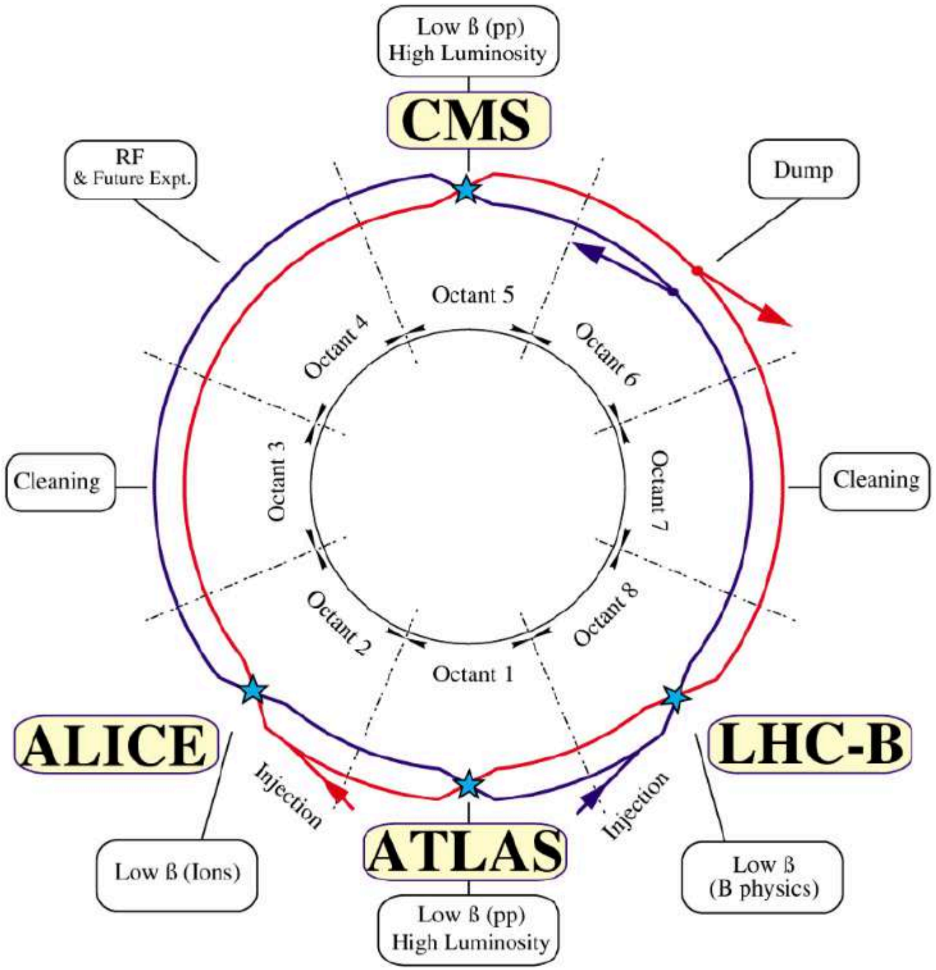
Most of the Energy is lost in the target, only a fraction is transformed into useful secondary particles.

$$E = E_{beam1} + E_{beam2}$$

All energy is available for the production of new particles.

Price to pay in a collider: event rate

LHC and its Experiments



LHC has **4 interaction points (IPs)** hosting particle physics experiments:

→ ATLAS, ALICE, CMS, LHCb

Therefore the two counterrotating **beams collide 4 times per turn**

When they collide the outer beam **cross over** to the inner circle and vise versa.

Particle Collisions

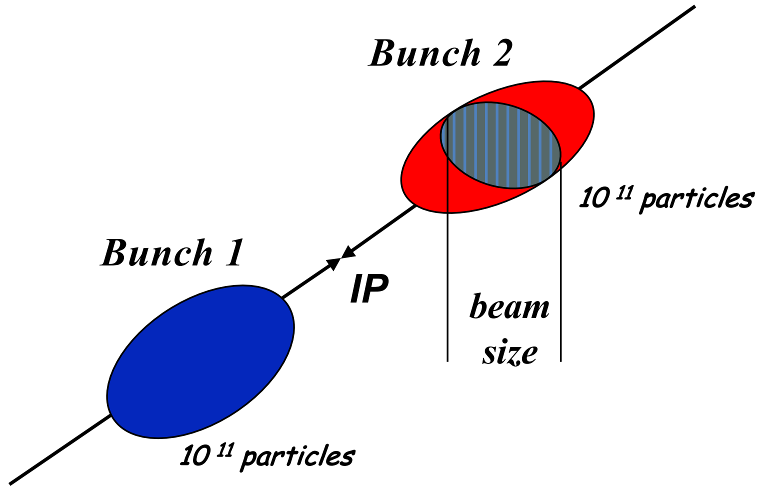
Experiments are interested in maximum number of interactions per second.
The event rate in an experiment is proportional to the collider luminosity.

Interaction Cross-section
(= constant probability for a given process to take place)

$$\boxed{\frac{dN_{evts}}{dt} = L \times \sigma_p}$$

Luminosity

Event rate
(= number of events per time)



"quality factor" of a Collider

The most important factor to describe the potential of a collider is the Luminosity.

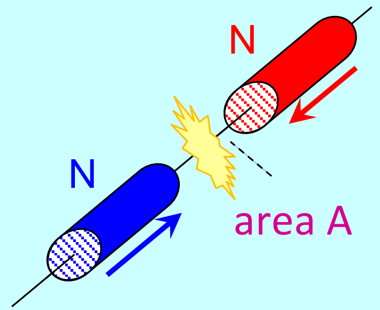
$$L = \frac{k N^2 f \gamma}{4\pi \beta^* \varepsilon} \cdot F$$

Defined by the injectors

N..... No. particles per bunch
k...... No. bunches
f....... revolution freq.
g...... rel. gamma
β^*.... beta-function at IPs
ε...... norm. trans. emit

$$\sigma = \sqrt{\varepsilon \beta}$$

$$\rightarrow \boldsymbol{\varepsilon\beta \approx A}$$



Limitation:
"Collective effects" cause beam instabilities for too high bunch intensities, too small bunch spacing,

too "bright" beams.

Overall Goal of an Collider: Maximizing Luminosity!

→ Many particles (N, k)

→ In a small transverse cross-section (ε, β)

Performance depends on the injectors:

→ Production of large N and small ε

→ Preservation of these parameters until collisions.

Optimizing Luminosity

Bunch properties (N & ϵ) are defined in the injectors.

But what can be done in the Collider?

$$L = \frac{kN^2 f\gamma}{4\pi\beta^*\varepsilon} \cdot F$$

N..... No. particles per bunch

k...... No. bunches

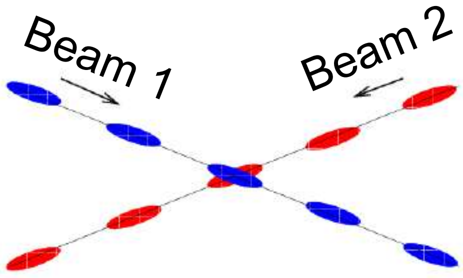
f....... revolution freq.

g...... rel. gamma

β^*.... beta-function at IPs

ε...... norm. trans. emit

$\boldsymbol{f_{rev}}, \boldsymbol{\gamma}$: defined by the design of the accelerator



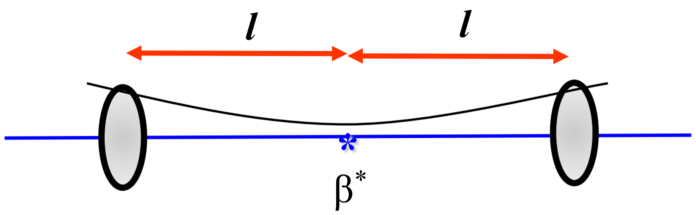
F [0,1]: When colliding with many bunches, a **crossing angle is needed** to avoid unwanted collisions. However this ***reduces the beam overlap*** and therefore the luminosity. Keep as *small as possible*! (at LHC ~0.8)
→Limited by beam-beam effects.

k: Optimize filling scheme and bunch spacing.

$\boldsymbol{\beta^*}$: Can be optimized by focusing!

Mini-Beta Insertions

Mini-beta insertion is a ***symmetric drift space*** with a ***waist of the β-function*** in the center of the insertion.



On each side of the symmetry point a quadrupole **doublet** or **triplet** is used to generate the waist.

They are not part of the regular lattice.

$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$

Collider experiments are located in mini-beta insertions: **smallest beam size possible** for the colliding beam to increase probability of collisions.

There is a price to pay: **The smaller β^*, the larger β at the triplet.**

Example: Mini-Beta Insertion at LHC

Example of the LHC
(design report values):

At the interaction point:

$\beta^* = 0.55$ m

$\sigma^* = 16\ \mu$m

That's smaller than a hair's diameter!

At the triplet:

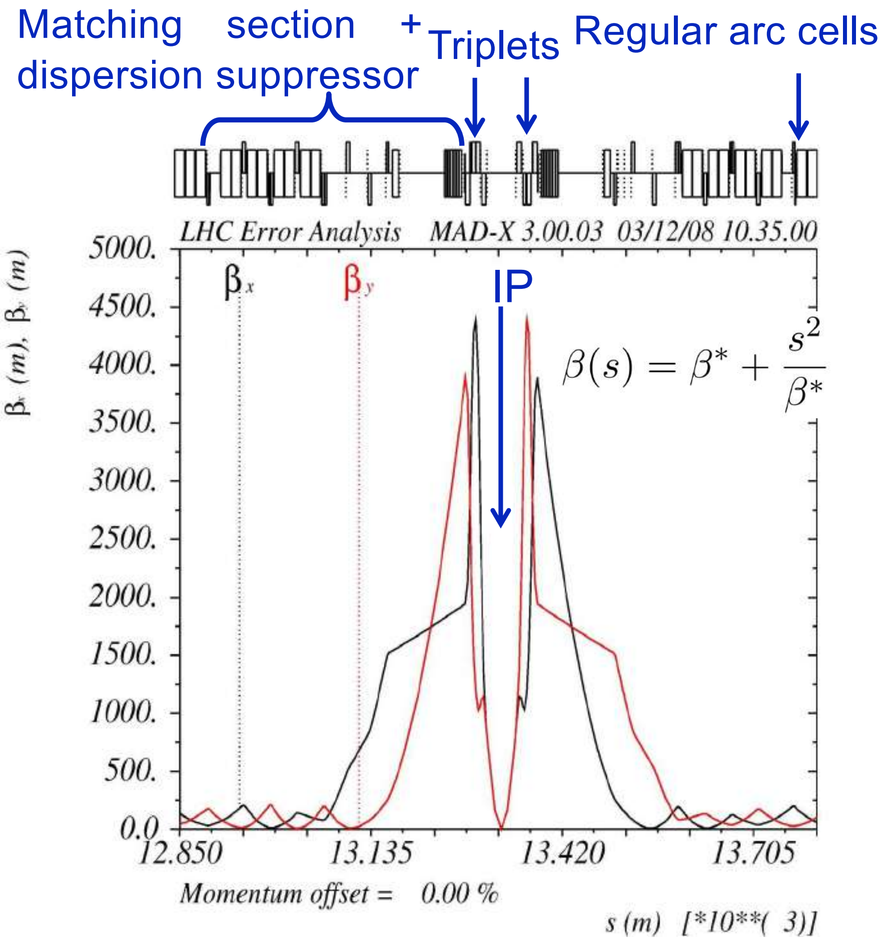
$\beta = 4500$ m

$\sigma = 1.5$ mm $= 1500\ \mu$m

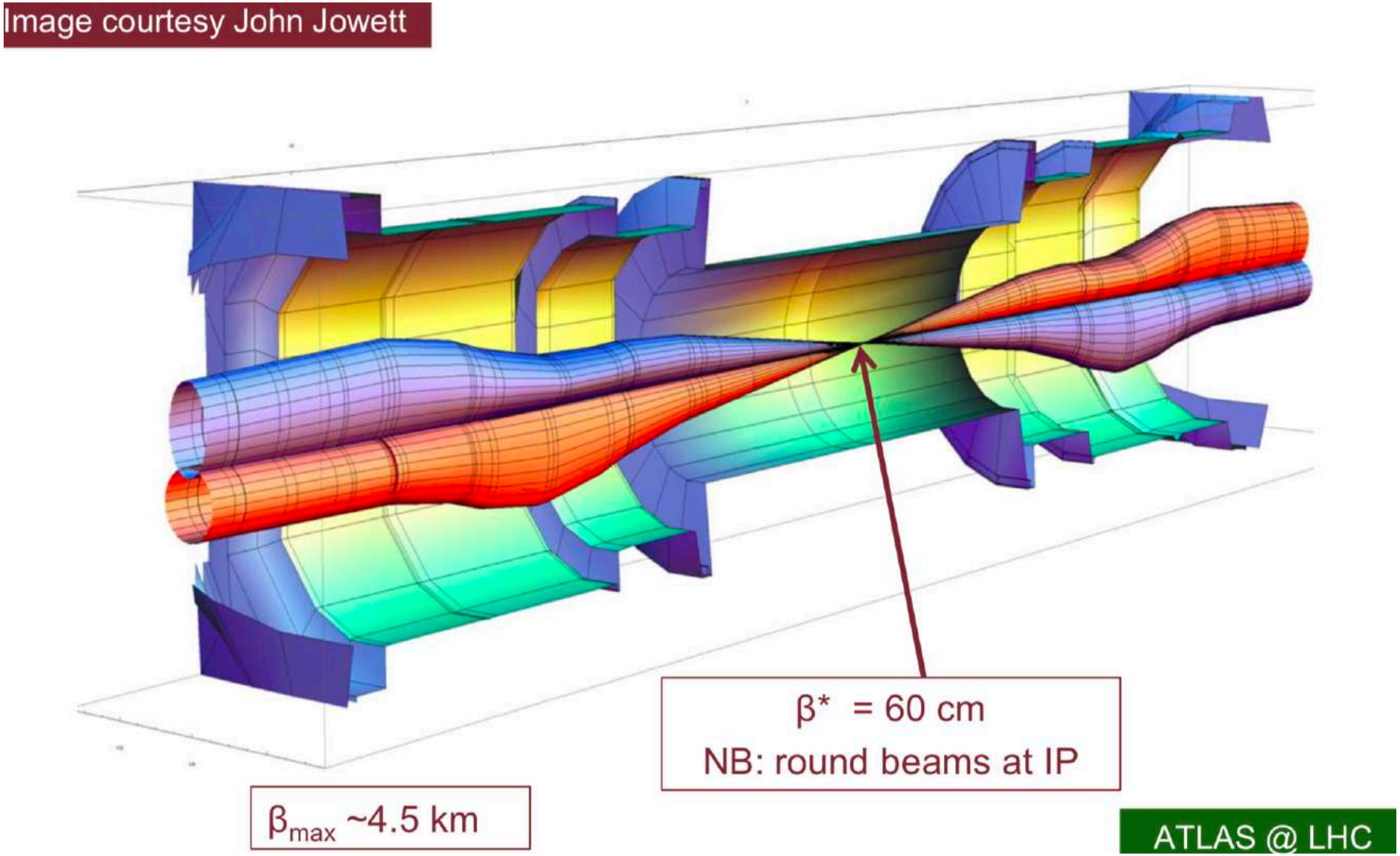
Largest beams size in the lattice!

Limitations:

- Tighter tolerances on field errors
- Triplet aperture limits β^* together with crossing angle.



Luminosity: beta squeeze



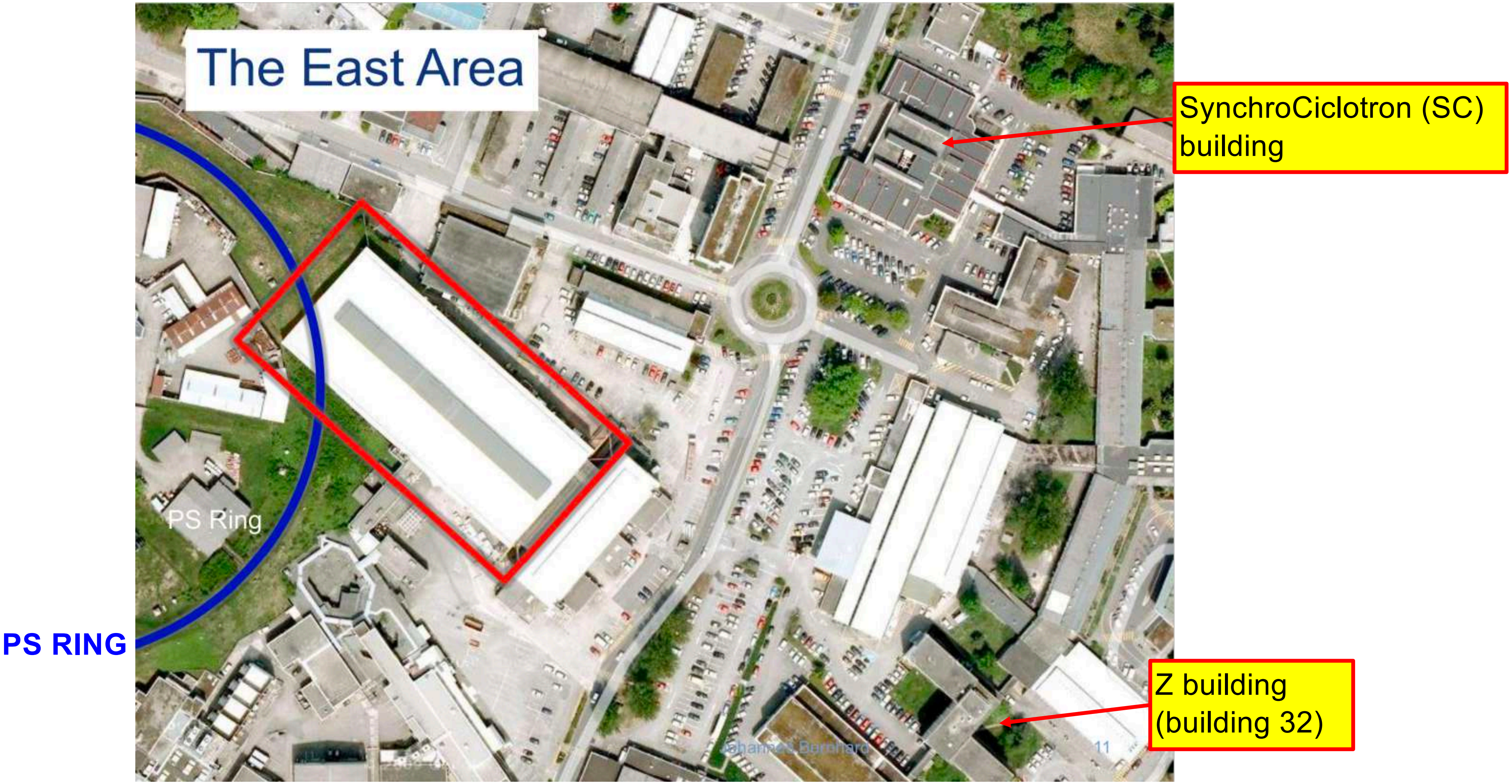
Let's open a parenthesis

(it is not part of the exam program)

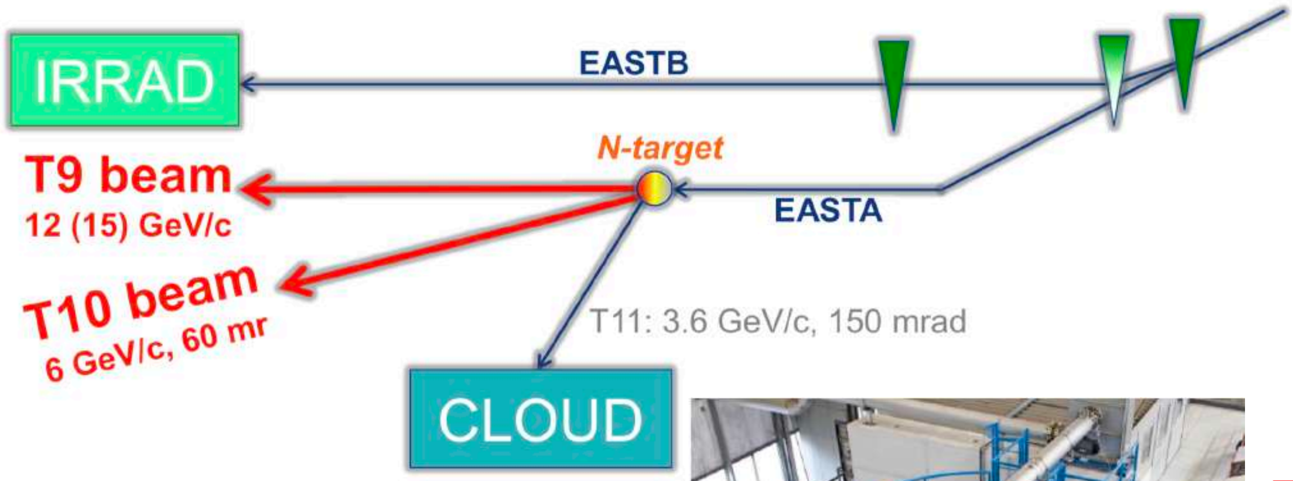
Beam lines

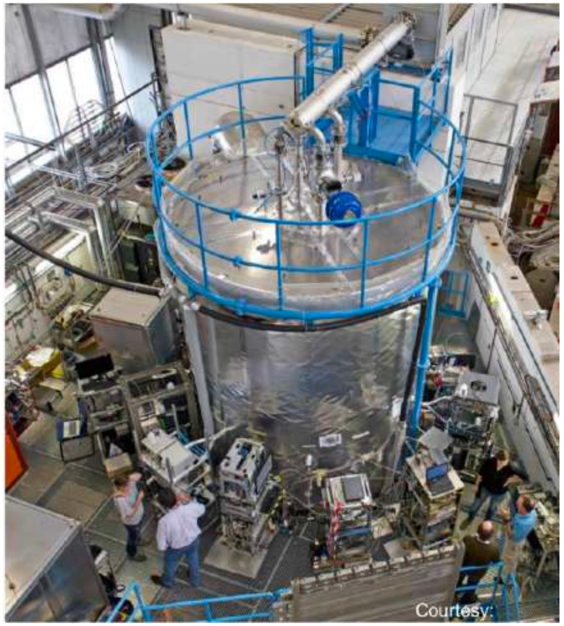
(It is not in the exam program but it will help us to better understand the problem with the antiprotons in the SppS collider)

Beam lines in the PS East area (today)



Beam lines in the PS East area (today)



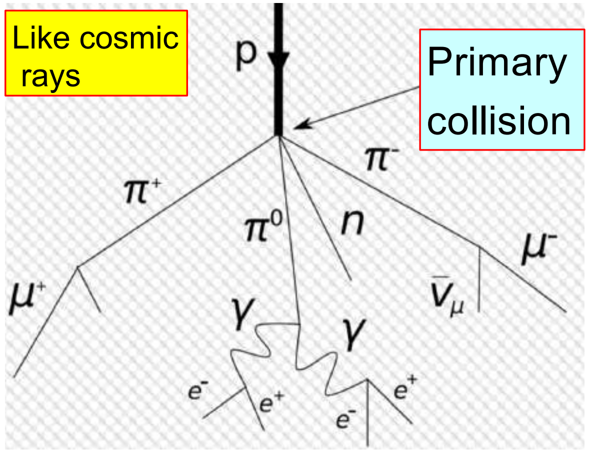


- Studies the influence of cosmic rays to cloud formation
- Cloud expansion chamber set-up with extensive instrumentation (mass spectrometers, particle counters, etc.)
- Uses PS beam as first and only particle beam experiment to study atmospheric and climate science
- Spectacular results achieved (several publications in Nature and Science)

Targets and particle production

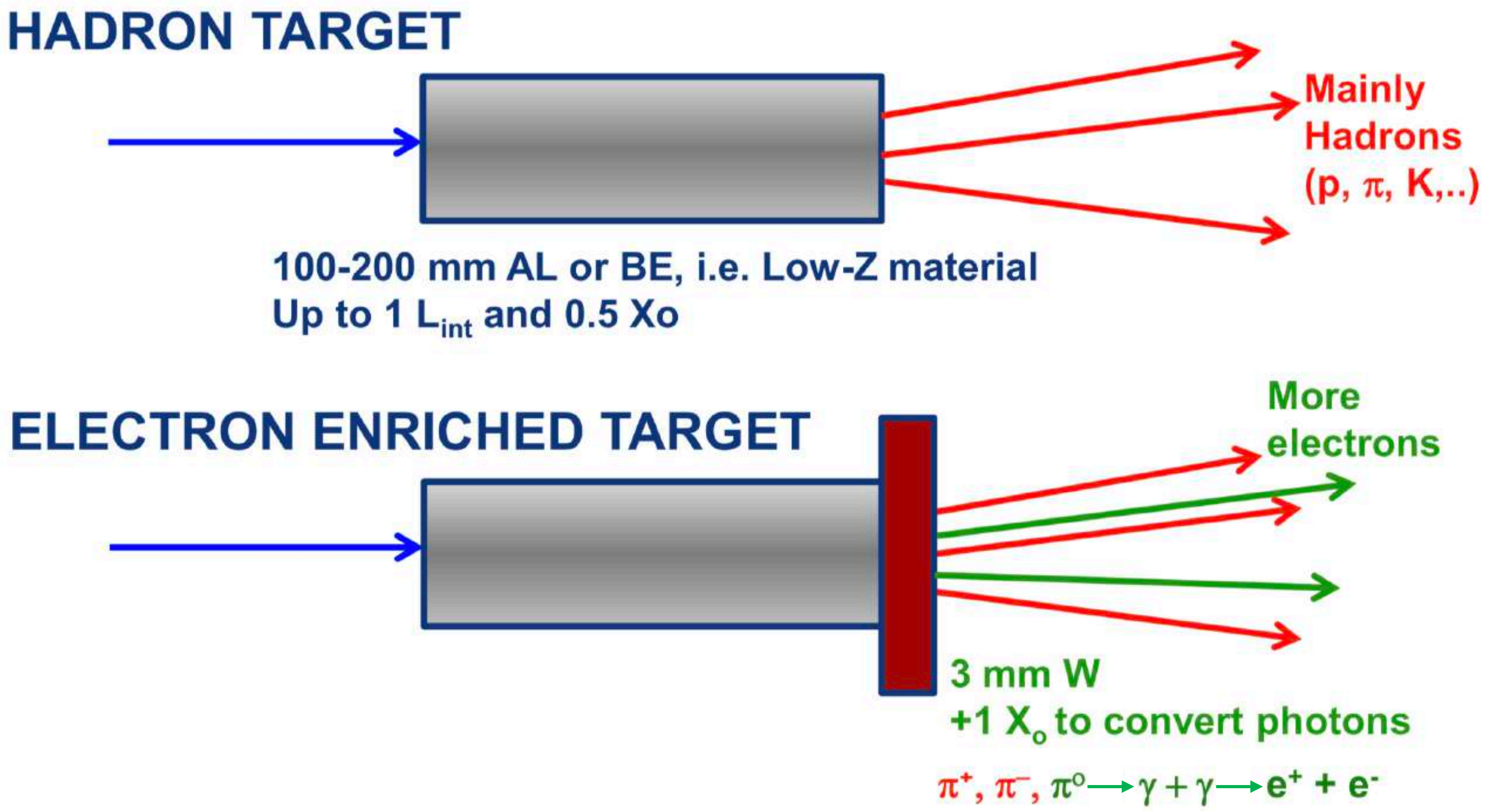
	Name		Q	Mass [MeV/c²]	Mean life (τ) [s]	cτ [m]	Mean decay distance [m/GeV/c]	Decays	
Leptons	Electron	e	±e	0.511	stable				
Leptons	Muon	μ	±e	105.6	2.2×10^{-6}	659.6	6.3×10^{3}	$\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$	(100%)
Hadrons – Mesons	Pion	π	±e	139.6	2.6×10^{-8}	7.8	56.4	$\pi^+ \to \mu^+ \nu_\mu$	(100%)
Hadrons – Mesons	Kaon	K	±e	493.6	1.23×10^{-8}	3.7	8.38	$K^+ \to \mu^+ \nu_\mu$ $\pi^0 e^+ \nu_e$ $\pi^0 \mu^+ \nu_\mu$ $\pi^+ \pi^0$ (...)	(63%) (5%) (3%) (28.9%)
Hadrons – Mesons	Kaon	K^0	0	497.6	K^0_S: 8.9×10^{-11}	0.02	0.060	$K^0_S \to \pi^0 \pi^0$ $\pi^+\pi^-$	(30.7%) (69.2%)
Hadrons – Mesons	Kaon	K^0	0	497.6	K^0_L: 5.12×10^{-8}	15.34	34.4	$K^0_L \to \pi^\pm e^\mp \nu_e$ $\pi^\pm \mu^\mp \nu_\mu$ $3\pi^0$ $\pi^+\pi^-\pi^0$	(40.5%) (27.0%) (19.5%) (12.5%)
Hadrons – Baryons	Proton	p	±e	938	stable				
Hadrons – Baryons	Lambda	Λ	0	1115.6	2.63×10^{-10}	0.079	0.237*	$\Lambda^0 \to p\,\pi^-$	(63.9%)
Hadrons – Baryons	Sigma Hyperons	Σ^+	+e	1189.3	8.02×10^{-11}	0.024	0.068*	$\Sigma^+ \to p\,\pi^0$	(51.57%)
Hadrons – Baryons	Sigma Hyperons	Σ^-	-e	1197.4	1.48×10^{-10}	0.044	0.125*	$\Sigma^- \to n\,\pi^-$	(99.84%)

(*) for 10 GeV/c



$c\tau$ is computed for a 10 GeV/p momentum

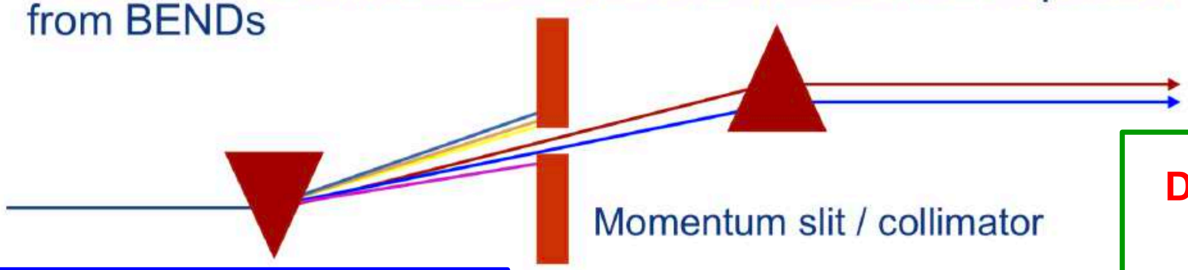
Targets and particle production



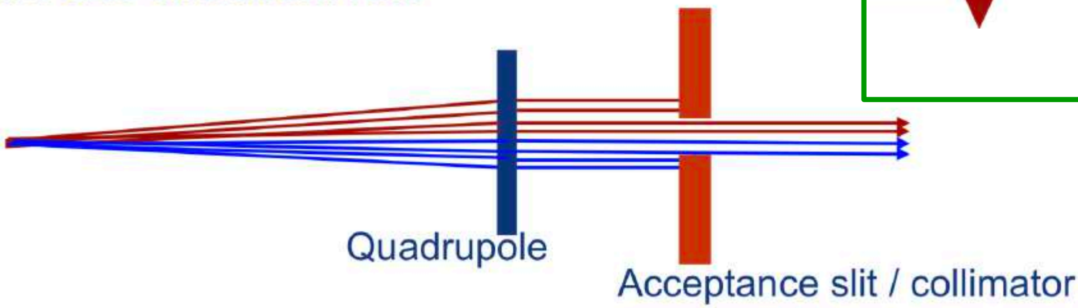
Secondary beam line - layout

Basic beam design

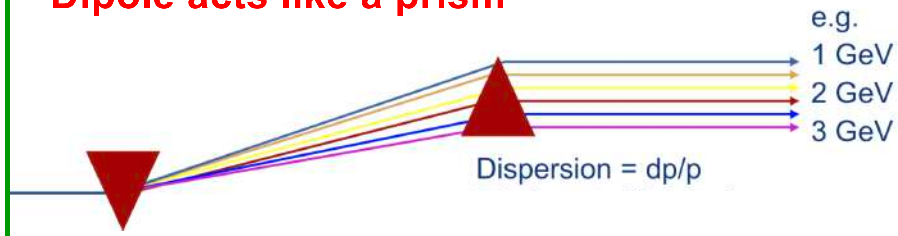
- Momentum selection and acceptance: collimators
  - Select small momentum band in combination with dispersion from BENDs



- Acceptance collimators
  - Select beam size and beam rate

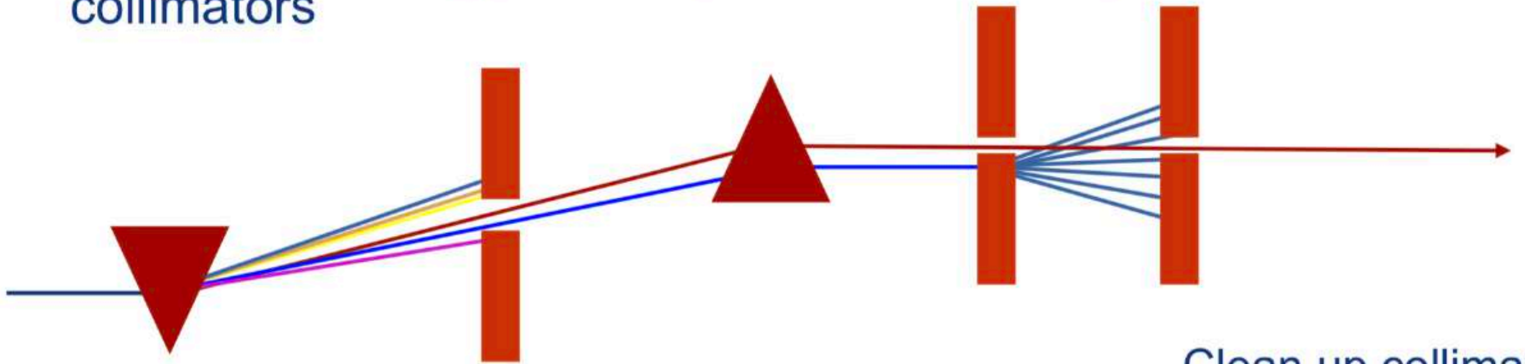


Dipole acts like a prism



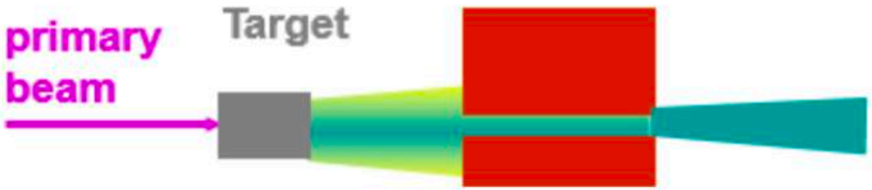
Secondary beam line - layout

- Clean up collimators
  - Absorb secondary particles produced in acceptance collimators



Clean up collimator

- TAX (Target attenuator)
  - Define initial acceptance of the beam line

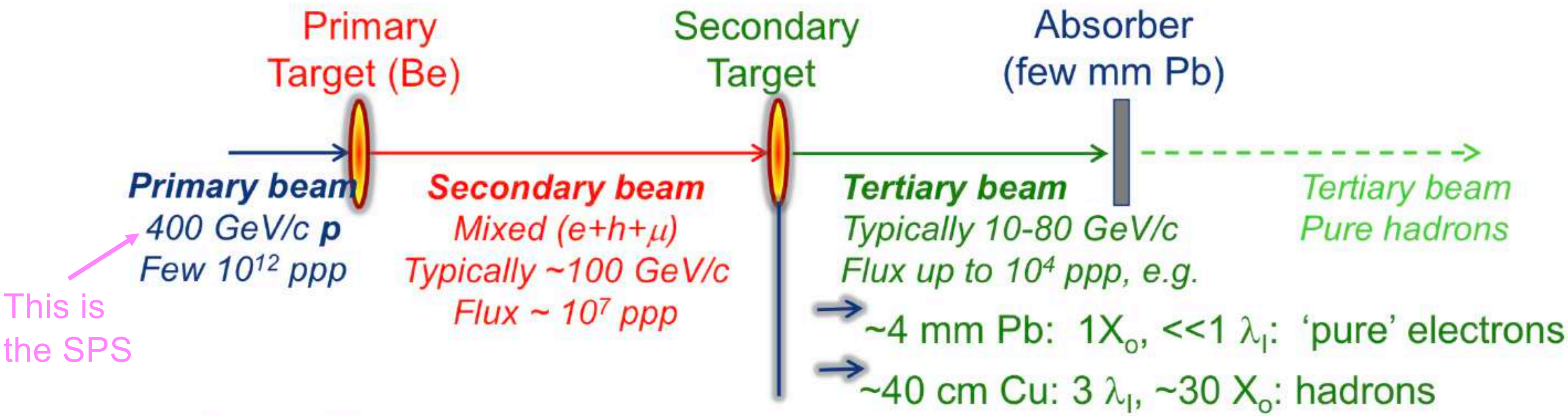


Acceptance defined by TAX

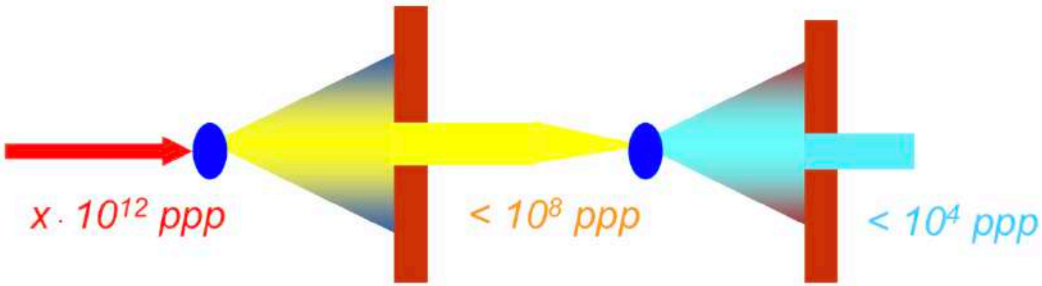
Secondary beam line - layout

Basic beam design

- Selection of particle types

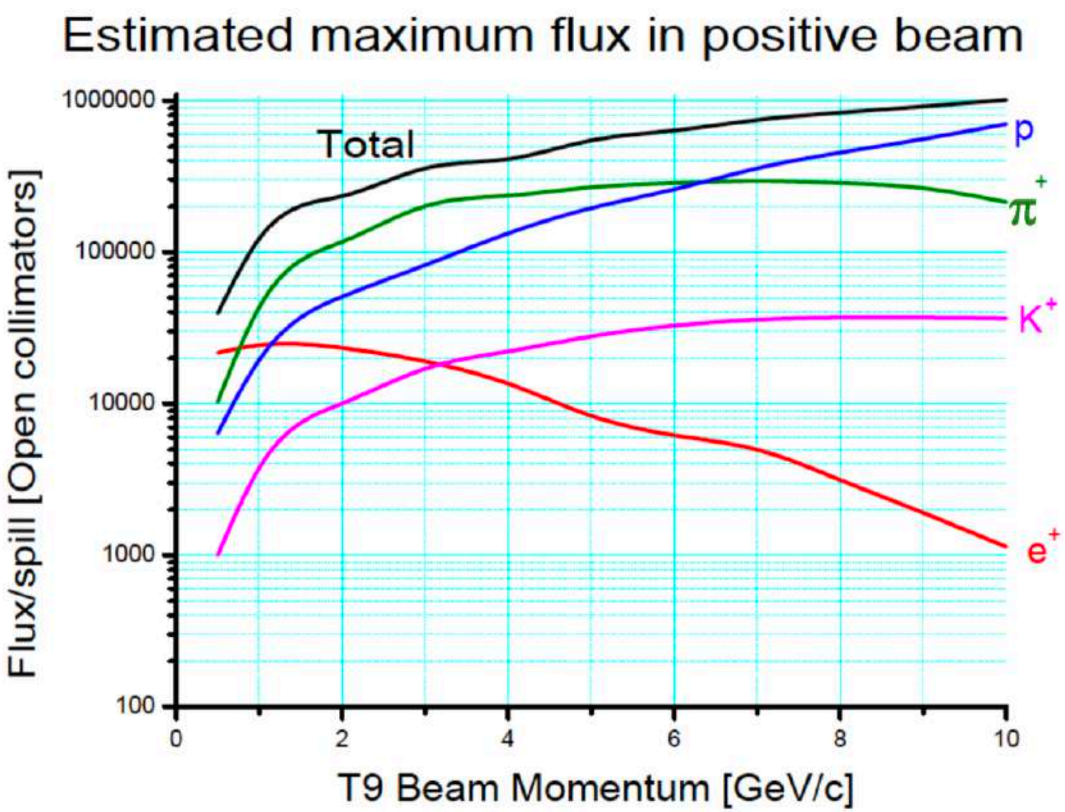


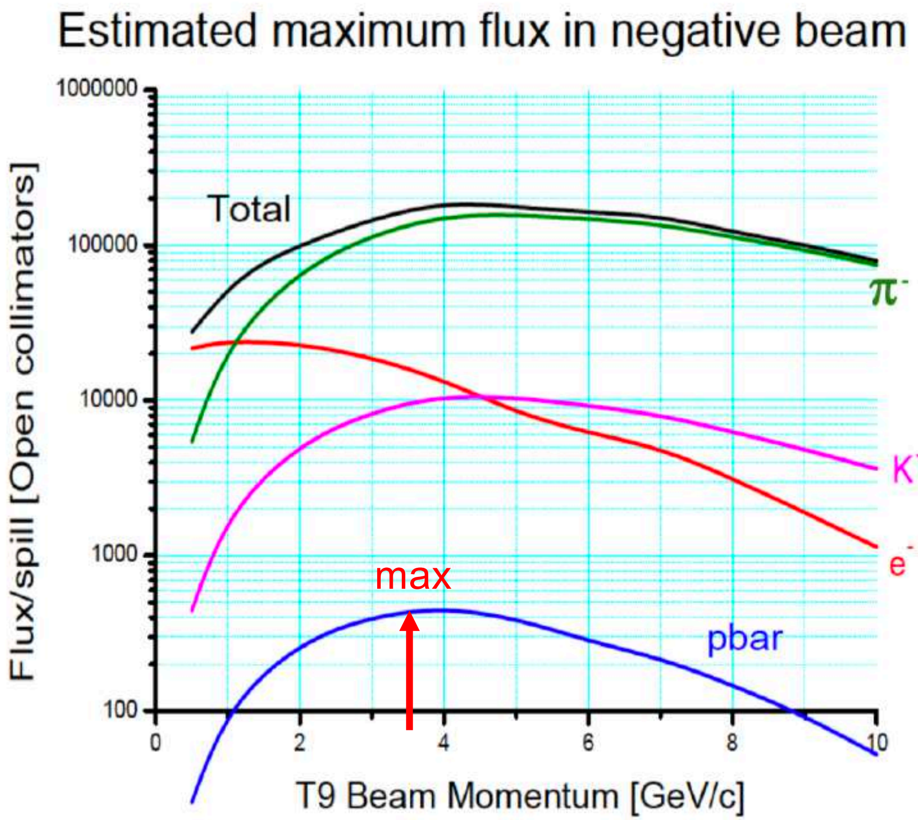
- Intensities



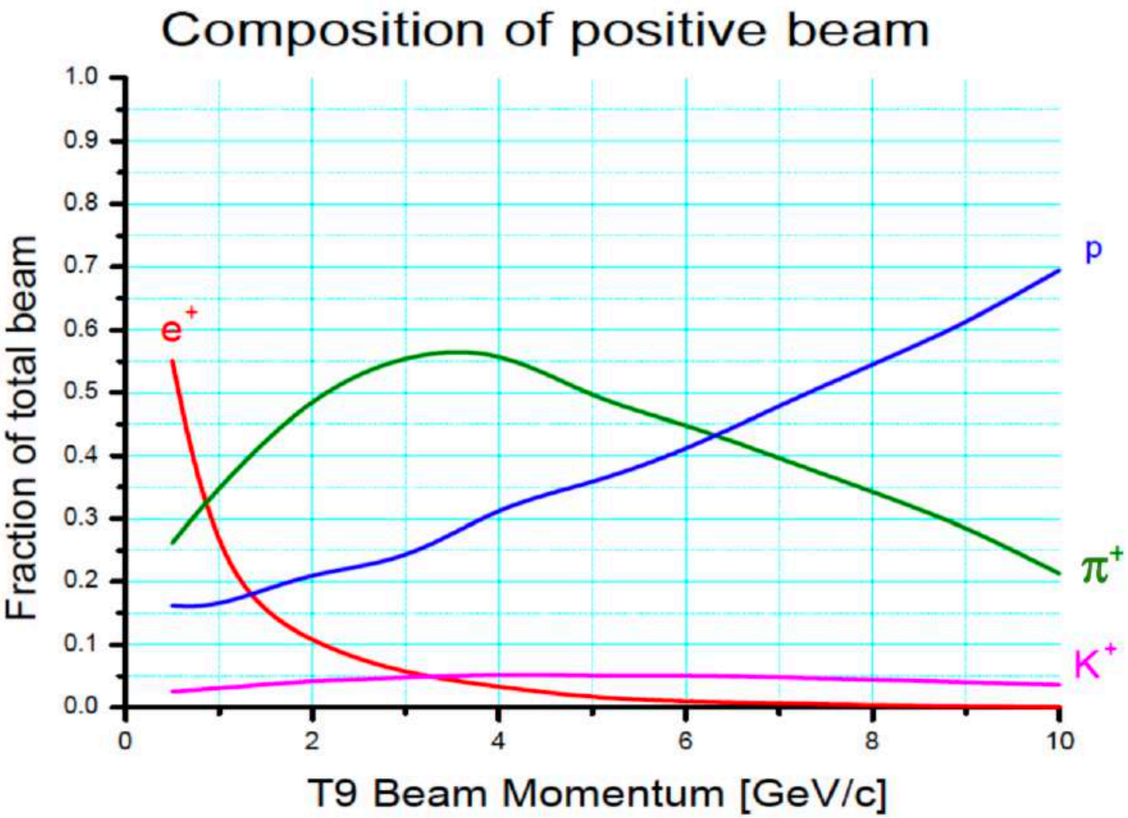
ppp = particles per pulse

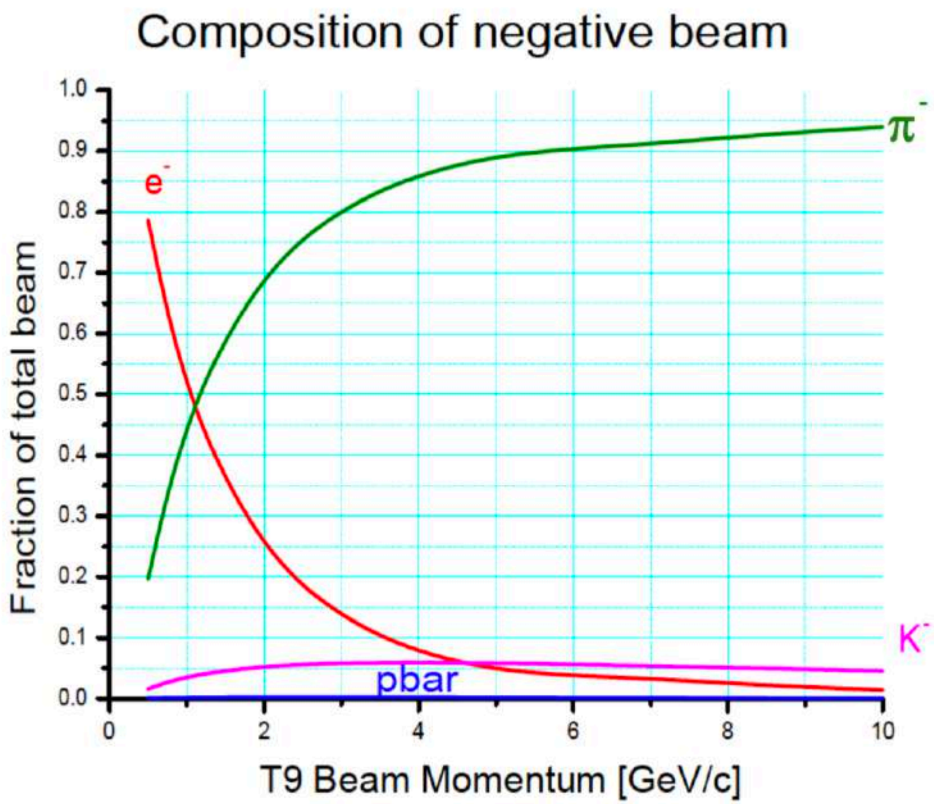
PS east area, T9 line: beam rates





PS east area, T9 line: beam composition





Very very few antiprotons

Let's close the parenthesis

End of chapter 1