Collider Particle Physics - Chapter 9 -

Introduction to Hera Collider: Physics and Experiments

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Chapter Summary

- deep inelastic scattering: a reminder
- □ Hera Collider and the experiments
- □ Results on Neutral and charge current scattering at high Q²
- **QCD** and deep inelastic scattering
- □ Parton Density Function determination
- **Extra topics** ?

Deep Inelastic Scattering: a Reminder



Notation: particle name with 4-momentum in parenthesis, e.g. l(k) means lepton with 4-momentum k

How many independent variables we need to describe the final state ?



Notation: particle name with 4-momentum in parenthesis, e.g. I(k) means lepton with 4-momentum k

How many independent variables we need to describe the final state ?

- 2 final states particles 4-mom. components = 8 unknowns
- 4-momentum conservation: -4
- Lepton on mass shell: -1

Total: 3

e.g. in polar coordinates, the lepton energy E_I , and two angles: polar (Θ) and azimuthal (φ).

Anyway due to invariance with respect to rotations around the incoming particles line (e.g. beamline) ϕ is irrelevant !

So 2 variables are sufficient to describe the final state system



Lorentz invariants

 $s = (k+P)^2 ,$ $M^{\!\scriptscriptstyle 2_X}\!\!= (q+P)^2 = p_{\prime}^2 \;,\; {
m Mass} \; {
m of} \; {
m final} \; {
m hadronic} \; {
m system} \; {
m X}$ $x = \frac{Q^2}{2P \cdot q} ,$ $= \frac{q \cdot P}{k \cdot P} ,$ y $\nu = \frac{q \cdot P}{\dots} \, .$ m_N $Q^2 = -q^2.$

Center-of-mass energy squared

Bjorken x

Inelasticity (fraction of energy lost by lepton in reference frame with p at rest)

energy lost by lepton in reference frame with p at rest

Absolute squared four-momentum transfer

Only 2 are independemnt (once s is fixed)



Only 2 are independemnt (once s is fixed)

lepton-hadron collision in fixed-target experiments

Proton rest frame



$$k = (E_l, 0, 0, E_l)$$

$$k' = (E'_l, E'_l \sin\Theta, 0, E'_l \cos\Theta),$$

$$P = (M_P, 0, 0, 0)$$

$$q^{2} = (k - k')^{2} = k^{2} + k'^{2} - 2k \cdot k'$$

Neglecting the lepton mass

$$q^{2} = -2k \cdot k' = -2(E_{l}E_{l}' - E_{l}E_{l}'\cos\Theta) = -2E_{l}E_{l}' (1 - \cos\Theta)$$





lepton-hadron collision in fixed-target experiments



 $\frac{l(k)}{p(P)} \xrightarrow{\gamma, Z, W(q=k-k')} X(p+q)$

Fixed target experiment Lab frame [notation: $p = (E_p, p_x, p_y, p_z)$]

$$k = (E_l, 0, 0, E_l)$$

$$k' = (E'_l, E'_l \sin\Theta, 0, E'_l \cos\Theta),$$

$$P = (M_P, 0, 0, 0)$$

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 $Q^{2} = -(k' - k)^{2} = 2E_{l}E_{l}'(1 - \cos\Theta)$ Large angle : large Q² $y = \frac{q \cdot P}{k \cdot P} = \frac{(E_{l} - E_{l}')M_{P}}{E_{l}M_{P}} = 1 - \frac{E_{l}'}{E_{l}}$ 0 < y < 1 1: all electron energy trasferred 0: no energy lost by lepton

Cross section can be expressed in terms of structure functions

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General expression for inelastic lepton-hadron cross section (assuming one photon exchange)

inelastic:

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{M^2 y^2}{Q^2} \right) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right]$$



Analogue of form factors of elstic scattering $ep \rightarrow ep$:







Cross section can be expressed in terms of structure functions

General expression for inelastic lepton-hadron cross section (assuming one photon exchange)



can be neglected for $Q^2 >> M^2_n$





Analogue of form factors of elstic scattering $ep \rightarrow ep$:







Deep Inelastic Scattering and SLAC data

Deep Inelastic Scattering (DIS) regime:		
$Q^2 \gg M_p^2$	Deep	
$M_X^2 \gg M_p^2$	Inelastic	
In this limit $Q^2 = xys$		

Behaviour or structure functions measured in DIS at SLAC in late 60s was unexpected:

In DIS regime, fixing x, structure functions were found to be almost constant with Q^2

In striking contradiction with expectation from a uniform charge cloud that would predict $\sim 1/Q^4$ similarly to elastic form factors

Parton model was proposed by Feynman to explain this observation...



Parton Model : meaning of x

The proton is made of pointlike constituents (quarks)

Let's assume that we are in a reference frame in which the proton is boosted with $P_z \gg M_p$

Considering that the scattered quark is on mass shell and that quark mass is close to zero :

 $\begin{aligned} (p_q+q)^2 &= (p_q')^2 \simeq 0 \\ q^2 + 2p_q \cdot q &= 0 \end{aligned}$

We can write the quark 4-momentum as $p_q = \xi P$ where ξ is the fraction of the proton momentum carried by the quark, then

$$q^2 + 2\xi P \cdot q = 0$$

$$\xi = \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2P \cdot q}$$



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In the Parton model the quark momentum fraction ξ is equal to the Bjorken-x variable defined as

$$x = \frac{Q^2}{2P \cdot q}$$





Parton Model : cross section



The cross section can be expressed as a sum of lepton-quark cross sections $\frac{d\sigma_q}{dQ^2}$, weighted with the probability density $f_q(x)$ for finding a quark of a given type with a fraction $\xi = x$ of the proton momentum:

$$lp \rightarrow l'X$$
:
 $\frac{d\sigma}{dxdQ^2} = \sum_{q=u,d,\dots} \frac{d\sigma_q}{dQ^2} f_q(x)$

Parton Model : meaning of x

Cross section for lepton-quark scattering $|q \rightarrow |q$ QED process, the calculation gives: $\frac{d\sigma_q}{dQ^2} = \frac{2\pi\alpha^2}{Q^4} e_q^2 [1 + (1 - y)^2]$ $\mathfrak{M} = -e^2 \,\overline{u} \, (k') \gamma^{\mu} \, u(k) \frac{1}{q^2} \overline{u} \, (p') \gamma_{\mu} \, u(p).$

And thus

$$\frac{d\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4} \frac{1}{2} [1 + (1 - y)^2] \sum_i e_i^2 f_i(x)$$
$$= \frac{4\pi\alpha^2}{Q^4} \Big[(1 - y) + \frac{1}{2} y^2 \Big] \sum_i e_i^2 f_i(x)$$

Parton Density Functions and Bjorken scaling

Comparing this result:

$$\frac{d\sigma}{dxdQ^{2}} = \frac{4\pi\alpha^{2}}{Q^{4}} \left(1 - y + \frac{1}{2}y^{2}\right) \sum_{q} e_{q}^{2} f_{q}(x)$$

with the generic expression for e-p scattering

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^2} \left[(1-y)F_2(x,Q^2)/x + y^2F_1(x,Q^2) \right]$$



We can identify : $F_2(x, Q^2) = x \sum_q e_q^2 f_q(x)$ $F_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 f_q(x)$

Bjorken scaling: structure functions has no dependence on Q^2

 $2xF_1(x) = F_2(x)$: the Callan-Gross relation, -> holds for spin 1/2 patons

$f_q(x)$ is the Parton Density Function (PDF) for Parton q

Structure functions F(x) are wighted sums of PDFs f(x) with weights given by the quark couplings relevant for the particular process, in the case of photon exchange e_q^2

What can we expect for parton densities ?



Going back to partonic cross-sections

Cross section for lepton-quark scattering lq -> lq

QED process, analogue to $e\mu \rightarrow e\mu$ scattering,

Mandelstam Variables

$$\hat{s} = (e + p_q)^2 = 2e.p_q$$

 $\hat{u} = (p_q - e')^2 = -2p_q e'$
 $\hat{t} = (e - e')^2 = -Q^2$

$$\frac{d\sigma}{d\hat{t}} = \frac{d\sigma}{dQ^2} = \frac{1}{16\pi\hat{s}^2} |M|^2$$

$$|M|^2 = 2e_q^2 (4\pi\alpha)^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$$

$$\frac{d\sigma}{dQ^2} = \left(2\pi\alpha^2 e_q^2\right) \frac{1}{Q^4} \left(1 + (1-y)^2\right)$$

$$\overset{k}{\longrightarrow} \overset{e^-}{\longrightarrow} \overset{k'}{\mathbb{N}} = -e^2 \,\overline{u} \,(k') \gamma^{\mu} \,u(k) \frac{1}{q^2} \overline{u} \,(p') \gamma_{\mu} \,u(p).$$

TABLE 6.1	
Leading Order Contributions to	Representative QED Processes



Helicity, angular distributions and y

$$\frac{d\sigma_i}{dQ^2} = \frac{2\pi\alpha}{Q^4} e_i^2 [1 + (1 - y)^2]$$

$$\frac{d\sigma_i}{dQ^2} \propto |M|^2$$

 $y = \frac{1 - \cos \theta^*}{2}$ θ^* is the scattering angle in e-q frame

For vector exchange (γ, Z, W) helicity conservation gives these relations :

$$e_L q_L$$
, $e_R q_R \rightarrow |M|^2 \propto 1$ (isotropic)

$$e_R q_L, e_L q_R \rightarrow |M|^2 \propto (1 + \cos \theta^*)^2 \propto (1 - y)^2$$
 (fwd peak)

In unpolarised photon exchange we have both terms with same weight, thus $1+(1-y)^2$





Charged Currents: W exchange

Applying the same approach :



$$W^{-} \text{selects } e_{L}^{-}, q_{L}, \overline{q}_{R} :$$

$$e_{L}^{-} + q_{L} : \propto 1$$

$$e_{L}^{-} + \overline{q}_{R} : \propto (1-y)^{2}$$

$$e^{-}p: \quad \widetilde{\sigma}^{CC}(x) = x \left[u(x) + c(x) + (1-y)^2 (\overline{d}(x) + \overline{s}(x)) \right]$$

Similarly using a positron beam: $e^+p: \tilde{\sigma}^{CC}(x) = x[\overline{u}(x) + \overline{c}(x) + (1-y)^2(d(x) + s(x))]$

Charged and Neutral currents processes are sensitive to different combination of quarks



DIS as a way to probe small distances

DIS : a microscope with resolution :

$$\Delta b \sim \frac{\hbar c}{\sqrt{Q^2}} = \frac{0.197}{\sqrt{Q^2}} \,\mathrm{GeV} \;\mathrm{fm}$$

The maximum Q² reachable in an experiment is given by the center-of-mass energy.

If quarks had a structure it would be resolved going to higher energy

Several DIS experiments in the 70's and 80's using electrons, muons and neutrinos, with increasing energy. Since the 70s it was clear that to make a significant step forward a collider was needed.

Beyond probing possible parton substructures, a collider would also improve enormously the knowledge of PDF

After several proposal have been considered, it materialised in early '80s as the proposal for HERA



DIS experiments and HERA collider

Historical Fixed target DIS experiments

BCDMS and EMC/NMC (CERN, SPS) up to 280 GeV muon beam



E665, NuTeV (Ferimlab, Tevatron) up to 470 GeV muon beam



Muon beams: low intensity, high acceptance spectrometers

SLAC: 20 GeV electron beam (from 1968) High intensity beams, low acceptance spectrometers



HERA the (so far) only ep collider

After the success of Doris and Petra e⁺e⁻ colliders, DESY (Hamburg, Germany) decided to build an ep collider. [with key contributions to the accelerators from Italy and later France]

HERA (Hadron Electron Ring Anlage)

Two accelerators in one 6.3 km tunnel

Proton ring: max energy E_p = 920 GeV ("similar" to Tevatron)

Electron/positron ring : max energy $E_e = 27.5 \text{ GeV}$ (A small LEP)

Center-of-mass energy : $\sqrt{s} = \sqrt{4E_eE_p} = 318$ GeV

Two large collider experiments: H1 and ZEUS

Two fixed-target experiments:

HERMES (polarised electrons on polarised gas target) HERA-b (for b physics, using p-beam halo on wire targets)





HERA Phase Space

(remember: $Q^2 = x y s$ and 0 < y < 1)

- Maximum Q^2 is given by center-of-mass energy: $Q^2 < s$
- Minimum x values are limited by $x > Q^2/s$
 - => HERA expands *x*, *Q*² range to low *x* by about 2 orders of magnitude

Very different beam energies -> need for asymmetric detectors !



The eq scattering is ~central il lab frame for x=0.03



HERA Detectors: requirements

- The same relations that we found for fixed target expt. link electron energy and angle to *x*, Q^2 . (note different convention of theta, it is measured from proton direction: $\theta_e = \pi \Theta_e$)
- Most events will have low Q^2 and thus small e deflection angle Θ_e



$$Q^{2} = 2E_{e}E'_{e}(1 + \cos \theta_{e})$$
$$y = 1 - \frac{E'_{e}}{2E_{e}}(1 - \cos \theta_{e})$$
$$x = \frac{Q^{2}}{sy}$$

Detectors should be able to reconstruct:

- Neutral Current : reconstruct electron
- => good electron reconstruction at all angles (track+calo)
- Charged Current : reconstruct hadronic system X
- => good hadronic calorimeter for high energy jets in central/forward
- => but also reconstruction of not so energetic hadrons
- Hadronic final states: reconstriction of light/charm/beauty hadrons in the final state (central tracker, vertex detectors, muon chambers)

HERA Detectors: H1

Asymmetric experiments convention: Z axis points towards p beam direction "forward" is proton beam direction

H1 : large German / French / UK components

Sub detectors:

- Silicon strip vertex detector
- "Jet Chamber" gaseous tracker
 - Liquid Argon EM calorimeter (barrel / forward)
 - Spacal EM calorimeter (spaghetti calorimeter) (rear)
- Liquid Argon hadronic calorimeter
- Large superconducting solenoid (1.16 T axial field)
- Muon Chambers inside iron return coil (streamer tubes)

LAr EM : $\sigma(E)/E \simeq 11\%/sqrt(E/GeV) \oplus 1\%$ LAr Had : $\sigma(E)/E \simeq 50\%/sqrt(E/GeV) \oplus 2\%$



A Neutral Current event in H1

Electron method, same as for fixed target (Note θ_e measured respect to Z axis ($\theta_e = -\Theta_e$)

$$Q^{2} = 2E_{e}E'_{e}(1 + \cos \theta_{e})$$
$$y = 1 - \frac{E'_{e}}{2E_{e}}(1 - \cos \theta_{e})$$
$$x = \frac{Q^{2}}{sy}$$



HERA Detectors: ZEUS

ZEUS : large German / US / ITA / UK / JP components (+ Russia, Poland, Canada...)

Main Sub detectors:

- Silicon Strip Vertex detector (from 2000)
- Drift Chamber (CTD)
- Thin superconducting solenoid 1.4 T
- pre-samplers (scintillators)
- Compensating Uranium/scintillator calorimeter
- Instrumented return coils (backing calorimeter)
- Muon Chambers (streamer tubes)

Main focus on hadronic calorimetry

EM : $\sigma(E)/E \simeq 18\%/sqrt(E/GeV)$ Had : $\sigma(E)/E \simeq 35\%/sqrt(E/GeV)$



A Charged Current event in ZEUS

In **Charged-Current** DIS the reconstruction method is the hadronic method, also known as Jaquet-Blondel (JB) method, exploits energy-momentum conservation to obtain the neutrino angle and energy:

Transverse momentum conservation :

$$p_{T,X} = p'_{T,\nu} = E'_{\nu} \sin\theta_{\nu}$$

Longitudinal momentum conservation :

$$(E - P_Z)_{\text{tot}} = 2E_e = (E - P_Z)_X + E_v(1 - \cos\theta_v)$$
 •

Then

$$y_{JB} = \frac{(E - P_Z)_X}{2E_e} \qquad Q_{JB}^2 = \frac{p_{T,X}^2}{1 - y_{JB}}$$

e p -> ν X



Hig-Q2 DIS Charged Current event

Part of the hadronic system escapes undetected in the forward beam hole, but gives negligible contribution to $(E - P_Z)_X$, (which is the reason for using this particular combination) Compare energy-conservation constraints for LEP (4), LHC (2) and HERA (3)

ZEUS hadronic calorimeter

Hadronic showers:

a fraction f_e of energy is released as EM (ionization, $\pi^0 \rightarrow \gamma \gamma$ etc.) the rest (1- f_e) is lost in nuclear interactions. The calorimeter response to EM (e) and hadron (h) interactions is different typically h < e

e.g. response to a pion:

 $\pi^{+} = f_{e}e + (1 - f_{e})h$

EM fraction f_e fluctuates, degrading the calorimeter resolution

Trick: find a special recipe to obtain compensation: e/h=1

Uranium + plastic Scintillator : large response to neutrons f_e can be tuned in sampling calorimeters: higher absorber fraction -> lower e.m. component

ZEUS used this structure :



Sü

2.6

ZEUS hadronic calorimeter

A unique compensating calorimeter !

Same response to EM and hadrons





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More on hadronic system reconstruction

Once we have a compensating hadron calorimeter we can reconstruct the hadroni system simply summing up the vectors of all calorimeter cell with an energy deposit above noise:

$$\vec{P} = \sum_{i \in cells} E^{i} \vec{r^{i}}$$

where the sum runs over all cells above threshold (not associated to the scattered electron) E^i is the cell energy and r^i is the unit vector pointing from event vertex (as reconstructed by central tracker) to the cell.

Various improvement were used: the sum was done over energy clusters corrected for energy lost in front of calorimeter based on dead material maps. Moreover "particle flow" objects (ZUFOs = ZEUS Unidentified objects) were used, using tracking rather than calorimetry for low-p_T charged particles.



Electron identification and reconstruction (ZEUS)

The electron is identified as a small calorimetric cluster with small radius and low energy deposit in the hadronic section.

A neural-network feed with 55 PMT signals from the 60x60 cm² region was used to discriminate hadrons and electrons.

In addition requirements on isolation and (within the central tracker acceptance) the presence of a charged track.

Scintillator pre-samplers placed in front of the calorimeter allow to correct for the energy lost in the dead material before








HERA Operations

Two main periods HERA-1 (1992 - 2000) and HERA-2 (2003-2007)

HERA-2: luminosity upgrade with low- β^* insertions near interaction points

Luminosity of HERA-1 was below expectations in particular with electrons, most luminosity was collected with positrons

HERA-2 had initial difficulties related to high backgrounds in the experiments, finally an integrated luminosity of 0.5 fb⁻¹ per experiment was collected (not very succesfull)

210 bunches (176 colliding) 96 ns between bunches

Peak luminosity in HERA-2 5x10³¹ cm⁻² s⁻¹

During the HERA-1 -> HERA-2 transition the detectors have been also upgraded. In particular new vertex detectors were introduced in both experiments to improve c and b tagging capabilities



Results on Neutral and Charged current scattering at high Q²

Results : Neutral Current cross section vs Q² (integrated over x)

- The NC cross section falls approximately like 1/Q⁴
- Eventually at $Q^2 \simeq M_W^2$ CC and NC become similar: weak and e.m. interaction unify at EW scale !
- CC cross section in e⁻p is larger than in e⁺p because sensitive to u quarks rather than d.
- At high-Q² a difference between e⁻p and e⁺p is also observed in NC : this is the effect of the Z exchange that introduces a charge dependence (C and P not conserved individually)



Results : limit on quark substructure

Do we see have any hint of quark substructures ?

Assuming a gaussian charge distribution with radius R for the quark:

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma^{\rm SM}}{dQ^2} \left(1 - \frac{R^2}{6}Q^2\right)^2$$

Current limit is 0.43×10^{-3} fm : less than thousand times smaller than a proton !



Results : Charged Current cross section vs y

The Helicity structure works !

$$e^+p:\tilde{\sigma}^{CC}(x) = x[\overline{u}(x) + \overline{c}(x) + (1-y)^2(d(x) + s(x))]$$



Results : Charged Current with polarized lepton beams

During HERA-2 period, HERA provided polarised lepton beams.

Electrons are polarised transversally (spin perpendicular to accelerator plane by syncrotron light emission in dipole magnets (Solokov-Ternov effect)

Special spin-rotator magnets rotate the spin just before/after the interaction points to obtain longitudinal polarization.

Polarisation $P_e = \frac{N_R - N_L}{N_R + N_L}$ was about 0.4 and was switched in sign every few months.

This permits to select leptons with "right helicity"

Charge current should not be present for e_R^- and e_L^+ Constraint on right-handed W



ZEUS

Results : Leptoquarks

Leptoquarks are possible I-q resonances Appear naturally in GUT theories

It would show-up as peak at a given value of x: $M^2 = (xP + k)^2 = 4xE_pE_e = xs$

A scalar LQ has flat y distribution (isotropic) as opposed to the low-y peaking of normal DIS.

Excess found by H1 in 1994-1997 data. ZEUS also had some mild excess at higher mass. Great excitement, but also "Ample reasons for doubt" (G. Altarelli)

In fact the excesses disappeared in following data taking periods, probably just statistical fluctuations.

HERA LQ limits are now mostly superseded by LHC



Higher orders and PDFs evolutions

QCD correction to parton model, $O(\alpha_S)$

The corrections for $e p \rightarrow e' X$ at $O(\alpha_S)$ are:



The diagram in red is divergent when the gluon transverse momentum κ_T tends to zero

Collinear approximation

Let's have a look at the "QCD Compton" diagrams

New propagator term $1/(p_{virt})^2 = 1/(p_q - k_g)^2$

For gluon transverse momentum $\kappa_T \rightarrow 0$

 $1/(p_{virt})^2 = 1/(\kappa_T)^2$

The propagartor term diverges when the gluon is "collinear"

Doing the calculation properly it is found that the cross section diverges as $1/(\kappa_T)^2$ (not as $1/(\kappa_T)^4$ as expected from the pure propagator term)

The cross section will be dominated by the region around the pole $\kappa_T \rightarrow 0$, so, instead of calculating exactly the new contribution we can concentrate to the region around the pole: collinear approximation

$$\int_{k_T^{min} \to 0}^{k_T^{max}} dk_T \, \frac{1}{k_T^2} \, f(k_T) \, \simeq \, f(0) \, \int_{k_T^{min} \to 0}^{k_T^{max}} \frac{1}{k_T^2}$$









QCD correction to eq scattering

Neglecting terms that are relevant only at high κ_T the cross section for this process can be written as :

$$\frac{d^2 \sigma(eq \to e'gq')}{dQ^2 d\kappa_T^2} = \frac{d\sigma(eq \to e'q')}{dQ^2} \frac{\alpha_S}{2\pi} \frac{1}{\kappa_T^2} P_{qq}(z)$$

Where $P_{qq}(z)$ is the quark-quark splitting function: the probability to find a quark with a fraction z of the initial quark after gluon emission :

$$P_{qq}(z) = \frac{4}{3} \frac{1+z^2}{1-z}$$

Integrating over κ_T^2 and exploiting the relation $(\kappa_T^2)_{\text{max}} \simeq Q^2$:

$$\frac{d^2 \hat{\sigma}(eq \to e'gq')}{dQ^2} = \frac{d \hat{\sigma}(eq \to e'q')}{dQ^2} \frac{\alpha_S}{2\pi} \log \frac{Q^2}{\mu^2} P_{qq}(z)$$

Where μ^2 is a cutoff and the parton cross section is divergent for $\mu^2
ightarrow 0$







QCD correction to parton model

To obtain the ep cross section we should integrate on all the possible momenta of the incoming quark $p_q = \xi P$:

$$\frac{d^2\sigma}{dxdQ^2} = \sum_{q=u,d,\dots} \frac{d\sigma^0}{dQ^2} \int_x^1 \frac{d\xi}{\xi} f_q(\xi) \left(\delta(1-z) + \frac{\alpha_S}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2} \right)$$

^

Leadin order term QCD Ccompton term

$$eq \rightarrow e'gq'$$

$$p'_{q}$$

$$p'_{q}$$

$$p_{virt} = zp_{q} = z\xi P = xP$$

$$k_{g} = (1-z)p_{q}$$

Note *x* is the momentum fraction of the quark seen by the electron, not the one from the proton proton density !

 $z = x/\xi$

QCD factorization and PDF dependence on Q²

The divergence for $\mu^2 \to 0$ which corresponds to collinear emission and is not physical when the KTof the emitted gluon is of of the order of typical hadronic scales, the perturbative expansion in $\alpha_s(\mu)$ breaks down and non-perturbative effects (e.g. confinement) become effective

The divergence can be absorbed in a re-definition of $f_i(x) \rightarrow f_i(x, Q^2)$:

$$\frac{d^2\sigma}{dxdQ^2} = \sum_{q=u,d,\dots} \frac{d\sigma^0}{dQ^2} fq(x,Q^2)$$

$$f_q(x,Q^2) = \int_x^1 \frac{d\xi}{\xi} f_q(\xi,Q_0^2) \left(\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi}P_{qq}\left(\frac{x}{\xi}\right)\log\frac{Q^2}{Q_0^2}\right)$$

 Q_0^2 is some reference "starting" value of Q^2 where we like to parametrize the PDF

Altarelli-Parisi (DGLAP) PDF evolution equation:

$$\frac{df_q(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} f_q(\xi,Q^2) P_{qq}\left(\frac{x}{\xi}\right)$$

QCD correction to parton model: gluon initiated diagram

Adding also the "boson-gluon fusion" diagram



$$\frac{d^2\sigma}{dxdQ^2} = \sum_{q=u,d,\dots} \frac{d\sigma^0}{dQ^2} \int_x^1 \frac{d\xi}{\xi} \left[f_q(\xi,\mu^2) \left(\delta(1-z) + \frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2} \right) + f_g(\xi,\mu^2) \frac{\alpha_s}{2\pi} P_{gq}(z) \log \frac{Q^2}{\mu^2} \right]$$

Even if the gluon is not charged, the gluon density enters in the cross section

Full DGLAP evalution equations

$$\frac{df_q(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left(P_{qq}(z) f_q(\xi,Q^2) + P_{gq}(z) f_g(\xi,Q^2) \right)$$
$$\frac{df_g(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left(P_{qg}(z) f_q(\xi,Q^2) + P_{gg}(z) f_g(\xi,Q^2) \right)$$

The variation of the PDFs (and structure functions) with Q² is given by DGLAP

Given the PDFs at a starting scale, Q_0^2 , they can be re-calculated at any scale Q^2 by solving DGLAP equations.

$$P_{qq} = \frac{4}{3} \left(\frac{1+z^2}{1-z} \right)$$

$$P_{gq} = \frac{4}{3} \left(\frac{1+(1-z)^2}{z} \right)$$

$$P_{qg} = \frac{1}{2} \left(z^2 + (1-z)^2 \right)$$

$$P_{gg} = 6 \left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z) \right)$$



DGLAP insight

Physical interpretation: as the scale increases we see inside the proton with higher resolution and we can observe more partons.





HERA measurement: e⁺p Neutral Current



x

A zoom into few bins





At medium/low x F_2 increases with Q² (more gluons, more sea) At high *x*, F_2 decreases with Q₂ (valence quarks irradiate) Scaling at *x* = 0.1 (by chance where was measured originally at SLAC !)

Extraction of parton densities

Extracting parton density functions from DIS at HERA

The parton densities at a fixed scale Q_0^2 can not be calculated from first principles, should be fitted from data

Parametrisation at starting scale Q_0^2

Then parametrisation is adjusted to minimise the difference between calculations and predictions.

Constraints and sum rules

 $\int dx(u-\overline{u}) = 2$, $\int dx(d-\overline{d}) = 1$, $s = \overline{s}$, $c = \overline{c}$

 $\int dx x (u + \overline{u} + d + \overline{d} + \dots + g) = 1$

Different processes are needed to distinguish the contribution of different quark/antiquark flavours and gluons

HERA meas	surements and sensitivity to PDFS
NC low/mid Q ² :	$4/9(u + c + \overline{u} + \overline{c}) + 1/9(d + s + \overline{d} + \overline{s})$

dF²/dlnQ²: $\alpha_S g$

 $F_L = (F_2 - 2xF_1): \quad \alpha_S g$

NC High Q2 e⁻ - e⁺: $g_1(u + c - \overline{u} - \overline{c}) + g_2(d + s - \overline{d} - \overline{s})$

CC e⁻p: $(u + c), (\overline{d} + \overline{s})$

CC e⁺p: $(d + s), (\overline{u} + \overline{c})$

Tagging struck quarks in final states: charm and beauty

The production of heavy quarks (c and b) can be tagged from the presence in the final state of D mesons, muons or displaced vertices. Various techniques have been used at HERA to determine $F_{2^{c,b}}$ the component of F_{2} involving c or b quarks.

 $c\overline{c}$ pairs are abundantly generated from gluon splitting, and are responsible for 40% of cross section at low-x and high-Q2, In standard PDF fits they are not considered as part of the proton PDF at scales $Q_0^2 < m_c^2$





PDFs based on HERA data



Х

Something strange at low x ?

At Low Q² and low x the PDF fits to HERA data are not so good

The reason is unclear, various possibilities have been proposed:

- Low-x resummation effects (it is known that for very low x the standard DGLAP evolution breaks down and large logs (1/x) should be resummed (BFKL evolution)
- Parton saturation: when gluon density is too high we should include recombination in evolution equations
- Some non-perturbative effect at low Q2



 χ^2 of fit to HERA data vs minimum Q2 to include a data point in the fit



PDFs and hadronic collisions: going beyond DIS

QCD factorization theorem for hadronic collisions :

$$\sigma_{h_a h_b \to X} = \sum_{i=u,d,\dots,g} \sum_{j=u,d,\dots,g} \iint \frac{dx_a}{x_a} \frac{dx_b}{x_b} f_i(x_a, \mu_F^2) f_j(x_b, \mu_F^2) \hat{\sigma}_{ij \to X}(s x_a x_b, \mu_F^2)$$
Partonic cross section
Partonic cross section

The PDFs are evaluated at the factorization scale μ_F which gives the scale below which initial state splittings are absorbed in the PDFs evolution. The result should not change when the scale is changed, because the change in PDF is compensated by the change in the hard-scattering cross section, anyway there is typically a residual dependence due to missing higher orders.

Traditional approach : extract PDFs from DIS data => use to calculate cross sections at hadron colliders

=> nice test of QCD, if we find deviations it may be new physics

=> Historically (apart from Drell-Yan) processes in hh were less precise both theoretically and experimentally than DIS

Moder approach: add hadron collision data as input to global PDF fits

=> experimentally very precise measurements from LHC for many different processes

=> some PDF (e.g. gluon) poorly constrained by DIS experiments at high / low x

=> theoretical calculations for many hh processes (beyond Drell Yan) are now very precise (e.g. top pair production, Z+j)

=> Possible drawback : we should be careful not to absorb new physics in PDFs !

Global fits to PDFs

Global fits include many measurements from HERA, fixed target experiments and hadron colliders

Important to select a subset of measurement with well controlled experimental and theoretical uncertainties

Experiment	Process	Measurement	Proton PDF combination	
HERA + fixed target DIS	ep -> e X	NC inclusive mid Q2	4/9(u+c+ ⁻ u+ ⁻ c)+1/9(d+s+ ⁻ d+ ⁻ s)	sum of quarks weighted with electric charge
		dF2/dlnQ2	g	scaling violation
		F_L=(F_2-2xF_1)	g	longitudinal structure function
HERA	e+p -> e+X vs e-p -> e- X	NC high-Q2 e e+	g_1 (u+cuc)+g_2 (d+sds)	Zexchange
	e-p -> nu X	CC e-	(u+c),(⁻ d+ ⁻ s)	W-
	e+p -> nu X	CC e+	(d+s),(Tu+Tc)	W+
	ep -> e c X	semi-inclusive charm NC	C	charm tagging
	ep -> e b X ep -> e iet X	semi-inclusive beauty NC	b	b tagging iets
			5	
fixed targer DIS NC (I = e, mu)	lp->eX vs ID->eX	DIS on deuterium low Q2	u/p	deuterium target
fixed target DIS CC (nu)	vu N -> mu X	CC		large nuclear uncertainties
fixed target DIS CC (nu)	vu N -> mu c X	CC (dimuon)	S	large nuclear uncertainties
LHC	pp -> l+l-	Low Mass DY	4/9 (uīu+cīc)+1/9 (dīd+sīs)	
	pp -> l+l-	Z pole DY	Σ ci q ⁻ q	
	pp -> l+l-	High Mass DY	Σc'iqīq	
	pp -> Z + j	ZpT	gq, q ⁻ q	
Tevatron	p-antip -> I+I-	Low Mass DY	4/9 (uu+cc+ ⁻ u ⁻ u+ ⁻ c ⁻ c)+1/9 (dd+ss+ ⁻ d ⁻ d+ ⁻ s ⁻ s)	
	p-antip -> I+I-	Z pole DY	qq + ¯q¯q	
	p-antip -> I+I-	High Mass DY	qq + ¯q¯q	
LHC	pp -> l+nu X	W+	uīd + c īs	
	pp -> l- nu X	W-	dīu+ sīc	
	pp -> l- nu c X	W+c	sg	
Tevatron	p-antip -> l nu	W+/-	ud + cs +	
Fixed target DY	p/pi N -> I+ I-	NC	4/9 (uu+cc+ ⁻ u ⁻ u+ ⁻ c ⁻ c)+1/9 (dd+ss+ ⁻ d ⁻ d+ ⁻ s ⁻ s)	
	p/pi H/D -> I+ I-	NC	u/d	deuterium target
LHC/Tevatron	pp -> gamma + jet	prompt photon	gq, qq	
	pp -> jets	jets	99	
LHC	pp->t tbar	top production	99	
	pp->t X	top production	b u + b d	

LHC : Drell-Yan production

LHC KINEMATICS

In terms of parton energies



Leading order relation between x, Q2 and mass and rapidity of the lepton pair

Di-lepton invariant mass

Lepton-pair rapidity

$$M^2 = x_1 x_2 S = x_1 x_2 4 E_{\text{beam}}^2$$

$$y = \frac{1}{2} \log \frac{x_1}{x_2}$$
$$x_1 = \frac{M}{\sqrt{S}} e^y \quad x_2 = \frac{M}{\sqrt{S}} e^{-y}$$



Kinematic plane and data points used in NNPDF4.0 global fit



Kinematic coverage

Kinematic plane and data points used in NNPDF4.0 global fit



Charm and beauty PDFs: intrinsic charm ?

At low/medium *x* the c and b cross sections are compatible with QCD predictions that assume that all heavy quarks are produced by gluon splitting, i.e. $f_c(x,Q^2 < m_c^2) = 0$

In principle an intrinsic heavy quark component may be present in the proton wave function despite the fact that the charm mass ($m_c = 1.5$ GeV) is higher than the proton mass ($m_p = 0.94$ GeV).

$$|p\rangle = |uud\rangle + |u\overline{u}\rangle + |d\overline{d}\rangle + |s\overline{s}\rangle + |c\overline{c}\rangle$$

A global analysis of PDF data (including LHC and fixed target data), *Nature* 608 (2022) 7923, finds evidence for an intrinsic charm content of the proton that does not vanish at low Q²



Challenges and Prospects on PDFs

PDFs enter in the calculation of all the processes at colliders: uncertainty on PDFs is (one of) the limiting uncertainty on several measurements:

- Precision SM parameters (m_W, sin²θ_W)
- Higgs absolute branching ratios
- BSM processes !

How to improve ?

1) make measurements at LHC that can be used for improving PDF fits

2) New ep collider ?

- EIC Electron Ion Collider is going to being built at Brookhaven
- 275 GeV p + 10 GeV e, luminosity 10³⁴ cm⁻²s⁻¹ (200 times HERA)
- Polarized beams (to study spin structure)
- Possibility to use lons (saturation at large parton densities etc)



Diffraction

Hard diffraction: something unespected* found at HERA

In about 5-10% of all DIS events the proton does not break-up or breaks into a very low mass state, separated from the rest of the hadronic system by a large rapidity gap without hadronic activity.

Two experimental signatures:

- "Large rapidity gap"
- Forward leading proton (measured with roman pots) with $X_L = p_Z'/p_Z \simeq 1$





(* first evidence of hard diffractive events was found by UA8 at SppS that found "exclusive" p p -> p p' jet-jet events)

Hard diffraction: interpretation

The proton fluctuates into colourless objects (cfr. Regge theory), the dominant object at high energy is the Pomeron, with the same quantum numbers as vacuum.

Diffractive factorisation :

 $x_{\rm IP} = 1 - X_L$: fraction of proton momentum carried by the Pomeron $\beta = x/x_{\rm IP}$: fraction of the Pomeron momentum carried by struck quark

 $F_2^{D(4)} = f_{\text{IP}}(x_{\text{IP}}, t)F_2^{\text{IP}}(\beta, Q^2)$ (+ subleading terms)

 $f_{\rm IP}(x_{\rm IP}, t)$ is the probability to find a Pomeron in the proton and $F_2^{\rm IP}(\beta, Q^2)$ Is the structure function F2 of the Pomeron.

The Pomeron flux is consistent with that obtained from soft physics (e.g. total cross section):

$$f_{\rm IP}(x_{\rm IP},t) \propto x_{\rm IP}^{1-2lpha_{\rm IP}(0)}$$
 with $lpha_{\rm IP}(0) \simeq 1.1$

While the Pomeron PDFs can be obtained from fits to diffractive data similarly to proton PDF. This model works remarkably well and can explain all HERA diffractive DIS data.





End of Chapter 9



H1 and ZEUS




How to reconstruct a NC DIS event at HERA 2

In **Neutral-Current** DIS the measurement is overconstrained: various methods have been used with different systematics and different sensitivity to QED corrections.

- electron method (from E'_e , θ_e)
- Hadronic method (from $p_{T,X}$ and $(E P_Z)_X$)
- Double Angle method (from θ_e , $\cos\gamma_{\text{DA}} = \frac{p_{T,X}^2 (E-P_Z)_X^2}{p_{T,X}^2 + (E-P_Z)_X^2}$)
- Other methods mixing electron and hadronic variables: Sigma method and PT method



