

Collider Particle Physics - Chapter 9 -

Introduction to Hera Collider: Physics and Experiments

Lectures by Massimo Corradi (INFN)

Claudio Luci



SAPIENZA
UNIVERSITÀ DI ROMA

Chapter Summary

- ☐ deep inelastic scattering: a reminder
- ☐ Hera Collider and the experiments
- ☐ Results on Neutral and charge current scattering at high Q^2
- ☐ QCD and deep inelastic scattering
- ☐ Parton Density Function determination
- ☐ Extra topics ?

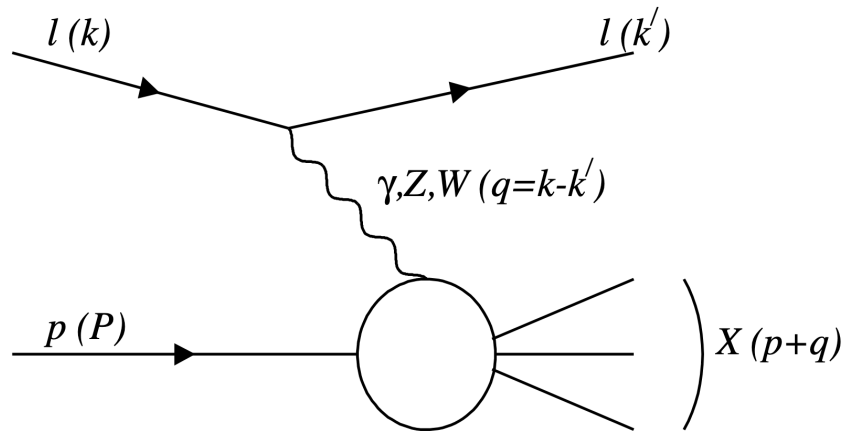
Not following a specific book

See e.g. Hanzel Martin

Or (more advanced) «QCD and collider physics» by Ellis, Stirling, Webber

Deep Inelastic Scattering: a Reminder

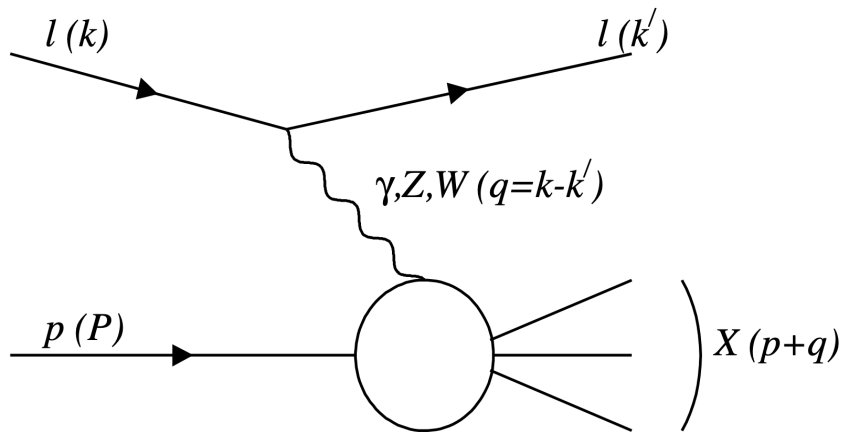
Kinematics of lepton-hadron scattering



Notation: particle name with 4-momentum in parenthesis, e.g. $l(k)$ means lepton with 4-momentum k

How many independent variables we need to describe the final state ?

Kinematics of lepton-hadron scattering



Notation: particle name with 4-momentum in parenthesis, e.g. $l(k)$ means lepton with 4-momentum k

How many independent variables we need to describe the final state ?

- 2 final states particles 4-mom. components = 8 unknowns
- 4-momentum conservation: -4
- Lepton on mass shell: -1

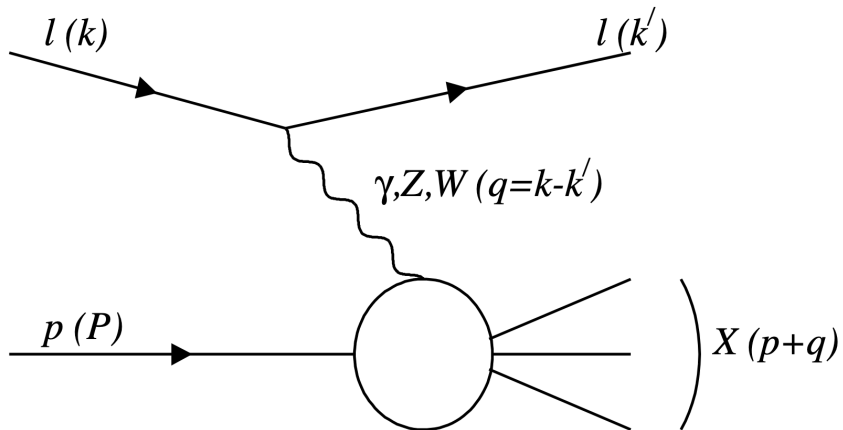
Total : 3

e.g. in polar coordinates, the lepton energy E_l , and two angles: polar (θ) and azimuthal (ϕ).

Anyway due to invariance with respect to rotations around the incoming particles line (e.g. beamline) ϕ is irrelevant !

So 2 variables are sufficient to describe the final state system

Kinematics of lepton-hadron scattering



Lorentz invariants

$$s = (k + P)^2, \quad \text{Center-of-mass energy squared}$$

$$M_X^2 = (q + P)^2 = p_X^2, \quad \text{Mass of final hadronic system X}$$

$$x = \frac{Q^2}{2P \cdot q}, \quad \text{Bjorken x}$$

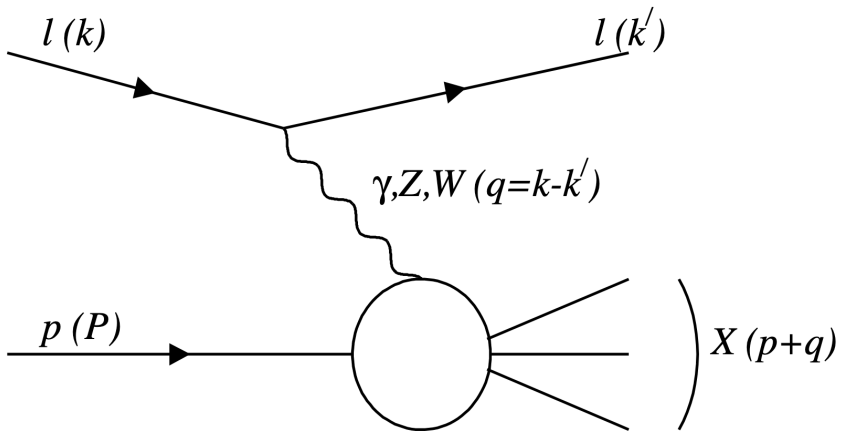
$$y = \frac{q \cdot P}{k \cdot P}, \quad \text{Inelasticity (fraction of energy lost by lepton in reference frame with p at rest)}$$

$$\nu = \frac{q \cdot P}{m_N}, \quad \text{energy lost by lepton in reference frame with p at rest}$$

$$Q^2 = -q^2. \quad \text{Absolute squared four-momentum transfer}$$

Only 2 are independent (once s is fixed)

Kinematics of lepton-hadron scattering



Lorentz invariants

$$s = (k + P)^2,$$

Center-of-mass energy squared

$$M_X^2 = (q + P)^2 = p_f^2, \quad \text{Mass of final hadronic system X}$$

$$x = \frac{Q^2}{2P \cdot q},$$

Bjorken x

$$y = \frac{q \cdot P}{k \cdot P},$$

Inelasticity (fraction of energy lost by lepton in reference frame with p at rest)

$$\nu = \frac{q \cdot P}{m_N}.$$

energy lost by lepton in reference frame with p at rest

$$Q^2 = -q^2.$$

Absolute squared four-momentum transfer

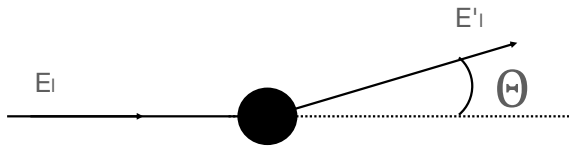
In the following we will use mainly these 4

Only 2 are independent (once s is fixed)

lepton-hadron collision in fixed-target experiments

Proton rest frame

Fixed target experiment Lab frame [notation: $p = (E_p, p_x, p_y, p_z)$]



$$k = (E_l, 0, 0, E_l)$$

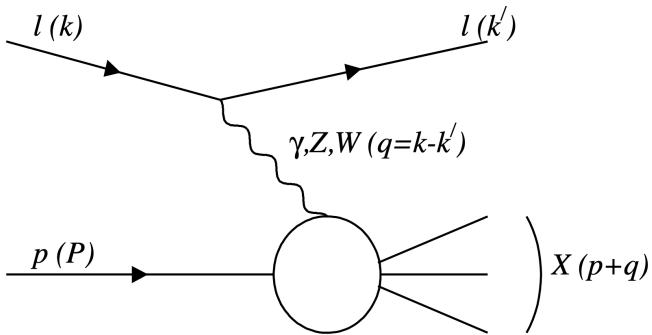
$$k' = (E_l', E_l' \sin \Theta, 0, E_l' \cos \Theta),$$

$$P = (M_p, 0, 0, 0)$$

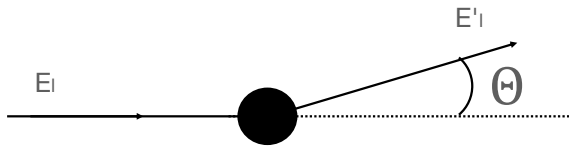
$$q^2 = (k - k')^2 = k^2 + k'^2 - 2k \cdot k'$$

Neglecting the lepton mass

$$q^2 = -2k \cdot k' = -2(E_l E_l' - E_l E_l' \cos \Theta) = -2E_l E_l' (1 - \cos \Theta)$$



lepton-hadron collision in fixed-target experiments



Fixed target experiment Lab frame [notation: $p = (E_p, p_x, p_y, p_z)$]

$$k = (E_l, 0, 0, E_l)$$

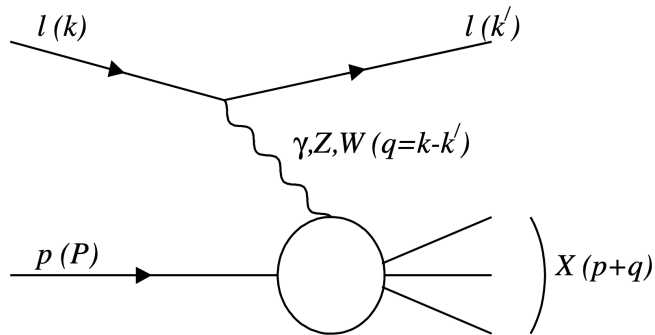
$$k' = (E'_l, E'_l \sin\theta, 0, E'_l \cos\theta),$$

$$P = (M_P, 0, 0, 0)$$

$$q^2 = (k - k')^2 = k^2 + k'^2 - 2k \cdot k'$$

Neglecting the lepton mass

$$q^2 = -2k \cdot k' = -2(E_l E'_l - E_l E'_l \cos\theta) = -2E_l E'_l (1 - \cos\theta)$$



$$Q^2 = -q^2$$

Large angle : large Q^2

$$y = \frac{q \cdot P}{k \cdot P} = \frac{(E_l - E'_l)M_P}{E_l M_P} = 1 - \frac{E'_l}{E_l}$$

$0 < y < 1$ relative electron energy loss

1: all electron energy transferred

0: no energy lost by lepton

Cross section can be expressed in terms of structure functions

General expression for inelastic lepton-hadron cross section
(assuming one photon exchange)

inelastic:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{M^2 y^2}{Q^2}\right) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right]$$

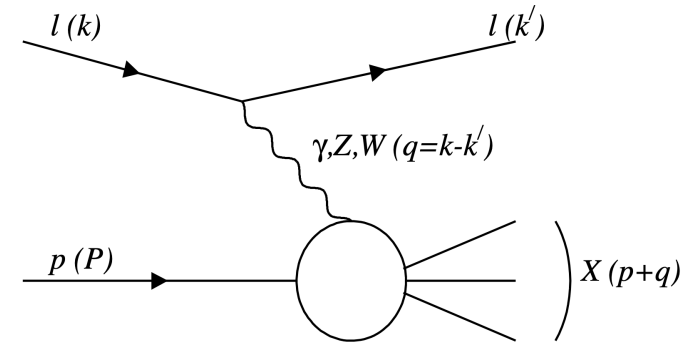
structure functions

Analogue of form factors of elastic scattering $ep \rightarrow ep$:

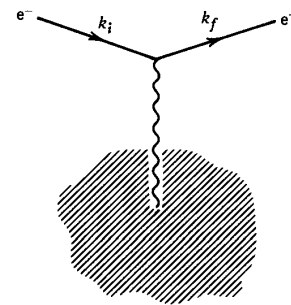
elastic:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[f_2(Q^2) \left(1 - y - \frac{M^2 y^2}{Q^2}\right) + \frac{1}{2} y^2 f_1(Q^2) \right]$$

Form factors



Elastic form factors are the Fourier transform of the charge distribution



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{point}} |F(q)|^2$$

$$F(\mathbf{q}) = \int \rho(\mathbf{x}) e^{i\mathbf{q} \cdot \mathbf{x}} d^3x$$

E.g. a density $\rho(x) = e^{-|x|} \Rightarrow F(q) = 1/q^2$

Cross section can be expressed in terms of structure functions

General expression for inelastic lepton-hadron cross section
(assuming one photon exchange)

inelastic:

can be neglected for $Q^2 \gg M_p^2$

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \cancel{\frac{M^2 y^2}{Q^2}} \right) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right]$$

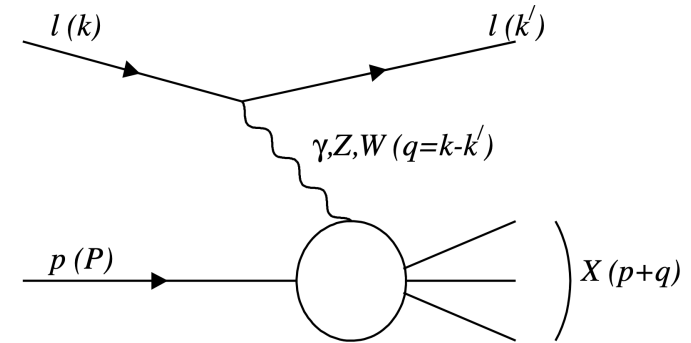
structure functions

Analogue of form factors of elastic scattering $ep \rightarrow ep$:

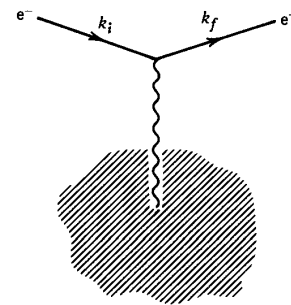
elastic:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[f_2(Q^2) \left(1 - y - \cancel{\frac{M^2 y^2}{Q^2}} \right) + \frac{1}{2} y^2 f_1(Q^2) \right]$$

Form factors



Elastic form factors are the Fourier transform of the charge distribution



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{point}} |F(q)|^2$$

$$F(\mathbf{q}) = \int \rho(\mathbf{x}) e^{i\mathbf{q} \cdot \mathbf{x}} d^3x$$

E.g. a density $\rho(x) = e^{-|x|} \Rightarrow F(q) = 1/(1 + q^2)$

Deep Inelastic Scattering and SLAC data

Deep Inelastic Scattering (DIS) regime:

$$Q^2 \gg M_p^2 \quad \text{Deep}$$

$$M_X^2 \gg M_p^2 \quad \text{Inelastic}$$

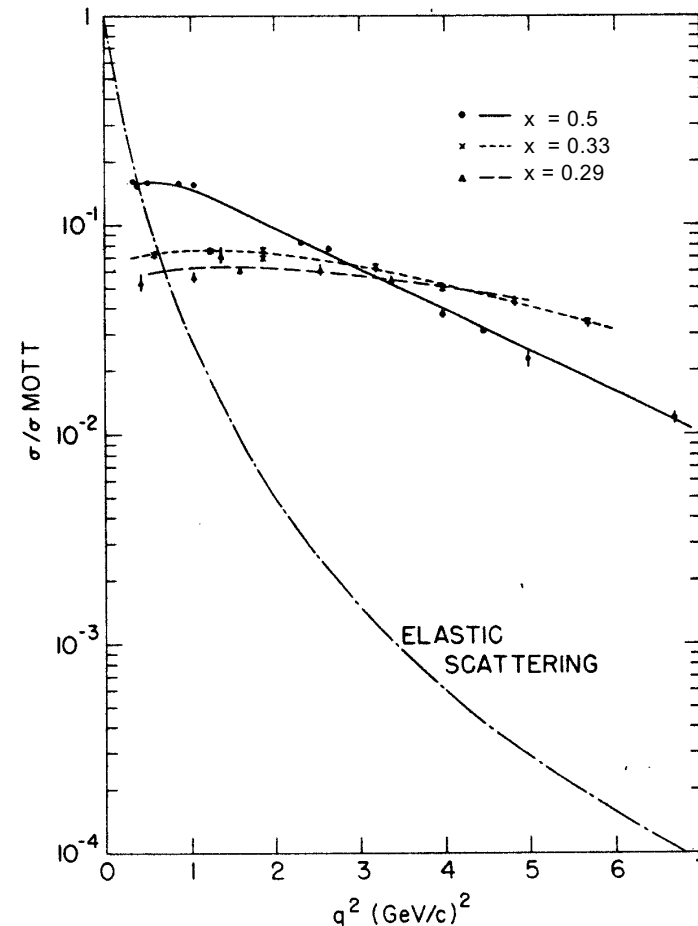
In this limit $Q^2 = xys$

Behaviour of structure functions measured in DIS at SLAC in late 60s was unexpected:

In DIS regime, fixing x , structure functions were found to be almost constant with Q^2

In striking contradiction with expectation from a uniform charge cloud that would predict $\sim 1/Q^4$ similarly to elastic form factors

Parton model was proposed by Feynman to explain this observation...



Parton Model : meaning of x

The proton is made of pointlike constituents (quarks)

Let's assume that we are in a reference frame in which the proton is boosted with $P_z \gg M_p$

Considering that the scattered quark is on mass shell and that quark mass is close to zero :

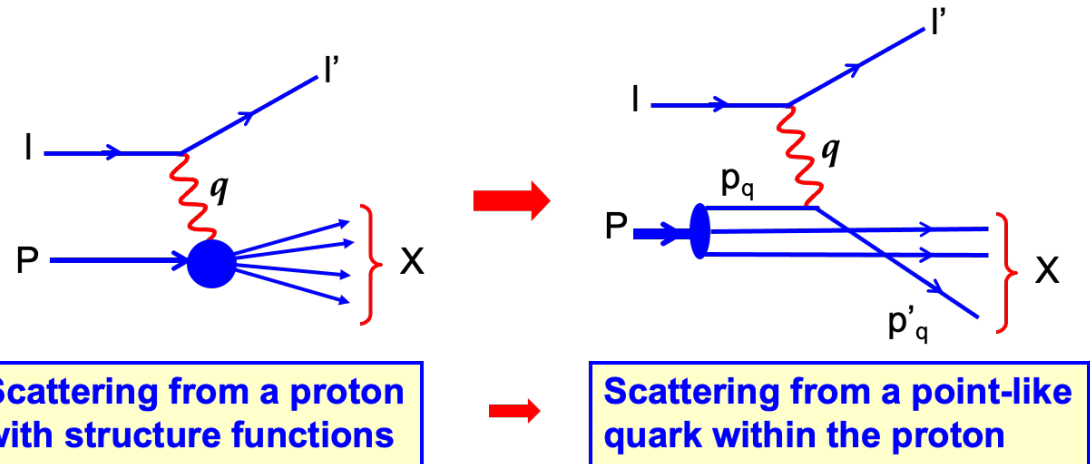
$$(p_q + q)^2 = (p'_q)^2 \simeq 0$$

$$q^2 + 2p_q \cdot q = 0$$

We can write the quark 4-momentum as $p_q = \xi P$ where ξ is the fraction of the proton momentum carried by the quark, then

$$q^2 + 2\xi P \cdot q = 0$$

$$\xi = \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2P \cdot q}$$



Parton Model : meaning of x

The proton is made of pointlike constituents (quarks)

Let's assume that we are in a reference frame in which the proton is boosted with $P_z \gg M_p$

Considering that the scattered quark is on mass shell and that quark mass is close to zero :

$$(p_q + q)^2 = (p'_q)^2 \simeq 0$$

$$q^2 + 2p_q \cdot q = 0$$

We can write the quark 4-momentum as $p_q = \xi P$ where ξ is the fraction of the proton momentum carried by the quark, then

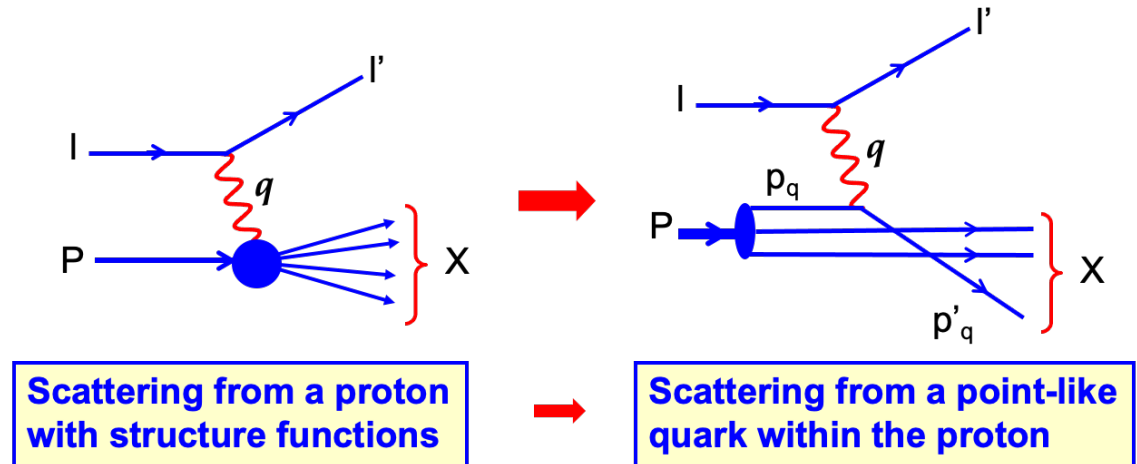
$$q^2 + 2\xi P \cdot q = 0$$

$$\xi = \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2P \cdot q}$$

In the Parton model the quark momentum fraction ξ is equal to the Bjorken- x variable defined as

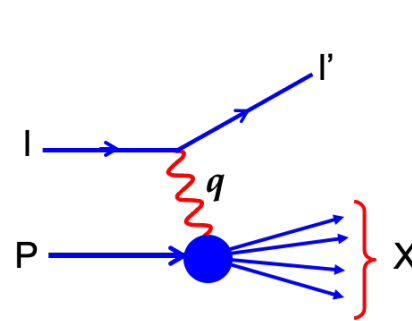
$$x = \frac{Q^2}{2P \cdot q}$$

this is true only at leading order in QCD

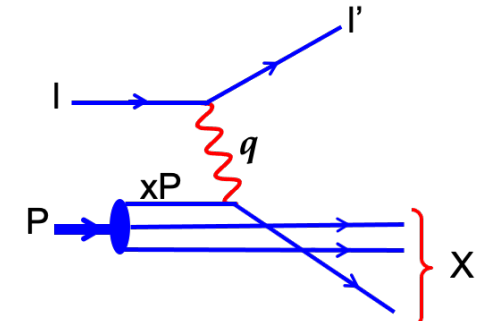


Parton Model : cross section

The cross section can be expressed as a sum of lepton-quark cross sections $\frac{d\hat{\sigma}_q}{dQ^2}$, weighted with the probability density $f_q(x)$ for finding a quark of a given type with a fraction $\xi = x$ of the proton momentum:



Scattering from a proton with structure functions



Scattering from a point-like quark within the proton

$lp \rightarrow l'X :$

$$\frac{d\sigma}{dx dQ^2} = \sum_{q=u,d,\dots} \frac{d\hat{\sigma}_q}{dQ^2} f_q(x)$$

Parton Model : cross section

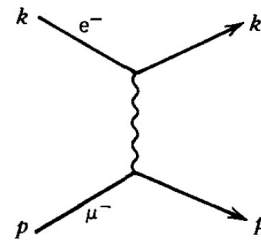
Cross section for lepton-quark scattering $lq \rightarrow lq$

QED process, the calculation gives:

$$\frac{d\hat{\sigma}_q}{dQ^2} = \frac{2\pi\alpha^2}{Q^4} e_q^2 [1 + (1 - y)^2]$$

And thus

$$\begin{aligned} \frac{d\sigma}{dx dQ^2} &= \frac{4\pi\alpha^2}{Q^4} \frac{1}{2} [1 + (1 - y)^2] \sum_i e_i^2 f_i(x) \\ &= \frac{4\pi\alpha^2}{Q^4} \left[(1 - y) + \frac{1}{2} y^2 \right] \sum_i e_i^2 f_i(x) \end{aligned}$$



$$\mathcal{M} = -e^2 \bar{u}(k') \gamma^\mu u(k) \frac{1}{q^2} \bar{u}(p') \gamma_\mu u(p).$$

Parton Density Functions and Bjorken scaling

Comparing this result:

$$\frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left(1 - y + \frac{1}{2}y^2\right) \sum_q e_q^2 f_q(x)$$

with the generic expression for e-p scattering

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^2} [(1-y)F_2(x, Q^2)/x + y^2 F_1(x, Q^2)]$$

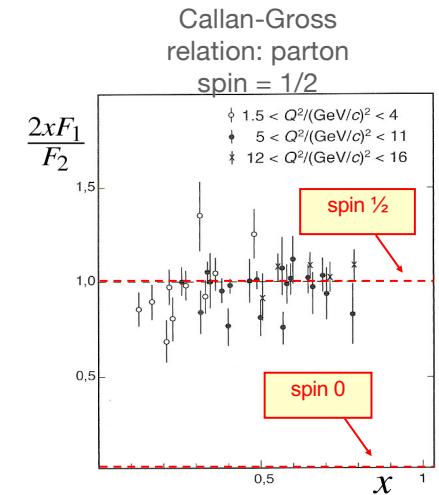
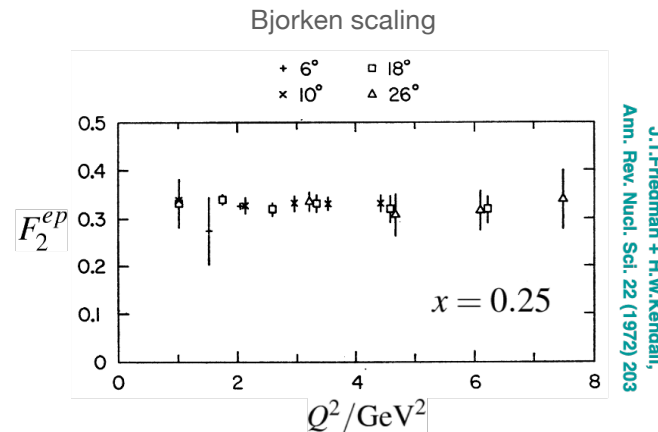
We can identify :

$$F_2(x, Q^2) = x \sum_q e_q^2 f_q(x)$$

$$F_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 f_q(x)$$

Bjorken scaling: structure functions has no dependence on Q^2

$2xF_1(x) = F_2(x)$: the Callan-Gross relation,
-> holds for spin 1/2 partons



$f_q(x)$ is the Parton Density Function (PDF) for Parton q

Structure functions $F(x)$ are weighted sums of PDFs $f(x)$ with weights given by the quark couplings relevant for the particular process, in the case of photon exchange e_q^2

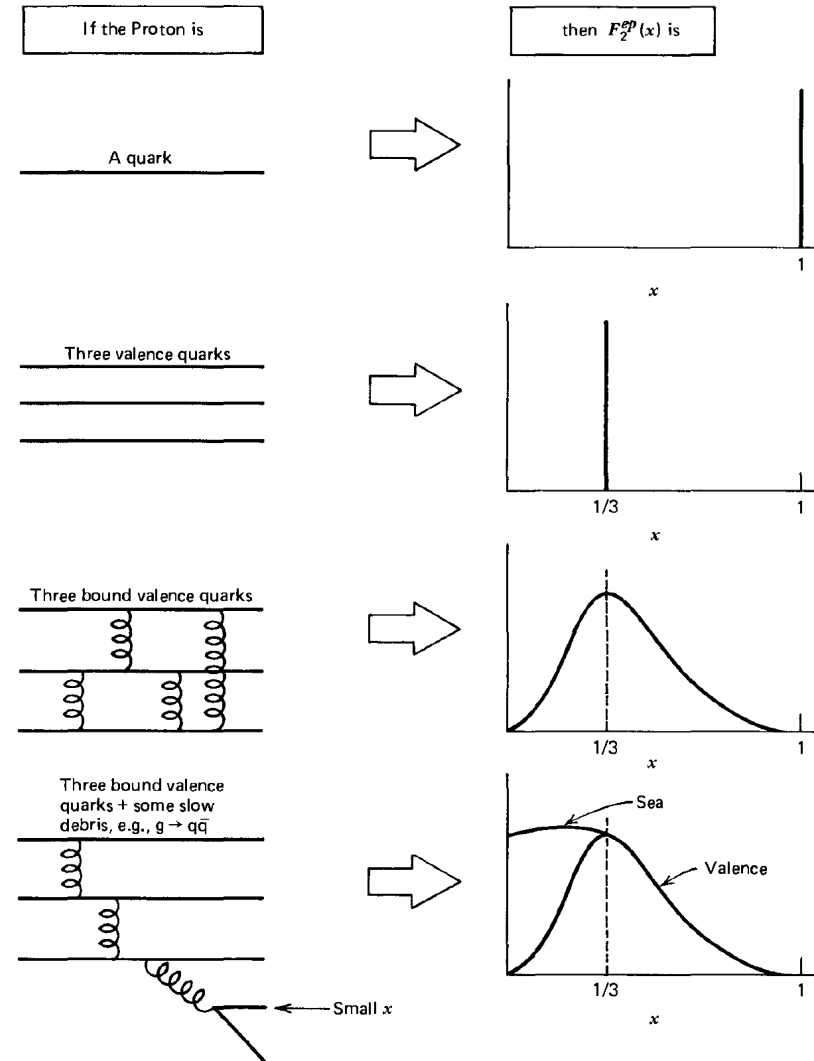
What can we expect for parton densities ?

What can we expect for parton densities ?

PDFs cannot be computed in perturbative QCD

Lattice and other non-perturbative methods
have been tried but so far large theoretical uncertainties

In practice PDFs need to be measured experimentally...



Going back to partonic cross-sections

Cross section for lepton-quark scattering $lq \rightarrow lq$

QED process, analogue to $e\mu \rightarrow e\mu$ scattering,

Mandelstam Variables

$$\hat{s} = (e + p_q)^2 = 2e \cdot p_q$$

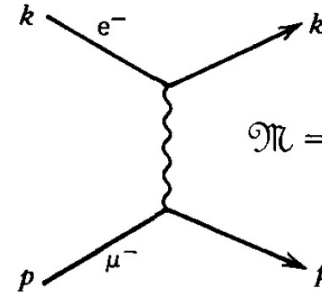
$$\hat{u} = (p_q - e')^2 = -2p_q e'$$

$$\hat{t} = (e - e')^2 = -Q^2$$

$$\frac{d\sigma}{d\hat{t}} = \frac{d\sigma}{dQ^2} = \frac{1}{16\pi\hat{s}^2} |M|^2 \quad (\text{mediato sugli spin iniziali})$$

$$|M|^2 = 2e_q^2 (4\pi\alpha)^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$$

$$\frac{d\sigma}{dQ^2} = (2\pi\alpha^2 e_q^2) \frac{1}{Q^4} (1 + (1-y)^2)$$



$$\mathcal{M} = -e^2 \bar{u}(k') \gamma^\mu u(k) \frac{1}{q^2} \bar{u}(p') \gamma_\mu u(p).$$

TABLE 6.1
Leading Order Contributions to Representative QED Processes

	Feynman Diagrams		$ \mathcal{M} ^2/2e^4$		
	Forward peak	Backward peak	Forward	Interference	Backward
Møller scattering $e^- e^- \rightarrow e^- e^-$			$\frac{s^2 + u^2}{t^2} + \frac{2s^2}{tu} + \frac{s^2 + t^2}{u^2}$		
(Crossing $s \leftrightarrow u$)			$(u \leftrightarrow t \text{ symmetric})$		
Bhabha scattering $e^- e^+ \rightarrow e^- e^+$	Forward 	"Time-like" 	Forward	Interference	Time-like
			$\frac{s^2 + u^2}{t^2} + \frac{2u^2}{ts} + \frac{u^2 + t^2}{s^2}$		
$e^- \mu^- \rightarrow e^- \mu^-$			$\frac{s^2 + u^2}{t^2}$		
(Crossing $s \leftrightarrow t$) $e^- e^+ \rightarrow \mu^- \mu^+$			$\frac{u^2 + t^2}{s^2}$		

Helicity, angular distributions and y

$$\frac{d\hat{\sigma}_i}{dQ^2} = \frac{2\pi\alpha}{Q^4} e_i^2 [1 + (1-y)^2]$$

$$\frac{d\hat{\sigma}_i}{dQ^2} \propto |M|^2$$

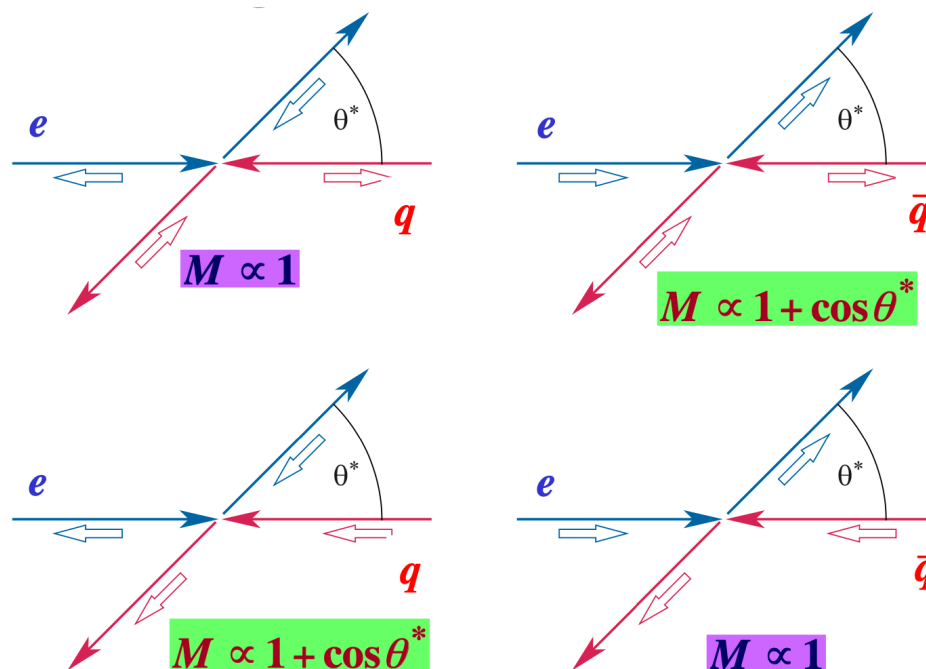
$$y = \frac{1 - \cos\theta^*}{2} \quad \theta^* \text{ is the scattering angle in e-q frame}$$

For vector exchange (γ, Z, W) helicity conservation gives these relations :

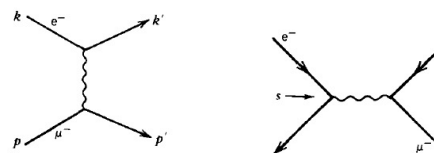
$$e_L q_L, e_R q_R \rightarrow |M|^2 \propto 1 \text{ (isotropic)}$$

$$e_R q_L, e_L q_R \rightarrow |M|^2 \propto (1 + \cos\theta^*)^2 \propto (1-y)^2 \text{ (fwd peak)}$$

In unpolarised photon exchange we have both terms with same weight, thus $1+(1-y)^2$



Connection with e+e- colliders:
«crossed diagram» $e^+ e^- \rightarrow f \bar{f}$
 $t \leftrightarrow s \quad \propto (1 + \cos^2 \theta^*)$



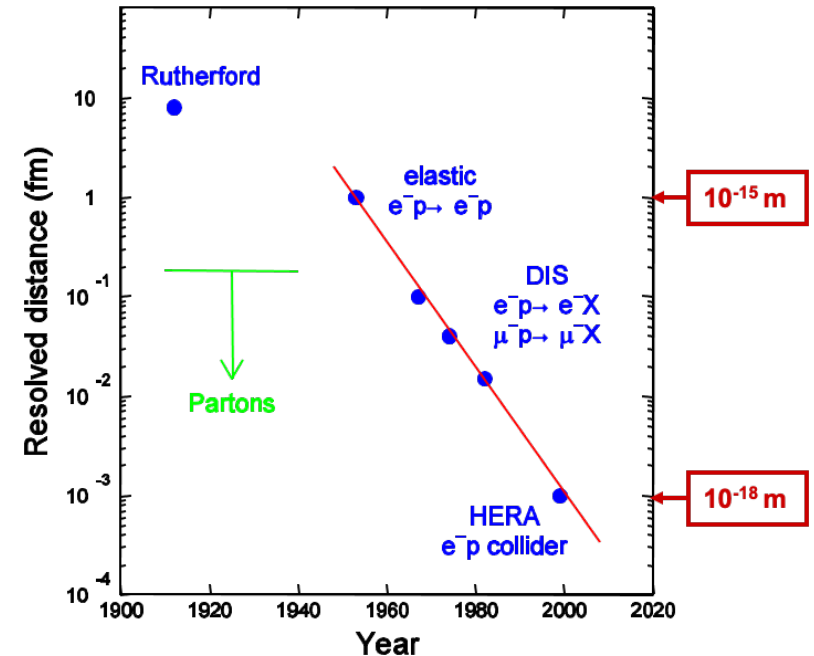
DIS as a way to probe small distances

DIS : a microscope with resolution :

$$\Delta b \sim \frac{\hbar c}{\sqrt{Q^2}} = \frac{0.197}{\sqrt{Q^2}} \text{ GeV fm}$$

The maximum Q^2 reachable in an experiment is given by the center-of-mass energy.

If quarks had a structure it would be resolved going to higher energy



Several DIS experiments in the 70's and 80's using electrons, muons and neutrinos, with increasing energy.

Since the 70s it was clear that to make a significant step forward a collider was needed.

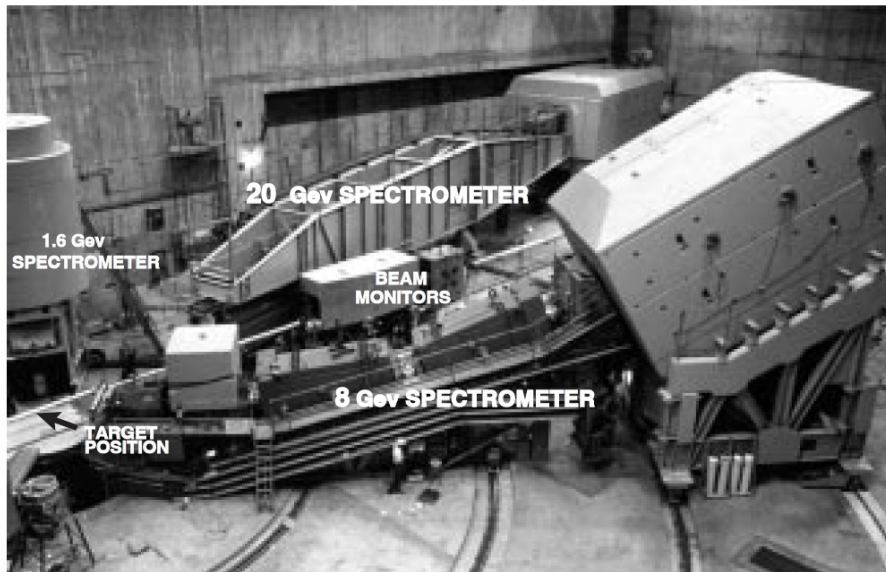
Beyond probing possible parton substructures, a collider would also improve enormously the knowledge of PDF

After several proposal have been considered, it materialised in early '80s as the proposal for HERA

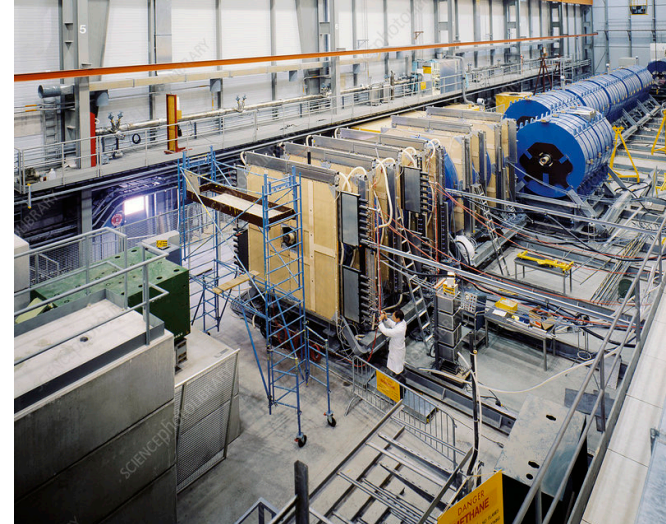
DIS experiments and HERA collider

Historical Fixed target DIS experiments

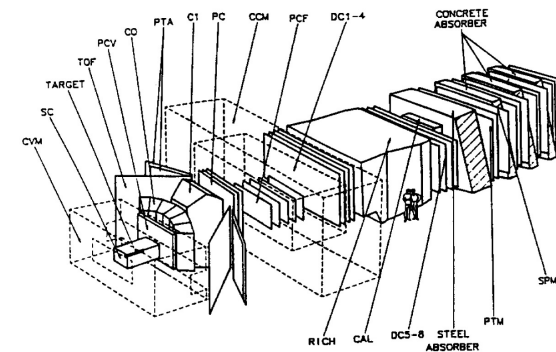
SLAC: 20 GeV electron beam (from 1968)
High intensity beams, low acceptance spectrometers



BCDMS and EMC/NMC (CERN, SPS) up to 280 GeV muon beam



E665, NuTeV (Fermilab, Tevatron) up to 470 GeV muon beam



Muon beams: low intensity, high acceptance spectrometers

HERA the (so far) only ep collider

After the success of Doris and Petra e^+e^- colliders, DESY (Hamburg, Germany) decided to build an ep collider. [with key contributions to the accelerators from Italy and later France]

HERA (Hadron Electron Ring Anlage)

Two accelerators in one 6.3 km tunnel

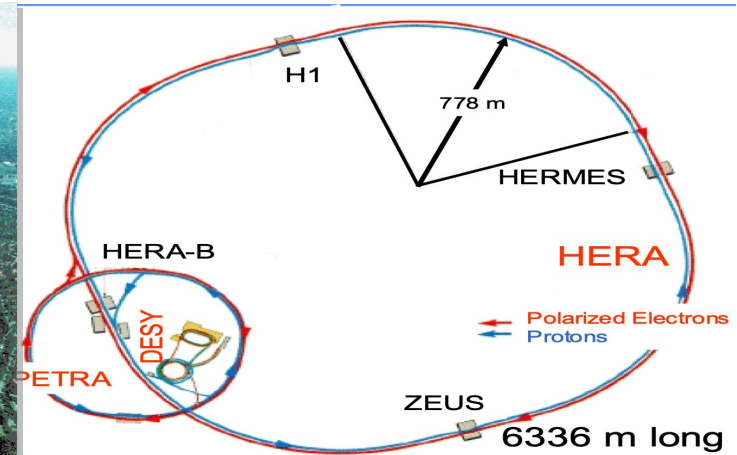
Proton ring: max energy $E_p = 920$ GeV
("similar" to Tevatron)

Electron/positron ring : max energy $E_e = 27.5$ GeV
(A small LEP)

Center-of-mass energy : $\sqrt{s} = \sqrt{4E_e E_p} = 318 \text{ GeV}$

Two large collider experiments: H1 and ZEUS

Two fixed-target experiments:
HERMES (polarised electrons on polarised gas target)
HERA-b (for b physics, using p-beam halo on wire targets)



HERA Phase Space

(remember: $Q^2 = x y s$ and $0 < y < 1$)

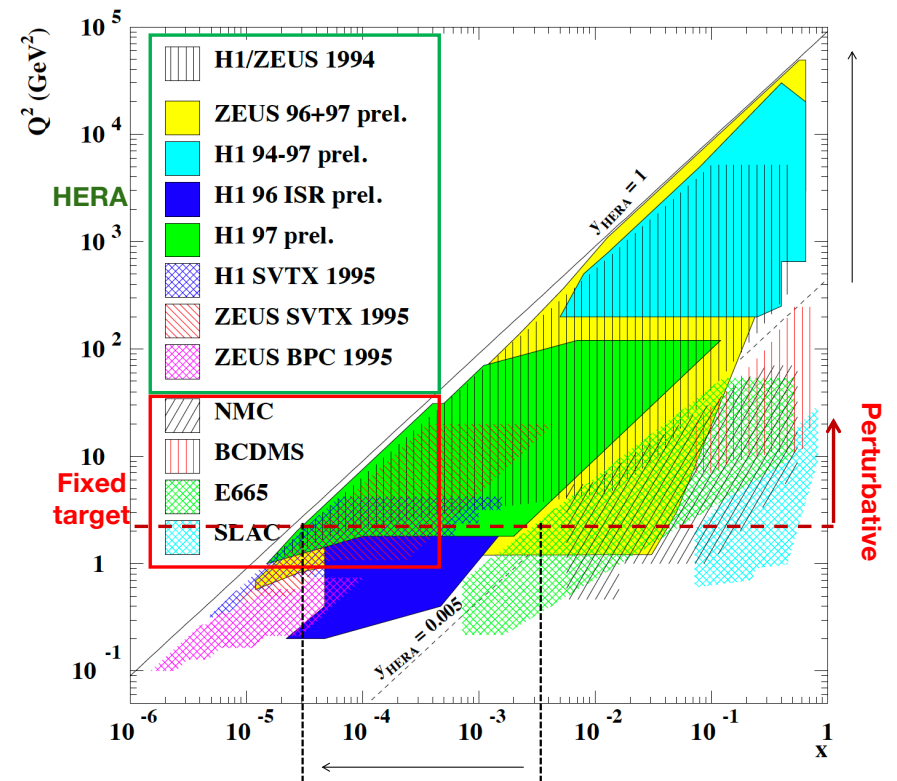
- Maximum Q^2 is given by center-of-mass energy: $Q^2 < s$
- Minimum x values are limited by $x > Q^2/s$

=> HERA expands x , Q^2 range to low x
by about 2 orders of magnitude

Very different beam energies -> need for asymmetric detectors !



The eq scattering is ~central in lab frame for $x=0.03$



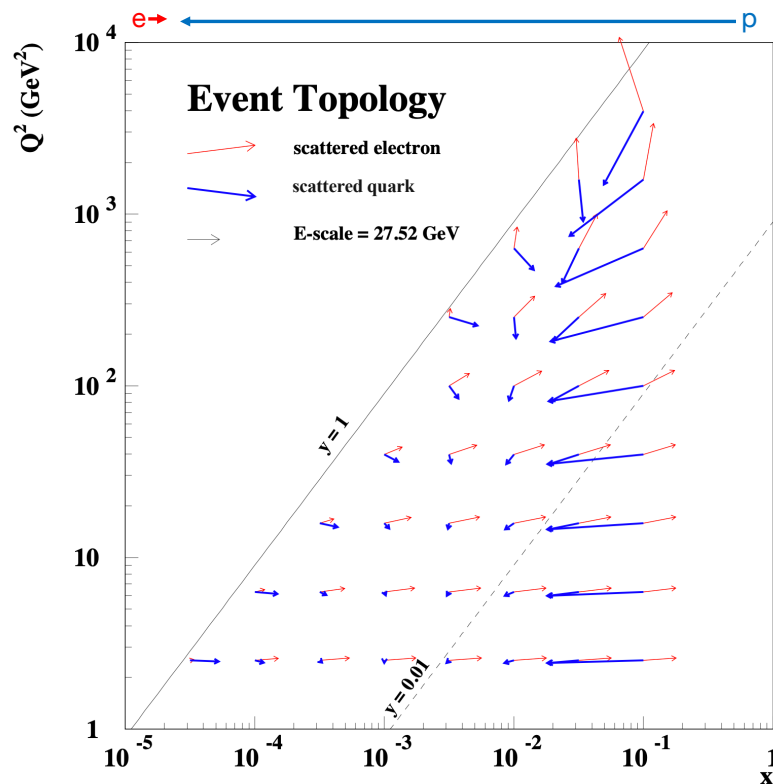
HERA Detectors: requirements

- The same relations that we found for fixed target expt. link electron energy and angle to x , Q^2 . (note different convention of theta, it is measured from proton direction: $\theta_e = \pi - \Theta_e$)
- Most events will have low Q^2 and thus small e deflection angle Θ_e

$$Q^2 = 2E_e E'_e (1 + \cos \theta_e)$$

$$y = 1 - \frac{E'_e}{2E_e} (1 - \cos \theta_e)$$

$$x = \frac{Q^2}{sy}$$



Detectors should be able to reconstruct:

- **Neutral Current** : reconstruct electron
- => good electron reconstruction at all angles (track+calo)
- **Charged Current** : reconstruct hadronic system X
- => good hadronic calorimeter for high energy jets in central/forward
- => but also reconstruction of not so energetic hadrons
- **Hadronic final states**: reconstruction of light/charm/beauty hadrons in the final state (central tracker, vertex detectors, muon chambers)

HERA Detectors: H1

Asymmetric experiments

convention: Z axis points towards p beam direction

“forward” is proton beam direction

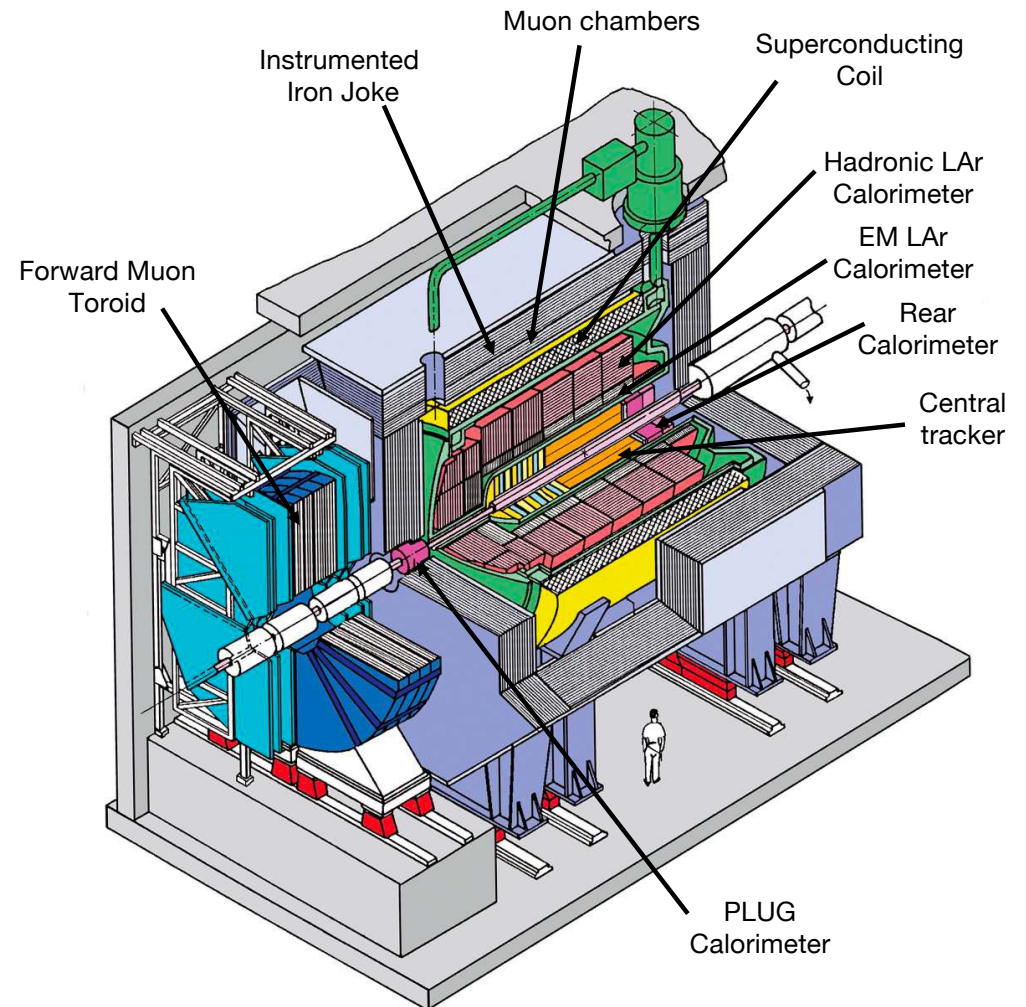
H1 : large German / French / UK components

Sub detectors:

- Silicon strip vertex detector
- “Jet Chamber” gaseous tracker
- Liquid Argon EM calorimeter (barrel / forward)
- Spacal EM calorimeter (spaghetti calorimeter) (rear)
- Liquid Argon hadronic calorimeter
- Large superconducting solenoid (1.16 T axial field)
- Muon Chambers inside iron return coil (streamer tubes)

LAr EM : $\sigma(E)/E \approx 11\%/\sqrt{E/\text{GeV}} \oplus 1\%$

LAr Had : $\sigma(E)/E \approx 50\%/\sqrt{E/\text{GeV}} \oplus 2\%$



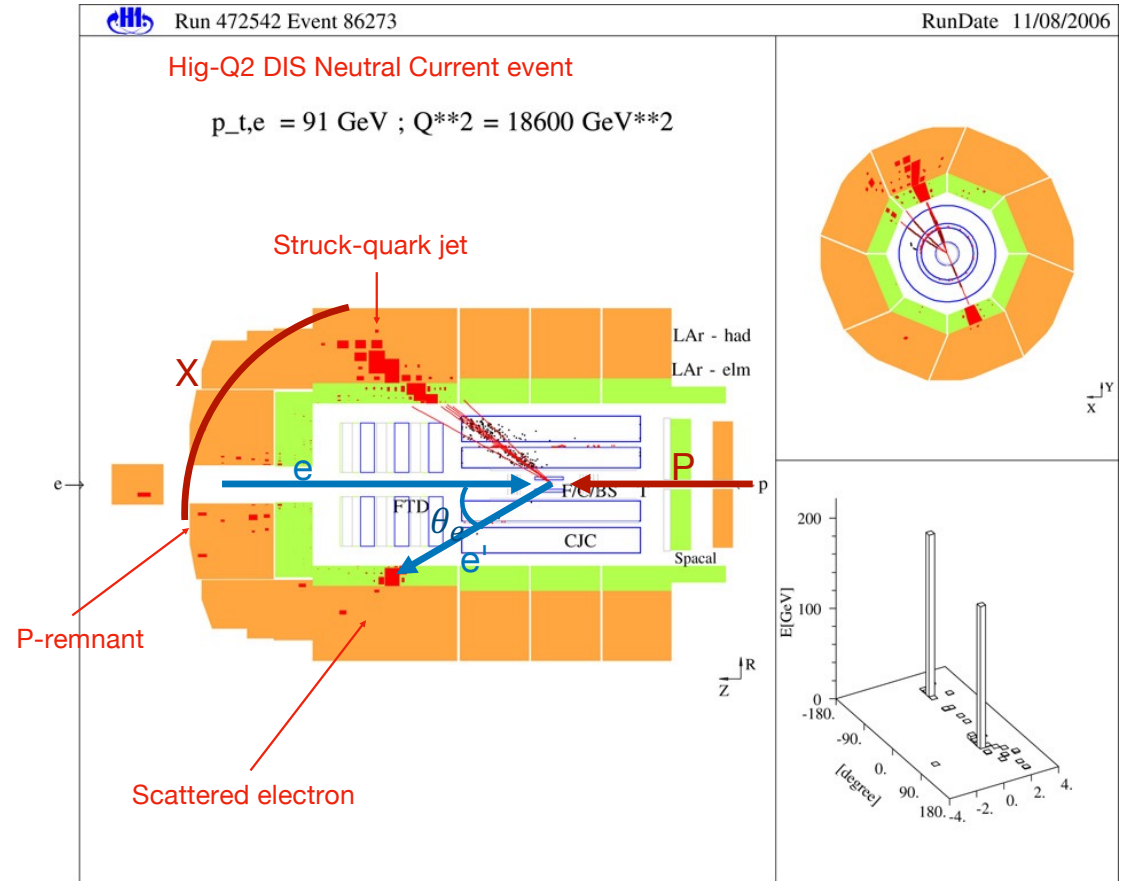
A Neutral Current event in H1

Electron method, same as for fixed target
(Note θ_e measured respect to Z axis ($\theta_e = -\Theta_e$))

$$Q^2 = 2E_e E'_e (1 + \cos \theta_e)$$

$$y = 1 - \frac{E'_e}{2E_e} (1 - \cos \theta_e)$$

$$x = \frac{Q^2}{sy}$$



HERA Detectors: ZEUS

ZEUS : large German / US / ITA / UK / JP components
(+ Russia, Poland, Canada...)

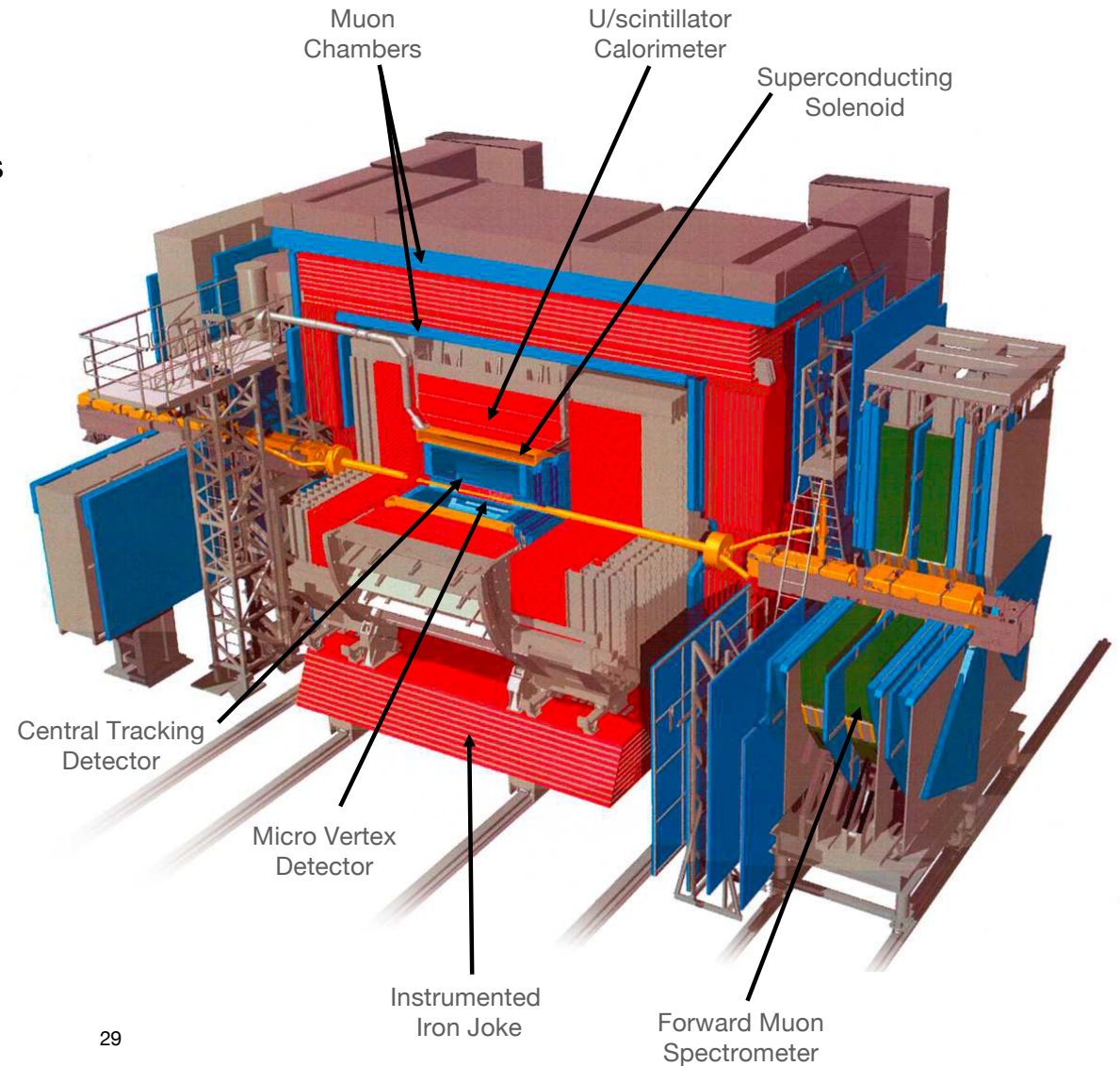
Main Sub detectors:

- Silicon Strip Vertex detector (from 2000)
- Drift Chamber (CTD)
- Thin superconducting solenoid 1.4 T
- pre-samplers (scintillators)
- Compensating Uranium/scintillator calorimeter
- Instrumented return coils (backing calorimeter)
- Muon Chambers (streamer tubes)

Main focus on hadronic calorimetry

EM : $\sigma(E)/E \approx 18\%/\sqrt{E/\text{GeV}}$

Had : $\sigma(E)/E \approx 35\%/\sqrt{E/\text{GeV}}$



A Charged Current event in ZEUS

In **Charged-Current** DIS the reconstruction method is the hadronic method, also known as Jaquet-Blondel (JB) method, exploits energy-momentum conservation to obtain the neutrino angle and energy:

Transverse momentum conservation :

$$p_{T,X} = p'_{T,\nu} = E'_\nu \sin\theta_\nu$$

Longitudinal momentum conservation :

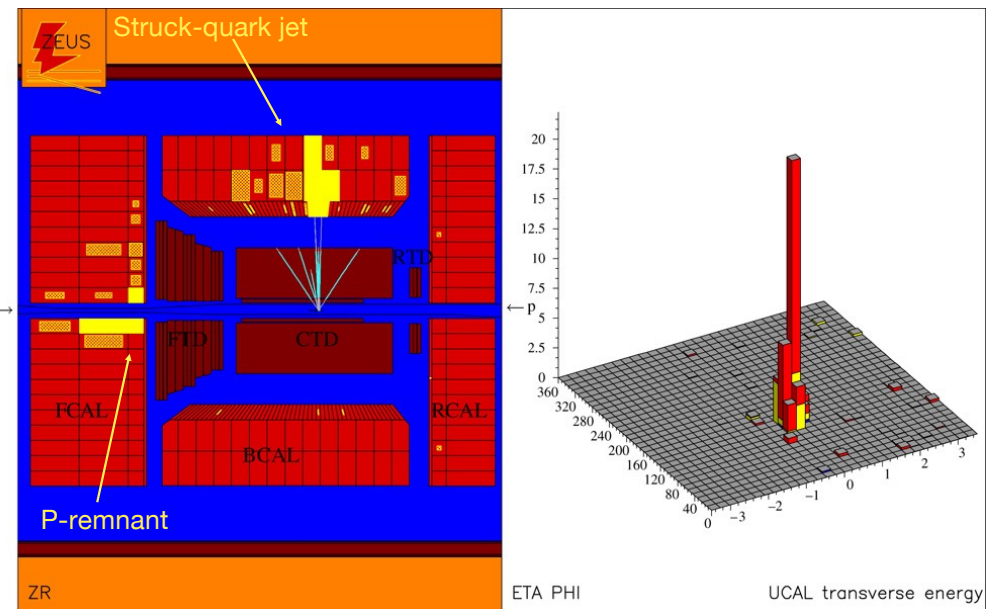
$$(E - P_Z)_{\text{tot}} = 2E_e = (E - P_Z)_X + E_\nu(1 - \cos\theta_\nu) \quad e \rightarrow$$

Then

$$y_{JB} = \frac{(E - P_Z)_X}{2E_e} \quad Q_{JB}^2 = \frac{p_{T,X}^2}{1 - y_{JB}}$$

$e p \rightarrow \nu X$

Hig-Q2 DIS Charged Current event



Part of the hadronic system escapes undetected in the forward beam hole, but gives negligible contribution to $(E - P_Z)_X$, (which is the reason for using this particular combination)

Compare energy-conservation constraints for LEP (4), LHC (2) and HERA (3)

ZEUS hadronic calorimeter

Hadronic showers:

a fraction f_e of energy is released as EM (ionization, $\pi^0 \rightarrow \gamma\gamma$ etc.)

the rest $(1 - f_e)$ is lost in nuclear interactions.

The calorimeter response to EM (e) and hadron (h) interactions is different typically $h < e$

e.g. response to a pion:

$$\pi^+ = f_e e + (1 - f_e) h$$

EM fraction f_e fluctuates, degrading the calorimeter resolution

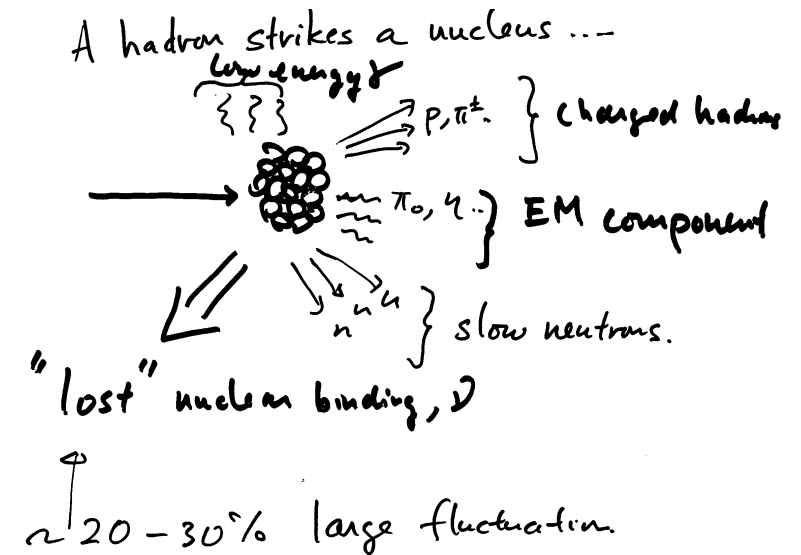
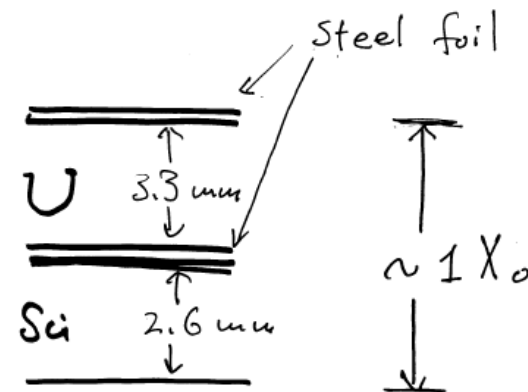
Trick: find a special recipe to obtain compensation: $e/h = 1$

Uranium + plastic Scintillator : large response to neutrons

f_e can be tuned in sampling calorimeters:

higher absorber fraction $\rightarrow$ lower e.m. component

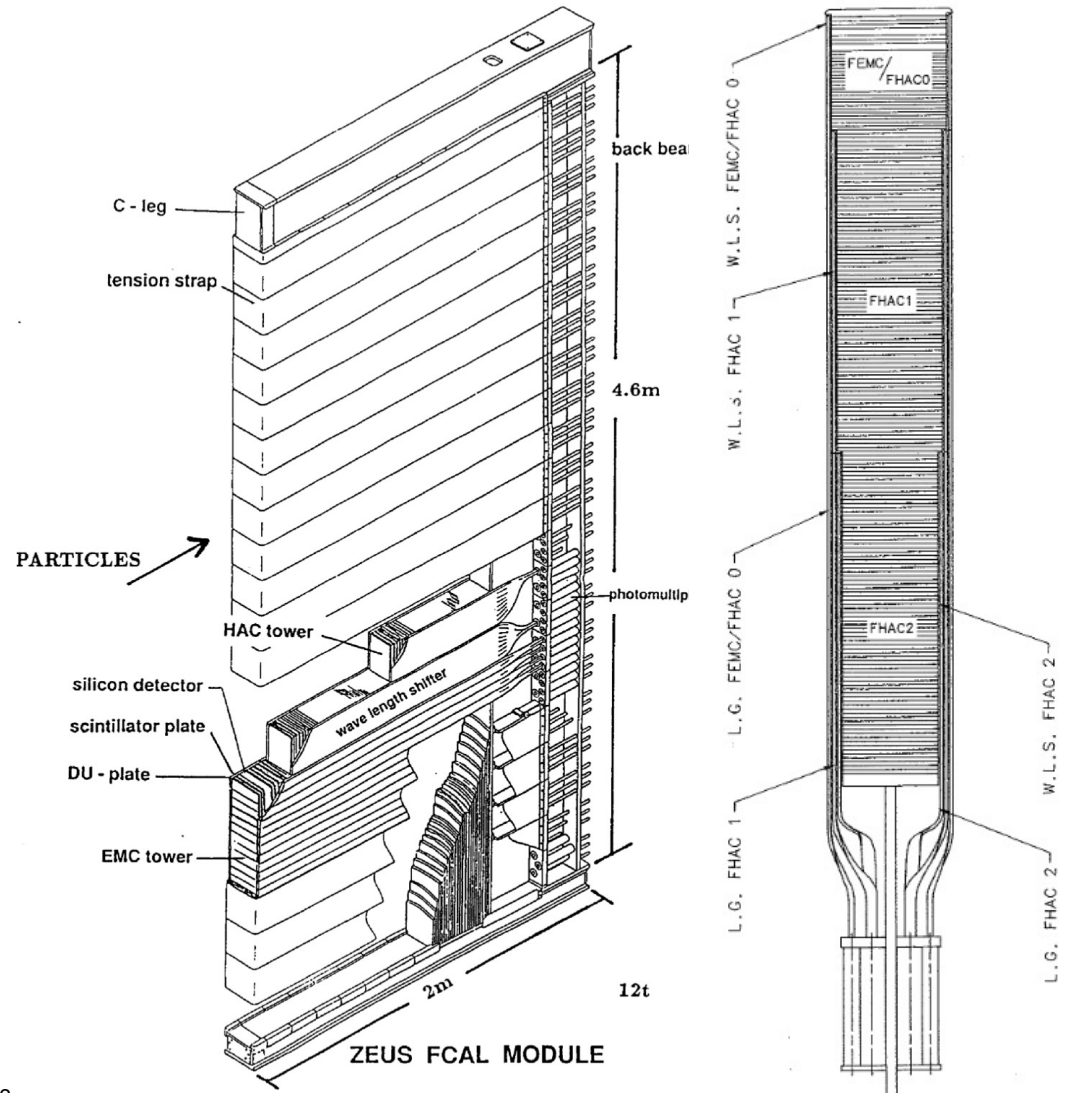
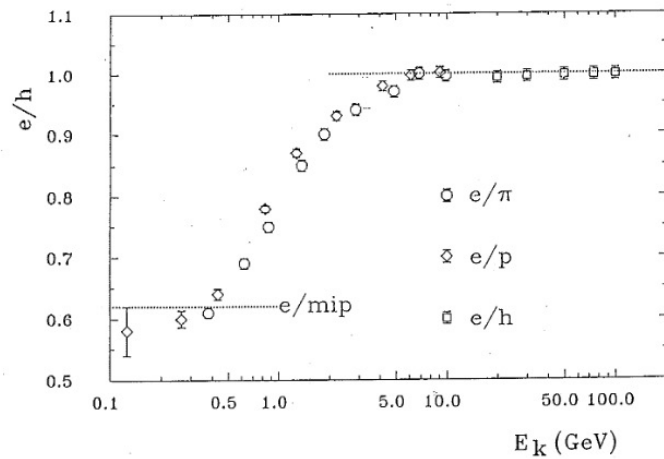
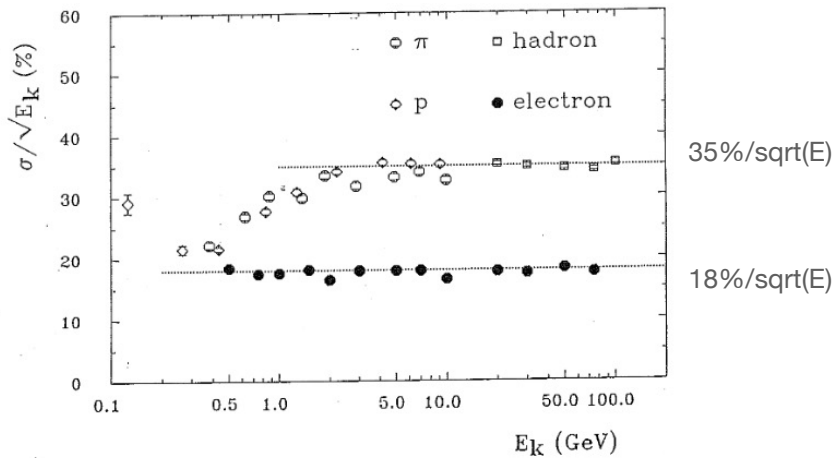
ZEUS used this structure :



ZEUS hadronic calorimeter

A unique compensating calorimeter !

Same response to EM and hadrons



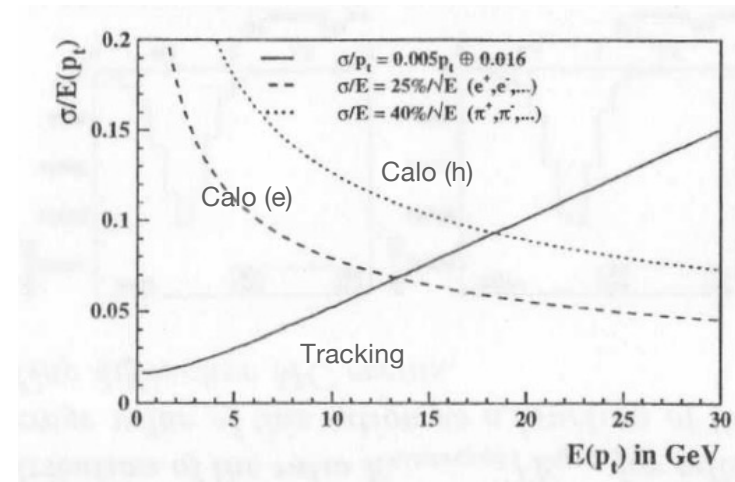
More on hadronic system reconstruction

Once we have a compensating hadron calorimeter we can reconstruct the hadronic system simply summing up the vectors of all calorimeter cell with an energy deposit above noise:

$$\vec{P} = \sum_{i \in \text{cells}} E^i \vec{r}^i$$

where the sum runs over all cells above threshold (not associated to the scattered electron) E^i is the cell energy and $\vec{r}^i$ is the unit vector pointing from event vertex (as reconstructed by central tracker) to the cell.

Various improvements were used: the sum was done over energy clusters corrected for energy lost in front of calorimeter based on dead material maps. Moreover “particle flow” objects (ZUFOS = ZEUS Unidentified objects) were used, using tracking rather than calorimetry for low- p_T charged particles.



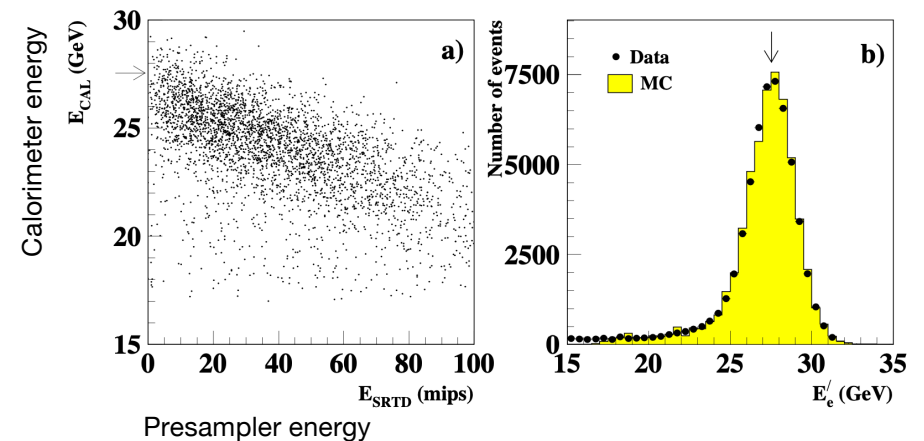
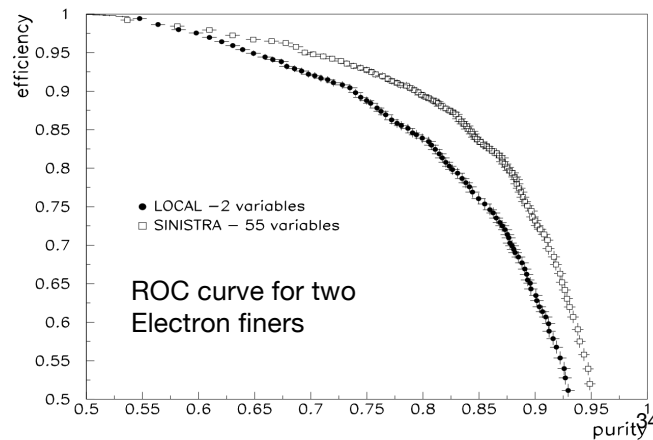
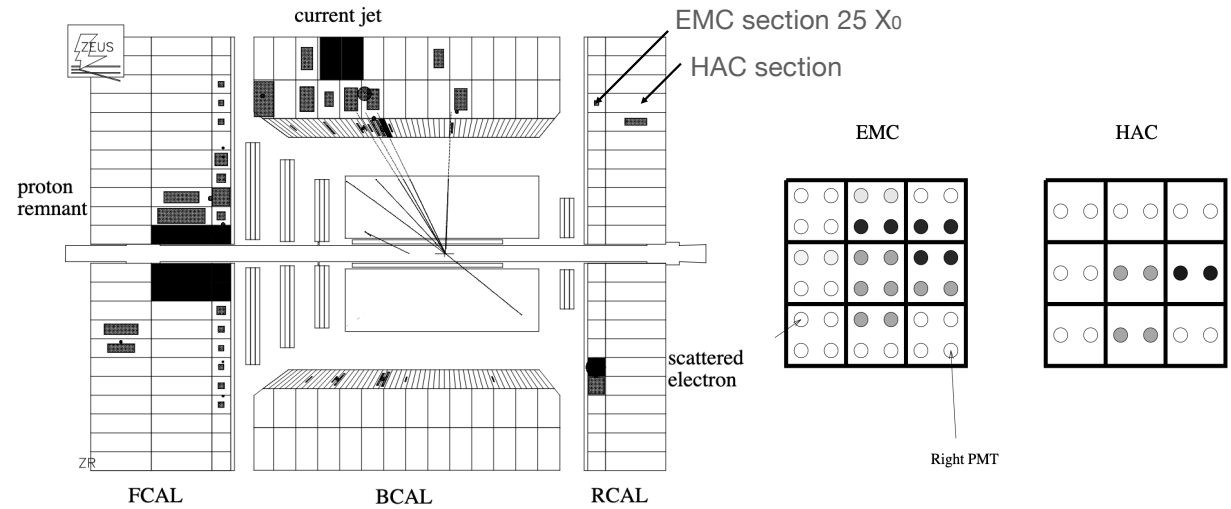
Electron identification and reconstruction (ZEUS)

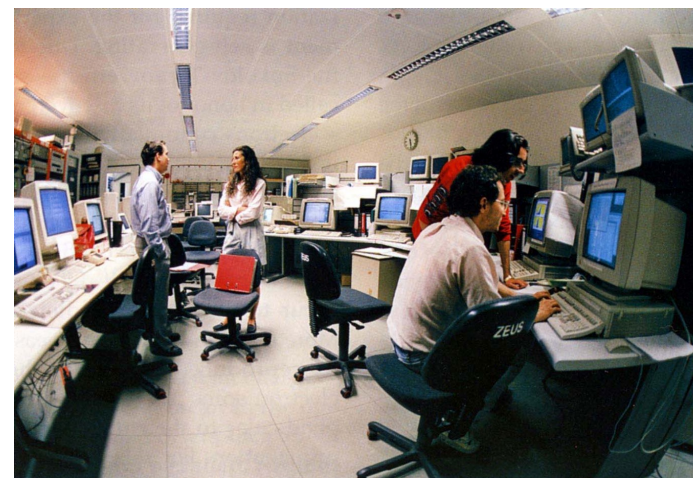
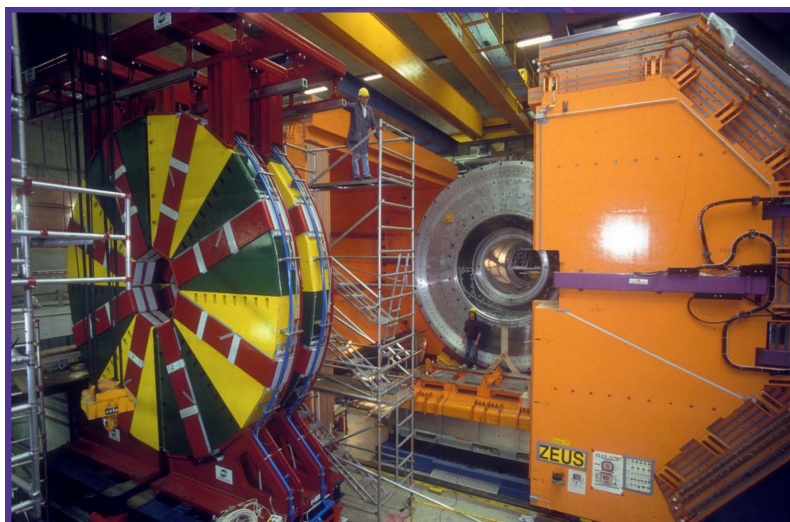
The electron is identified as a small calorimetric cluster with small radius and low energy deposit in the hadronic section.

A neural-network feed with 55 PMT signals from the $60 \times 60 \text{ cm}^2$ region was used to discriminate hadrons and electrons.

In addition requirements on isolation and (within the central tracker acceptance) the presence of a charged track.

Scintillator pre-samplers placed in front of the calorimeter allow to correct for the energy lost in the dead material before





HERA Operations

Two main periods HERA-1 (1992 - 2000) and HERA-2 (2003-2007)

HERA-2: luminosity upgrade with low- β^* insertions near interaction points

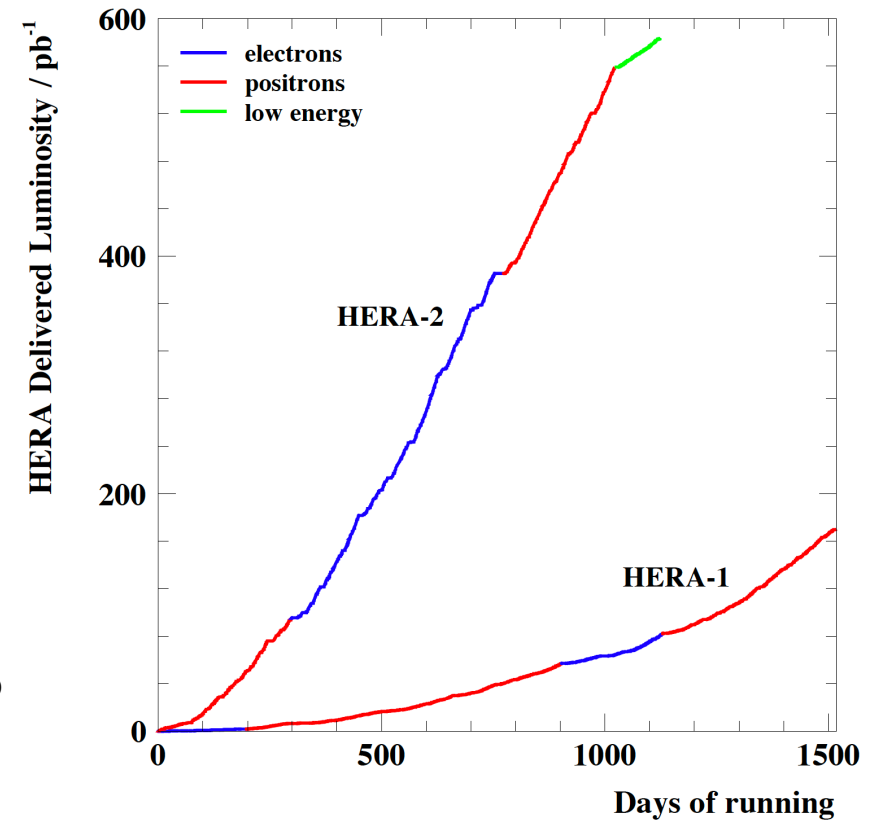
Luminosity of HERA-1 was below expectations in particular with electrons, most luminosity was collected with positrons

HERA-2 had initial difficulties related to high backgrounds in the experiments, finally an integrated luminosity of 0.5 fb^{-1} per experiment was collected (not very successful)

210 bunches (176 colliding) 96 ns between bunches

Peak luminosity in HERA-2 $5 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$

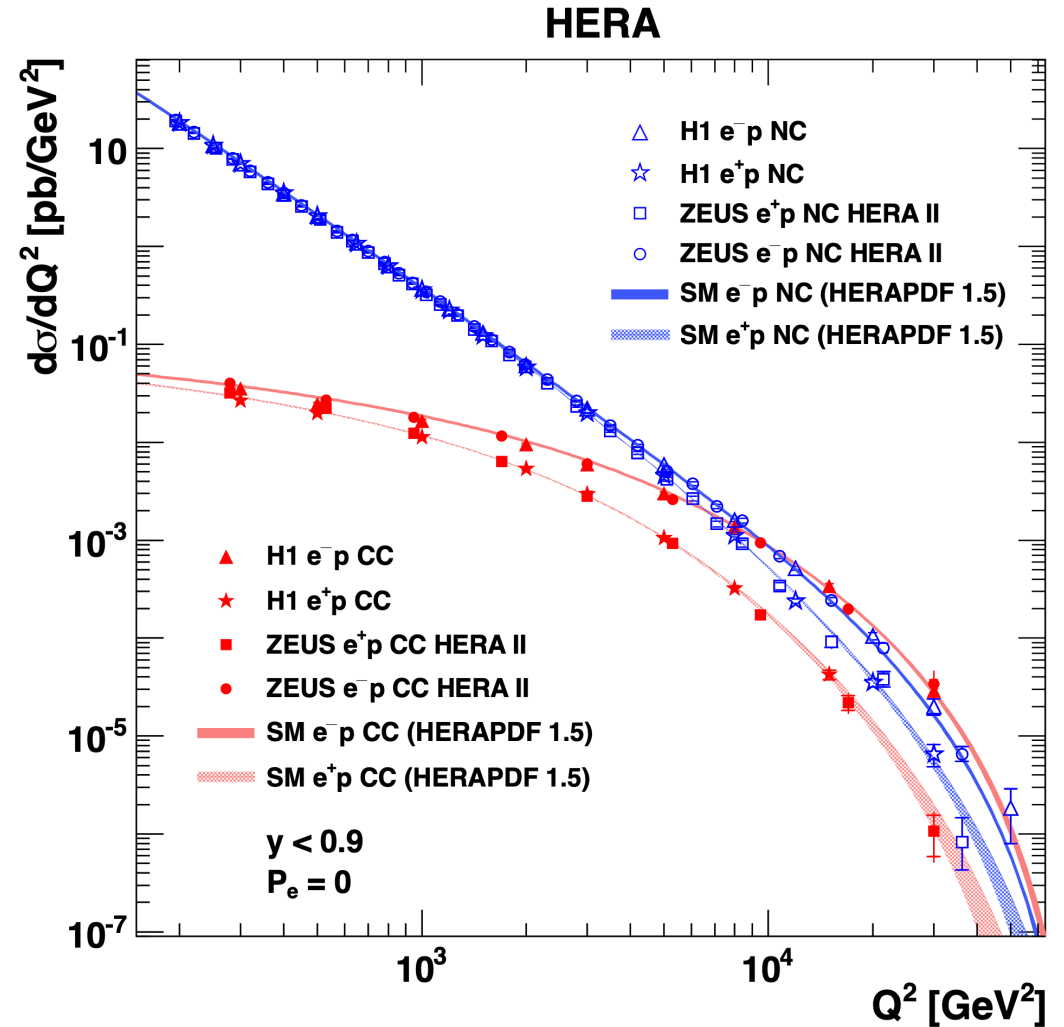
During the HERA-1 $\rightarrow$ HERA-2 transition the detectors have been also upgraded. In particular new vertex detectors were introduced in both experiments to improve c and b tagging capabilities



Results on Neutral and Charged current scattering at high Q^2

Results : Neutral Current cross section vs Q^2 (integrated over x)

- The NC cross section falls approximately like $1/Q^4$
- Eventually at $Q^2 \simeq M_W^2$ CC and NC become similar: weak and e.m. interaction unify at EW scale !
- CC cross section in e^-p is larger than in e^+p because sensitive to u quarks rather than d.
- At high- Q^2 a difference between e^-p and e^+p is also observed in NC : this is the effect of the Z exchange that introduces a charge dependence (C and P not conserved individually)



Results : limit on quark substructure

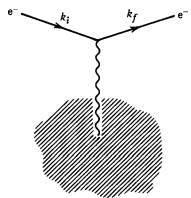
Do we see have any hint of quark substructures ?

RMS quark radius:

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma^{\text{SM}}}{dQ^2} \left(1 - \frac{R^2}{6} Q^2 \right)^2$$

Current limit is $0.43 \times 10^{-3} \text{ fm}$:
less than thousand times smaller than a proton !

Elastic form factors are the fourier transform
of the charge distribution



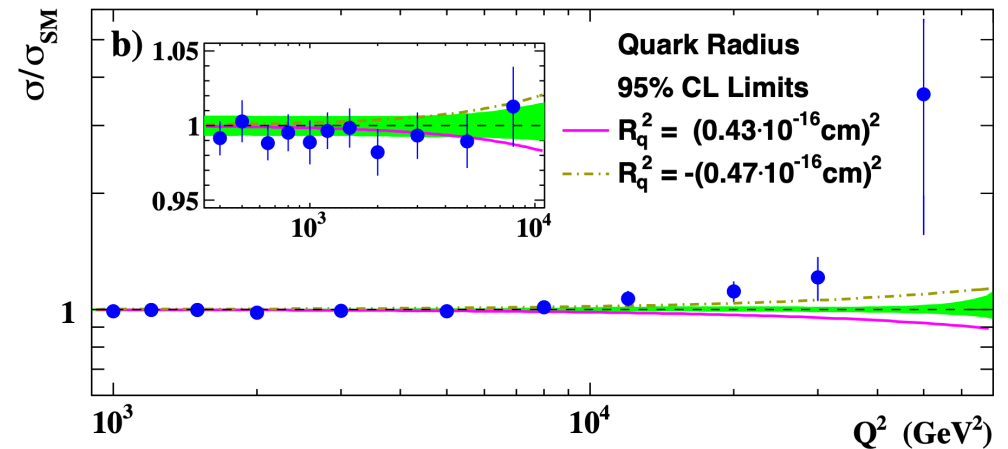
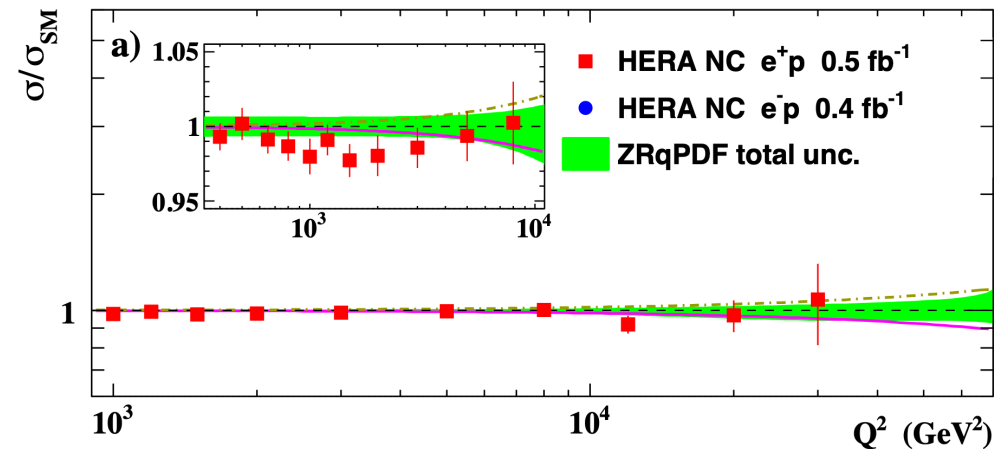
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{point}} |F(q)|^2$$

$$F(\mathbf{q}) = \int \rho(\mathbf{x}) e^{i\mathbf{q} \cdot \mathbf{x}} d^3x$$

$$F(\mathbf{q}) = \int \left(1 + i\mathbf{q} \cdot \mathbf{x} - \frac{(\mathbf{q} \cdot \mathbf{x})^2}{2} + \dots \right) \rho(\mathbf{x}) d^3x$$

$$= 1 - \frac{1}{6} |\mathbf{q}|^2 \langle r^2 \rangle + \dots,$$

ZEUS



Charged Currents: W exchange

Applying the same approach :

$$\frac{d\sigma^{CC}}{dx dQ^2} = \overset{\text{coupling}}{\frac{G_F}{2\pi x}} \left(\overset{\text{propagator}}{\frac{M_W^2}{M_W^2 + Q^2}} \right)^2 \overset{\text{Reduced cross-section}}{\tilde{\sigma}^{CC}(x)}$$

W^- selects $e_L^-, u_L, \bar{d}_R \dots$:

$$e_L^- + q_L : \propto 1$$

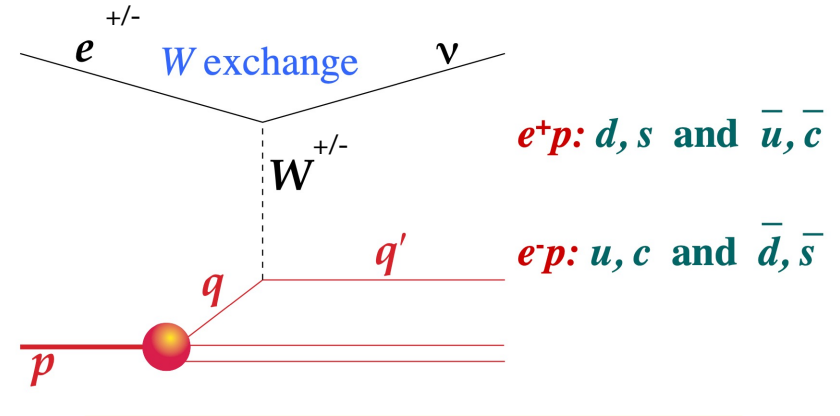
$$e_L^- + \bar{q}_R : \propto (1-y)^2$$

$$e^- p: \tilde{\sigma}^{CC}(x) = x[u(x) + c(x) + (1-y)^2(\bar{d}(x) + \bar{s}(x))]$$

Similarly using a positron beam:

$$e^+ p: \tilde{\sigma}^{CC}(x) = x[\bar{u}(x) + \bar{c}(x) + (1-y)^2(d(x) + s(x))]$$

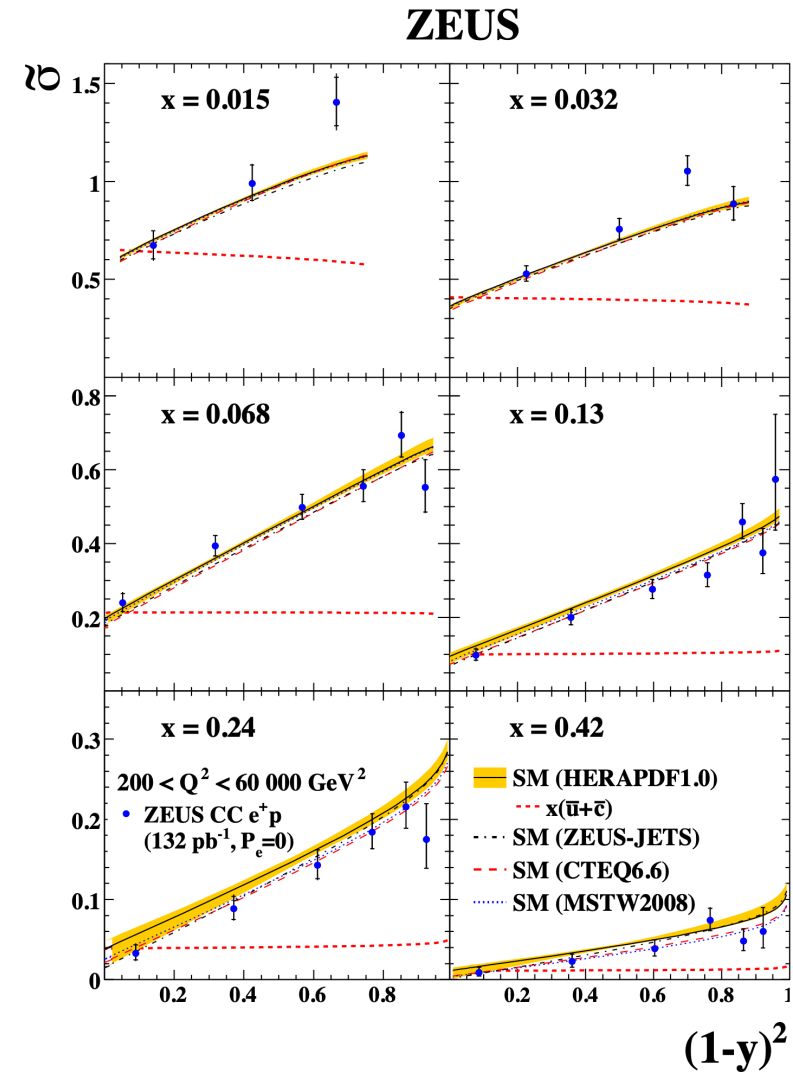
Charged and Neutral currents processes are sensitive to different combination of quarks



Results : Charged Current cross section vs y

The Helicity structure works !

$$e^+p: \tilde{\sigma}^{CC}(x) = x[\bar{u}(x) + \bar{c}(x) + (1-y)^2(d(x) + s(x))]$$



Results : Charged Current with polarized lepton beams

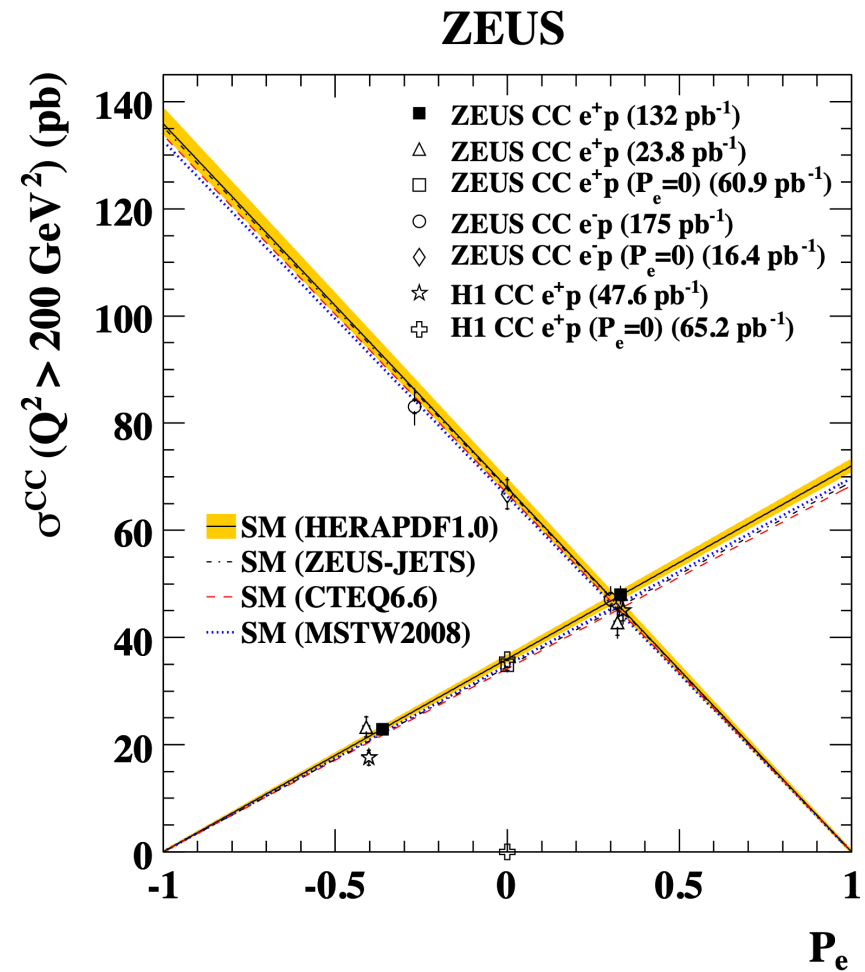
During HERA-2 period, HERA provided polarised lepton beams.

Electrons are polarised transversally (spin perpendicular to accelerator plane by synchrotron light emission in dipole magnets (Solokov-Ternov effect)
Special spin-rotator magnets rotate the spin just before/after the interaction points to obtain longitudinal polarization.

Polarisation $P_e = \frac{N_R - N_L}{N_R + N_L}$ was about 0.4 and was switched in sign every few months.

This permits to select leptons with “right helicity”

Charge current should not be present for e^-_R and e^+_L
Constraint on right-handed W



Z exchange and F_3

$$\tilde{\sigma}^{e^\pm p} = \frac{xQ^4}{2\pi\alpha^2} \frac{1}{Y_+} \frac{d^2\sigma(e^\pm p)}{dx dQ^2} = \tilde{F}_2(x, Q^2) \mp \frac{Y_-}{Y_+} x\tilde{F}_3(x, Q^2) - \frac{y^2}{Y_+} F_L(x, Q^2) \quad F_L \text{ is zero at LO}$$

$$\tilde{F}_2^\pm = F_2^\gamma - (v_e \pm P_e a_e) \chi_Z F_2^{\gamma Z} + (v_e^2 + a_e^2 \pm 2P_e v_e a_e) \chi_Z^2 F_2^Z,$$

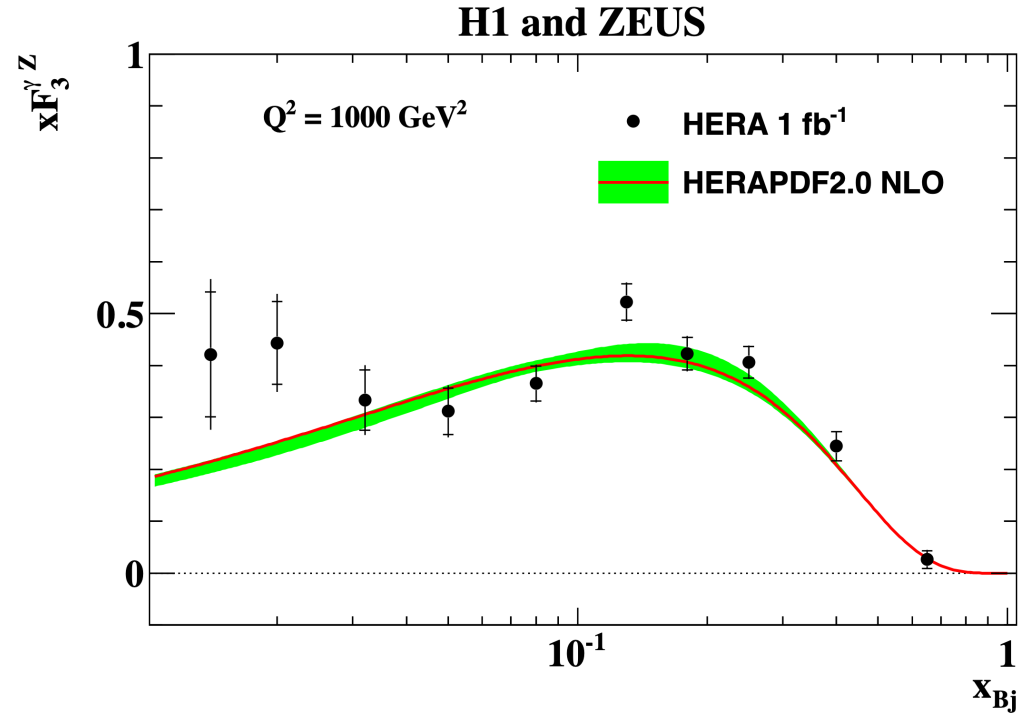
$$x\tilde{F}_3^\pm = -(a_e \pm P_e v_e) \chi_Z xF_3^{\gamma Z} + (2v_e a_e \pm P_e(v_e^2 + a_e^2)) \chi_Z^2 xF_3^Z$$

$$x\tilde{F}_3 = \frac{Y_+}{2Y_-} (\tilde{\sigma}^{e^- p} - \tilde{\sigma}^{e^+ p})$$

$$\begin{aligned} v_f &= (T_3^f - 2e_f \sin^2 \theta_w) \\ a_f &= T_3^f, \end{aligned} \quad \chi_Z = \frac{1}{\sin^2 2\theta_W} \frac{Q^2}{M_Z^2 + Q^2}.$$

$$[F_2^\gamma, F_2^{\gamma Z}, F_2^Z] = \sum_q [e_q^2, 2e_q v_q, v_q^2 + a_q^2] x(q + \bar{q}),$$

$$[xF_3^{\gamma Z}, xF_3^Z] = \sum_q [e_q a_q, v_q a_q] 2x(q - \bar{q}),$$



Very low Q^2 : “photoproduction”

The NC cross section behaves as $1/Q^4$: large cross at small Q^2 .

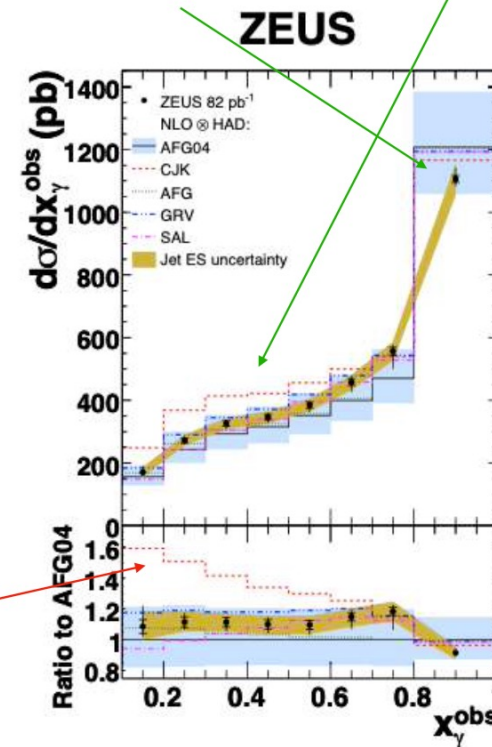
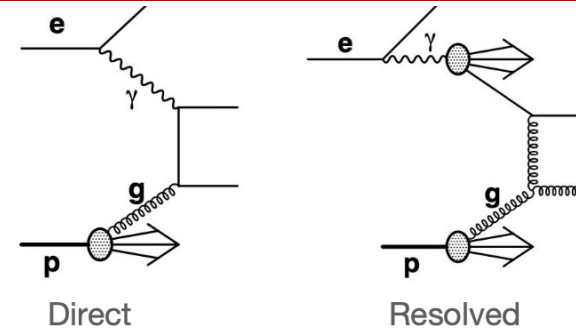
Experimentally in low- Q^2 events ($Q^2 < 1 \text{ GeV}^2$) the scattered electron escapes undetected in the rear beam-pipe hole.

Equivalent-photon approximation: at low Q^2 the ep cross-section can be factored as the product of a photon flux in the electron times gamma-p cross section,

$$\sigma(ep) = f_{\gamma/e} \sigma(\gamma p).$$

Measuring e.g. jet production at low Q^2 is equivalent to measure it in gamma-proton interactions : thus the name photo-production.

The photon itself can fluctuate into q-qbar pairs and develops its own internal structure and PDFs: so photoproduction processes can be divided in direct and “resolved” photoproduction.



Dijets

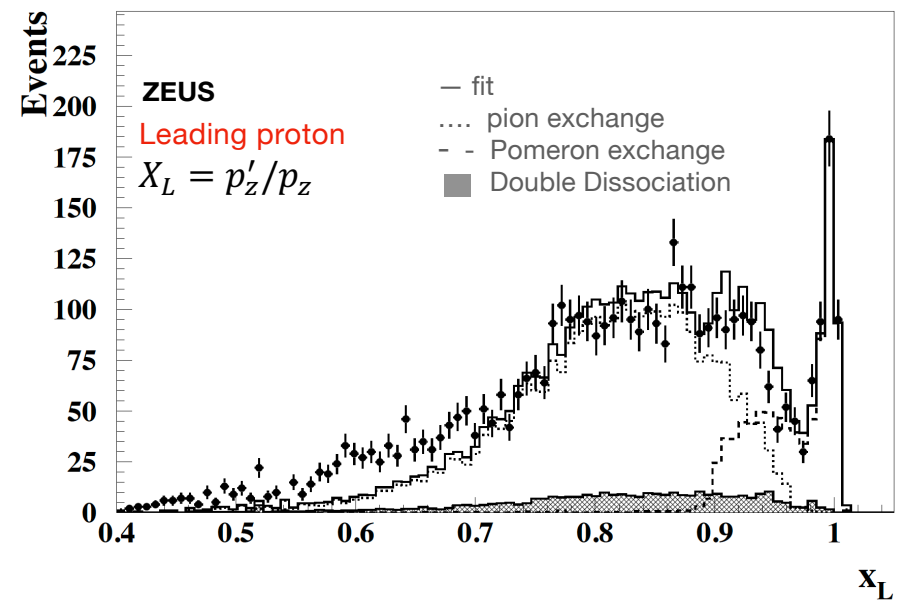
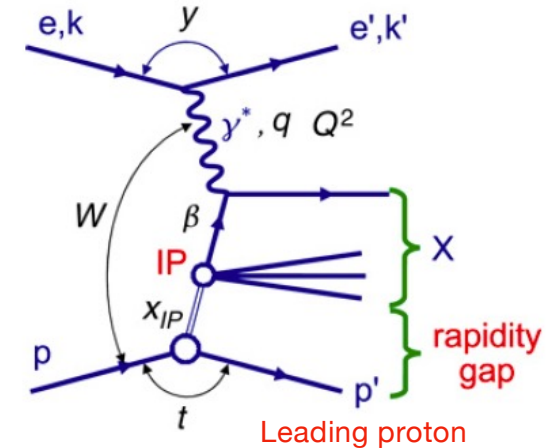
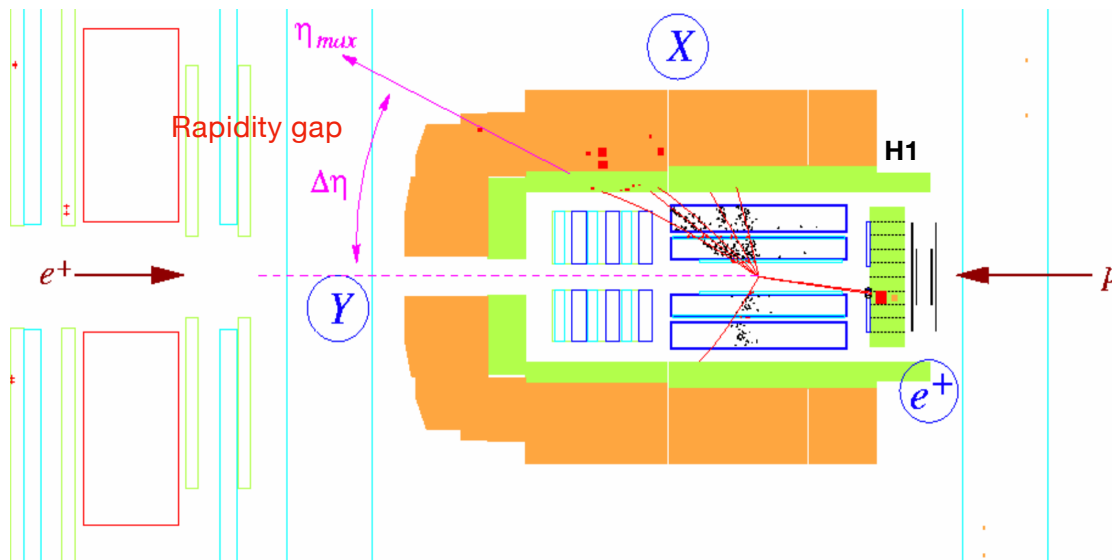
$$x_\gamma^{\text{obs}} = \frac{(E - P_Z)_{\text{jets}}}{(E - P_Z)_X}$$

Hard diffraction: something unexpected* found at HERA

In about 5-10% of all DIS events the proton does not break-up or breaks into a very low mass state, separated from the rest of the hadronic system by a large rapidity gap without hadronic activity.

Two experimental signatures:

- "Large rapidity gap"
- Forward leading proton (measured with roman pots) with $X_L = p'_z/p_z \simeq 1$



(* first evidence of hard diffractive events was found by UA8 at SpS that found "exclusive" $p p \rightarrow p p' \text{ jet-jet}$ events)

Hard diffraction: interpretation

The proton fluctuates into colourless objects (cfr. Regge theory), the dominant object at high energy is the Pomeron, with the same quantum numbers as vacuum.

Diffractive factorisation :

$x_{IP} = 1 - X_L$: fraction of proton momentum carried by the Pomeron

$\beta = x/x_{IP}$: fraction of the Pomeron momentum carried by struck quark

$$F_2^{D(4)} = f_{IP}(x_{IP}, t) F_2^{IP}(\beta, Q^2) \text{ (+ subleading terms)}$$

$f_{IP}(x_{IP}, t)$ is the probability to find a Pomeron in the proton and

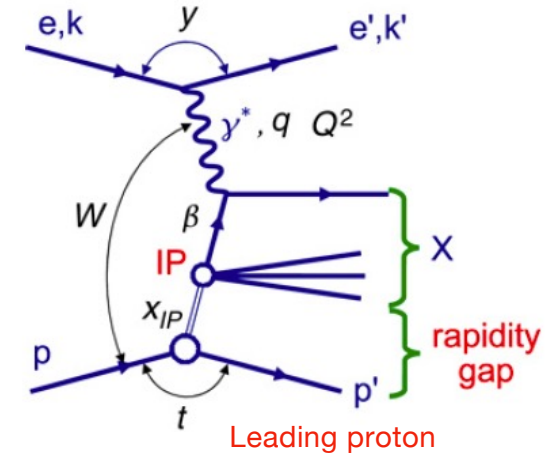
$F_2^{IP}(\beta, Q^2)$

is the structure function F2 of the Pomeron.

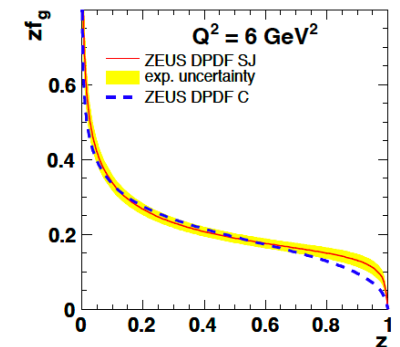
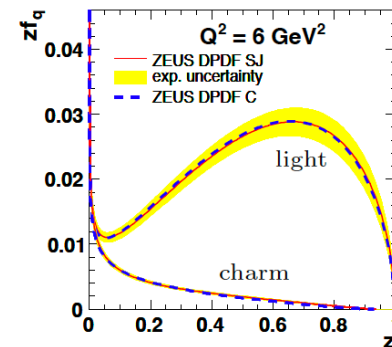
The Pomeron flux is consistent with that obtained from soft physics (e.g. total cross section):

$$f_{IP}(x_{IP}, t) \propto x_{IP}^{1-2\alpha_{IP}(0)} \text{ with } \alpha_{IP}(0) \simeq 1.1$$

While the Pomeron PDFs can be obtained from fits to diffractive data similarly to proton PDF. This model works remarkably well and can explain all HERA diffractive DIS data.



Pomeron PDFs



Something new NOT found at HERA : Leptoquarks

Leptoquarks are possible l-q resonances
Appear naturally in GUT theories

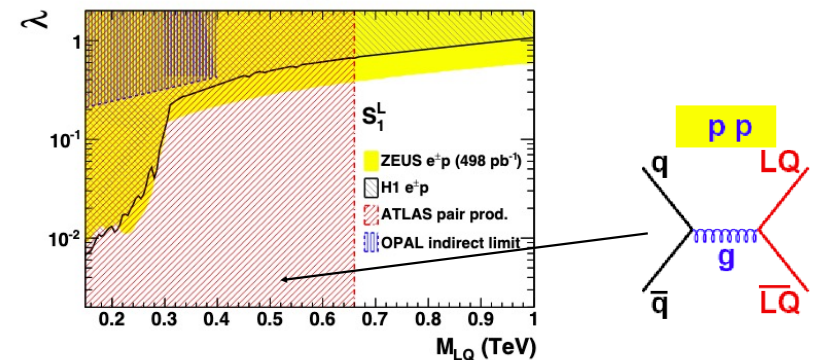
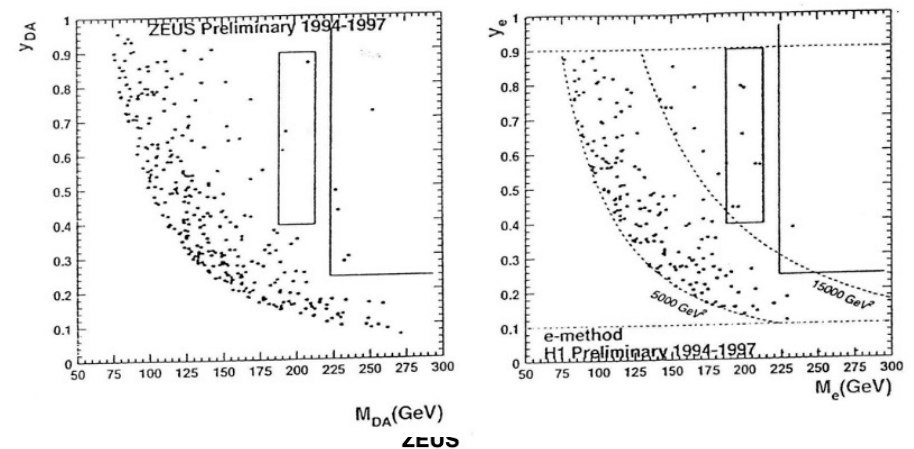
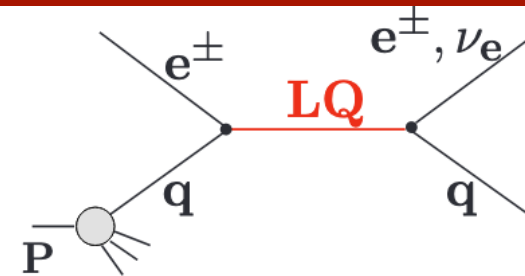
It would show-up as peak at a given value of x:
 $M^2 = (xP + k)^2 = 4xE_p E_e = xs$

A scalar LQ has flat y distribution (isotropic) as opposed to the low-y peaking of normal DIS.

Excess found by H1 in 1994-1997 data. ZEUS also had some mild excess at higher mass. Great excitement, but also “Ample reasons for doubt” (G. Altarelli)

In fact the excesses disappeared in following data taking periods, probably just statistical fluctuations.

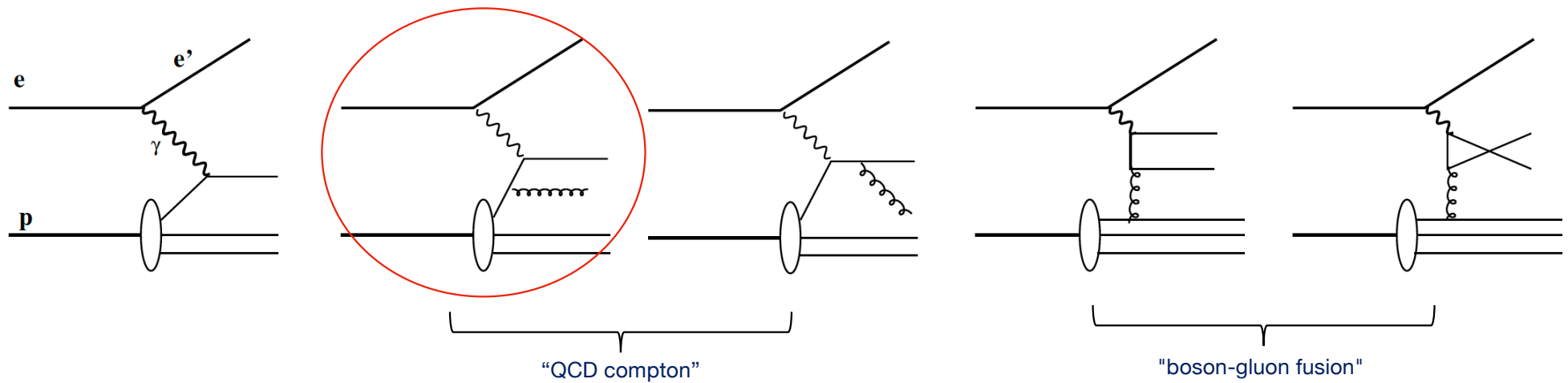
HERA LQ limits are now superseded by LHC



Higher orders and PDFs evolutions

QCD correction to parton model, $O(\alpha_s)$

The corrections for $e p \rightarrow e' X$ at $O(\alpha_s)$ are:



The diagram in red is divergent when the gluon transverse momentum k_T tends to zero

Collinear approximation

Let's have a look at the "QCD Compton" diagrams

New propagator term $1/(p_{virt})^2 = 1/(p_q - k_g)^2$

For gluon transverse momentum $\kappa_T \rightarrow 0$

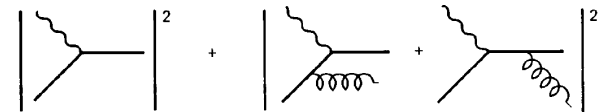
$$1/(p_{virt})^2 = 1/(\kappa_T)^2$$

The propagator term diverges when the gluon is "collinear"

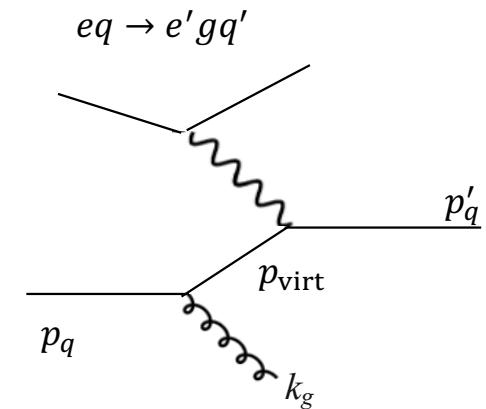
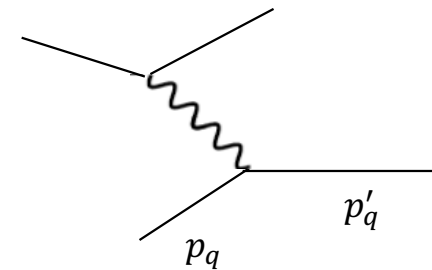
Doing the calculation properly it is found that the cross section diverges as $1/(\kappa_T)^2$ (not as $1/(\kappa_T)^4$ as expected from the pure propagator term)

The cross section will be dominated by the region around the pole $\kappa_T \rightarrow 0$, so, instead of calculating exactly the new contribution we can concentrate to the region around the pole: collinear approximation

$$\int_{k_T^{min} \rightarrow 0}^{k_T^{max}} dk_T \frac{1}{k_T^2} f(k_T) \simeq f(0) \int_{k_T^{min} \rightarrow 0}^{k_T^{max}} \frac{1}{k_T^2}$$



$eq \rightarrow e' q'$



QCD correction to eq scattering

Neglecting terms that are relevant only at high κ_T
the cross section for this process can be written as :

$$\frac{d^2\hat{\sigma}(eq \rightarrow e' g q')}{dQ^2 d\kappa_T^2} = \frac{d\hat{\sigma}(eq \rightarrow e' q')}{dQ^2} \frac{\alpha_S}{2\pi} \frac{1}{\kappa_T^2} P_{qq}(z)$$

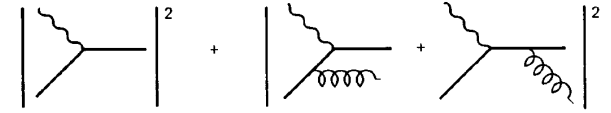
Where $P_{qq}(z)$ is the quark-quark splitting function: the probability to find a quark with a fraction z of the initial quark after gluon emission :

$$P_{qq}(z) = \frac{4}{3} \frac{1+z^2}{1-z}$$

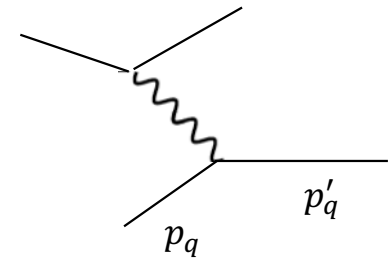
Integrating over κ_T^2 and exploiting the relation $(\kappa_T^2)_{\max} \simeq Q^2$:

$$\frac{d^2\hat{\sigma}(eq \rightarrow e' g q')}{dQ^2} = \frac{d\hat{\sigma}(eq \rightarrow e' q')}{dQ^2} \frac{\alpha_S}{2\pi} \log \frac{Q^2}{\mu^2} P_{qq}(z)$$

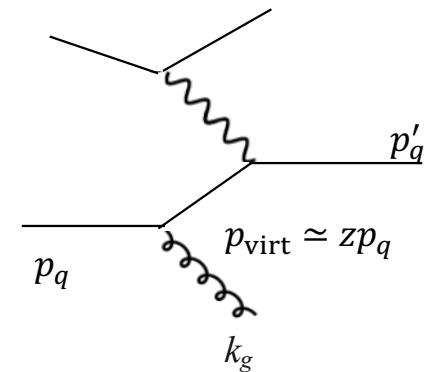
Where μ^2 is a cutoff and the parton cross section is divergent for $\mu^2 \rightarrow 0$



$eq \rightarrow e' q'$



$eq \rightarrow e' g q'$

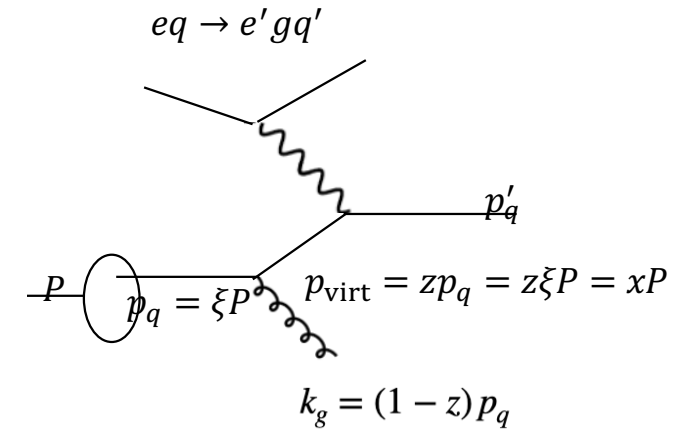


QCD correction to parton model

To obtain the ep cross section we should integrate on all the possible momenta of the incoming quark $p_q = \xi P$:

$$\frac{d^2\sigma}{dx dQ^2} = \sum_{q=u,d,\dots} \frac{d\hat{\sigma}^0}{dQ^2} \int_x^1 \frac{d\xi}{\xi} f_q(\xi) \left(\delta(1-z) + \frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2} \right)$$

$\uparrow$ $\uparrow$
 Leadin order term QCD Ccompton term



Note x is the momentum fraction of the quark seen by the electron, not the one from the proton density !

$$z = x/\xi$$

QCD factorization and PDF dependence on Q^2

The divergence for $\mu^2 \rightarrow 0$ which corresponds to collinear emission is not physical, when the $1/\kappa_T$ of the emitted gluon is of the size of typical hadronic scales non-perturbative effects (e.g. confinement) become effective

The divergence can be absorbed in a re-definition of $f_i(x) \rightarrow f_i(x, Q^2)$:

$$\frac{d^2\sigma}{dx dQ^2} = \sum_{q=u,d,\dots} \frac{d\hat{\sigma}^0}{dQ^2} f_q(x, Q^2)$$

$$f_q(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} f_q(\xi, \mu_0^2) \left(\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \log \frac{Q^2}{\mu_0^2} \right)$$

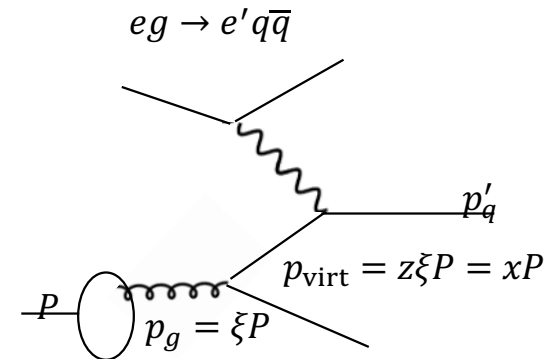
μ_0^2 is an arbitrary value of Q^2 where we like to “fix” the PDF

Altarelli-Parisi (DGLAP) PDF evolution equation (from eq. above, choose $\mu_0^2 = Q^2$ and derivate in $\log Q^2$):

$$\frac{df_q(x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} f_q(\xi, Q^2) P_{qq}\left(\frac{x}{\xi}\right)$$

QCD correction to parton model: gluon initiated diagram

Adding also the “boson-gluon fusion” diagram



$$\frac{d^2\sigma}{dx dQ^2} = \sum_{q=u,d,\dots} \frac{d\hat{\sigma}^0}{dQ^2} \int_x^1 \frac{d\xi}{\xi} \left[f_q(\xi, \mu_0^2) \left(\delta(1-z) + \frac{\alpha_S}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2} \right) + f_g(\xi, \mu^2) \frac{\alpha_S}{2\pi} P_{gq}(z) \log \frac{Q^2}{\mu^2} \right]$$

Even if the gluon is not charged, the gluon density enters in the cross section

Full DGLAP evolution equations

$$\frac{df_q(x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} (P_{qq}(z)f_q(\xi, Q^2) + P_{gq}(z)f_g(\xi, Q^2))$$

$$\frac{df_g(x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} (P_{qg}(z)f_q(\xi, Q^2) + P_{gg}(z)f_g(\xi, Q^2))$$

The variation of the PDFs (and structure functions) with Q^2 is given by DGLAP

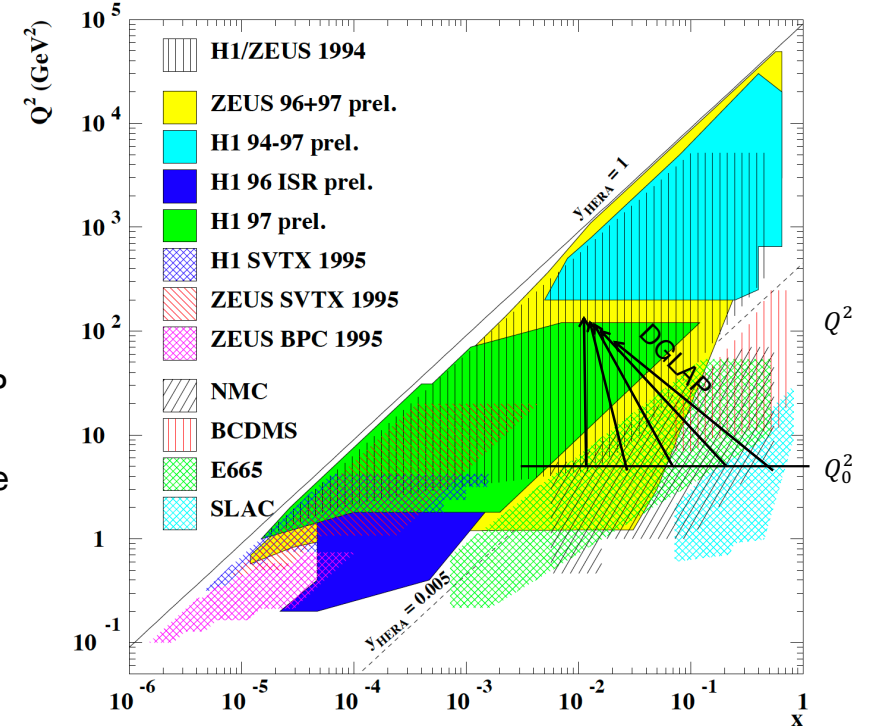
Given the PDFs at a starting scale, Q_0^2 , they can be re-calculated at any scale Q^2 by solving DGLAP equations.

$$P_{qq} = \frac{4}{3} \left(\frac{1+z^2}{1-z} \right) \quad \text{LO}$$

$$P_{gq} = \frac{4}{3} \left(\frac{1+(1-z)^2}{z} \right)$$

$$P_{qg} = \frac{1}{2} (z^2 + (1-z)^2)$$

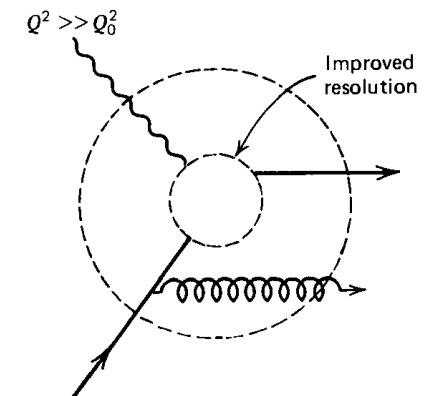
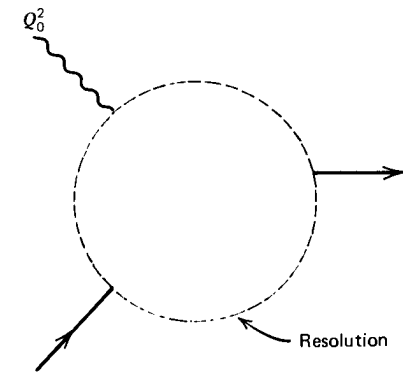
$$P_{gg} = 6 \left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z) \right)$$



DGLAP insight

Physical interpretation: as the scale increases we see inside the proton with higher resolution and we can resolve more partons.

$$\Delta b \sim \frac{\hbar c}{\sqrt{Q^2}} = \frac{0.197}{\sqrt{Q^2}} \text{ GeV fm}$$



How To measure Structure Functions

What can be measured directly are cross sections and structure functions, not PDFs !

How to measure:

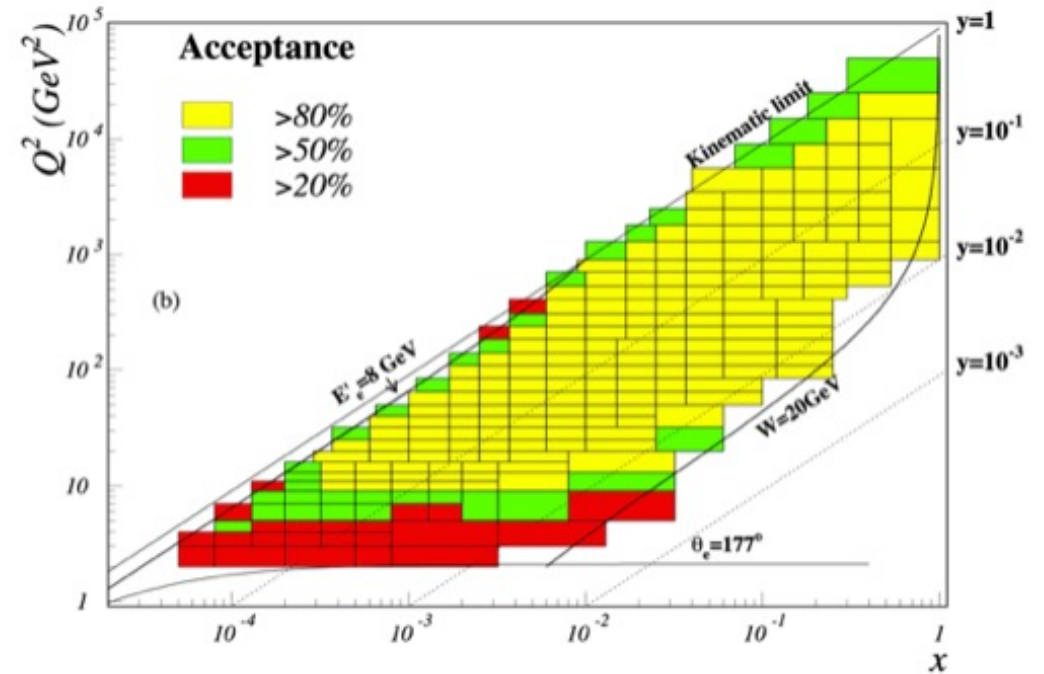
- Divide the x, Q^2 plane in bins with size $\Delta x_i \Delta Q_i^2$
- Reconstruct x, Q^2 of each event and assign it to a bin i
- Then the average cross section in the i^{th} bin is given by

$$\sigma_i = \frac{N_i^{\text{rec}} - N_i^b}{A_i L}$$

where N_i^{rec} is the number of events reconstructed in the bin, N_i^b is the estimated background, L is the data-set luminosity and A_i is the bin acceptance that can be obtained by Monte Carlo as the ratio between reconstructed and generated events:

$$A_i = \left(\frac{N_i^{\text{reco}}}{N_i^{\text{gen}}} \right)_{\text{MC}}$$

- then the double-differential cross section at the center of the bin $x_{\text{ref}}, Q_{\text{ref}}^2$ is obtained by the bin average with a bin-centering correction C_i



$$\frac{d^2\sigma}{dx dQ^2}(x_{\text{ref}}, Q_{\text{ref}}^2) = \frac{\sigma_i}{\Delta x_i \Delta Q_i^2} \cdot C_i$$

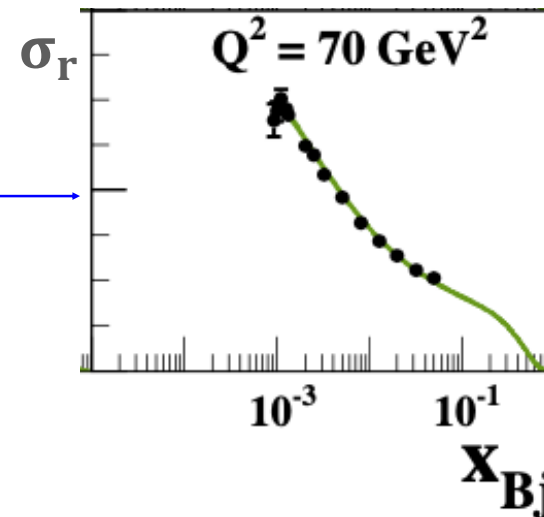
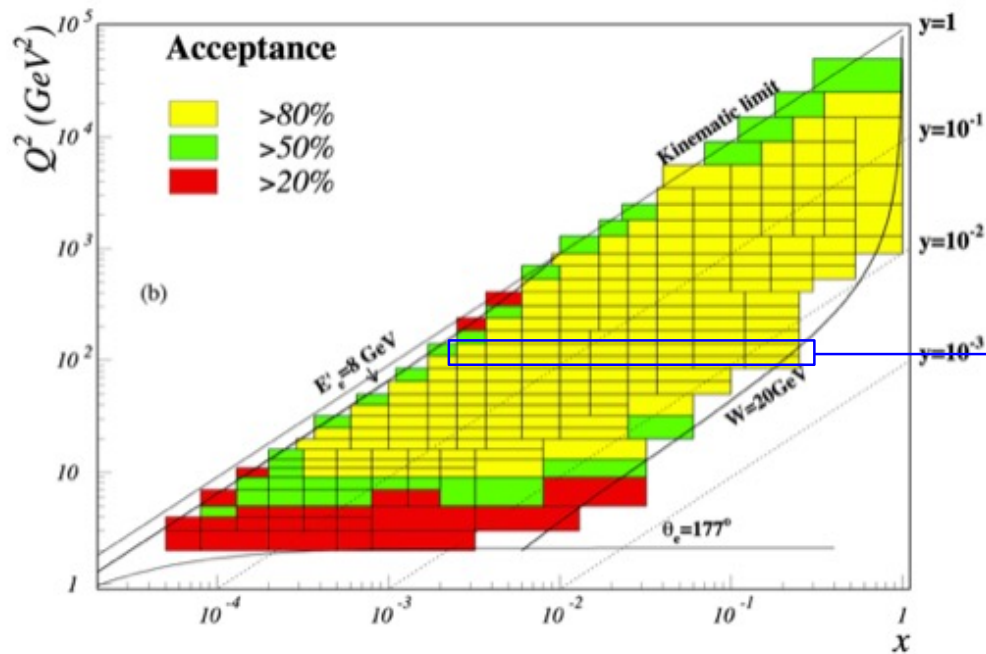
How To measure Structure Functions

- Often what is shown is a “reduced cross section” σ_r which is equivalent to F_2 in leading order“

Reminder, at LO and neglecting Z exchange:

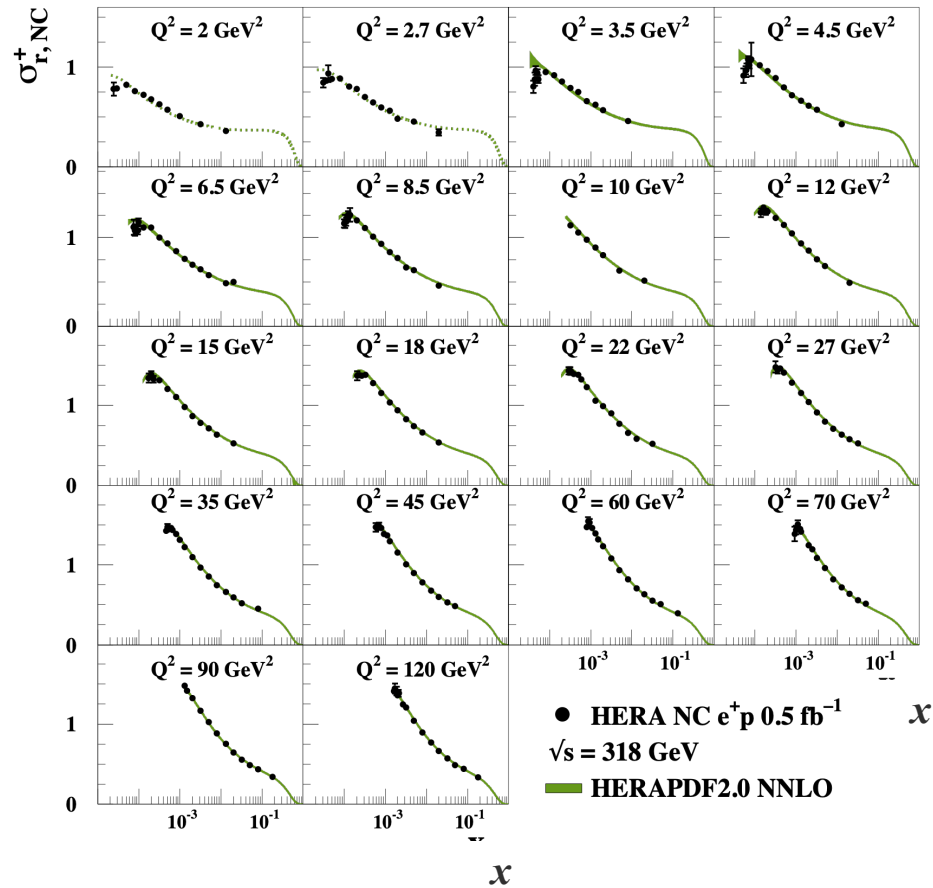
$$\frac{d\sigma}{dx dQ^2} = \frac{2\pi\alpha}{Q^4} \left[1 + (1-y)^2 \right] F_2(x, Q^2)$$

$$\sigma_r = \frac{d^2\sigma}{dx dQ^2} \left[\frac{2\pi\alpha}{Q^4} \left[1 + (1-y)^2 \right] \right]^{-1}$$

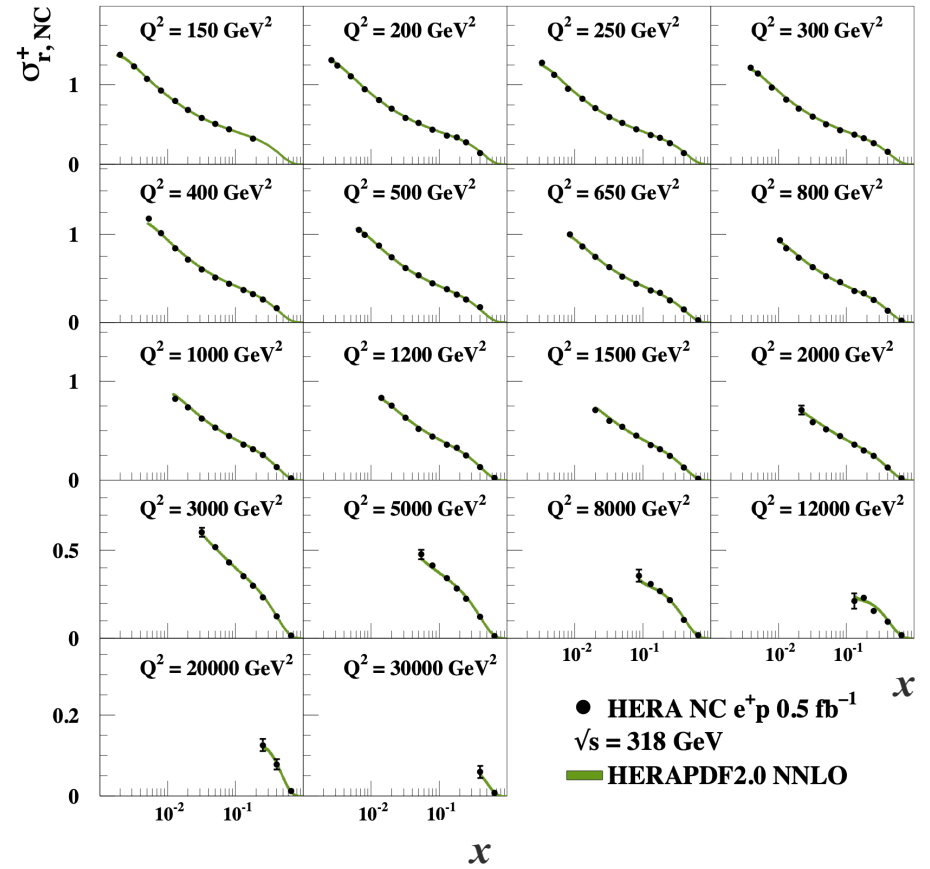


HERA measurement: e^+p Neutral Current

H1 and ZEUS

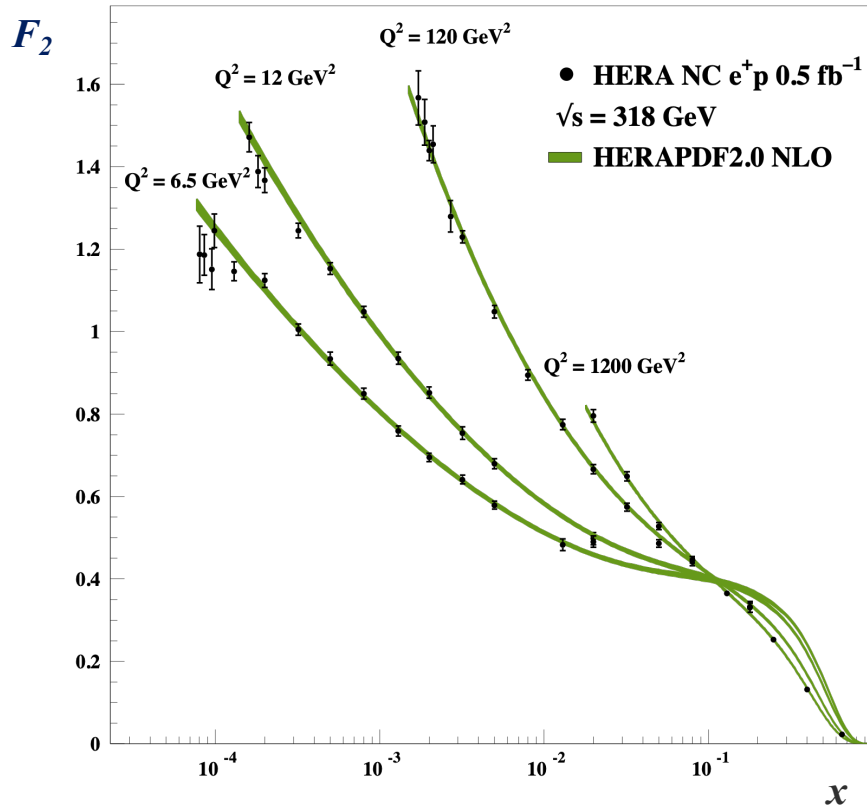


H1 and ZEUS

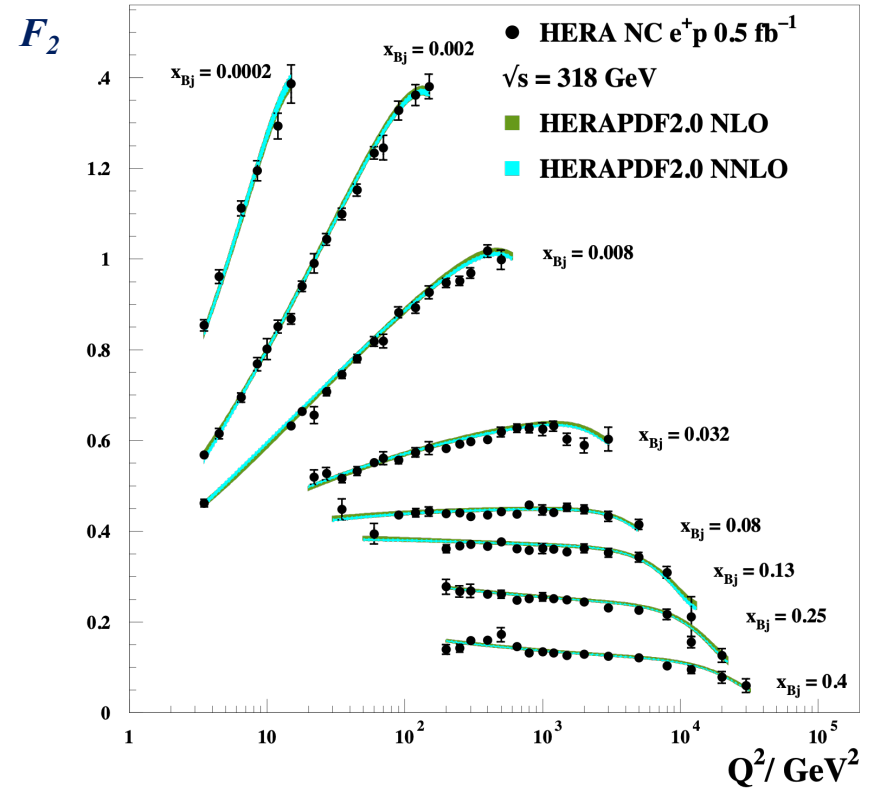


A zoom into few bins

H1 and ZEUS



H1 and ZEUS



At medium/low x F_2 increases with Q^2 (more gluons, more sea)
 At high x , F_2 decreases with Q^2 (valence quarks irradiate)
 Scaling at $x = 0.1$ (by chance where was measured originally at SLAC !)

Extraction of parton densities

Extracting the parton density functions

The parton densities at a fixed scale Q_0^2 can not be calculated from first principles, should be extracted from data.

How many independent functions ?

At low scale Q_0^2 , lower than charm mass squared, the only partons are $u, d, s, \bar{u}, \bar{d}, \bar{s}, g$

(Charm and beauty are produced dynamically from gluon splitting when $Q^2 > m_c^2, m_b^2$ respectively)

To reduce the freedom we can make some assumptions and put together the sea quarks:

$$S(x) = \bar{u}(x) + \bar{d}(x) + \bar{s}(x) \quad (\text{total sea})$$

$$\Delta S = \bar{u}(x) - \bar{d}(x) \quad (\text{u-d sea asymmetry})$$

and assume $s(x) = \bar{s}(x) = f_s \bar{d}(x)$ where f_s is a strange suppression factor taken by other measurements.

Thus we reduce to 5 PDFs: $u_v(x), d_v(x), S(x), \Delta S(x), g(x)$

where subscript v is for "valence": $u(x) = u_v(x) + \bar{u}(x)$ and $d(x) = d_v(x) + \bar{d}(x)$

PDF parametrization and sum rules

Typical (simple) parametrization at starting scale Q_0^2

$$xf(x, Q_0^2; \mathbf{p}) = p_1 x^{p_2} (1-x)^{p_3} (1+p_4 x)$$

normalization
low x
high x
intermediate

Some parameters are fixed by sum rules:

$$\int dx (u - \bar{u}) = 2, \quad \int dx (d - \bar{d}) = 1, \quad s = \bar{s}, \quad c = \bar{c} \quad \text{Proton quark content}$$

$$\int dx x (u + \bar{u} + d + \bar{d} + \dots + g) = 1 \quad \text{Momentum conservation}$$

From a parametrization at a starting scale Q_0^2
we can obtain the PDFs at any scale by solving
numerically DGLAP equations :

$$f(x, Q_0^2; \mathbf{p}) \text{ -- DGLAP ---> } f(x, Q^2; \mathbf{p})$$

Sensitivity of different inclusive DIS processes to PDFs

Different DIS processes are sensitive to different PDF combinations:

$$\text{NC} \quad \propto \quad 4/9 (u + c + \bar{u} + \bar{c}) + 1/9 (d + s + \bar{d} + \bar{s} + b + \bar{b})$$

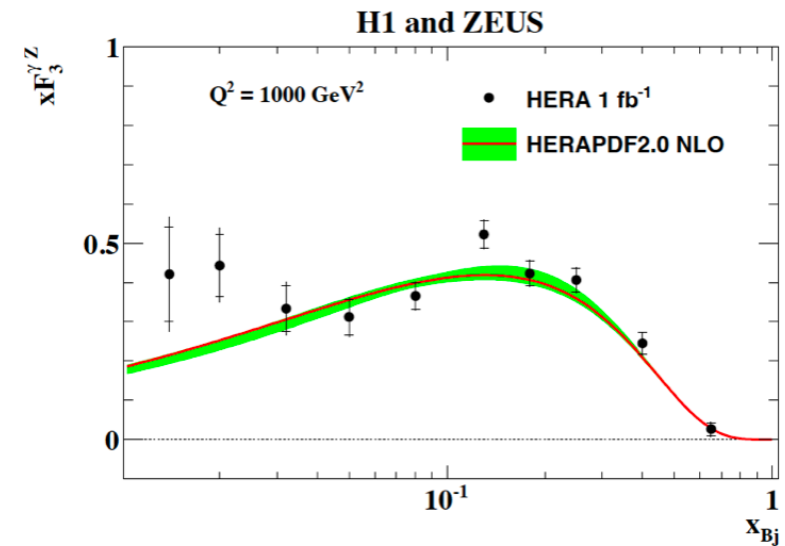
$$\text{CC e-p} \quad \propto \quad u + c + (1 - y)^2 (\bar{d} + \bar{s} + \bar{b})$$

$$\text{CC e+p} \quad \propto \quad \bar{u} + \bar{c} + (1 - y)^2 (d + s + b)$$

we can separate 5 different quark combinations

In addition the parity violating part of NC ($Z\gamma$ interference) is sensitive to a different quark combination:

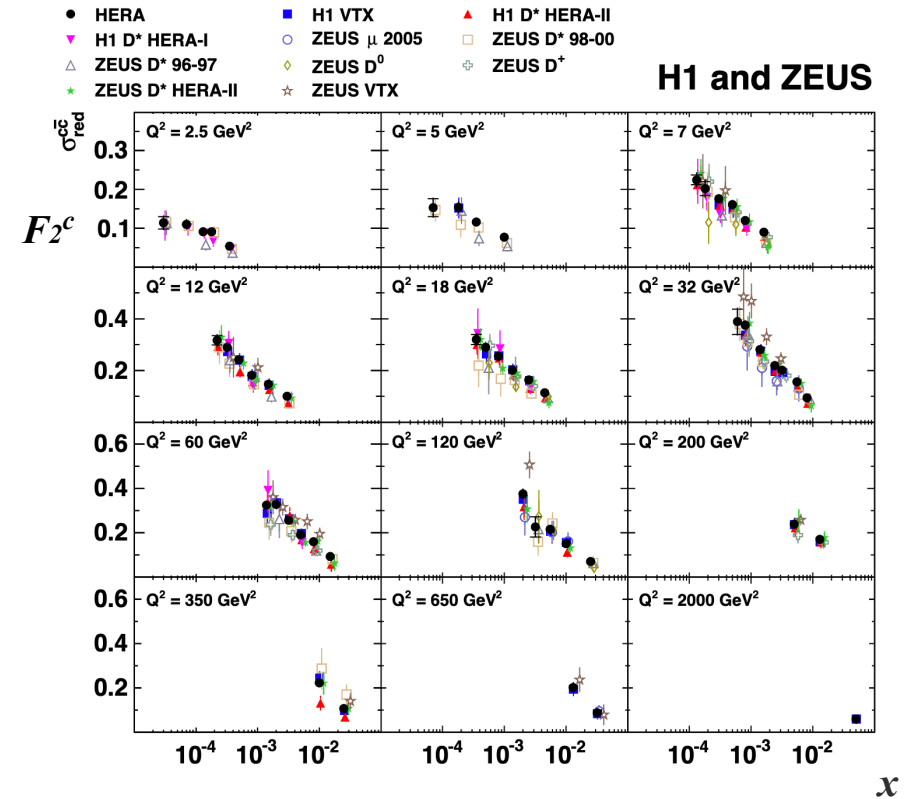
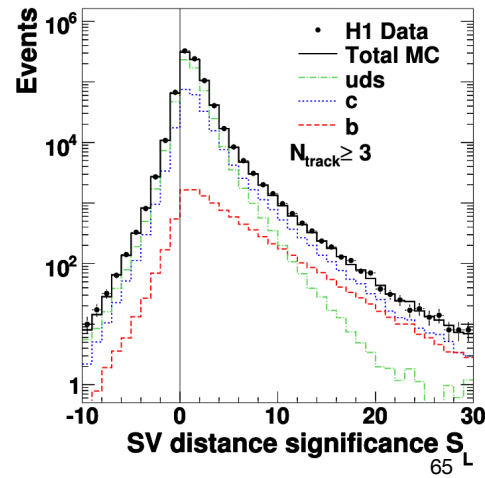
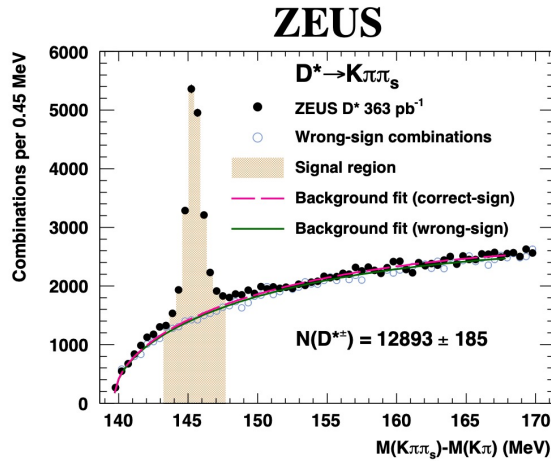
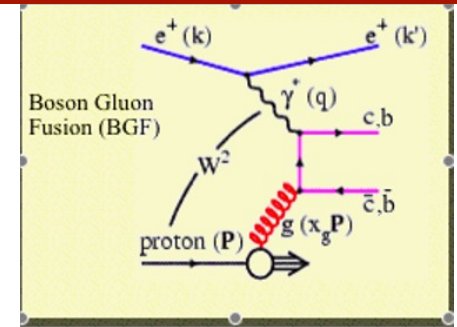
$$\text{High-}Q^2 \text{ NC} : \quad xF_x \propto \sigma(e^-p) - \sigma(e^+p) \propto 2u_v + d_v$$



Tagging struck quarks in final states: charm and beauty PDFs

The production of heavy quarks (c and b) can be tagged from the presence in the final state of D mesons, muons or displaced vertices. Various techniques have been used at HERA to determine $F_2^{c,b}$ the component of F_2 involving c or b quarks.

$c\bar{c}$ pairs are abundantly generated from gluon splitting, and are responsible for 40% of cross section at low-x and high- Q^2 ,
In standard PDF fits they are not considered as part of the proton PDF at scales $Q_0^2 < m_c^2$



PDF Fitting

The method for extracting the PDFs consists in building a global χ^2

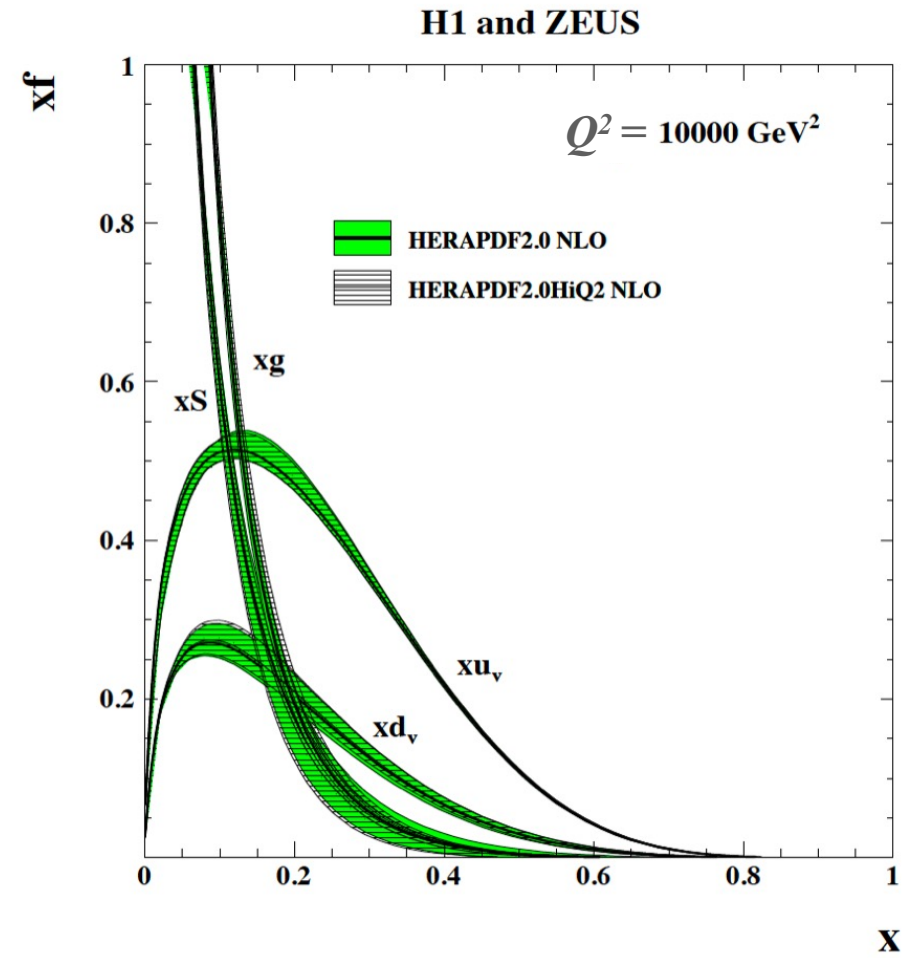
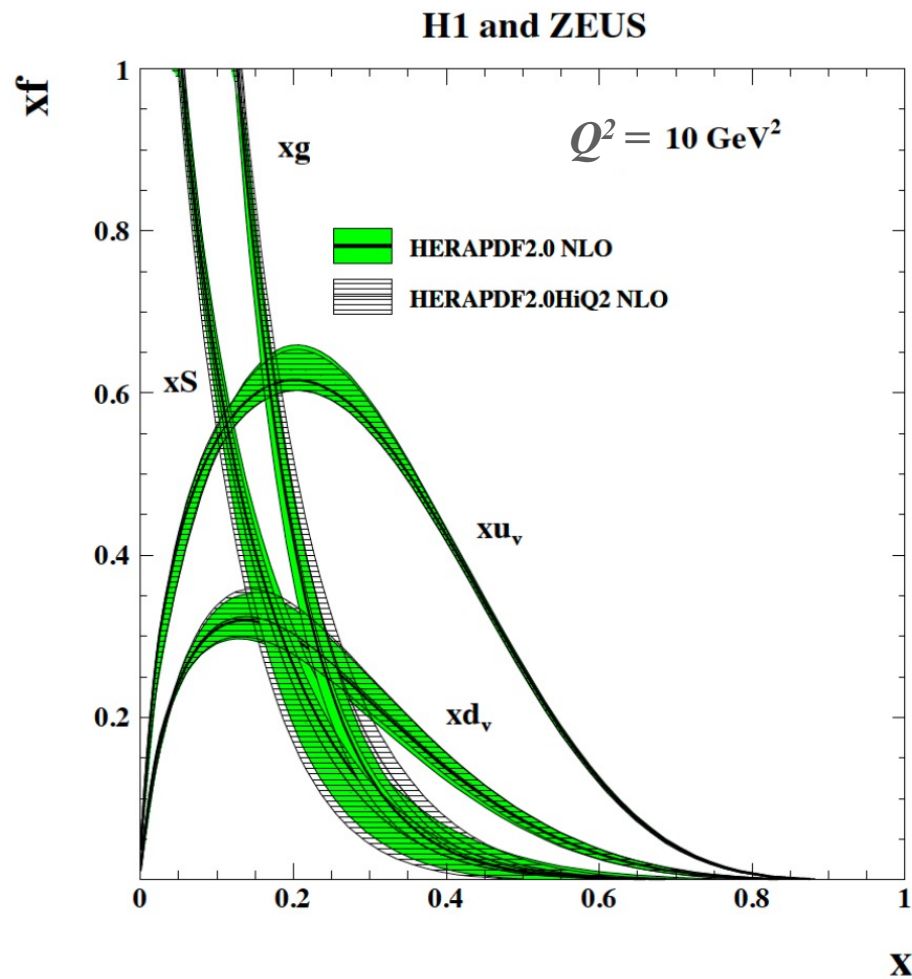
For each measured observable o_i^{meas} (e.g. a NC differential cross section bin), with uncertainty δ_i , we can calculate a theoretical prediction $o_i^{\text{theo}}(\mathbf{p})$ based on a parametrization of the PDFs at the starting scale ($Q_0^2 = 1.9 \text{ GeV}^2$) with a set of parameters $\mathbf{p}$.

$$\chi^2 = \sum_i \frac{(o_i^{\text{meas}} - o_i^{\text{theo}}(\mathbf{p}))^2}{\delta_o^2}$$

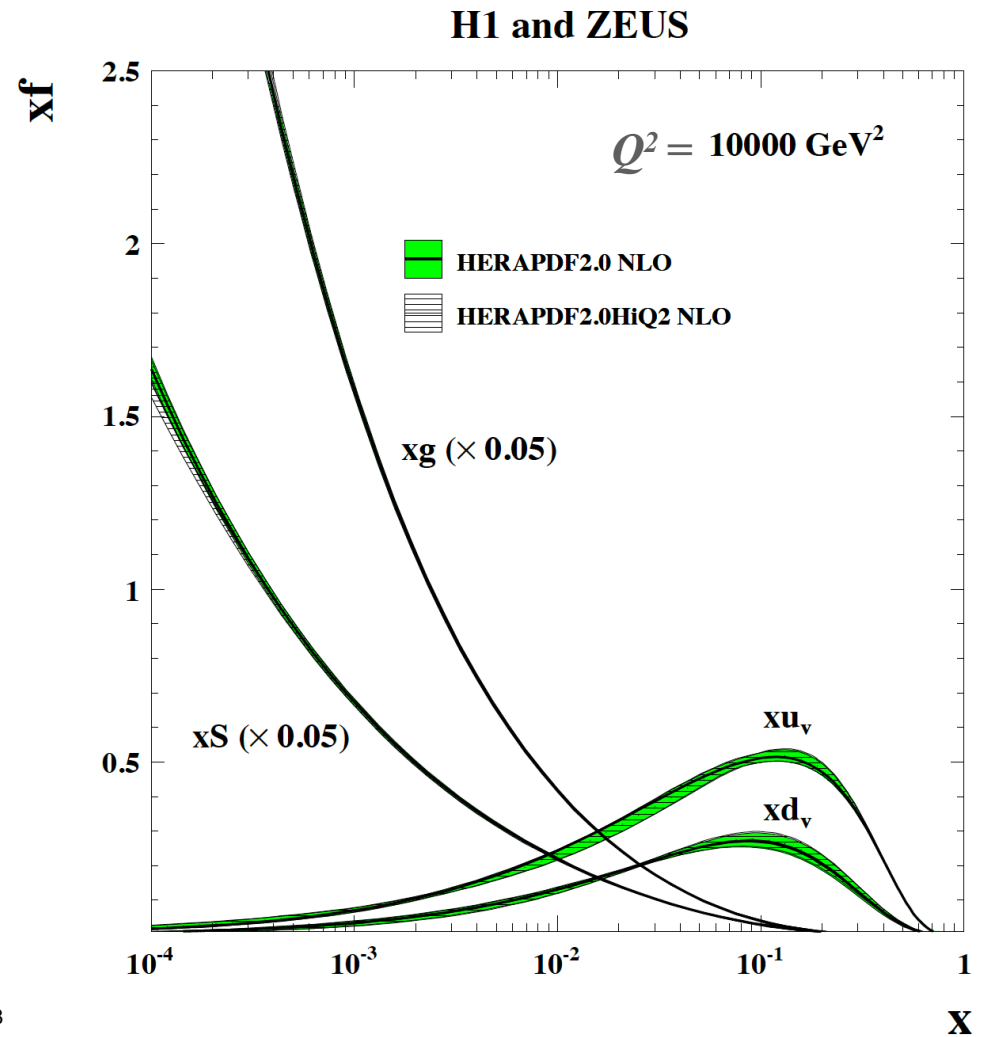
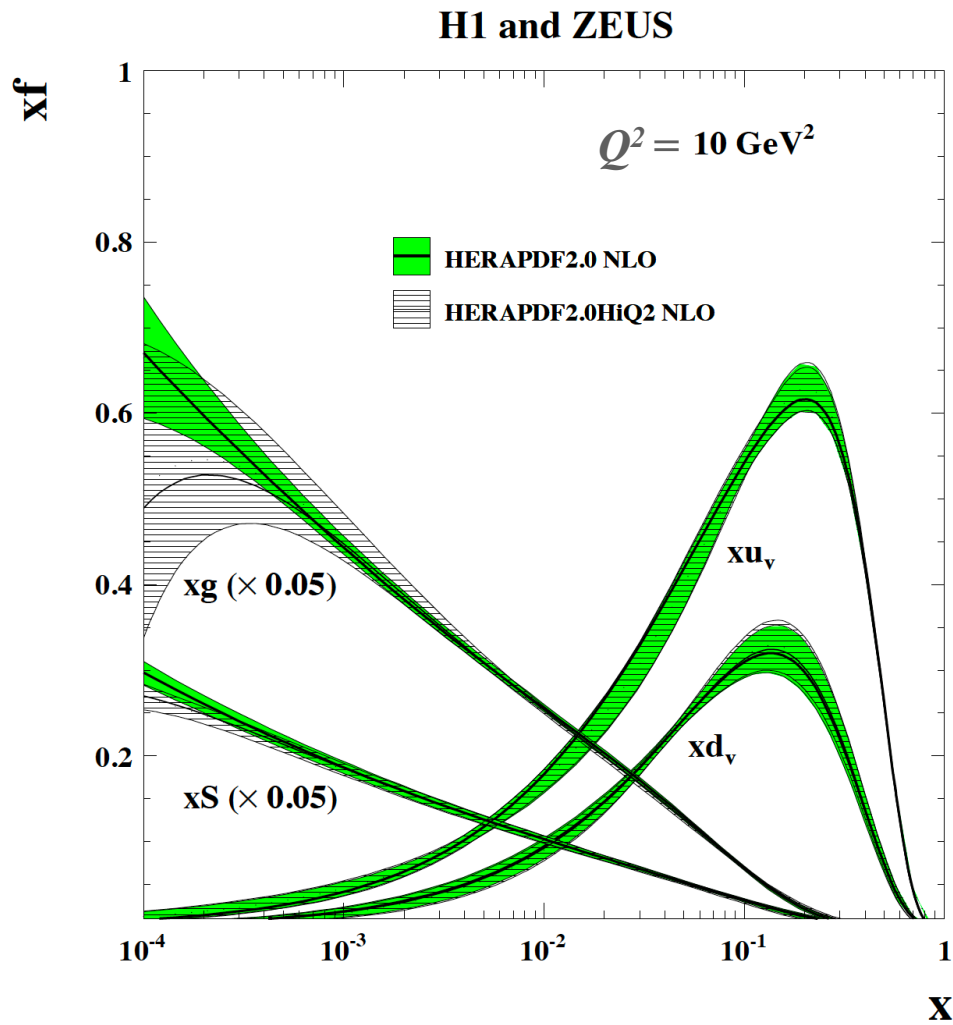
(actually to take into account systematics a more complex definition of χ^2 is used)

Then the set of parameters $\mathbf{p}$ that minimizes χ^2 is found.

PDFs based on HERA data (linear x scale)



PDFs based on HERA data (log x scale)

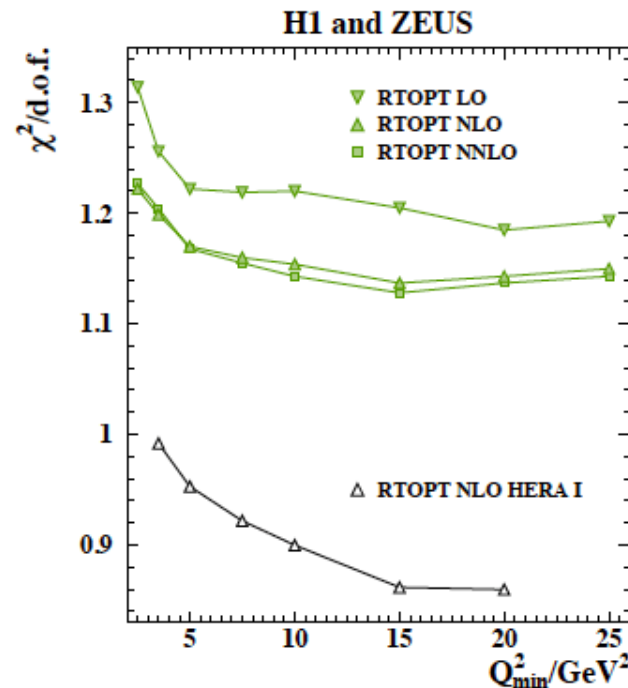


Something strange at low x ?

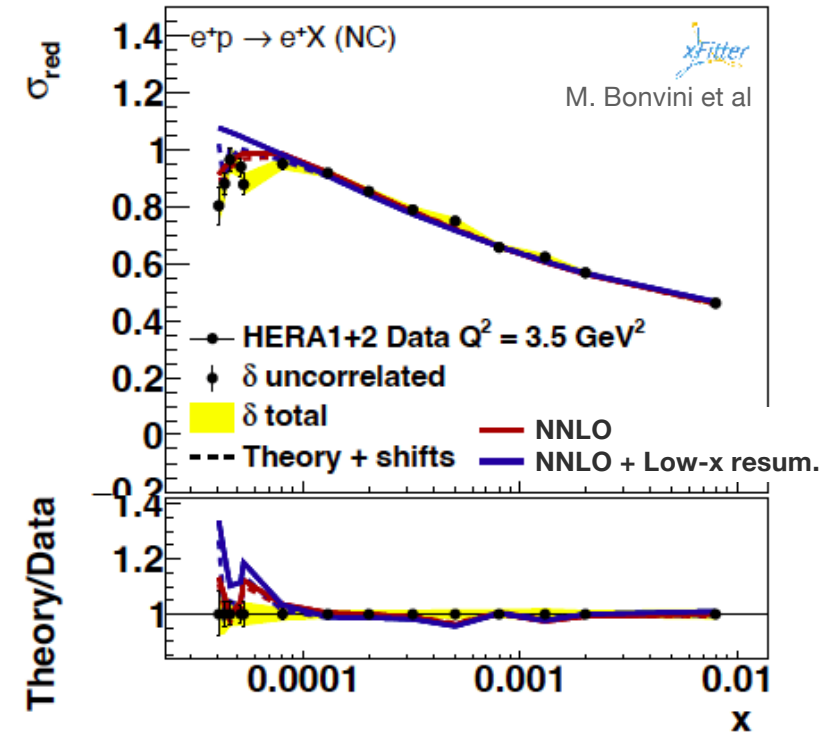
At Low Q^2 and low x the PDF fits to HERA data are not so good

The reason is unclear, various possibilities have been proposed:

- Low-x resummation effects (it is known that for very low x the standard DGLAP evolution breaks down and large logs ($1/x$) should be resummed (BFKL evolution))
- Parton saturation: when gluon density is too high we should include recombination in evolution equations
- Some non-perturbative effect at low Q^2



χ^2 of fit to HERA data vs minimum Q^2 to include a data point in the fit



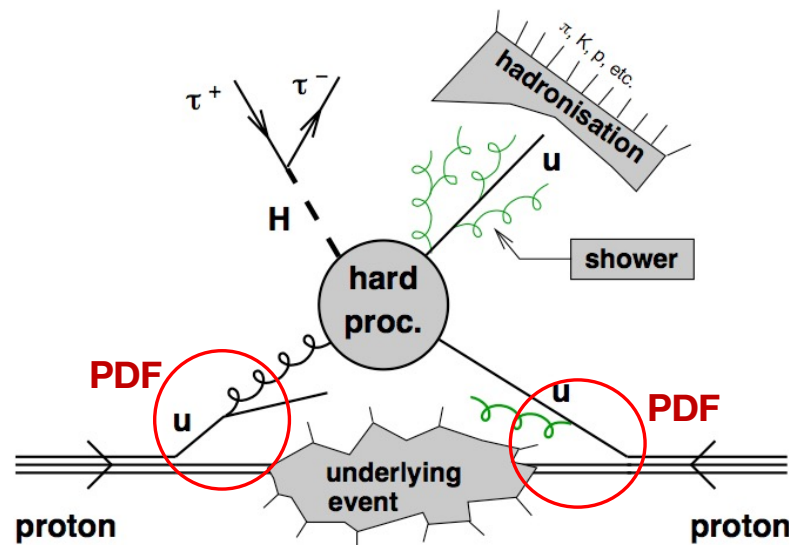
PDFs and hadronic collisions: going beyond DIS

QCD factorization theorem for hadronic collisions :

$$\sigma_{h_a h_b \rightarrow X} = \sum_{i=u,d,\dots,g} \sum_{j=u,d,\dots,g} \iint \frac{dx_a}{x_a} \frac{dx_b}{x_b} f_i(x_a, \mu_F^2) f_j(x_b, \mu_F^2) \hat{\sigma}_{ij \rightarrow X}(s x_a x_b, \mu_F^2)$$

Parton densities
Partonic cross section

The PDFs are evaluated at the factorization scale μ_F which gives the scale below which initial state splittings are absorbed in the PDFs evolution. The result should not change when the scale is changed, because the change in PDF should be compensated by the change in the hard-scattering cross section. There is a residual dependence due to missing higher orders.



PDFs and hadronic collisions: Global PDF Fits

Traditional approach : extract PDFs from DIS data => use to calculate cross sections at hadron colliders

=> nice test of QCD, if we find deviations it may be new physics

=> Historically (apart from Drell-Yan) processes in hh were less precise both theoretically and experimentally than DIS

Global approach: add hadron collision data as input to global PDF fits

=> experimentally we can exploit very precise measurements from LHC for many different processes

=> some PDF (e.g. gluon) poorly constrained by DIS experiments at high / low x

=> theoretical calculations for many hh processes are now very precise (e.g. DY, top pair production, Z+j)

=> Possible drawback : we should be careful not to absorb new physics in PDFs !

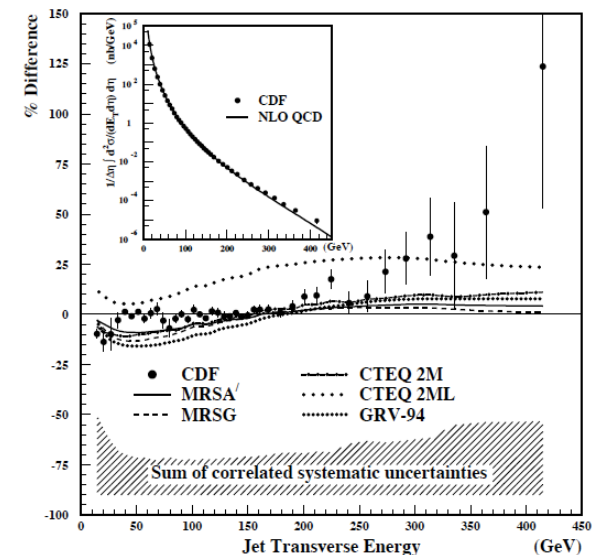
=> Don't use processes that could contain BSM physics oin PDF fits.

CDF excess in high E_T jets in 1995

New physics or PDFs ?

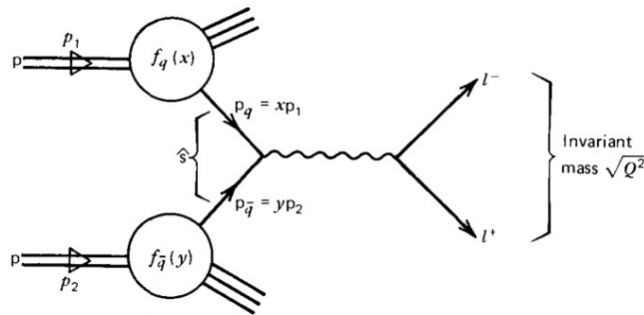
Eventually it was found that the standard PDFs were didn't have enough constraints on gluon density at high-x

Important to assess the uncertainty associated to the PDFs



LHC kinematics

Example: Drell-Yan process

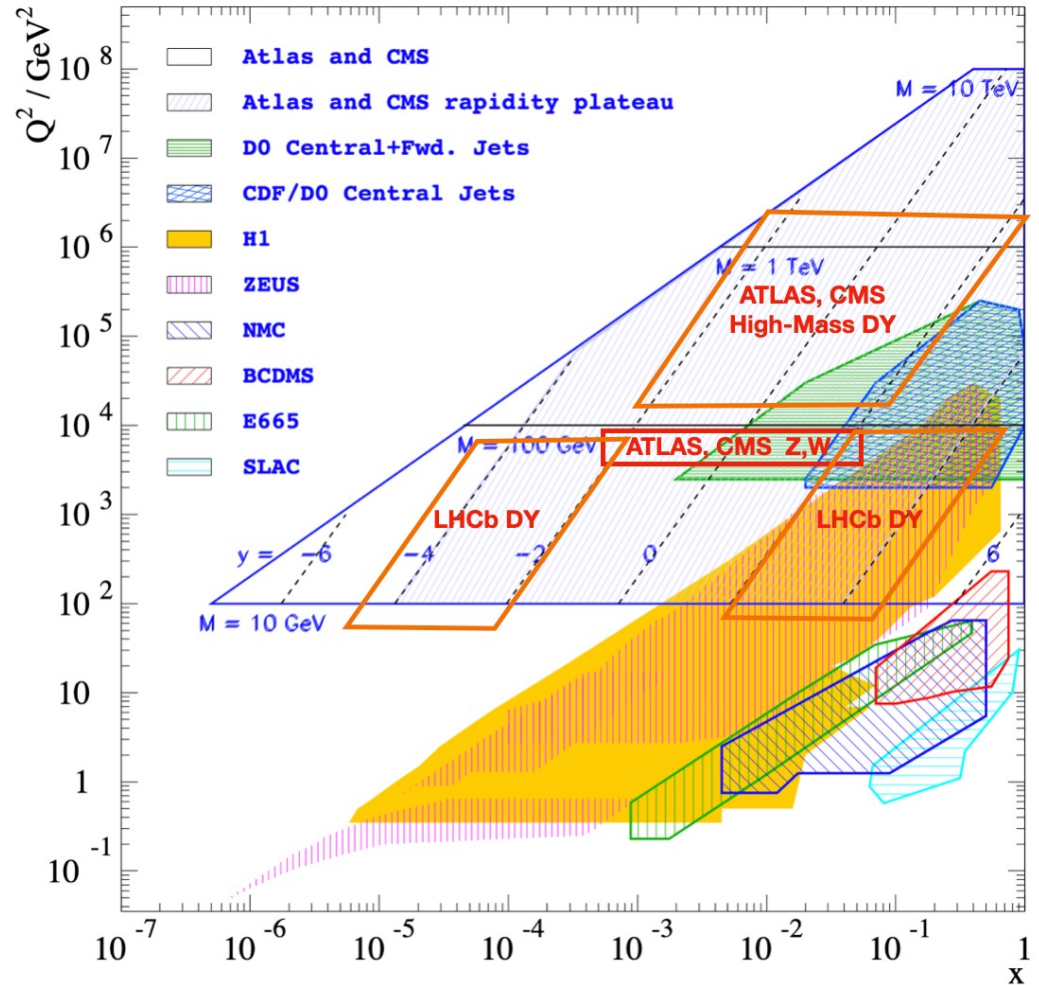


At LO there is a relation between mass M and rapidity y (that are measured observables) and x , Q^2 of the PDFs:

$$Q^2 = M^2 = \hat{s} = x_1 x_2 s$$

$$x_{1,2} = \frac{M}{\sqrt{s}} \exp(\pm y)$$

$$y = \frac{1}{2} \log \frac{E + p_z}{E - p_z} = \frac{1}{2} \log \frac{x_1}{x_2}$$



Global fits to PDFs

Global fits include many measurements from HERA, fixed target experiments and hadron colliders

Important to select a subset of measurement with well controlled experimental and theoretical uncertainties

Experiment	Process	Measurement	Proton PDF combination	
HERA + fixed target DIS	ep → e X	NC inclusive mid Q2	$4/9(u+c+\bar{u}+\bar{c})+1/9(d+s+\bar{d}+\bar{s})$	sum of quarks weighted with electric charge
		$dF_2/d\ln Q^2$	g	scaling violation
		$F_L=(F_2-2xF_{1,1})$	g	longitudinal structure function
HERA	e+p → e+X vs e-p → e- X	NC high-Q2 e- - e+	$g_1(u+c+\bar{u}+\bar{c})+g_2(d+s+\bar{d}+\bar{s})$	Z exchange
	e-p → ν X	CC e-	(u+c),(d+s)	W-
	e+p → ν X	CC e+	(d+s),(u+c)	W+
	ep → e c X	semi-inclusive charm NC	c	charm tagging
	ep → e b X	semi-inclusive beauty NC	b	b tagging
	ep → e jet X		g	jets
fixed target DIS NC (l = e, μ)	lp→eX vs lD→eX	DIS on deuterium low Q2	u/p	deuterium target
fixed target DIS CC (ν)	ν N → μ X	CC		large nuclear uncertainties
fixed target DIS CC (ν)	ν N → μ c X	CC (dimuon)	s	large nuclear uncertainties
LHC	pp → l+l-	Low Mass DY	$4/9(\bar{u}u+c\bar{c})+1/9(\bar{d}d+s\bar{s})$	
	pp → l+l-	Z pole DY	$\sum c_i q_i^2$	
	pp → l+l-	High Mass DY	$\sum c_i^2 q_i^2$	
	pp → Z + j	ZpT	gq, $\bar{q}q$	
Tevatron	p-antip → l+l-	Low Mass DY	$4/9(uu+cc+\bar{u}\bar{u}+\bar{c}\bar{c})+1/9(dd+ss+\bar{d}\bar{d}+\bar{s}\bar{s})$	
	p-antip → l+l-	Z pole DY	qq + $\bar{q}q$	
	p-antip → l+l-	High Mass DY	qq + $\bar{q}q$	
LHC	pp → l+ν X	W+	$\bar{u}d + c\bar{s}$	
	pp → l- ν X	W-	$d\bar{u} + s\bar{c}$	
	pp → l- ν c X	W+c	sg	
Tevatron	p-antip → l ν	W+/-	ud + cs + ...	
Fixed target DY	p/π N → l+ l-	NC	$4/9(uu+cc+\bar{u}\bar{u}+\bar{c}\bar{c})+1/9(dd+ss+\bar{d}\bar{d}+\bar{s}\bar{s})$	
	p/π H/D → l+ l-	NC	u/d	deuterium target
LHC/Tevatron	pp → γ + jet	prompt photon	gq, qq	
	pp → jets	jets	gg	
LHC	pp → t t̄	top production	gg	
	pp → t X	top production	b u + b̄ d	

Kinematic plane and data points used in NNPDF4.0 global fit

Order 500 data points

Available Global PDF fits :

- NNPDF
- MSTH
- CT

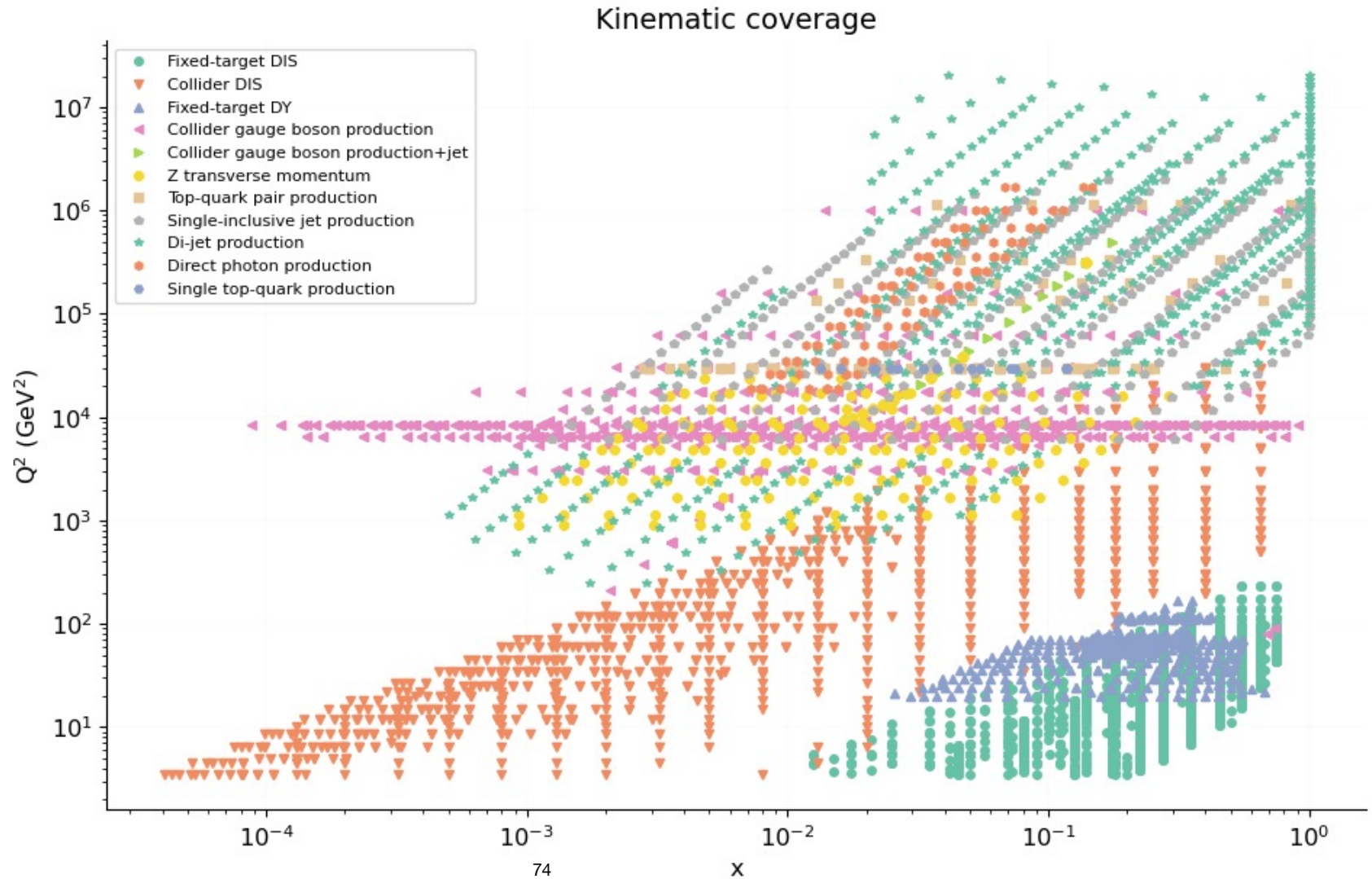
«reduced data set» PDF fits

- HERAPDF2.0
- ATLASWZ

State of the art is

Next-to-next-to-leading order
in α_s for PDFs and partonic
cross sections.

Very recently we had some fit
At N3LO



Kinematic plane and data points used in NNPDF4.0 global fit

Global PDF fits :

- NNPDF
- MSTH
- CT

«reduced data set» PDF fits

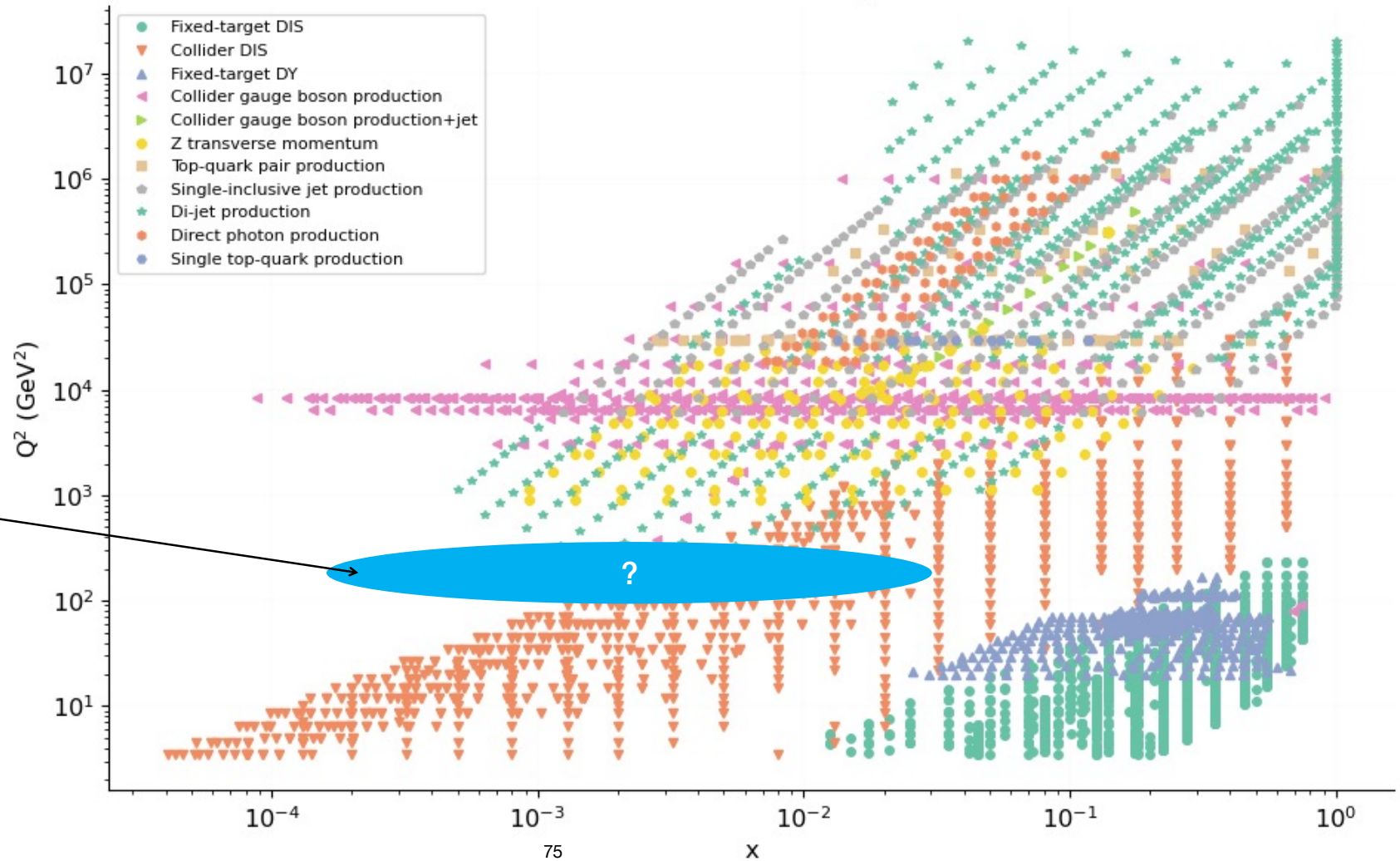
- HERAPDF2.0
- ATLASWZ

Area covered by
ongoing ATLAS
Low-mass DY
analysis

$7 < M < 56 \text{ GeV}$
 $|Y| < 2.5$

Will it help to
solve the low- x
puzzle ?

Kinematic coverage



Challenges and Prospects on PDFs

PDFs enter in the calculation of all the processes at colliders:
uncertainty on PDFs is (one of) the limiting uncertainty on several measurements:

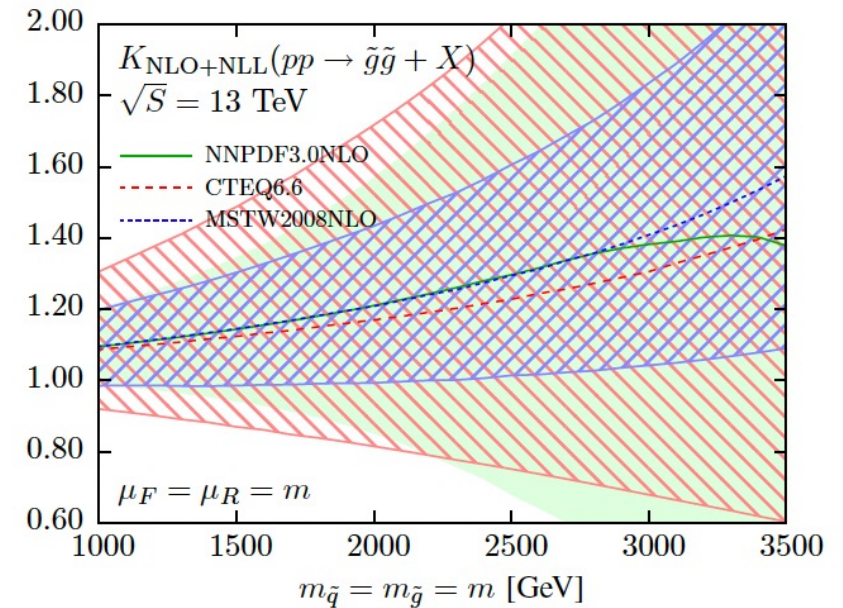
- Precision SM parameters (m_W , $\sin^2\theta_W$)
- Higgs absolute branching ratios
- BSM processes !

How to improve ?

1) make precise measurements at LHC that can be used for improving PDF fits

2) New ep collider

- EIC Electron Ion Collider planned to be built at Brookhaven (USA)
- 275 GeV p + 10 GeV e ($\sqrt{s} = 105$ GeV)
- Luminosity $10^{34} \text{ cm}^{-2}\text{s}^{-1}$ (200 times HERA)
- Polarized beams (to study spin structure)
- Possibility to use ions (saturation at large parton densities etc)



Cross section ratio to reference cross-section with PDF uncertainty band for production of gluino pairs

End of Chapter 9

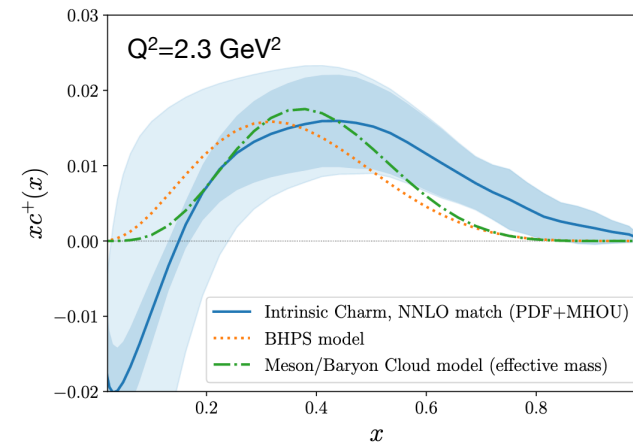
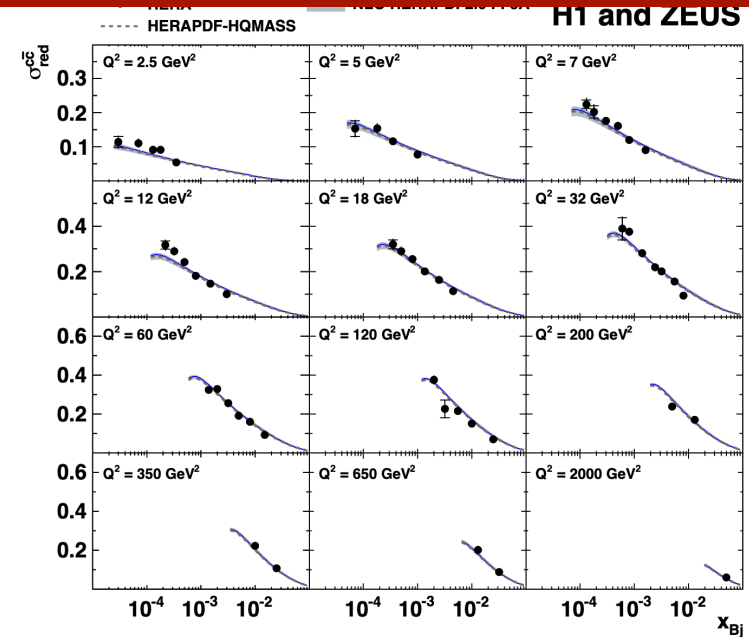
Charm and beauty PDFs: intrinsic charm ?

At low/medium x the c and b cross sections are compatible with QCD predictions that assume that all heavy quarks are produced by gluon splitting, i.e. $f_c(x, Q^2 < m_c^2) = 0$

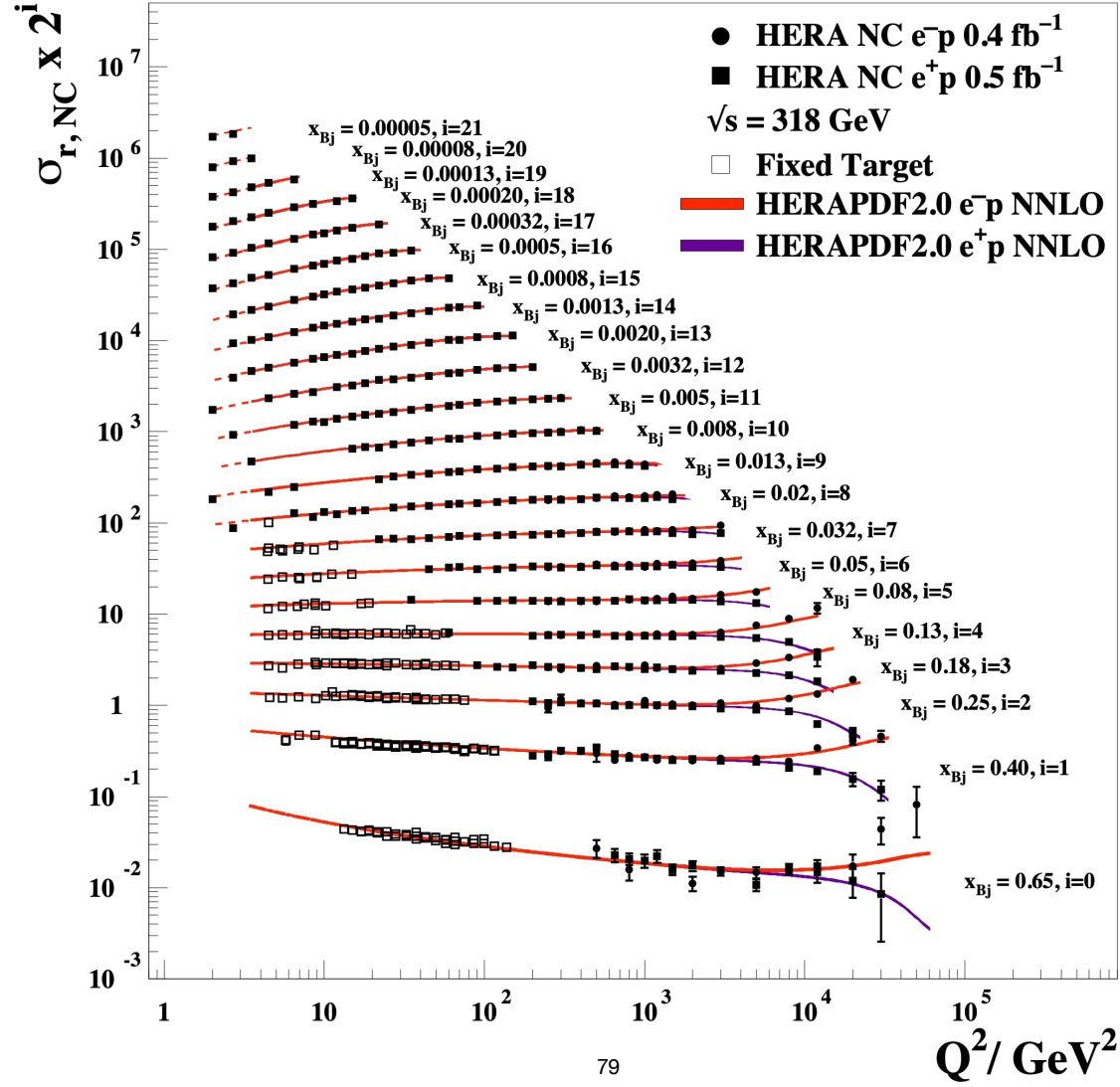
In principle an intrinsic heavy quark component may be present in the proton wave function despite the fact that the charm mass ($m_c = 1.5$ GeV) is higher than the proton mass ($m_p = 0.94$ GeV).

$$|p\rangle = |uud\rangle + |u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle + |c\bar{c}\rangle$$

A global analysis of PDF data (including LHC and fixed target data), *Nature* 608 (2022) 7923, finds evidence for an intrinsic charm content of the proton that does not vanish at low Q^2



H1 and ZEUS



How to reconstruct a NC DIS event at HERA 2

In **Neutral-Current** DIS the measurement is over-constrained: various methods have been used with different systematics and different sensitivity to QED corrections.

- electron method (from E'_e, θ_e)
- Hadronic method (from $p_{T,X}$ and $(E - P_Z)_X$)
- Double Angle method (from $\theta_e, \cos\gamma_{DA} = \frac{p_{T,X}^2 - (E - P_Z)_X^2}{p_{T,X}^2 + (E - P_Z)_X^2}$)
- Other methods mixing electron and hadronic variables: Sigma method and PT method

