Collider Particle Physics
- Chapter 9 -

Introduction to Hera Collider: Physics and Experiments

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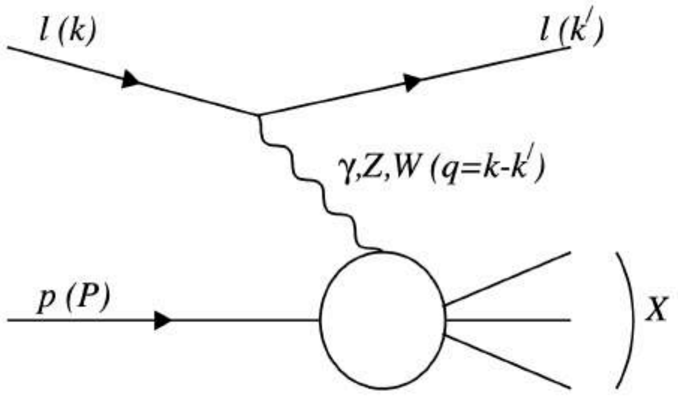
Chapter Summary

- deep inelastic scattering: a reminder
- Hera Collider and the experiments
- Results on Neutral and charge current scattering at high Q^2
- QCD and deep inelastic scattering
- Parton Density Function determination
- Extra topics ?

Deep Inelastic Scattering: a reminder

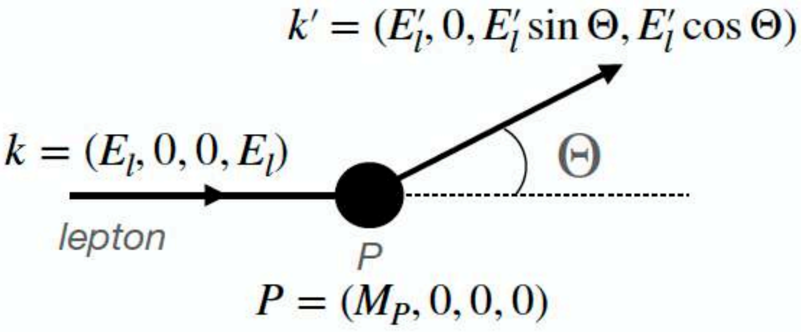
Kinematics of lepton-hadron scattering



Different possible
Lorentz invariants

$M_p^2 = P^2$	Proton mass
$s = (k + P)^2$	Center-of-mass energy squared
$Q^2 = -q^2 = -(k - k')^2$	Squared four-momentum transfer
$W^2 = (q + P)^2 = M_X^2$	Mass of final hadronic state
$x = \dfrac{Q^2}{2P \cdot q}$	Bjorken x
$y = \dfrac{q \cdot P}{k \cdot P}$	Inelasticity (energy lost by lepton in reference frame with p at rest)
$\nu = p \cdot q$	Energy transfer in frame with p at rest (mostly used in old/fixed targed expts.)

Proton rest frame



Fixed target experiment Lab frame [notation: $p = (E_p, p_x, p_y, p_z)$] :

$$q^2 = (k - k')^2 = k^2 + k'^2 - 2kk' = -2E_lE'_l + 2E_lE'_l \cos\Theta = 2E_lE'_l(\cos\Theta - 1)$$

$$Q^2 = -q^2 = 2\,E_lE'_l(1 - \cos\Theta)$$

Large scattering angle : large Q^2

$$y = \frac{q \cdot P}{k \cdot P} = \frac{(E_l - E'_l)\,M_P}{E_l\,M_P} = 1 - \frac{E'_l}{E_l}$$

0 < y < 1
1: all electron energy lost in collision
0: no energy lost by lepton

$$\nu = q \cdot P = M_p\,(E_l - E'_l)$$

energy lost by lepton

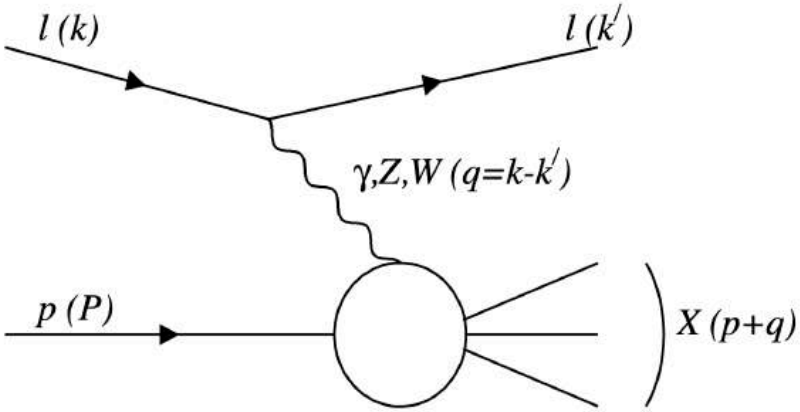
Cross-sections in terms of structure functions

Deep Inelastic Scattering (DIS) regime:

$Q^2 \gg M_p^2$ Deep

$M_X^2 \gg M_p^2$ Inelastic

In this limit $Q^2 = xys$



General expression for single-photon exchange cross section have the form (can be derived from QED):

$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\pi\, \alpha^2}{x\, Q^4}\left[(1-y)\, F_2(x,Q^2) \;+\; y^2\, xF_1(x,Q^2)\right]$$

F_1 and F_2 are Structure Functions that parametrize our knowledge of the hadronic structure

[note: F_1 parametrizes the cross section with the exchange of transversely polarized photons and F_2-$2xF_1$ of longitudinally polarized photons.]

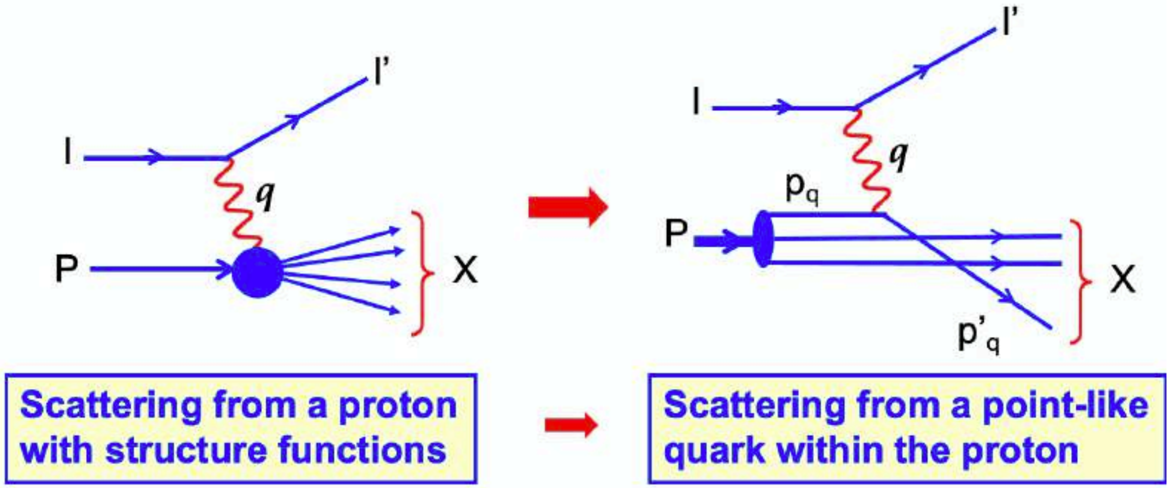
Parton model: meaning of Bjorken x

The proton is made of pointlike constituents (quarks)

Let's assume that we are in a reference frame in which the proton is very boosted with $P_z \gg M_p$

Considering that the scattered quark is on mass shell and that quark mass is close to zero :

$$(p_q + q)^2 = (p'_q)^2 = 0$$
$$q^2 + 2p_q \cdot q = 0$$



We can write the incoming quark 4-momentum as
$p_q = \xi P$
where ξ is the fraction of the proton momentum carried by the quark, then

$$q^2 + 2\xi P \cdot q = 0$$

$$\xi = \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2P \cdot q} = x$$

In the Parton model we can identify Bjorken x with the quark momentum fraction

Parton model: cross-section

The cross section can be expressed as an incoherent sum of lepton-quark elastic cross sections, weighted with the probability density function (PDF) for finding a quark of a given type i with a fraction x of the proton momentum:

$$l\,p \to l'\,X: \qquad \frac{d\sigma}{dx\,dQ^2} = \sum_{i=u,d,\dots} \frac{d\hat{\sigma}_i}{dQ^2} f_i(x)$$

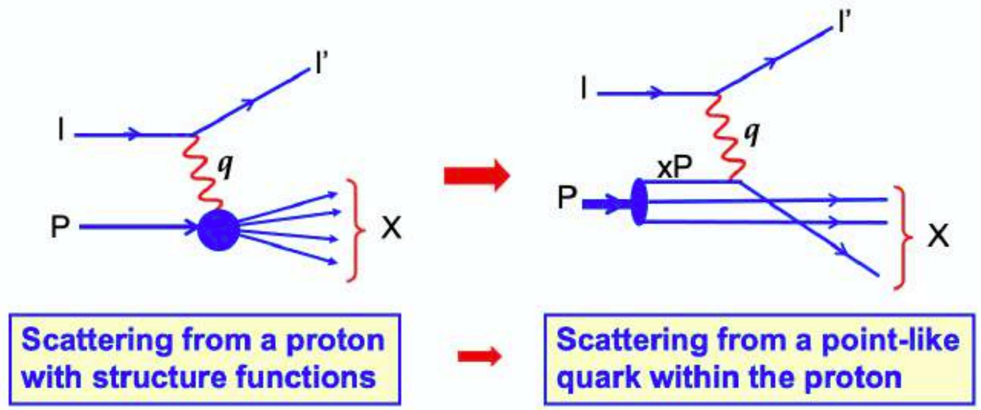
Lepton-quark elastic cross sections are of the form:

$$l\,q_i \to l'\,q_i': \qquad \frac{d\hat{\sigma}_i}{dQ^2} = \frac{2\pi\alpha}{Q^4} e_i^2 \left[1 + (1-y)^2\right]$$

Where e_i is the quark charde in electron charge units

And thus

$$\frac{d\sigma}{dx\,dQ^2} = \frac{2\pi\alpha}{Q^4}\left[1 + (1-y)^2\right] \sum_i e_i^2\, f_i(x)$$



Parton densities and structure functions

Comparing this result (rewritten by expanding the squared term):

$$\frac{d\sigma}{dx\, dQ^2} = \frac{4\pi\,\alpha}{Q^4}\left(1 - y - \frac{1}{2}y^2\right)\sum_i e_i^2\; f_i(x)$$

with the general expression for e-p scattering:

$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\pi\;\alpha^2}{x\;Q^4}\left[\left(1-y\right)F_2(x,Q^2)\; + \; y^2\; xF_1(x,Q^2)\right]$$

We can see identify:

1) $xF_2(x,Q^2) = \sum_i e_i^2 f_i(x)$

2) $2xF_1(x) = F_2(x)$

The structure functions are given by the sum of quark probability densities functions (PDFs) weighted with their charge squared

$F_1(x)$ and $F_2(x)$ are linked by a particular relation that is valid for partons with spin s=1/2 partons (Callan-Gross relation)
For s=0 we would have $F_2(x)$=0

[we will see later that this relation is only valid in parton model, QCD introduces deviations of order α_S due to gluon-initiated processes]

Scaling violation

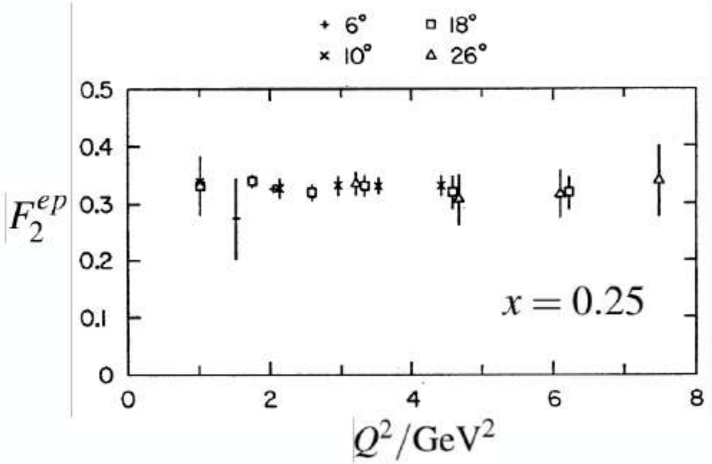
$$xF_2(x,Q^2) = \sum_i e_i^2 f_i(x)$$

In the parton model the structure functions don't depend on Q^2 : Bjorken scaling

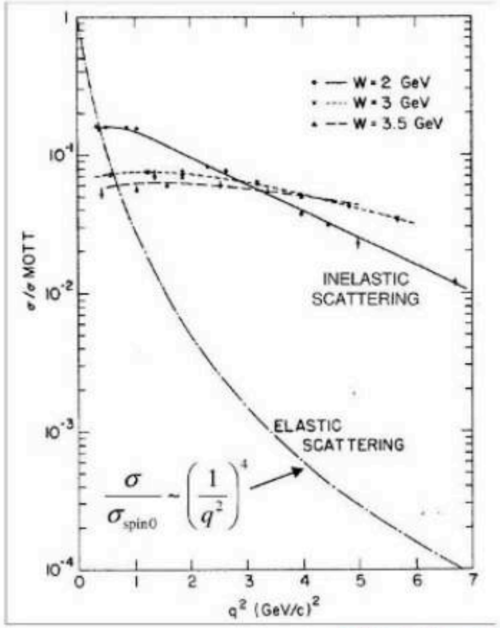
This is a consequence of the presence of pointlike constituents

In the case of a diffuse charge distribution, the structure function would decrease quickly with Q^2 similarly to elastic form factors

Measurement of F_2 at SLAC showing Bjorken scaling



J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. 22 (1972) 203



M.Breidenbach et al., Phys. Rev. Lett. **23** (1969) 935

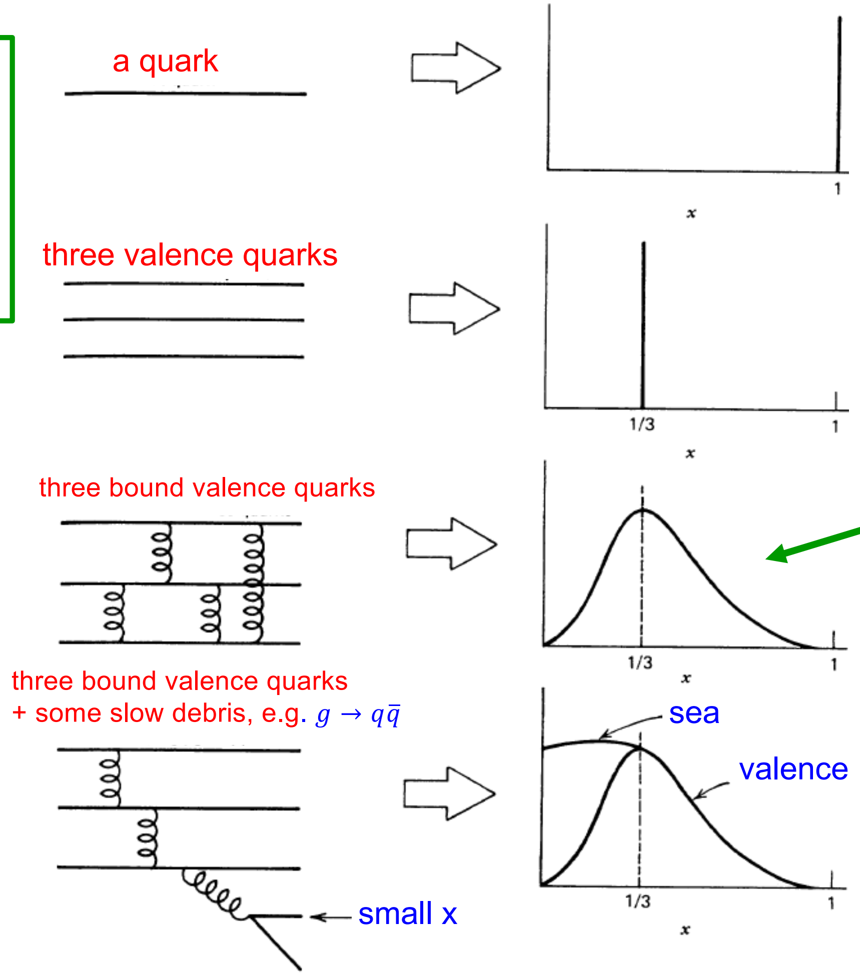
What can we expect for parton densities?

PDFs cannot be computed by perturbative QCD

Lattice and other non-perturbative methods
Have been used but so far large theoretical uncertainties

In practice PDFs need to be measured experimentally…

If the proton is: | Then $F_2^{ep}(x)$ is:



The interactions between quarks smear the momentum among them

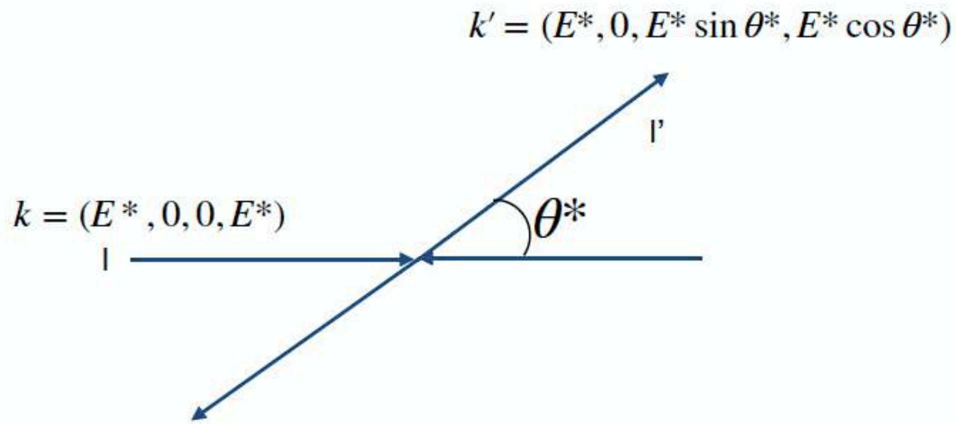
Parton model: meaning of y

$$y = \frac{q \cdot P}{k \cdot P}$$

$$q = k - k' = (0, 0, -E^* \sin\theta^*, E^*(1 - \cos\theta^*))$$

$$P = (E_p^*, 0, 0, -E_p^*)$$

$$y = \frac{E_p^* \, E^* \, (1 - \cos\theta^*)}{2 \, E_p^* \, E^*} = \frac{1 - cos\theta^*}{2}$$



y is closely related to the scattering angle in the lepton-quark rest frame

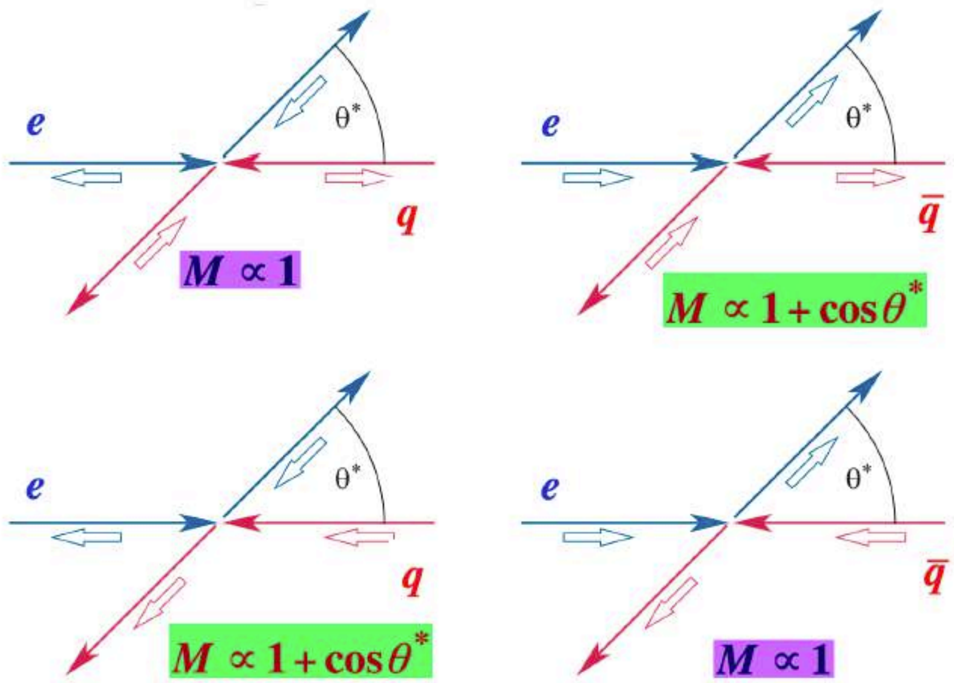
Helicity structure and y

$$y = \frac{1 - \cos\theta^*}{2}$$

What was the origin of the therm in square brackets ?

$$\frac{d\hat{\sigma}_i}{dQ^2} = \frac{2\pi\alpha}{Q^4} e_i^2 \left[1 + (1-y)^2\right]$$

$$\frac{d\hat{\sigma}_i}{dQ^2} \propto |M|^2$$



For vector exchange (γ, Z, W) helicity conservation gives these relations :

$e_L q_L,\ e_R q_R \rightarrow |M|^2 \propto (1-y)^2$ (forward peak)

$e_R q_L,\ e_L q_R \rightarrow |M|^2 \propto 1$ (isotropic)

Analogy with e+e-

In unpolarised photon exchange we have both terms with same weight, thus $1+(1-y)^2$

HERA collider
and
the experiments

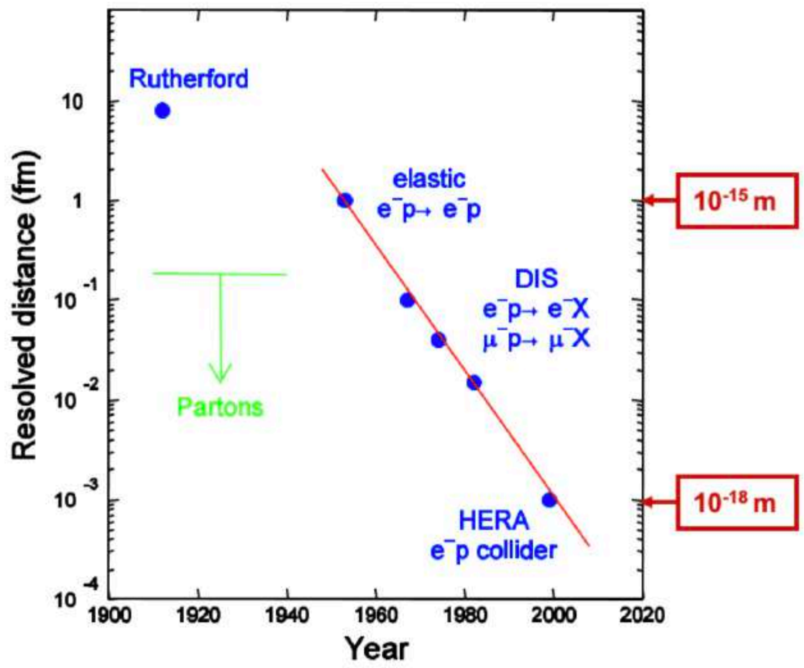
High-Q^2: DIS is a way to probe small distances

DIS : a microscope with resolution :

$$\Delta b \sim \frac{\hbar c}{\sqrt{Q^2}} = \frac{0.197}{\sqrt{Q^2}} \text{GeV fm}$$

The maximum Q^2 reachable in an experiment is given by the center-of-mass energy.

If quarks had a structure it would be resolved going to higher energy



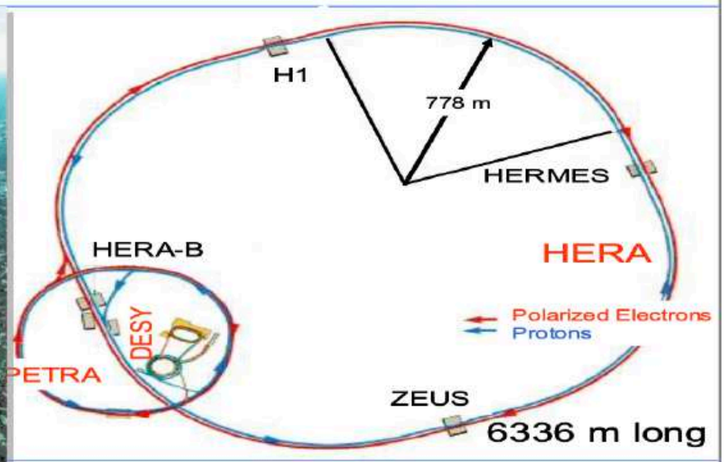
Several DIS experiments in the 70's and 80's using electrons, muons and neutrinos, with increasing energy. Since the 70s it was clear that to make a significant step forward a collider was needed.

Beyond probing quark substructure it would also improve enormously the knowledge of PDF

After several proposal have been considered, it materialised in early '80s as the proposal for HERA

HERA: the (so far) only ep collider

After the success of Petra e+e- collider, DESY (Hamburg, Germany) decided to build an ep collider. [with key contributions to the accelerators from Italy and later France]



Two accelerators in one 6.3 km tunnel

Proton ring: max energy E_p = 920 GeV ("similar" to Tevatron)

Electron/positron ring : max energy E_e = 27.5 GeV (A small LEP)

Center-of-mass energy : $\sqrt{s} = \sqrt{4E_eE_p} = 318\ \text{GeV}$

Two large collider experiments: H1 and ZEUS

Two fixed-target experiments:
HERMES (polarised electrons on polarised gas target)
HERA-b (for b physics, using p-beam halo on wire targets)

HERA operations

Two main periods HERA-1 (1992 - 2000) and HERA-2 (2003-2007)

HERA-2: luminosity upgrade with low-β^* insertions near interaction points

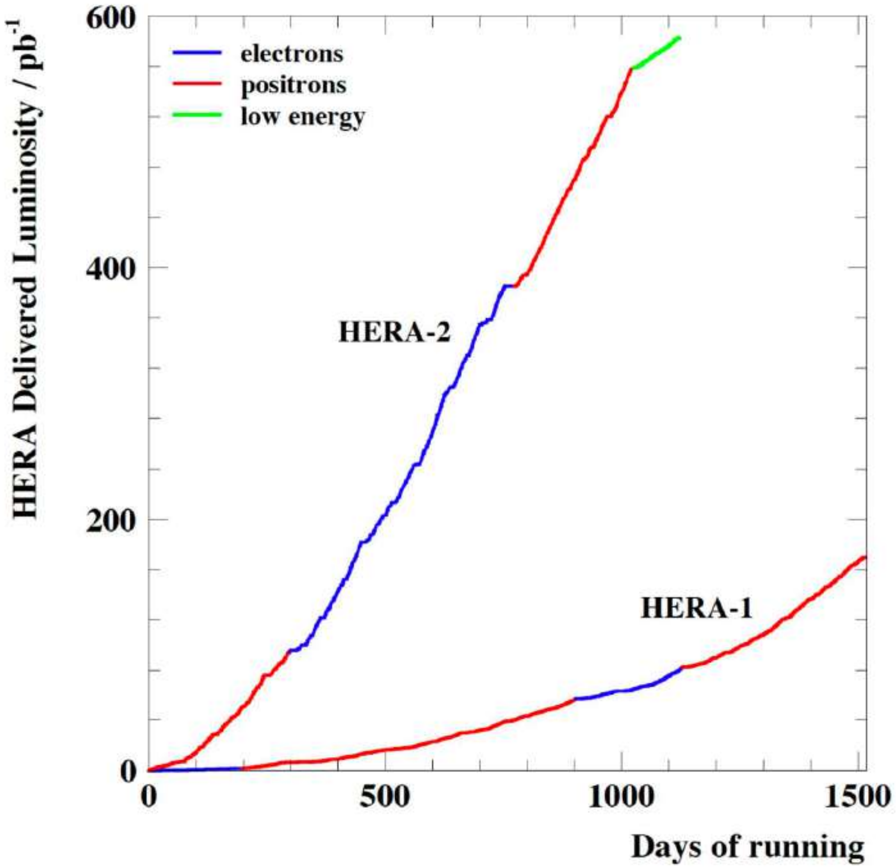
Luminosity of HERA-1 was below expectations in particular with electrons, most luminosity was collected with positrons

HERA-2 had initial difficulties related to high backgrounds in the experiments but finally an integrated luminosity of 0.5 fb^{-1} per experiment was collected.

210 bunches (176 colliding) 96 ns between bunches

Peak luminosity in HERA-2 $5\times10^{31}\ cm^{-2}\ s^{-1}$

During the HERA-1 -> HERA-2 transition the detectors have been also upgraded. In particular new vertex detectors were introduced in both experiments to improve c and b tagging capabilities

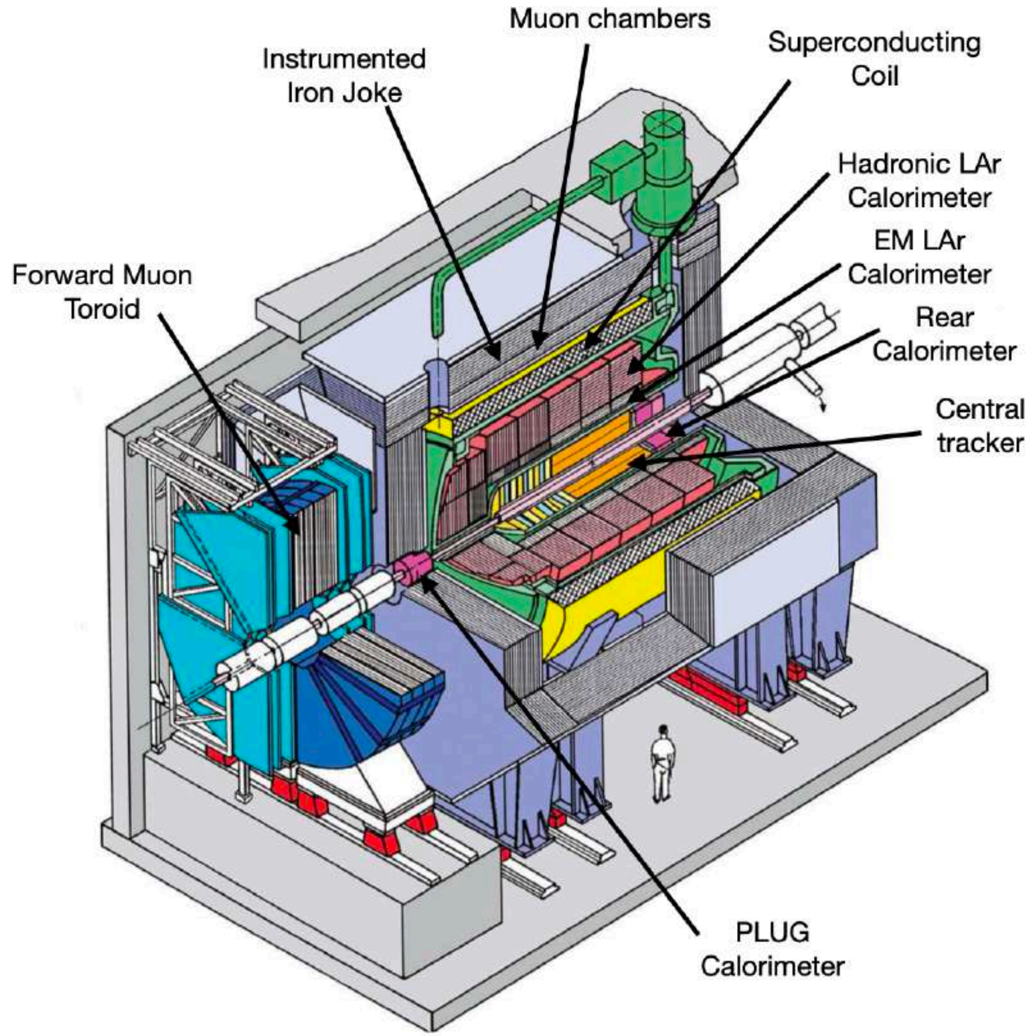


Detectors: H1

Asymmetric experiments
convention: Z axis points towards p beam direction
“forward” is proton beam direction

H1 : large German / French / UK components

Sub detectors:
- Silicon strip vertex detector
- “Jet Chamber” gaseous trackes
  - Liquid Argon EM calorimeter (barrel / forward)
  - Spacal EM calorimeter (spaghetti calorimeter) (rear)
- Liquid Argon hadronic calorimeter
- Large superconducting solenoid (1.16 T axial field)
- Muon Chambers inside iron return coil (streamer tubes)



LAr EM : $\sigma(E)/E \simeq 11\%/\mathrm{sqrt}(E/GeV) \oplus 1\%$
LAr Had : $\sigma(E)/E \simeq 50\%/\mathrm{sqrt}(E/GeV) \oplus 2\%$

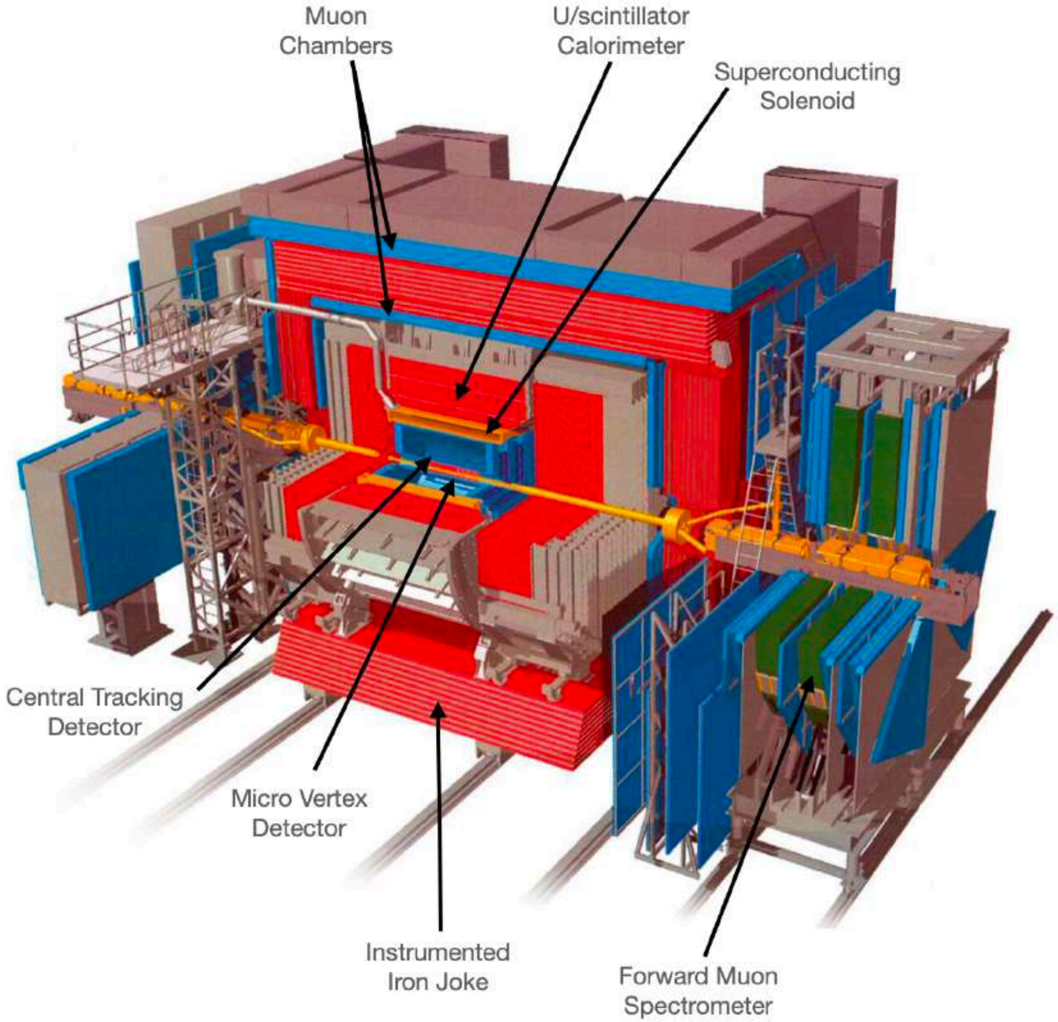
Detectors: ZEUS

ZEUS : large German / US / ITA / UK / JP components (+ Russia, Poland, Canada…)

Main Sub detectors:
- Silicon Strip Vertex detector (from 2000)
- Drift Chamber (CTD)
- Thin superconducting solenoid 1.4 T
- pre-samplers (scintillators)
- Compensating Uranium/scintillator calorimeter
- Instrumented return coils (backing calorimeter)
- Muon Chambers (streamer tubes)

Main focus on hadronic calorimetry

EM : σ(E)/E ≃18%/sqrt(E/GeV)
Had : σ(E)/E ≃ 35%/sqrt(E/GeV)



ZEUS: a few pictures

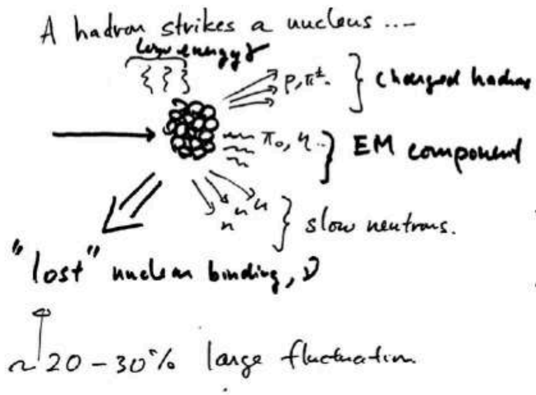

ZEUS Hadronic Calorimeter

Response to pion:

$$\pi^+ = f_e e \ + \ (1 - f_e) h$$

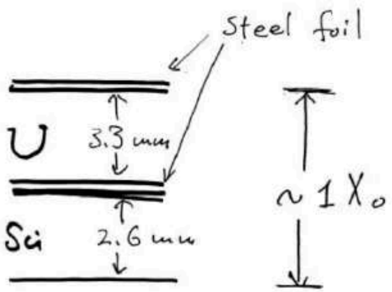
EM fraction f_e fluctuates,
degrading the calorimeter resolution

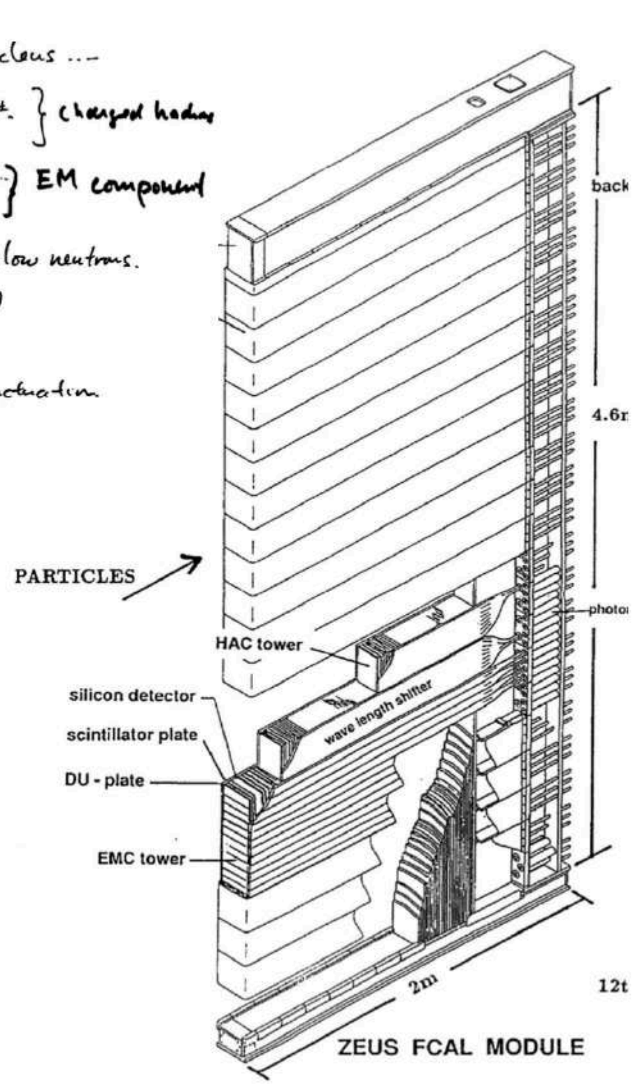
Trick: obtain compensation: $f_e = 1$

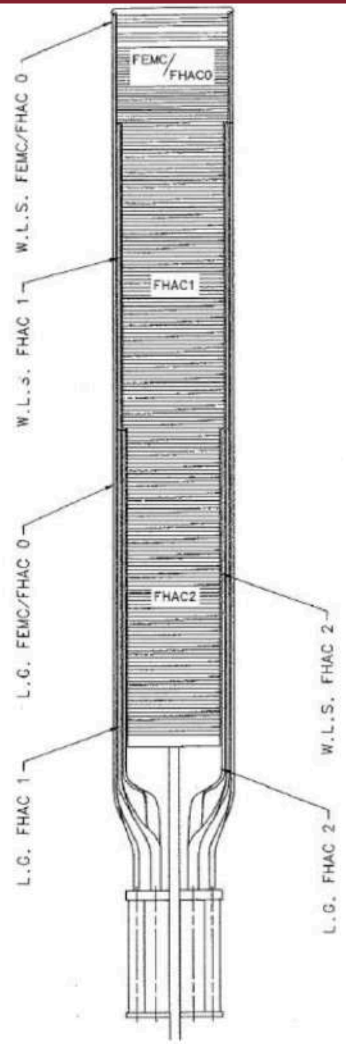


Uranium + plastic Scintillator : large response to neutrons
f_e can be tuned in sampling calorimeters:
higher absorber fraction -> lower e.m. component

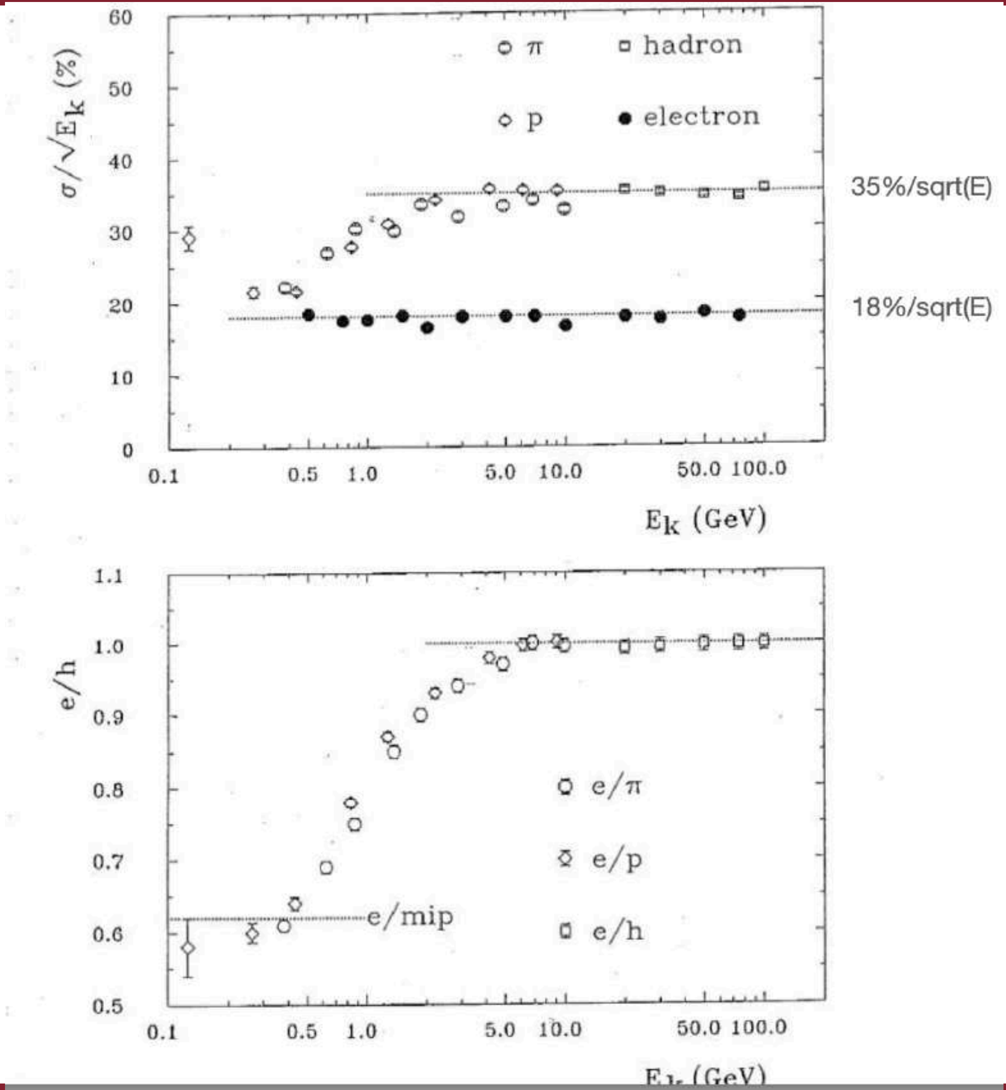
ZEUS used this structure obtaining :







ZEUS Hadronic Calorimeter



A unique compensating calorimeter !

Same response to EM and hadrons

Less sensitive to the fluctuation of the type of particles inside a jet, for instance on the relative content of $\pi^0/\pi^\pm$

Deep Inelastic Scattering at HERA

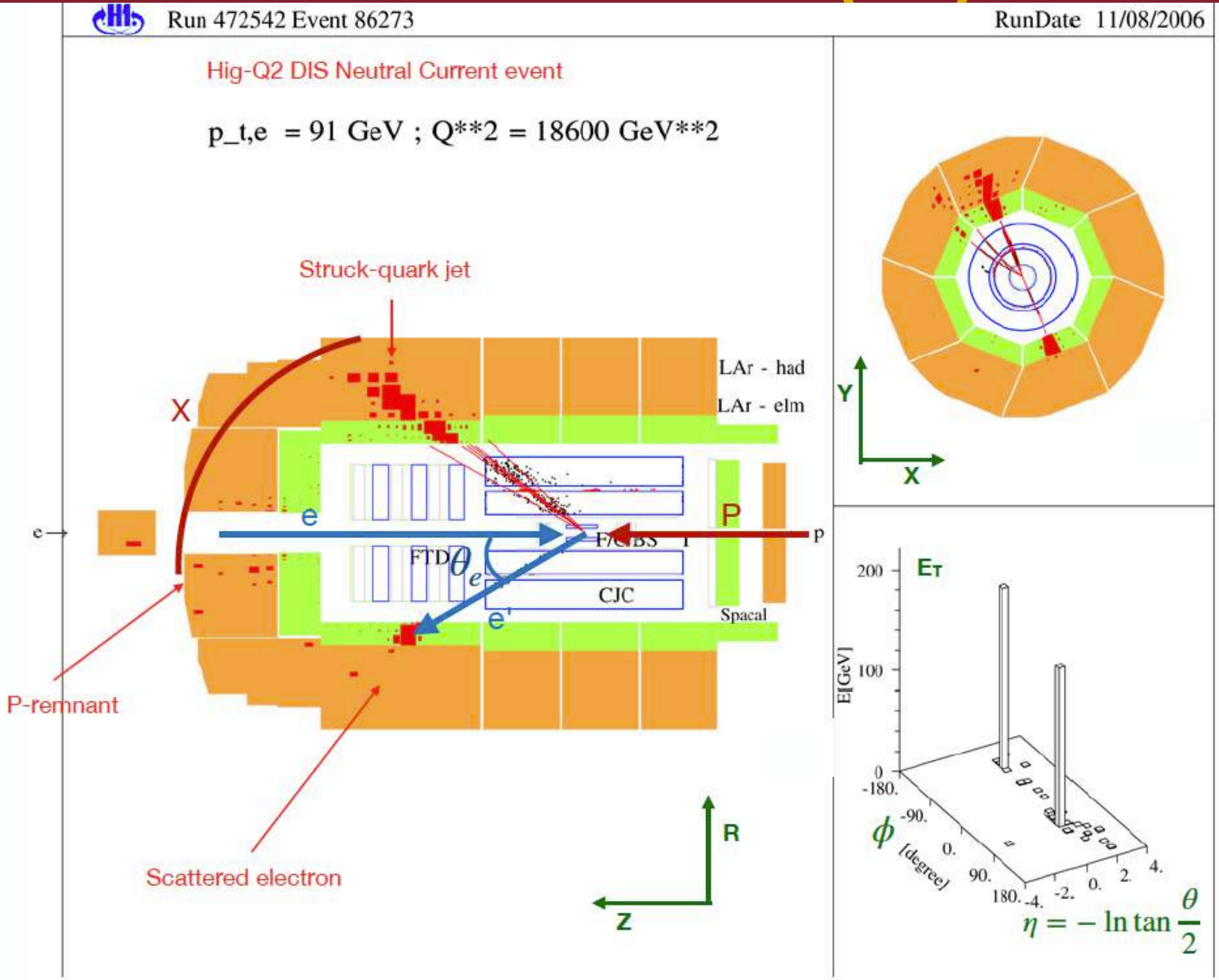
Neutral Current DIS reconstruction (H1)

Electron method, same as for fixed target
(Note θ_e measured
respect to Z axis ($\theta_e = -\Theta_e$)

$$Q^2 = 2E_e E'_e (1 + \cos\theta_e)$$

$$y = 1 - \frac{E'_e}{2E_e}(1 - \cos\theta_e)$$

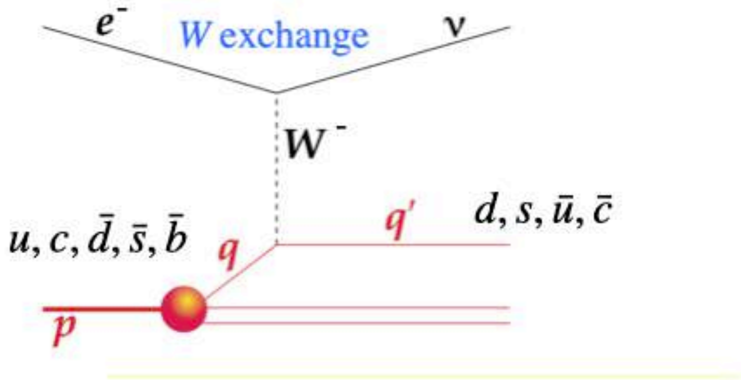
$$x = \frac{Q^2}{sy}$$

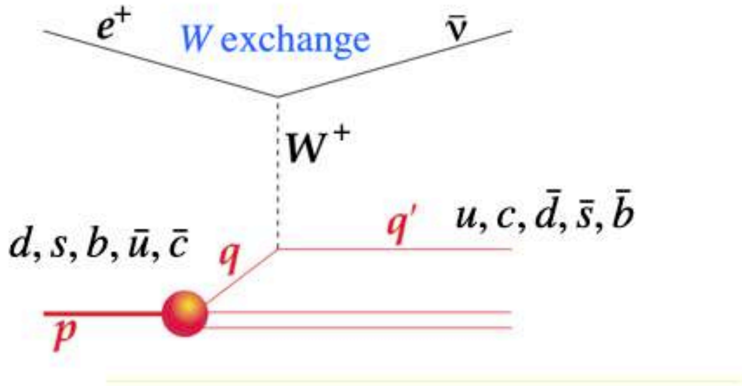


Charge Current DIS

Charge Current: W exchange

e^+ and e^- beams select different quark combinations

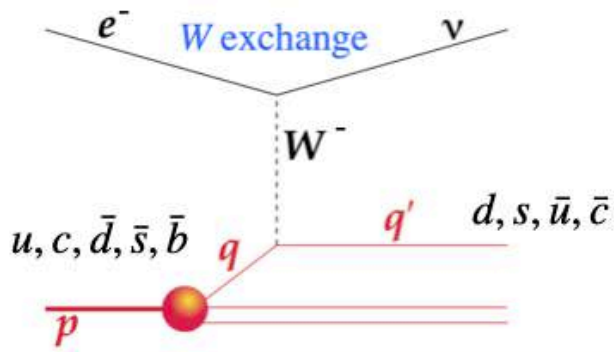




Charge Current DIS cross sections and PDFs

Applying the same approach of parton model as for photon exchange :

$$\frac{d\sigma^{CC}}{dx\,dQ^2} = \frac{G_F}{2\pi x}\left(\frac{M_W^2}{M_W^2+Q^2}\right)^2 \tilde{\sigma}^{CC}(x)$$



electron beam: W^- exchange selects $e_L^-\ q_L;\ e_L^-\ \bar{q}_R$ that have different helicity structures:

$$e^-p\ :\ \tilde{\sigma}^{CC}(x) = x\left[u(x) + c(x) + (1-y)^2\,(\bar{d}(x) + \bar{s}(x))\right]$$

The neutrino escapes undetected: we can't use the scattered lepton to reconstruct the event kinematics

positron beam: W^+ exchange selects $e_R^+\ \bar{q}_R,\ e_R^+ q_L$:

$$e^+p\ :\ \tilde{\sigma}^{CC}(x) = x\left[\bar{u}(x) + \bar{c}(x) + (1-y)^2\,(d(x) + s(x))\right]$$

Charged and Neutral currents processes are sensitive to different combination of quarks

How to reconstruct a CC event at HERA ?

In **Charged-Current** DIS the only possible reconstruction method is the hadronic method, also known as Jaquet-Blondel (JB) method

Transverse momentum conservation:
the transverse momentum of the hadronic system X is equal to that of the lepton:

$$E'_\nu \sin\theta_\nu = (p_T)_X$$

Longitudinal momentum conservation :

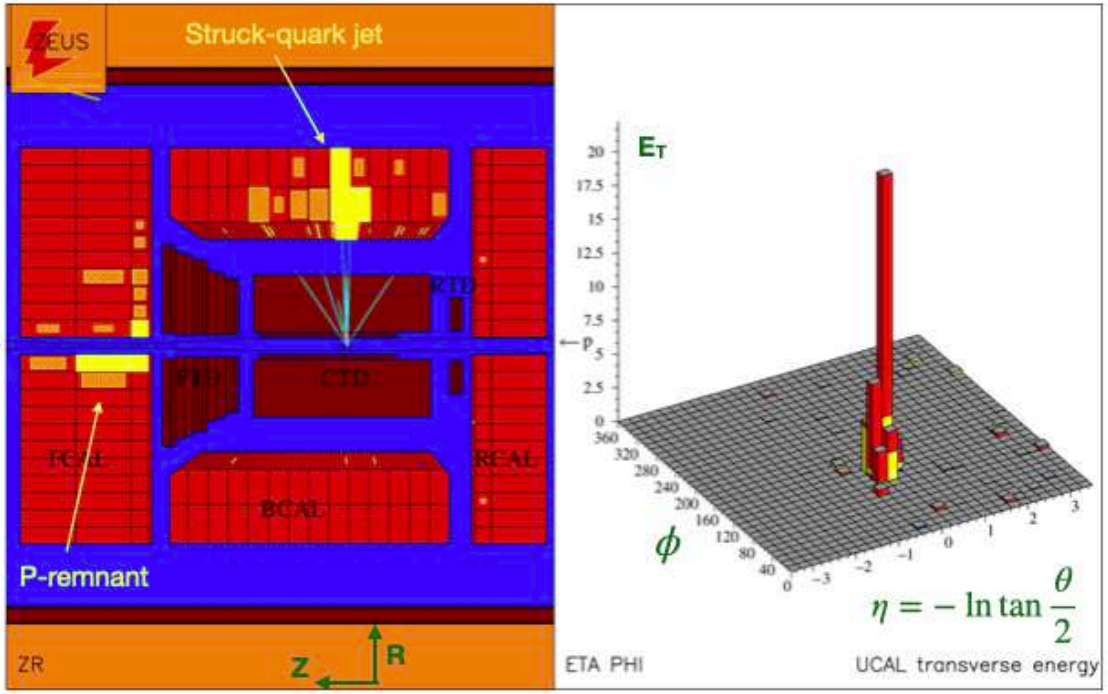
$$(E - P_Z)_{\text{initial}} = (E - P_Z)_e + (E - P_Z)_P$$
$$= E_e + E_e + E_p - E_p = 2E_e$$
$$(E - P_Z)_{\text{final}} = E'_\nu(1 - \cos\theta_\nu) + (E - P_Z)_X$$

Therefore:

$$E'_\nu(1 - \cos\theta_\nu) = 2E_e - (E - P_Z)_X$$

The neutrino kinematics can be reconstructed from two quantities measured from the hadronic system:
$(p_T)_X$, $(E - P_Z)_X$

Hig-Q2 DIS Charged Current event



It can be shown that kinematics can be reconstructed as:

$$y_{JB} = \frac{(E - P_Z)_X}{2E_e} \qquad Q^2_{JB} = \frac{p^2_{T,X}}{1 - y_{JB}}$$

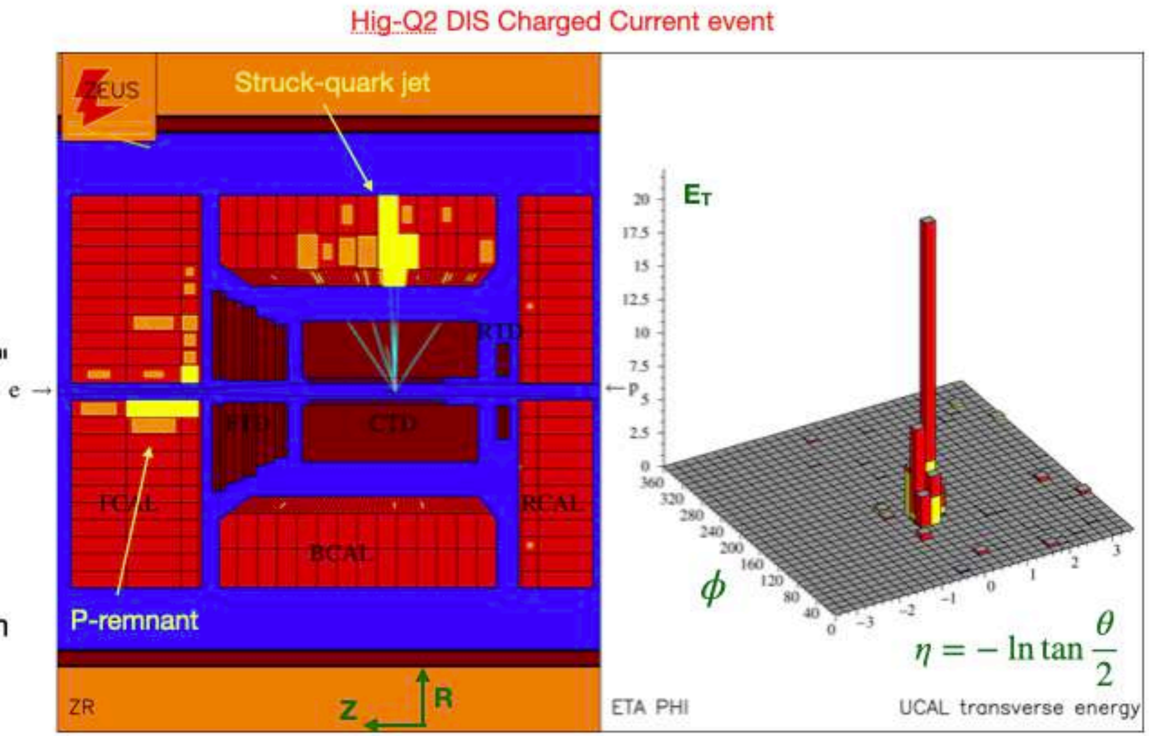
How to reconstruct a CC event at HERA ?

The hadronic system can be reconstructed by summing up energy deposits from all energy deposits in the calorimeters:

$$\vec{P}_X = \sum_{i \in cells} E^i \vec{r}^i$$

In a typical CC event there is a jet from the struck quark and hadrons produced in "forward direction" from the proton remnant that partly escape in the forward beampipe hole and partly leave energy deposits at small angles.

Hadrons in very forward directions give negligible contribution to $(p_T)_X$, $(E - P_Z)_X$ so it is not a problem to loose hadrons in the beampipe.



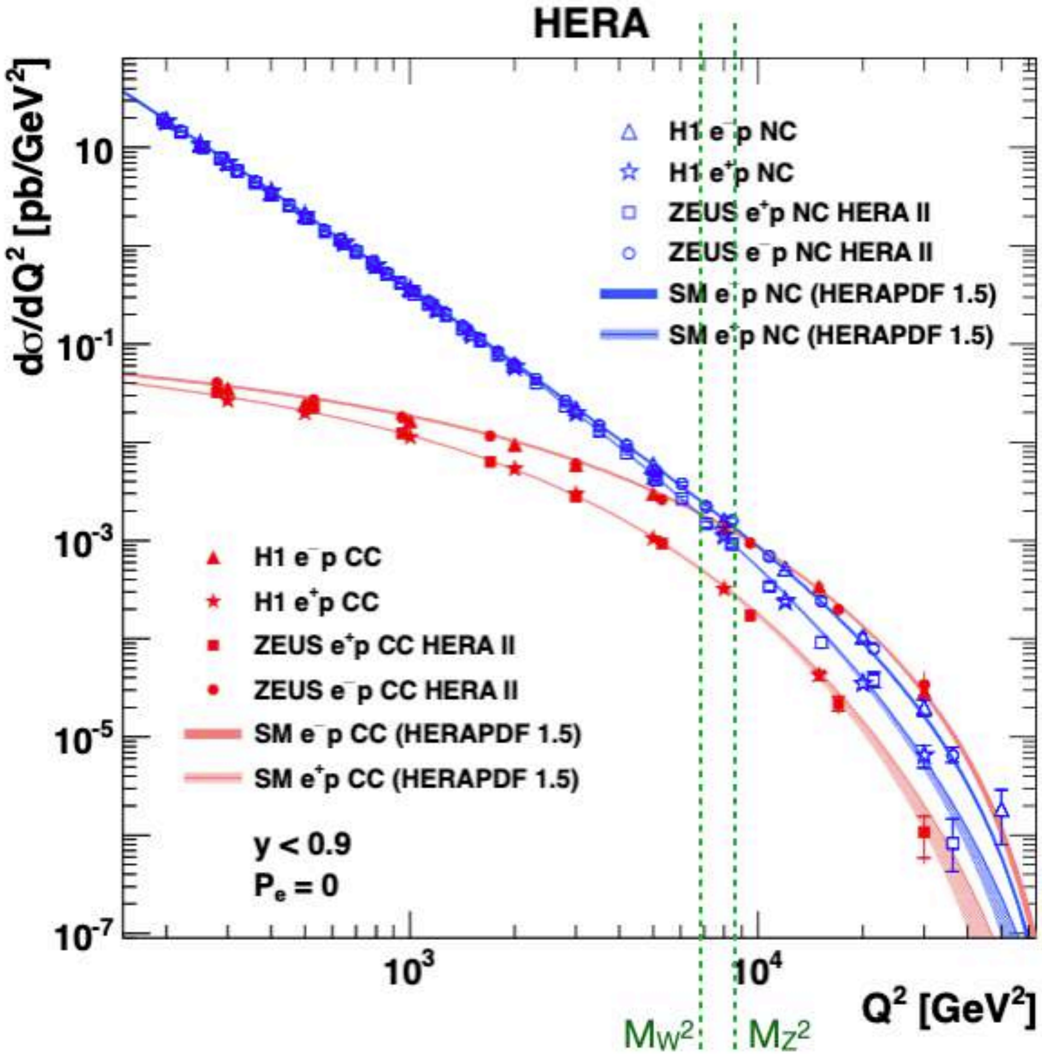
Result: cross-section versus Q^2 (integrated over x)

- The NC cross section falls approximately like $1/Q^4$

- Eventually at $Q^2 \simeq M_W^2$ CC and NC become similar: weak and e.m. interaction unify at EW scale !

- CC cross section in e^-p is larger than in e^+p because sensitive to u quarks rather than d.

- At high-Q^2 a difference between e^-p and e^+p is observed in NC : this is the effect of the Z exchange that introduces a charge dependence (C and P not conserved individually)



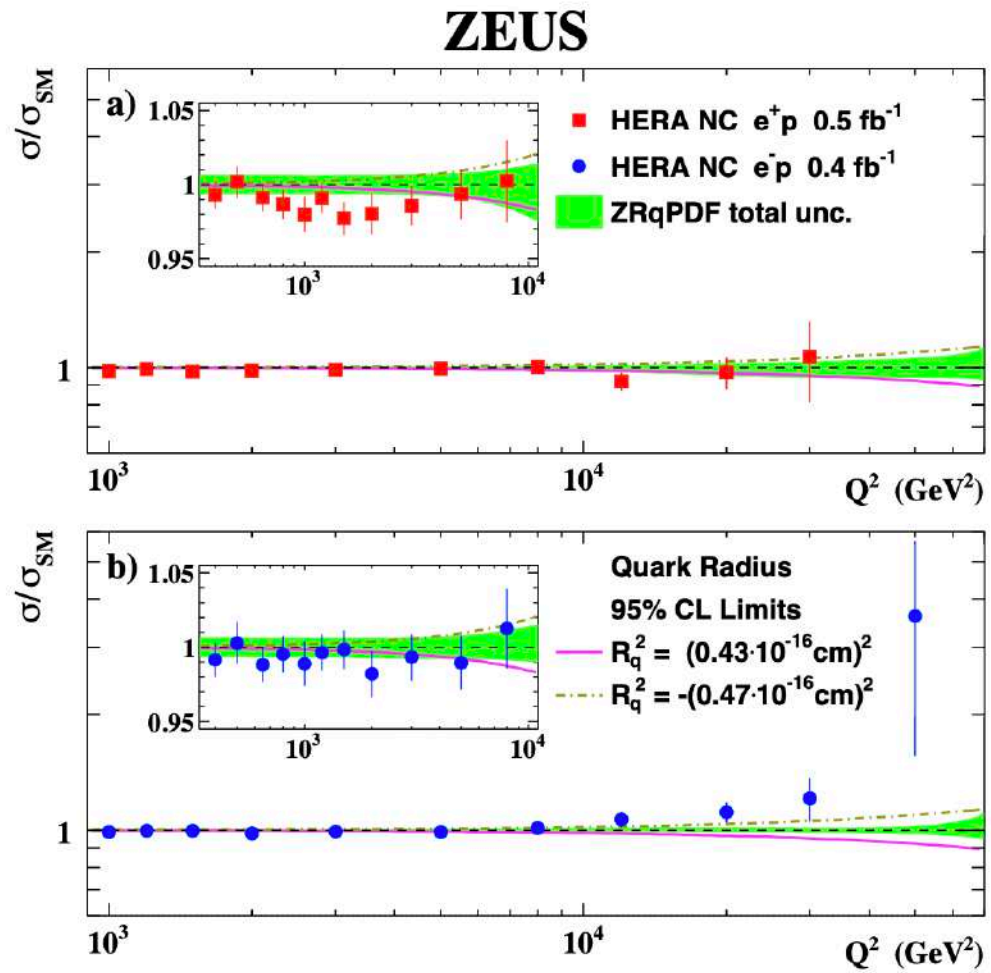
Limits of quarks substructure

Do we see have any hint of quark substructures ?

Assuming a gaussian charge distribution with radius R for the quark:

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma^{SM}}{dQ^2}\left(1 - \frac{R^2}{6}Q^2\right)$$

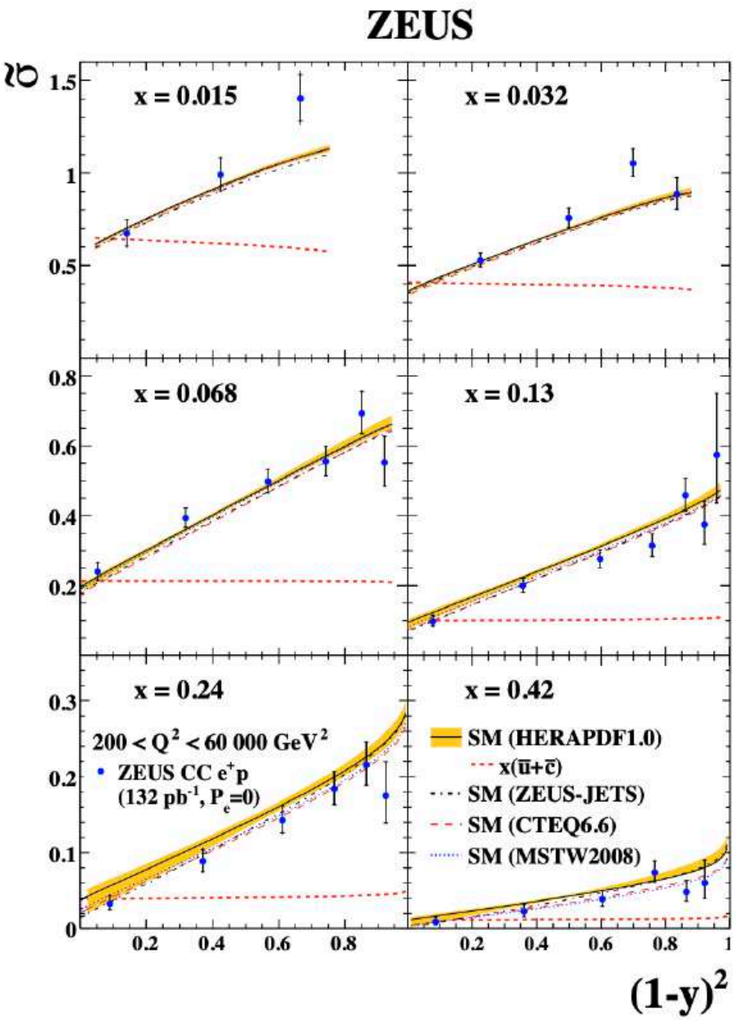
Current limit is 0.43×10^{-3} fm :
less than thousand times smaller than a proton !



Results: charged current

The Helicity structure works !

$$e^+p \ : \ \tilde{\sigma}^{CC}(x) = x\left[\bar{u}(x) + \bar{c}(x) + (1-y^2)\,(d(x) + s(x))\right]$$



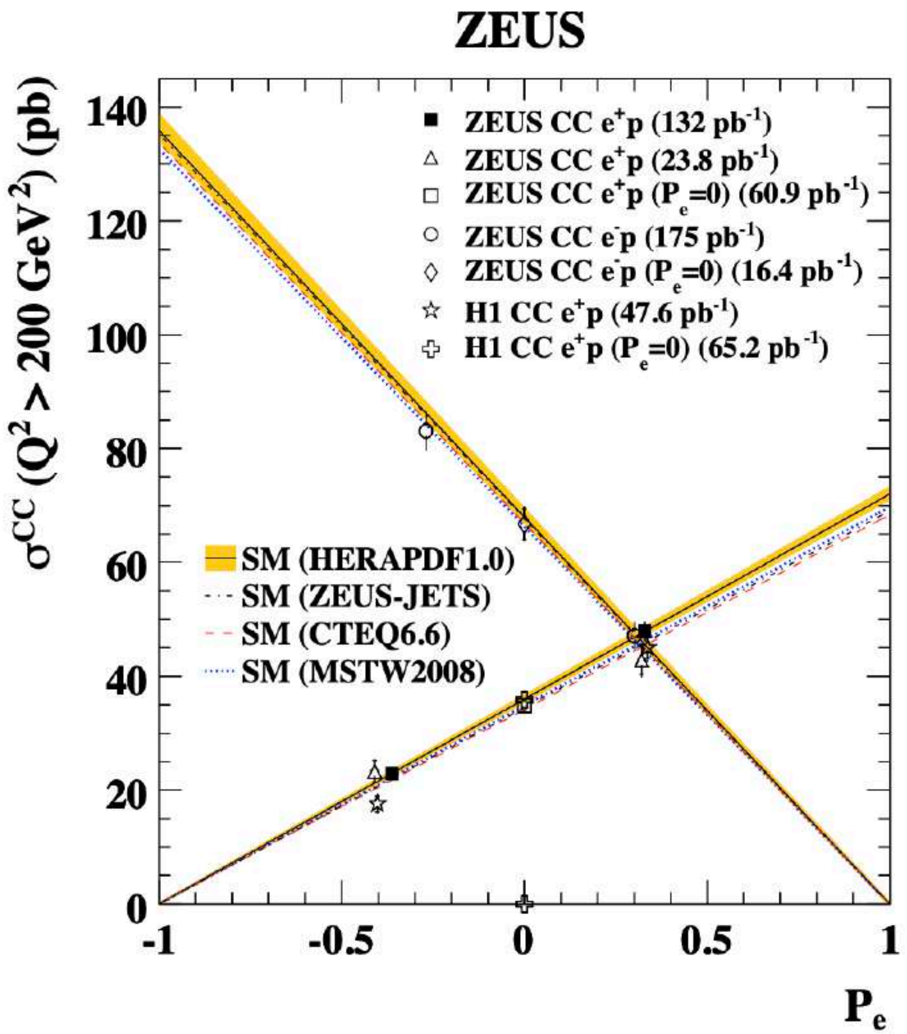
Charged currents: polarised leptons

During HERA-2 period, HERA had polarised leptons.

Electrons are polarised transversally (spin perpendicular to accelerator plane by syncrotron light emission in dipole magnets (Solokov-Ternov effect)
Special spin-rotator magnet rotate the spin just before/after the collision point to obtain longitudinal polarization.

Polarisation $P_e = \dfrac{N_R - N_L}{N_R + N_L}$ was about 0.4 and was switched in sign every few months.

This permits to select leptons with "right helicity"



Parton model and QCD

QCD corrections to parton model

Perturbative approach

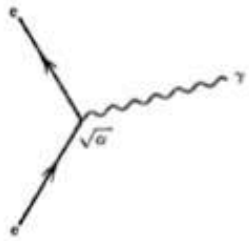
Cross sections are written as a power series in the em, α, and in the strong force coupling constant α_S

$$\sigma(ep \to eX) = \alpha\,\sigma_0 + \alpha\,\alpha_S\,\sigma_1 + \alpha\,\alpha_S^2\,\sigma_2 + \dots$$

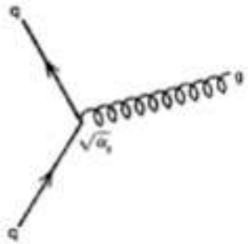
σ_0 is the order zero in in term (equivalent to parton model)

σ_1 is the lowest order QCD correction to parton model

(higher orders EM corrections α^2 also exist, not discussed here)

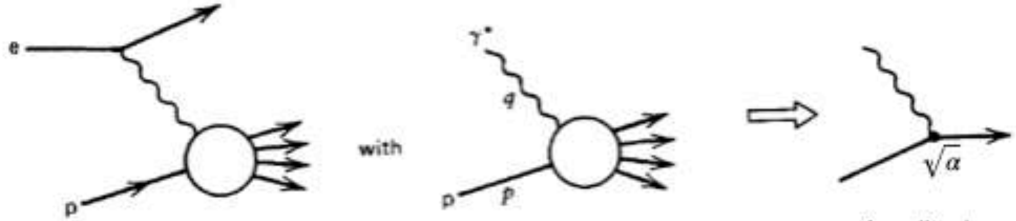


QED: probablilty $\propto \alpha$



QCD: probablilty $\propto \alpha_S$

parton model: $O(\alpha)$

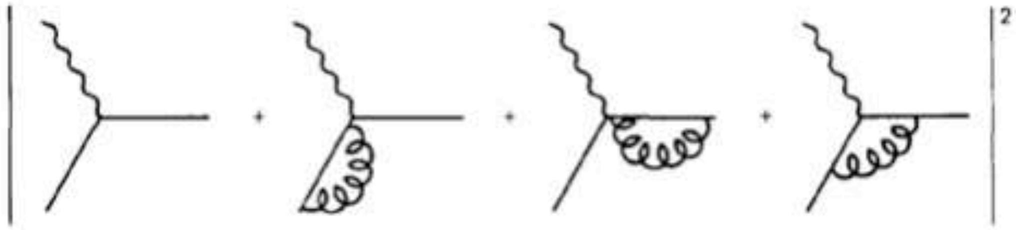


Amplitude

QCD corrections to parton model

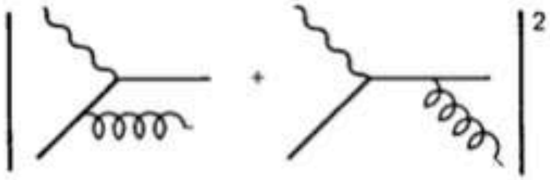
Vertex corrections
squaring these diagrams we obtain:
the $O(\alpha)$ term from the square of the first diagram and terms of order $O(\alpha\alpha_S)$ from the interference terms

$$\gamma^* q \rightarrow q'$$



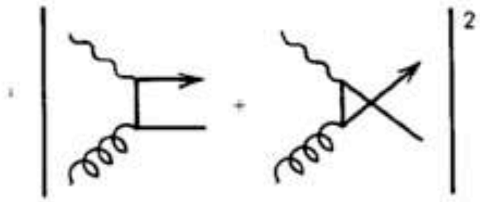
Gluon emission:
QDC-compton
new process $O(\alpha\alpha_S)$

$$\gamma^* q \rightarrow q' g$$



Gluon-initiated:
boson-gluon fusion
new process $O(\alpha\alpha_S)$

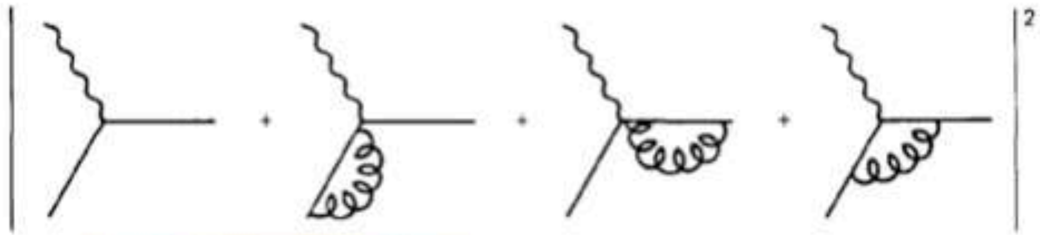
$$\gamma^* g \rightarrow q\bar{q}$$



QCD corrections to parton model

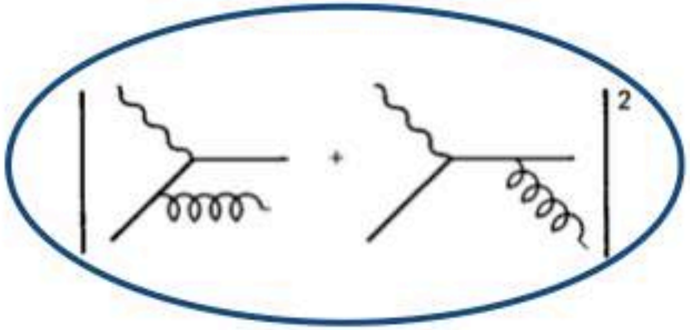
Vertex corrections
squaring these diagrams we obtain:
the $O(\alpha)$ term from the square of the first diagram and terms of orde $O(\alpha\alpha_S)$ from the interference terms

$\gamma^* q \to q'$



Gluon emission:
QDC-compton
new process $O(\alpha\alpha_S)$

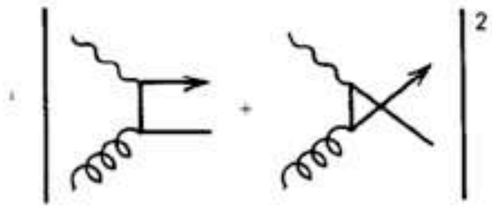
$\gamma^* q \to q' g$



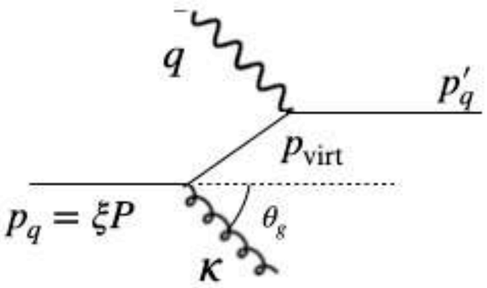
We will consider this part in next slides

Gluon-initiated:
boson-gluon fusion
new process $O(\alpha\alpha_S)$

$\gamma^* g \to q\bar{q}$



Let's look at the gluon emission diagram



$$z = \frac{Q^2}{2 p_q \cdot q}$$

κ is the 4-momentum of the outgoing gluon
$p_{\rm virt}$ is the momentum of the virtual quark
z is equivalent to Bjorken-x but replacing the incoming proton with the incoming quark.

For gluon emission at small angles (collinear with the incoming quark), z is the fraction of the incoming quark carried by the virtual quark:

$$p_{\rm virt} = z p_q = z \xi P \; ; \qquad \kappa = (1-z) p_q$$

In this approximation the "virtuality" t of the propagator is:

$$t \equiv p_{\rm virt}^2 = (p_q - \kappa)^2 = 2 E_q E_k (1 - \cos\theta_g) \simeq (1-z) E_\kappa^2 \sin^2\theta_g = (1-z)\,\kappa_T^2$$

(where we used : $2\sin^2\theta_g/2 \simeq \sin^2\theta_g$)

Gluon emission: partonic cross section

The cross section for the gluon emission diagram is proportional to $\frac{1}{-t}$:
due to the propagator term the cross section diverges when the for $t \to 0$ that corresponds to $\kappa_T \to 0$: collinear emission

It can be shown that (neglecting terms that are relevant only at high κ_T) the cross section for $eq \to e'gq'$ is

$$\frac{d^2\hat{\sigma}(e\,q \to e'\,g\,q')}{dQ^2\,d\kappa_T^2} = \frac{d\hat{\sigma}_0(e\,q \to e'\,q')}{dQ^2}\,\frac{\alpha_S}{2\pi}\,\frac{1}{\kappa_T^2}\,P_{qq}(z)$$

Where $d\hat{\sigma}_0/dQ^2$ is the $eq \to eq$ cross section

$P_{qq}(z)$ is the quark-quark splitting function: the probability to find a quark with a fraction z of the initial quark:

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$$

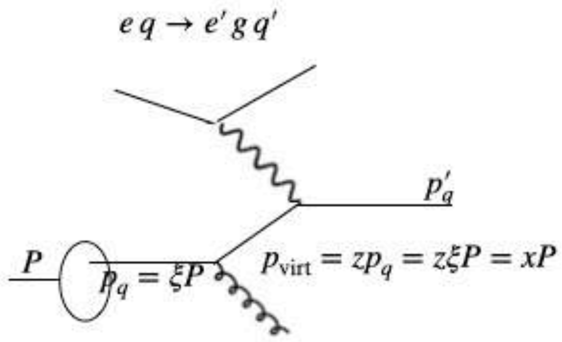
Gluon emission: partonic cross section

Integrating over κ_T^2 and exploiting the fact that the maximum virtuality *-t* in the propagator is the virtuality of the exchanged photon: $(\kappa_T^2)_{\max} \simeq |-t|_{\max} = Q^2$ we get:

$$\frac{d^2\hat{\sigma}(e\,q \to e'\,g\,q')}{dQ^2} = \frac{d\hat{\sigma}_0(e\,q \to e'\,q')}{dQ^2}\,\frac{\alpha_S}{2\pi}\,P_{qq}(z)\,\log\frac{Q^2}{\mu^2}$$

Where μ^2 is a cutoff and the parton cross section is divergent for $\mu^2 \to 0$

Gluon emission: hadronic cross section



To obtain the ep cross section we should integrate on all the possible momenta of the incoming quark ξP:

$$\frac{d^2\sigma(ep)}{dx\,dQ^2}(x,Q^2) = \sum_{i=u,d,\dots} \int_0^1 d\xi\; f_i(\xi) \int_0^1 dz\; \delta(x - z\xi)\, \frac{d^2\hat{\sigma}}{dQ^2}(z,Q^2)$$

$$= \sum_{i=u,d,\dots} \int_x^1 \frac{d\xi}{\xi}\, f_i(\xi)\, \frac{d^2\hat{\sigma}}{dQ^2}(x/\xi, Q^2)$$

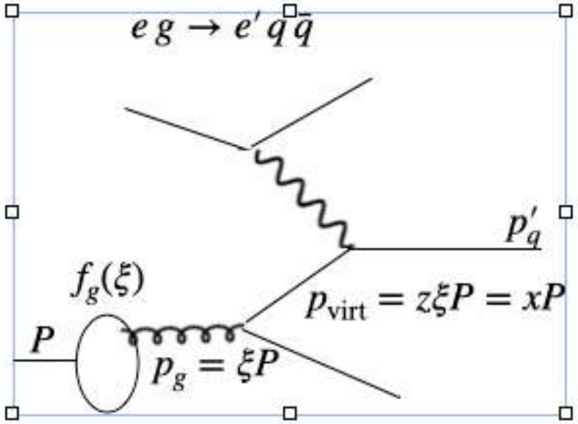
and therefore:

$$\frac{d^2\sigma(ep)}{dx\,dQ^2} = \sum_{i=u,d,\dots} \frac{d\hat{\sigma}_0}{dQ^2} \int_x^1 \frac{d\xi}{\xi}\, f_i(\xi) \left(\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_S}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \log\frac{Q^2}{\mu^2} \right)$$

Parton-model term

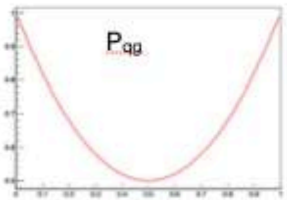
Gluon emission term

Adding gluon initiated term



Adding also the "boson-gluon fusion" diagrams

$$\frac{d^2\sigma(ep)}{dx\,dQ^2} = \frac{d\hat{\sigma}_0}{dQ^2}\int_x^1 \frac{d\xi}{\xi}\left[\sum_{i=u,d,\dots} f_i(\xi)\left(\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_S}{2\pi}P_{qq}\left(\frac{x}{\xi}\right)\log\frac{Q^2}{\mu^2}\right) + f_g(\xi)\,\frac{\alpha_S}{2\pi}P_{qg}\left(\frac{x}{\xi}\right)\log\frac{Q^2}{\mu^2}\right]$$



gluon probability density

gluon-quark splitting function: probability to find a quark with fraction $z = x/\xi$ of the initial gluon

$$P_{qg}(z) = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$

QCD factorisation

The divergency for $\mu^2 \to 0$ which corresponds to collinear emission can be absorbed in a re-definition of $f_i(x)$:
$f_i(x) \to f_i(x, \mu^2)$

such that the cross sections (or equivalently the structure functions F) are independent of the arbitrary choice of the cutoff parameter μ^2:

$$\sigma^{ep}(Q^2) = \sum_{i=u,d,\ldots,g} \int \frac{d\xi}{\xi} \, f_i(\xi, \mu^2) \; \hat{\sigma}^{eq_i}(\xi s, Q^2/\mu^2)$$

PDF — parton cross section

The collinear-divergent parts are absorbed in the PDFs: as the scale μ^2 increases, the PDF increase as $\log \mu^2$ to compensate the $\log Q^2/\mu^2$ term in the partonic cross section.

This is known as the QCD factorization theorem and is a general ansatz used to calculate all high-energy cross-sections at colliders with hadron beam(s).

It has been proved rigorously for few process like DIS or Drell-Yan.

QCD factorisation in DIS

A typical practical choice for performing calculations is $\mu^2 = Q^2$: in this case the DIS cross section reduces to

$$\sigma^{ep}(Q^2) = \sum_{i=u,d,\ldots,g} \int \frac{d\xi}{\xi} f_i(\xi, Q^2)\ \hat{\sigma}^{ei}(\xi s)$$

[There are also finite terms that appear at higher orders in α_S that we did not consider here. This gives some arbitrariness in the definition of the PDFs (terms can be incorporated in the PDFs or in the partonic cross sections.
This brings to different "factorisation schemes" anyway this is purely technical: if both PDFs and partonic cross-sections are calculated in the same scheme then the cross section does not change.]

In particular in the "DIS" scheme

$$F_2(x, Q^2) = x \sum_{i=u,d,\ldots} e_i^2 f_i(x, Q^2)$$

the same equation as parton model but now PDFs depend on Q^2

So the ep cross section is actually violating the Bjorken scaling and has a logarithmic dependence on Q^2

DGLAP* Equations

The full dependency of PDFs from the factorization scale is given by:

$$\frac{df_q(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left(P_{qq}(z)\, f_q(\xi,\mu^2) \;+\; P_{gq}(z)\, f_g(\xi,\mu^2)\right)$$

$$\frac{df_g(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left(\sum_{i=u,d,\ldots} P_{qg}(z)\, f_i(\xi,\mu^2) \;+\; P_{gg}(z)\, f_g(\xi,\mu^2)\right)$$

These are the DGLAP equations

Gluon and quark PDFs are coupled

Knowing the PDFs at a starting scale, μ_0^2, they can be re-calculated at any scale μ^2 by solving DGLAP equations.

Physical interpretation

As the scale increases we see inside the proton with higher resolution and we can resolve more partons.

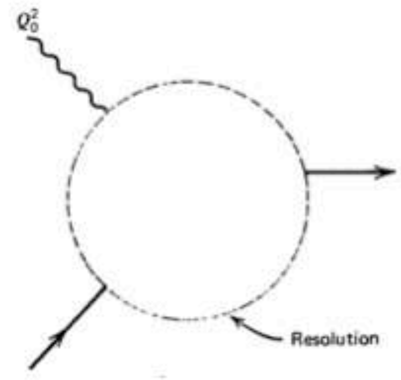
The same process can be seen

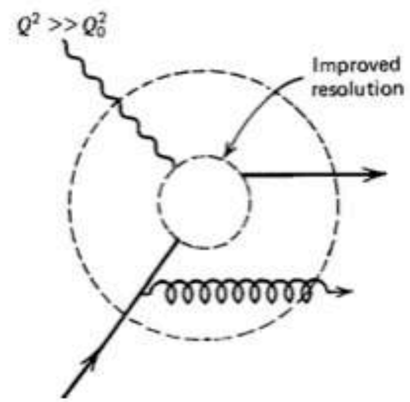
- at low resolution (Q_0^2) a quark with fraction x of the proton momentum originating from the proton structure interacting with the exchanged photon or
- at high resolution ($Q^2 \gg Q_0^2$) as a quark coming from the proton structure with proton momentum fraction $\xi > x$ that splits into a gluon and a quark and the latter interacts with the exchanged photon

Based on the choice of scale we can move splittings from the hard parton scattering to the PDFs (and vice-versa)

What happened to the collinear divergence ?
The divergent part was actually coming from a perturbative calculation extrapolated at very low scale, $Q^2 \to 0$, beyond the applicability of the perturbative approximation: at long-range breaks we face the non-perturbative proton structure.

$$\Delta b \sim \frac{\hbar c}{\sqrt{Q^2}} = \frac{0.197}{\sqrt{Q^2}} \text{GeV fm}$$



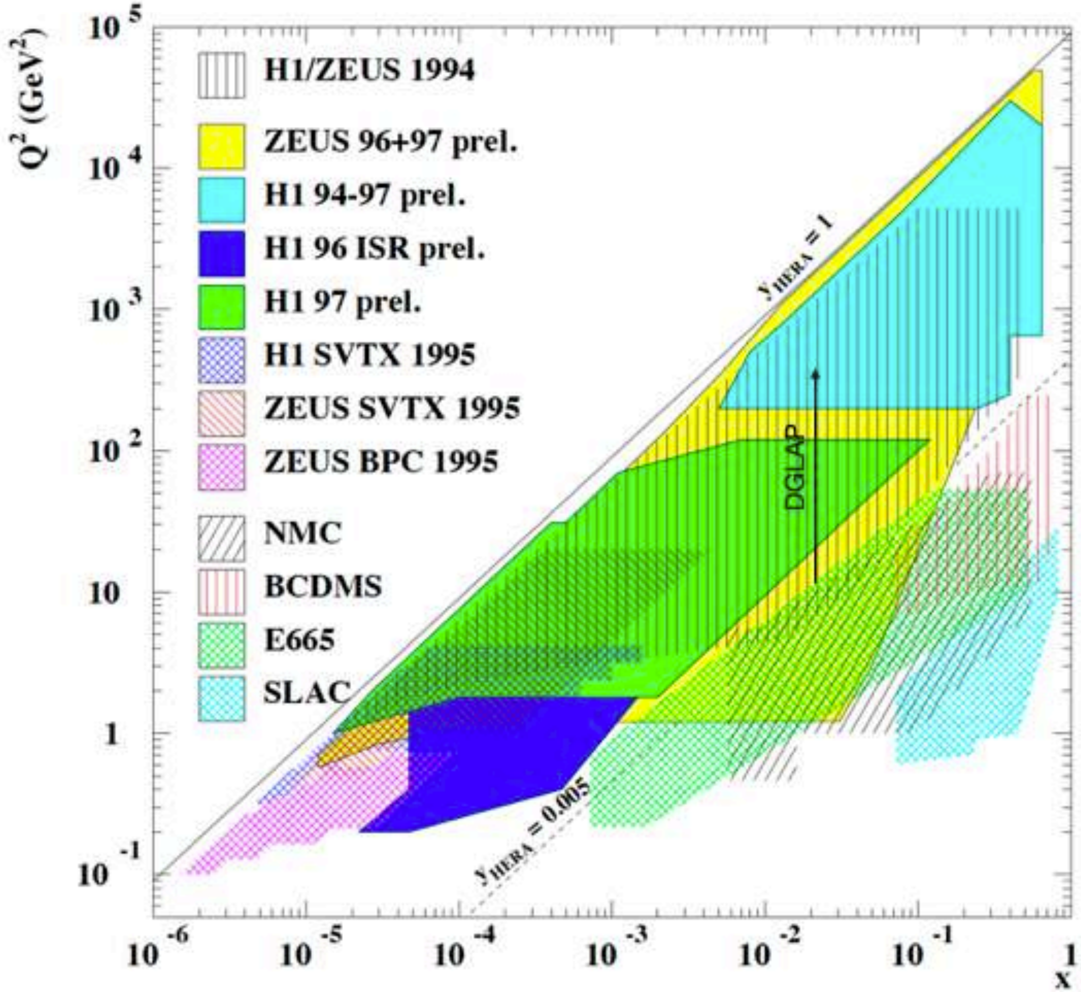


HERA Measurements of DIS cross-sections and structure functions

HERA Kinematic plane compared to fixed target

HERA extended the x, Q2 coverage from previous DIS experiments by 2 orders of magnitude towards lower x and higher Q2.

Measurements of cross sections at different Q^2 can be compared directly.



How to measure structure functions

What can be measured directly are cross sections and structure functions, not PDFs !

How to measure:

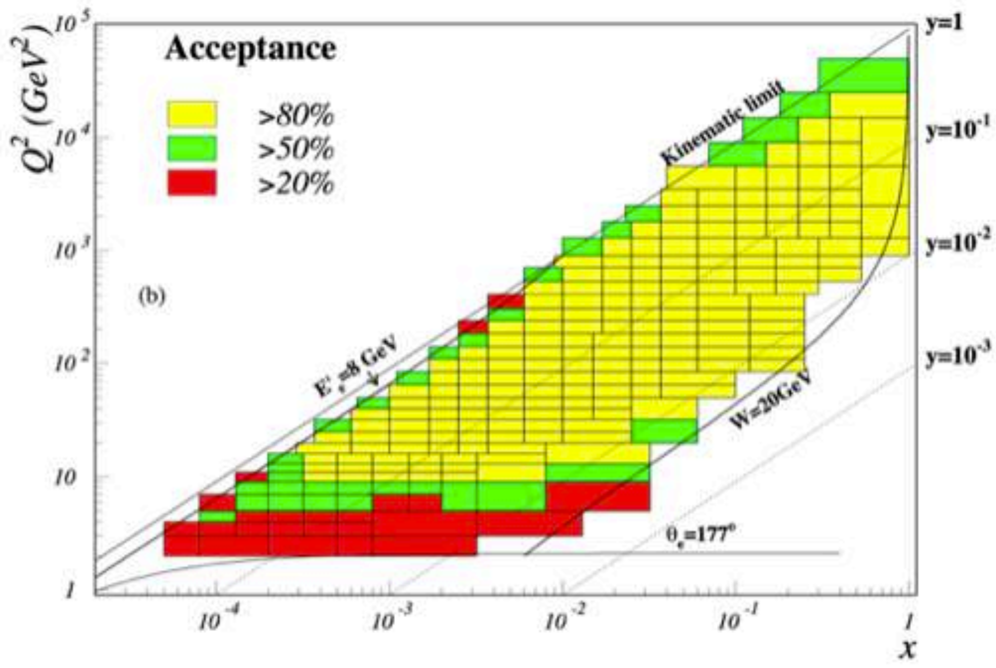
- Divide the x,Q² plane in bins with size $\Delta x_i \; \Delta Q_i^2$
- Reconstruct x, Q^2 of each event and assign it to a bin i
- Then the average cross section in the i^th^ bin is given by

$$\sigma_i = \frac{N_i^{\text{rec}} - N_i^b}{A_i \; L}$$

where N_i^{rec} is the number of events reconstructed in the bin, N_i^b is the estimated background, L is the data-set luminosity and A_i is the bin acceptance that can be obtained by Monte Carlo as the ratio between reconstructed and generated events:

$$A_i = \left(\frac{N_i^{\text{reco}}}{N_i^{\text{gen}}} \right)_{\text{MC}}$$

- then the double-differential cross section at the center of the bin $x_{\text{ref}}, Q^2_{\text{ref}}$ is obtained by the bin average with a bin-centering correction C_i



$$\frac{d^2\sigma}{dx\, dQ^2}(x_{\text{ref}}, Q^2_{\text{ref}}) = \frac{\sigma_i}{\Delta x_i \, \Delta Q_i^2} \cdot C_i$$

How to present results...

Reminder: neglecting deviations from Callan-Gross relation and Z exchange:

$$\frac{d\sigma}{dx\, dQ^2} = \frac{2\pi\alpha}{Q^4}\left[1 + (1-y)^2\right] F_2(x, Q^2)$$

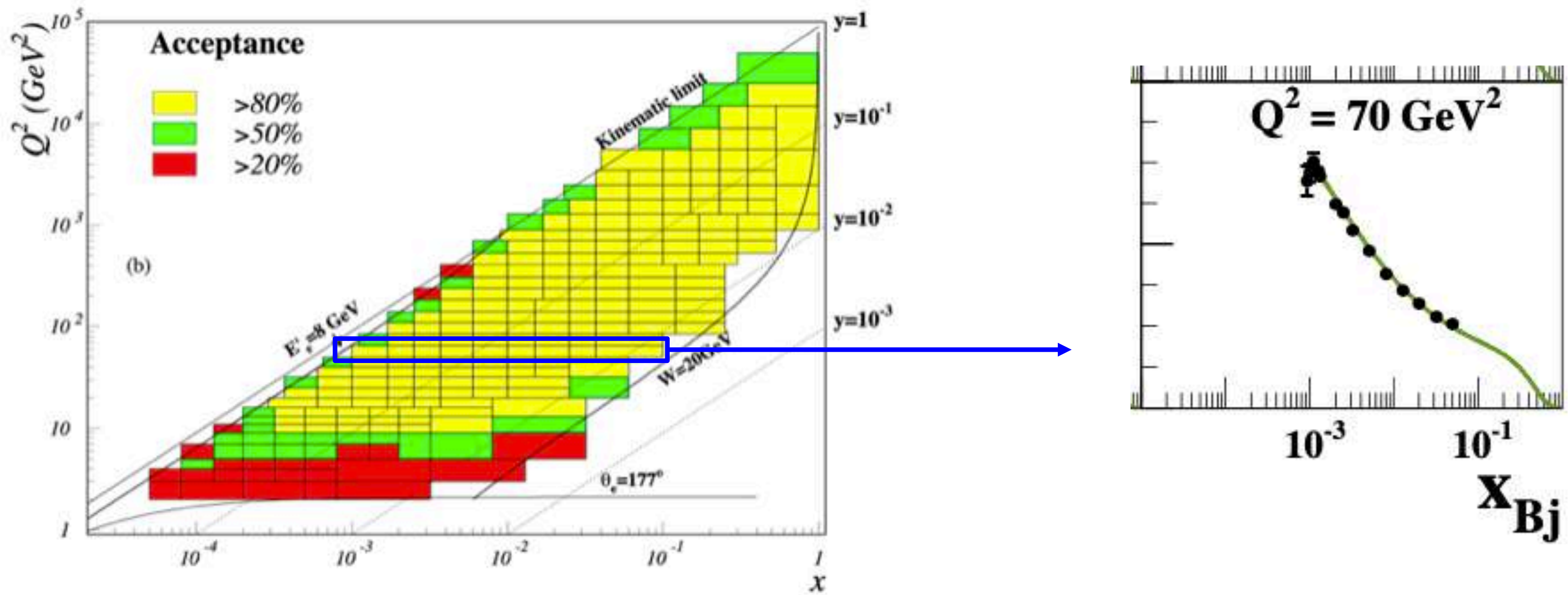
What was typically measured at HERA was actually the "reduced cross-section" σ_r

$$\sigma_r = \frac{d^2\sigma}{dx\, Q^2}\left[\frac{2\pi\alpha}{Q^4}\left[1 + (1-y)^2\right]\right]^{-1}$$

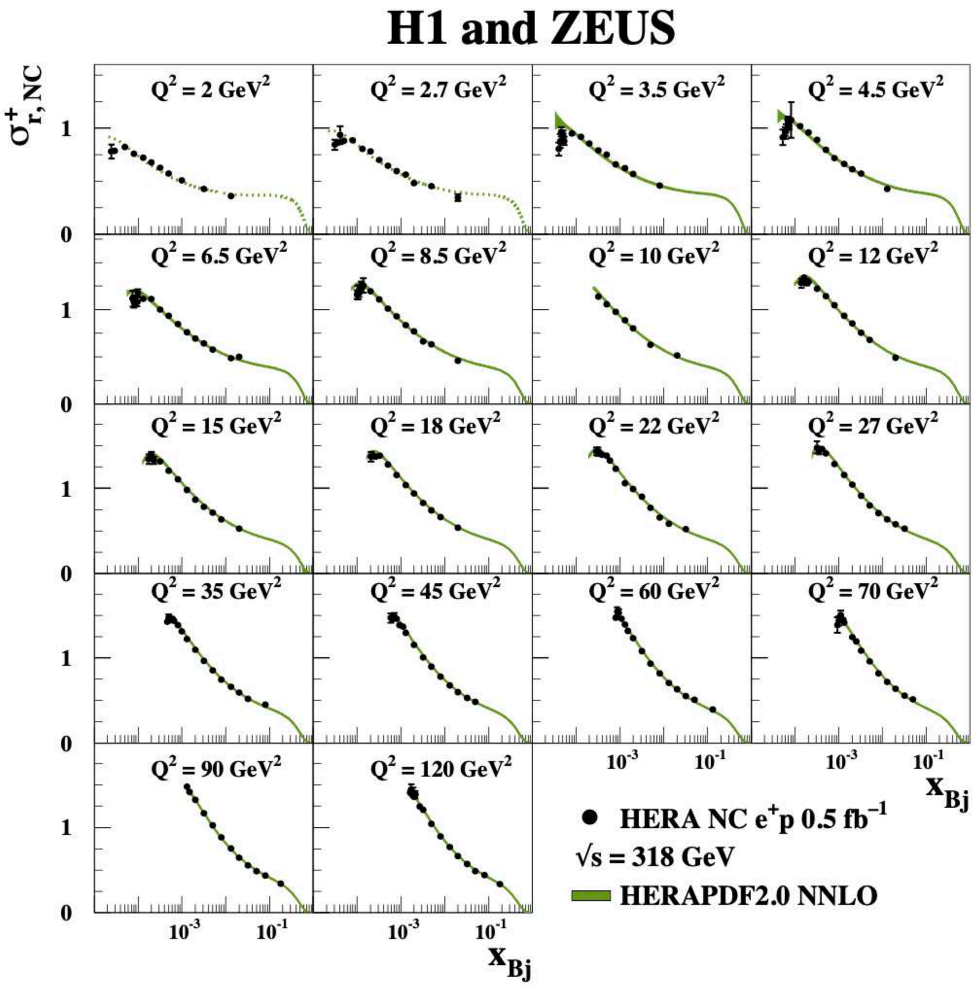
that is $\sigma_r \simeq F_2$ if we neglect and Z exchange

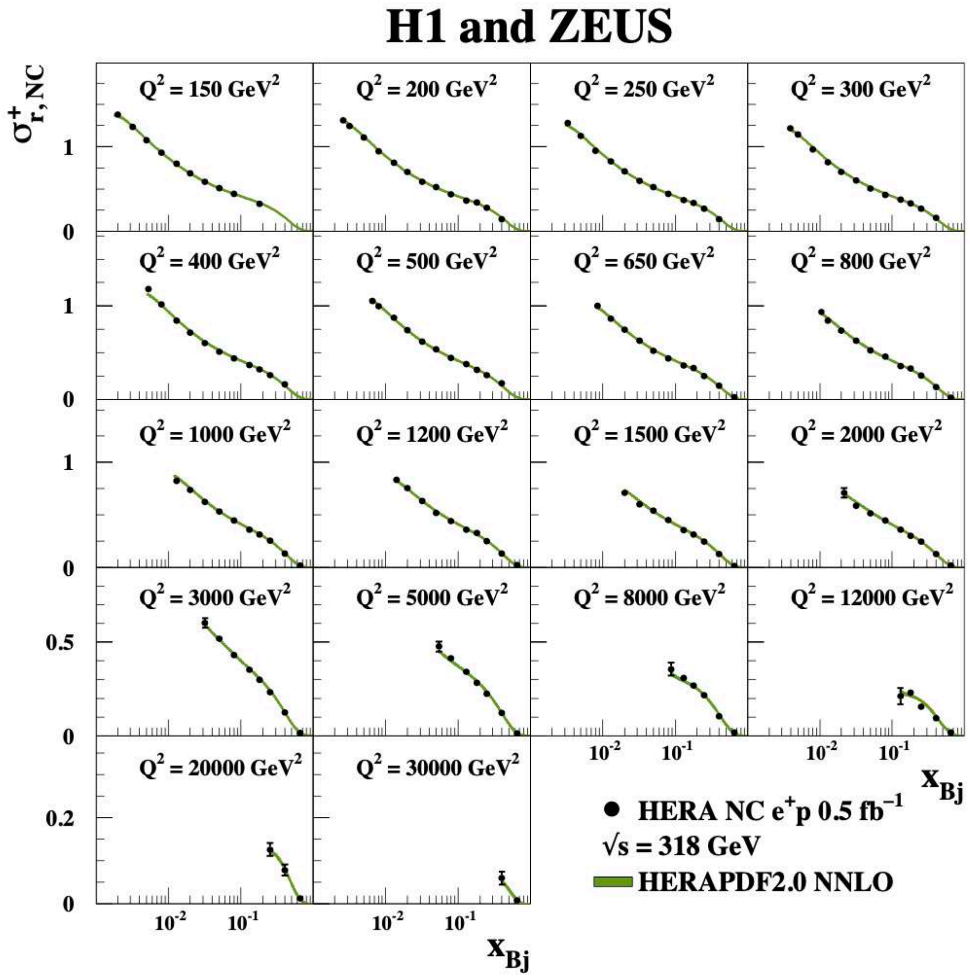
DIS plots

❑ **The cross-sections are presented as "slices" in Q^2 as a function of x**

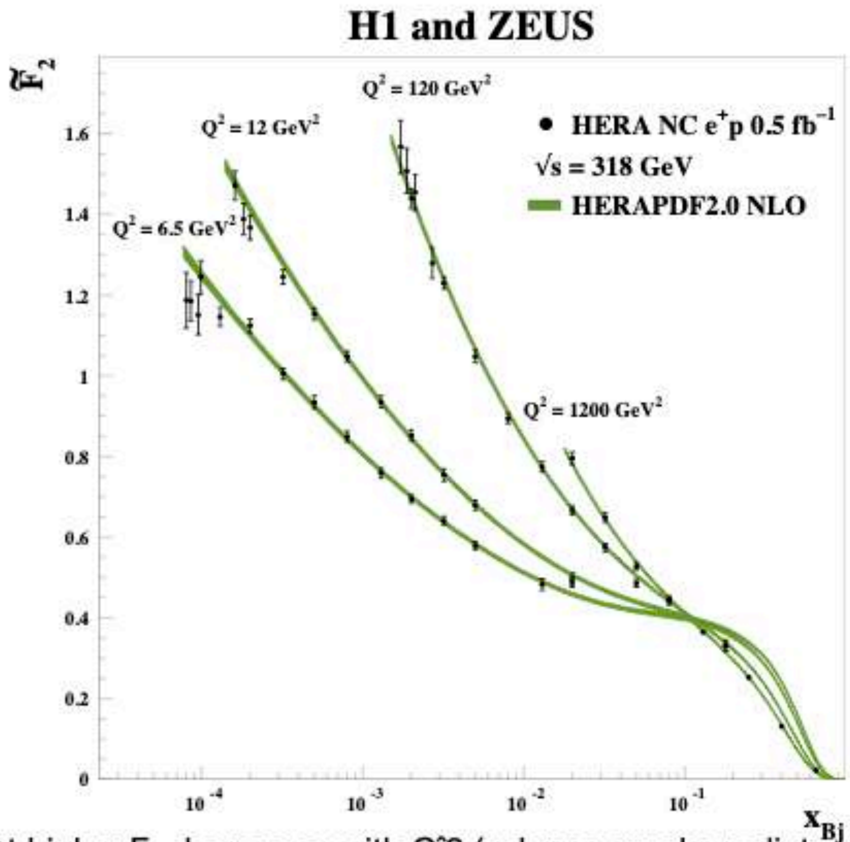


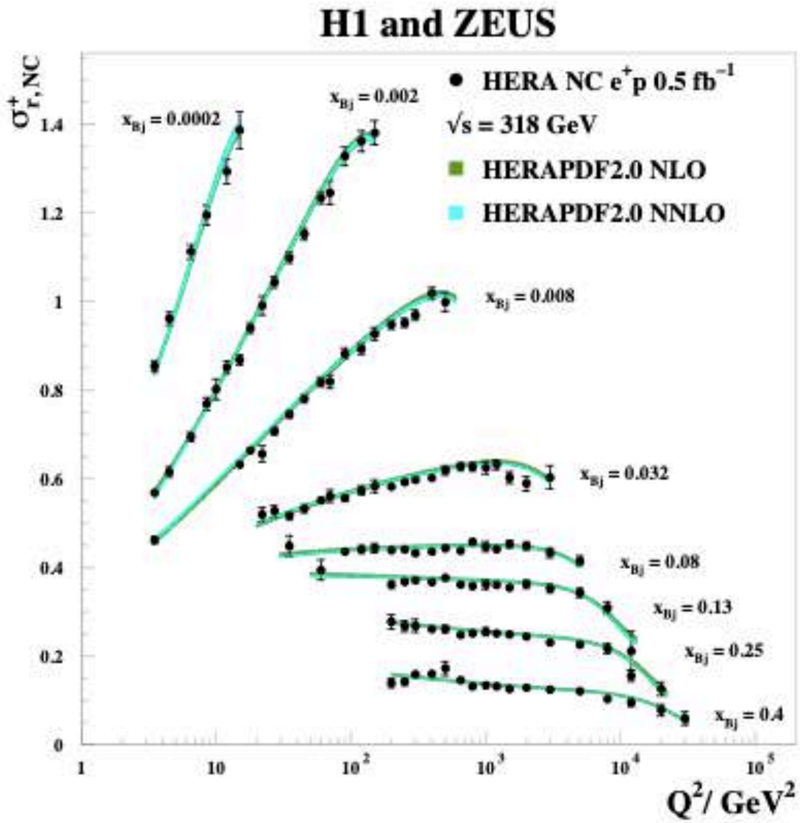
DIS cross-sections: example e^+p NC





A zoom into a few bins

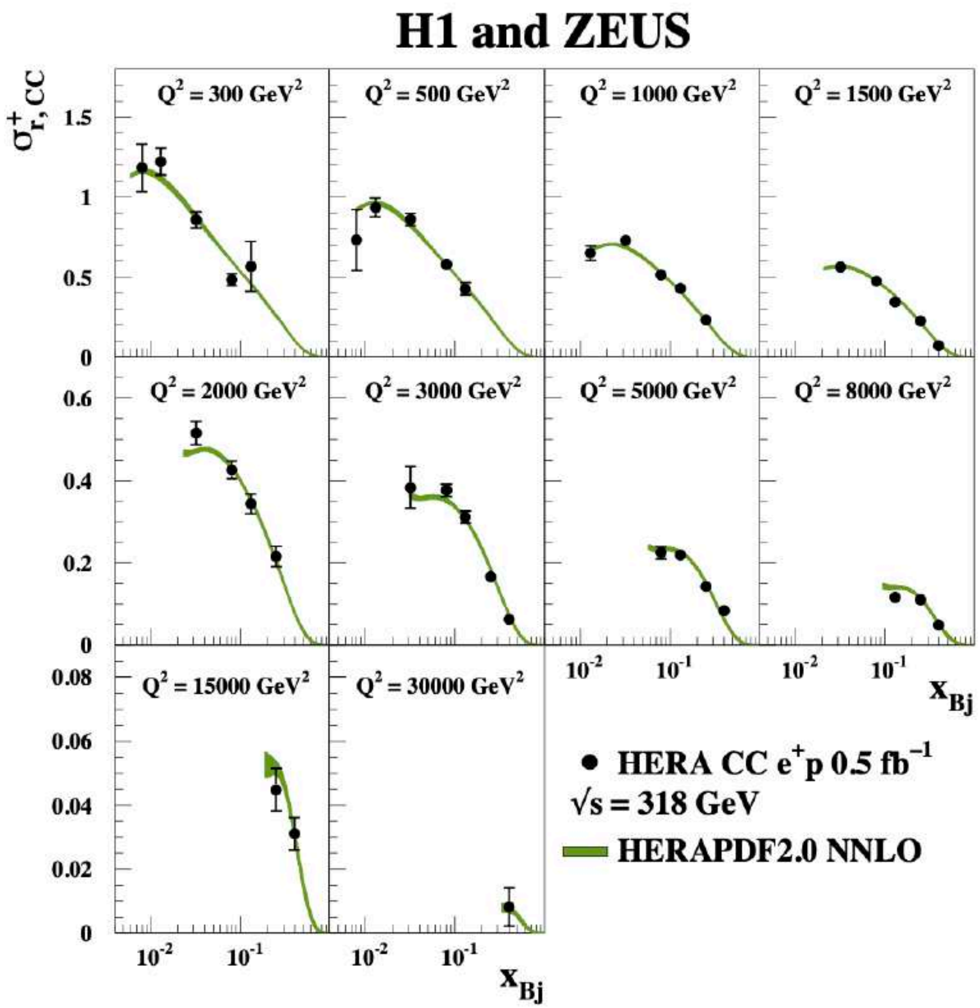




At high x F_2 decreases with Q^2 (valence quarks radiate gluons)
At low x F_2 increases with Q^2 (more gluons, more sea)

"Bjorken scaling" at x = 0.1 (by chance where was measured originally at SLAC !)

HERA Measurement: example e^+p CC



Extraction of PDFs

Extracting the parton density functions

The parton densities at a fixed scale Q_0^2 can not be calculated from first principles, should be extracted from data.

How many independent functions ?
At low scale Q_0^2, lower than charm mass squared, the only partons are $u, d, s, \bar{u}, \bar{d}, \bar{s}, g$
(Charm and beauty are produced dynamically from gluon splitting when $Q^2 > m_c^2, \quad m_b^2$ respectively)

To reduce the freedom we can make some assumptions and put together the sea quarks:
$S(x) = \bar{u}(x) + \bar{d}(x) + \bar{s}(x)$ (total sea)
$\Delta S = \bar{u}(x) - \bar{d}(x)$ (u-d sea asymmetry)
and assume $s(x) = \bar{s}(x) = f_S \, \bar{d}(x)$ where f_s is a strange suppression factor taken by other measurements.

Thus we reduce to 5 PDFs: $u_v(x), d_v(x), S(x), \Delta S(x), g(x)$
where subscript *v* is for "valence": $u(x) = u_v(x) + \bar{u}(x)$ and $d(x) = d_v(x) + \bar{d}(x)$

Extracting the PDFs: parametrization

Typical (simple) parametrization at staring scale Q_0^2

$$xf(x, Q_0^2; \mathbf{p}) = p_1 x^{p_2} (1-x)^{p_3} (1 + p_4 x)$$

normalization (p_1), low x (p_2), high x (p_3), intermediate (p_4)

Some parameters are fixed by sum rules:

$$\int dx\,(u - \bar{u}) = 2, \quad \int dx\,(d - \bar{d}) = 1, \quad s = \bar{s}, \quad c = \bar{c}$$ Proton quark content

$$\int dx\, x\,(u + \bar{u} + d + \bar{d} + \cdots + g) = 1$$ Momentum conservation

From a parametrization at a starting scale Q_0^2 we can obtain the PDFs at any scale by solving numerically DGLAP equations :

$f(x, Q_0^2; \mathbf{p})$ -- DGLAP ---> $f(x, Q^2; \mathbf{p})$

Extracting the PDFs: DIS data

Different DIS processes are sensitive to different PDF combinations:

NC $\propto 4/9\,(u + c + \bar{u} + \bar{c}) + 1/9\,(d + s + \bar{d} + \bar{s} + b + \bar{b})$

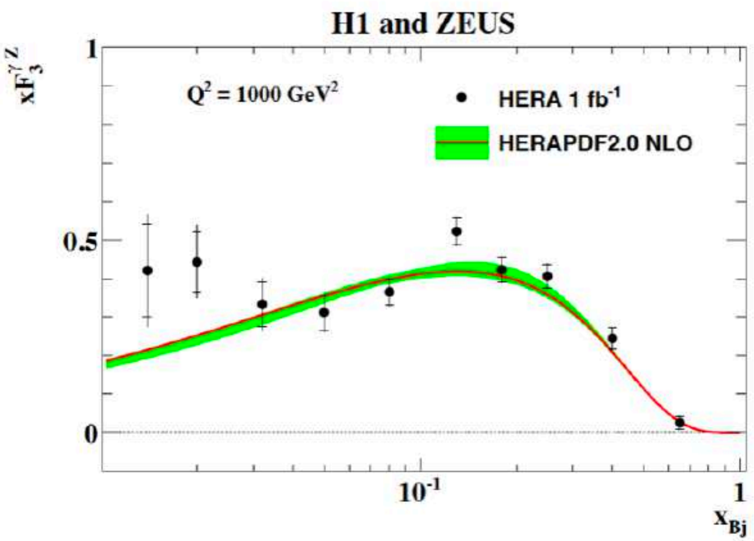
CC e^-p $\propto u + c + (1-y)^2\,(\bar{d} + \bar{s} + \bar{b})$

CC e^+p $\propto \bar{u} + \bar{c} + (1-y)^2\,(d + s + b)$

we can separate 5 different quark combinations

In addition the parity violating part of NC ($Z\gamma$ interference) is sensitive to a different quark combination:

High-Q^2 NC : $xF_x \propto \sigma(e^-p) - \sigma(e^+p) \propto 2u_v + d_v$



Different processes are needed to distinguish different quarks

The method for extracting the PDFs consists in building a global χ^2

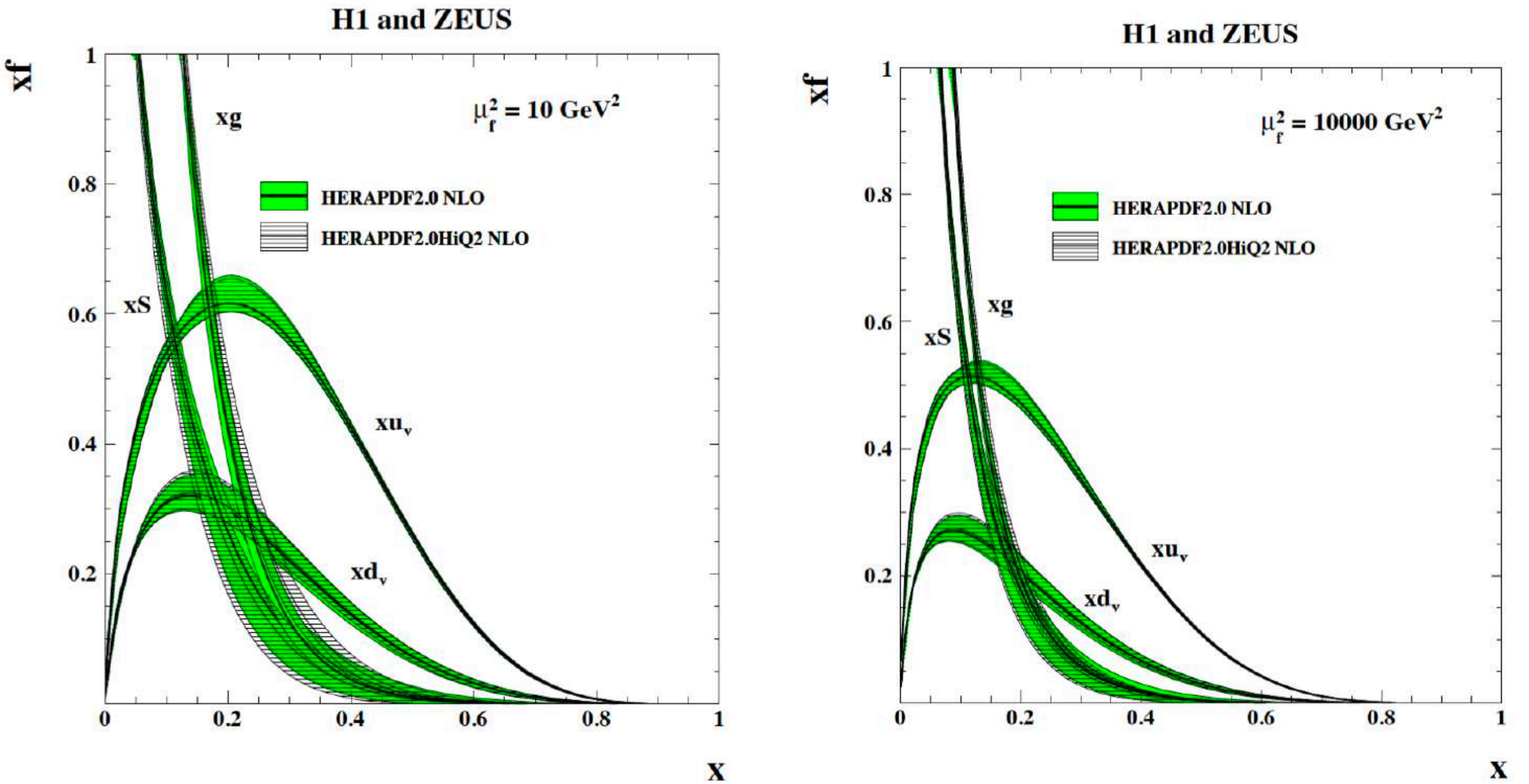
For each measured observable o_i^{meas} (e.g. a NC differential cross section bin), with uncertainty δ_i, we can calculate a theoretical prediction $o_i^{\text{theo}}(\mathbf{p})$ based on a parametrization of the PDFs at the starting scale ($Q_0^2 = 1.9\ \text{GeV}^2$) with a set of parameters **p.**

$$\chi^2 = \sum_i \frac{\left(o_i^{\text{meas}} - o_i^{\text{theo}}(\mathbf{p}) \right)^2}{\delta_o^2}$$

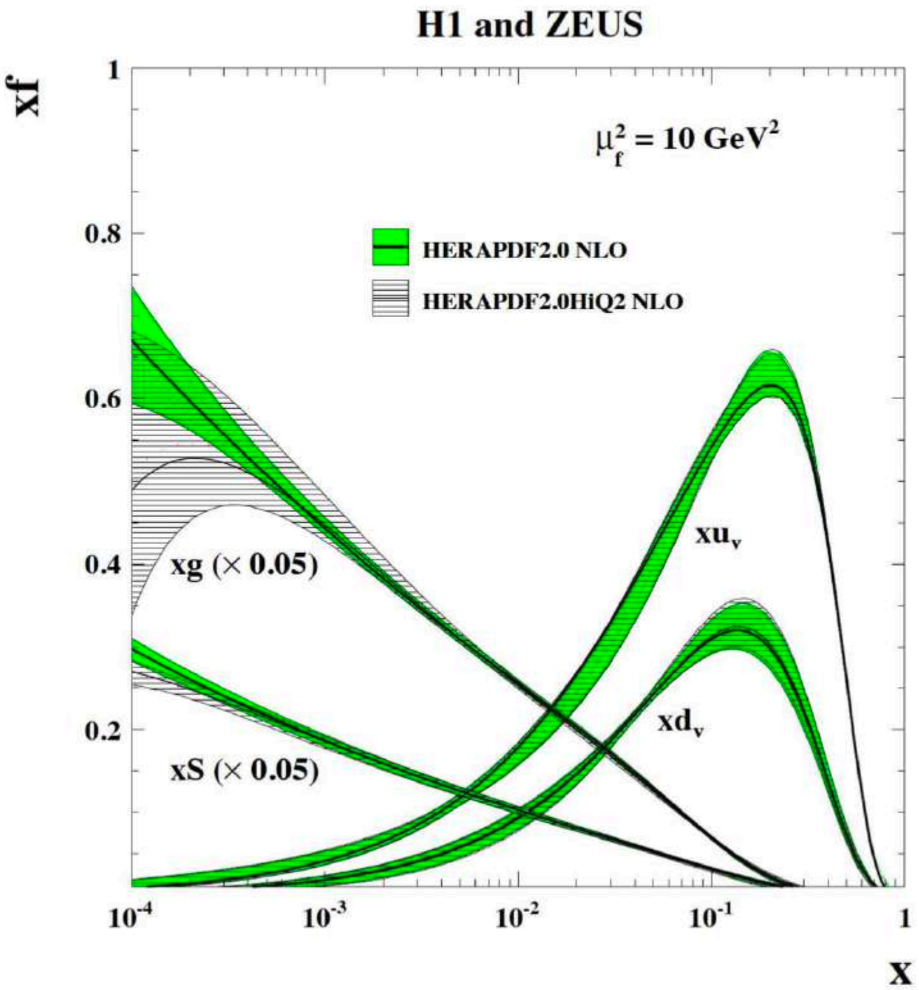
(actually to take into account systematics a more complex definition of is used)

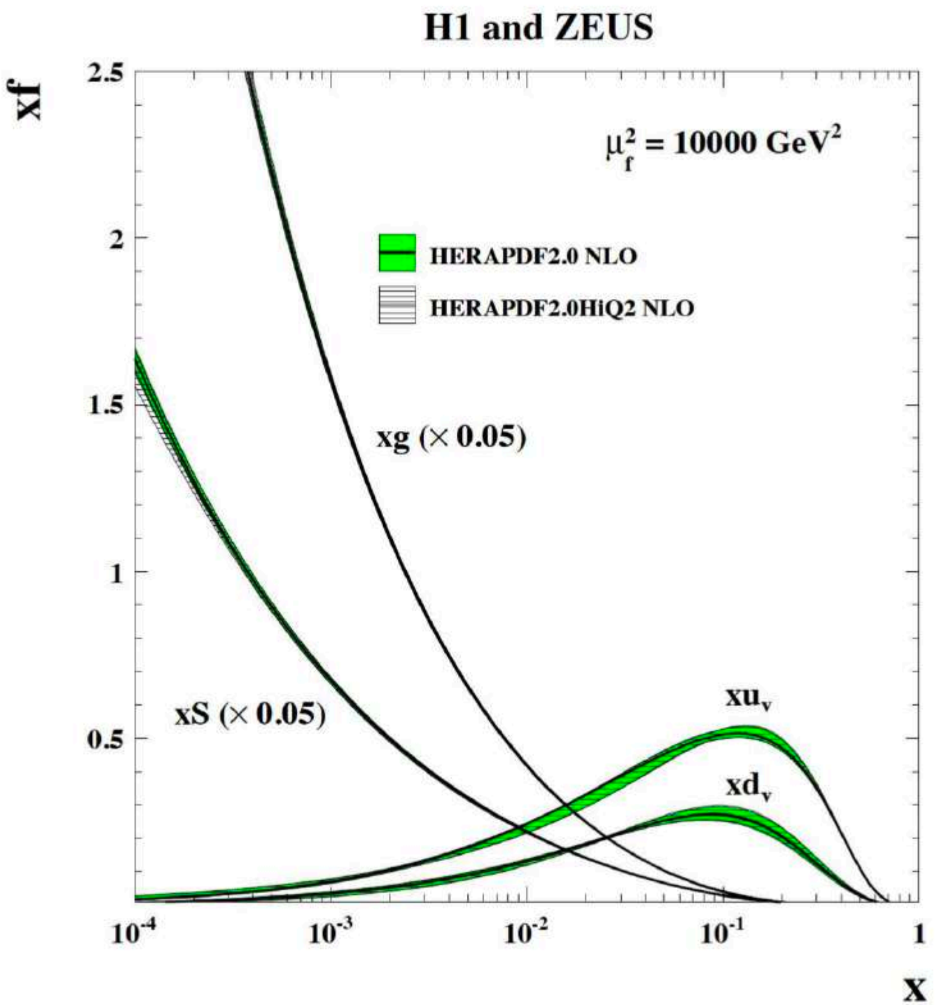
Then the set of parameters **p** that minimizes χ^2 is found.

HERAPDF: parton density fit based on HERA measurement only



HERAPDF: log-x scale





and the gluon?

Inclusive DIS is only sensitive to quarks

Nevertheless we obtained a measure of the gluon PDF from the fit

This is mainly because the gluon and the sea PDFs are coupled through evolution of the DGLAP equations.

In particular for low-x NC the slope of the variation with log Q^2 is proportional to the gluon density.

$$\frac{dF_2}{d\log Q^2} \propto \alpha_S\, g(x, Q^2)$$

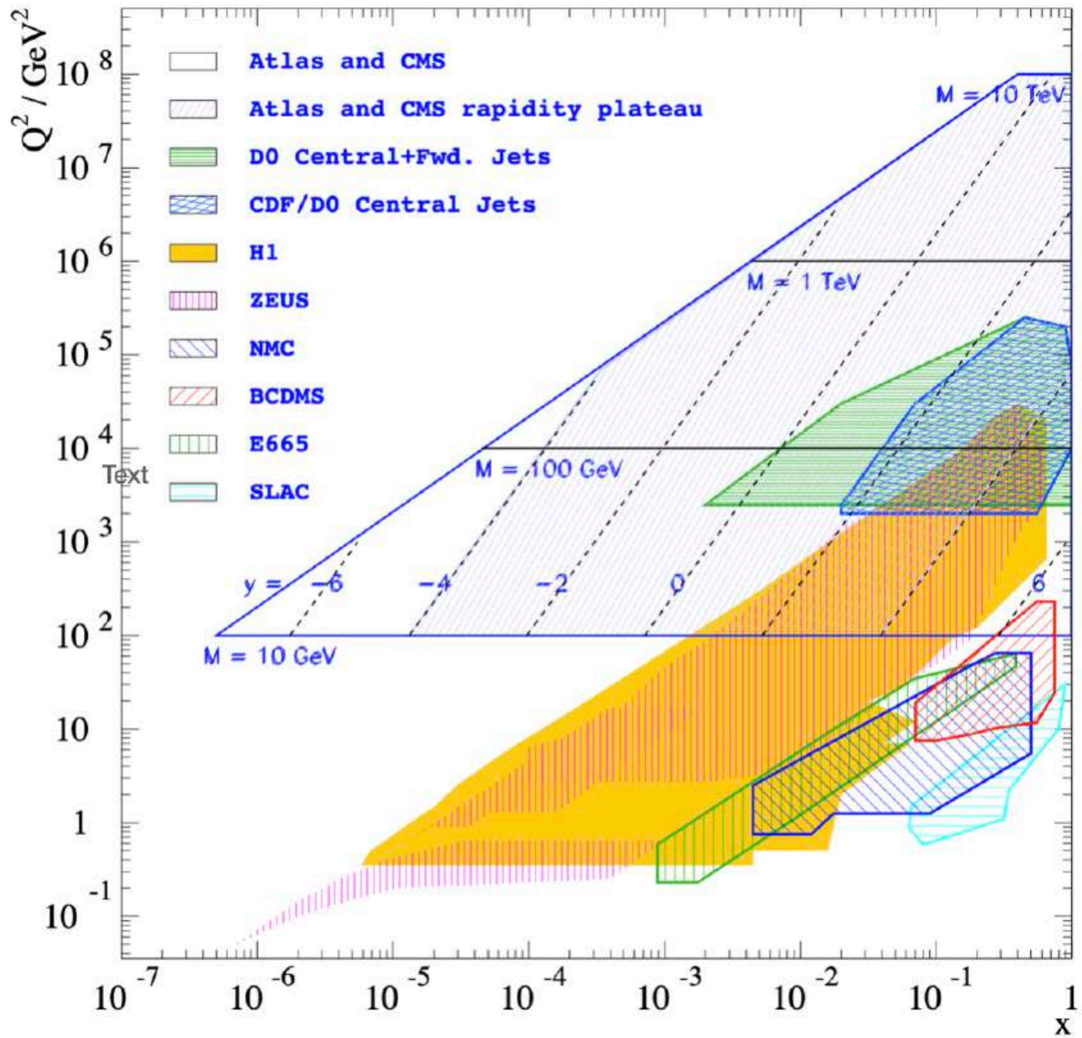
To improve the sensitivity on g(x) we can also add some "non-inclusive" DIS data :
- DIS Jets
- DIS with production of c or b quarks

HERA, PDFs and LHC

HERA extended the x, Q2 coverage from previous DIS experiments by 2 orders of magnitude towards lower x and higher Q2.

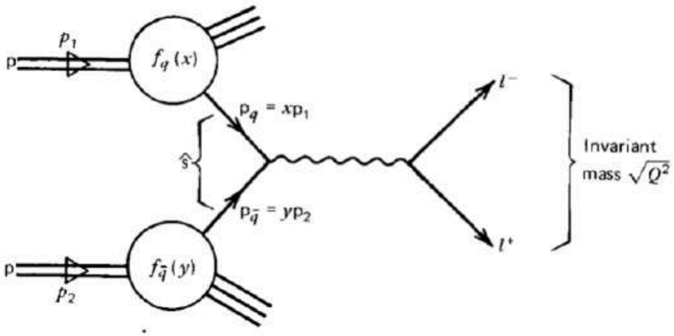
We can use PDFs extracted from DIS to to calculate all processes at hadron colliders.

In reality LHC data can also be used to improve the knowledge on PDFs: this is done by "global" PDF fit groups that include typically data on W and Z production at Tevatron and LHC and in some cases also jet production.



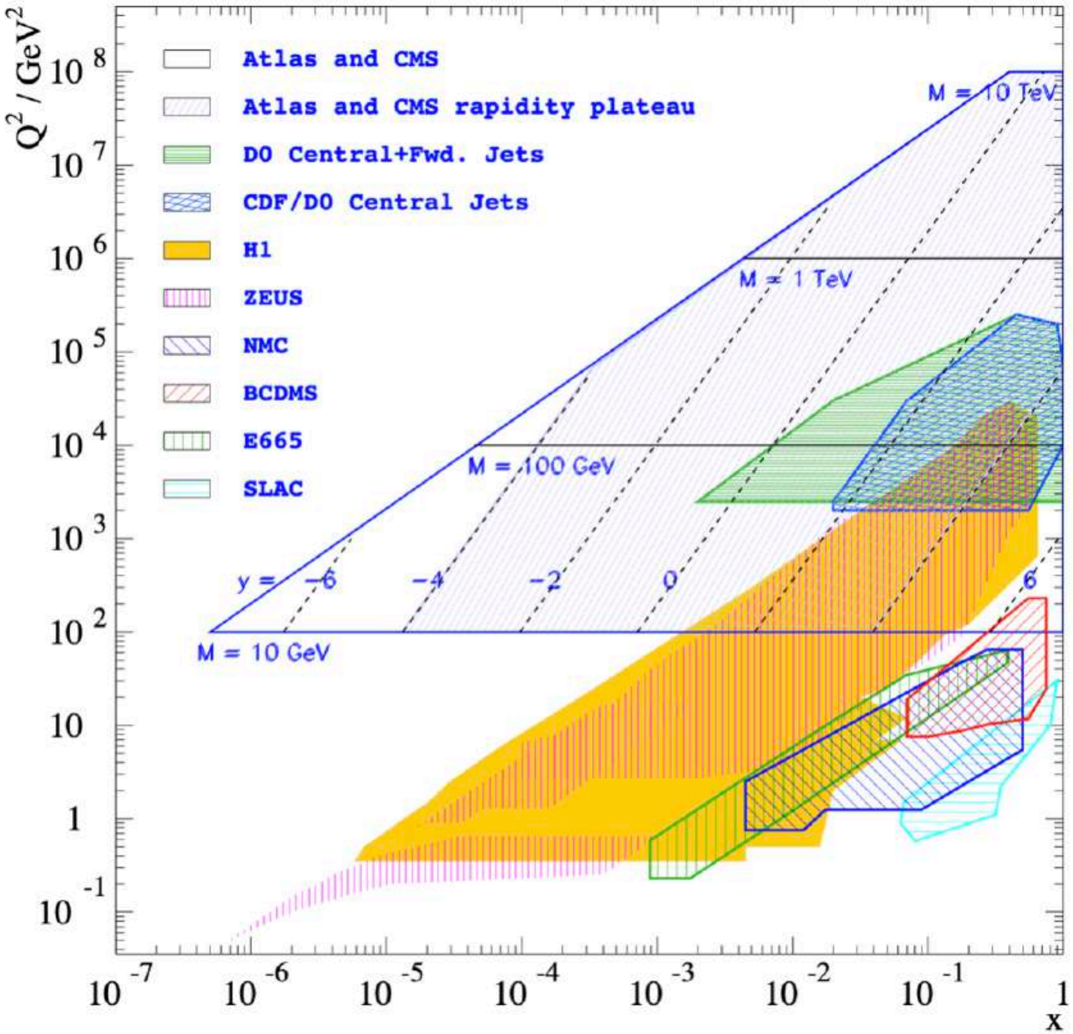
DY processes and x,Q^2

Example: Drell-Yan process



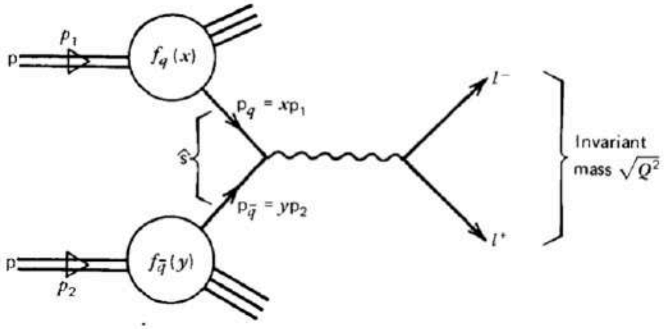
At LO there is a relation between mass M and rapidity y (that are measured observables) and x, Q2 of the PDFs:

$$Q^2 = M^2 = \hat{s} = x_1 x_2 s$$
$$x_1,2 = \frac{M}{\sqrt{s}} \exp(\pm y)$$
$$y = \frac{1}{2}\log\frac{E+p_z}{E-p_z} = \frac{1}{2}\log\frac{x_1}{x_2}$$



DY processes and x,Q^2

Example: Drell-Yan process

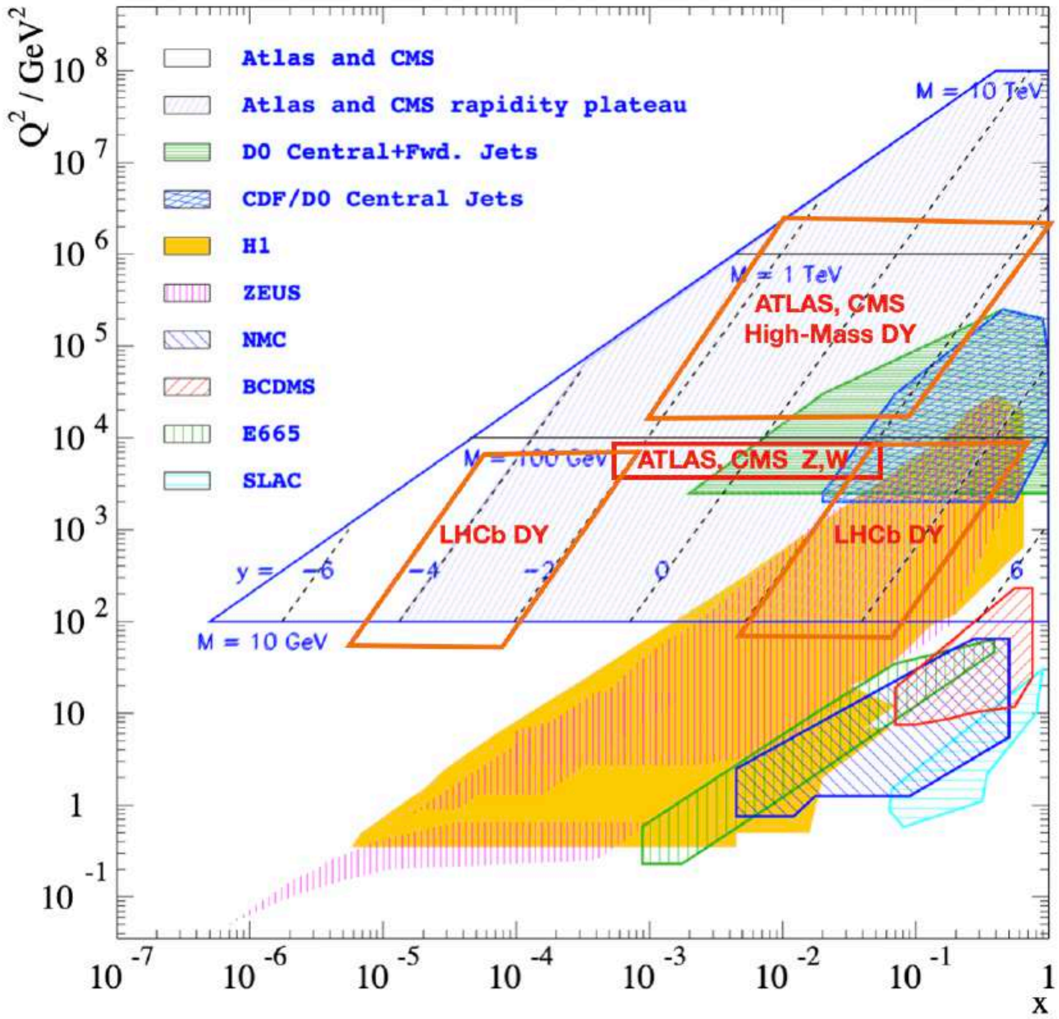


At LO there is a relation between mass M and rapidity y (that are measured observables) and x, Q2 of the PDFs:

$$Q^2 = M^2 = \hat{s} = x_1 x_2 s$$

$$x_1,2 = \frac{M}{\sqrt{s}} \exp(\pm y)$$

$$y = \frac{1}{2}\log\frac{E+p_z}{E-p_z} = \frac{1}{2}\log\frac{x_1}{x_2}$$



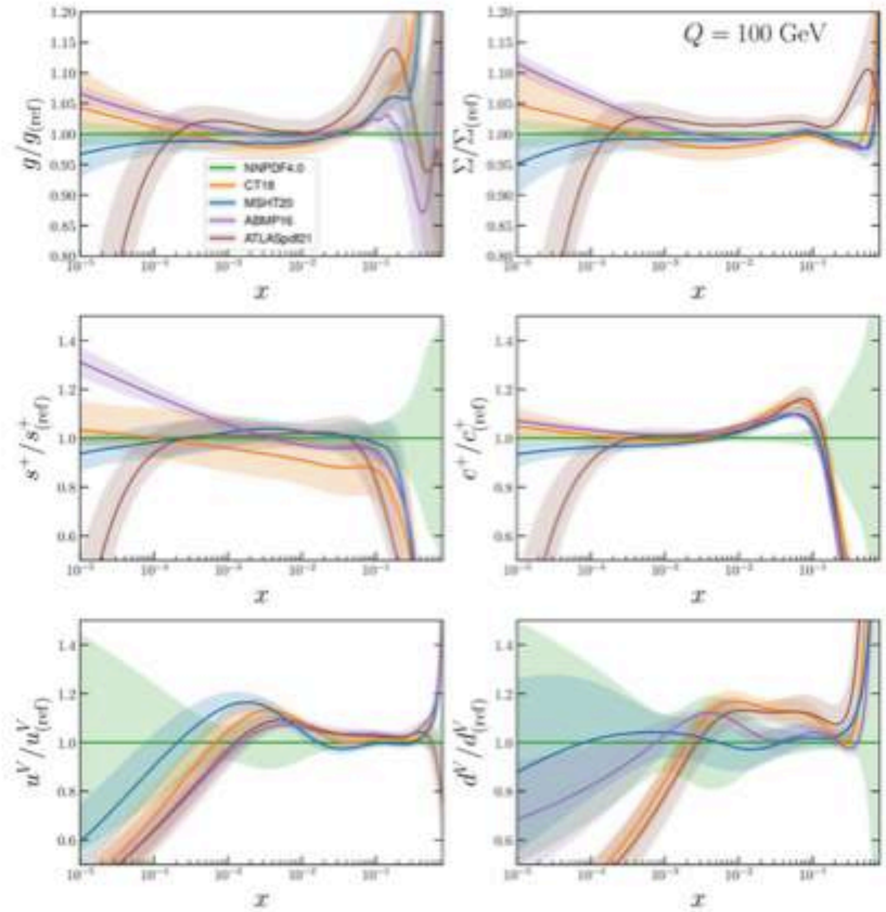
Status and outlook on PDFs

Currently we have several PDF parametrisations extracted by different teams. Typically Fits include from 2000 to 5000 measurements

Different choices about perturbative order of calculations, choice ofdata sets to include etc. introduce difference beyond those of estimated uncertainties.

The uncertainty on several precision observables measured at LHC is actually limited by PDFs that are the main source of uncertainty, e.g. W mass, weak mixing angle $\sin^2\theta_W$

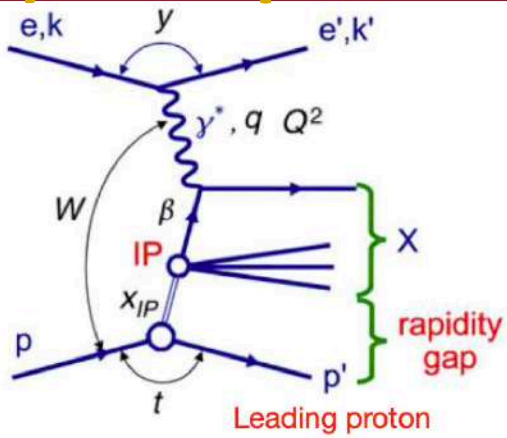
So there is a big theoretical and phenomenological interest in improving further



Comparison of the PDFs at Q = 100 GeV. The PDFs shown are the N2LO sets of NNPDF4.0, CT18, MSHT20, ABMP16 with αs(MZ) = 0.118, and ATLASpdf21. The ratio to the NNPDF4.0 central value and the relative 1σ uncertainty are shown for the gluon g, singlet Σ, total strangeness s + = s + ¯ s, total charm c + = c + ¯ c, up valence u V and down valence d V PDFs.

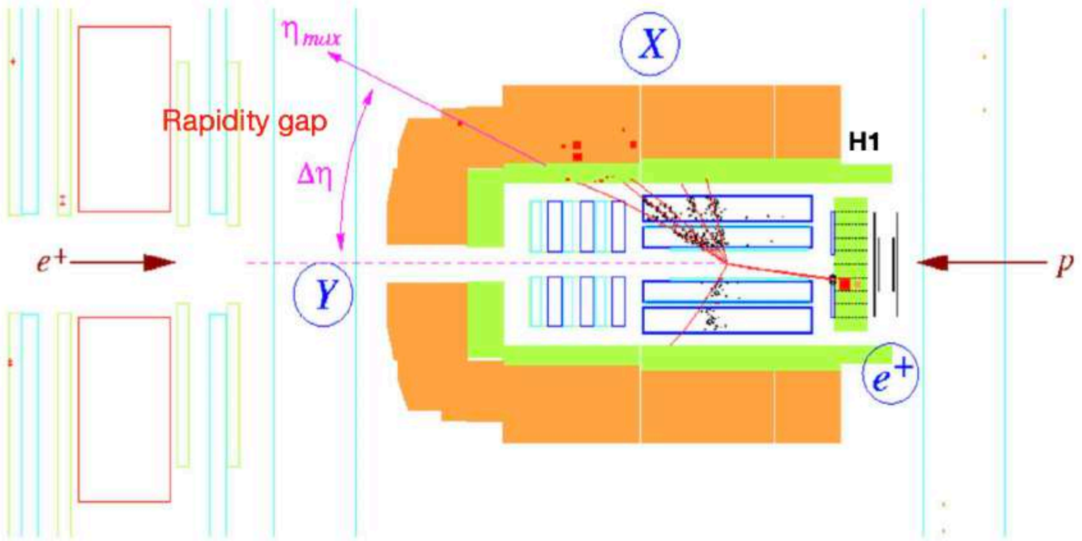
Hard diffraction

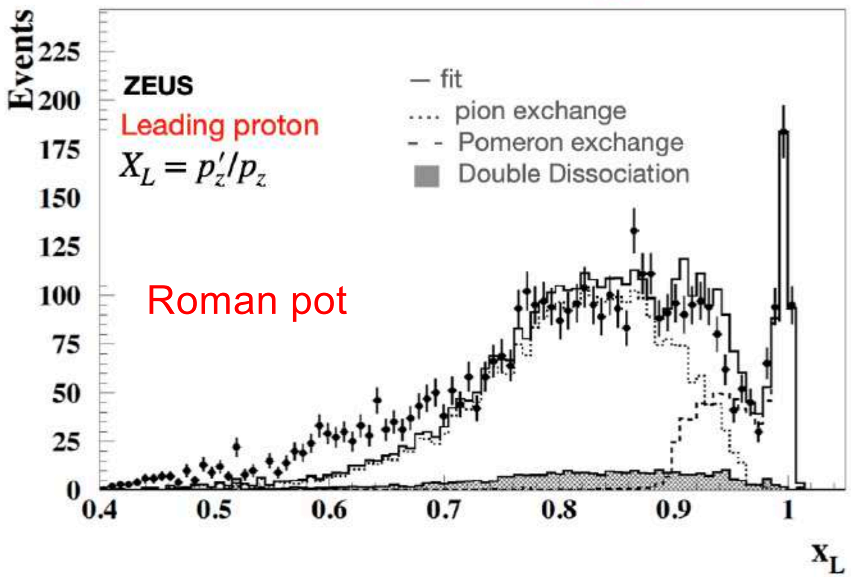
Hard diffraction: something (almost*) unexpected!



In about 5-10% of all DIS events the proton does not break-up or breaks into a very low mass state, separated from the rest of the hadronic system by a large rapidity gap without hadronic activity.

Two experimental signatures:
- "Large rapidity gap"
- Forward leading proton (measured with roman pots) with $X_L = p_z'/p_z \simeq 1$

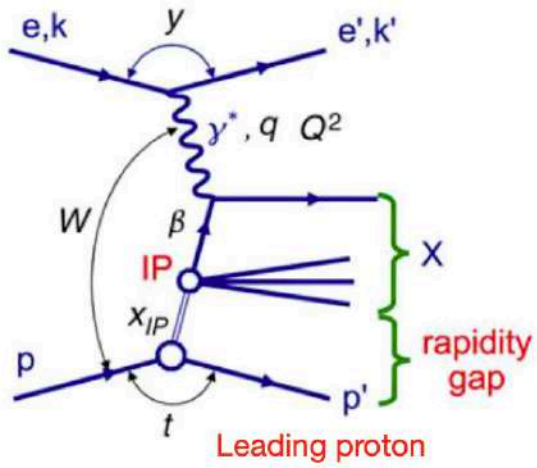




(* first evidence of hard diffractive events was found by UA8 at SppS that found "exclusive" *p p -> p p' jet-jet* events)

Hard diffraction: interpretation

The proton fluctuates into colourless objects (cfr. Regge theory), the dominant object at high energy is the Pomeron, with the same quantum numbers as vacuum.



Diffractive factorisation :

$x_{\mathrm{IP}} = 1 - X_L$: fraction of proton momentum carried by the Pomeron

$\beta = x/x_{\mathrm{IP}}$: fraction of the Pomeron momentum carried by struck quark

$F_2^{D(4)} = f_{\mathrm{IP}}(x_{\mathrm{IP}}, t)\, F_2^{\mathrm{IP}}(\beta, Q^2)$ (+ subleading terms)

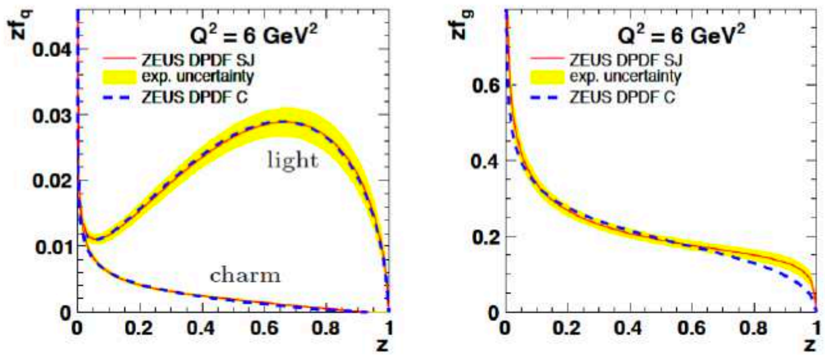
$f_{\mathrm{IP}}(x_{\mathrm{IP}}, t)$ is the probability to find a Pomeron in the proton and $F_2^{\mathrm{IP}}(\beta, Q^2)$
Is the structure function F2 of the Pomeron.

The Pomeron flux is consistent with that obtained from soft physics (e.g. total cross section):

$f_{\mathrm{IP}}(x_{\mathrm{IP}}, t) \propto x_{\mathrm{IP}}^{1-2\alpha_{\mathrm{IP}(0)}}$ with $\alpha_{\mathrm{IP}}(0) \simeq 1.1$

While the Pomeron PDFs can be obtained from fits to diffractive data similarly to proton PDF. This model works remarkably well and can explain all HERA diffractive DIS data.

Pomeron PDFs

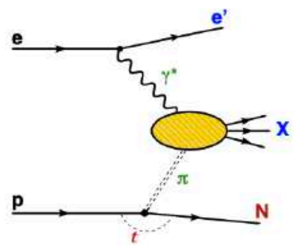


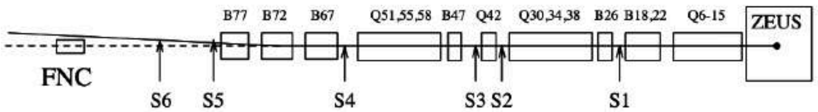
One pion exchange

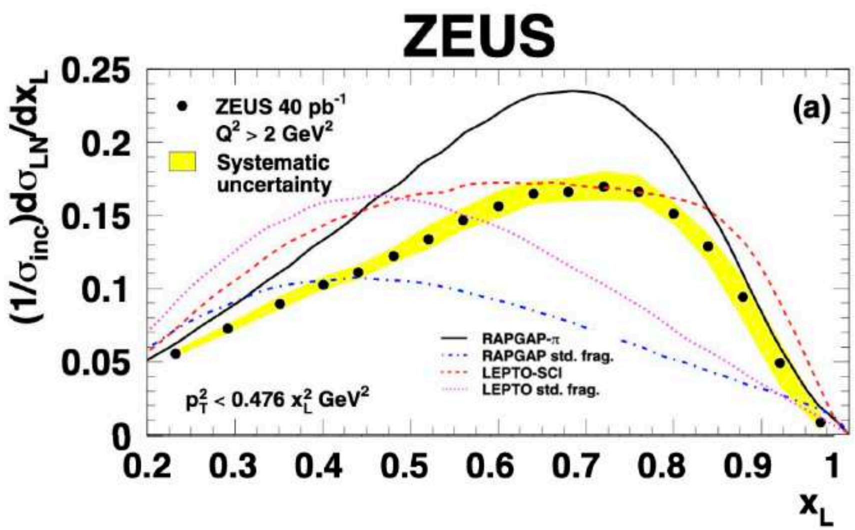
Similarly to "leading protons" in diffraction, very often a neutron with large x_L is observed in the forward neutron calorimeter.

Standard fragmentation MC models fail to describe this behaviour, One-Pion-Exchange models give a good description of the observation.

=> this gives a complementary view of hard scattering wrt to standard PDF approach: the proton is fluctuating in various states IP, π, …

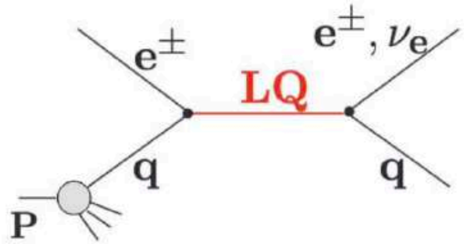






Extra Topics

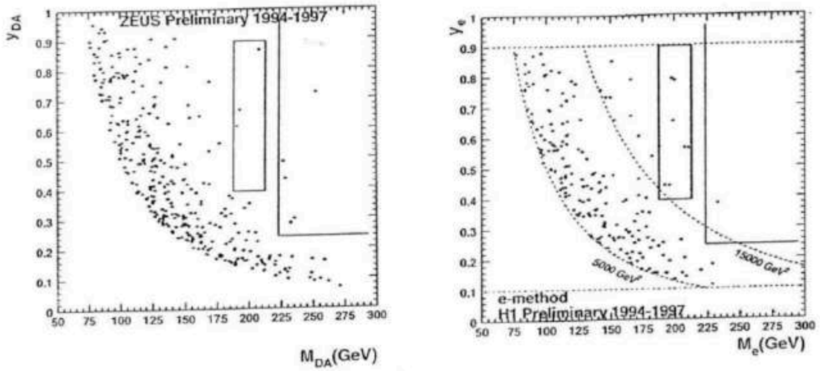
Leptoquarks



Leptoquarks are possible l-q resonances
Appear naturally in GUT theories

It would show-up as peak at a given value of x:
$M^2 = (xP + k)^2 = 4xE_pE_e = xs$

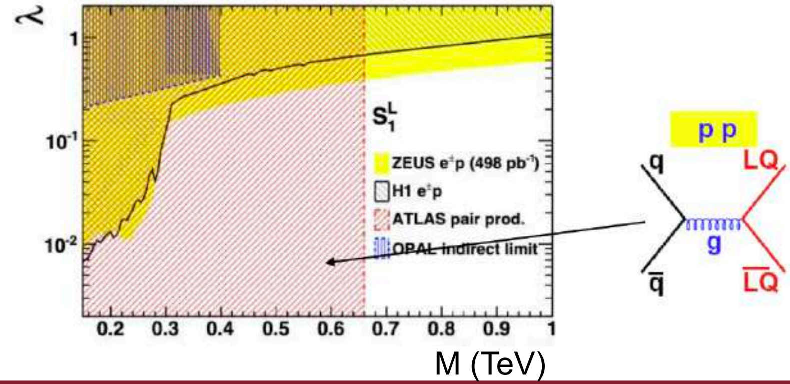
A scalar LQ has flat y distribution (isotropic) as opposed to the low-y peaking of normal DIS.



Excess found by H1 in 1994-1997 data. ZEUS also had some mild excess at higher mass. Great excitement, but also “Ample reasons for doubt” (G. Altarelli)

In fact the excesses disappeared in following data taking periods, probably just statistical fluctuations.

HERA LQ limits are now mostly superseded by LHC



ALLA FINE

Very low Q²: photoproduction

The NC cross section behaves as 1/Q⁴ : large cross at small Q2.

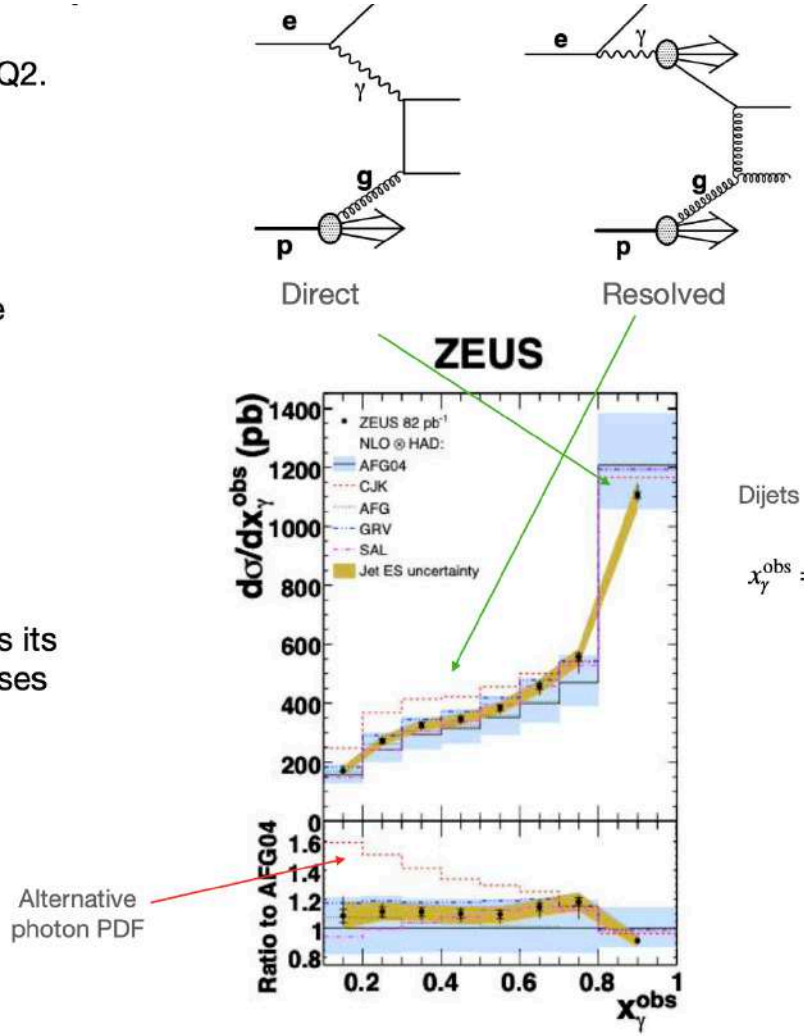
Experimentally in low-Q² events (Q²<1 GeV²) the scattered electron escapes undetected in the rear beam-pipe hole.

Equivalent-photon approximation: at low Q² the ep cross-section can be factored as the product of a photon flux in the electron times gamma-p cross section,

$$\sigma(ep) = f_{\gamma/e}\,\sigma(\gamma p).$$

Measuring e.g. jet production at low Q2 is equivalent to measure it in gamma-proton interactions : thus the name photo-production.

The photon itself can fluctuate into q-qbar pairs and develops its own internal structure and PDFs: so photoproduction processes can be divided in direct and "resolved" photoproduction.



Dijets

$$x_\gamma^{obs} = \frac{(E-P_Z)_{jets}}{(E-P_Z)_X}$$

How to reconstruct a NC DIS event at HERA?

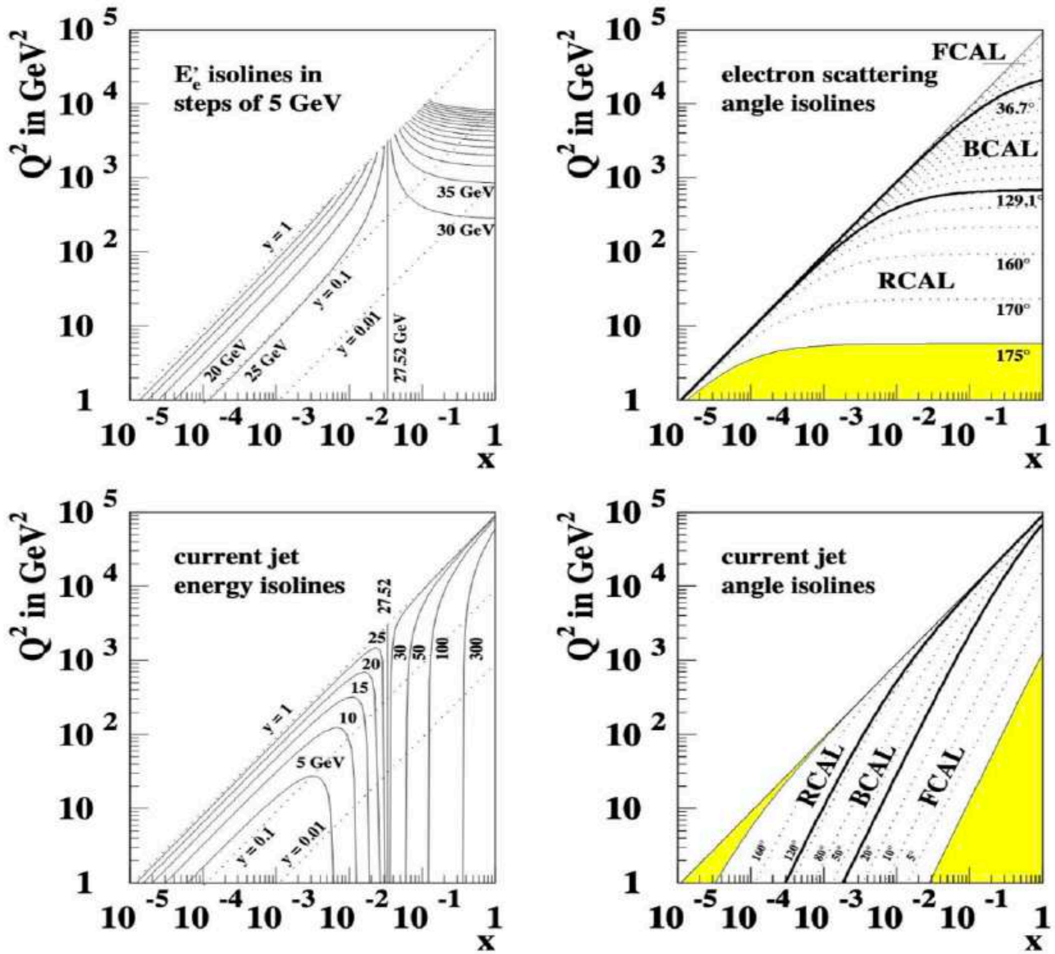
In **Neutral-Current** DIS the measurement is over-constrained: various methods have been used with different systematics and different sensitivity to QED corrections.

- electron method (from E'_e, θ_e)

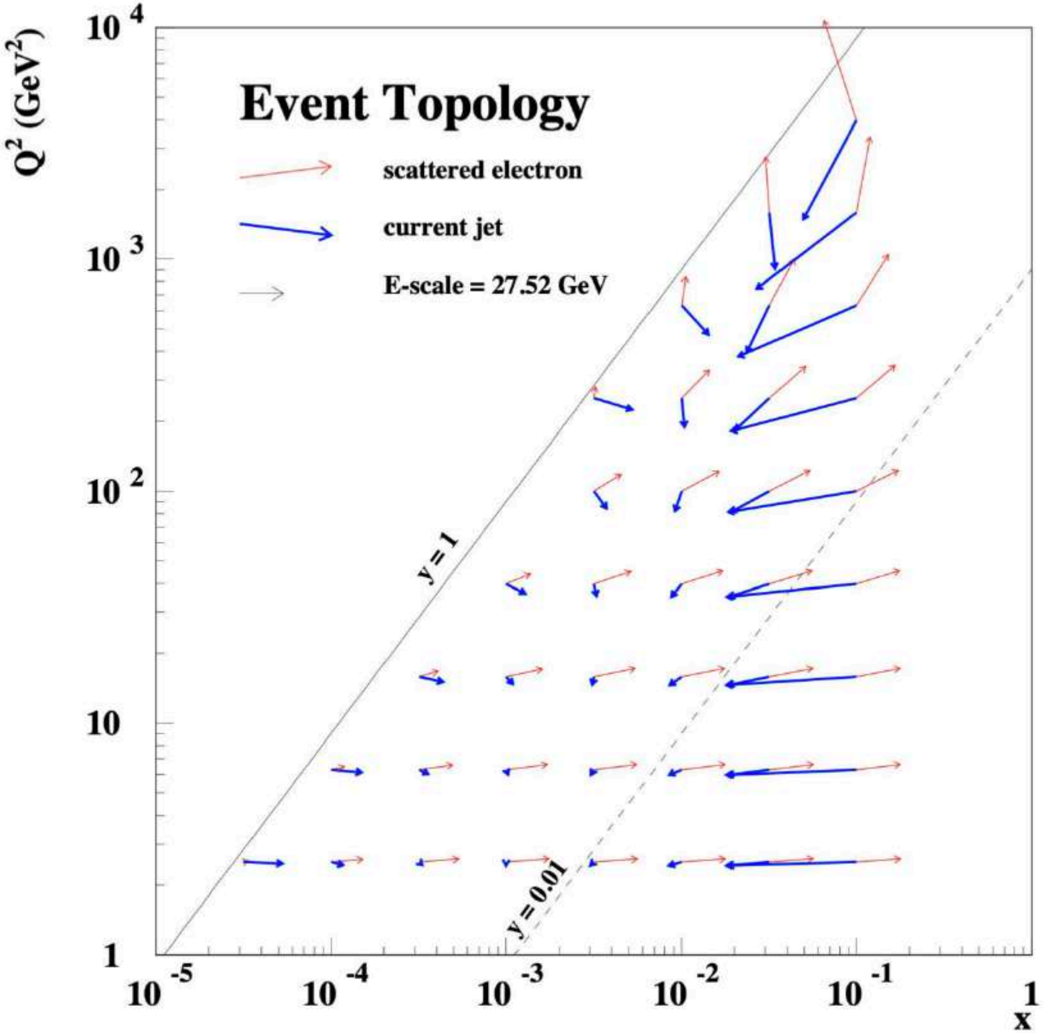
- Hadronic method (from $p_{T,X}$ and $(E - P_Z)_X$)

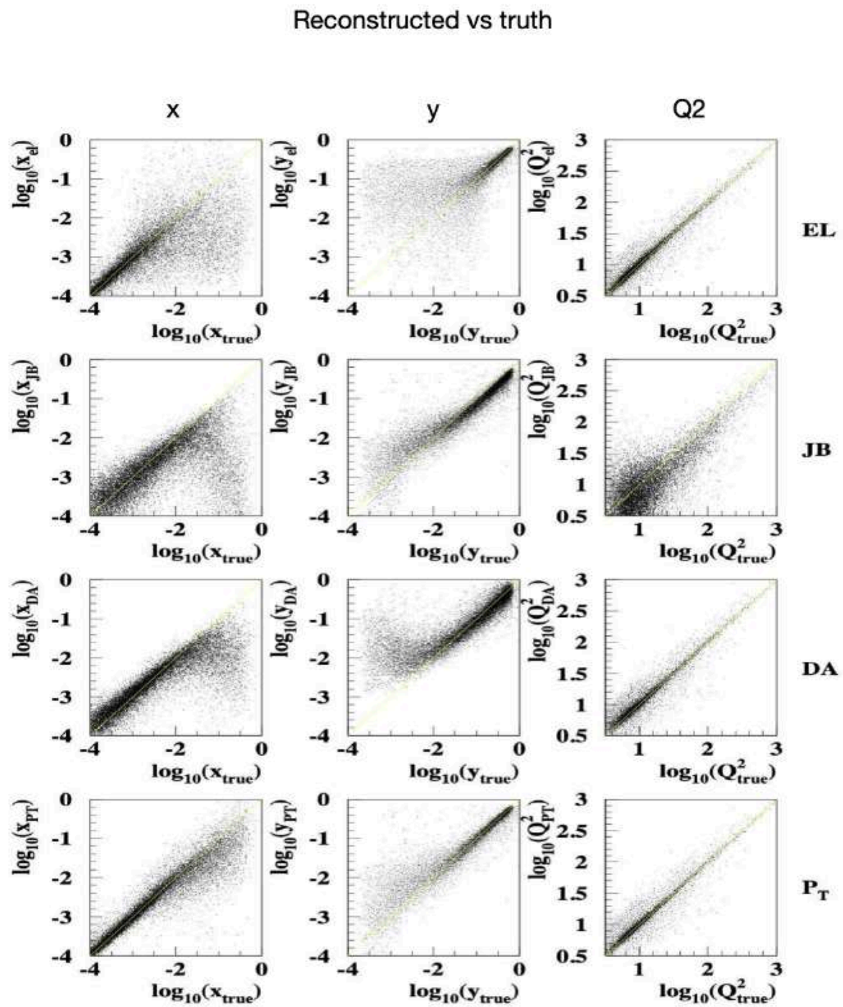
- Double Angle method (from θ_e, $\cos\gamma_{DA} = \dfrac{p^2_{T,X} - (E - P_Z)^2_X}{p^2_{T,X} + (E - P_Z)^2_X}$)

- Other methods mixing electron and hadronic variables: Sigma method and PT method



A few DIS plots



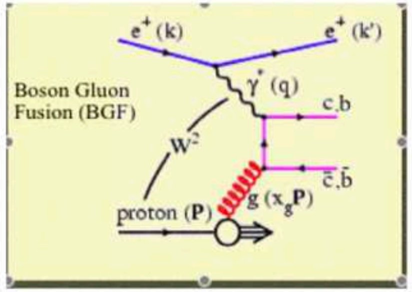


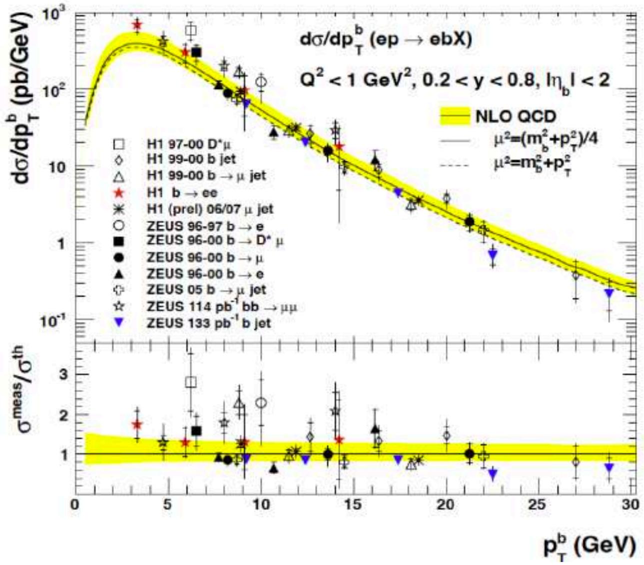
Final states can be used to tag specific quarks: c and b

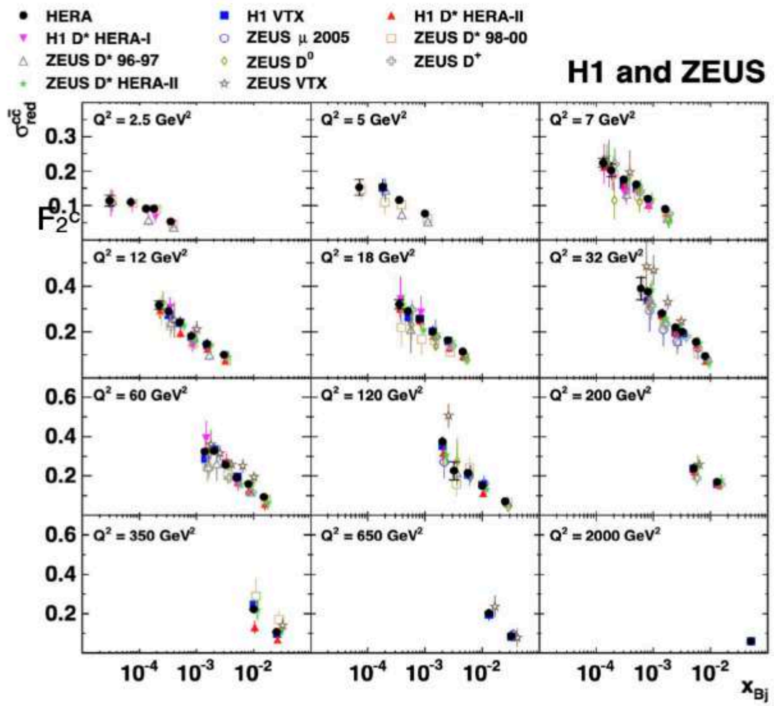
Various techniques have been used at HERA to determine $F_2^{c,b}$ the component of F2 involving with c or b quarks.

$c\bar{c}$ pairs are abundantly generated from gluon splitting, and are responsible for 40% of cross section at low-x and high-Q2, In standard PDF fits they are not considered as part part of the proton PDF at scales $Q_0^2 < m_c^2$

b

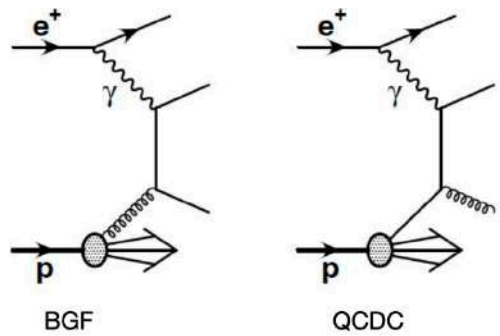






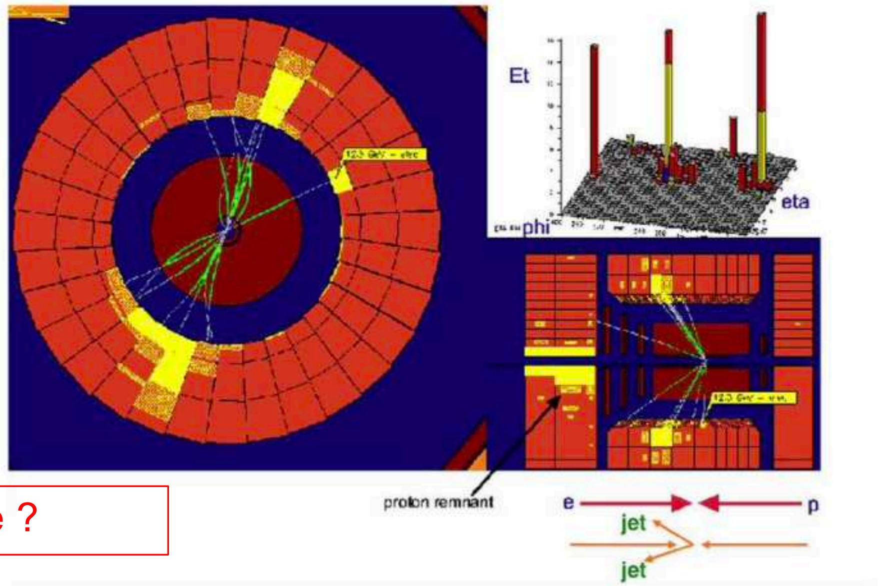
More handles on the gluon: jets

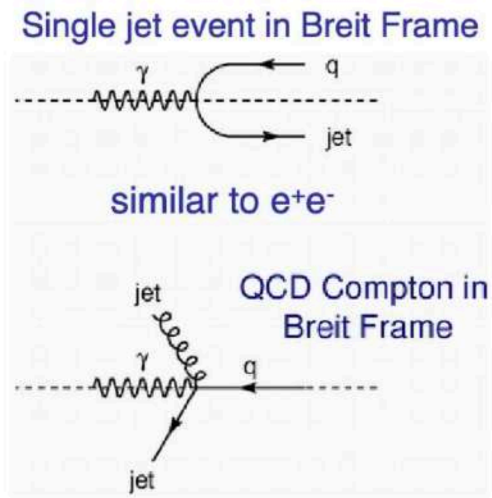
The gluon density can be also obtained by looking at the final states => beyond inclusive DIS, in general less precise theory calculations, larger systematics



Jets are reconstructed in the Breit frame : $q + 2xP = 0$
"QPM" jet has zero p_T QCDC and BGF have 2 jets with high pT

Jet events select a sample enriched in Boson-Gluon-Fusion events





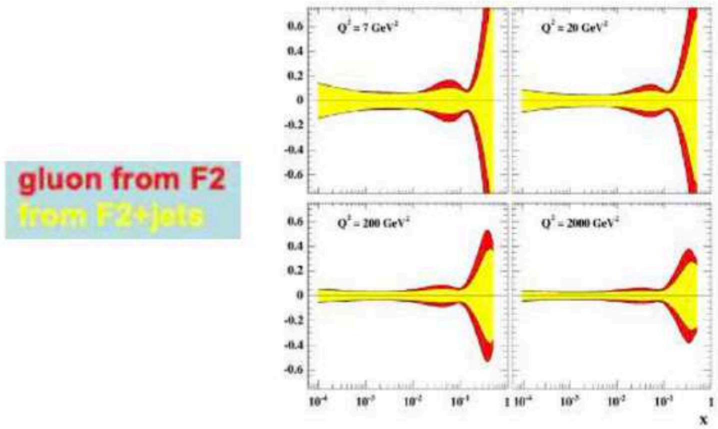
Rimuovere ?

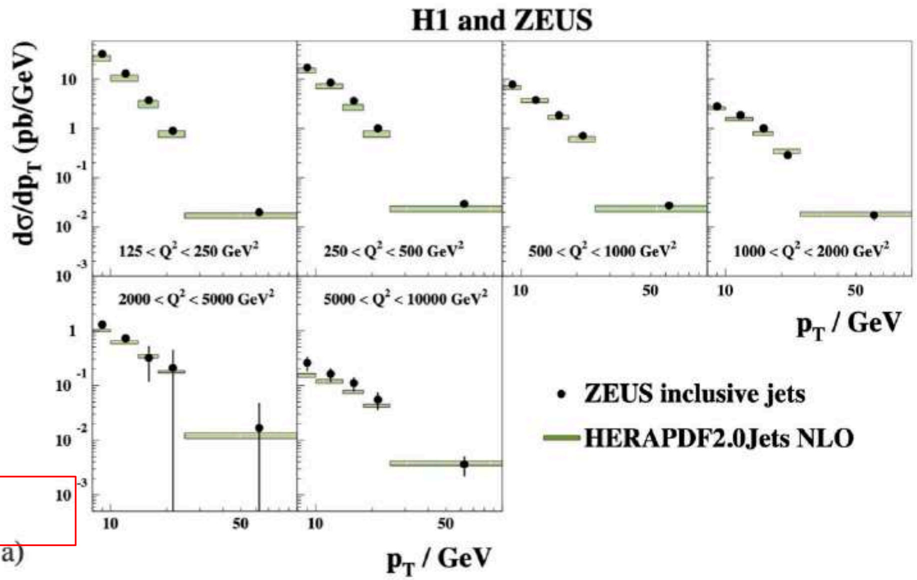
Jets, PDFs and α_s

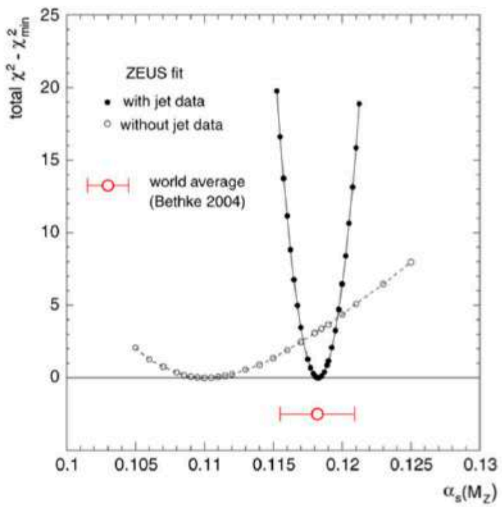
Jet cross sections can be added to PDF fits

Without including jets there is a large anti-correlation between the gluon PDF and α_S (as scaling violation is sensitive to the product $\alpha_S\, g(x, Q^2)$).

When jets are included the anti-correlation is reduced, uncertainty on gluon is reduced and α_S can also be extracted from the fit with error competitive to world average.







Rimuovere ?

Charm and Beauty PDFs

At HERA the c and b cross sections are compatible with QCD predictions assuming that all heavy quarks are produced by gluon splitting.

Different QCD schemes for c and b PDFs:
- Fixed-Flavour-Number scheme : c and b are not part of proton PDF, only generated by higher order
- Variable Flavour-Number scheme : c and b are treated like other quarks with their PDFs at $Q^2 \gg m_c^2$

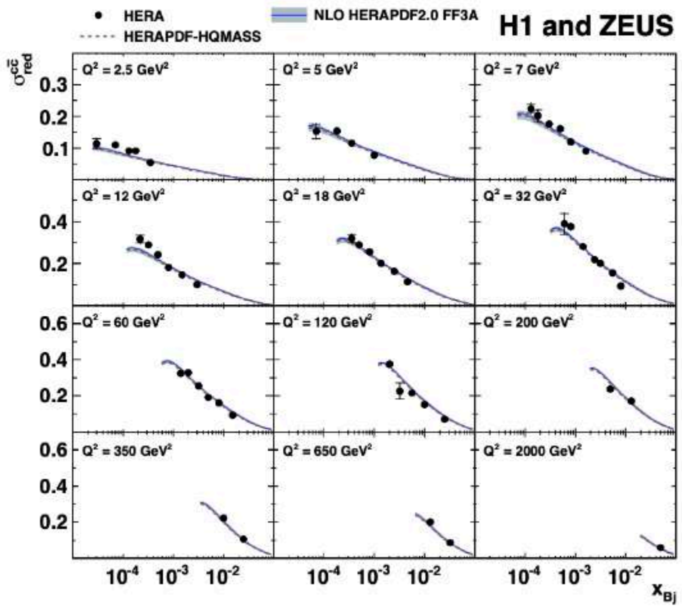
These measurements provide a further constraint on gluon and flavour separation in sea.

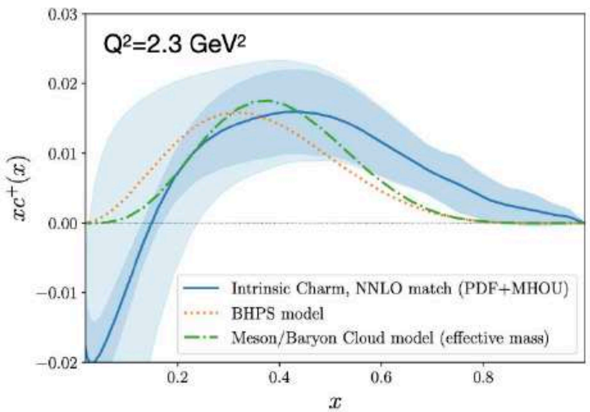
In principle an intrinsic heavy quark component may be present in the proton wave function :

$$|p\rangle = |uud\rangle + |u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle + |c\bar{c}\rangle$$

A recent global analysis of PDF data (including LHC and fixed target data), *Nature* 608 (2022) 7923, finds evidence for an intrinsic charm content of the proton that does not vanish at low Q2 despite the fact that the charm mass ($m_c = 1.5$ GeV) is higher than the proton mass ($m_p = 0.94$ GeV).

Rimuovere ?





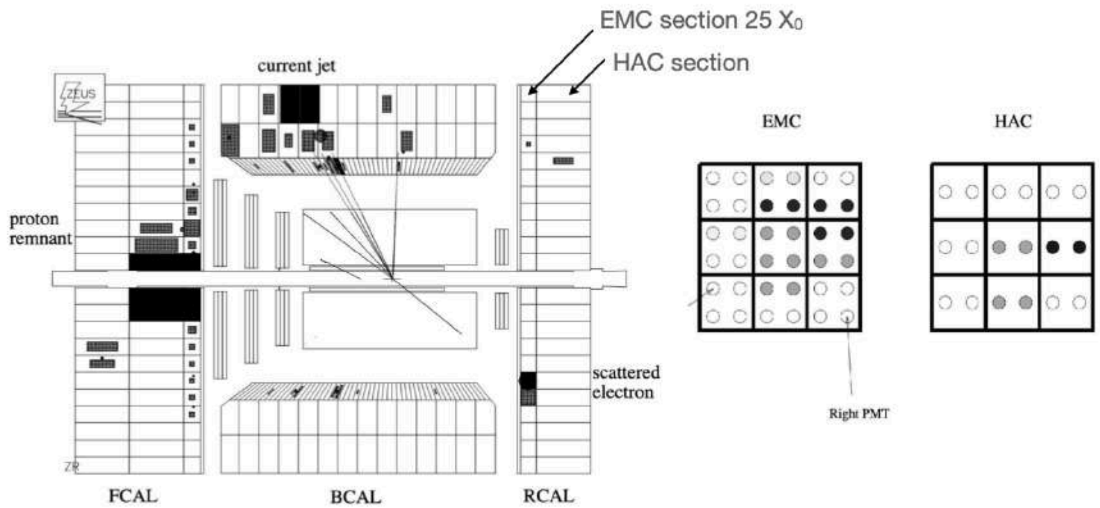
Electron Reconstruction (ZEUS case)

The electron is identified as a small calorimetric cluster with small radius and low energy deposit in the hadronic section.

A neural-network with in input 55 PMT signals from the 60x60 cm^2 region was used to discriminate hadrons and electrons.

In addition requirements on isolation and (within the central tracker acceptance) the presence of a charged track.

Scintillator pre-samplers placed in front of the calorimeter allow to correct for the energy lost in the dead material before



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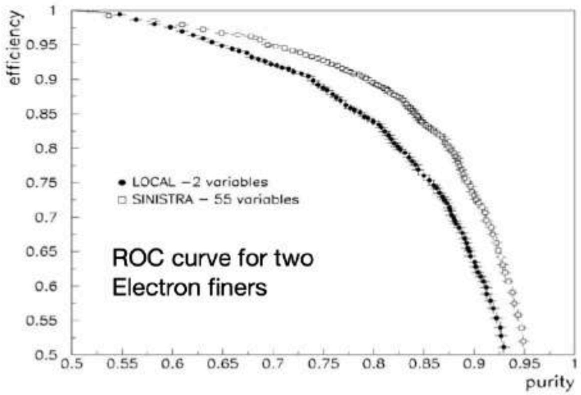

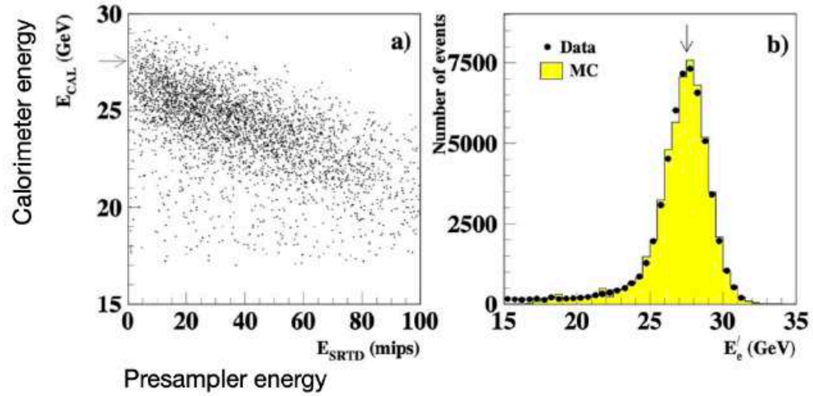

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End of chapter 9