

Collider Particle Physics

- Chapter 1 -

Accelerators



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Chapter Summary

- ☐ Electrostatic accelerators
- ☐ LINAC
- ☐ Circular accelerators
- ☐ Bending and focusing in circular accelerators
- ☐ Particle dynamics in the transvers plane
- ☐ Beam injection and extraction
- ☐ Acceleration and phase stability
- ☐ Luminosity in a collider

Accelerators in the world

where accelerators are used

Industry

- Material studies and processing
- Food sterilization
- Ion implantation

Security

- Airports & borders
- Nuclear security
- Imaging

Energy

- Destroying radioactive waste
- Energy production
- Nuclear fusion
- Thorium fuel amplifier



World wide about **>30'000** particle accelerators are in operation with a large variety of applications.

Health

- Diagnostic and imaging
- X-rays
- Cancer therapy
- Radioisotope production

Research (<1%)

- Particle Physics
- Storage rings & Colliders
- Material science
- Light sources
- R&D

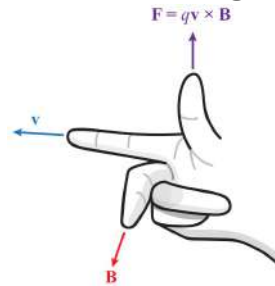
PhD in Physics of the Accelerators at the Sapienza University: more information [here](#)

How can we accelerate particles?

How can we increase the energy of a particle?

A *charged particles* that travels through an electro-magnetic field feels the **Lorentz force**:

$$\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$$



Magnetic field B:

Force acts perpendicular to path.

→ Can change direction of particle

→ cannot accelerate

Electric field E:

Force acts parallel to path.

→ Can accelerate

→ not optimal for deflection

Numeric Example:

$$v = c, B = 1\text{T}$$

$$E = vB = 3 \times 10^8 \text{ m/s} \times 1\text{T}$$

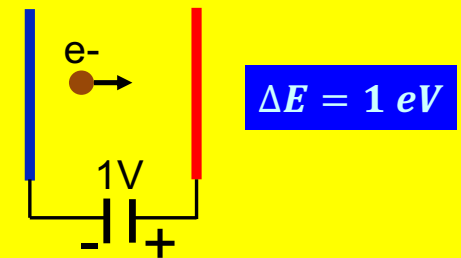
$$E = 300 \text{ MV/m}$$

Technical limit for el. field:

$$E \propto 1\text{MV/m}$$

$$\Delta E = q \int_{r_1}^{r_2} (\vec{v} \times \vec{B} + \vec{E}) d\vec{r}$$
$$= q \int_{r_1}^{r_2} \vec{E} d\vec{r} = qU.$$

$$(\vec{v} \times \vec{B}) d\vec{r} = 0$$

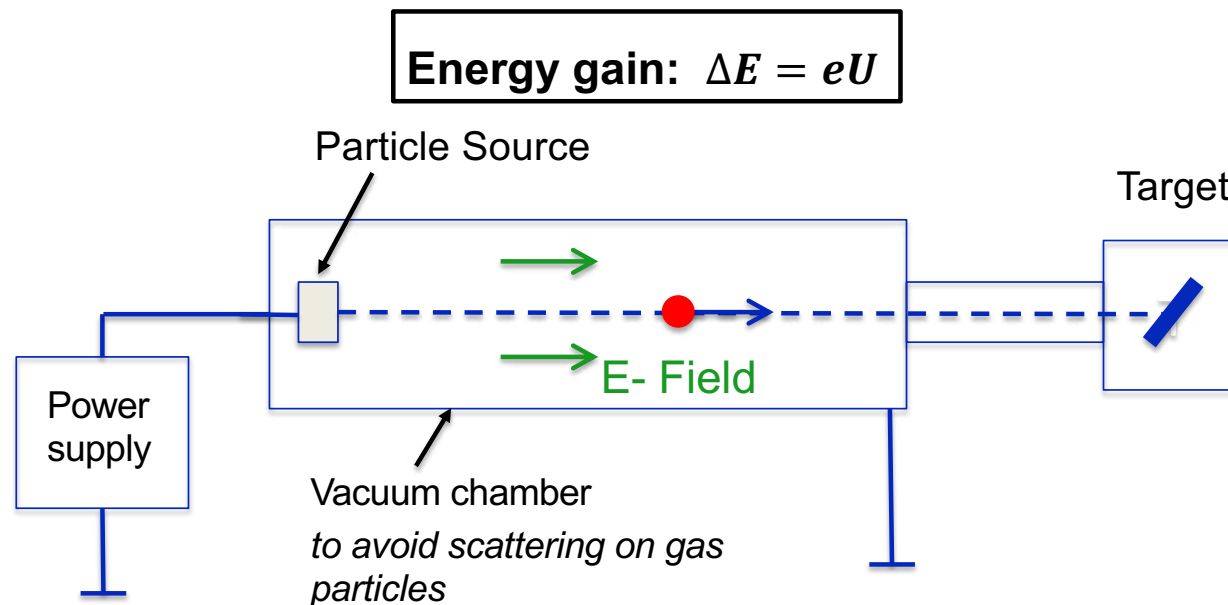


Which types of accelerators exists? And how do they work?

Basic accelerator

Electro-static accelerator (most basic accelerator)

→ Charged particle travels through a fixed high voltage U



Final particle energy is limited by a maximum reachable voltage.

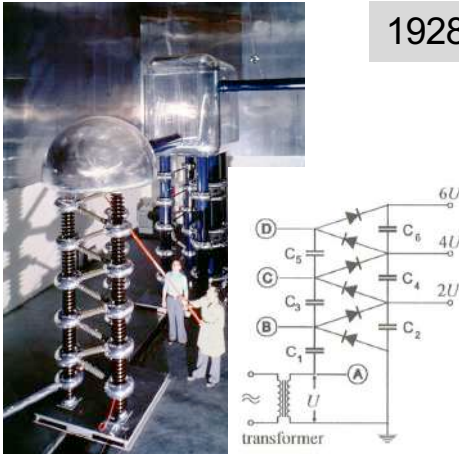
Max. voltage limited by corona formation and discharge to ~10MV.

↑
ionization of the air surrounding a conductor

Electrostatic accelerators: ~ 1930

Cockcroft-Walton cascade generator

1928



Concept:

rectifier circuit, built of capacitors and diodes (Greinacker circuit)

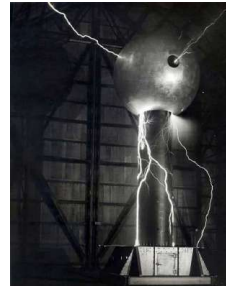
Limitation:

Electrical discharge in air (Paschen Law)

Max. Voltage ~ 1 MV

Van de Graaff accelerator

1930



Concept:

mechanical transport of charges via rotating belt

Electrode in high pressure gas to suppress discharge (SF_6)

Max. Voltage ~ 1- 10 MV

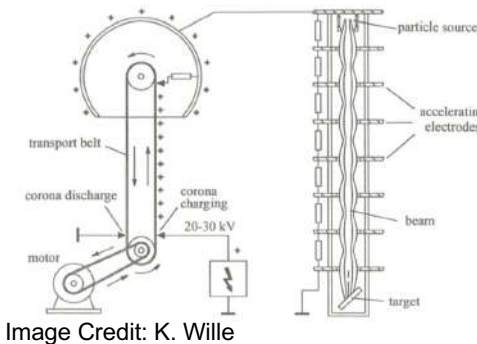


Image Credit: K. Wille

Tandem Van de Graaff accelerator

1936



at MPI Heidelberg

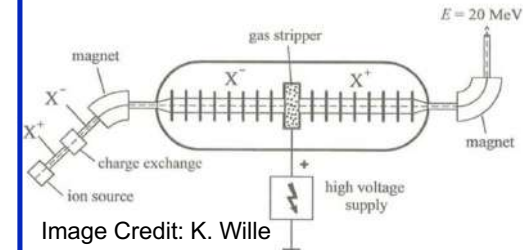


Image Credit: K. Wille

Concept:

Generate negative ions, strip off electrons in the center, use voltage a 2nd time with now positive ions

Max. Voltage ~ 25 MV

Electrostatic Accelerator Limitation



Electrostatic

Limitation:

Generation of max. (direct) voltage before sparking.

Acceleration over one stage or gap.



Radio Frequency

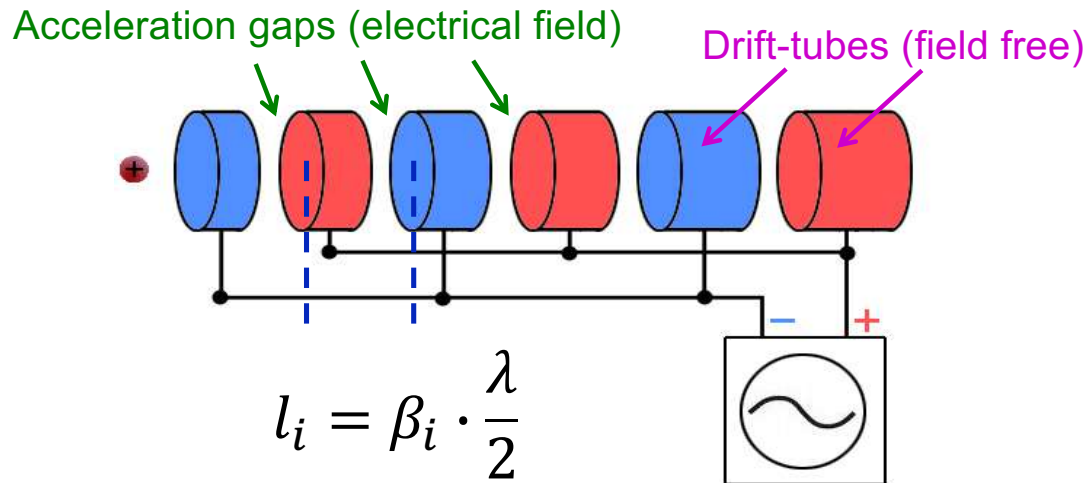
Solution:

Use alternating (RF) voltages and pass the particles through many acceleration gaps of the same voltage.

1925 idea by Ising

1928 first working RF accelerator by Wideroe

LINear ACcelerator (LINAC): functionalities



Energy gain after n gaps:

$$E = n q V_{RF} \sin \phi_s$$

n No. of acceleration gaps

q Charge of the particle

V_{RF} Peak voltage of RF System

ϕ_s synchronous phase w.r.t. RF field

Question

Once build, can we use the LINAC to accelerate any particle we like?

- High-frequency RF field (turn-over frequency MHz): $\lambda = c/f_{RF}$
- Particle should only feel the field when the field direction is synchronized.
- Drift-tubes screen the field as long as the field has the reversed polarity.
 - The more energy the particle gains, the faster it becomes (non-relativistic regime)

→ Drifts have to increase in length.

→ Particles have to be clustered into packages (bunches). →

Question: why are in bunches?



Exercise: LINAC

Question

Once build, can we use the LINAC to accelerate any particle we like?

Drift tubes provide shielding of the particles during the negative half wave of the RF.

Time span of the negative half wave: $\tau_{RF}/2$

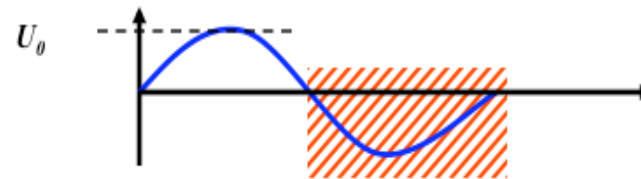
Length of the Drift Tube: $l_i = v_i * \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles

$$E_i = \frac{1}{2} m v^2$$

This question could be rephrased to:

How does the drift tube length l_i depend on the particle type?



$$v_i = \sqrt{2E_i/m} \quad E_i = i q V_{RF} \sin \phi_s$$

$$l_i = \frac{1}{f_{RF}} \sqrt{\frac{i q V_{RF} \sin \phi_s}{2m}}$$

valid for *non-relativistic* particles ...

So the answer is **no**. The drift tube length depends on the charge-to-mass-ratio (q/m) of the particle and the RF system. For a given RF system bandwidth only a certain range of q/m leads to a synchronized acceleration. One knob to play could be the charge state for ions, which may allow to get closer to the design q/m .

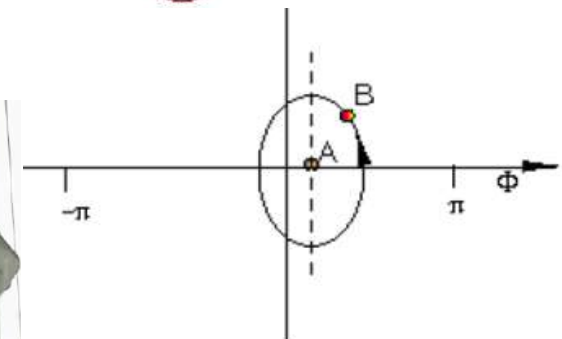
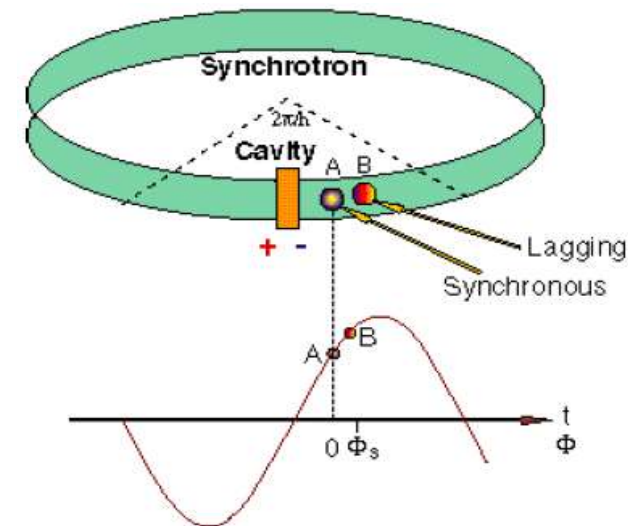
Exercise: phase focusing

- ❑ Developed independently by McMillan and Veksler (1945)
- ❑ The RF cavity is set such as the particle at the centre of the bunch (synchronous particle) receives the needed energy
- ❑ Voltage in the cavity: $V = V_0 \sin(2\pi\omega_{RF}t) = V_0 \sin(\varphi(t))$
- ❑ For no acceleration, synchronous particle phase: $\varphi_s = 0$
- ❑ For acceleration, synchronous particle phase: $0 < \varphi_s < \pi$ in order to achieve: $\Delta E = V_0 \sin(\varphi(t))$
- ❑ Particles arriving late: $\varphi > \varphi_s \rightarrow$ Energy increase *larger* than the synchronous particle
- ❑ Particles arriving early: $\varphi < \varphi_s \rightarrow$ Energy increase *smaller* than the synchronous particle

→ Particles are grouped – *bunches*!

Pay attention: this region of phase stability is valid only for not relativistic particles. We will see later what happens for relativistic particles.

Example is with the synchrotron, but for the LINAC is the same.



LINAC limitation



LINAC

Linear

Consists of a chain of **many accelerating gaps** placed on a straight line.

Particles pass the accelerator only **ONCE**.

The final energy is limited by length.

Circular



Synchrotron



Cyclotron

Use **magnets** that bend particles on a **circular orbit**.

Particles circulate over **MANY turns** and can gain more energy at each passage through the acceleration gap.

PSI cyclotron, currently doing first class physics

Cyclotron – “spiral version of a LINAC”

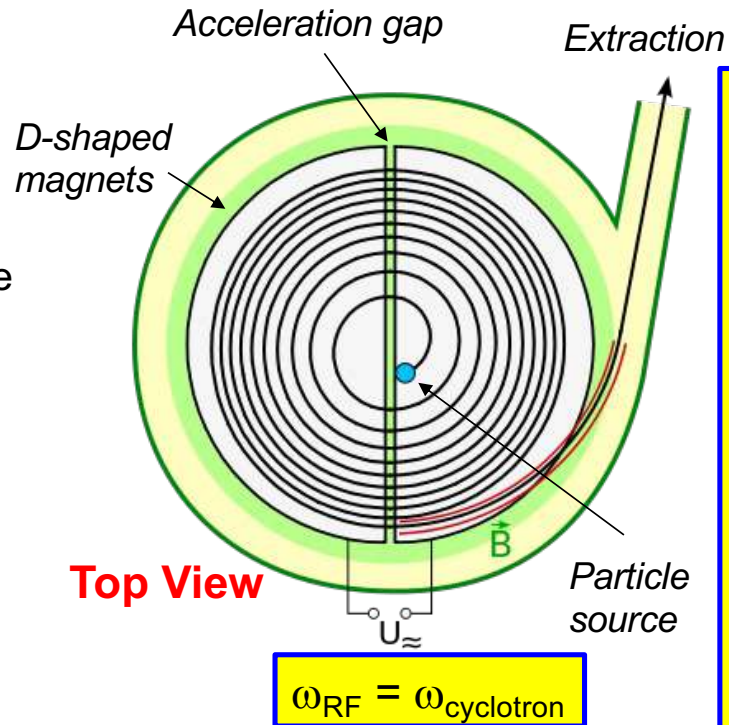
1929 proposed E.O. Lawrence
1931 built by Livingston

- **Particle Source in the middle**
- Acceleration gap connected to RF source between the two D-shaped magnets.
- **Constant vertical magnetic field** to guide the particles in the horizontal plane. **The radius of particle trajectory becomes larger and larger with larger energy.**
- Particles extracted with a deflector magnet or an electrode.

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \longrightarrow F_L = q v B \longrightarrow \begin{matrix} \text{Vertical B} \\ \text{No E} \end{matrix}$$

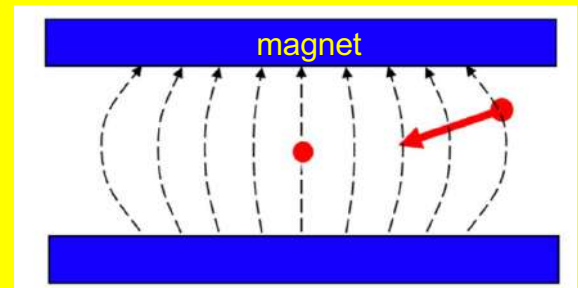
$$F_c = m \frac{v^2}{r} \longrightarrow \text{centrifugal force}$$

$$F_L = F_c \longrightarrow \omega = \frac{v}{r} = \frac{qB}{m} \longrightarrow \text{revolution period}$$



Weak focusing

Side View



B field is decreasing moving outward from the center.

A component of the Lorentz force prevents the particles to hit the magnet walls

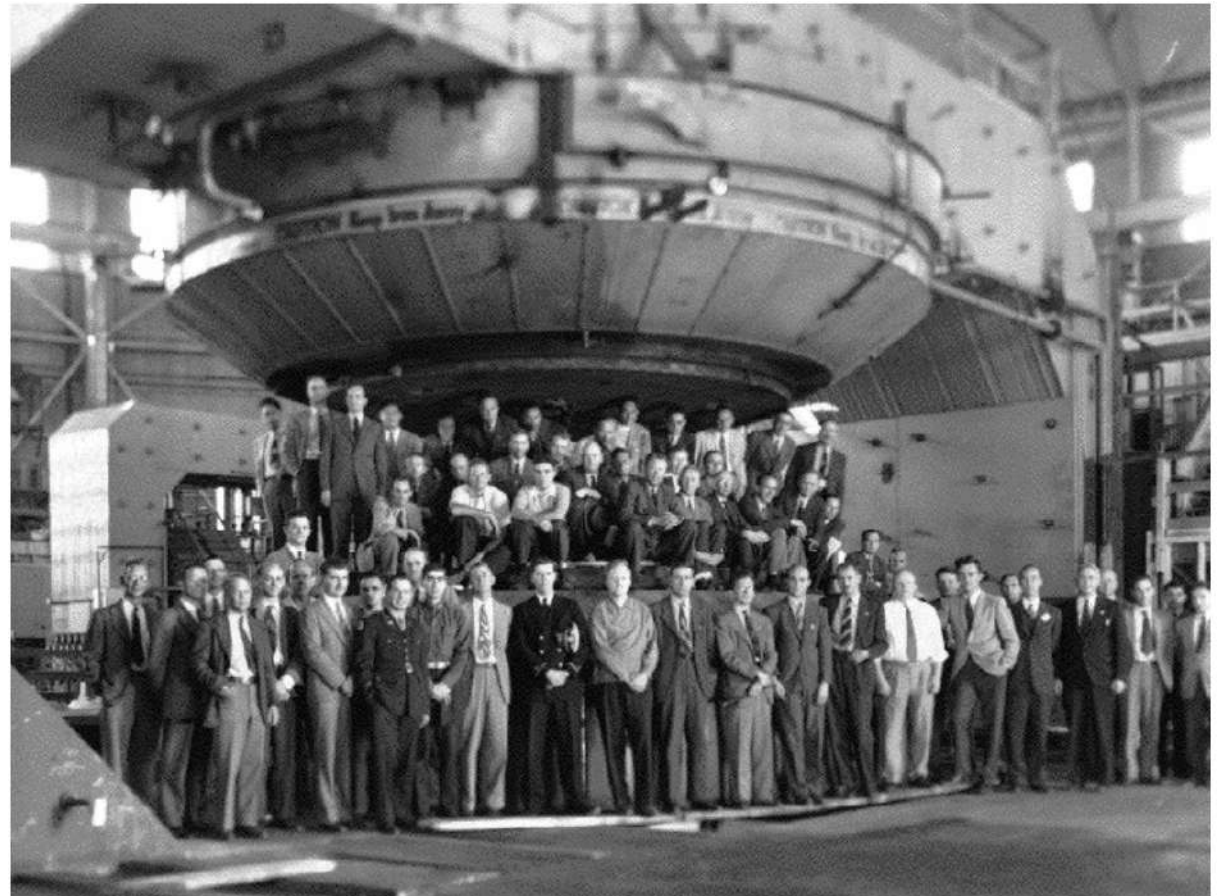
Same principle of weak focusing is working in the dipole magnets

Cyclotron: an example

First cyclotron built at Berkeley
by E.O. Lawrence



- α particles in radioactive decays: 1÷5 MeV
- 1939: cyclotron 1.5 m diameter: 19 MeV.
- Maximum cyclotron energy: 25 MeV
- next steps: synchrocyclotron
and then synchrotron



Berkeley campus, 1942

184-inch cyclotron, 4000 tons: 1 single dipole with 467 cm diameter
Designed to reach 100 MeV, it was converted in 1946 in a synchrocyclotron

Cyclotron limitation

Constant revolution frequency
for constant mass:

$$\omega = \frac{v}{r} = \frac{Bq}{m} = \frac{Bq}{m(E)}$$

$$\begin{aligned} f_{RF} &= \text{const.} \\ B &= \text{const.} \end{aligned}$$

But, for relativistic particles the mass is not constant!

[Well ... it is the relation between p and v ,
or p and E that is different]

The classical cyclotron only valid for particles up to few % of speed of light.

→ **Not useful for electrons** ... already relativistic at ~500 keV.

Modifications:

Synchro-cyclotron

$$\begin{aligned} f_{RF} &(E) \\ B &(E) \text{ or } B = \text{const.} \end{aligned}$$

Isochronous cyclotron

$$\begin{aligned} f_{RF} &= \text{const.} \\ B &(r) \end{aligned}$$

Common accelerator for *medium energy protons and ions* up to ~60 MeV/n,
used for nuclear physics, radio isotope production, hadron therapy.

Modern "cyclotrons" can reach > 500 MeV (PSI, TRIUMF, RIKEN)

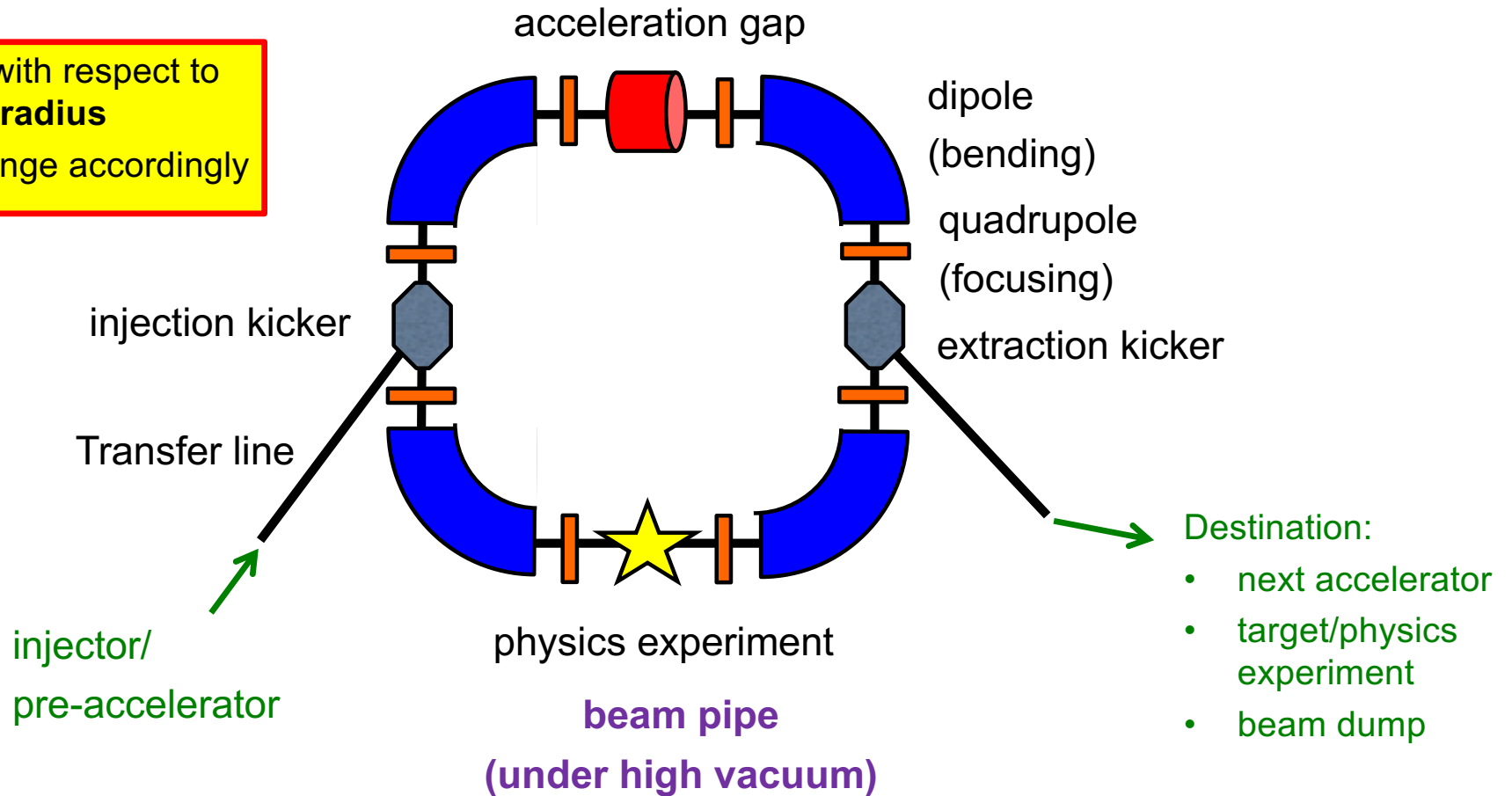
The synchrotron

Basic Synchrotron

Synchrotrons are THE accelerators to reach highest particle energies
and are able to store the beam over many hours.

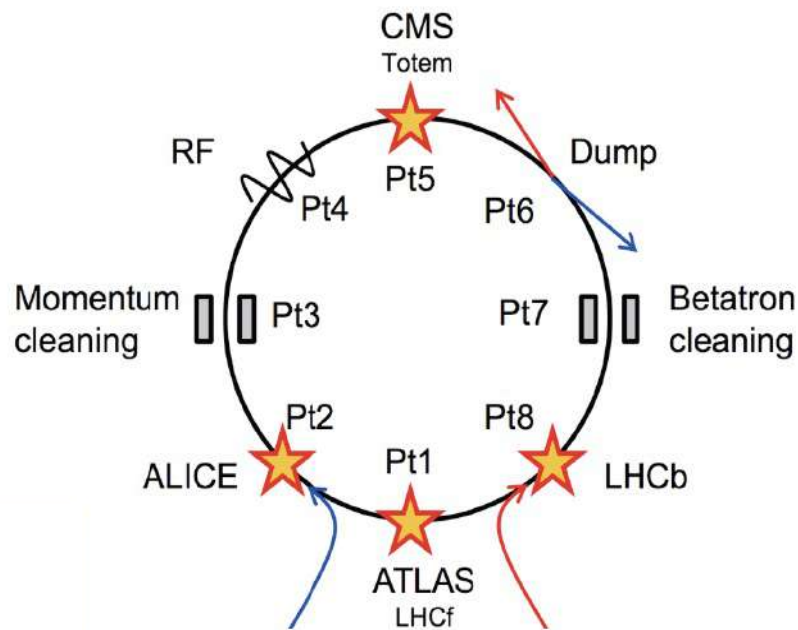
Main difference with respect to
Cyclotron: **fixed radius**
→ B and RF change accordingly

$$R = \frac{p}{qB}$$



Most famous example

The largest machine in the world The Large Hadron Collider (LHC)



27 km circumference

100m underground

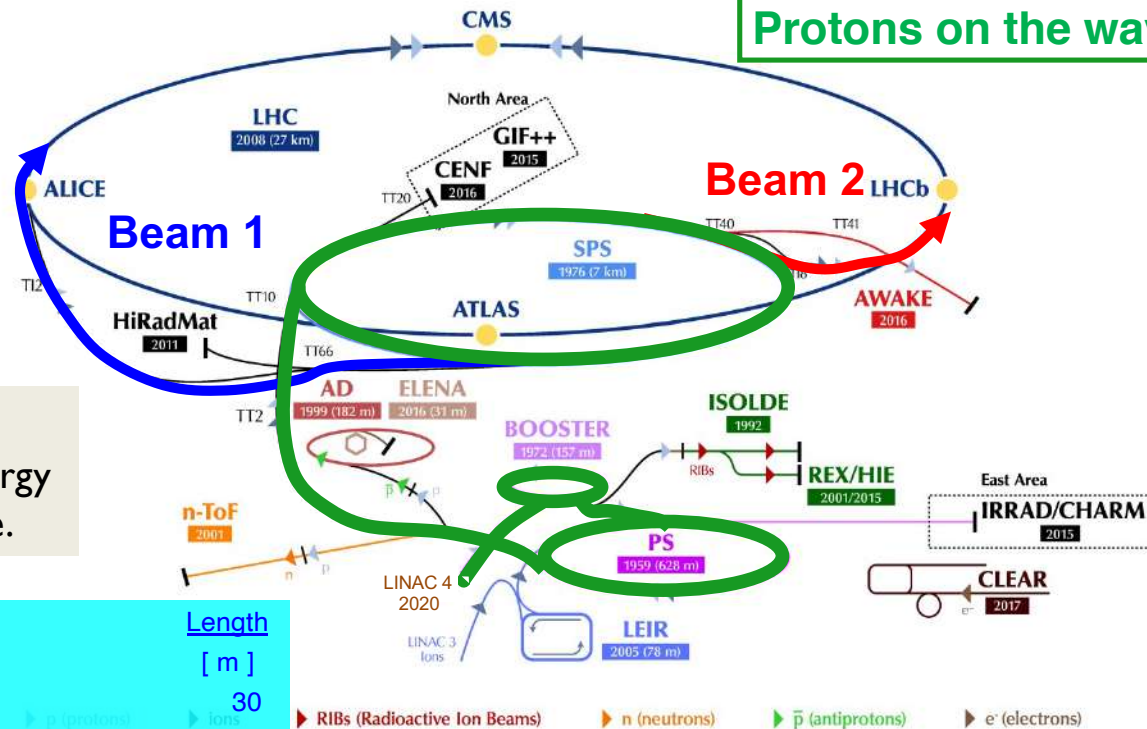
Accelerates protons and heavy-ions
to $E = 6.8 \text{ TeV}$ (2022).

Collides 2 counter-rotating beams
in 4 physics experiments.

Getting particles into the LHC

The CERN accelerator complex
Complexe des accélérateurs du CERN

Protons on the way to LHC



- 1) Linear → Circular
- 2) The higher the particle's energy the larger the circumference.

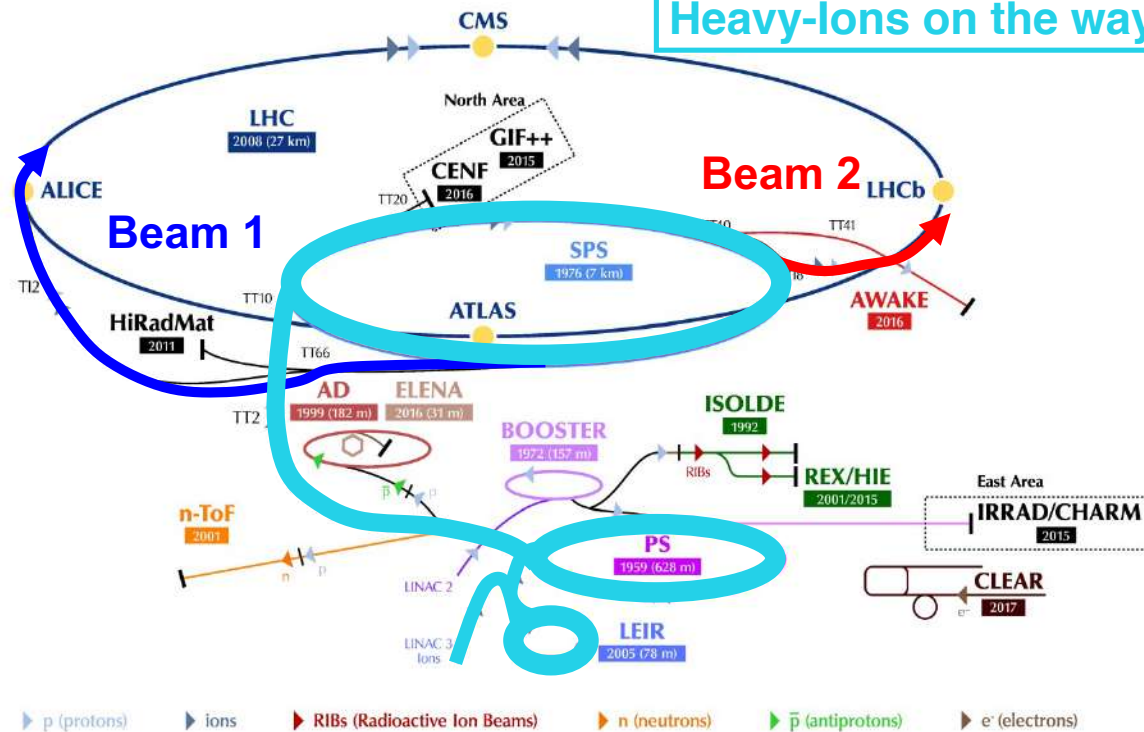
	Year	Top energy [GeV]	Length [m]
Linac 4	2020	0.05	30
PSB	1972	1.4	157
PS	1959	26.0	628
SPS	1976	450.0	6911
LHC	2008	7000.0	26657

LHC - Large Hadron Collider // SPS - Super Proton Synchrotron // PS - Proton Synchrotron // AD - Antiproton Decelerator // CLEAR - CERN Linear Accelerator for Heavy Ions // AWAKE - Advanced WAKEfield Experiment // ISOLDE - Isotope Separator OnLine // REX/HIE - Radioactive Experiment/High Intensity Heavy Ion Experiment // ISOLDE // LEIR - Low Energy Ion Ring // LINAC - LINear ACcelerator // n-ToF - Neutrons Time Of Flight

Getting particles into the LHC

The CERN accelerator complex
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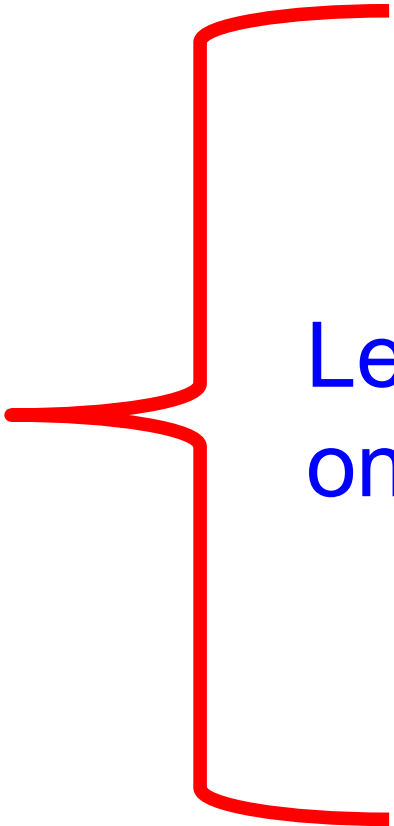
Heavy-Ions on the way to LHC



LHC - Large Hadron Collider // SPS - Super Proton Synchrotron // PS - Proton Synchrotron // AD - Antiproton Decelerator // CLEAR - CERN Linear Electron Accelerator for Research // AWAKE - Advanced WAKEfield Experiment // ISOLDE - Isotope Separator OnLine // REX/HIE - Radioactive EXperiment/High Intensity and Energy ISOLDE // LEIR - Low Energy Ion Ring // LINAC - LINear ACcelerator // n-ToF - Neutrons Time Of Flight //

Question

Why so many accelerators in cascade and not using only a single synchrotron?



Let's open a parenthesis on synchrotron history

(it is not part of the exam program)

*Fatti non foste a viver come bruti
ma per seguir virtute e canoscenza*

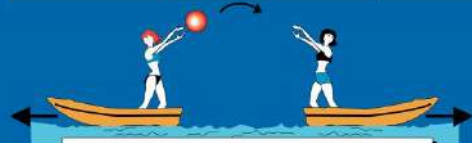
What we knew in ... 1935

- ❑ Atoms are composed by three particles: electron, proton and neutron
- ❑ neutrino hypothesis (1930 by Pauli, discovered later in 1956)
- ❑ there are four fundamental forces:

- Strong force: it acts only on nucleons (hadrons), range $\sim 10^{-15}$ m
- e.m. force: it acts on all charged particles, infinit range
- Weak force: it acts on all particles, range $\sim 10^{-15}$ m
- Gravitational force: it acts on all particles, infinit range

The forces in Nature

TYPE	INTENSITY OF FORCES (DECREASING ORDER)	BINDING PARTICLE (FIELD QUANTUM)	OCCURS IN:
STRONG NUCLEAR FORCE	~ 1	GLUONS (NO MASS)	ATOMIC NUCLEUS
ELECTRO-MAGNETIC FORCE	$\sim 10^{-3}$	PHOTONS (NO MASS)	ATOMIC SHELL, ELECTROTECHNIQUE
WEAK NUCLEAR FORCE	$\sim 10^{-5}$	BOSONS Z^0, W^+, W^- (HEAVY)	RADIOACTIVE BETA DISINTEGRATION
GRAVITATION	$\sim 10^{-38}$	GRAVITONS (?)	HEAVENLY BODIES



THE EXCHANGE OF PARTICLES IS RESPONSIBLE FOR THE FORCE

CERN AC 204 - V22/8/1992

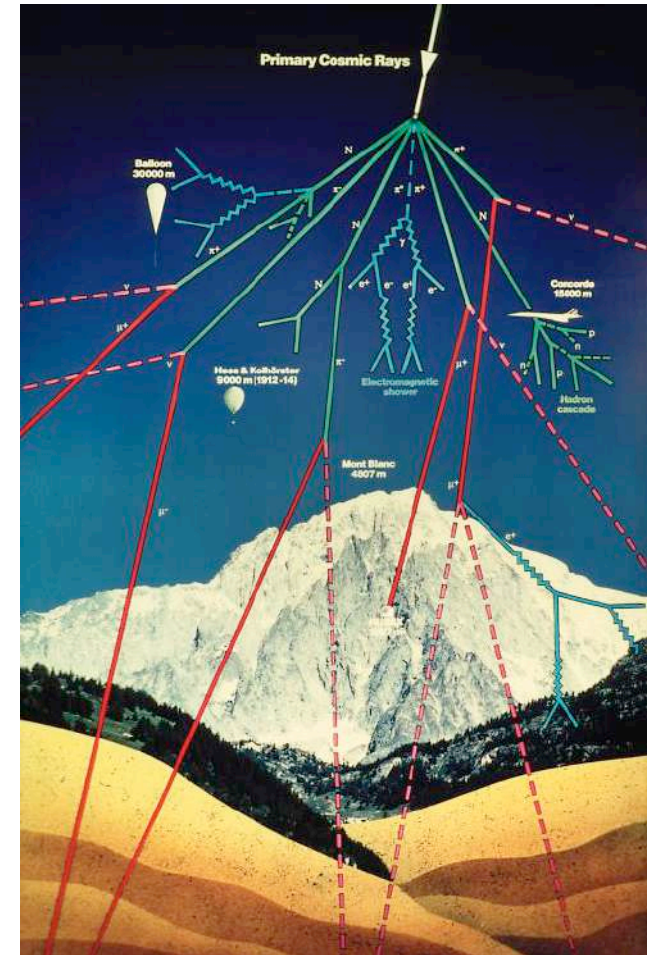
- ❑ Positron discovery (anti-electron) in 1932, predicted by Dirac in 1928
- ❑ Mesotron discovery, predicted by Yukawa as mediator of the strong force

We understood everything (almost!)



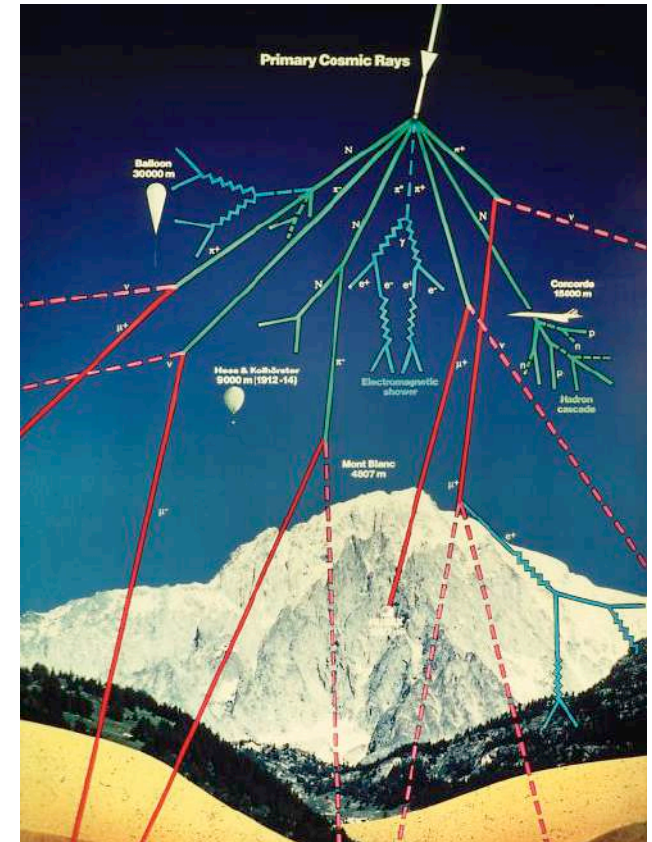
Cosmic rays

- ❑ Discovered by V.Hess in 1912. They are composed by proton (86%), alpha (12%) and other nuclei.
- ❑ Positron discovery (1932)
- ❑ Mesotron discovery (1937), later identified (1947) by Conversi, Pancini and Piccioni as the “muon”, an heavier replica of the electron, unstable.



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- ❑ Pion discovered in 1947 followed by the discovery of “strange” particles, someone heavier than proton



Cosmic rays studies were difficult: experiments up in the mountains or in balloons; flux and energy not under control. People wanted to reproduce the primary interaction in the Lab by accelerating protons (or electrons) and make them colliding with fixed targets.

And man created cosmic rays

❑ 1952: BNL (Brookhaven National Laboratory, Long Island), COSMOTRON

➤ 3 GeV protons; 2000 Tons of iron. 20 m diameter.

➤ it confirms associated production of strange particles: $\pi + p \rightarrow \Lambda + K$



And man created cosmic rays

❑ 1952: BNL (Brookhaven National Laboratory, Long Island), COSMOTRON

➤ 3 GeV protons; 2000 Tons of iron. 20 m diameter.

➤ it confirms associated production of strange particles: $\pi + p \rightarrow \Lambda + K$

❑ 1954: LBL (Lawrence-Berkeley Laboratory, California), BEVATRON

➤ 6 GeV protons; 10000 Tons of iron.

➤ E. Segré discovers the antiproton (Nobel prize in 1959). $p + p \rightarrow p + p + p + \bar{p}$

❑ 1957: Dubna; SYNCHROPHASATRON. 10 GeV protons , 36000 Tons of iron!

In order to go at higher energy it was needed a smart idea to reduce the amount of iron needed to build the magnets.

Christofilos and **Courant**, **Livingston** and **Snyder** discovered/invented/found/designed the **strong focusing** in 1952.

[In the same year (1952) D.Glaser invents the bubble chamber]

The birth of CERN

- ❑ After the war, Europe was in ruins. The physicists had been missing. Scientific knowledge and technical skills had moved to the USA.
- ❑ In December 1949, Louis de Broglie recommended an international research laboratory at a UN cultural conference.
- ❑ In 1950 UNESCO approved a resolution by I. Rabi and in 1952, 11 European countries participated in CERN (European Council for Nuclear Research).
P. Auger and E. Amaldi were the spiritual fathers of CERN.
- ❑ It was chosen as the site of the laboratory Meyrin, a town near Geneva, close to the French border.
- ❑ On 29 September 1954 the European Organization for Nuclear Research (CERN) was born
- ❑ **August 8, 1951: the National Institute of Nuclear Physics (INFN) was established with decree number 599 of the CNR president Gustavo Colonnetti.**

Sur le terrain du futur institut nucléaire



Sous la conduite de M. A. Picot, les membres du Conseil européen pour la recherche nucléaire se sont rendus hier à Meyrin pour reconnaître le terrain où s'élèvera le Centre nucléaire (voir en Dernière heure)
(Photo Freddy Bertrand, Genève)

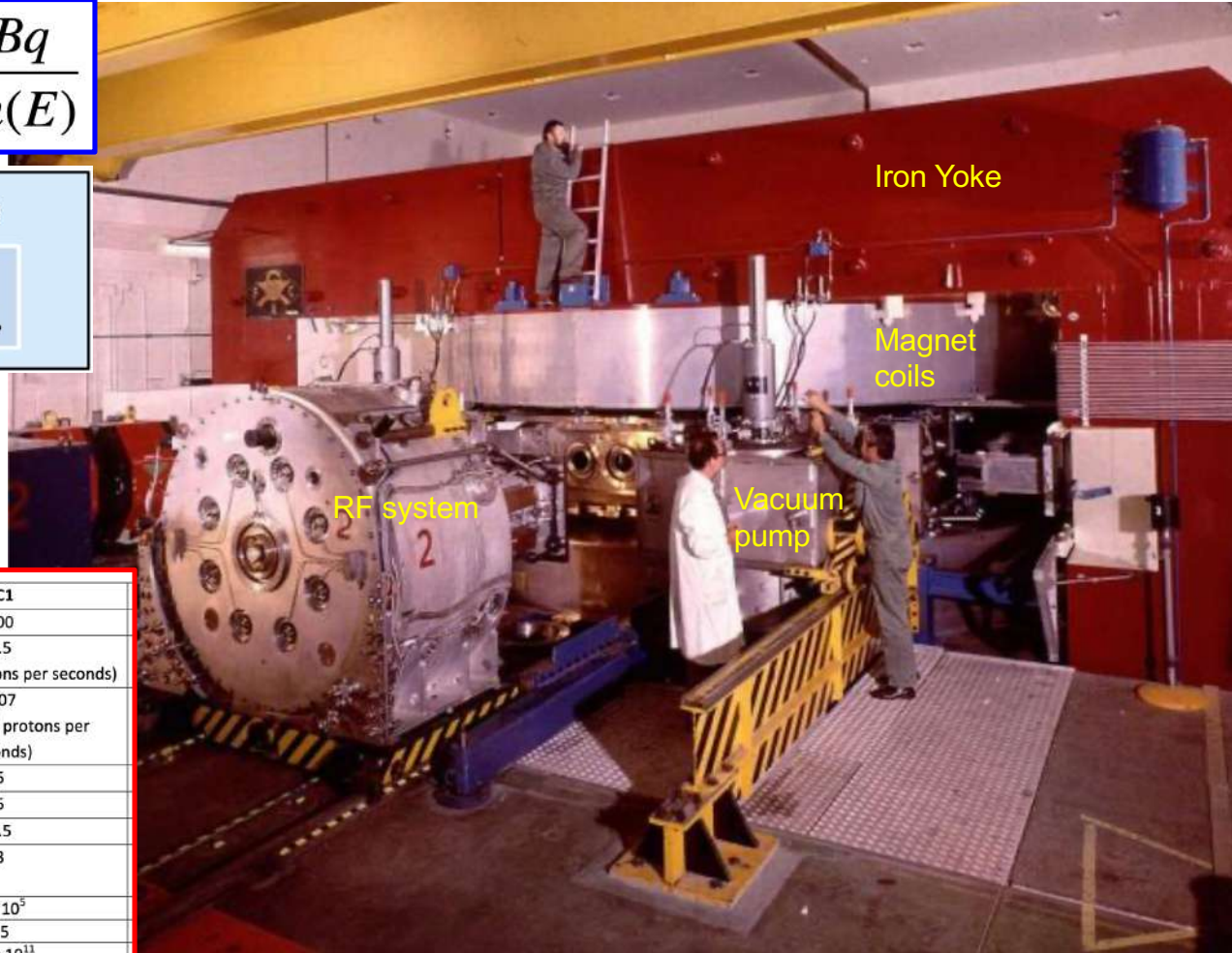
La Suisse du 30 octobre 1953

The first CERN accelerator: the 600 MeV synrocyclotron

$$\omega = \frac{v}{r} = \frac{Bq}{m} = \frac{Bq}{m(E)}$$

Synchro-cyclotron

$$\frac{f_{RF}(E)}{B(E) \text{ or } B = \text{const.}}$$



- 1954: construction started
- 1957: first beam delivered
- 1958: start experimental program
- 1974: new SC2 commissioned
- 1990: end of operation
- 2012: it became a visit point

1958: first important discovery



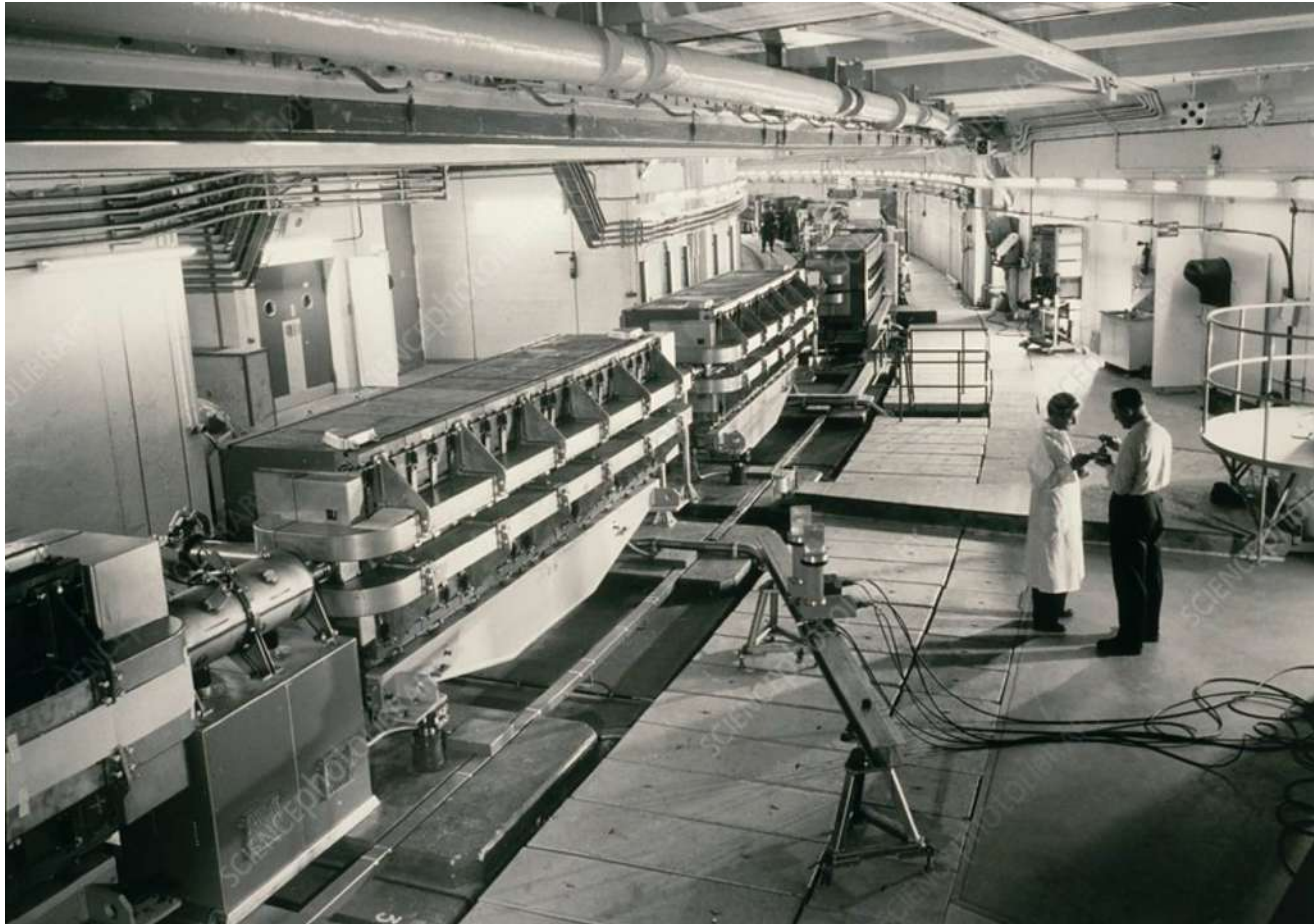
B.R. $(1.22 \pm 0.30) \cdot 10^{-4}$
in agreement with the
V-A theory

today: $(1.230 \pm 0.004) \cdot 10^{-4}$

Machine parameters	SC1
Proton kinetic energy (MeV)	600
Internal proton beam (μA)	1.5
	($\sim 6 \times 10^{12}$ protons per seconds)
Extracted proton beam (μA)	0.07
	($\sim 4.4 \times 10^{11}$ protons per seconds)
Extraction efficiency (%)	5
Energy spread (FWHM) (MeV)	5
Acceleration time (ms)	8.5
Average energy gain per turn (keV)	3
Number of revolutions	2×10^5
Repetition rate (Hz)	55
Protons per pulse	1.2×10^{11}
RF frequency swing	30.6 – 16.6 MHz

CERN comes in the game... and strong focusing too!

❑ 1959: CERN. Protosynchrotron PS, 24 GeV, 3200 Tons, 200 m diameter.

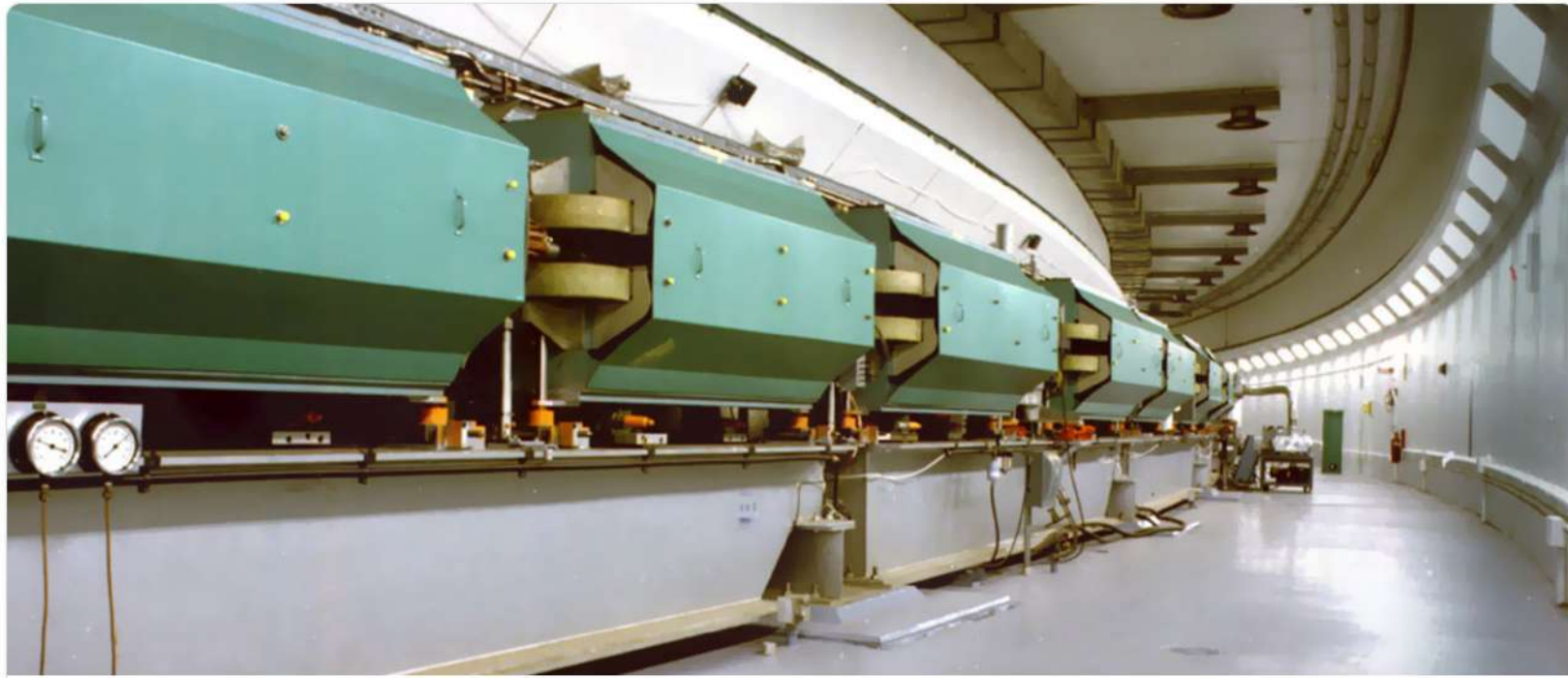


This was the first accelerator to use the strong focusing principle

Actually, bending (dipole) and focusing (quadrupole) are embedded in the same magnet.

CERN comes in the game... and strong focusing too!

- ❑ 1959: CERN. Protosynchrotron PS, 24 GeV, 3200 Tons, 200 m diameter.
- ❑ 1960: BNL. Alternate Gradient Synchrotron AGS, 33 GeV, 4000 Tons, 257 m diameter.



The construction of the two machines was a challenge for both laboratories, so the two teams were exchanging information to successfully complete the two accelerators using this novel technology.

CERN comes in the game... and strong focusing too!

- ❑ 1959: CERN. Protosynchrotron PS, 24 GeV, 3200 Tons, 200 m diameter.
- ❑ 1960: BNL. Alternate Gradient Synchrotron AGS, 33 GeV, 4000 Tons, 257 m diameter.
- ❑ Three nobel prize experiments at AGS:
 - 1962: L. Lederman, M.Schwartz and J. Steinberger: discovery of neutrino μ .
 - 1964: J. Cronin and V. Fitch: discovery of the CP violation in the K_0 system.
 - 1974: S. Ting: discovery of the J particle and quark charm (discovered also by B. Richter at SLAC)
- ❑ 1963: N. Samios: discovery of the Ω^- at the AGS with a 80-inch (~ 200 cm) bubble chamber, validating the eightfold way model of M.Gell-Mann and Y.Ne'eman
- ❑ The first important discovery at PS was the weak neutral current in 1973 at the bubble chamber Gargamelle by a team led by A. Lagarrigue that proved the existence of the Z boson foreseen by the Standard Model.

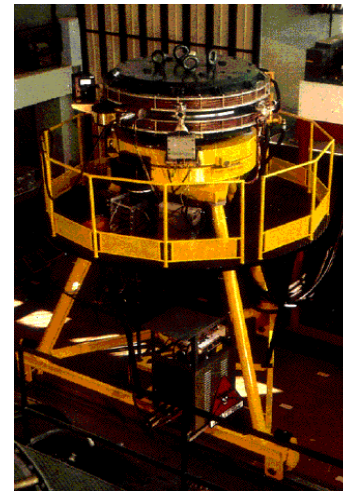
Question: why the PS took so long to have a major discovery?

ADA: the first e^+e^- collider

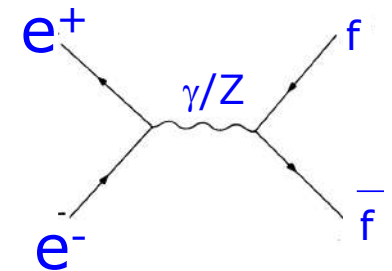
- In 1961, in the Frascati National Laboratories, a small group of young physicists and engineers led by Bruno Touschek conceived and built AdA, the first particle-antiparticle accelerator. In the same ring, electrons and positrons circulated in opposite directions with equal speed, annihilating and transforming all the initial energy into new particles.



ADA – 1961 - LNF

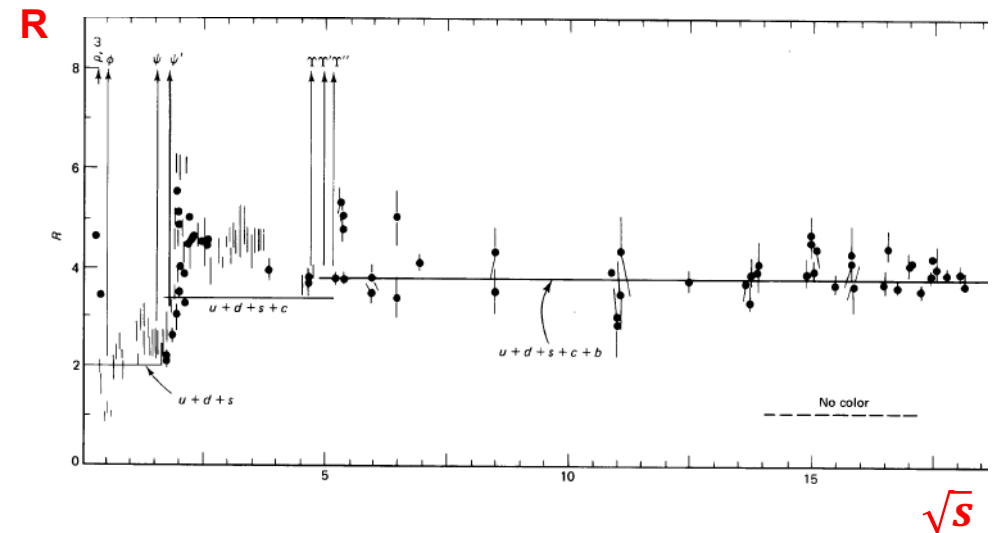
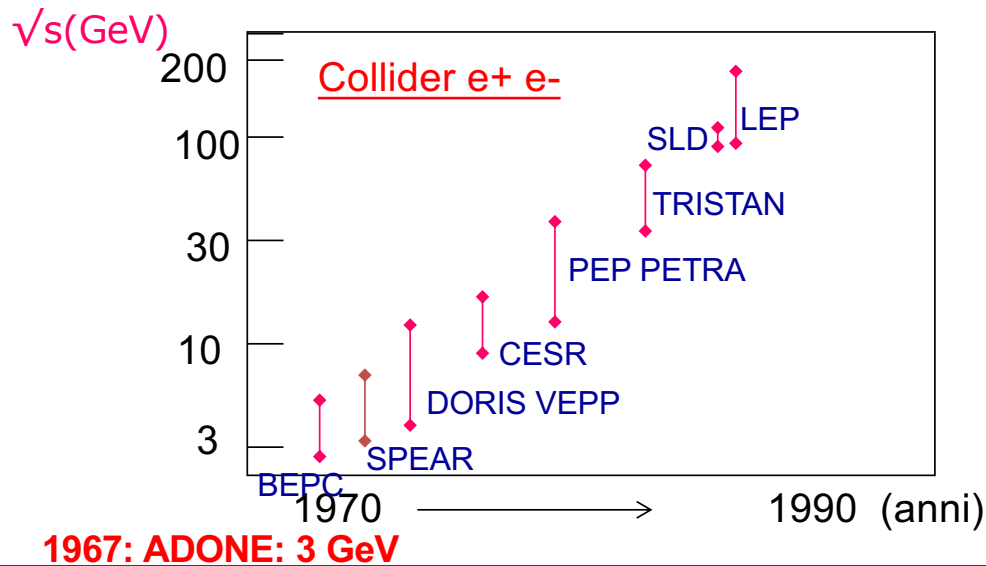


$\sqrt{s} = 500 \text{ MeV}$

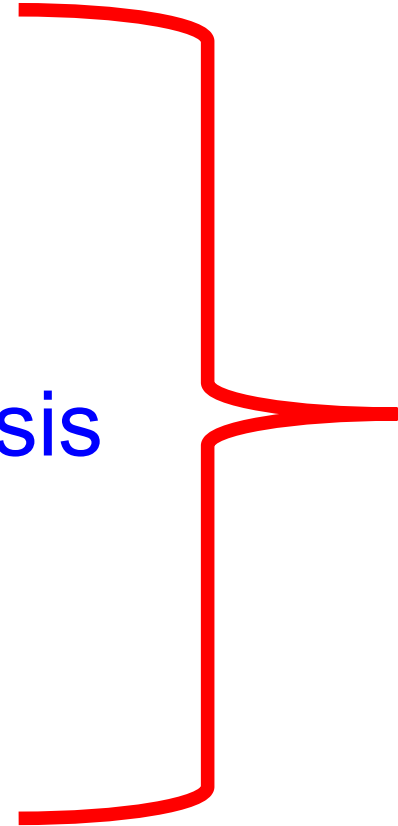


ADA: the first e^+e^- collider

- In 1961, in the Frascati National Laboratories, a small group of young physicists and engineers led by Bruno Touschek conceived and built AdA, the first particle-antiparticle accelerator. In the same ring, electrons and positrons circulated in opposite directions with equal speed, annihilating and transforming all the initial energy into new particles.
- AdA, despite having had a short scientific life, remains a milestone in the history of science. As a prototype of the many accumulation rings that followed, it showed to the Particle Physics community the concrete feasibility of electron-positron colliders.



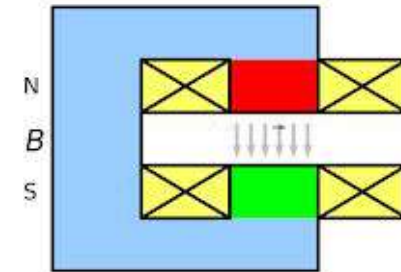
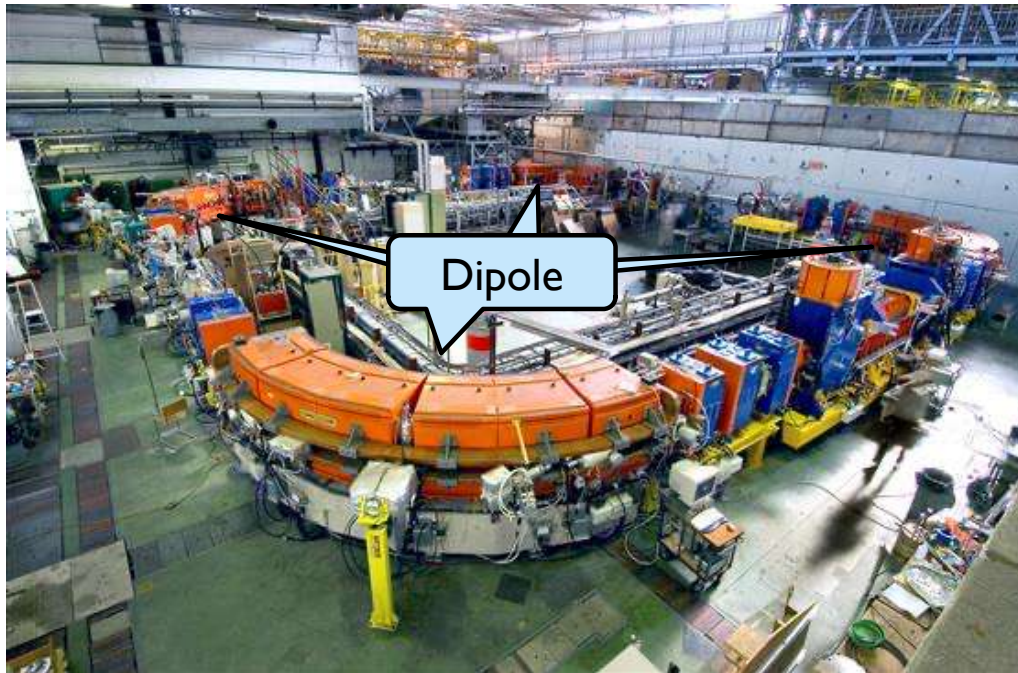
Let's close the parenthesis



Synchrotron: bending and focusing

Bending

Vertical magnetic field to bend in horizontal plane.



LEIR has 4 dipoles, each with 90° bending angle, to keep particles on a circular orbit

LEIR (Low Energy Ion Ring)

- 78m circumference
- first circular accelerator for CERN's heavy-ions on the way to LHC
- 2.5 sec to accelerate ion bunches from 4.2 MeV/n to 72 MeV/n

Bending at LHC

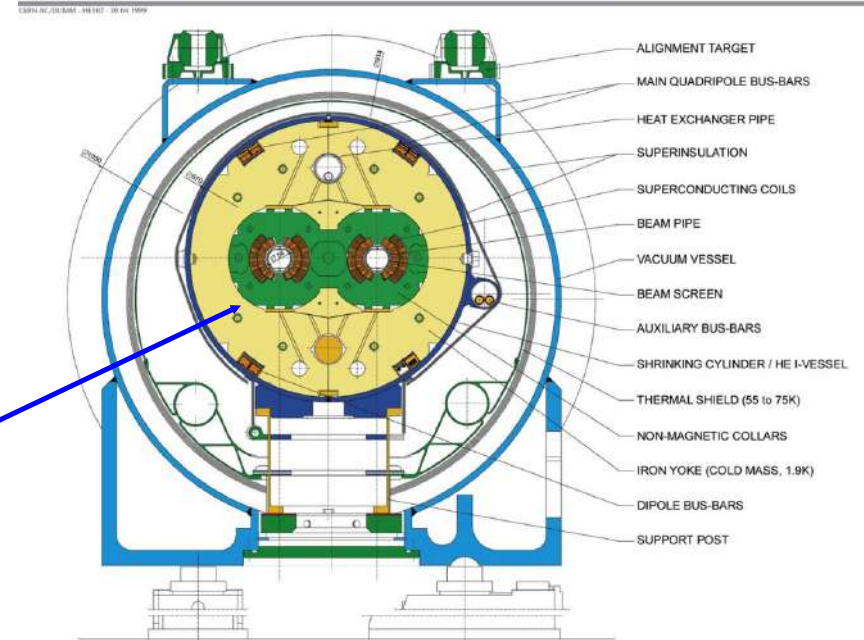


The superconducting coils are cooled to 1.9 K (the cosmic background radiation is at 2.7 K). LHC is the coldest point in the Universe (on a large scale).

LHC has 1232 superconducting dipole magnets, each 15 m long and able to deflect the beam by 0.29° .

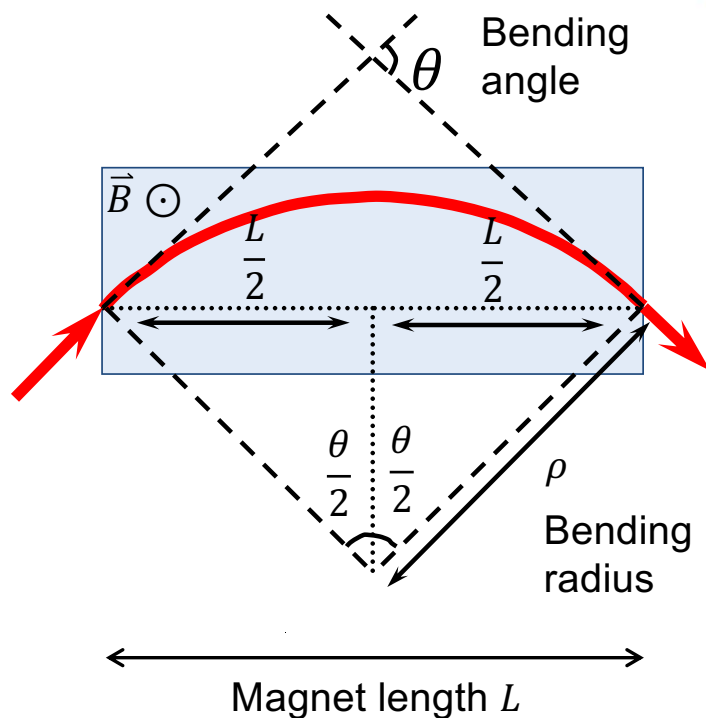
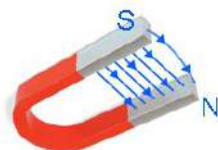
8.33 Tesla (max 2 T in iron)
11.7 kA (superconducting coil)

LHC DIPOLE : STANDARD CROSS-SECTION



Deflection of a charged particle

Charged particles are deflected in a magnetic field



The ideal circular orbit

Lorentz Force $F_L = q v B$

Centrifugal Force $F_{centr} = \frac{\gamma m_0 v^2}{\rho}$

$$F_L = F_{centr}$$

$$\frac{p}{q} = B \rho$$

$B \rho = \text{Beam rigidity}$

q = charge

$p = \gamma m_0 v$ momentum

B = mag. Field strength

ρ = bending radius

Required Magnetic Field Strength

Full circle

$$\alpha = \int \frac{dl}{\rho} = \int \frac{Bdl}{B\rho} = 2\pi \quad \xrightarrow[\frac{p}{e} = B\rho]{\int B dl \approx N l B}$$

$$B = 2\pi p / (qNl)$$

N: number of magnets
l: length of a magnet

Example SPS:

- Particle:
 - $p = 450 \text{ GeV}/c$
 - $q = +1e$ (proton)
- Dipole magnets:
 - $l = 6.2\text{m}$
 - $\rho = 735\text{m}$
 - $N = 744$



$$B \approx \frac{2\pi \times 450 \text{ GeV}}{744 \times 6.2 \text{ m} \times 3 \times 10^8 \frac{\text{m}}{\text{s}} \times e} = 2.0 \text{ T}$$

normal conducting magnet

Example LHC:

- Particle:
 - $p = 7000 \text{ GeV}/c$
 - $q = +1e$ (proton)
- Dipole magnets:
 - $l = 15\text{m}$
 - $\rho = 2803\text{m}$
 - $N = 1232$

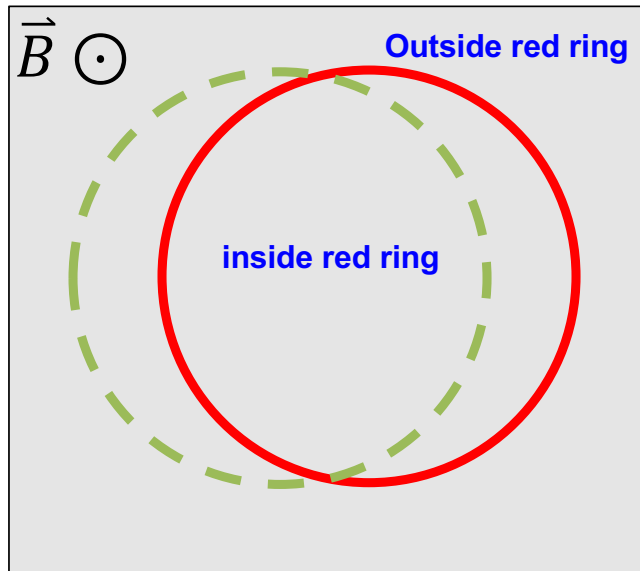


$$B \approx \frac{2\pi \times 7000 \text{ GeV}}{1232 \times 15 \text{ m} \times 3 \times 10^8 \frac{\text{m}}{\text{s}} \times e} = 8.3 \text{ T}$$

superconducting magnet

Particles oscillation

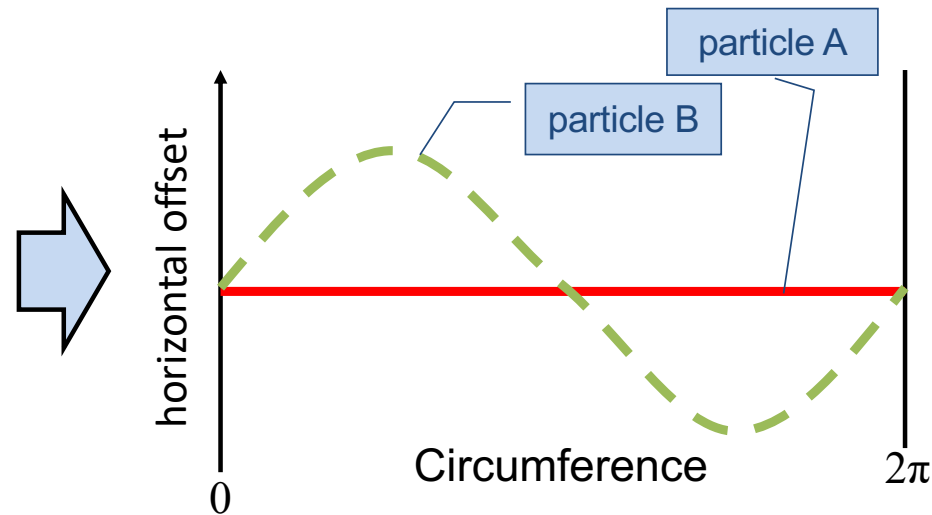
Example: two charged particles, with the same momentum, in a homogeneous magnetic field



— particle A
- - - particle B

“horizontal” movement (distance between the two orbits)

Particle B, while it is turning, go outside and inside the trajectory of the particle A

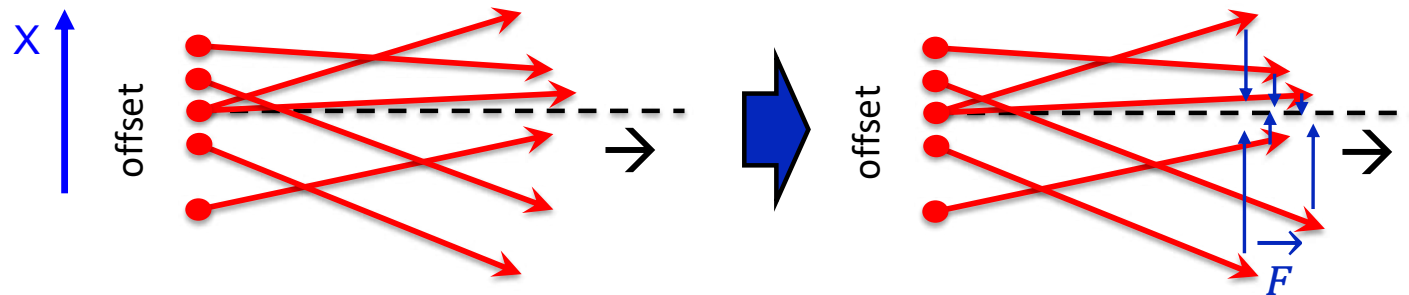


In a homogeneous magnetic field, particles with varying initial conditions fulfil oscillations around the design orbit → **Betatron-Oscillation**

design orbit = trajectory of ideal particle → defined by dipole magnets

Beam focusing

A bunch contains many particles with different initial conditions.



Many different positions, angles and energy offsets

We need a focusing force that keeps the particles close to the design orbit.

Focusing force should rise as a function of the distance to the design orbit.

Beam focusing

Requirement:

Lorentz force linearly increasing as a function of distance from design orbit.

→ Linearly increasing magnetic field.

$$F(x) = q \cdot v \cdot B(x)$$

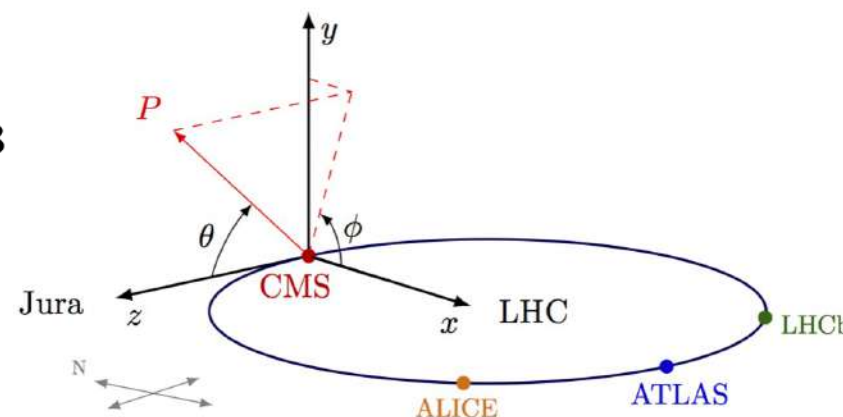
Taylor series as a function of distance from magnet center

$$B_y(x) = B_{y0} + \underbrace{\frac{\partial B_y}{\partial x} x}_{\text{dipole}} + \underbrace{\frac{1}{2} \frac{\partial^2 B_y}{\partial x^2} x^2}_{\text{quadrupole}} + \underbrace{\frac{1}{3!} \frac{\partial^3 B_y}{\partial x^3} x^3}_{\text{sextupole}} + \underbrace{\dots}_{\text{octupole}}$$

Normalize to p/q:

$$\frac{p}{q} = B \rho$$

$$\frac{B_y(x)}{p/q} = \frac{1}{\rho} + \underbrace{kx}_{\text{quadrupole}} + \frac{1}{2} m x^2 + \frac{1}{3!} n x^3 + \dots$$



N.B. if you need a force along x, B component has to be along y since particle velocity is along z

Beam focusing

Focusing of particles with quadrupoles: strong focusing

$$F(x) = q \cdot v \cdot B(x)$$

with the vertical (y) and horizontal (x) quadrupole fields

$$B_y = g \cdot x$$

$$B_x = g \cdot y$$

where g is the gradient

$$g = \frac{2\mu_0 n I}{r^2} \left[\frac{T}{m} \right]$$

Normalized gradient = **focusing strength**

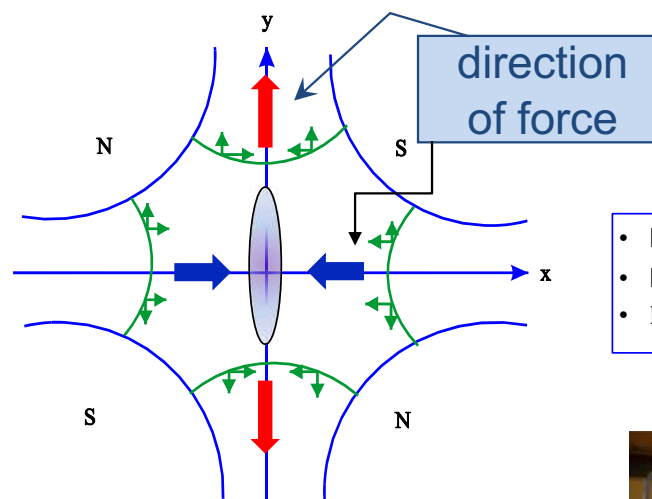
$$k = \frac{g}{p/q} [m^{-2}]$$

I coil current

n number of windings

r distance magnet center to pole

μ_0 permeability of free space



- $\mathbf{B}$ from N to S
- Beam is exiting the page
- $\mathbf{F} = q \mathbf{v} \times \mathbf{B}$

Do you see the problem with this?

Quadrupoles focus in one plane, but defocus in the other!



quadrupole magnet

Focusing analogous to geometrical optics

Focusing of particles with quadrupoles is similar to focusing of light with lenses.

A series of alternating focusing and defocusing lenses will focus:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

In a synchrotron quadrupoles are lenses with the focal length:

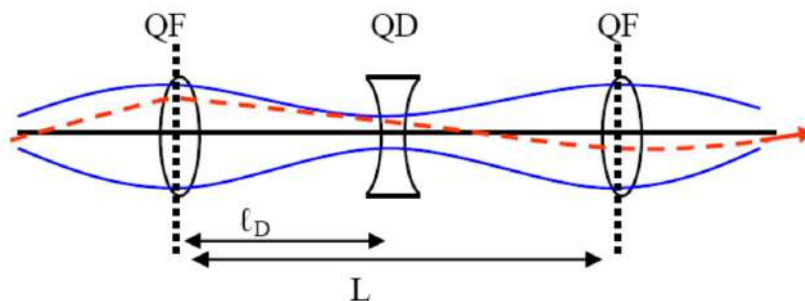
$$f = \frac{1}{k \cdot l_Q}$$

Consider:

$$\begin{aligned} f_1 &= f \\ f_2 &= -f \end{aligned}$$

Then:

$$F = \frac{f^2}{d} > 0$$

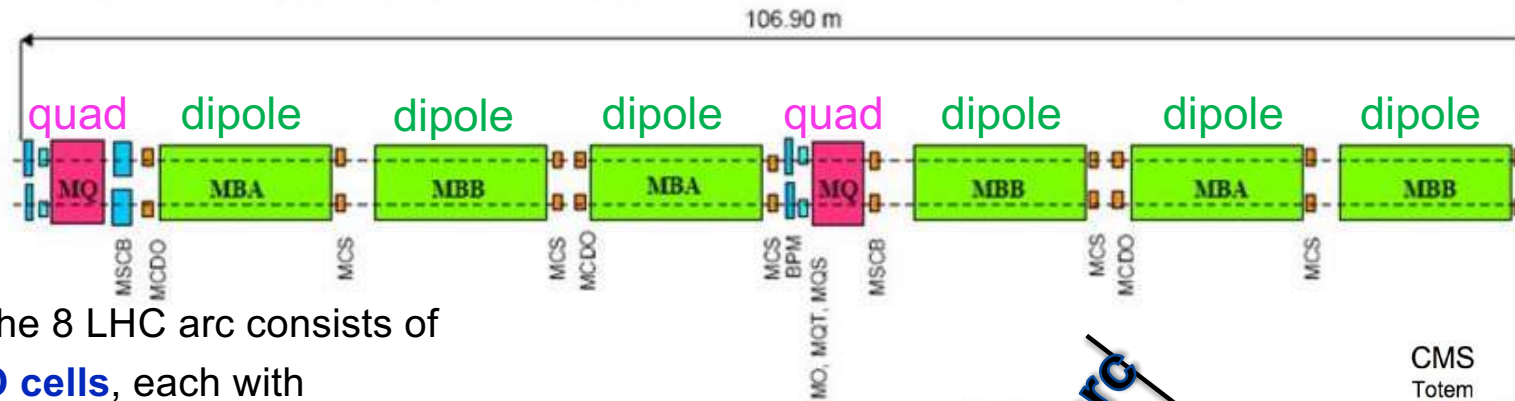


Typical alternating

F = focusing
0 = nothing (dipole, RF, ...)
D = defocusing
0

lattice of quadrupoles in an accelerator

The LHC FODO cells



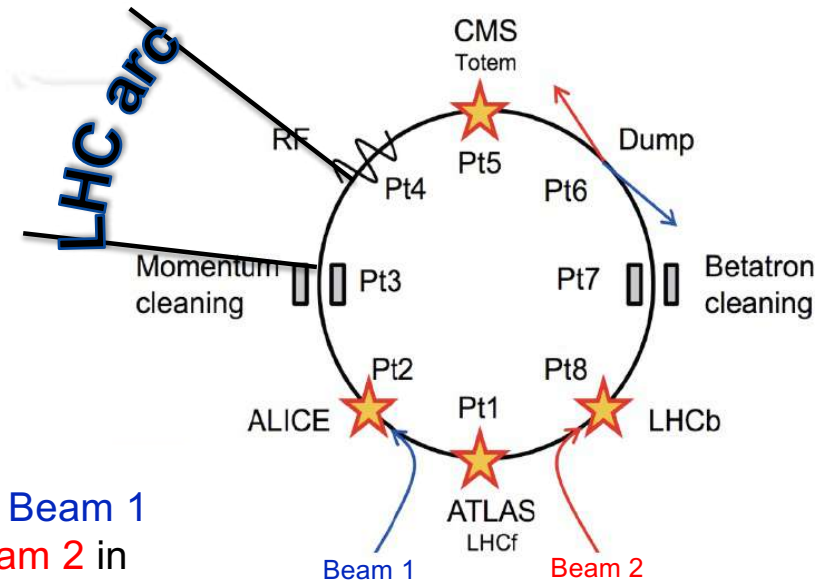
Each of the 8 LHC arc consists of **23 FODO cells**, each with

- **2 Quadrupoles**
- **6 Dipoles**
- Additional instrumentation and corrector magnets are installed in between for beam control.



LHC quadrupole

A **focusing** magnet for **Beam 1** is a **defocusing** for **Beam 2** in the same plane.



Example of magnets

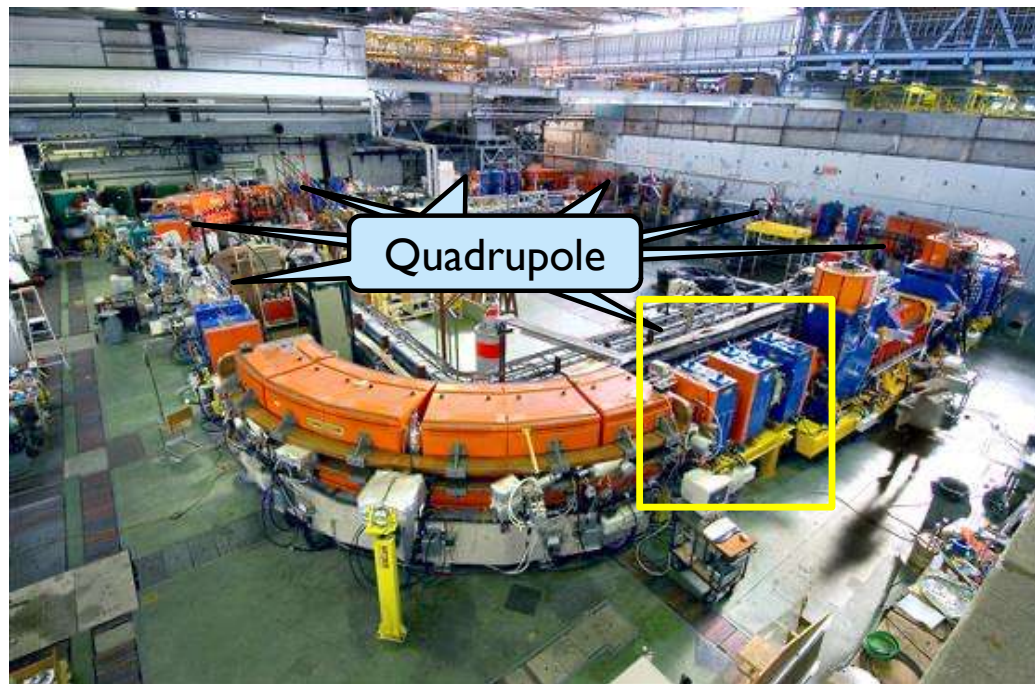


a sextupole in front of
a "C" dipole

a quadrupole of the
HERA accelerator



Beam focusing



LEIR – first circular accelerator for CERN's heavy-ions on the way to LHC

How does a particle move in an accelerator

(No need to remember all equations. This is only meant to give you the big picture and the “naming”)

Particle motion

Focusing force that keeps the particles close to the design orbit, which rises as a function of the distance.

$$F(x) = q \cdot v \cdot B(x)$$

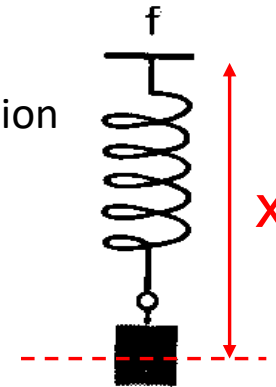
Classical free harmonic oscillator

→ experiences **restoring force proportional to the displacement x** when displaced from equilibrium position

Second law of motion:

$$\vec{F} = m\vec{a}$$

$$m \frac{d^2 x}{dt^2} = m\ddot{x} = -kx$$

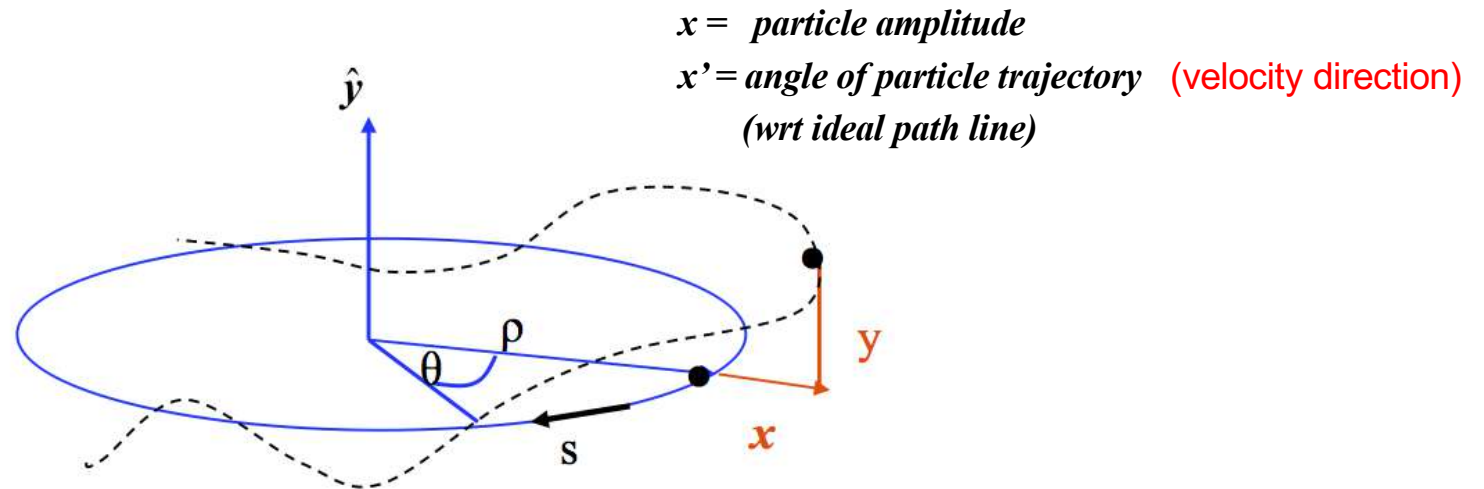


Solution of equation of motion is sinusoidal oscillation:

$$x(t) = A \cos(\omega t + \varphi)$$

Coordinate system

Use different coordinate system: **Frenet-Serret** rotating frame



- The ideal particle defines “design” trajectory: $x=0, y=0$
→ *travels through the center of all magnets.*
- $x, y \ll \rho$

Look at the particle motion along the path length s .

Toward the equation of motion

$$F_x = m \cdot \ddot{x}$$

Describes motion as a function of time.

But what we need is something like $F_x = Mx''$ $\dot{x} = \frac{dx}{dt}$

→ Replace free parameter time t by path length s .

$$x' = \frac{dx}{ds}$$

→ Compare to Lorentz force $F(x) = q \cdot v \cdot B(x)$

Taylor expansion of normalize magnetic field:

$$\frac{B_y(x)}{p/q} = \frac{1}{\rho} + kx + \frac{1}{2} \cancel{m} x^2 + \frac{1}{3!} \cancel{n} x^3 + \dots \begin{matrix} \text{higher} \\ \text{orders} \end{matrix}$$

dipole quadrupole sextupole octupole

Only consider **linear** terms: **dipole & quadrupole** fields!

$$\frac{B_y(x)}{p/q} \approx \frac{1}{\rho} + kx$$

Equation of motion

Equation of motion

Horizontal motion:

$$x'' + Kx = 0$$

Vertical motion:

$$y'' - ky = 0$$

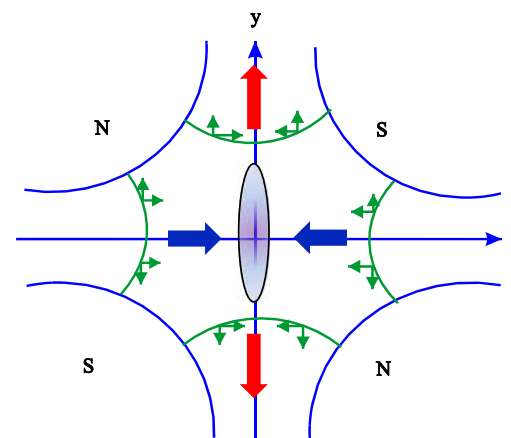
Where $K = \frac{1}{\rho^2} + k$

with k as the quadrupole focusing strength and ρ the bending radius.

In vertical:

→ In general, no dipoles: $\frac{1}{\rho^2} = 0$

→ Sign change of force direction: $k \Longleftrightarrow -k$



Assuming the motion in the horizontal and vertical plane are independent → **Particle motion in x & y is uncoupled**

Solving the equation of motion – focusing quadrupole

Equation of motion in horizontal plane

$$x'' + Kx = 0$$

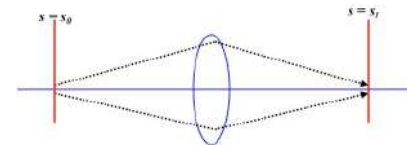
Equation of the **harmonic oscillator** with spring constant K.

Ansatz

For **K > 0 (focusing)** the solution can be found with this ansatz and boundary conditions:

$$x(s) = a_1 \cos(\omega s) + a_2 \sin(\omega s)$$

$$s = 0 \rightarrow \begin{cases} x(0) = x_0, \\ x'(0) = x'_0 \end{cases}$$



Solution

Inserting these into the equation of motion yields:

$$x(s) = x_0 \cos(\sqrt{K}s) + x'_0 \frac{1}{\sqrt{K}} \sin(\sqrt{K}s)$$

$$x'(s) = -x_0 \sqrt{K} \sin(\sqrt{K}s) + x'_0 \cos(\sqrt{K}s)$$

Transfer Matrix

Use matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = M_{foc} \cdot \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

Focusing Quadrupole

$$M_{foc} = \begin{pmatrix} \cos(\sqrt{K}s) & \frac{1}{\sqrt{K}} \sin(\sqrt{K}s) \\ -\sqrt{K} \sin(\sqrt{K}s) & \cos(\sqrt{K}s) \end{pmatrix}$$

Solving the equation of motion – defocusing quadrupole

Equation of motion in horizontal plane

$$x'' + Kx = 0$$

Equation of the **harmonic oscillator** with spring constant K.

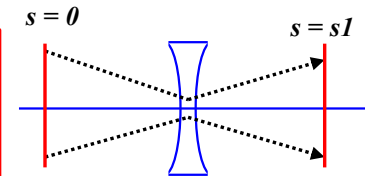
Defocusing
Quadrupole

For **K < 0 (defocusing)**
the new ansatz is:

$$x(s) = a_1 \cosh(\omega s) + a_2 \sinh(\omega s)$$

**Defocusing
Quadrupole**

$$M_{defoc} = \begin{pmatrix} \cosh(\sqrt{|K|}s) & \frac{1}{\sqrt{|K|}} \sinh(\sqrt{|K|}s) \\ \sqrt{|K|} \sinh(\sqrt{|K|}s) & \cosh(\sqrt{|K|}s) \end{pmatrix}$$



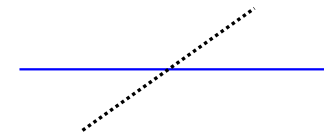
Drift Space

For **K = 0 (drift)** the ansatz is:

$$x(s) = x'_0 s$$

Drift Space

$$M_{drift} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$



Dipole

For **K = 1/ρ² (dipole)** use the result for a focusing dipole and insert K.

Dipole

$$M_{dipole} = \begin{pmatrix} \cos(\frac{s}{\rho}) & \rho \sin(\frac{s}{\rho}) \\ -\frac{1}{\rho} \sin(\frac{s}{\rho}) & \cos(\frac{s}{\rho}) \end{pmatrix}$$

Particle tracking

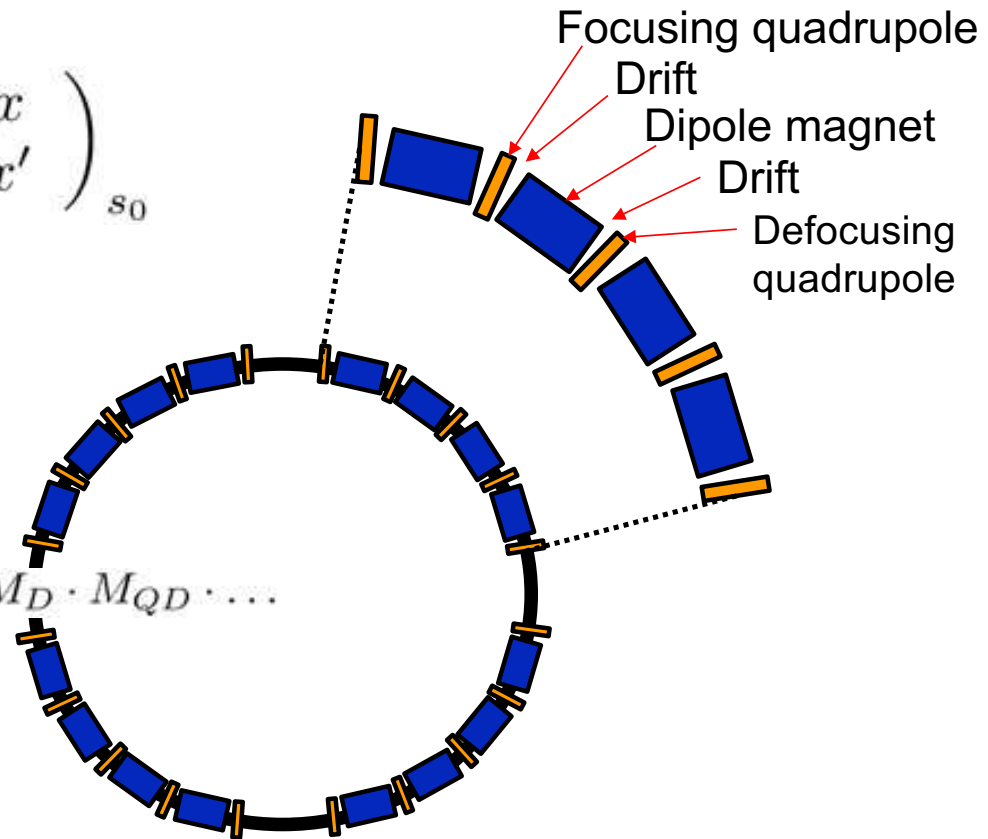
Knowing the initial coordinates at $s=s_0$, we can use the transfer matrix to calculate the effect of an element to the particle's trajectory and get its new coordinates at $s=s_1$.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

For a sequence of elements:

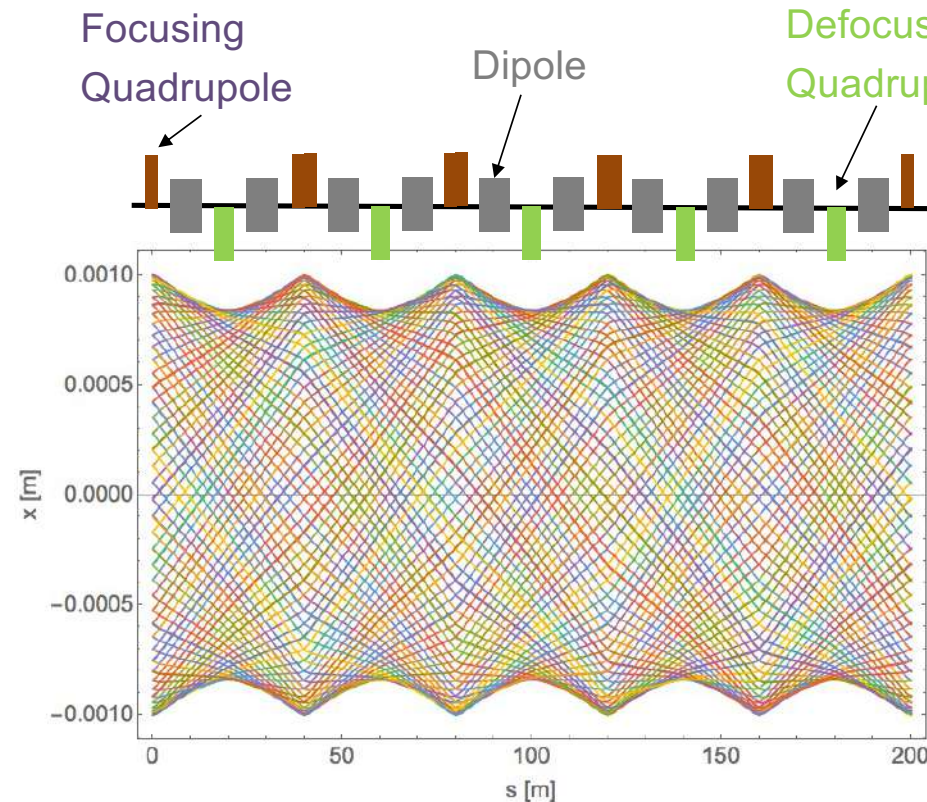
$$M_{total} = M_{QF} \cdot M_D \cdot M_{Bend} \cdot M_D \cdot M_{QD} \cdot \dots$$

Building up the particles path through the accelerator ...



How does a particle trajectory look like?

Initial coordinates
 $x_0 = 0.001\text{m}$ (1 mm)
 $x'_0 = 0$



The envelope of all trajectories has a periodicity that depends on the lattice

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

Hill's equation

We had ...

$$x'' + Kx = 0$$

But, around the accelerator K is not constant and does depend on s !

$$x''(s) + K(s)x(s) = 0$$

Hill's equation

- $K(s+L) = K(s) \rightarrow$ periodic function, where L is the “lattice period”
- General solution of Hill's equation:

$$x(s) = \sqrt{2J_x\beta_x(s)} \cos(\psi(s) + \phi)$$

It is a **quasi harmonic oscillation**, where **amplitude and phase depend on the position s** in the ring.

The Beta function

General solution of Hill's equation

$$x(s) = \sqrt{2J_x \beta_x(s)} \cos(\psi(s) + \phi)$$

Integration constants: determined by initial conditions

The **beta function** is a periodic function determined by the focusing properties of the lattice, i.e. quadrupoles

$$\beta(s + L) = \beta(s)$$

The **“phase advance”** of the oscillation between the point s_0 and point s in the lattice.

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

The Tune

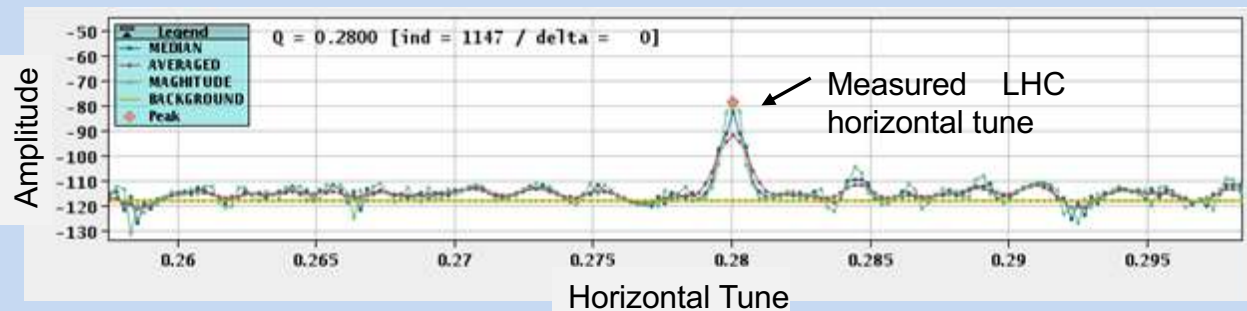
The number of oscillations per turn is called “**tune**”

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)} \xrightarrow{\text{full turn}} Q = \frac{1}{2\pi} \int \frac{ds}{\beta(s)}$$

The tune is an important parameter for the **stability of motion** over many turns. It has to be **chosen appropriately, measured and corrected**.

Tune Measurement

- 1) Measure beam position at one location turn by turn.
- 2) Beam position will change $\propto \cos(2\pi Qi)$.
- 3) Perform FFT to get frequency of oscillation $\rightarrow$ tune



Courant-Snyder Parameters: $\alpha(s), \beta(s), \gamma(s)$

General solution of Hill's equation: $x(s) = \sqrt{2J_x\beta_x(s)} \cos(\psi(s) + \phi)$

Define: $\alpha(s) = -\frac{1}{2}\beta'(s) \quad \gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$

$\alpha(s), \beta(s), \gamma(s)$ are called **Courant-Snyder parameters or Optics parameters**

Let's assume for $s(0) = s_0, \psi(0) = 0, \beta(0) = \beta_0$ and $\alpha(0) = \alpha_0$

Defines ϕ from initial conditions: x_0 and x'_0, β_0 and α_0 .

Re-write transfer matrix with optics parameters:

$$M = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}(\cos \psi + \alpha_0 \sin \psi) & \sqrt{\beta\beta_0} \sin \psi \\ \frac{(\alpha_0 - \alpha) \cos \psi - (1 + \alpha\alpha_0) \sin \psi}{\sqrt{\beta\beta_0}} & \sqrt{\frac{\beta_0}{\beta}}(\cos \psi - \alpha \sin \psi) \end{pmatrix}$$

Once we know α and β , we can compute the single particle trajectories between two locations without remembering the exact lattice structure and strength of each element!

Phase Space

General solution of Hill's equation: $x(s) = \sqrt{2J_x\beta_x(s)} \cos(\psi(s) + \phi)$

J_x is called **action** and can be written as:

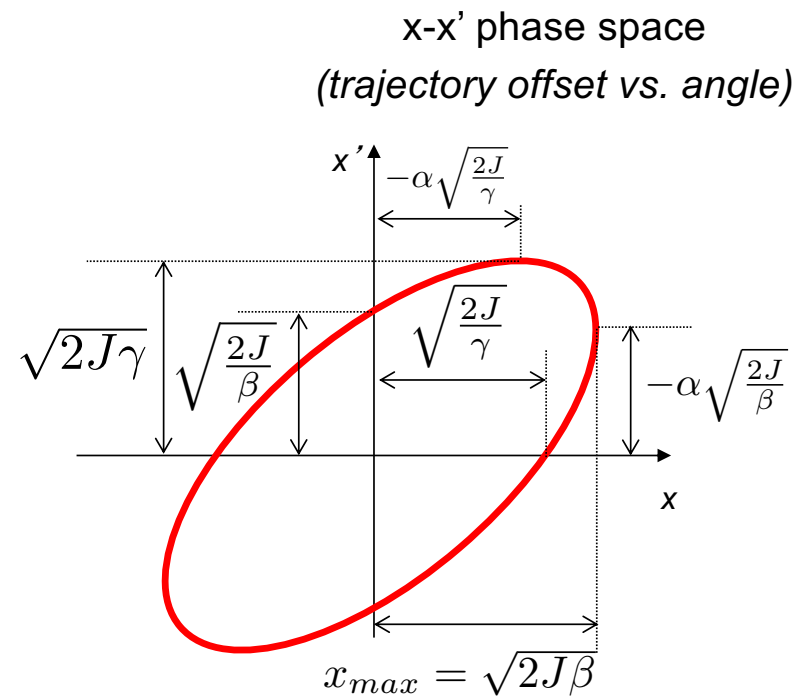
$$J_x = \frac{1}{2} (\gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2)$$

which is the equation of an **ellipse** in the **phase-space x, x'** .

The shape and orientation of ellipse are defined by the Courant-Snyder parameters.

The area of the ellipse is:

$$A = 2 \cdot \pi \cdot J_x$$



Emittance and beam size

At a given location: $x = \sqrt{2\beta_x J_x} \cos \psi_x$

The mean square value of this is:

$$\langle x^2 \rangle = 2\beta_x \langle J_x \cos^2 \psi_x \rangle = \beta_x \langle J_x \rangle = \beta_x \epsilon_x$$

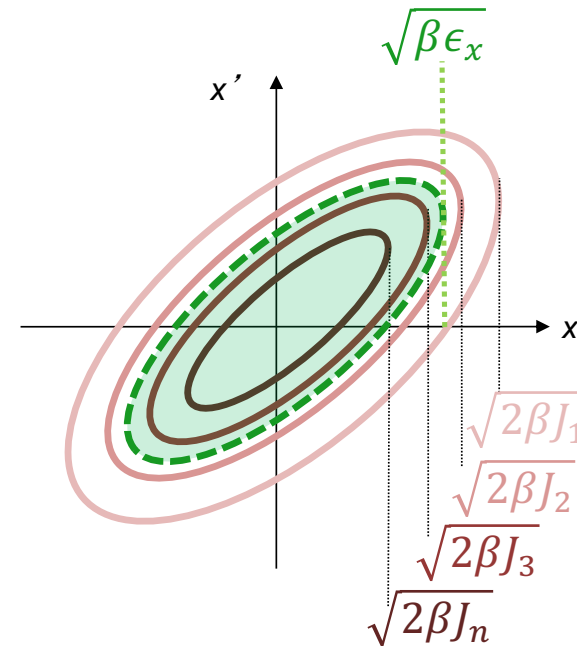
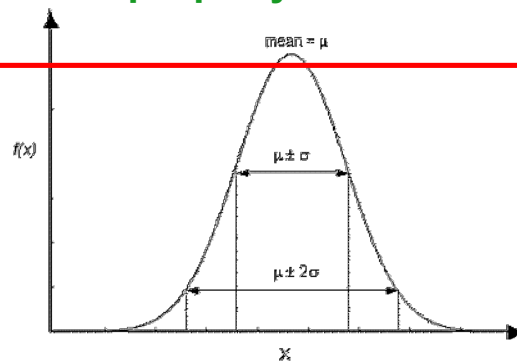
assumes action and phase uncorrelated, and uniform distribution in phase from 0 to 2π .

Defines **emittance** of particle distribution:

$$\langle J_x \rangle = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2} := \epsilon_x$$

ϵ_x is an **intrinsic beam property** that is defined at it's creation.

↑
In LHC it is defined by
the injector chain
properties



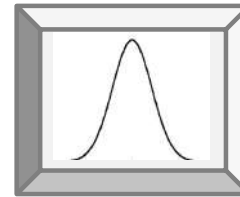
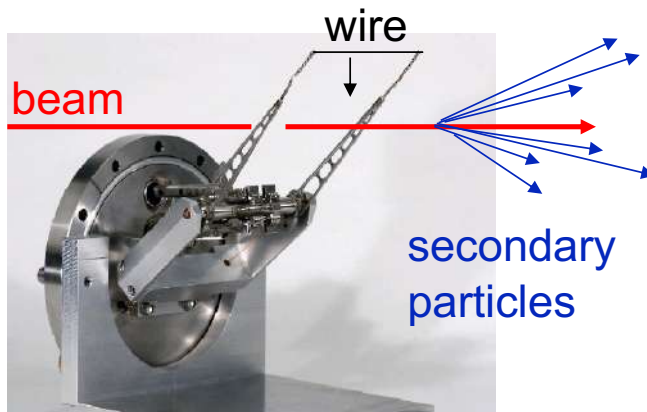
Typically the distribution of particles in a bunch follows a Gaussian shape:

$$\rho(x) = \frac{N}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{x^2}{2\sigma_x^2}}$$

Therefore, $\sigma_x = \sqrt{\langle x^2 \rangle} = \sqrt{\epsilon_x \beta_x}$ describes the one sigma **beam size**.

Beam size and emittance measurement

Principle of a wire-scanner beam size measurement

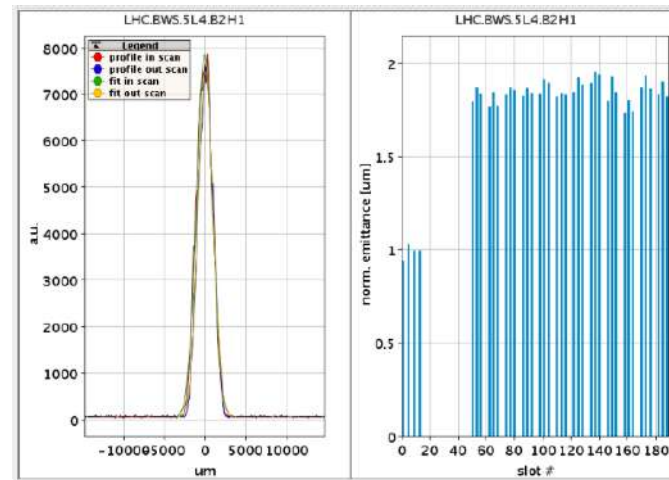


$$\sigma_x = \sqrt{\varepsilon \beta_x}$$

Gaussian fit to profile $\rightarrow$ beam size σ

Knowledge of β -function $\rightarrow$ emittance ε

Single
horizontal
bunch
profile



LHC measurement

*Emittance calculated from profile
measurement.*

All circulating bunches.

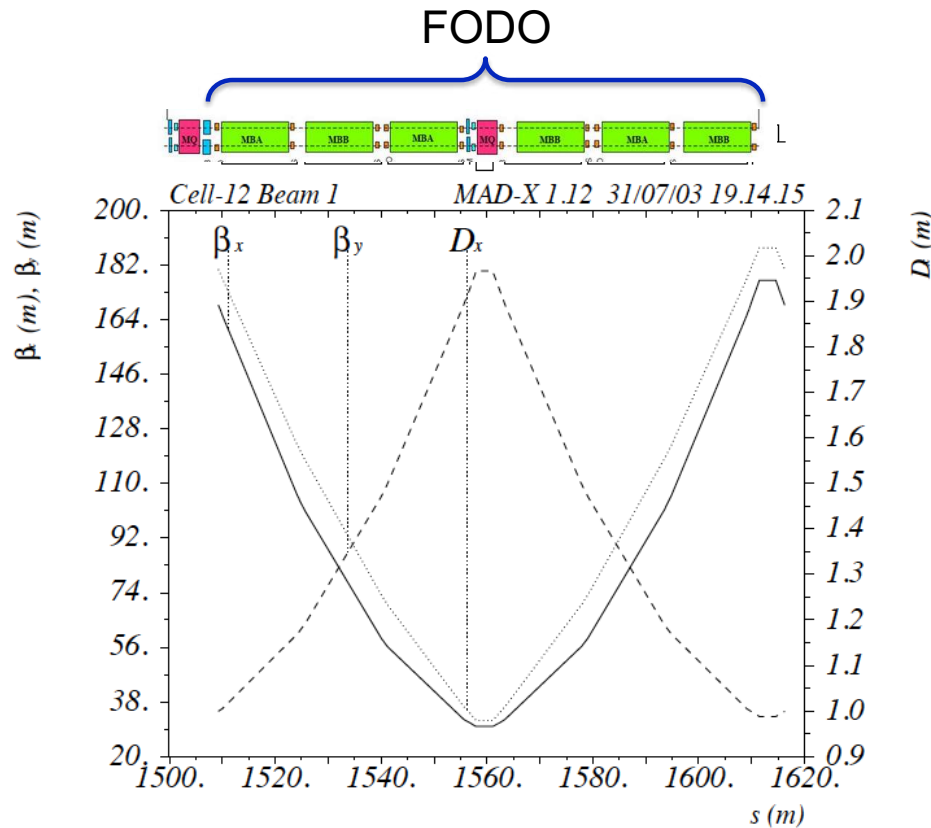
Beam size around the accelerator

The β -function is periodic

→ It changes along the cell.

→ **The beam size changes along the cell!**

$$\sigma = \sqrt{\varepsilon \beta}$$



Max. horizontal beam size in the focusing quadrupoles

Max. vertical beam size in the defocusing quadrupoles

The regular LHC FODO cell:

- Phase advance: 90°
- Maximum beta: 180 m

Things to remember

Phase space

A space that represents all possible states of a system.

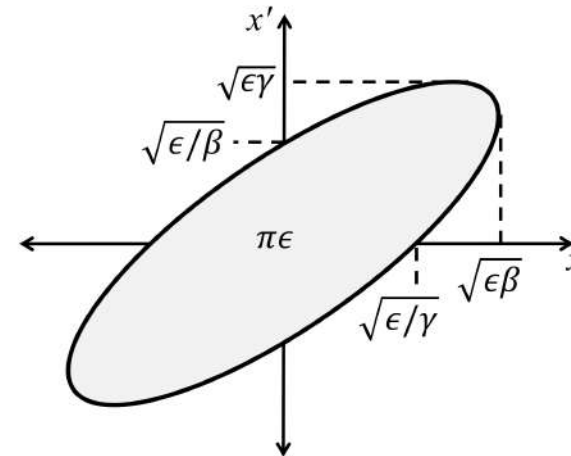
A particle's trajectory points or coordinates at a given element draw an **ellipse in phase space**.

The orientation and shape of that ellipse is described by the optical (Courant-Snyder) parameters. $\rightarrow \beta$ -function

The area of that ellipse is $\propto$ **emittance**.

Emittance is a beam property that cannot be changed by focusing.

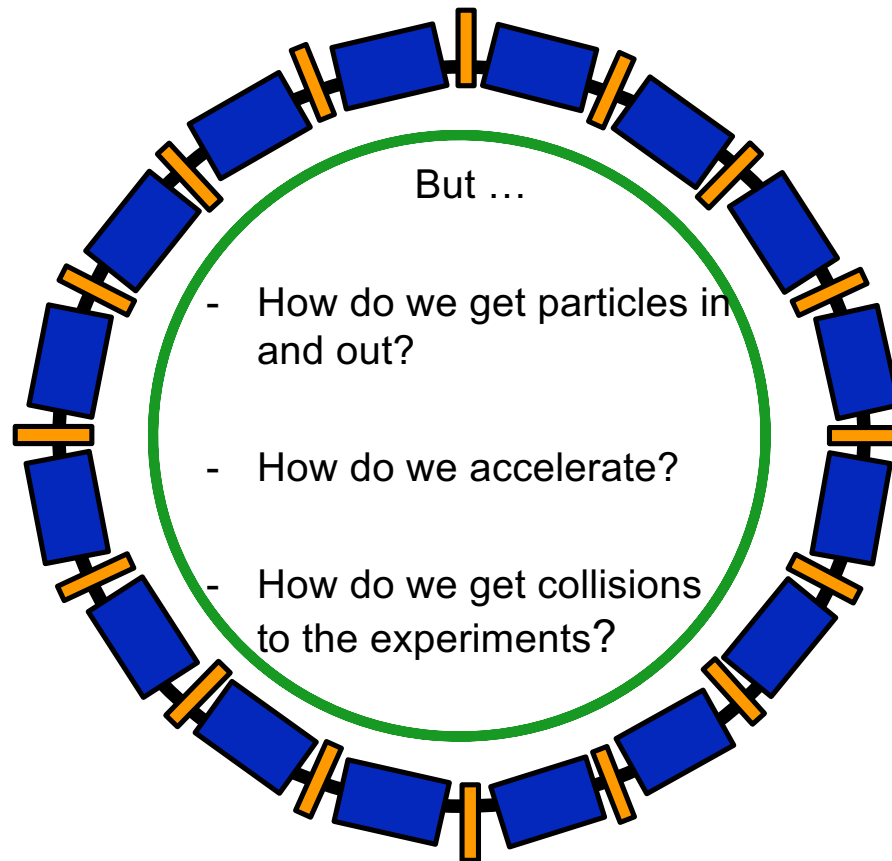
The **beam size** of a particle ensemble is defined by $\sigma = \sqrt{\epsilon\beta}$.



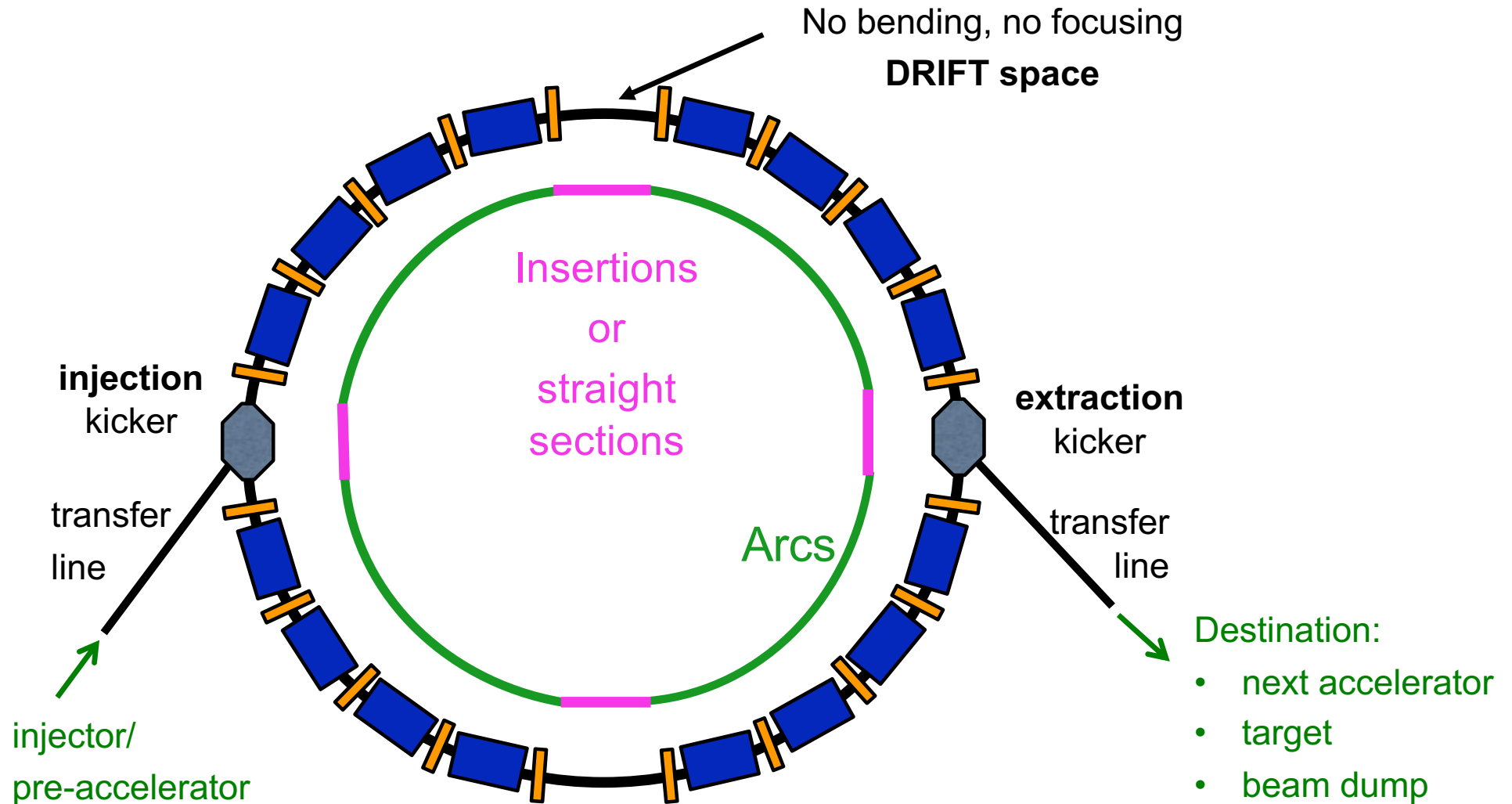
Beam Injection/extraction

What we learned so far?

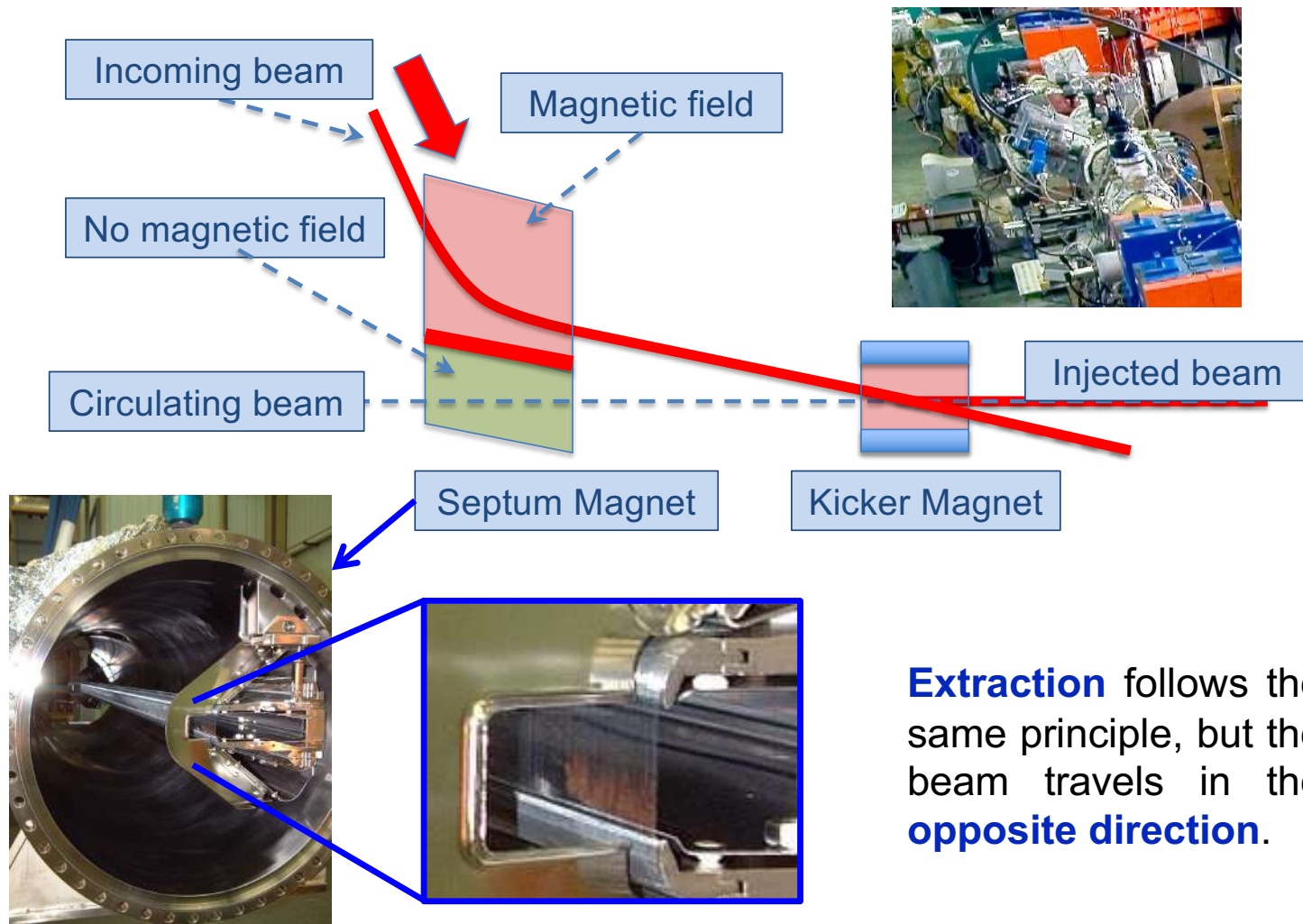
We know, how particles behave along the magnetic lattice of an accelerator.



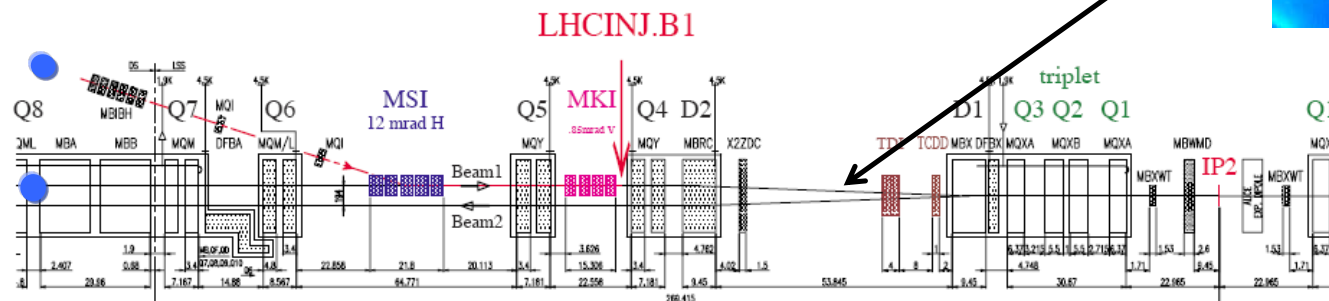
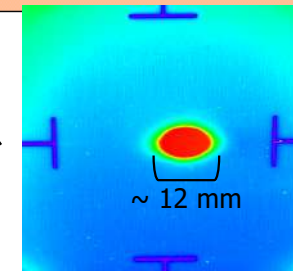
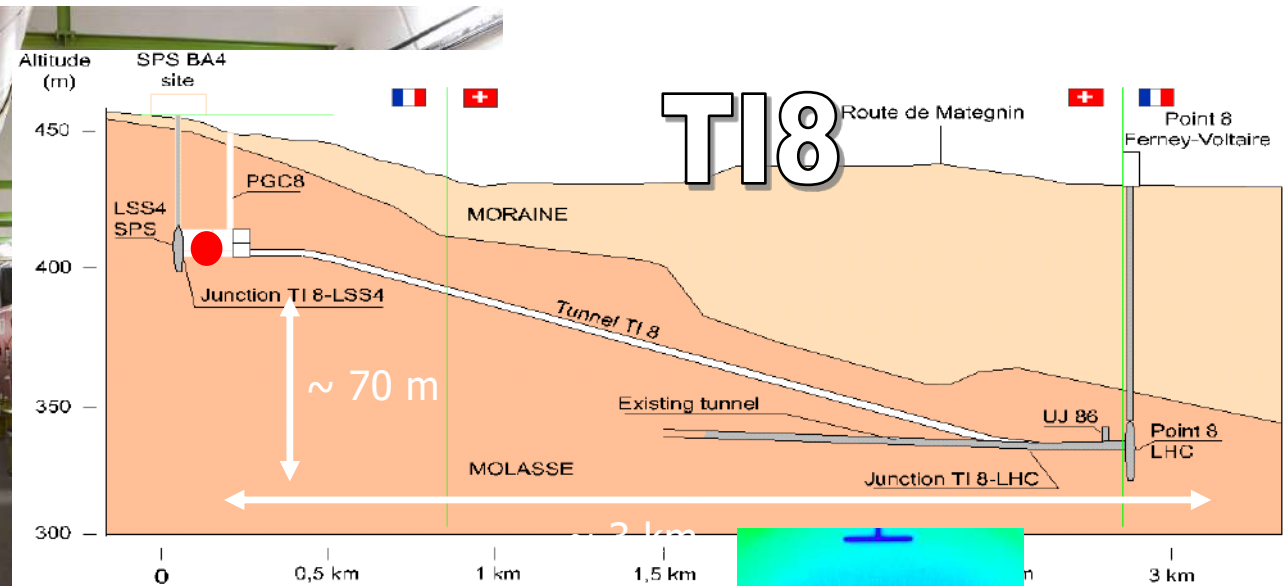
Straight Sections and Insertions



Injection and extraction

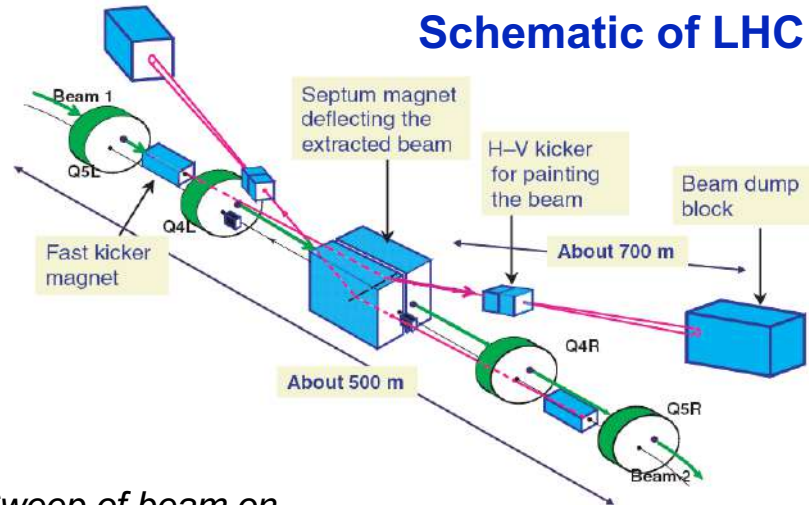


Injection of Beam 2 into LHC

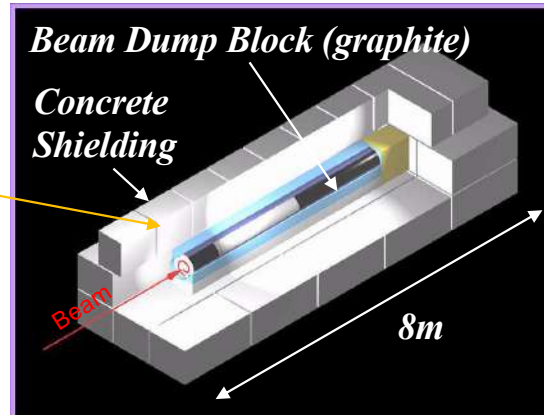
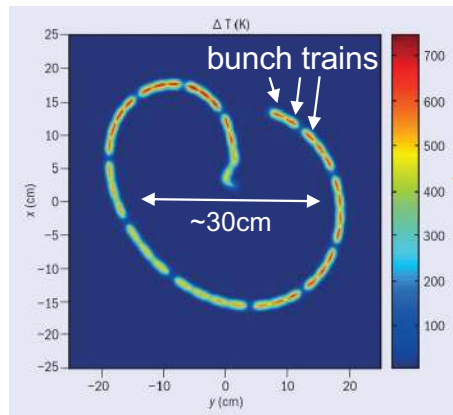


Beam dump – How to safely kill the LHC beam

Schematic of LHC beam dump system



Sweep of beam on
beam dump window



LHC beam stores ~360MJ energy.



Let's compare it to the kinetic energy of a frecciarossa train whose mass is **500 ton**

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 3.6 \cdot 10^8}{5 \cdot 10^5}} =$$

$$= 37.9 \frac{m}{s} \cong 140 \frac{km}{h}$$

Better be careful

acceleration

RF Acceleration and magnet field increase

What about the magnetic field during acceleration?

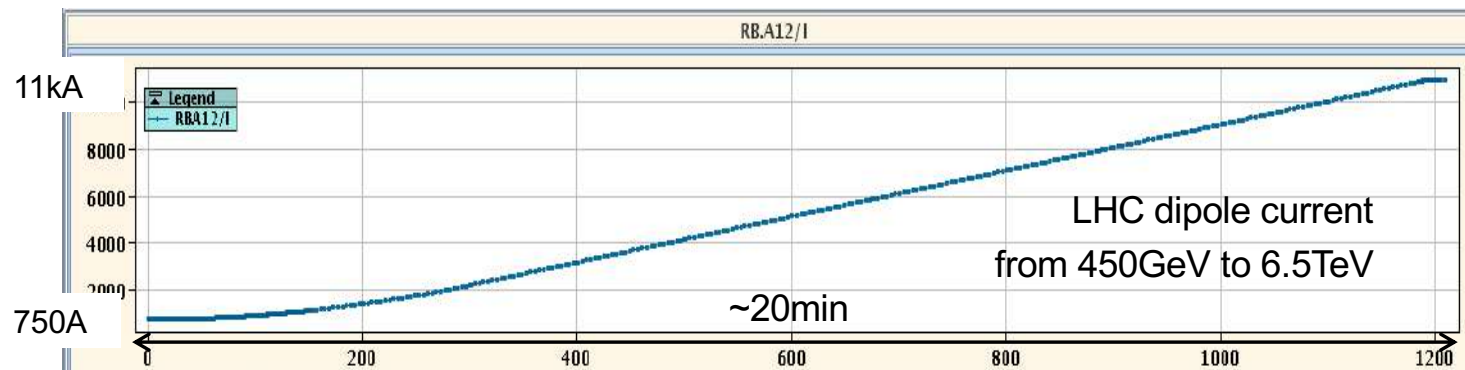
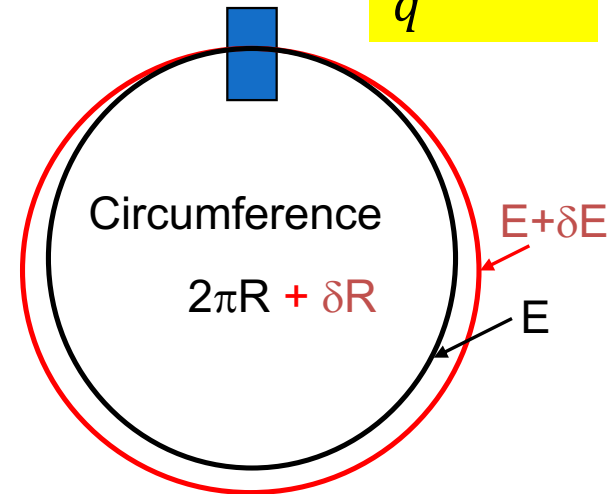
cavity

$$\frac{p}{q} = B \rho$$

Beam rigidity needs to be increased proportionally to increasing energy.

→ Machine radius is constant.

→ Need to increase dipole field accordingly!



Acceleration without magnetic field increase

LHC magnetic dipole field at 450 GeV:

$$B = \frac{p}{q\rho} = \frac{450 \text{ GeV}/c}{e \times 2803 \text{ m}} = 0.535 \text{ T}$$

Required bending radius at 7 TeV with $B_{\text{inj}}=0.5\text{T}$:

$$\rho = \frac{p}{qB} = \frac{7 \text{ TeV}/c}{e \times 0.535 \text{ T}} = 43.6 \text{ km}$$

Equivalent to **270km circumference**

(pure dipole field! without any insertions or quadrupoles)

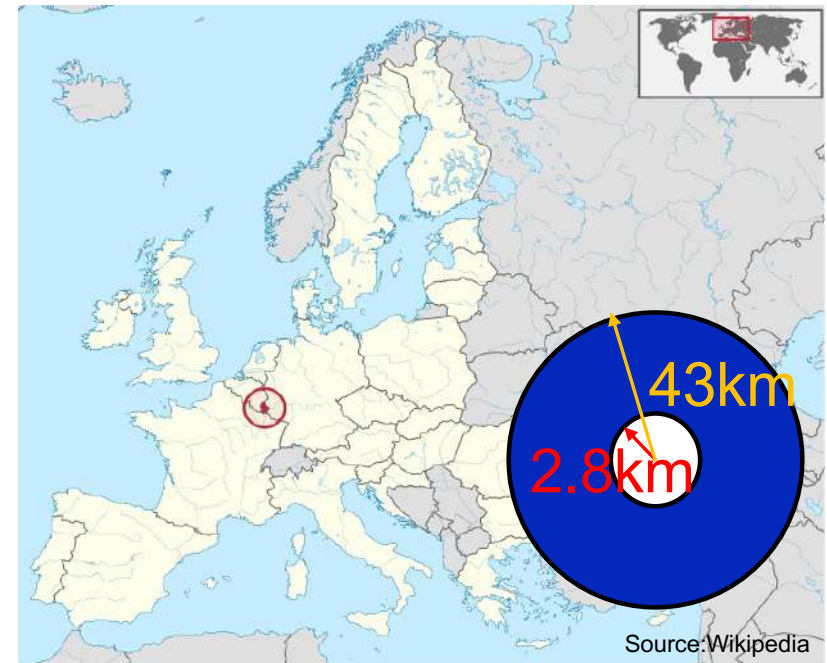
Magnet surface = 5800km^2

→ Area of Brunei (South-Eastern Asia)

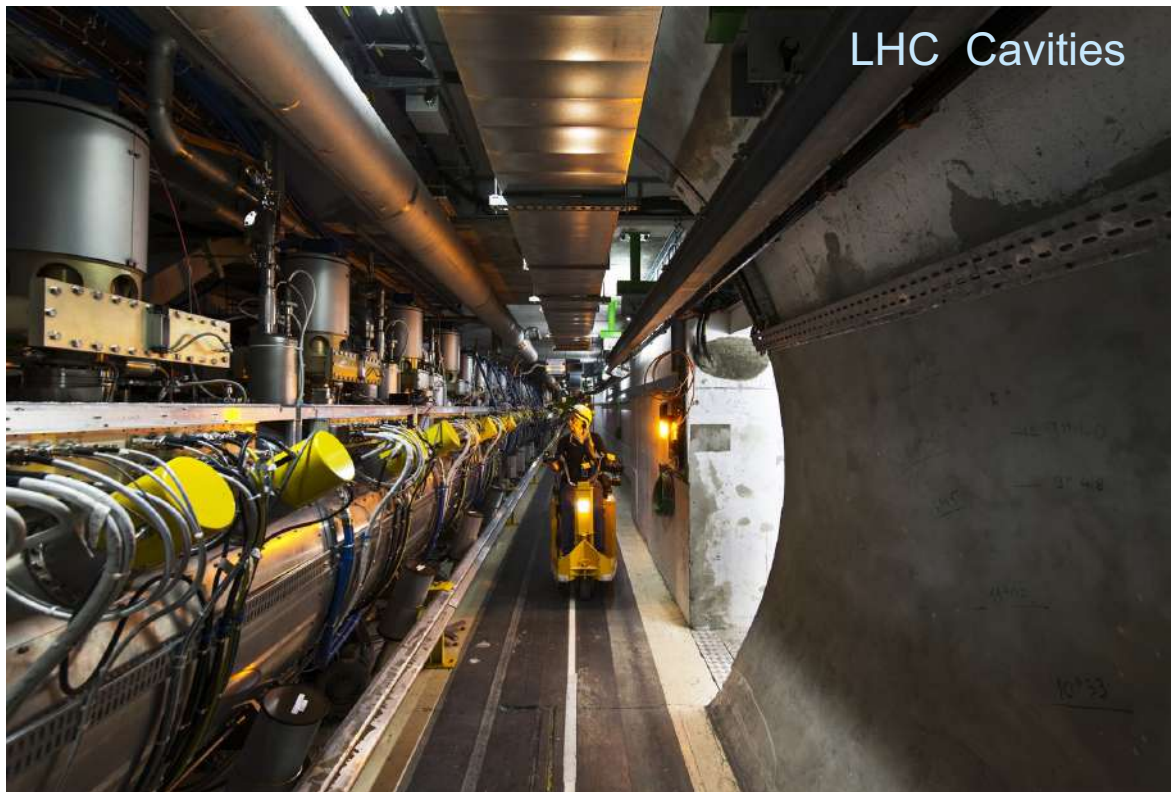
→ **Area of 2x Luxemburg**

How does the bending radius changes, when accelerating without adjusting the magnetic field?

$$\frac{p}{q} = B \rho$$



Example: LHC accelerating system



LHC has

- 8 superconducting cavities per beam
- Accelerating field 5 MV/m
- Can deliver 2 MV/cavity (peak voltage)
- Operating at 400 MHz
- Beam aperture (radius) $\sim 30\text{cm}$
- Energy gain/turn during ramp 485 keV (11245 turns/s)

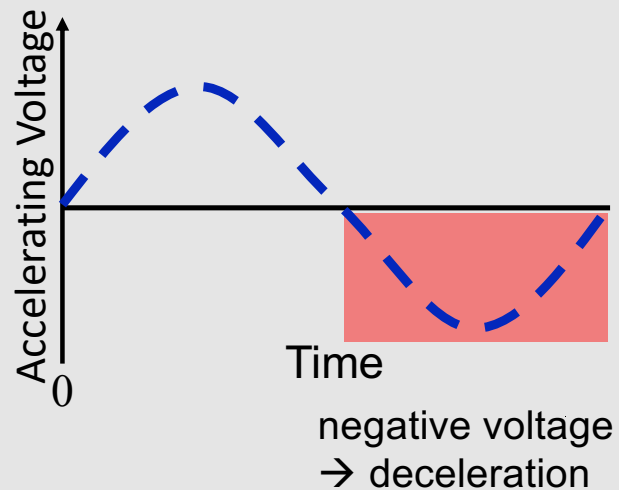
Going from 450 GeV (injection energy) up to 6.8 TeV (collision energy) takes about 20 minutes.

RF acceleration

Accelerating voltage is changing with time. That has two consequences:

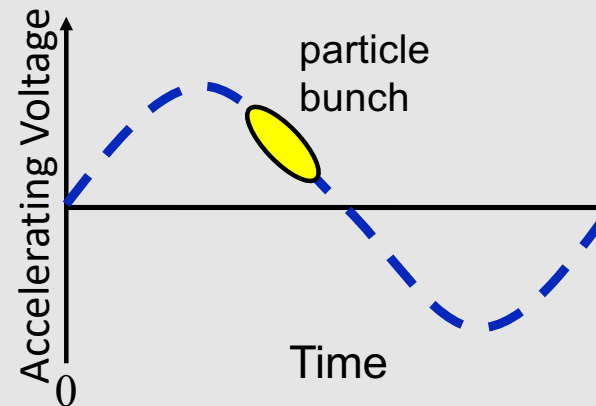
Need **synchronization** between beam and RF phase to gain energy.

There is a **synchronous RF phase** for which the energy gain fits the increase of the magnetic field.



Not all particles see the same voltage, because they arrive at different times.

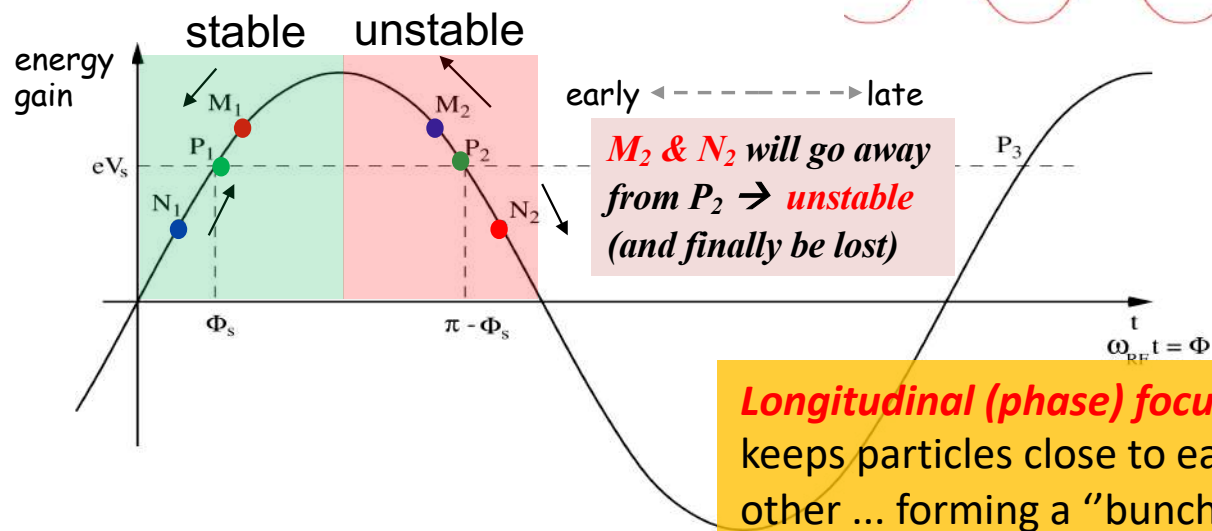
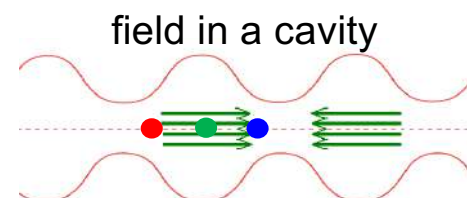
Not all particles gain the same energy.



Phase stability (non-relativistic regime)

Assume the situation where **energy increase is transferred into a velocity increase** (non-relativistic regime).

Particles P_1 , P_2 have the synchronous phase.



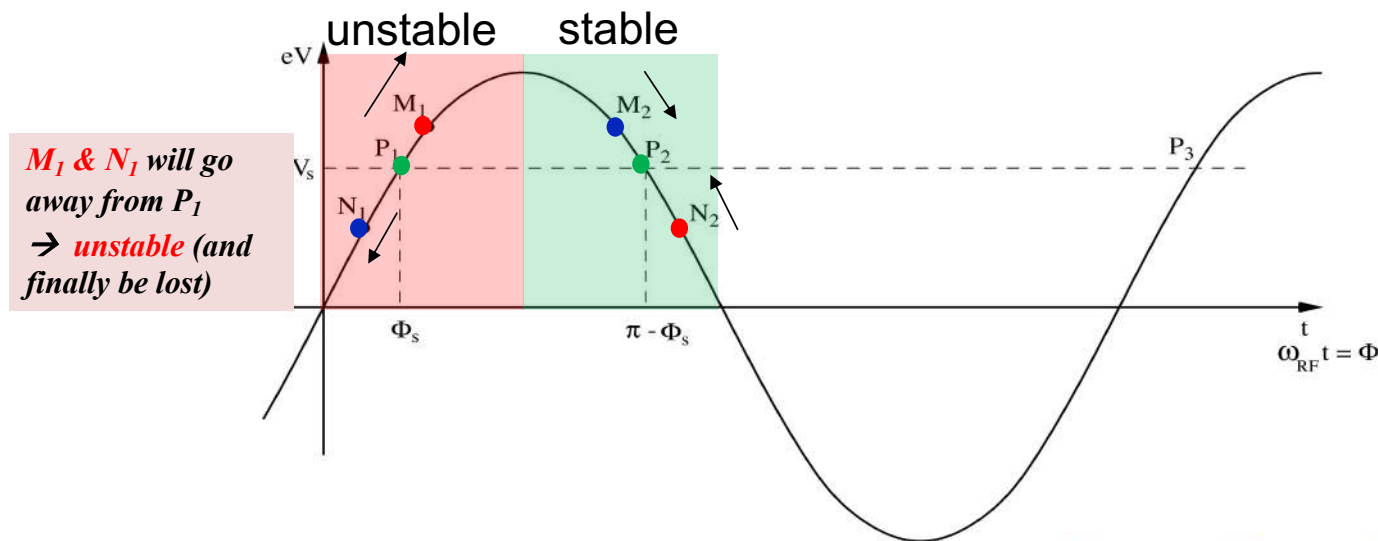
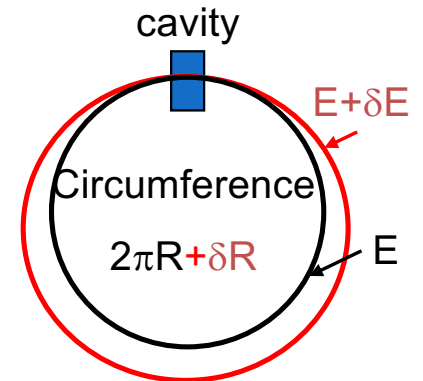
- Ideal particle
- Particle with $\Delta t < 0$ (early) $\rightarrow$ lower energy gain $\rightarrow$ gets slower
- Particle with $\Delta t > 0$ (late) $\rightarrow$ higher energy gain $\rightarrow$ gets faster
- $\rightarrow M_1$ & N_1 will move towards $P_1 \rightarrow$ **stable**

Phase stability (relativistic regime)

Now assume relativistic energies ($v \approx c$):

An **increase in momentum** transforms into a **longer orbit and thus a longer revolution time**.

$$\frac{p}{q} = B \rho$$



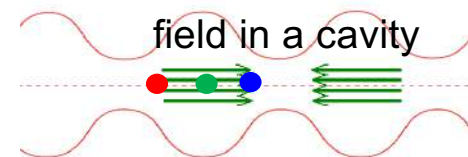
M_1 & N_1 will go away from P_1
 $\rightarrow$ **unstable** (and finally be lost)

● Ideal particle

● Particle with $\Delta t < 0 \rightarrow$ higher energy gain $\rightarrow$ gets longer orbit

● Particle with $\Delta t > 0 \rightarrow$ lower energy gain $\rightarrow$ gets shorter orbit

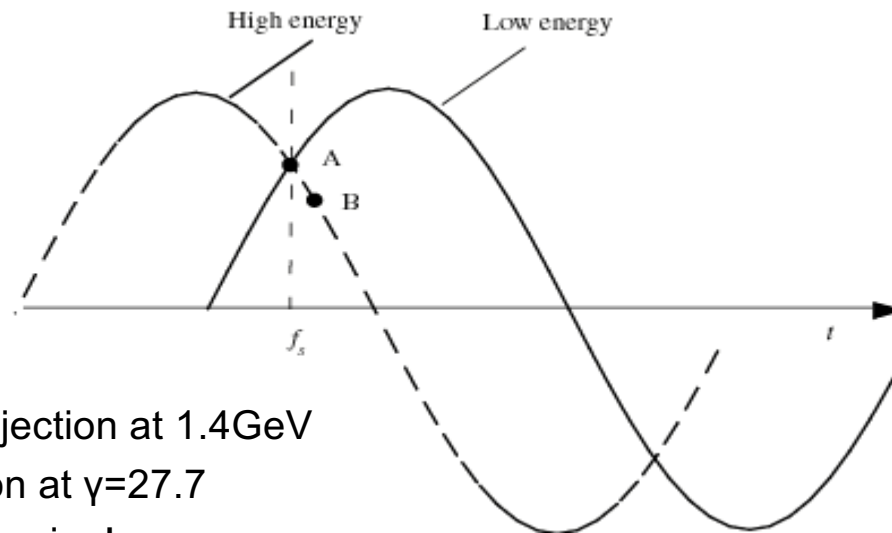
M_2 & N_2 will move towards $P_2 \rightarrow$ **stable**



Crossing transition

The previously stable synchronous phase becomes unstable when $v \Rightarrow c$ and the gain in path length overtakes the gain in velocity $\rightarrow$ **Transition**

Transition from one slope to the other during acceleration $\rightarrow$ **Crossing Transition**.
The RF system needs to make a rapid change of the RF phase, a 'phase jump'.



In the PS: γ_t is at ~ 6 GeV, injection at 1.4 GeV

In the SPS: $\gamma_t = 22.8$, injection at $\gamma = 27.7$

$\Rightarrow$ no transition crossing!

In the LHC: γ_t is at ~ 55 GeV, also far below injection energy

Transition crossing not needed in leptons machines, why?

Synchrotron Oscillation

Like in the transverse plane the particles are oscillating in longitudinal space.

Particles keep *oscillating around the stable synchronous particle* varying phase and dp/p .

Typically one synchrotron oscillation takes many turns (much slower than betatron oscillation)

Phase-space ellipse defines *longitudinal emittance*.

Separatrix is the trajectory separating stable and unstable motion.

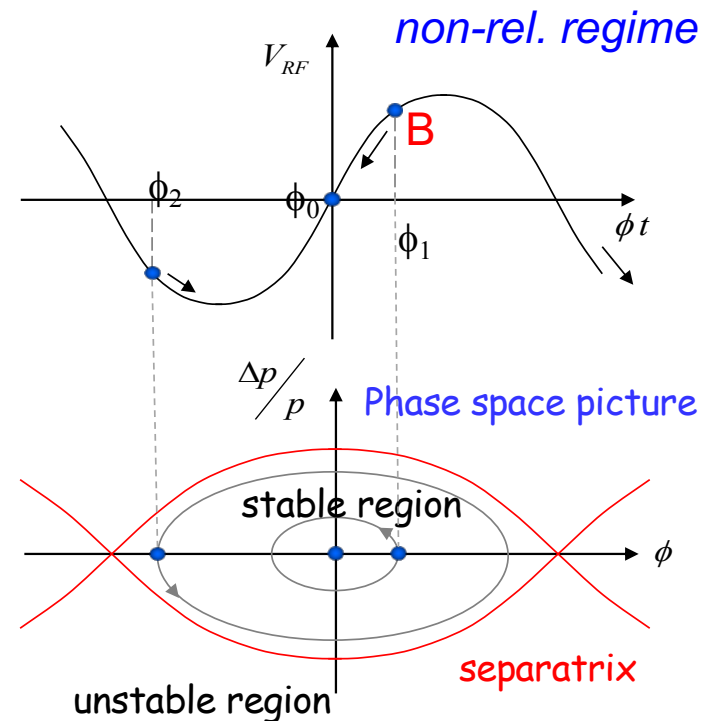
Stable region is also called **bucket**.

→ Harmonic number h = number of buckets:

$$f_{RF} = h f_{rev}$$

Simple case (no accel.): $B = \text{const.}$

- Stable phase: $\phi_0 = 0$
- Particle B oscillates around ϕ_0 .



Emittance during Acceleration

What happens to the emittance if the reference momentum P_0 changes?

Can write down transfer matrix for reference momentum change:

$$M_x = \begin{pmatrix} 1 & 0 \\ 0 & P_0/P_1 \end{pmatrix} \longrightarrow \epsilon_{x1} = \frac{P_0}{P_1} \epsilon_{x0}$$

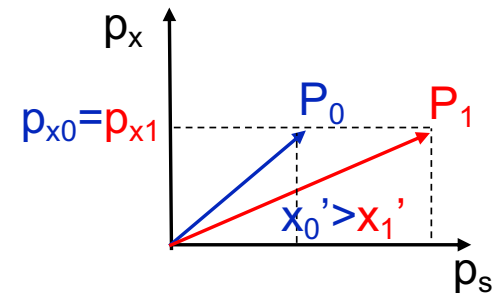
The emittance shrinks with acceleration!

With $P = \beta\gamma mc$ where γ, β are the relativistic parameters.

The conserved quantity is

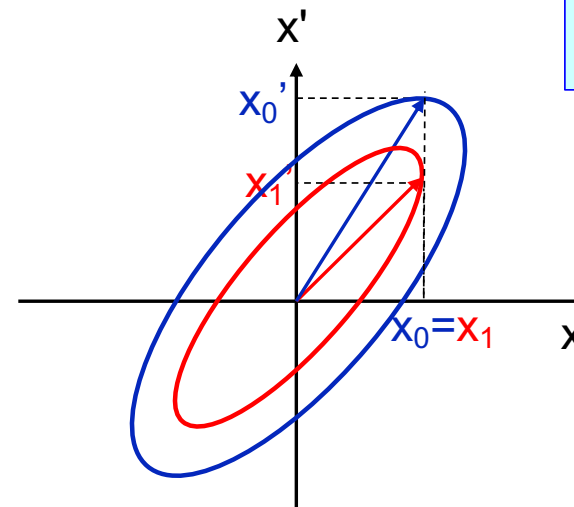
$$\beta_1 \gamma_1 \epsilon_{x1} = \beta_0 \gamma_0 \epsilon_{x0}$$

It is called **normalized emittance**.



Only longitudinal momentum changes during the acceleration

$$X'_0 \approx \sin\theta = \frac{P_{x0}}{P_0}$$



How big are the beams in the LHC?

Normalized emittance at LHC : $\varepsilon_n = 3.5 \mu\text{m}$

→ ε_n preserved during acceleration.

The **geometric emittance**:

- Injection energy of 450 GeV: $\varepsilon = 7.3 \text{ nm}$
- Top energy of 7 TeV: $\varepsilon = 0.5 \text{ nm}$

$$\varepsilon_{7\text{TeV}} = \varepsilon_{450\text{GeV}} \frac{\gamma_{450\text{GeV}}}{\gamma_{7\text{TeV}}}$$

The corresponding max. **beam sizes** in the arc,
at the location with the maximum beta function ($\beta_{\text{max}} = 180 \text{ m}$):

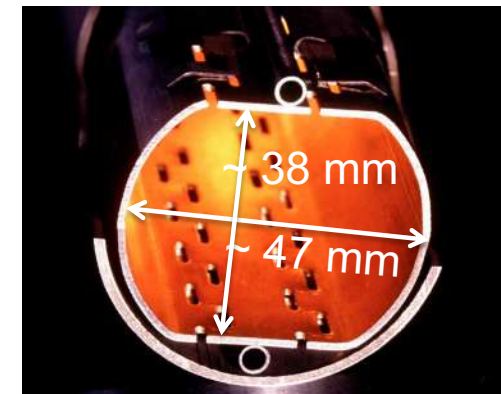
- $\sigma_{450\text{GeV}} = 1.1 \text{ mm}$
- $\sigma_{7\text{TeV}} = 300 \mu\text{m}$

Aperture requirement: $a > 10 \sigma$

LHC beam pipe radius:

- Vertical plane: $19 \text{ mm} \sim 17 \sigma @ 450 \text{ GeV}$
- Horizontal plane: $23 \text{ mm} \sim 20 \sigma @ 450 \text{ GeV}$

$$\sigma = \sqrt{\varepsilon \beta}$$

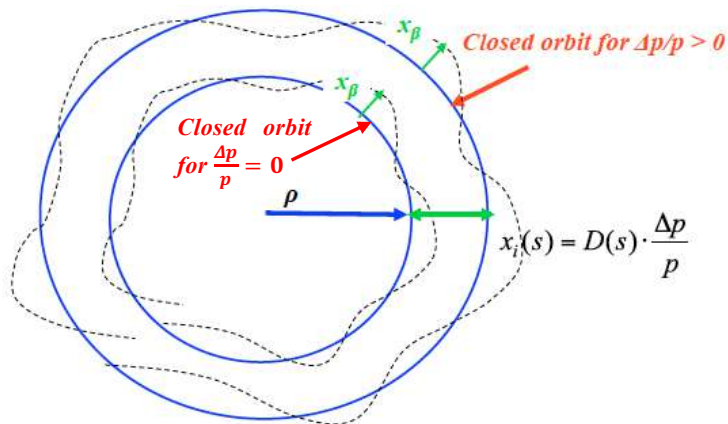
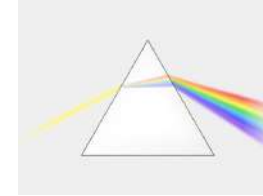


Transverse-Longitudinal Coupling: Dispersion

Dipole magnets generate dispersion:

→ Particles with different momentum are bent differently.

Due to the momentum spread in the beam $\frac{\Delta p}{p}$, this has to be taken into account for the particle trajectory.



$$x(s) = x_{\beta}(s) + D(s) \frac{\Delta p}{p}$$

Dispersion function $D(s)$

corresponds to the trajectory of a particle with momentum offset

$$\frac{\Delta p}{p} = 1.$$

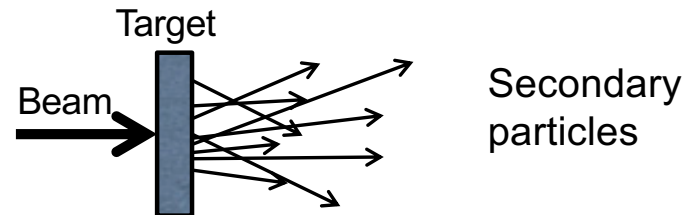
This also has an effect on the beam size:

$$\sigma = \sqrt{\beta \epsilon} \quad \longrightarrow \quad \sigma = \sqrt{\beta \epsilon + D^2 \left(\frac{\Delta p}{p} \right)^2}$$

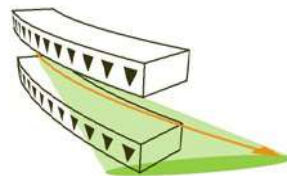
Experiments and Luminosity

Each accelerator and experiment requires specific beam properties.
Fundamentally different are:

Fixed Target:

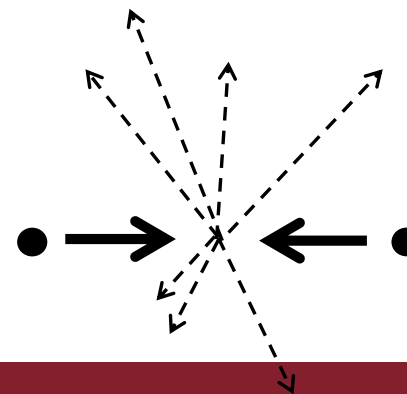
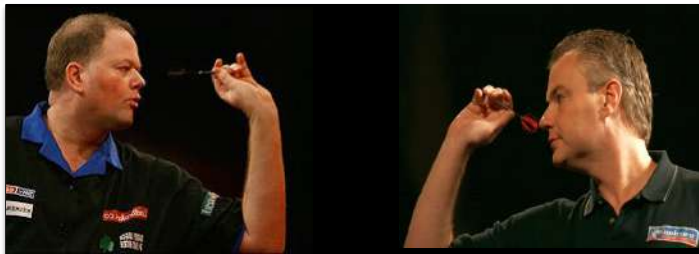


Light Sources:



Particles that are bent to a circular orbit emit energy/light.

Collider:



“Smashing” Modes and Center-of-Mass Energy

The **center-of-mass energy** defines the upper limit of the newly created particle’s mass.

Fixed Target



$$E \propto \sqrt{E_{beam}}$$

Most of the Energy is lost in the target, only a fraction is transformed into useful secondary particles.

Collider

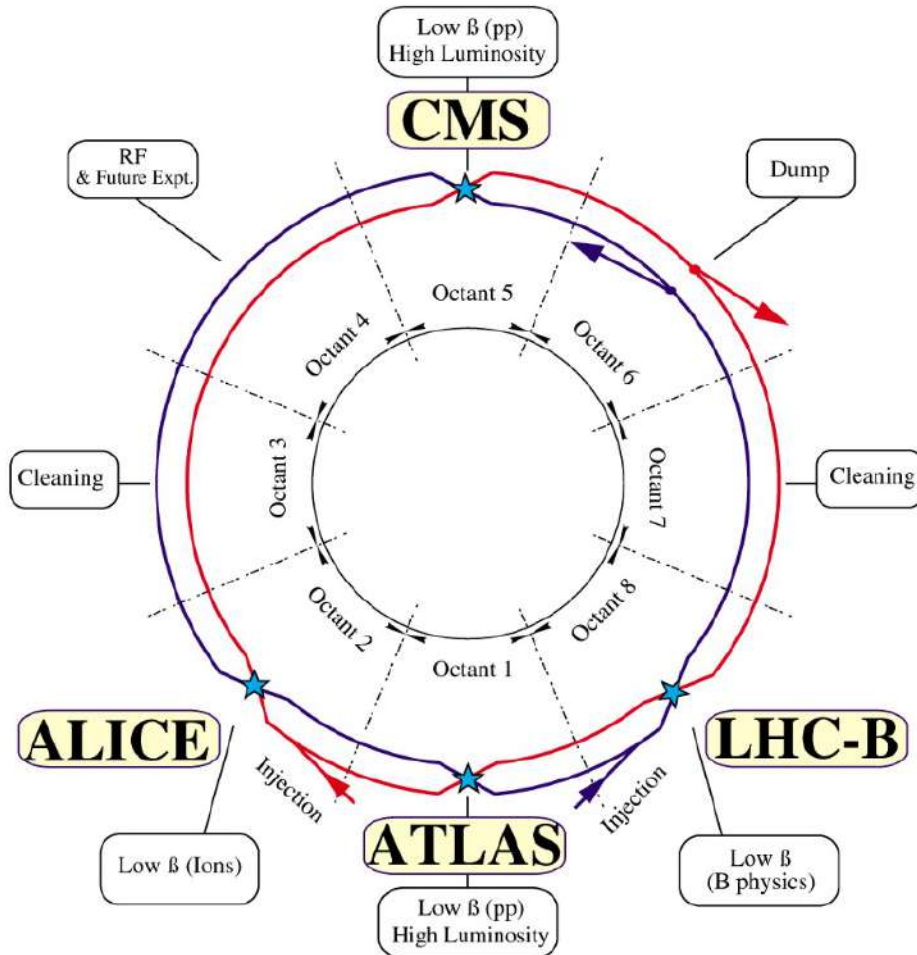


$$E = E_{beam1} + E_{beam2}$$

All energy is available for the production of new particles.

Price to pay in a collider: event rate

LHC and its Experiments



LHC has **4 interaction points (IPs)** hosting particle physics experiments:

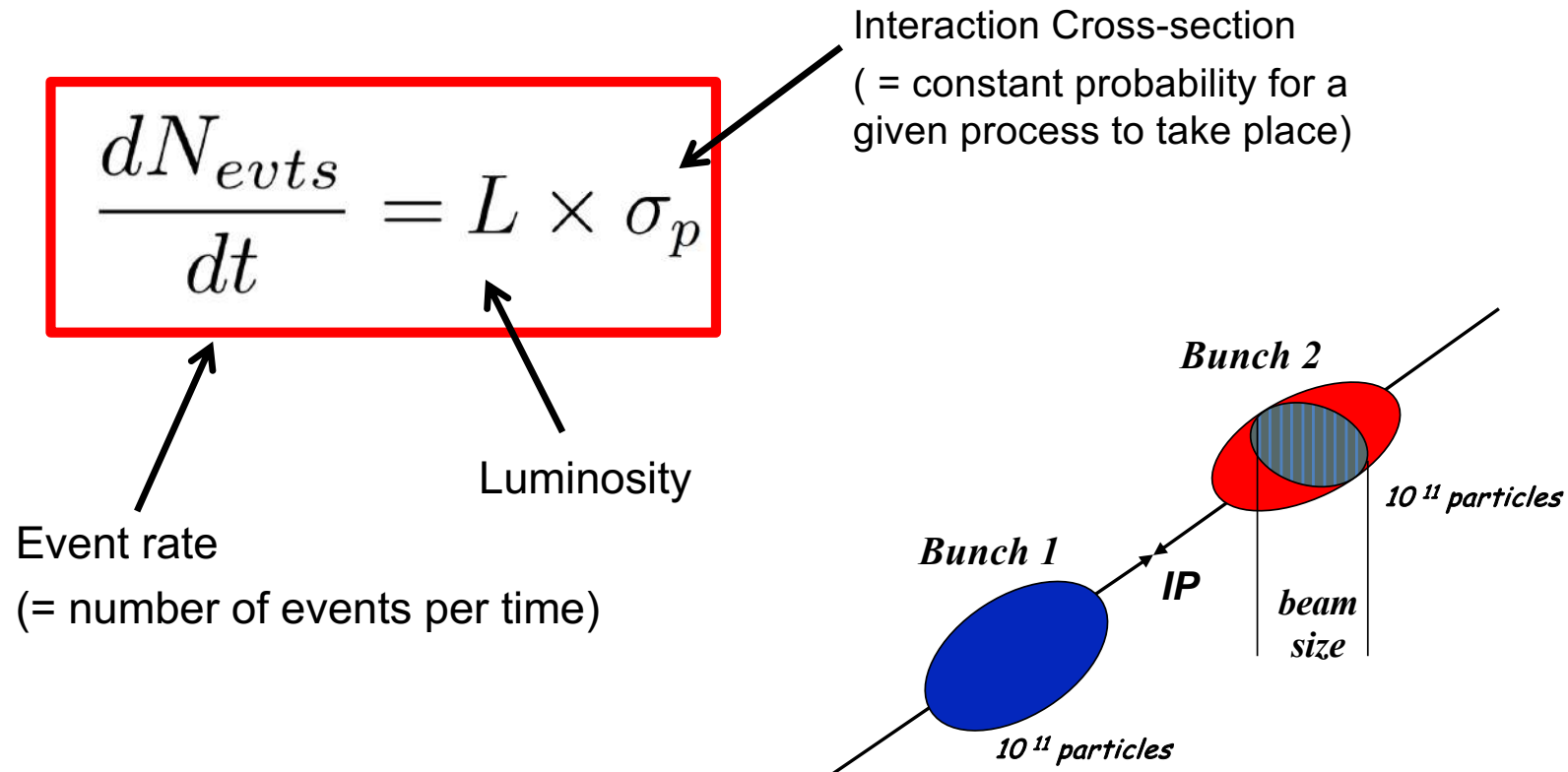
→ ATLAS, ALICE, CMS, LHCb

Therefore the two counterrotating **beams collide 4 times per turn**

When they collide the outer beam **cross over** to the inner circle and vice versa.

Particle Collisions

Experiments are interested in maximum number of interactions per second.
The event rate in an experiment is proportional to the collider luminosity.



“quality factor” of a Collider

The most important factor to describe the potential of a collider is the
Luminosity.

$$L = \frac{k N^2 f \gamma}{4 \pi \beta^* \epsilon} \cdot F$$

Defined by
the injectors

N..... No. particles per bunch
k..... No. bunches
f..... revolution freq.
g..... rel. gamma
 β^* beta-function at IPs
 ϵ norm. trans. emit

Limitation:

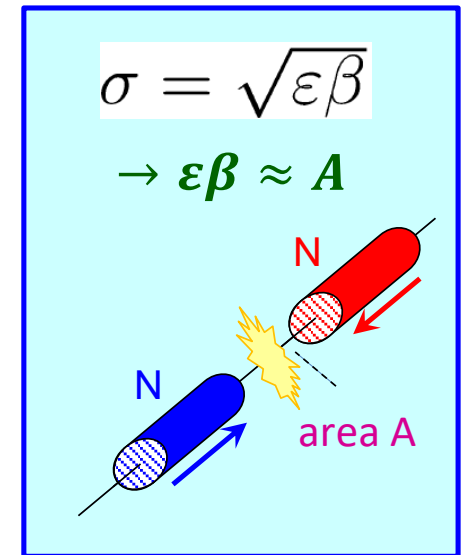
“Collective effects”
cause beam
instabilities for too high
bunch intensities, too
small bunch spacing,
too “bright” beams.

Overall Goal of an Collider: Maximizing Luminosity!

- Many particles (N, k)
- In a small transverse cross-section (ϵ , β)

Performance depends on the injectors:

- Production of large N and small ϵ
- Preservation of these parameters until collisions.



Optimizing Luminosity

Bunch properties (N & ϵ) are defined in the injectors.

But what can be done in the Collider?

$$L = \frac{k N^2 f \gamma}{4 \pi \beta^* \epsilon} F$$

N..... No. particles per bunch

k..... No. bunches

f..... revolution freq.

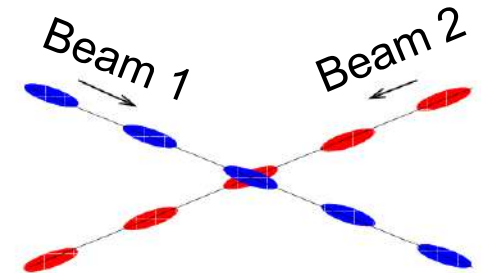
γ rel. gamma

β^* beta-function at IPs

ϵ norm. trans. emit

f_{rev}, γ : defined by the design of the accelerator

$F [0,1]$: When colliding with many bunches, a **crossing angle is needed** to avoid unwanted collisions. However this **reduces the beam overlap** and therefore the luminosity. Keep as *small as possible!* (at LHC ~0.8)
→ Limited by beam-beam effects.

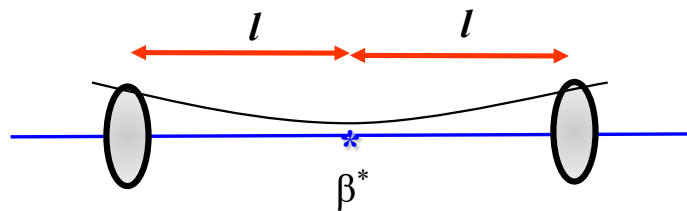


k : Optimize filling scheme and bunch spacing.

β^* : Can be optimized by focusing!

Mini-Beta Insertions

Mini-beta insertion is a **symmetric drift space** with a **waist of the β -function** in the center of the insertion.



$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$

On each side of the symmetry point a quadrupole **doublet** or **triplet** is used to generate the waist.

They are not part of the regular lattice.

Collider experiments are located in **mini-beta insertions: smallest beam size possible** for the colliding beam to increase probability of collisions.

There is a price to pay: **The smaller β^* , the larger β at the triplet.**

Example: Mini-Beta Insertion at LHC

Example of the LHC
(design report values):

At the interaction point:

$$\beta^* = 0.55 \text{ m}$$

$$\sigma^* = 16 \text{ } \mu\text{m}$$

That's smaller than a hair's diameter!

At the triplet:

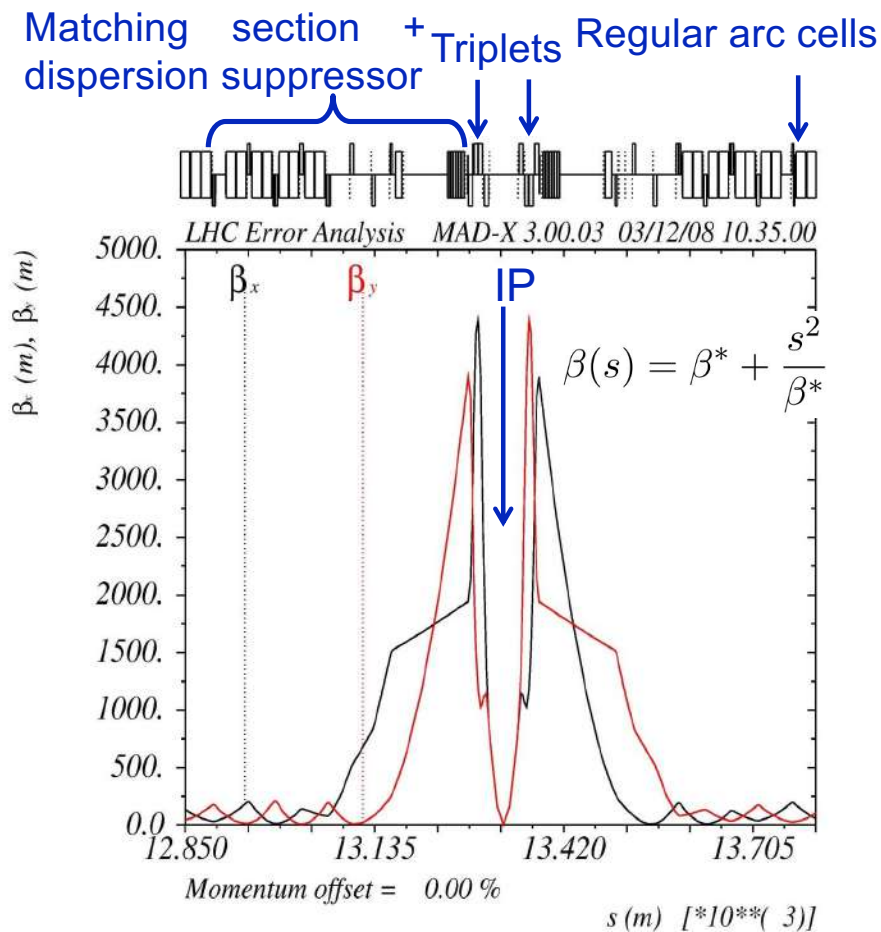
$$\beta = 4500 \text{ m}$$

$$\sigma = 1.5 \text{ mm} = 1500 \text{ } \mu\text{m}$$

Largest beams size in the lattice!

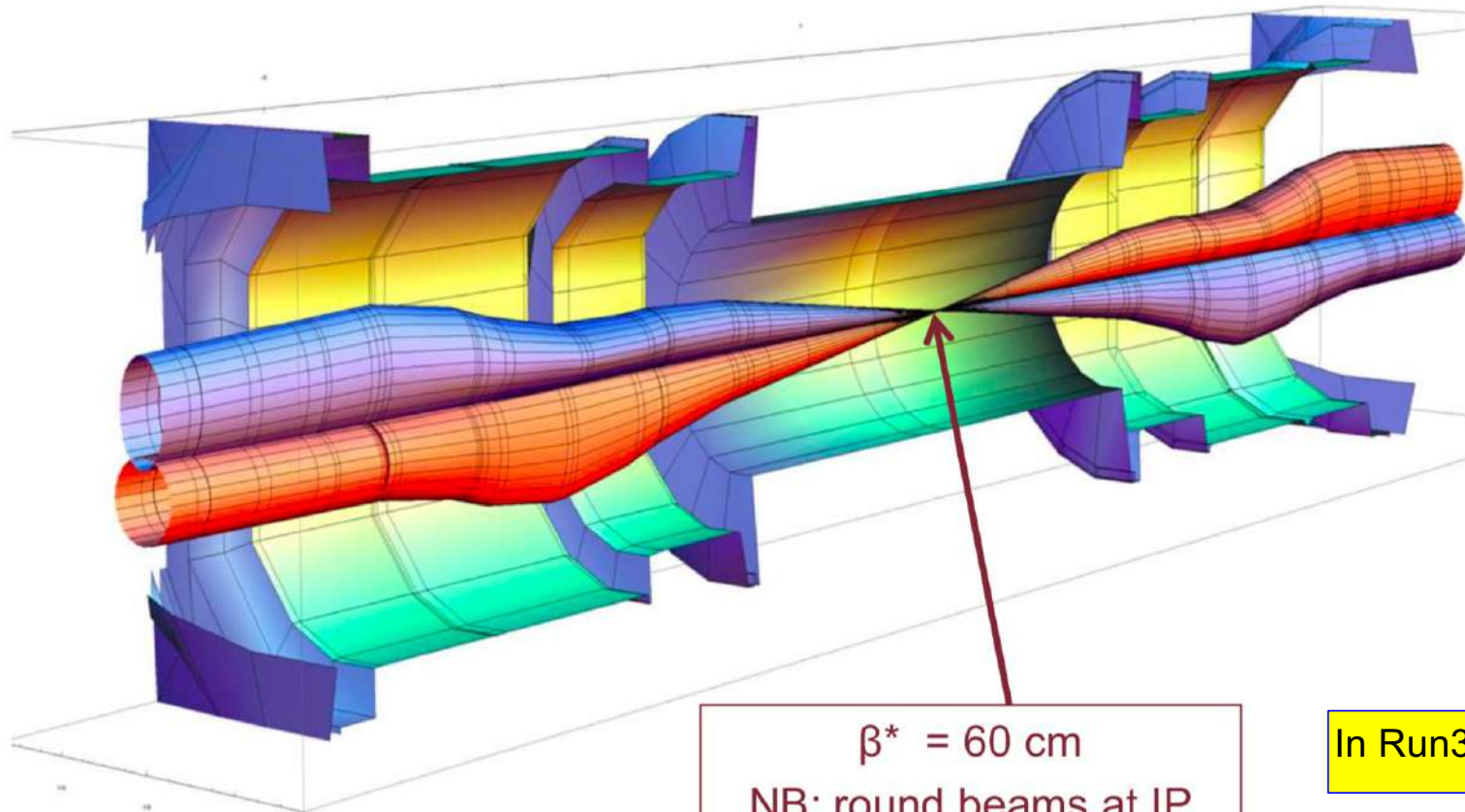
Limitations:

- Tighter tolerances on field errors
- Triplet aperture limits β^* together with crossing angle.



Luminosity: beta squeeze

Image courtesy John Jowett

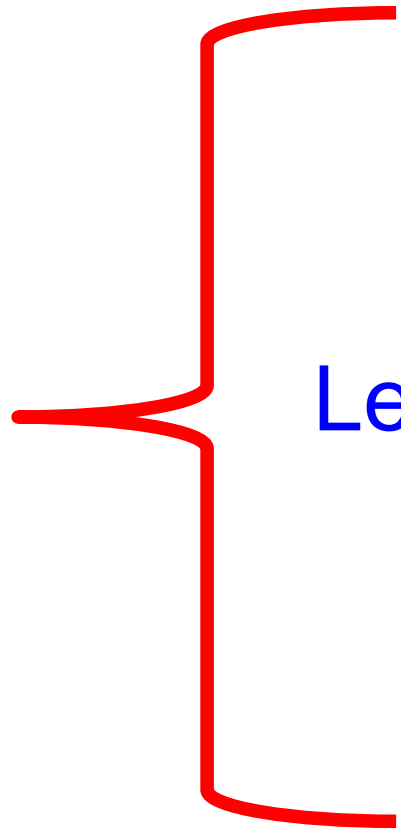


$\beta^* = 60 \text{ cm}$
NB: round beams at IP

In Run3 $\beta^* = 30 \text{ cm}$

$\beta_{\text{max}} \sim 4.5 \text{ km}$

ATLAS @ LHC



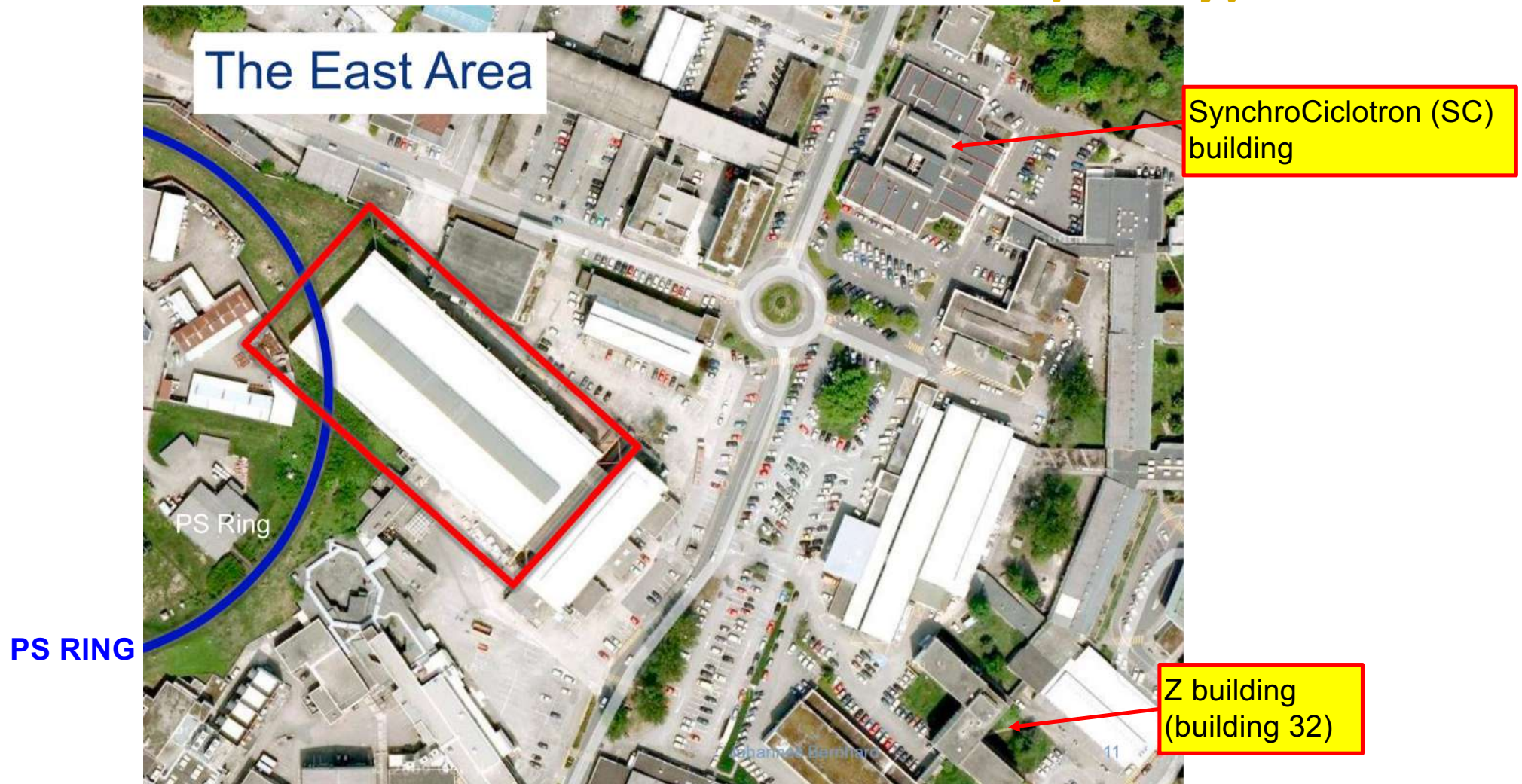
Let's open a parenthesis

(it is not part of the exam program)

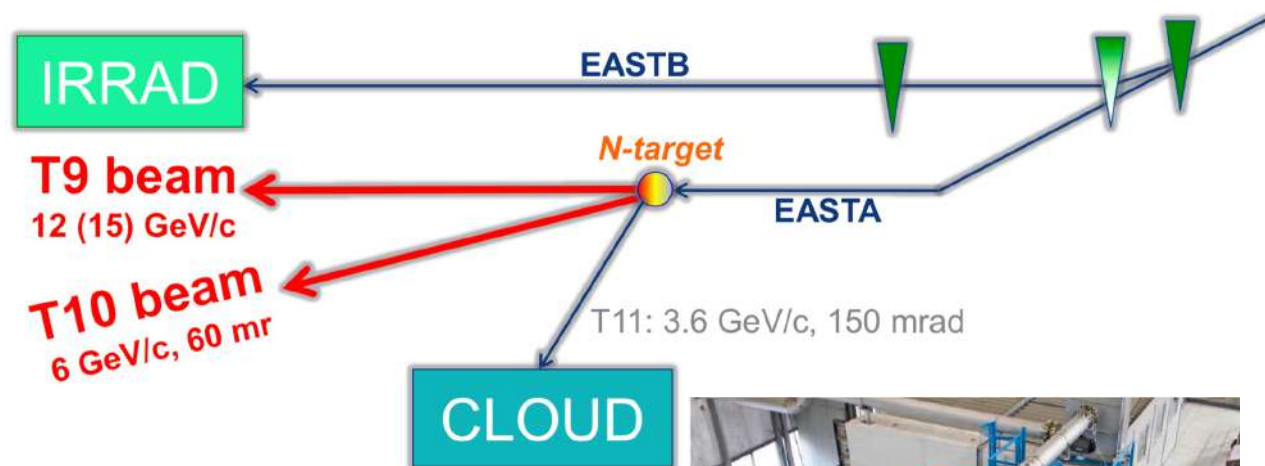
Beam lines

(It is not in the exam program but it will help us to better understand the problem with the antiprotons in the SppS collider)

Beam lines in the PS East area (today)



Beam lines in the PS East area (today)



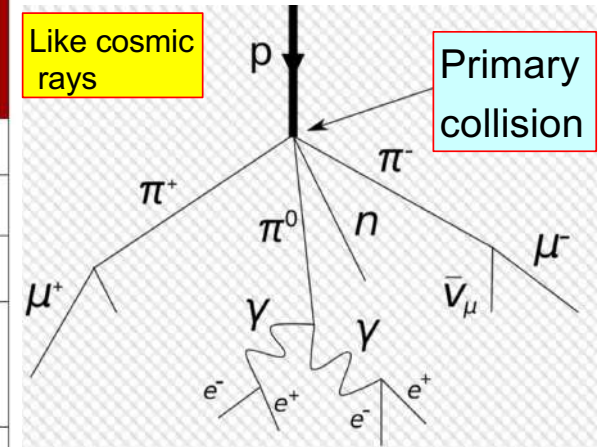
- Studies the influence of cosmic rays to cloud formation
- Cloud expansion chamber set-up with extensive instrumentation (mass spectrometers, particle counters, etc.)
- Uses PS beam as first and only particle beam experiment to study atmospheric and climate science
- Spectacular results achieved (several publications in Nature and Science)

Targets and particle production

		Name		Q	Mass	Mean life (τ)		cτ	Mean decay distance	Decays	
					[MeV/c ²]	[s]	[m]	[m/GeV/c]			
Leptons	Electron		e	±e	0.511	stable					
	Muon		μ	±e	105.6	2.2×10 ⁻⁶	659.6	6.3×10 ³	μ ⁺ → e ⁺ ν̄ _e ν _μ (100%)		
Hadrons	Mesons	Pion		π	±e	139.6	2.6×10 ⁻⁸	7.8	56.4	π ⁺ → μ ⁺ ν _μ (100%)	
		Kaon	K	±e	493.6	1.23×10 ⁻⁸	3.7	8.38	K ⁺ → μ ⁺ ν _μ (63%) π ⁰ e ⁺ ν _e (5%) π ⁰ μ ⁺ ν _μ (3%) π ⁺ π ⁰ (...) (28.9%)		
			K ⁰	0	497.6	K ⁰ _S	8.9×10 ⁻¹¹	0.02	0.060	K ⁰ _S → π ⁰ π ⁰ (30.7%) π ⁺ π ⁻ (69.2%)	
		K ⁰		0	497.6	K ⁰ _L	5.12×10 ⁻⁸	15.34	34.4	K ⁰ _L → π [±] e [∓] ν _e (40.5%) π [±] μ [∓] ν _μ (27.0%) 3π ⁰ (19.5%) π ⁺ π ⁻ π ⁰ (12.5%)	
			Proton			p	±e	938	stable		
		Baryons	Lambda		Λ	0	1115.6	2.63×10 ⁻¹⁰	0.079	0.237*	Λ ⁰ → p π ⁻ (63.9%)
	Sigma Hyperons		Σ ⁺	+e	1189.3	8.02×10 ⁻¹¹	0.024	0.068*	Σ ⁺ → p π ⁰ (51.57%)		
		Σ ⁻	-e	1197.4	1.48×10 ⁻¹⁰	0.044	0.125*	Σ ⁻ → n π ⁻ (99.84%)			

(*) for 10 GeV/c

$c\tau$ is computed for a 10 GeV/p momentum

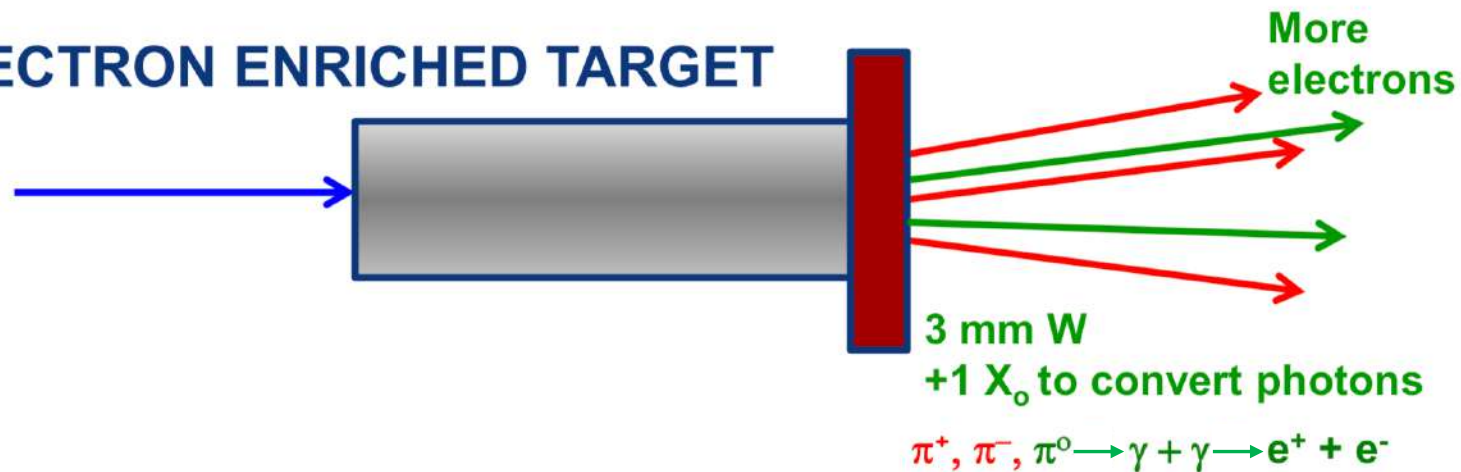


Targets and particle production

HADRON TARGET



ELECTRON ENRICHED TARGET

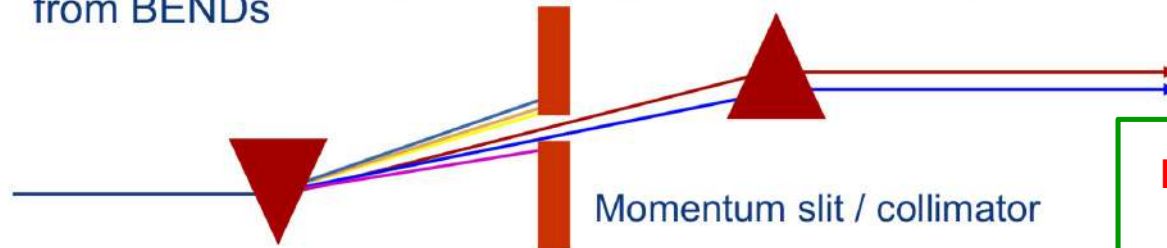


Secondary beam line - layout

Basic beam design

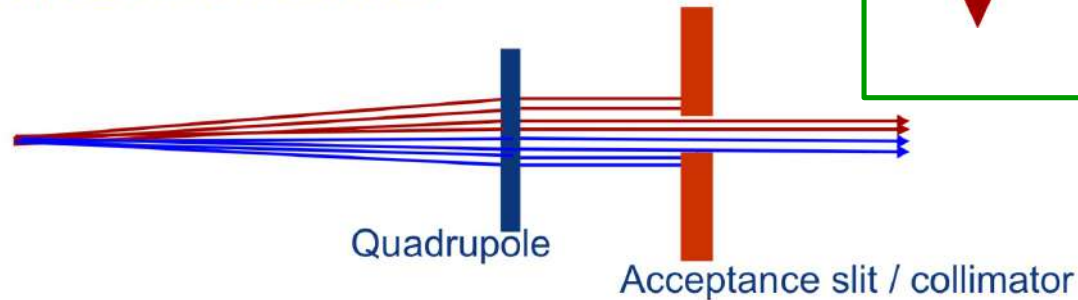
• Momentum selection and acceptance: collimators

- Select small momentum band in combination with dispersion from BENDs

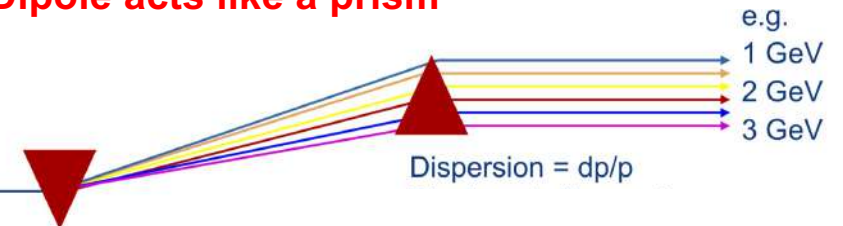


• Acceptance collimators

- Select beam size and beam rate

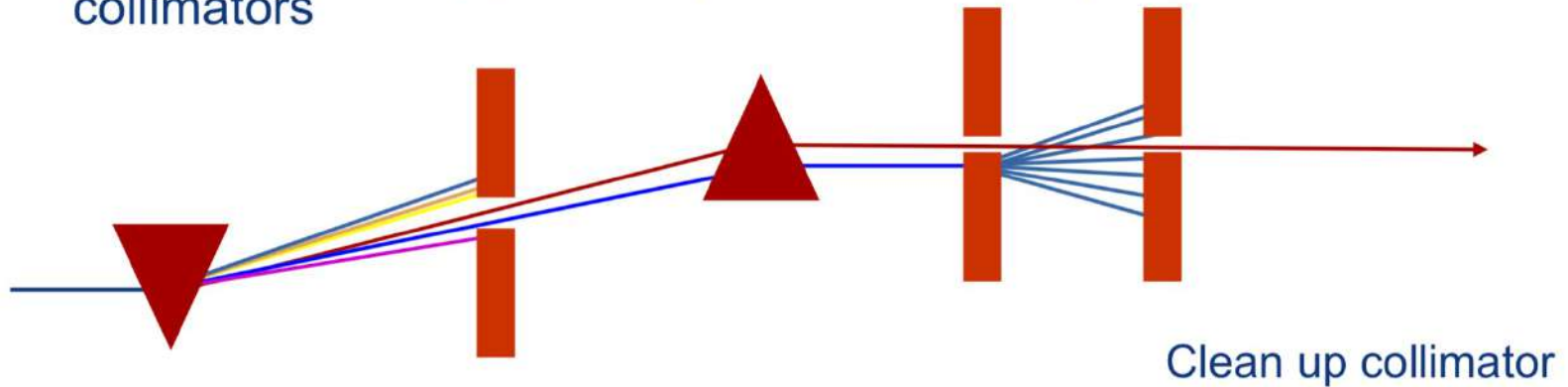


Dipole acts like a prism



Secondary beam line - layout

- Clean up collimators
 - Absorb secondary particles produced in acceptance collimators



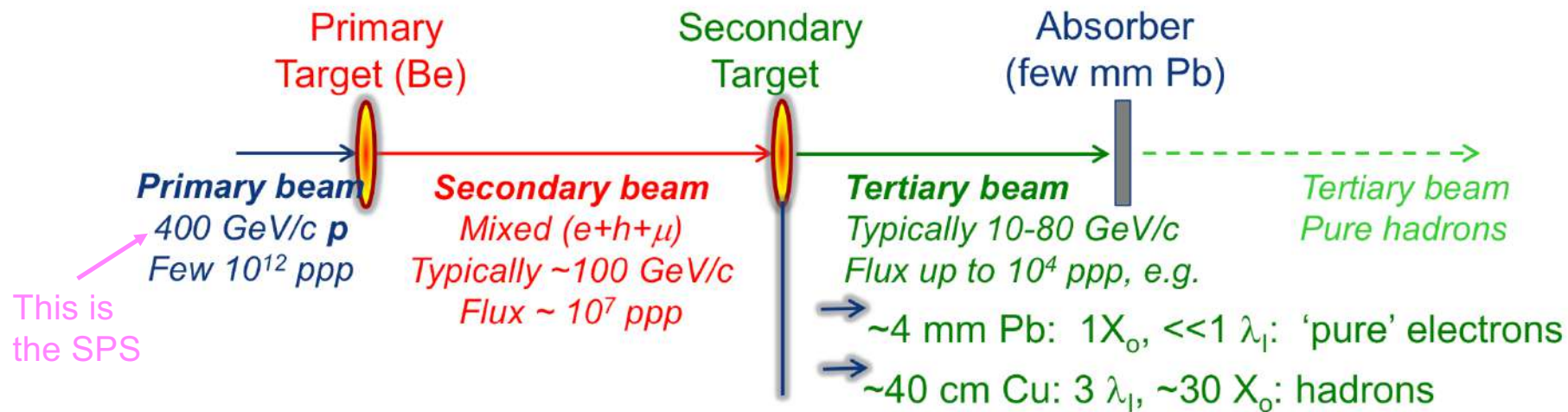
- TAX (Target attenuator)
 - Define initial acceptance of the beam line



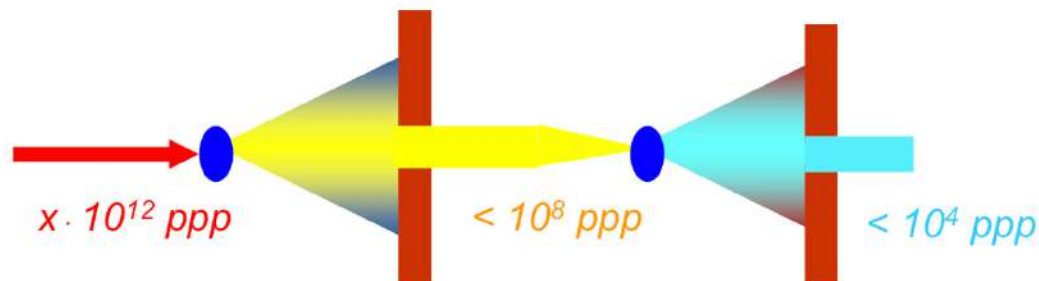
Secondary beam line - layout

Basic beam design

- Selection of particle types

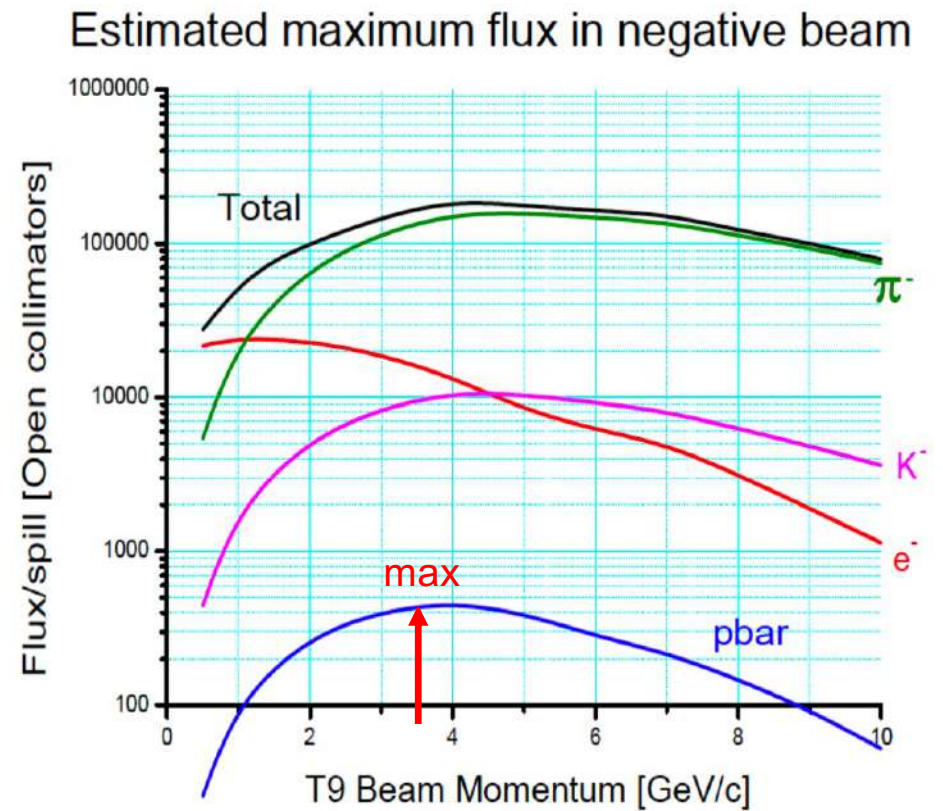
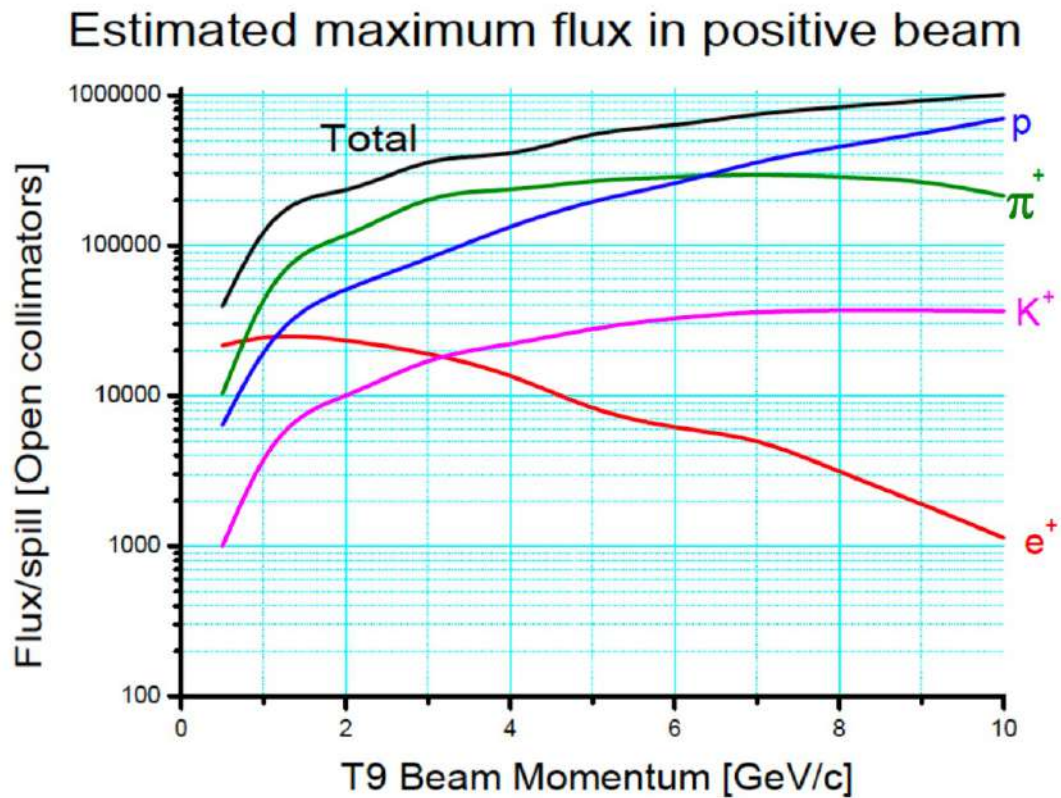


- Intensities

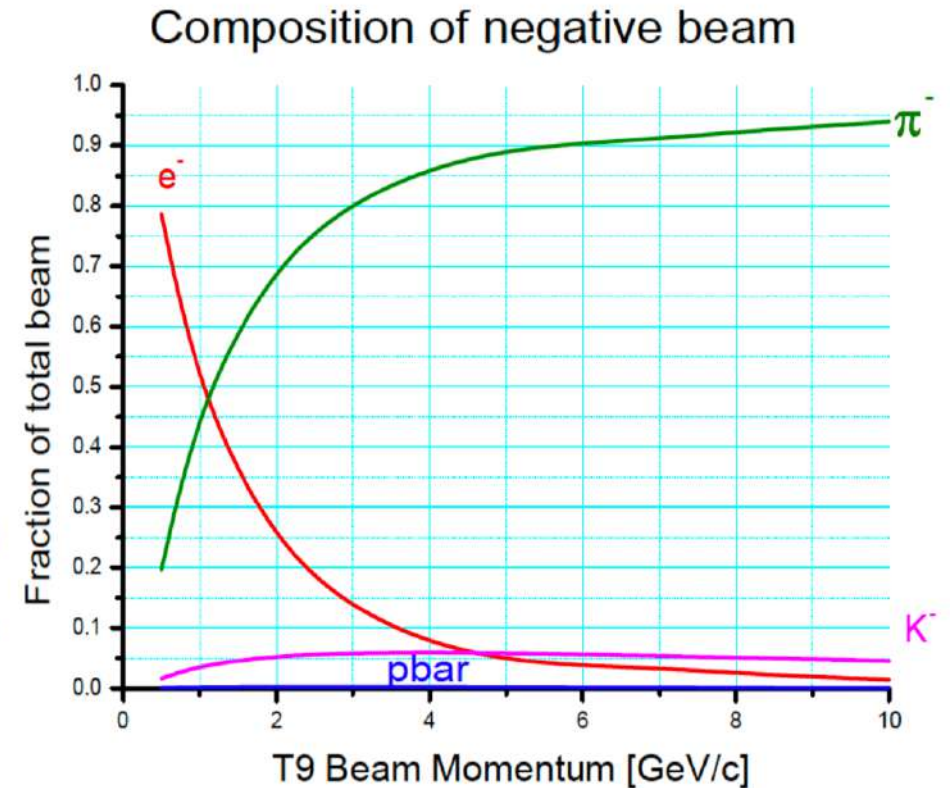
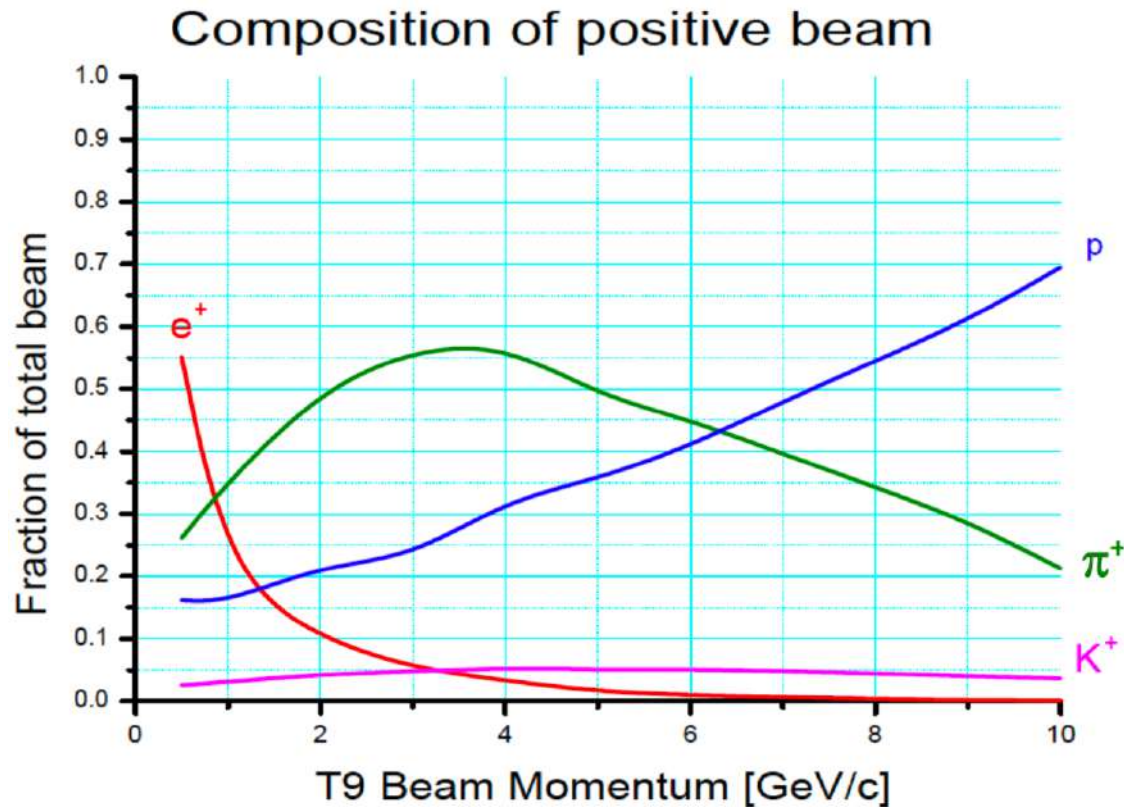


ppp = particles per pulse

PS east area, T9 line: beam rates

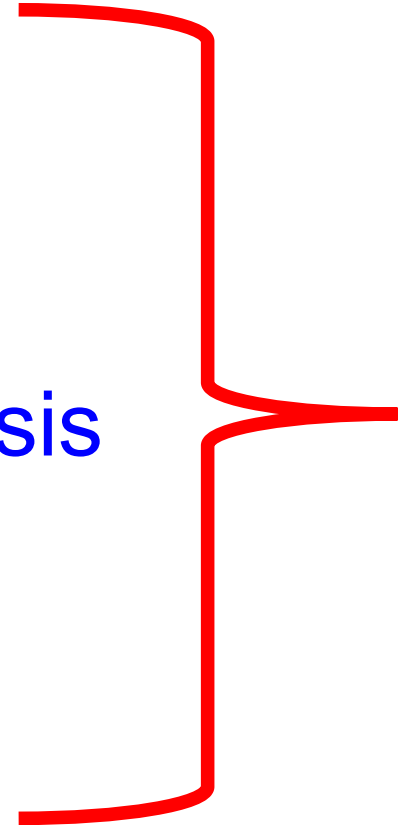


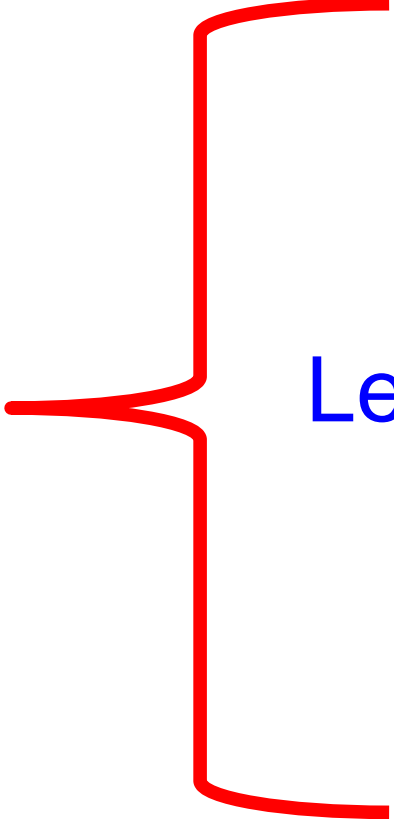
PS east area, T9 line: beam composition



Very very few antiprotons

Let's close the parenthesis





Let's open another parenthesis

(it is not part of the exam program)

What about the future?

A non exhaustive list of future projects

❑ e^+e^- linear colliders (1-2-3 TeV range):

- ILC (International Linear Collider) [Japan]
- CLIC (Cern Linear Collider) [CERN]

❑ e^+e^- circular colliders (90 – 365 GeV range):

- TLEP (Triple Large Electron Positron Collider) [CERN]
- CEPC (Circular Electron Positron Collider) [China]
- FCC-ee (Future Circular Collider – ee option) [CERN]

❑ Hadron Collider (100 TeV)

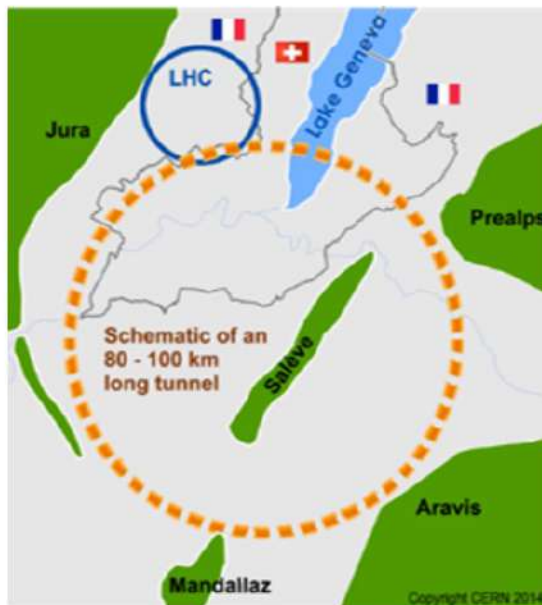
- FCC-hh (Future Circular Collider – hh option) [CERN]

❑ Muon Collider (10 TeV?) *(not yet technically feasible, but ... it could be the future)*

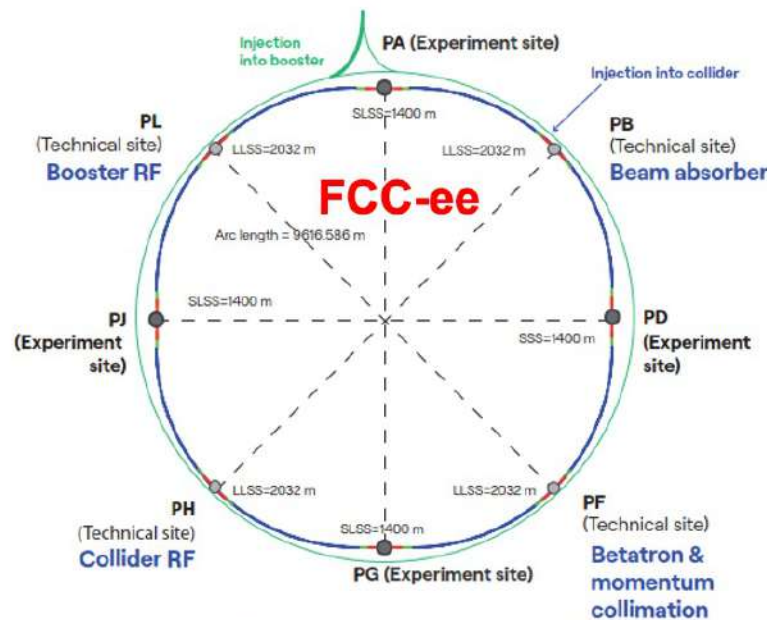
- IMCC (International Muon Collider Collaboration) [CERN]

FCC integrated program - scope

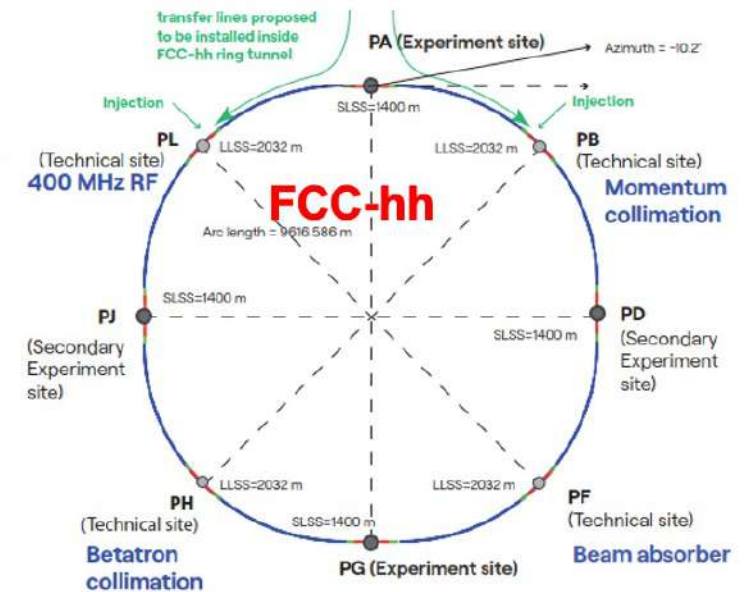
- stage 1: FCC-ee (Z, W, H, $t\bar{t}$) as Higgs factory, electroweak & top factory at highest luminosities
- stage 2: FCC-hh (~ 100 TeV) as natural continuation at energy frontier, pp & AA collisions; e-h option
- common civil engineering and technical infrastructures, building on and reusing CERN's existing infrastructure
- FCC integrated project allows the start of a new, major facility at CERN within few years of the end of HL-LHC exploitation



2020 - 2045

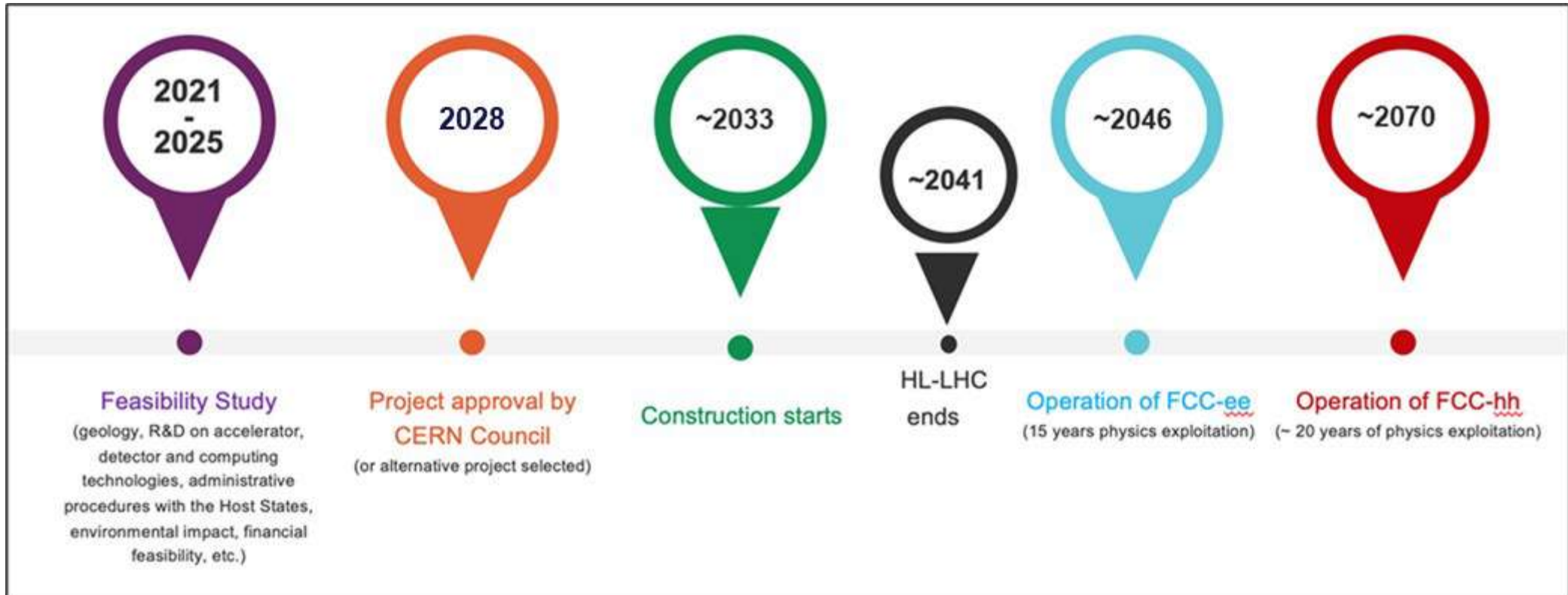


2046 - 2065



2070 - 2100

FCC timeline



Ambitious schedule taking into account:

- ☐ past experience in building colliders at CERN
- ☐ approval timeline: ESPP, Council decision
- ☐ that HL-LHC will run until 2041
- ☐ **project preparatory phase with adequate resources immediately after Feasibility Study**

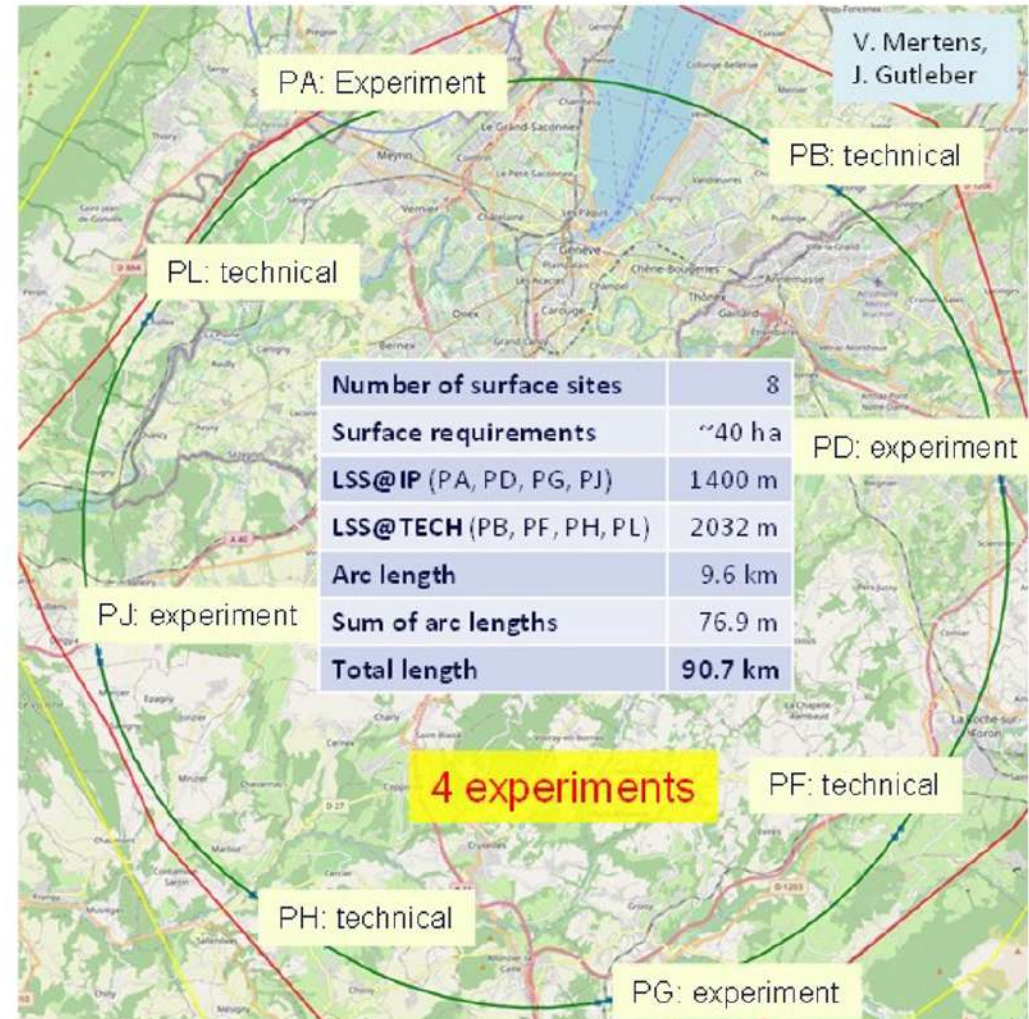
FCC reference layout and implementation – 90.7 km

Layout chosen out of ~ 100 initial variants, based on several criterias:

- **geology**,
- **surface constraints** (land availability, urbanistic, etc.),
- **environment**, (protected zones),
- **infrastructure** (electricity, transport),
- **machine performance**

“Avoid-reduce-compensate” principle of EU and French regulations.

**Overall lowest-risk baseline:
90.7 km ring, 8 surface points,
4-fold symmetry**



FCC-ee collider

FCC-ee collider concept for maximizing luminosity

double ring collider

- many bunches, high current, like LHC and B factories, different from LEP

top-up injection

- standard at modern light sources, like SLS
- used at recent e^+e^- colliders, PEP-II (USA), KEKB (Japan), BEPCII (China)

crab-waist collision scheme

- successfully demonstrated at DAΦNE (Italy) and SuperKEKB (Japan)

superconducting radiofrequency system

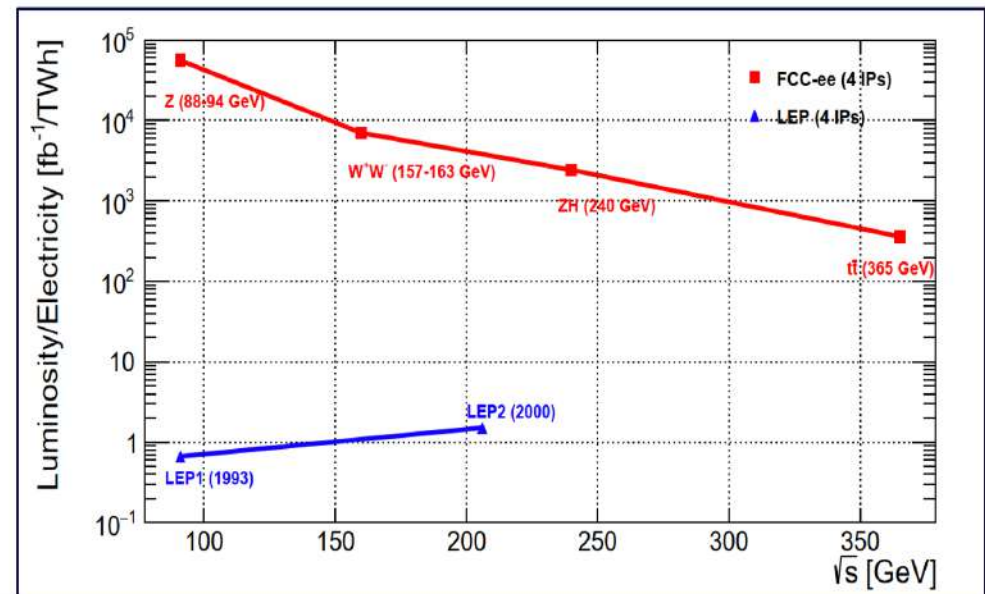
- Nb/Cu 400 MHz SC cavities pioneered at former CERN LEP
- bulk Nb 800 MHz SC cavities similar to ESS (Sweden), EuXFEL (Germany)
- revolutionary highly efficient RF power sources
- new operation scheme for flexible energy switching & reduced complexity

Combining concepts from past and present lepton colliders results in a giant step in efficiency:

→ $10^4 - 10^5 \times$ luminosity/energy of LEP

→ sustainable physics

(Lep-1 data sample taken in just 20 minutes)

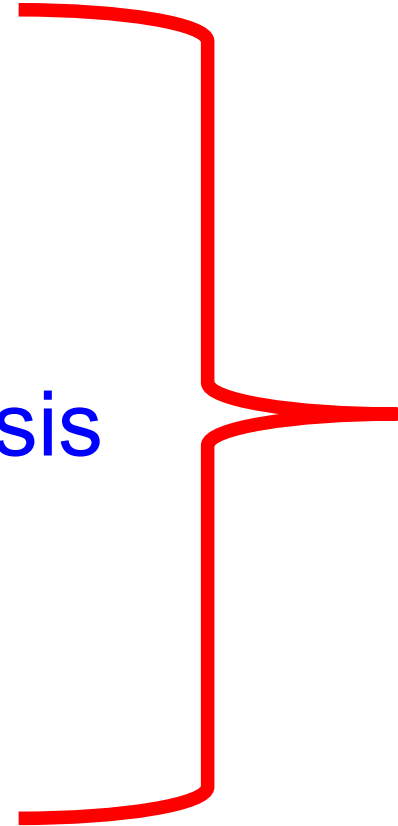


FCC-ee main parameters and operation plan

parameter	Z	WW	H (ZH)	$t\bar{t}$	
Collision energy $\sqrt{s}$ [GeV]	88, 91, 94	157, 163	240	340-350	365
synchrotron radiation/beam [MW]	50	50	50	50	50
beam current [mA]	1294	135	26.8	6.0	5.1
number bunches / beam	11200	1852	300	70	64
total RF voltage 400 / 800 MHz [GV]	0.08 / 0	1.0 / 0	2.1 / 0	2.1 / 7.4	2.1 / 9.2
luminosity / IP [$10^{34} \text{ cm}^{-2}\text{s}^{-1}$]	144	20	7.5	1.8	1.4
luminosity / year [ab^{-1}]	68	9.6	3.6	0.83	0.67
run time (including lumi ramp-up) [years]	4	2	3	1	4
total integrated luminosity [ab^{-1}]	205	19.2	10.8	0.4	2.7
total number of events	$6 \cdot 10^{12} \text{ Z}$	$2.4 \cdot 10^8 \text{ WW}$ (incl. WW at higher $\sqrt{s}$)	$2.2 \cdot 10^6 \text{ ZH}$ $65\text{k WW} \rightarrow \text{H}$	$2 \cdot 10^6 t\bar{t} + 370\text{k ZH}$ $+ 92\text{k WW} \rightarrow \text{H}$	

FCC-ee will be an incredible machine for high precision measurements: m_W , m_t , m_H , Higgs and Z couplings, etc...

Let's close the parenthesis





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End of chapter 1