Collider Particle Physics
- Chapter 12 -

Higgs properties measurements at LHC

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last update: Dec 2024

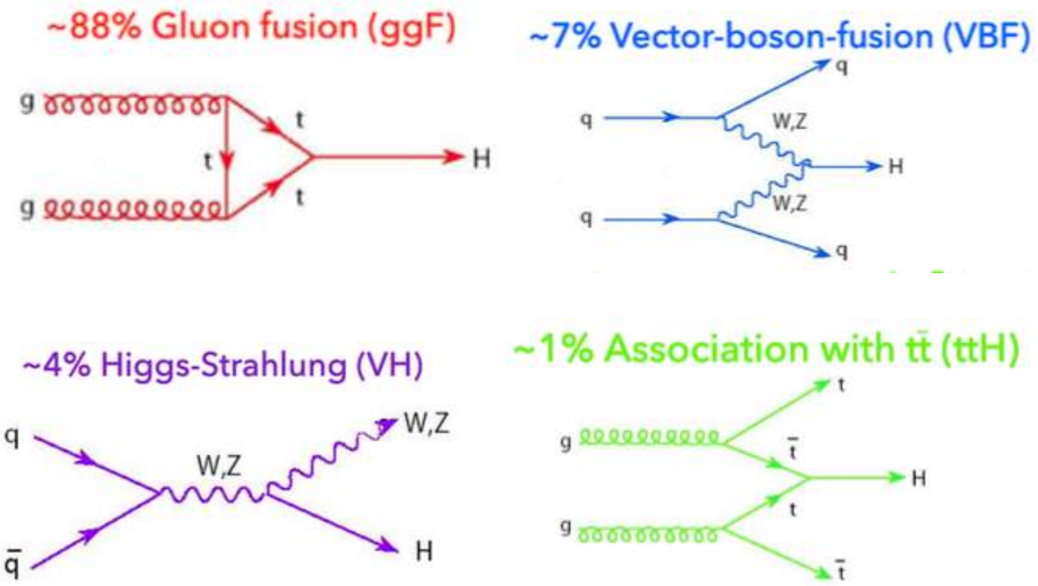
Chapter Summary

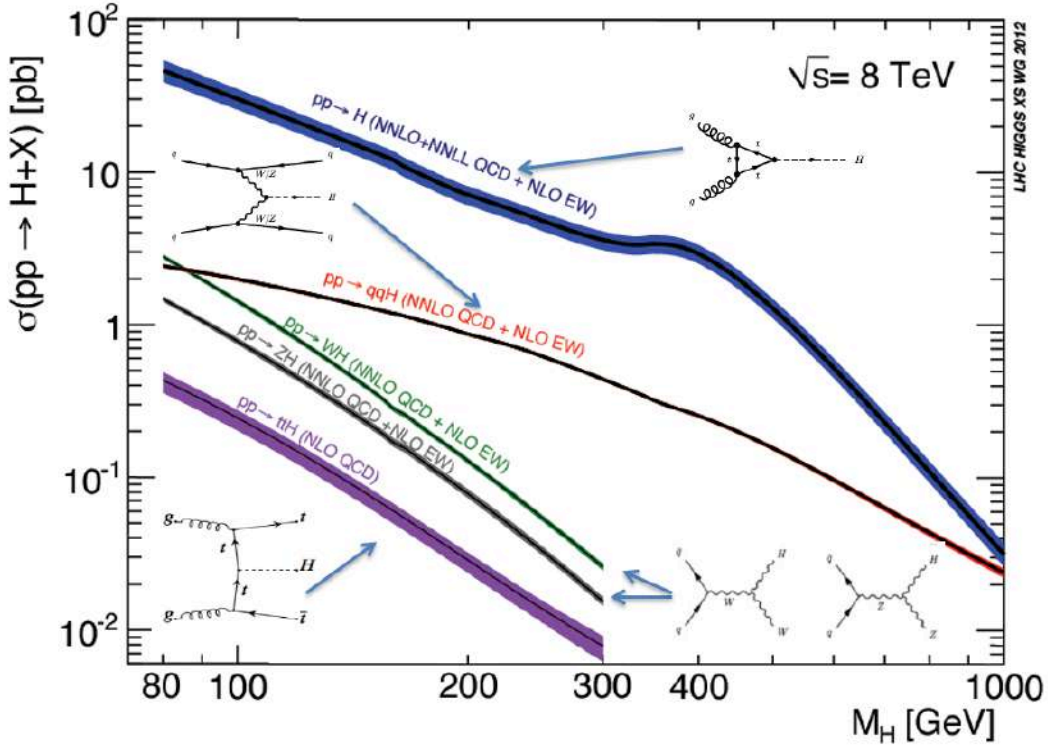
- A brief introduction on the Higgs at the Large Hadron Collider (LHC)
- An experimental overview of the discovery of the Higgs boson
  - Covering in some detail the two main discovery channels
- The first properties measurements
  - Mass and width
  - Spin and parity
- Status of the most recent measurements
  - Couplings and combination of all channels
  - Di-Higgs production
- Simplified template cross sections and coupling measurements
- BSM couplings in the Effective Field Theory
- Plans for the coming runs of LHC

DISCLAIMER: I am including a few more results from ATLAS just because I am more familiar with them, but of course CMS has a pretty similar set of results

The search for the Higgs boson at LHC

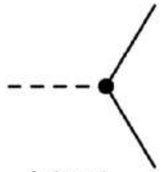
- In the Standard Model (SM), p-p collisions can lead to the production of an Higgs boson via the following diagrams
  - A loop is needed to couple the Higgs to massless gluons: dominated by top-quark loop because the Higgs coupling is proportional to the mass



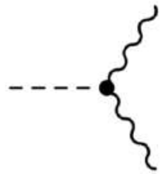


Higgs decays

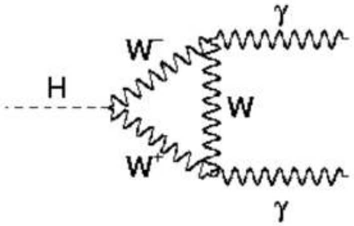
- What we ”see” in a detector are the Higgs decay products
- Higgs coupling to fermions (bosons) is proportional to the mass (mass squared)
  - NB, the two Z’s are identical particles → reduced BR with respect to H→W+W- because of phase space
- The photon is massless, but still the Higgs decays to $\gamma\gamma$ via loop diagrams with W and top

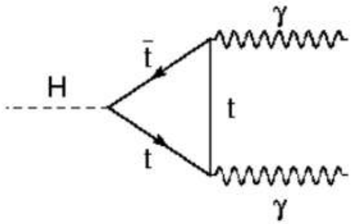


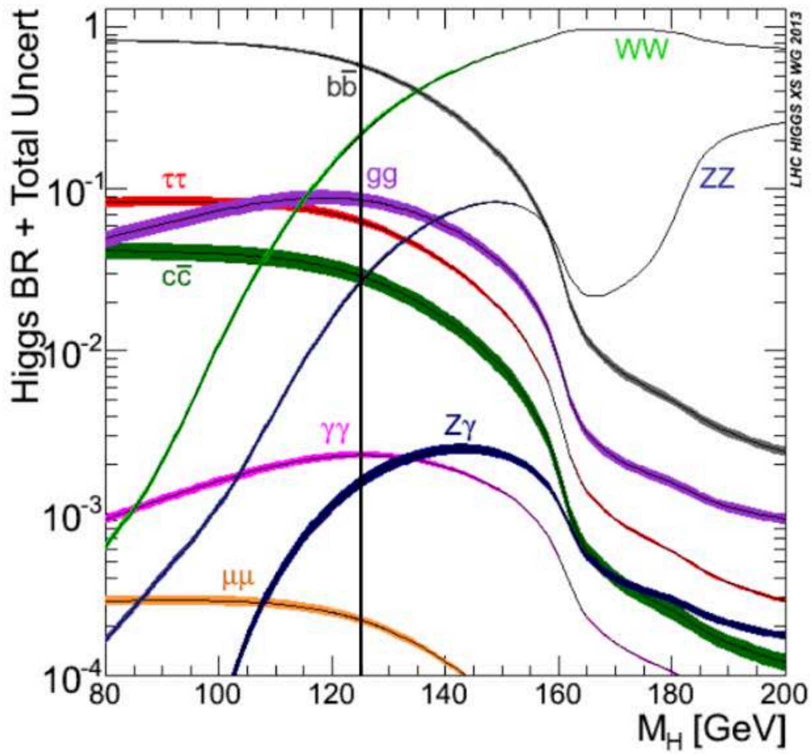
$$g_F = \sqrt{2}\frac{m_f}{v}$$



$$g_V = 2\frac{m_V^2}{v}$$



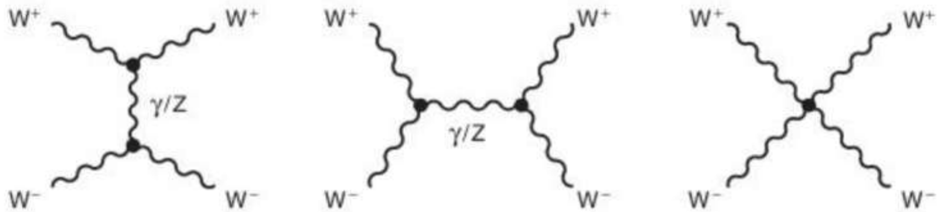




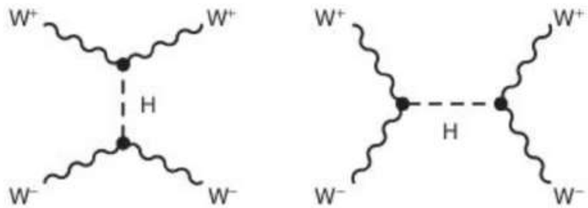
- In the case of the decays to W and Z bosons, one has also to consider the decay BR of the weak bosons into each possible final state

The Higgs boson search, before LHC

- An issue similar to the unitarity violation in e+e- →W+W- (solved by the diagram with the Z boson exchange) arises in the W+W- → W+W- scattering (from the longitudinal component WLWL→WLWL)



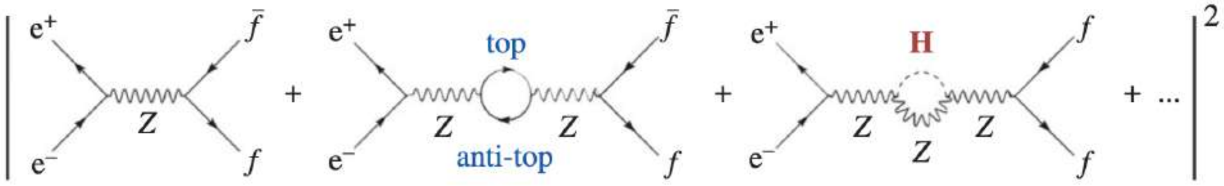
- It's canceled by the exchange of a scalar particle, that must have a mass M≲800 GeV: the Higgs boson in the case of the Standard Model



- Lower limits were set by previous experiments, whose results also allowed to estimate the Higgs mass in the Standard Model hypothesis

The situation after LEP

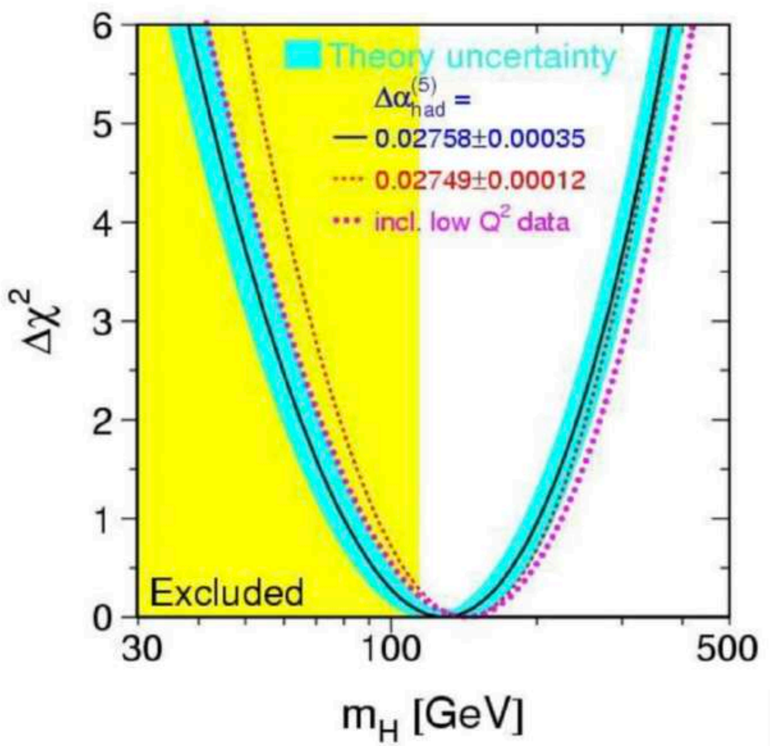
- Every electroweak observable can be expressed in terms of the Higgs and top mass



$$m_W^2 \sin^2\theta_W = m_Z^2 \cos^2\theta_W \sin^2\theta_W = \frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r)$$

$$\Delta r \propto \frac{3G_F}{8\pi^2\sqrt{2}} m_t^2 \, , \; \frac{\sqrt{2}G_F}{16\pi^2}\left(\frac{11}{3}\ln\frac{m_H^2}{m_Z^2} + \ldots\right)$$

Best fit for the Higgs mass was 129^{+74}_{-49} GeV. and upper limit at 95% CL was at 285 GeV

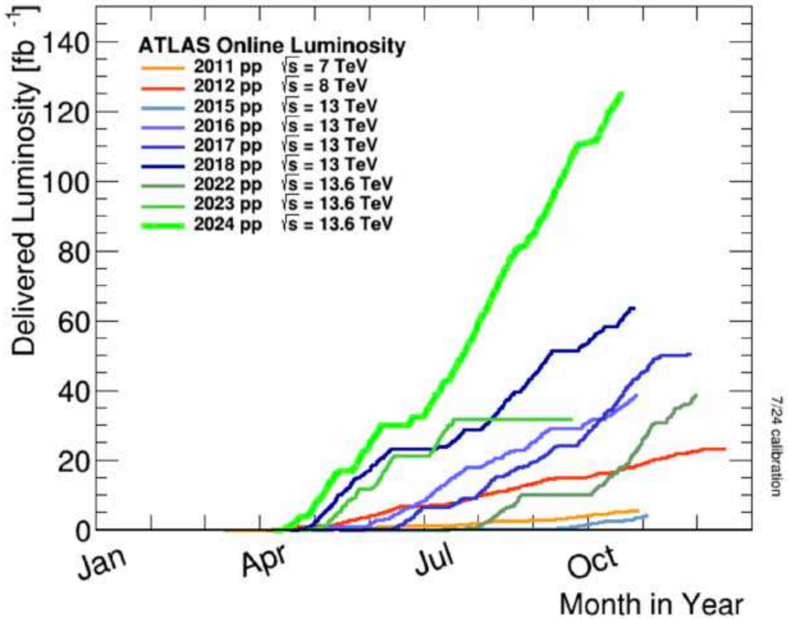


- Precision measurements of the Z (LEP), W (LEP and Tevatron), and top (Tevatron) allowed a fit of the Higgs mass
- Only valid in the SM hypothesis → the experiments were designed for a search over a broader mass range

LHC

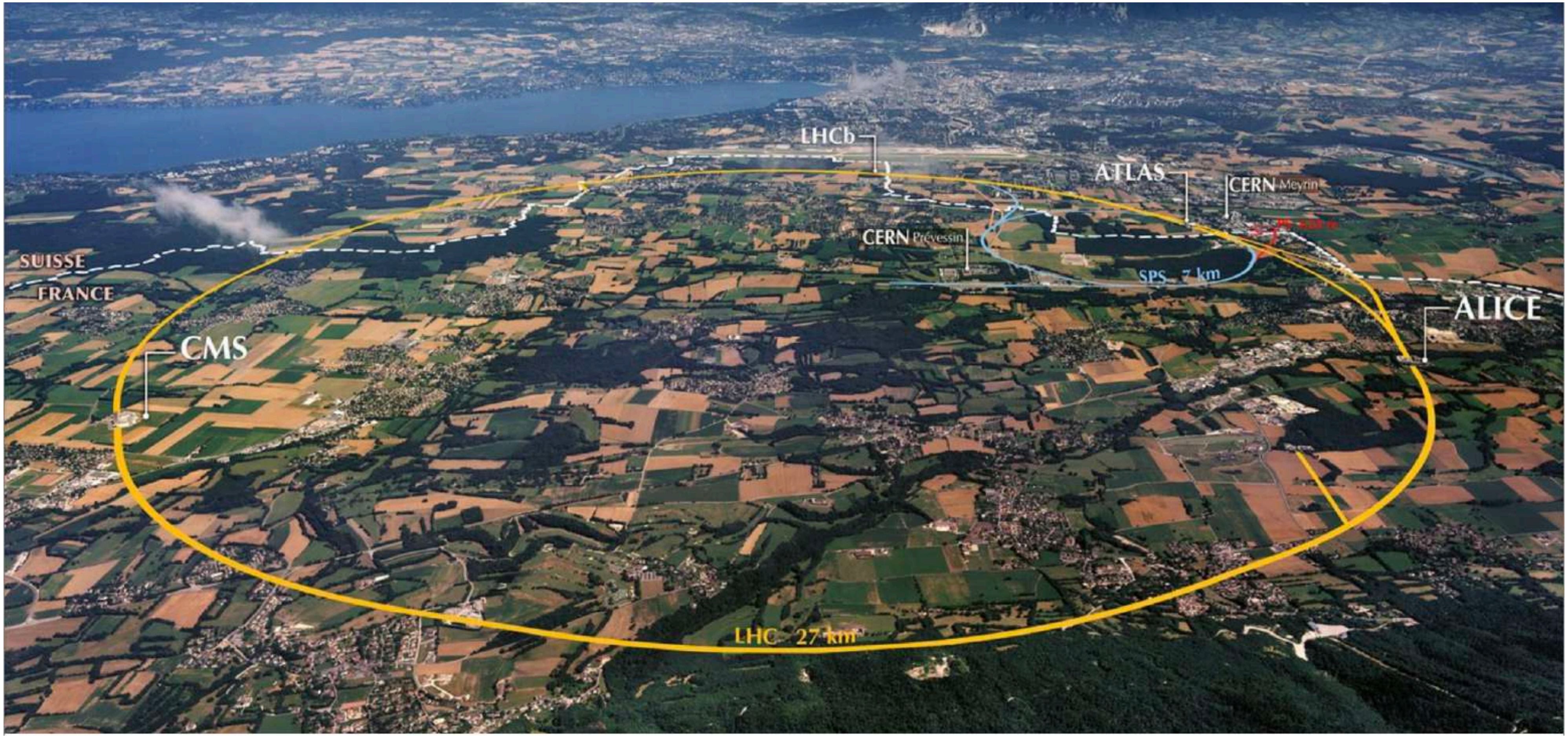
- LHC is a proton-proton collider that can operate at proton-proton center of mass energies of up to 14 TeV
  - Reminder: that is not the center of mass energy of the interaction between the partons
- The LHC experiments, ATLAS and CMS have taken data up to now in three periods:

- Run-1: 2010-2012
  - center of mass energies of 7 and 8 TeV, fb^{-1}
- Run-2: 2015-2018
  - center of mass energy of 13 TeV, 139 fb^{-1}
- Run-3: 2022-ongoing
  - center of mass energy of 13.6 TeV, at the moment ~180 fb^{-1}



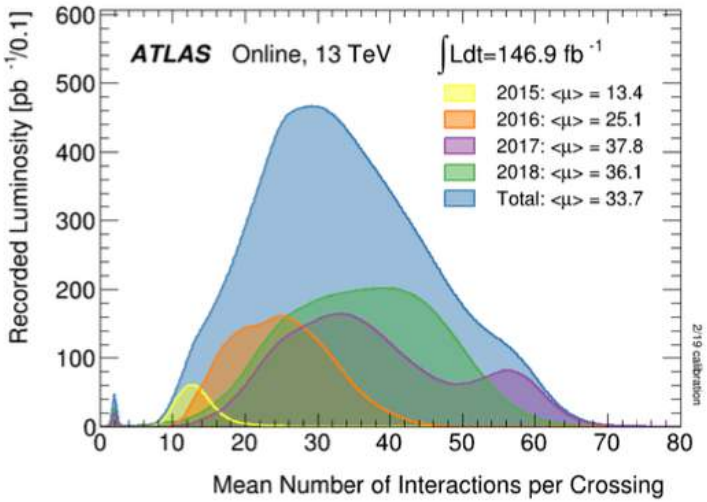
- When talking about the discovery I will show Run-1 results, while all the results on properties measurements will include the most recent Run-2 results

LHC

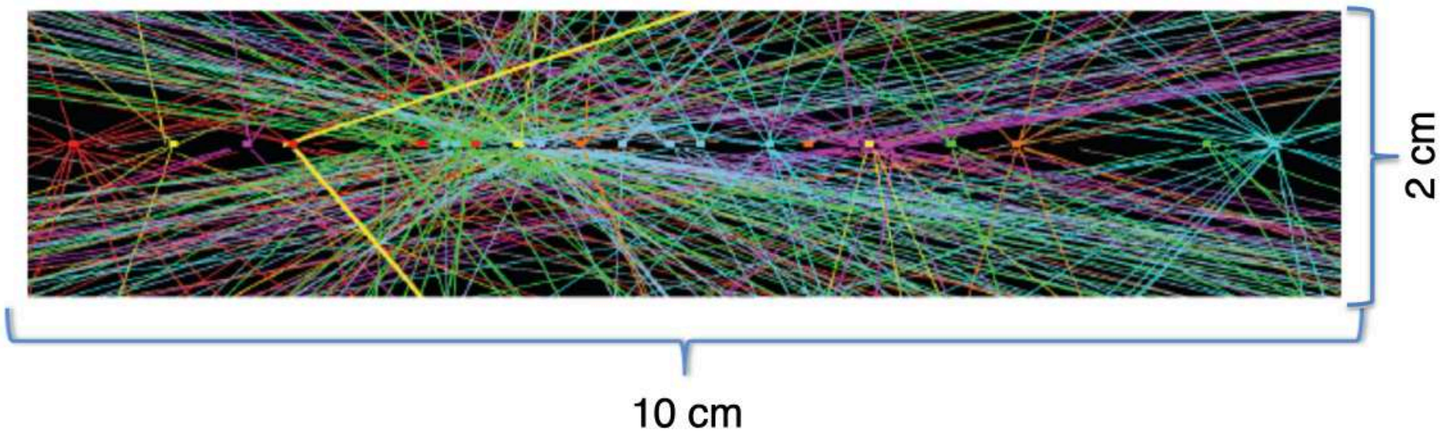

LHCb
ATLAS
CERN Meyrin
CERN Prévessin
SPS 7 km
SUISSE
FRANCE
ALICE
CMS
LHC 27 km

Luminosity and pileup

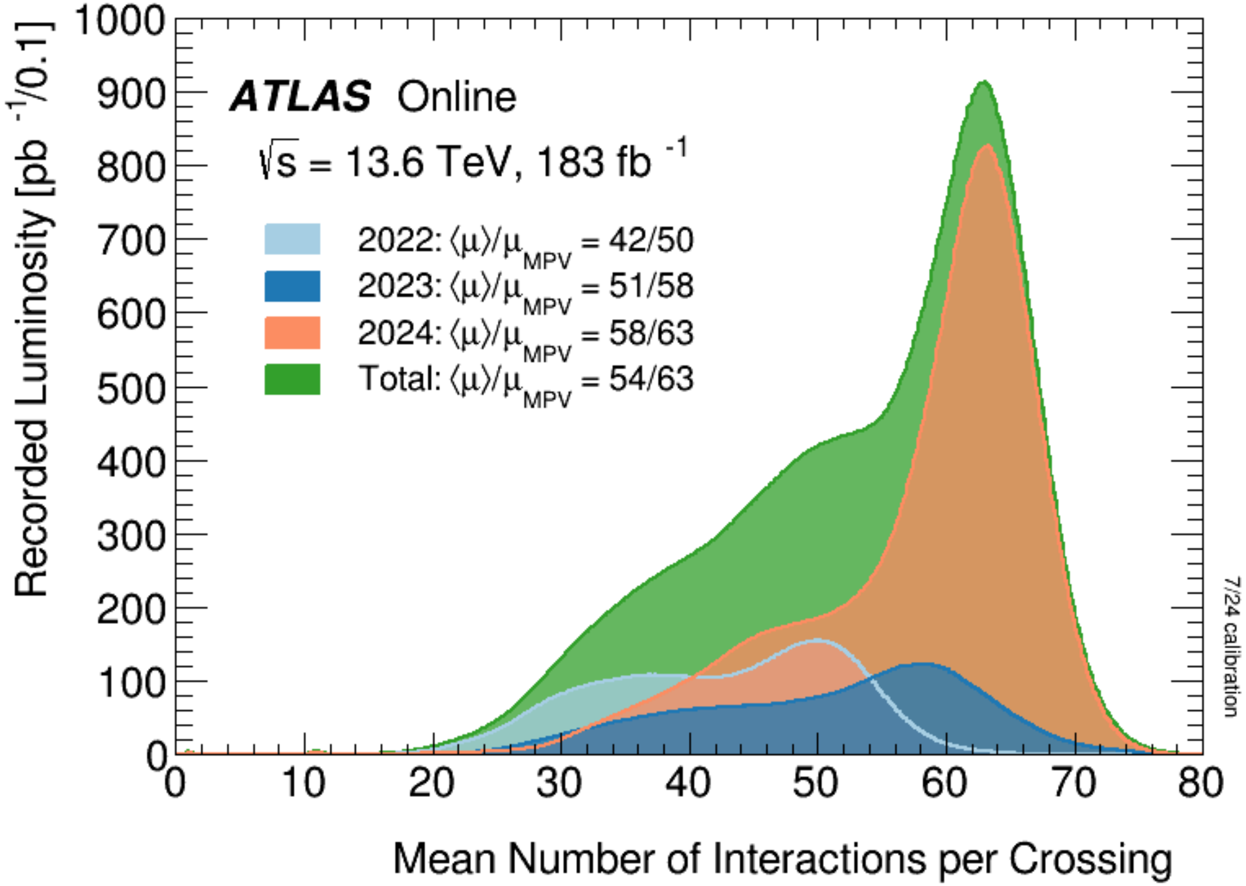
- In each bunch-crossing there are multiple proton-proton interactions occurring
- The interaction points are distributed along the z axis with a σ of ~ 5 cm
- Each primary vertex is reconstructed fitting a common position to groups of track



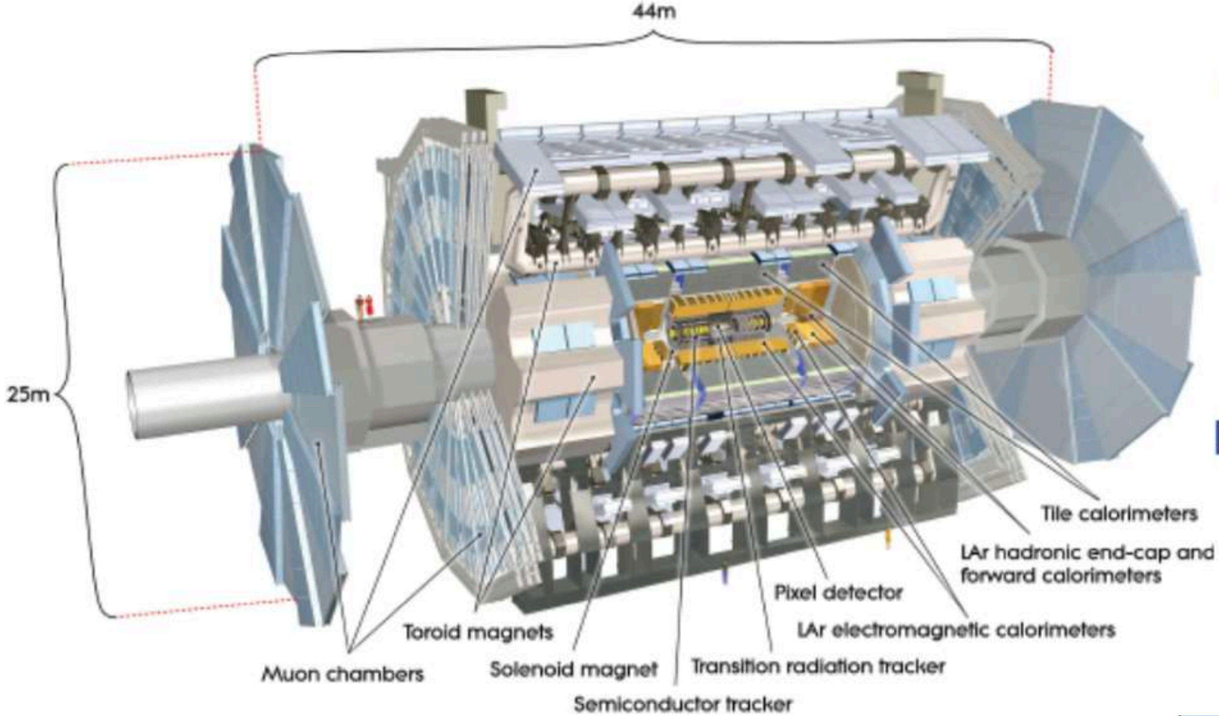
- The resulting resolution on the z of each primary vertex is ~90 microns thus making it possible to separate the vertices using the z
- We now reached 60 mean interactions, and will reach 200 in run-4



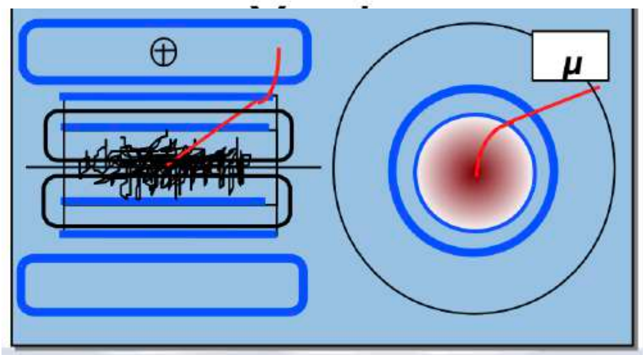
Pileup in run-3



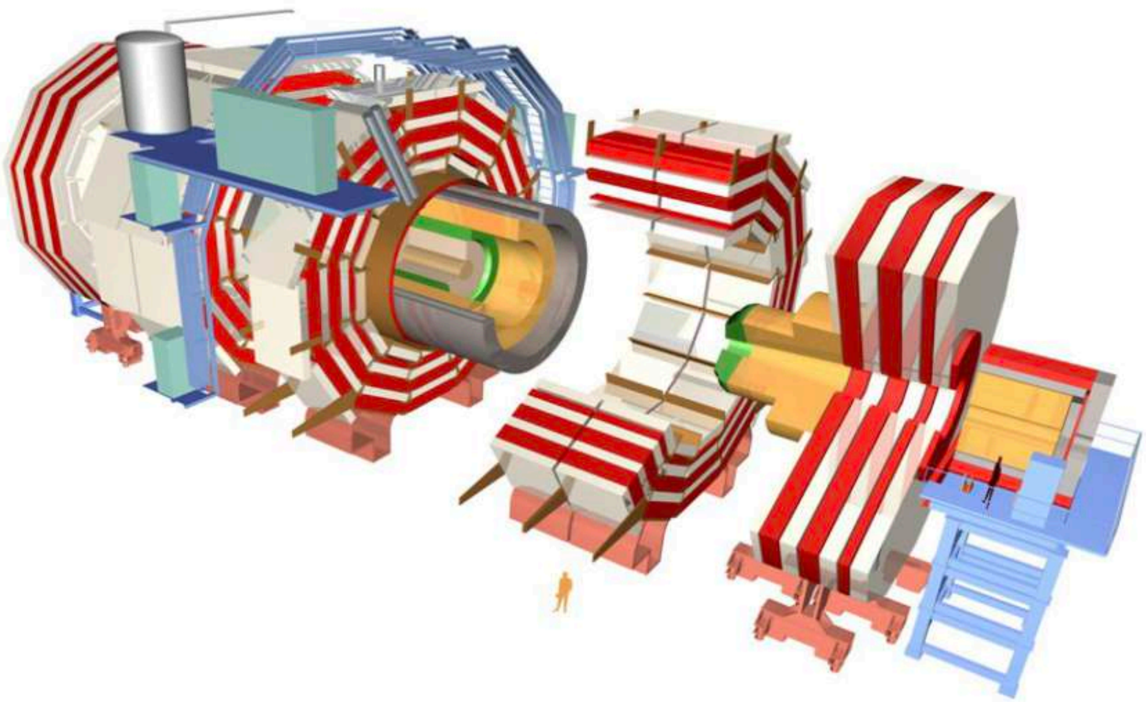
The LHC detectors: ATLAS



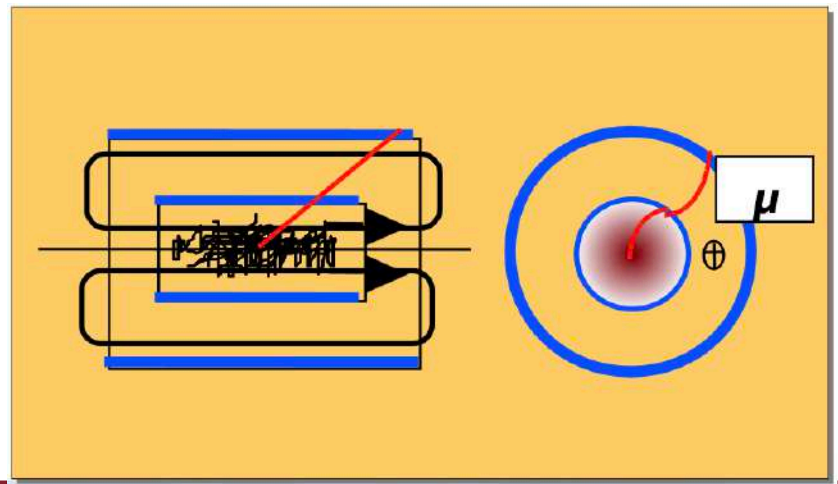
Magnets	1 Central Solenoid (2 T) + 3 air-core toroids
Tracking	Silicon+Transition radiation tracker
EM calo	Sampling LAr calo
HAD calo	Plastic scintillator (barrel) LAr technology (endcap)
Muon	Reco and trigger Standalone reco capabilities



The LHC detectors: CMS



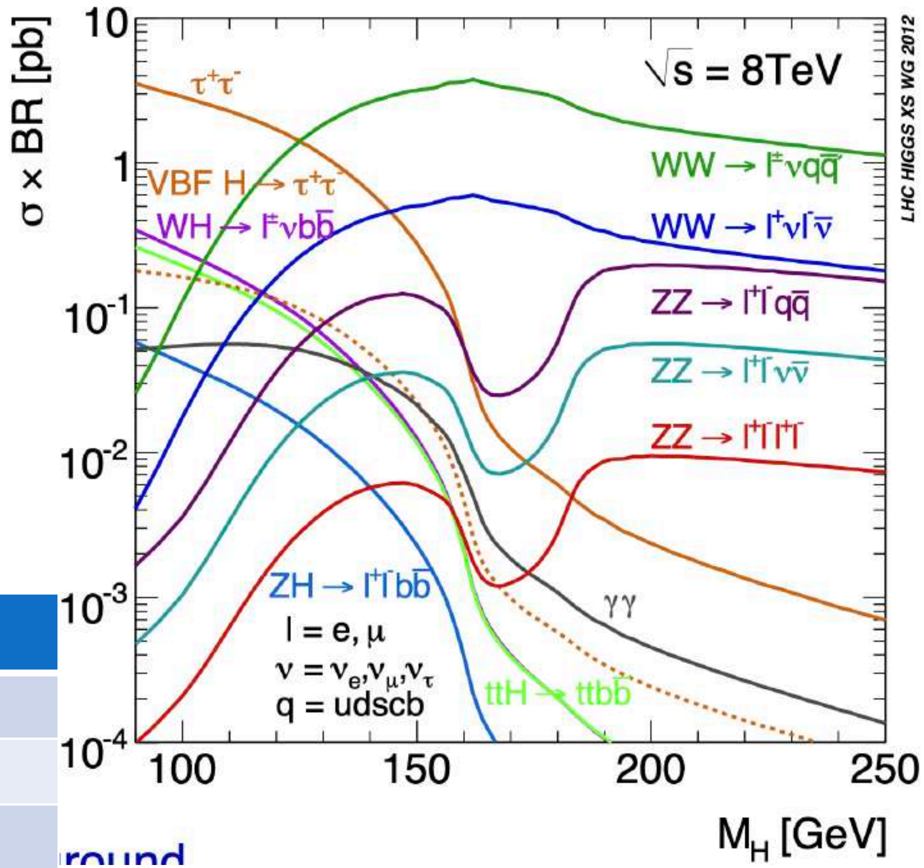
- 1 central solenoid with 4T field
- Muon chambers magnetic field in the solenoid return yoke
- Silicon trackers in the inner detector
- Crystal electromagnetic calorimeter
- Hadronic calo in brass + plastic scintillator



Searching the Higgs events

Some of the most sensitive channels

- The most sensitive channels depend on the Higgs mass
- In the low mass (mH < 150 GeV), in spite of the low branching ratios, the ZZ(*)→4l channel
- Channels including jets are in general difficult
  - Large backgrounds
  - Worse resolution in jet reconstruction
- Exception are the b-jets (and c-), that can be indentified thanks to their longer lifetime
- Leptons and photons provide a cleaner signature



Events expected in 20 fb^{-1} at $\sqrt{s}$ =8 TeV, for m_H=125 GeV

These are the events before any selection !

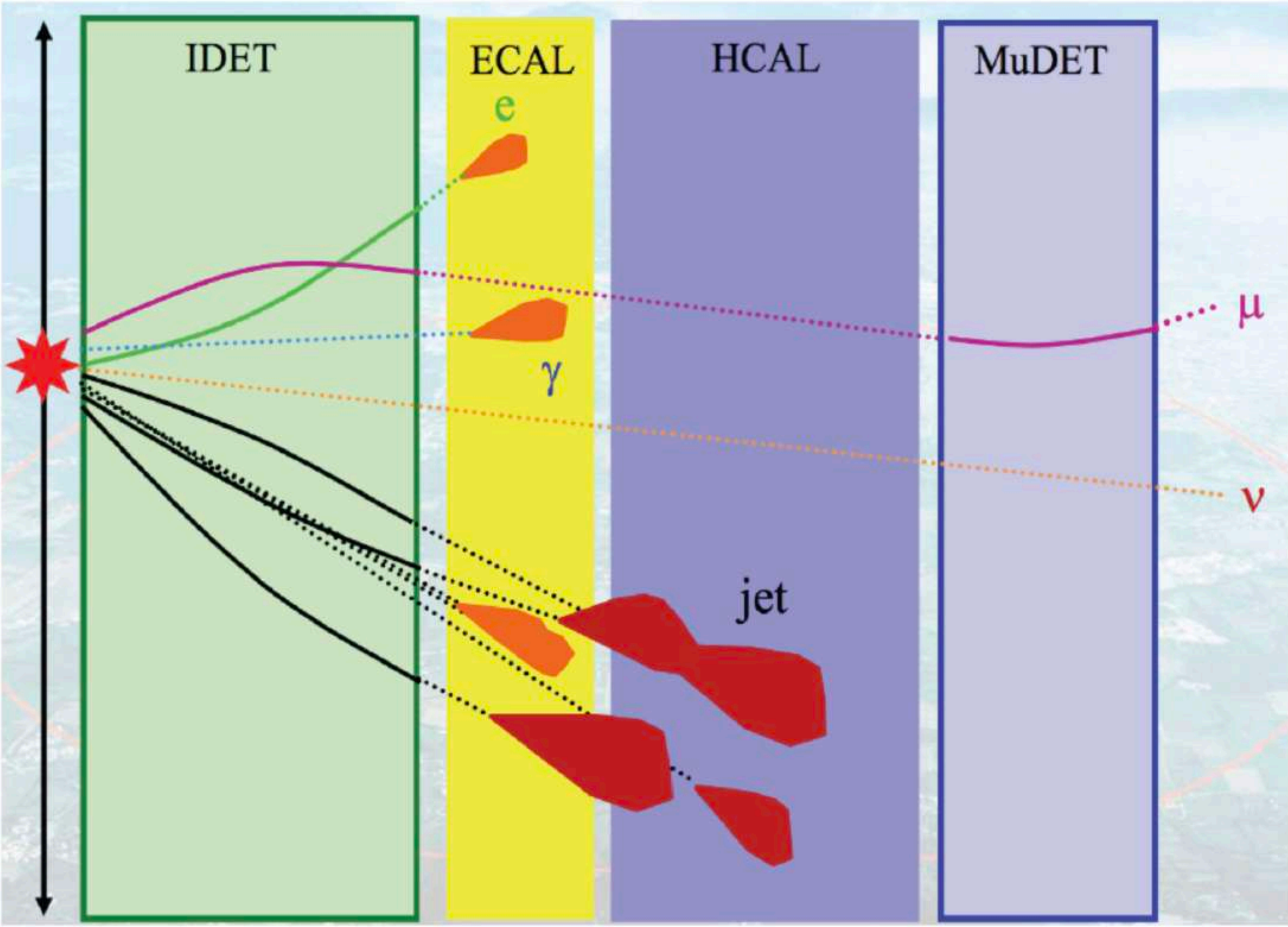
At 13 TeV the rates increase by a factor ~2

Decay Channel	Signal Events
H→$\gamma\gamma$	~1000
H→ZZ*→4l	~60
H→WW→lvlv	~4000
H→$\tau\tau$	~20000

Steps for building a selection

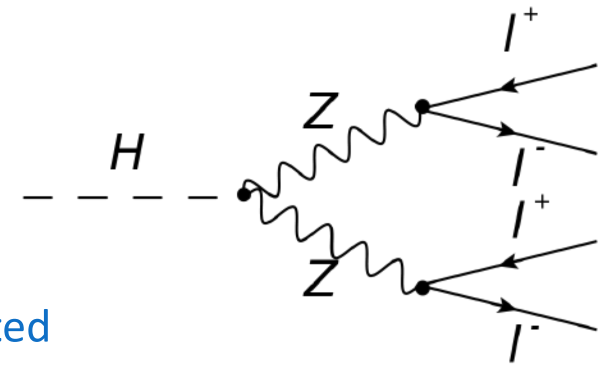
- In particular for searches, it is very important to develop the signal selection **without looking at the data** (blind analysis) in the signal region
- This avoids biases in the selection: looking at the data can cause even involountarily to pick excess regions and artificially increase the significance
- The analysis can be developed using MonteCarlo, and looking at the so-called control regions
  - Regions selected with cuts that remove completely the signal and only keep the backgrouns under study
- And, when looking for a signal, the most important thing is… the background:
  - Processes that give the same signature as the signal you are looking for
  - Can be either identical (same final state): irreducible background or a different final state that can be mis-identified as the signal
- For each signal a selection must be developed, aiming at maximizing the efficiency for the signal, and the rejection for the background

Identification and reconstruction

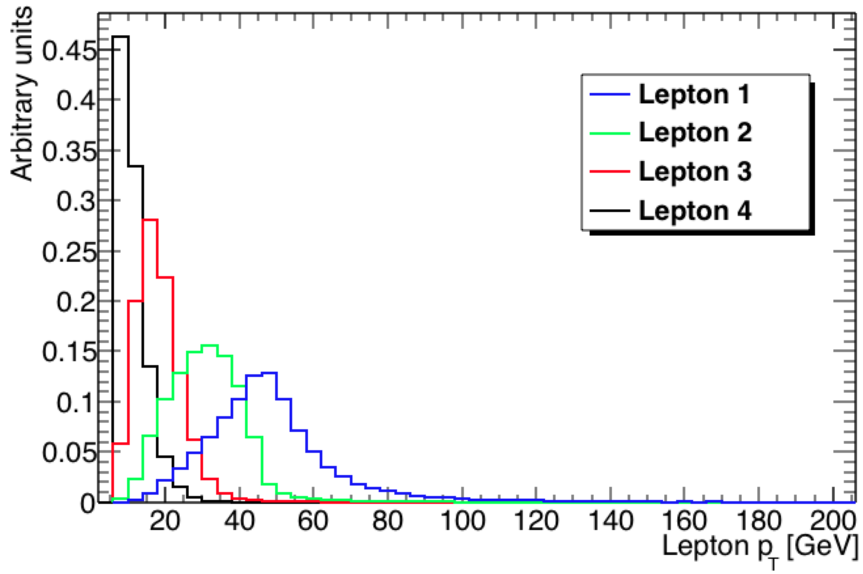


Schematic view of a generic detector parts and their usage for particle identification

The H→ZZ*→4-lepton channel

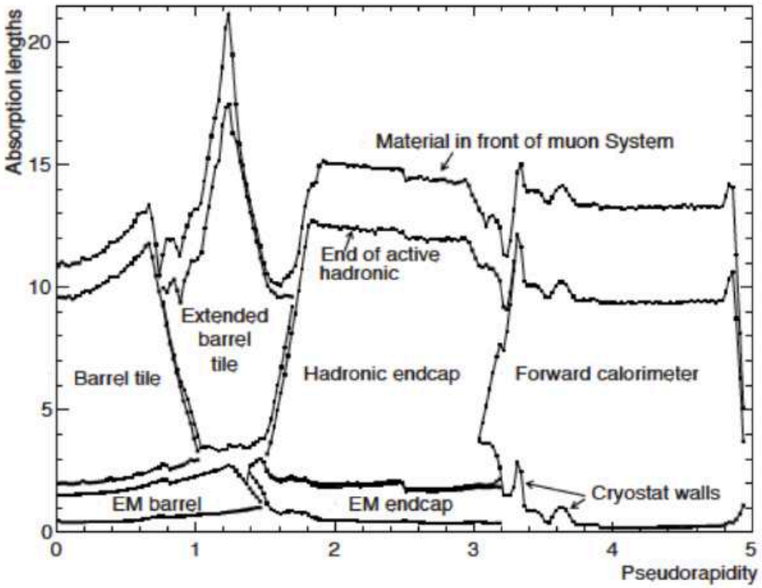


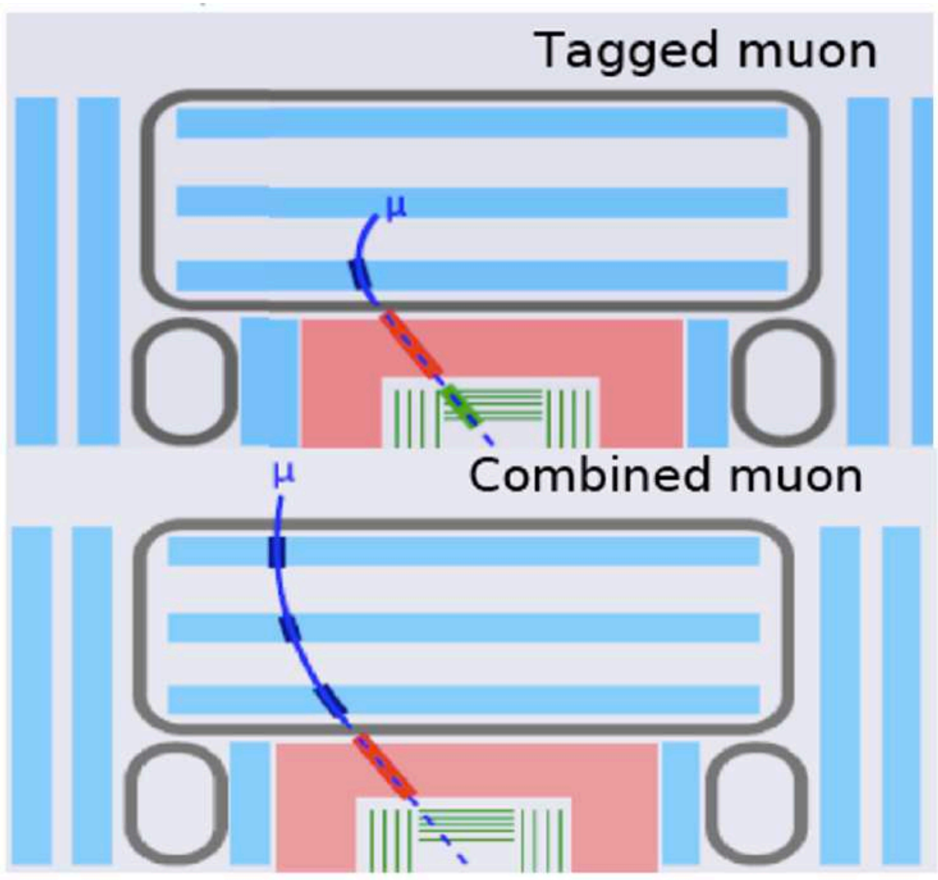
- Consider only electrons and muons because:
  - taus decay with neutrinos in the final state
  - channels with jets (llqq, qqqq) have larger BR but much larger backgrounds, and worse mass resolution
  - channels with neutrinos in the final state can't be fully recontructed
- Electrons and muons are the final state particles that can be reconstructed with the best resolution



- The Higgs mass defines whether the two Z's are both on shell or not
  - For masses above 114 GeV, at least one Z must be on-shell
- For the Higgs at 125 GeV, only one Z is on-shell, the other is a virtual Z* with mass above ~12 GeV

Identifying the muons



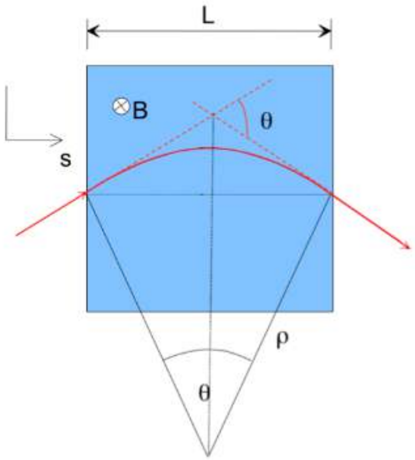


- ❑ If a particle reaches the muon spectrometer, it is already identified as a muon
  - ➢ Small fraction of punch-through
- ❑ Then the muons can be reconstructed in different ways, mostly as "combined" muons
  - ➢ This depends also on the experiment layout, i.e. CMS and ATLAS are a bit different in this aspect

Muon momentum measurement

- In case of a spectrometer with a uniform magnetic field:

  pT=qBρ = 0.3B(T)ρ(m)



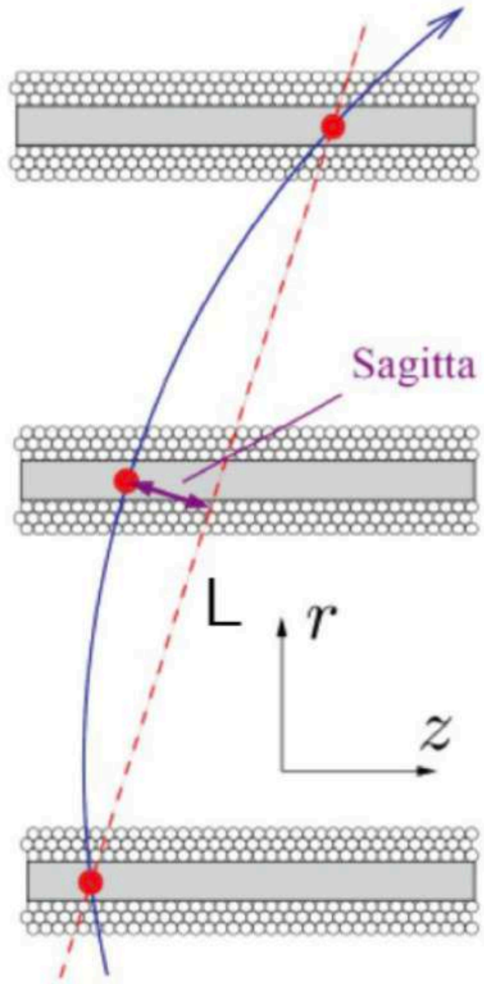
- What is usually measured is the "sagitta" s

$$s = \frac{0.3BL^2}{8p_T}$$

- For example, measuring s via 3 equidistant points, s=(x1+x3)/2-x2

$$\frac{\sigma(p_T)}{p_T} = \sqrt{\frac{3}{2}}\sigma_X \frac{8p_T}{0.3BL^2}$$

- The impact of the detector resolution on the momentum resolution increases vs pT

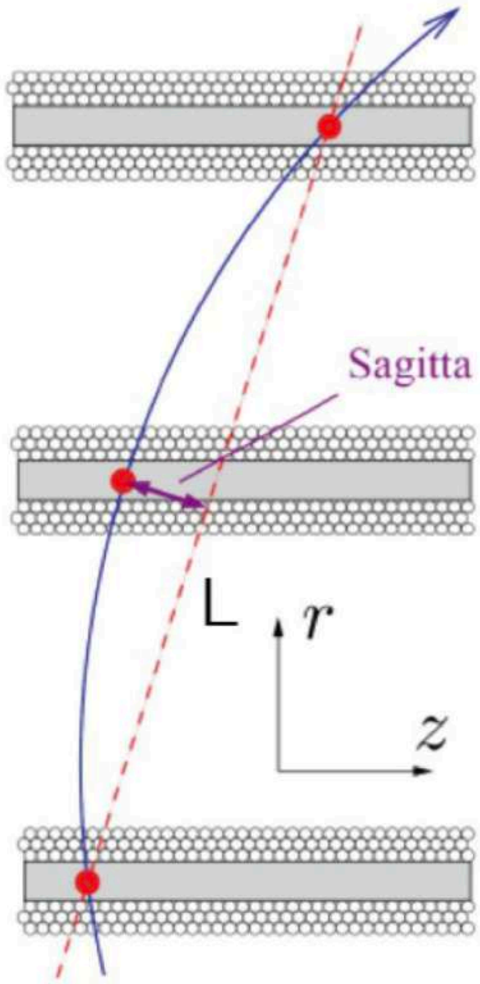


Muon momentum measurement

- In addition there is the contribution from the multiple scattering
  - stochastic scattering in the materials crossed by the muon
- The impact on the track direction decreases with the momentum

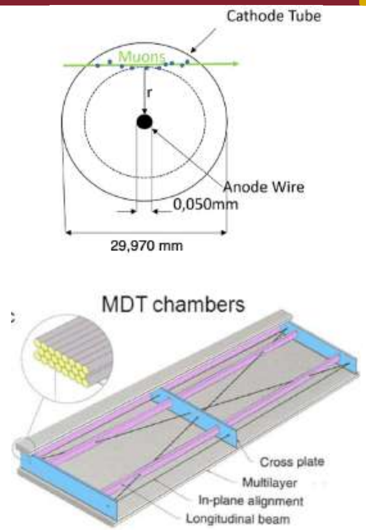
$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{\beta c p}\, z \sqrt{x/X_0}\left[1 + 0.038 \ln(x/X_0)\right]$$

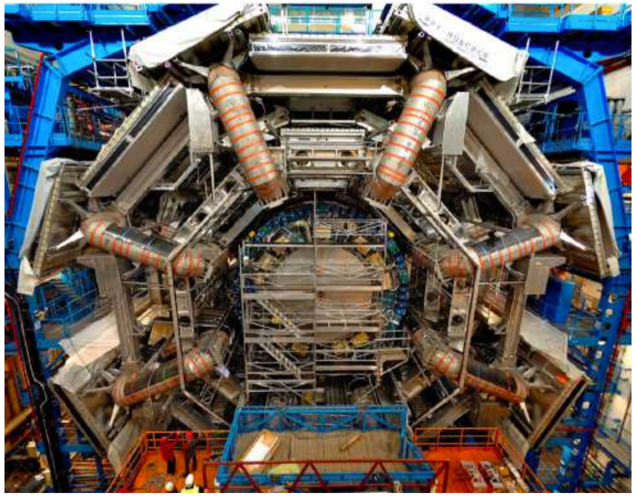
- So the final impact on the relative momentum resolution σ_{pT}/p_T is ~ constant
- In general the rsolution is a combination of the detector resolution and the multiple scattering, whose balance depends on the detector structure



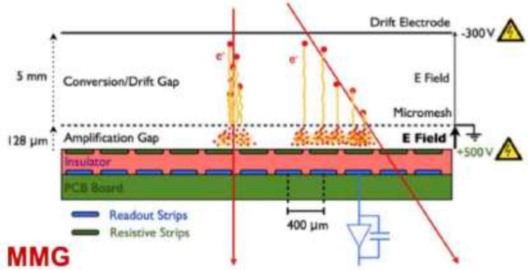
Muon Detectors examples

- ❑ Drift chambers
  - ➢ For precision tracking (and trigger)
  - ➢ In total ~ 340K tubes organized in tracking chambers
  - ➢ Coupled with dedicated trigger detectors
  - ➢ In ATLAS since the beginning



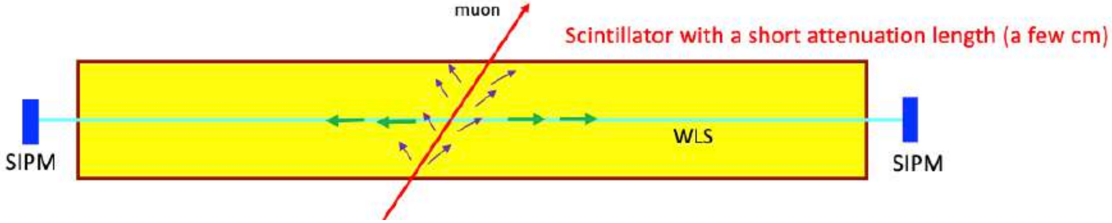


- ❑ Micro-Pattern Gaseous detectors
  - ➢ MicroMegas
    - ▪ Fast signal induction and charge collection for sustaining very high rates
  - ➢ Crucial for the current and the future LHC runs



Muon Detectors examples

- ❑Scintillator-based detectors
  - ➢ Scintillators with wavelngth-shifting fibers
  - ➢ Silicon Photo-Multipliers for signal formation



- ❑Trigger and readout electronics
  - ➢ ASICs for implementing the trigger coincidences
  - ➢ FPGAs programming for trigger algorithms

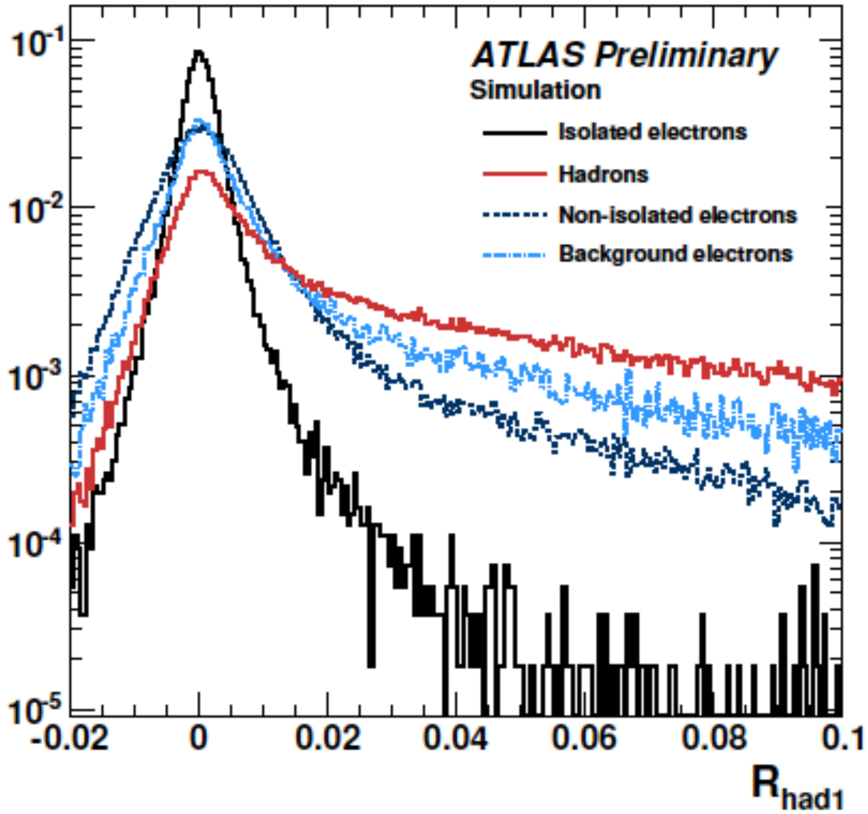
- ❑ The ATLAS Rome Group is having since the beginning of LHC a crucial role in the design, construction and operation of the muon spectrometer detectors
  - ➢ Also on trigger and reconstruction software development and on all simulation aspects
- ❑ Also: research and development for ATLAS upgrades and for future colliders

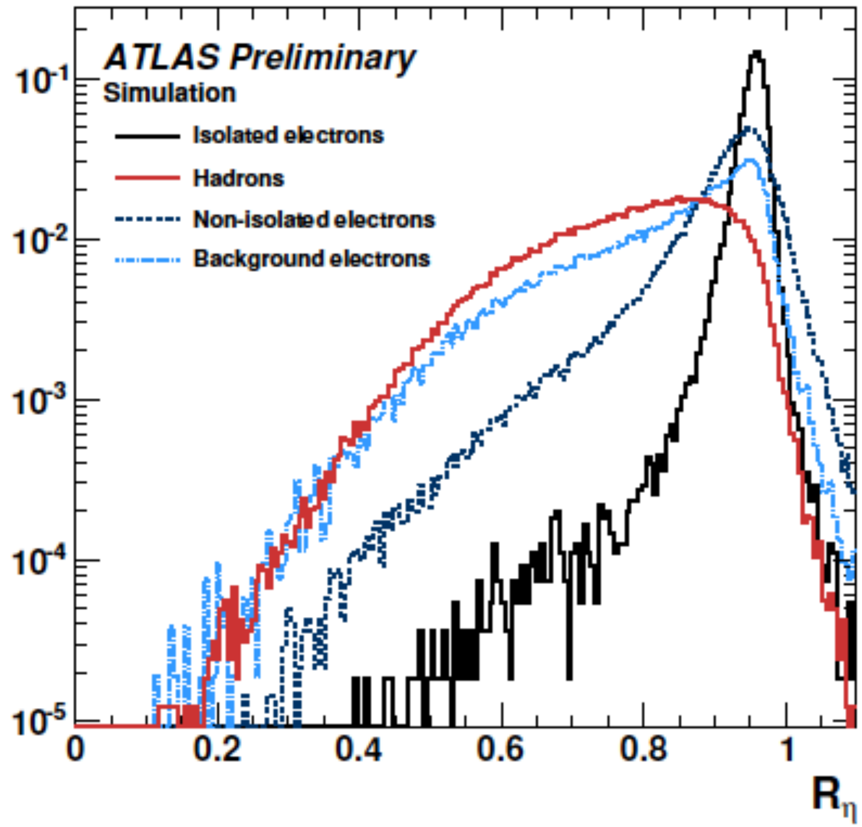
Identifying electrons and photons

- An electron is identified by a track in the inner detector, spatially matched to a cluster of energy cells in the EM calorimeter
- The criterium is in principle simple, but there are other objects that can create a fake electron
  - jets and hadrons from quarks hadronization have tracks and energy deposits → main background to electron identification
  - photons have clusters exactly like of the electrons but no track in the ID → but another track in the event can be mis-associated to the EM cluster
- Some criteria to identify the electrons are:
  - Low activity in the hadronic calorimeter in correspondence of the EM cluster
  - Shape of the EM cluster
  - Compatibility between the track and the cluster
- After the identification, the direction of the electron is given by the track, the energy by the EM cluster energy
- For the photons, the criteria are ~ the same as for the electrons on the EM clusters, but of course no requirement of a track and all the related variables

Some electron discriminating variables

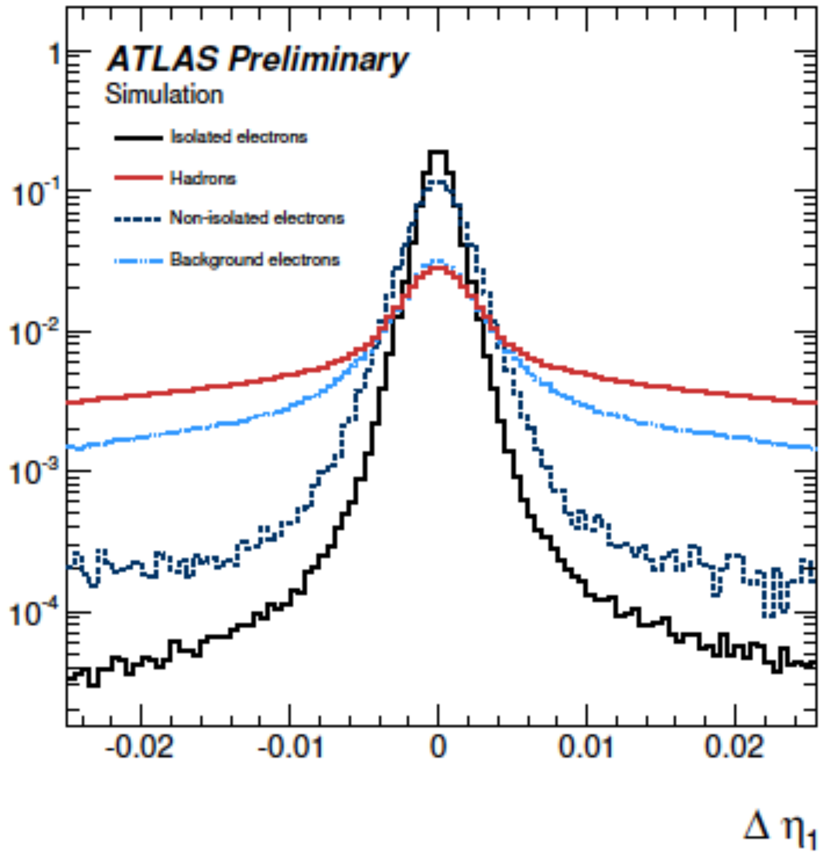
- Hadronic leakage and inner cells/total cluster energy

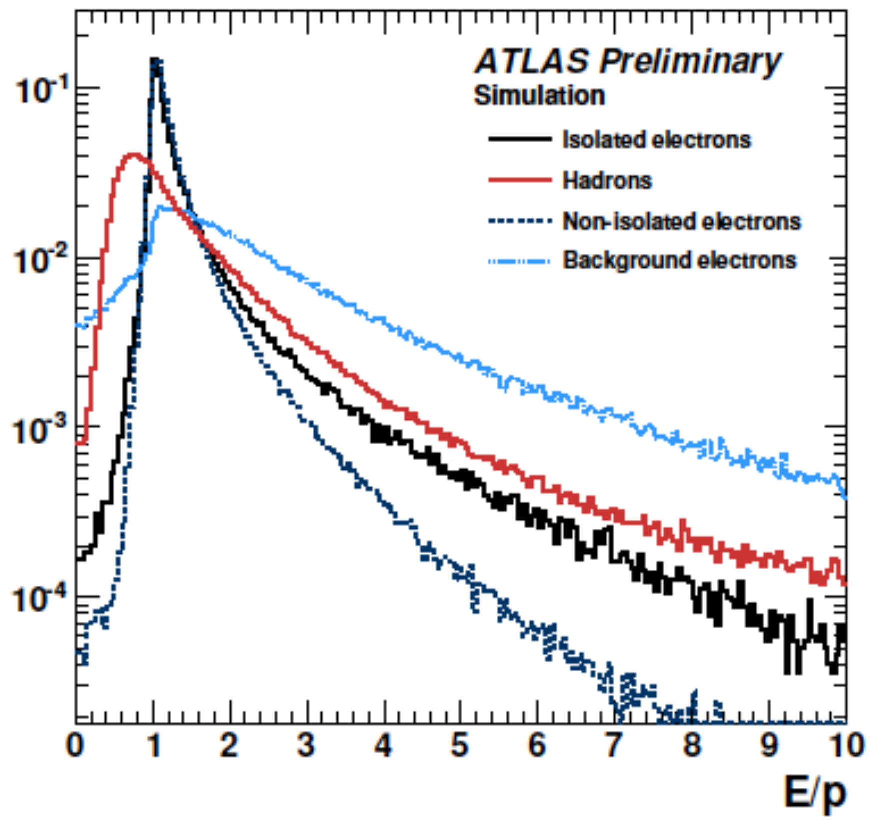




Some electron discriminating variables

- ❑ $\Delta\eta$ between track and cluster, and cluster energy / track momentum



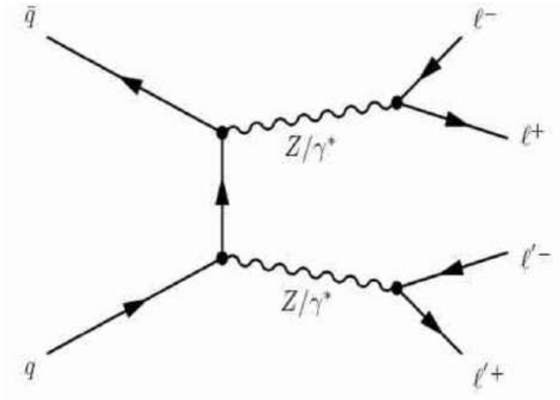


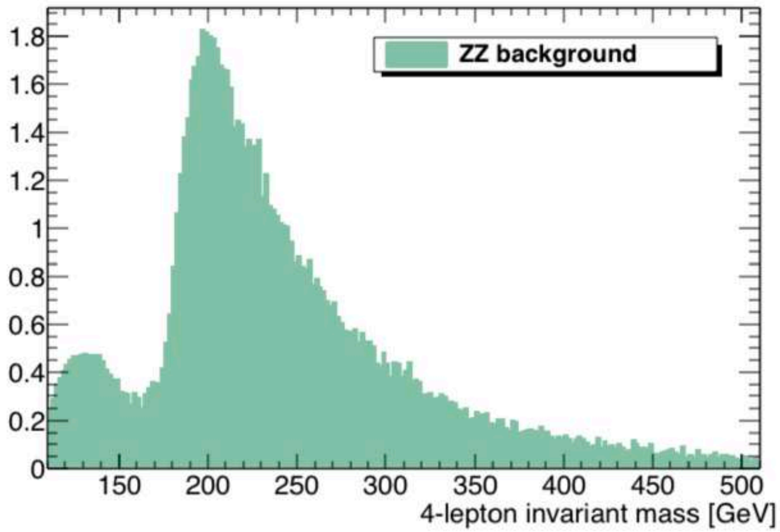
4-lepton irreducible background

- qq→ZZ(*)→4l has the same final state as the signal, but with a continuous mass distribution
- The shape comes from the onset of the two on-shell Z's production at ~ $2xM_Z$

- The variable that can be used to discriminate the signal from the background is in this case only the mass of the 4-leptons
  - Doing an assumption on the SM scalar Higgs hypothesis, also the decay products angular distributions are discriminating

- Here the plots are normalized to 20 fb-1 at 8 TeV
  - Expected SM distributions





Irreducible background

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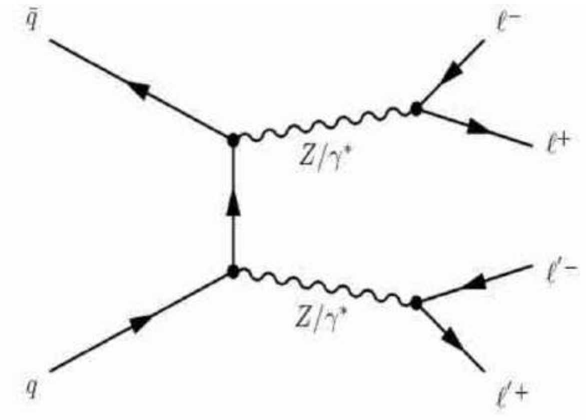

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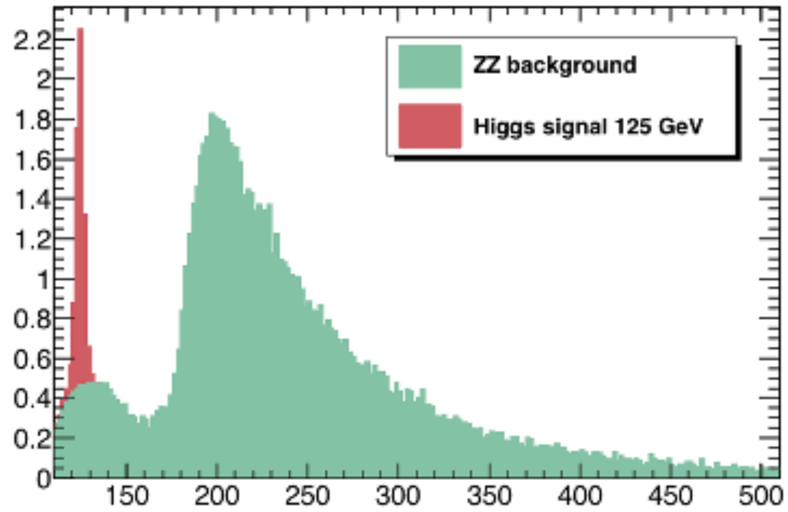

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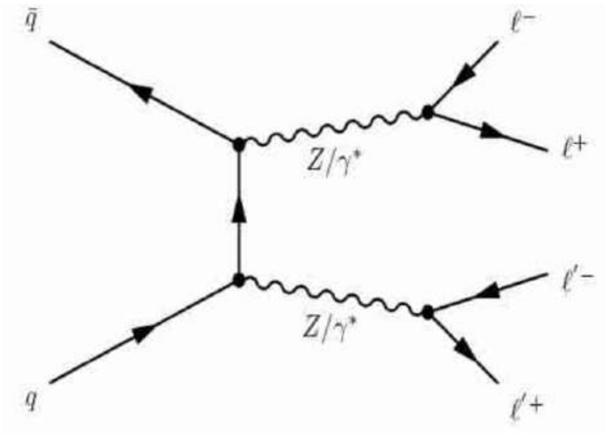
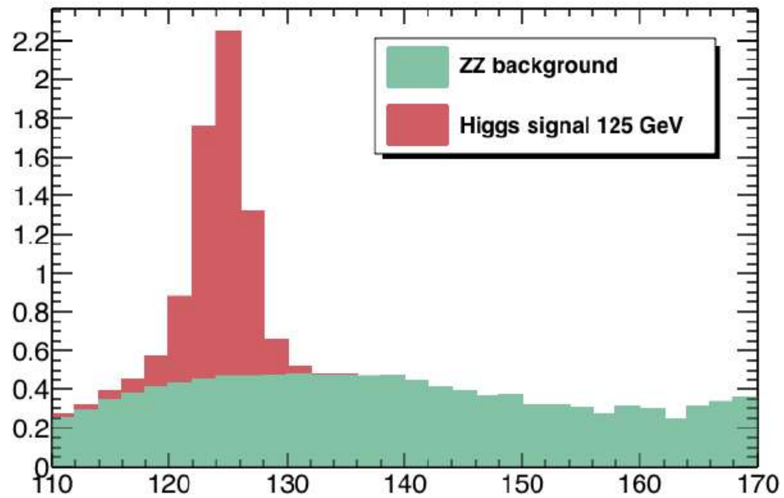
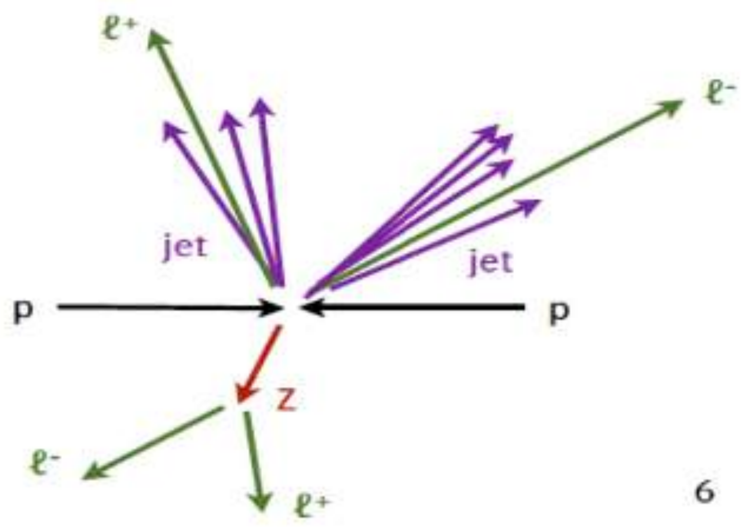


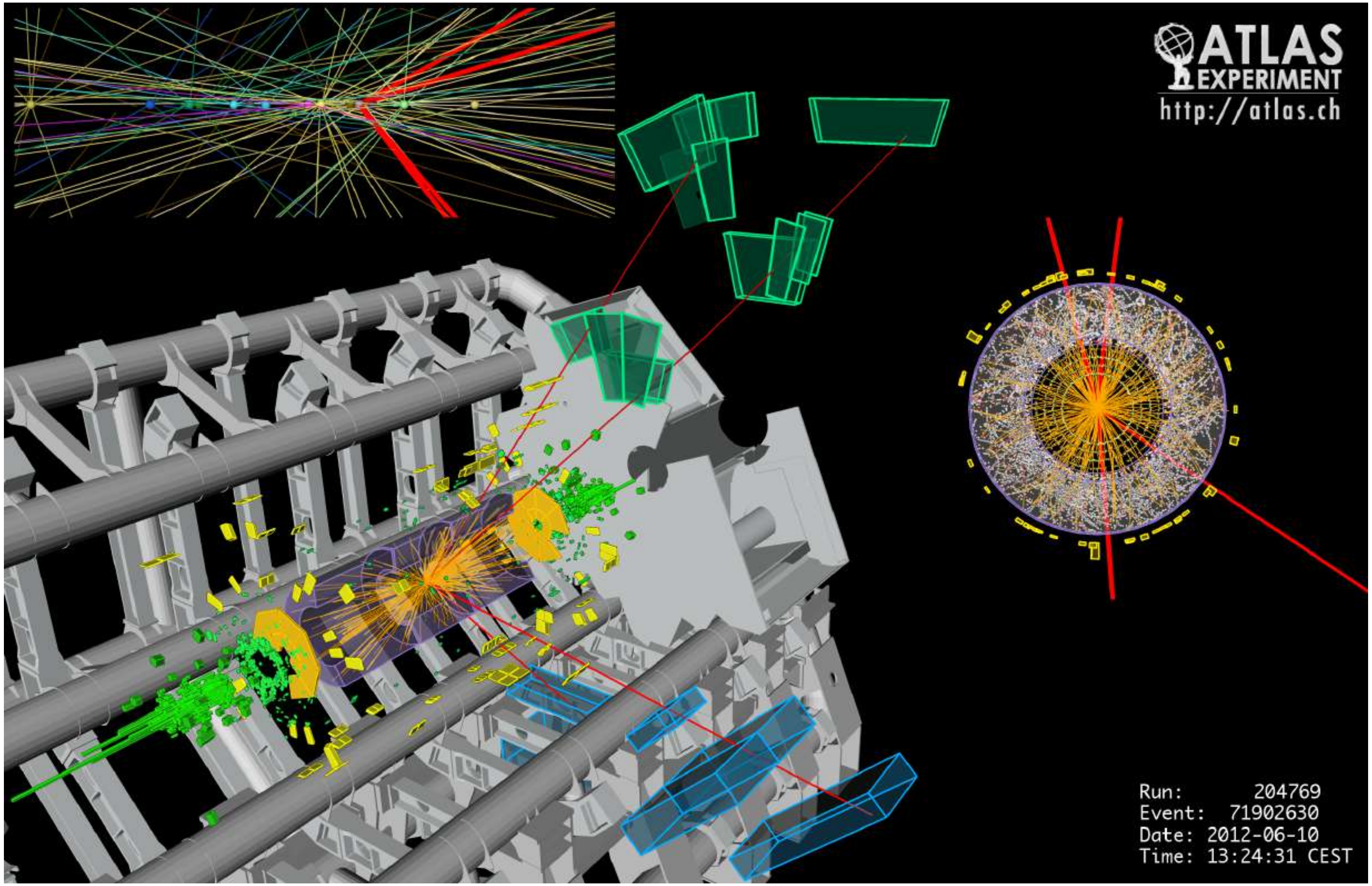


Reducible backgrounds

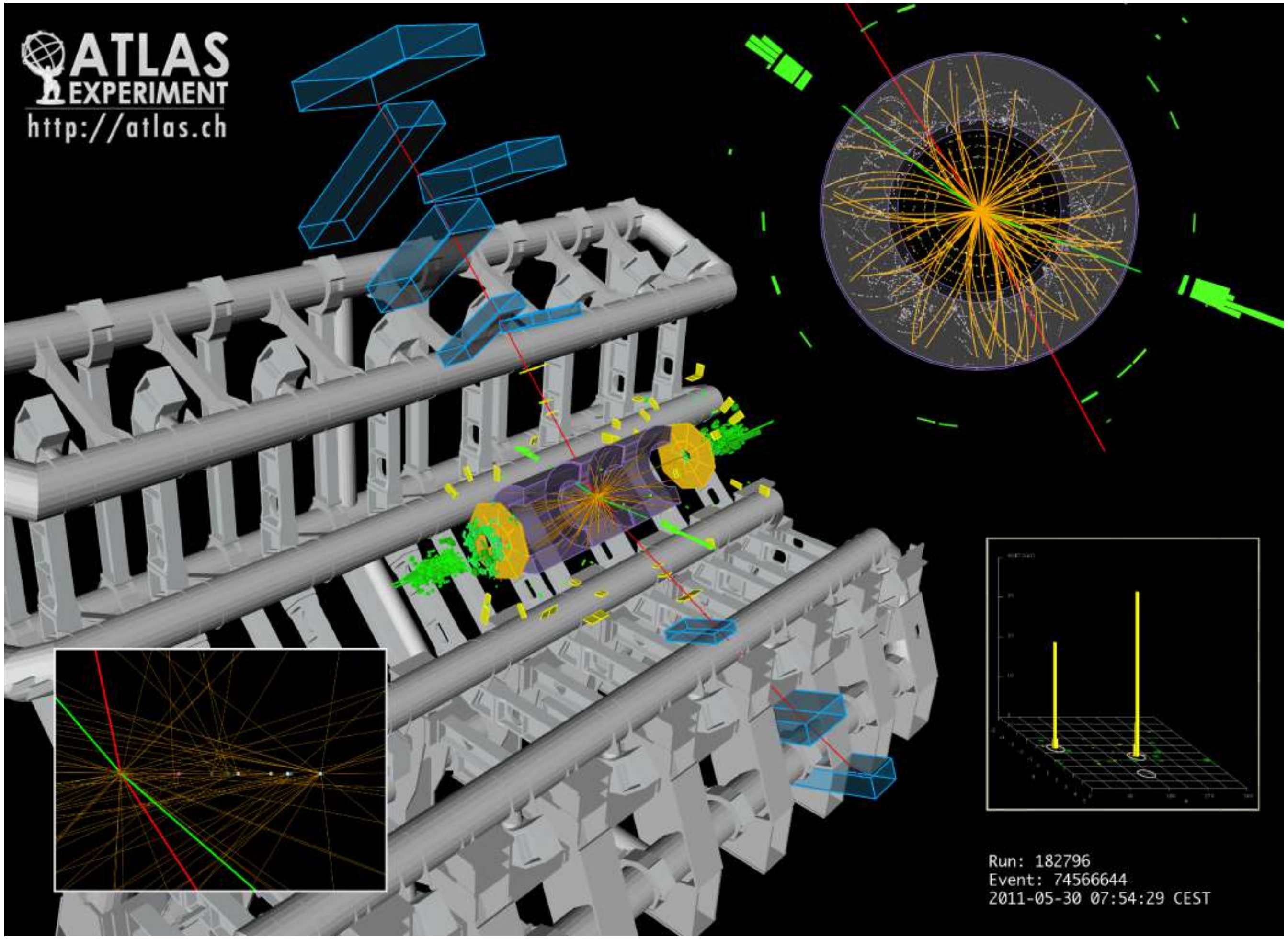
- In this case the final state is not exactly the same as for the signal, but there are objects that can "fake" the signal signature
- For H→4l the main reducible background is a single Z produced in association with two jets
- In case of jets from light-quarks (not b or c):
  - possible misidentification of the
  - decay in-flight of pions and kaons within the jet ($\pi \rightarrow \mu\nu$ $K \rightarrow \mu\nu$) that can produce a muon track
- Or, they can contain leptons produced via decays like:
  - $\pi \rightarrow \mu\nu$ $K \rightarrow \mu\nu$, easier to discriminate
  - decays of hadrons with heavy quarks
- All these occurrencies are very rare, and in general can be discriminated via cuts on the track impact parameter and on the isolation, but, the Z+jets cross section is ~ 5 orders of magnitude larger than the signal one



Some real events



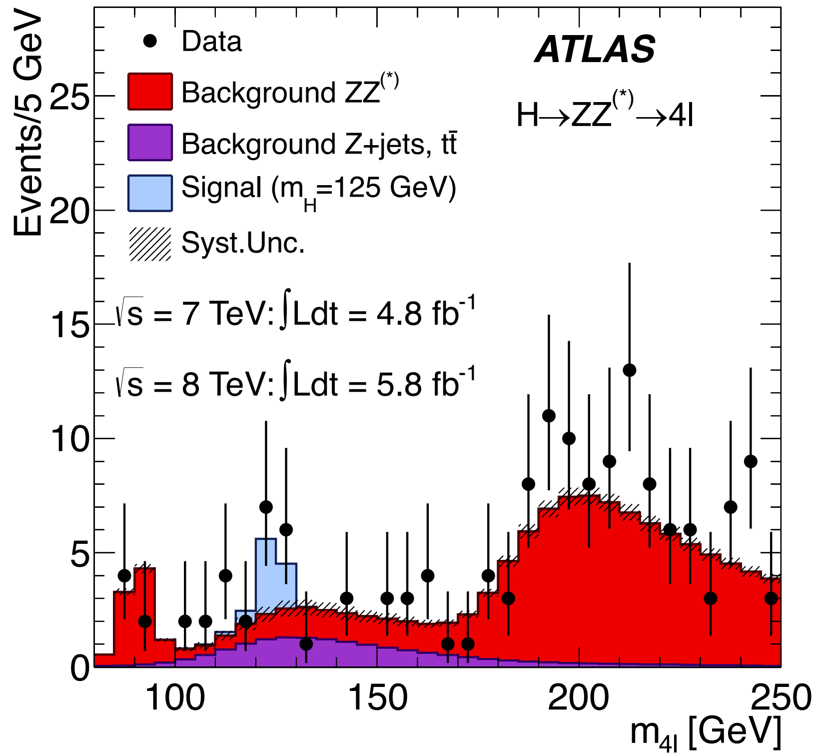
Some real events



July 2012: the discovery

- ❑ Number of events in a mass window around the signal peak 120-130 GeV

	Signal	$ZZ^{(*)}$	$Z+\text{jets}, t\bar{t}$	Observed
4μ	2.09±0.30	1.12±0.05	0.13±0.04	6
$2e2\mu/2\mu2e$	2.29± 0.33	0.80±0.05	1.27±0.19	5
$4e$	0.90±0.14	0.44±0.04	1.09±0.20	2



Total expected signal: 5.3 events

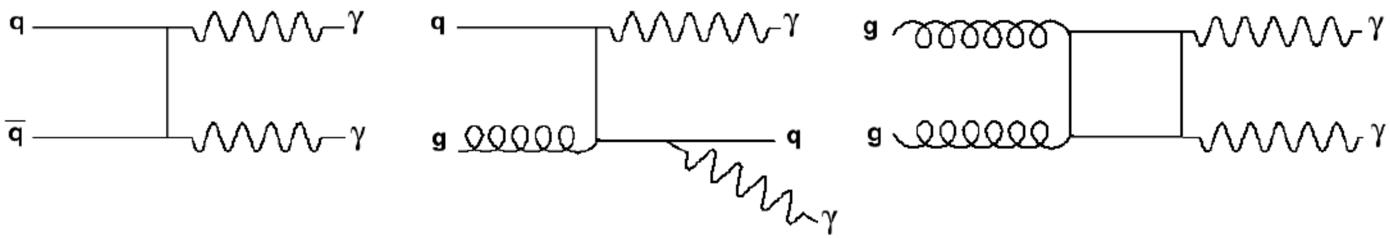
Total expected background: 4.9 events

Total observed events in data: 13 !

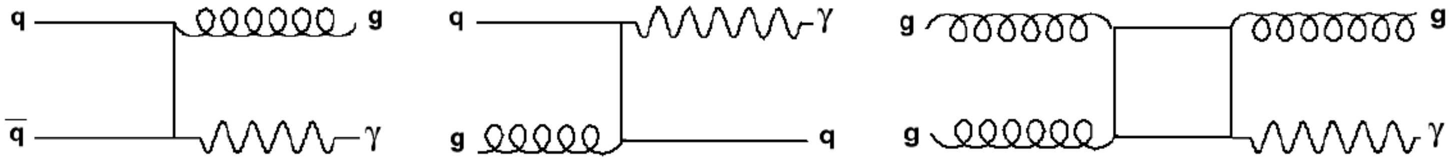
Already a simple calculation shows you something: probability to observe 13 events when you expect 4.9 is 0.17 % (of course no errors no syst here)

H→$\gamma\gamma$ selection

- In spite of the low BR (~0.2%) this channel has the clear signature of two energetic and isolated photons, with a narrow mass peak
- The main backgrounds are the irreducible 2-photons continuum:



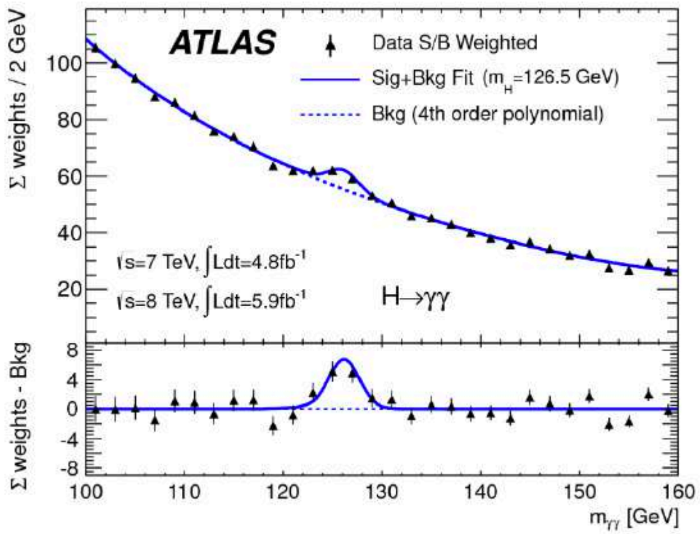
- And the +jet, or 2-jets, or 2-electrons, in which jets or electrons are misidentified as photons

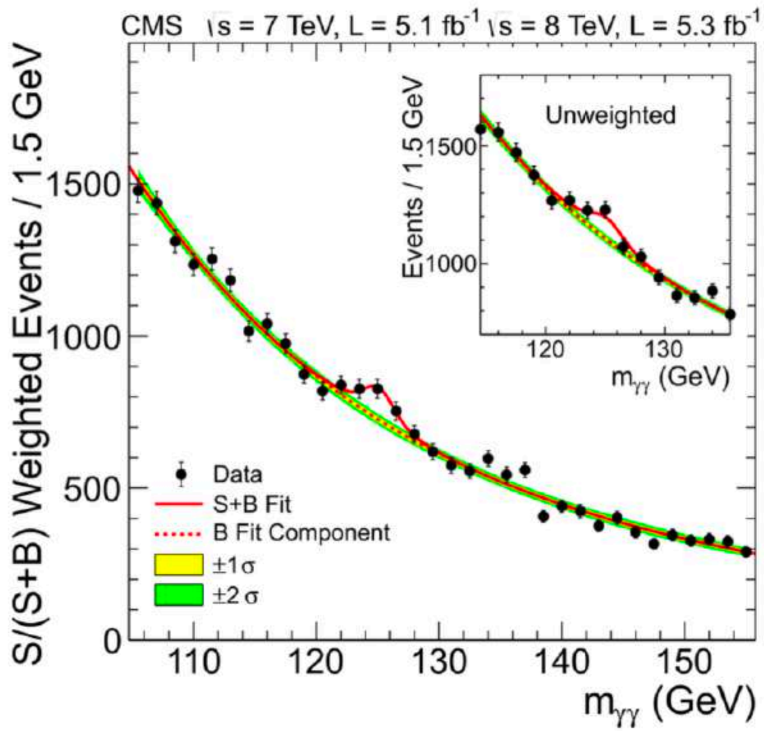


- The continuum background can be fitted with empirical functions (close to an exponential)

H→$\gamma\gamma$ results in July 2012

- ❑ In this case the background has to be fully extracted from a fit to the data distribution
  - ➢ The functions to be used in the fit can be studied on the MC, on a sample that contains a mix of th expected backgrounds
  - ➢ It's very important to avoid introducing fake signals with the fit
- ❑ Events are divided in categories based on number of jets, region of the photons
  - ➢ Purpose is to select the categories with the best S/B ratio, as this enhances the sensiti
  - ➢ categories optimized on MC+data





Exclusion or discovery ?

- Let’s consider the simple case of a counting experiment
  - One can always go back to this case selecting a narrow mass window around the signal peak
- First of all we need to consider the “expected” number of Signal (S) and Background (B)
  - These can be derived either from simulation (signal , and in some cases also the backgrounds), or from data (backgrounds)

- Then, we need to consider the number of events that pass the selection in data: N
- There are two possible cases:
  - N is close to B or smaller
  - N is close to S+B or larger
- Plus, all the cases in which N is somewhere in between B and S+B
- If we consider a mass window around the signal, and count the events in that window, our search becomes a counting experiment
- Let’s see how to deal with these cases in the frequentistic approach

Exclusion

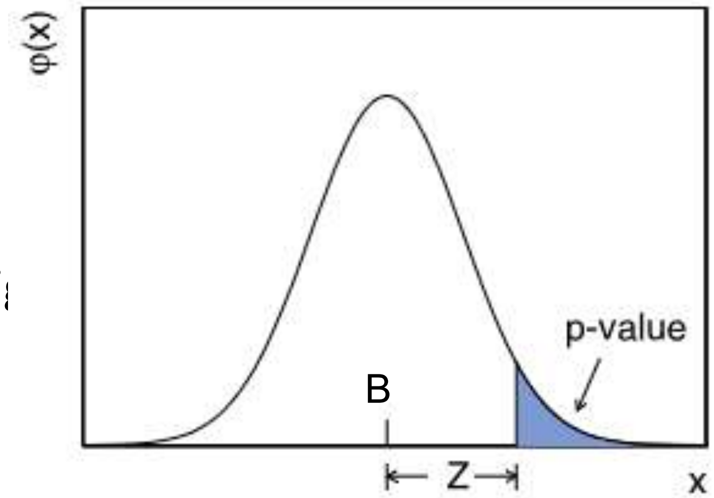
- If N is close to B, or smaller than that, we can probably exclude the presence of a signal, but how can we quantify the probability that a signal is excluded ?
- We must consider the hypothesis of S+B expected events, i.e. that a signal exists, and calculate the probability to observe n≤N events.
  - P(n≤N|S+B)
- In other words, we calculate the probability that S+B can fluctuate down to N or lower: if this probability is low enough, we can exclude the presence of a signal (CL_S)
- To set a limit at α confidence level, the probability must be:

P(n≤N|S+B)<1- α

- So for example if α=95% (the value normally used), the probability must be <5%
- The probability distribution to be considered for n depends on how large S+B is

Discovery

- When the number of observed events is instead close to S+B, it's possible that we are observing a signal... But how to quantify the observation ?
- In this case we must consider the hypothesis of only background B, and calculate the probability that the number of events can fluctuate to N or a number larger than that:
  p = P(n≥N|B)
- The significance Z is defined as **$Z=\Phi^{-1}(1-p)$** where Φ is the cumulative of the standard gaussian
- In other words when p is equal to the 1-sided probability content of a gaussian, corresponding to the integral from B+ Z si… significance is equal to Z sigmas
  - It becomes $S/\sqrt{B}$ for a gaussian distribution: the probability that B fluctuates to S+B



- A discovery is normally claimed at 5 sigmas, but there are intermediate situations, i.e. in which one can neither exclude nor claim the discovery of a signal
  - Exactly the situation in which we were in December 2011 !

1-sided 5-sigmas corresponds to a p value of 3×10^{-7}

Likelihood ratio

- The actual significance calculation is actually more complex, but based on the same principle of testing the (in)compatibility of the data with an hypothesis
  - Only background hypothesis for discovery
  - Signal+background hypothesis for exclusion
- Use distributions instead of pure counting experiment (binned or unbinned)
- Introduce systematic uncertainties
  - For example, one might have an uncertainty on the normalization of a given background, or on its shape and composition
  - Or, the uncertainties could come from detector performance, for example, how well do I know the efficiency of electron/muon/photon reconstruction?

- In this cases one introduces a set of “nuisance parameters” $\boldsymbol{\theta}$ (a vector) each component corresponding to a given systematic effect

Likelihood ratio

- So as a function of μ and $\boldsymbol{\theta}$ one can write a likelihood:

$$L(\mu, \boldsymbol{\theta}) = \prod_{j=1}^{N} \frac{(\mu s_j + b_j)^{n_j}}{n_j!} e^{-(\mu s_j + b_j)}$$

i.e. the product of the poisson probabilities over all bins of the distribution. μ is a scale factor for the signal, i.e. μ=1 means a SM signal

- To test a value of μ, letting the syst vary one can write the profile likelihood ratio

$$\lambda(\mu) = \frac{L(\mu, \hat{\hat{\boldsymbol{\theta}}})}{L(\hat{\mu}, \hat{\boldsymbol{\theta}})}$$

where $\hat{\hat{\boldsymbol{\theta}}}$ is the set of nuisance parameters that maximizes the likelihood function for a given value of μ (so the conditional set of nuisance parameters), that can be scanned

While the denominator is the maximum likelihood, obtained for the values $(\hat{\mu}, \hat{\boldsymbol{\theta}})$ fixed i.e. in this case the maximum is a normalization

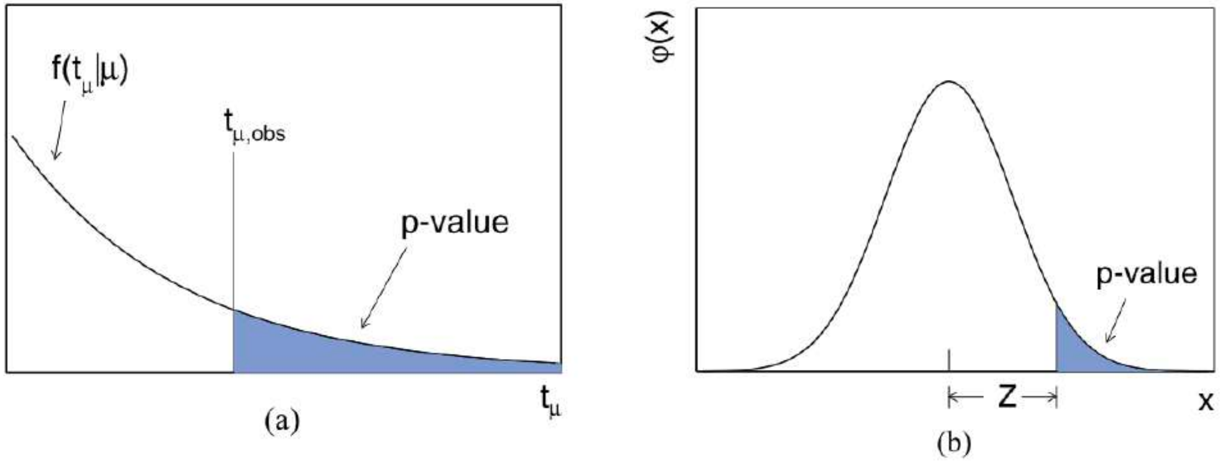
Likelihood ratio and significance

- So to calculate the discovery significance using the likelihood ratio one should refer to the case of background-only (as in the counting experiment) and use the "test-statistics": q0=-2lnλ(0) , or better:

$$q_0 = \begin{cases} -2\ln\lambda(0), & \hat{\mu} \geq 0, \\ 0, & \hat{\mu} < 0, \end{cases}$$

- Then the p-value to quantify the disagreement with the background-ony hypothesis is:

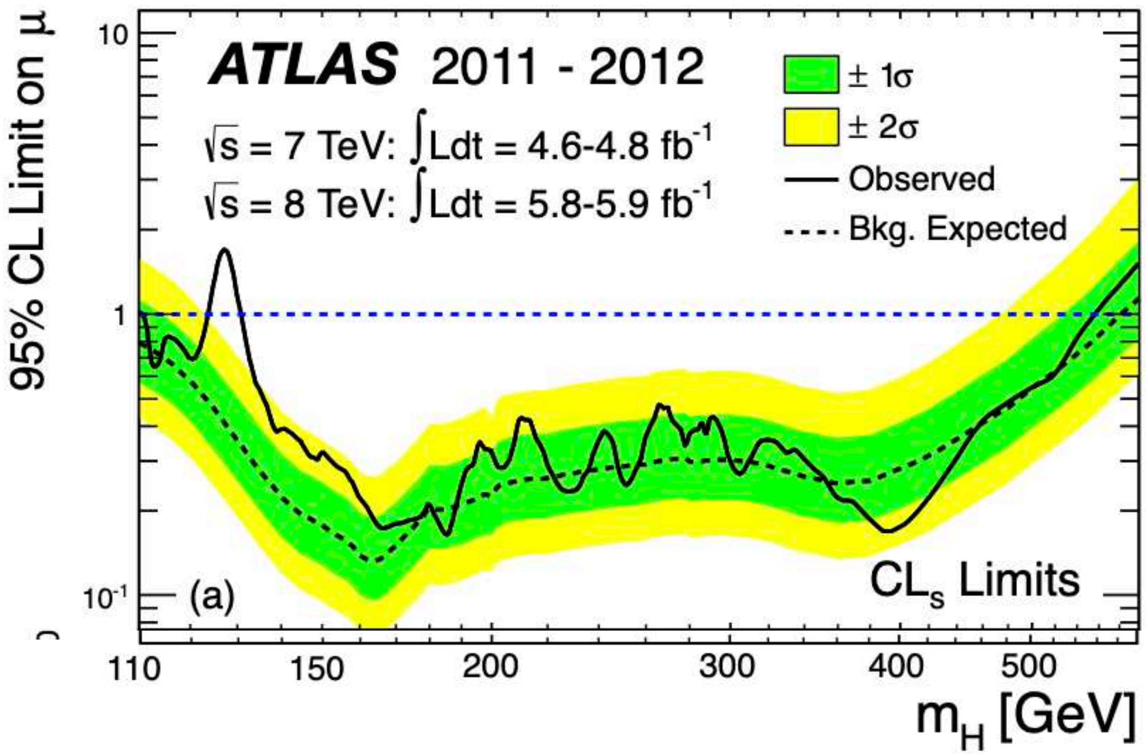
$$p_0 = \int_{q_{0,\mathrm{obs}}}^{\infty} f(q_0|0)\, dq_0$$



- The f(q0|0) can be analytically calculated using some approximations, or, derived from the simulation
- You can find more on that in: Eur. Phys. J. C (2011) 71: 1554

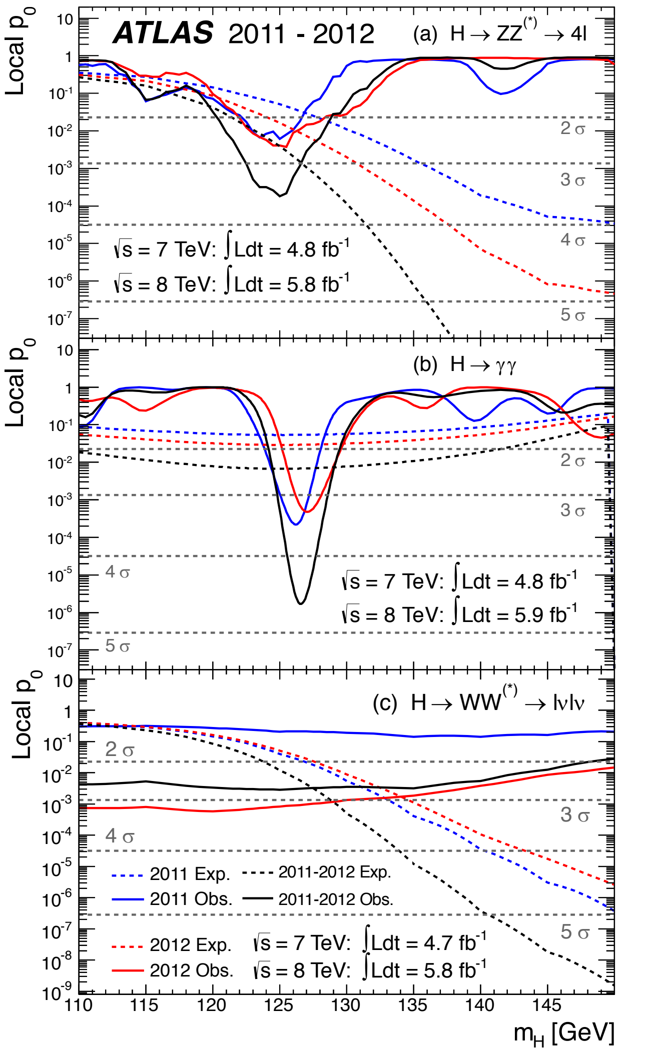
Exclusion plots

- The exclusion can be applied on the value of μ, i.e. say a value of μ about a given value is excluded at 95%



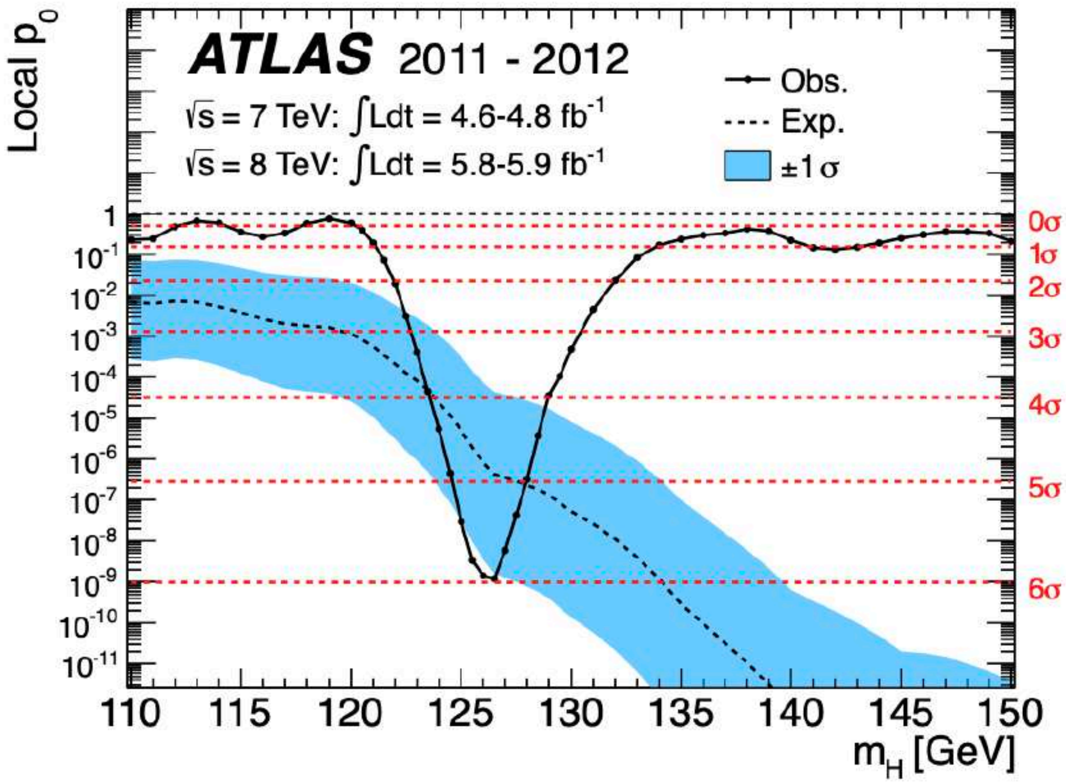
In fact, a clear region appears, where μ =1 can't be excluded

The actual discovery plots



p-values as a function of the tested Higgs mass

→ Discovery at mH ~ 125 GeV



Mass and Width

Mass measurements

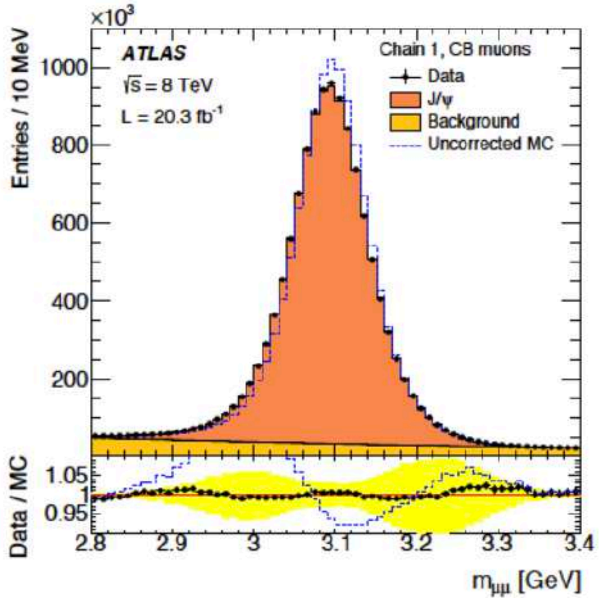
- You have seen the Z and W mass measurement techniques:
  - Z: measure the cross section at different e+e- center of mass energies
  - W mass: use direct reconstruction of the mass peak, or the W+W- cross section vs the center of mass energy

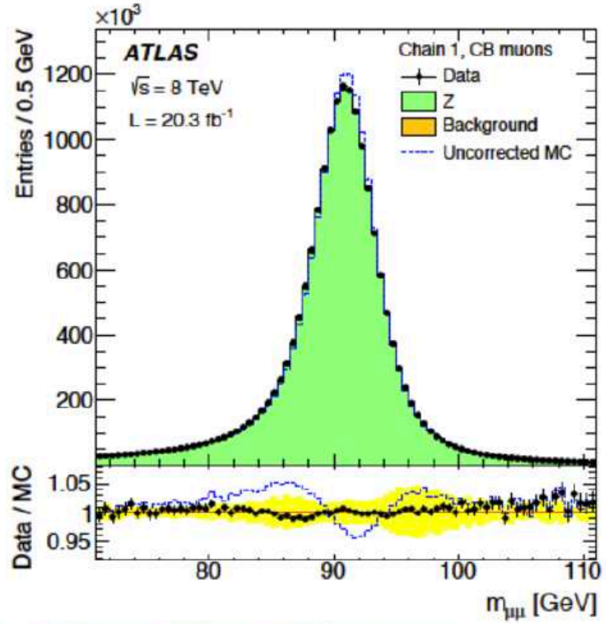
- In the case of the Higgs at LHC, the mass can be measured directly reconstructing the resonance invariant mass, in the channels where that's possible, in spite of their low BR
  - In the H → ZZ* → 4-leptons and H → $\gamma\gamma$ is visible as a peak on top of the background

- Crucial point is the calibration of the muons, electrons and photons energy scales
- In fact the detector response is affected by many experimental effects, that is usually difficult to model in the simulation
  - Detectors alignment, knowledge of the magnetic fields, active and passive materials

Calibrations for the 4-lepton channel

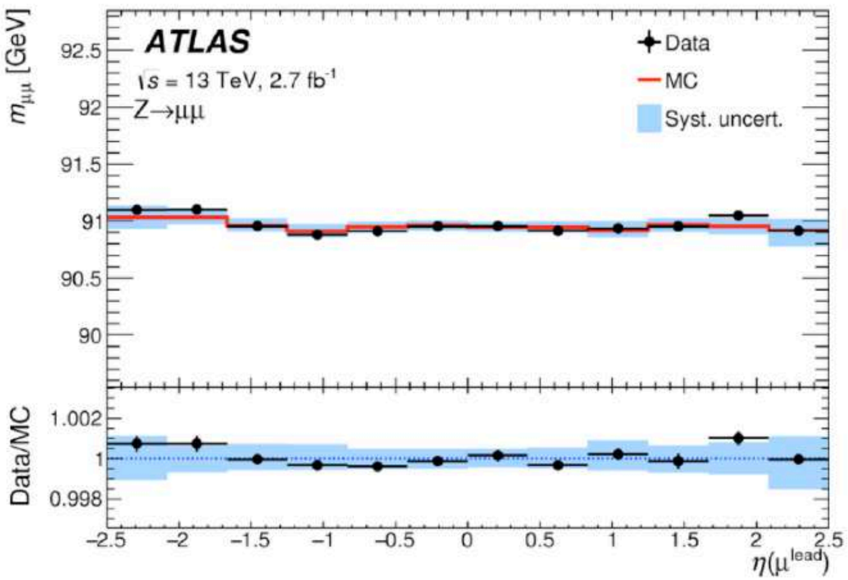
- ❑ Leptons energy scale and resolution need to be calibrated, to make sure that the simulation reproduces exactly the data
  - ➢ The peak of known resonances (Z, J/ψ) can be used on this purpose
- ❑ Get MC corrections in bins of η (one can also correct data)
  - ➢ scale: pT=s0+pT(1+s1)
  - ➢ resolution: add a smearing with sigmas Δr0 , Δr1 x pT
- ❑ The procedure is similar for muons and electrons

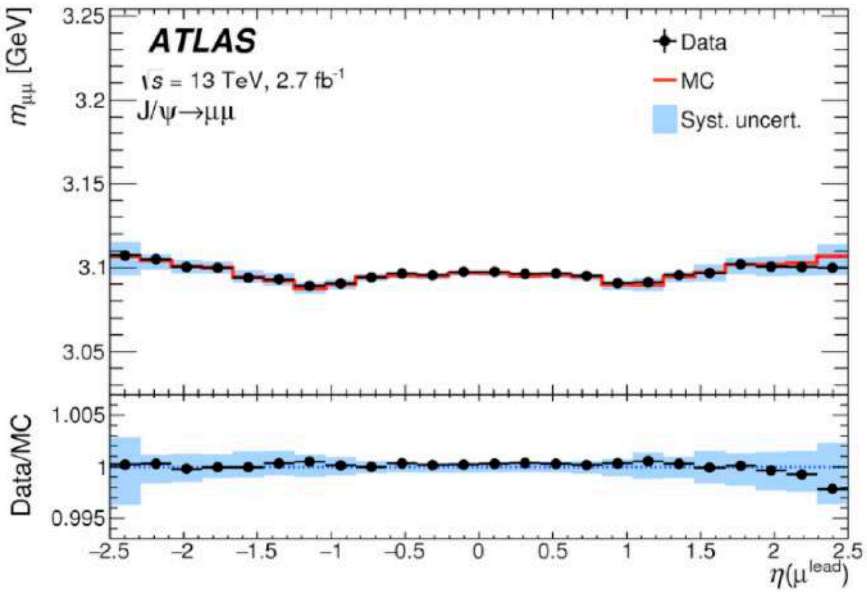




Systematics

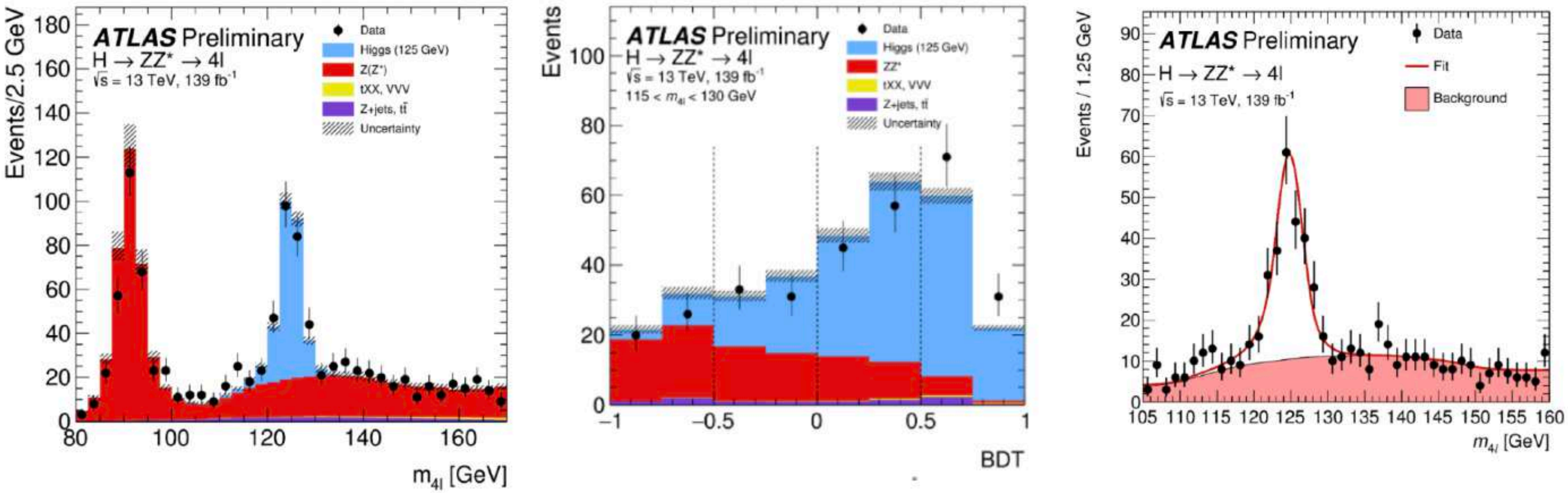
- Like every calibration, the momentum scale and resolution is affected by systematic uncertainties
- In the case of the 4-lepton channel, whose error is at the moment still dominated by the statistical component, systematics are not





4-lepton mass fit

- One lepton pair comes from an on-shell Z, so constraining their invariant mass to MZ leads to an improvement on the resolution
    - Likelihood fit constraining the di-lepton mass to the Z lineshape
    - Up to ~17% improvement in the resolution
- Recovering photons from final state radiations leads to a ~1% improvement
- Besides the 4l mass, more variables enter in the fit mass
    - Events categorized based on a BDT designed to separate the signal from the irreducible background
    - Event-by-event uncertainty on the 4l mass



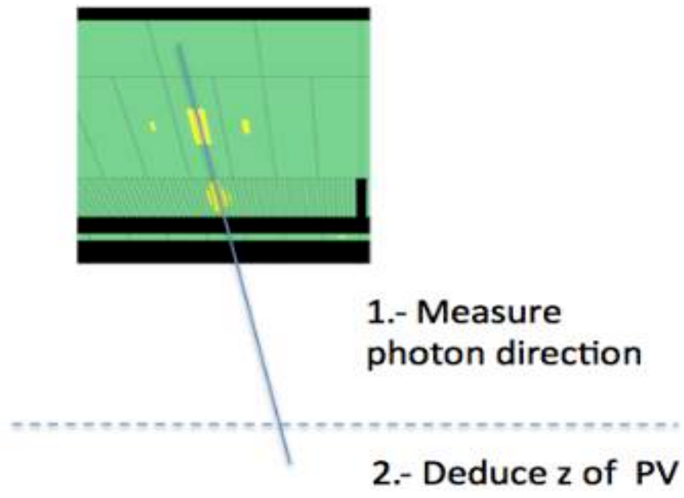
Mass in H→$\gamma\gamma$

- The resolution on the invariant mass depends on

$$m_{\gamma\gamma} = \sqrt{2\,E_1\,E_2\,(1-\cos\Delta\theta)} = 2\sqrt{E_1\,E_2}\sin\frac{\Delta\theta}{2}$$

$$\frac{\delta m_{\gamma\gamma}}{m_{\gamma\gamma}} = \left(\frac{1}{2}\frac{\delta E_1}{E_1}\right) \oplus \left(\frac{1}{2}\frac{\delta E_2}{E_2}\right) \oplus \left(\frac{\delta\sin(\frac{\Delta\theta}{2})}{\sin(\frac{\Delta\theta}{2})}\right)$$

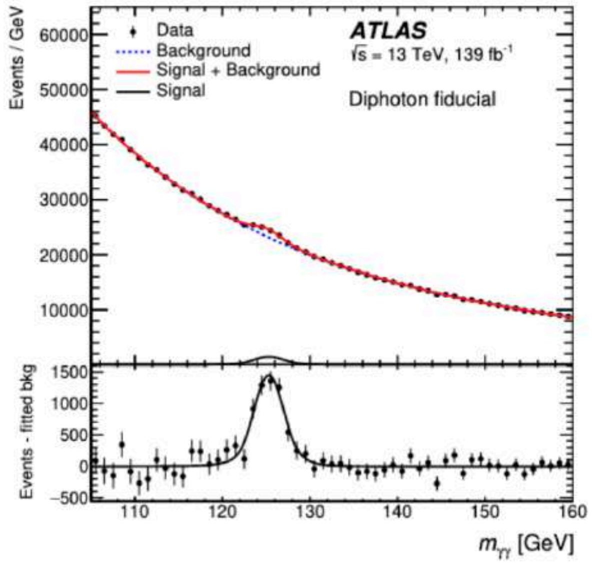
For the typical values of photons energy resolution (~ $3\%/\sqrt{E}$) and angular resolution (~ 5 mrad) one can see that the angular term is larger than the energy one → right vertex association is very relevant

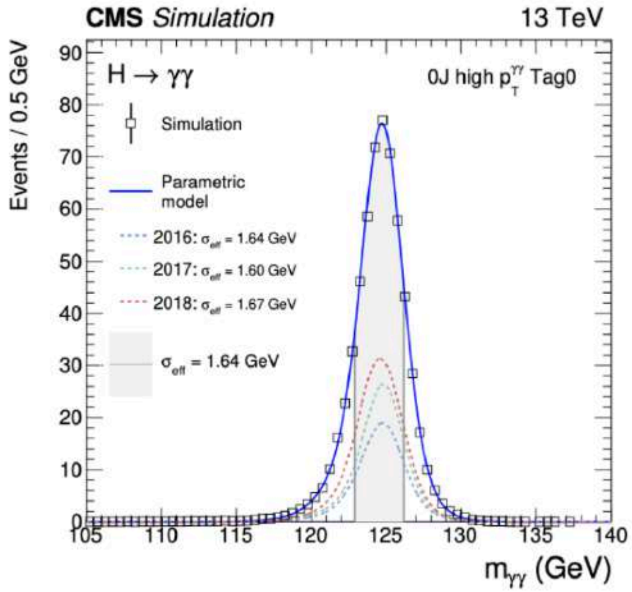


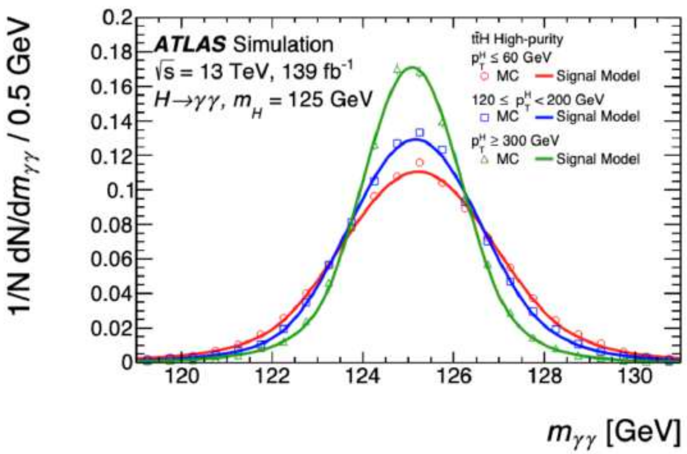
- Photon energy calibration more difficult than for the leptons, as no calibration resonance can be used
  - Still events like Z (or J/) → ee+γ , can be used, after having calibrated the electrons
- in general however the systematics on photon calibration are larger than those for electrons and muons

H→$\gamma\gamma$ mass fit

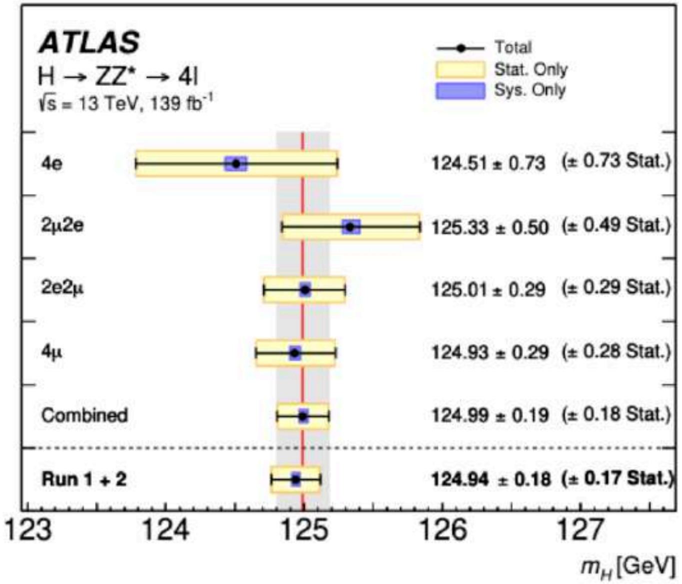
- Split selected events in categories (31) with different mass resolutions and S/B
- Background modeling (mainly γγ, γj, jj) with empirical functions
- Signal distribution models, either fit with double-sided Crystal Ball or with sums of gaussians
- Systematics mostly from E calibration

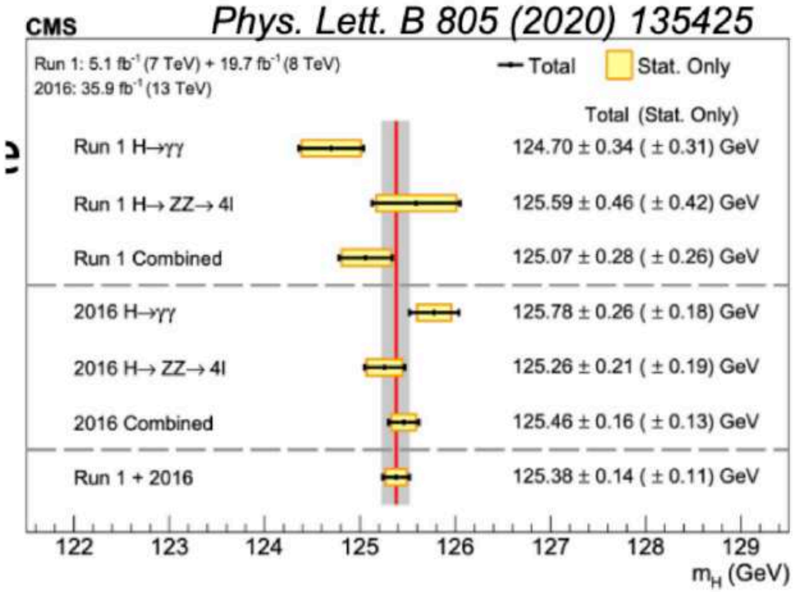


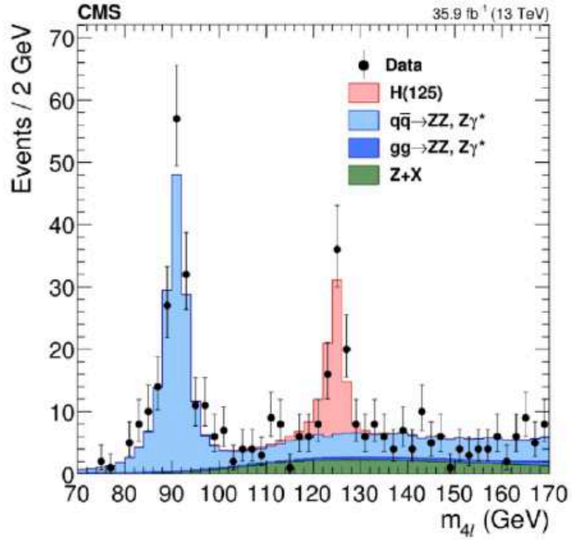




Summary of the current results on the mass







ATLAS 4l with 139 fb-1 at 13 TeV

m_H=124.94±0.17(stat)±0.03(syst)

CMS combined 4l and γγ

$$m_H = 125.38 \pm 0.11\ (\text{stat.}) \pm 0.08\ (\text{syst.})\ \text{GeV}$$

ATLAS γγ with 36 fb-1 at 13 TeV

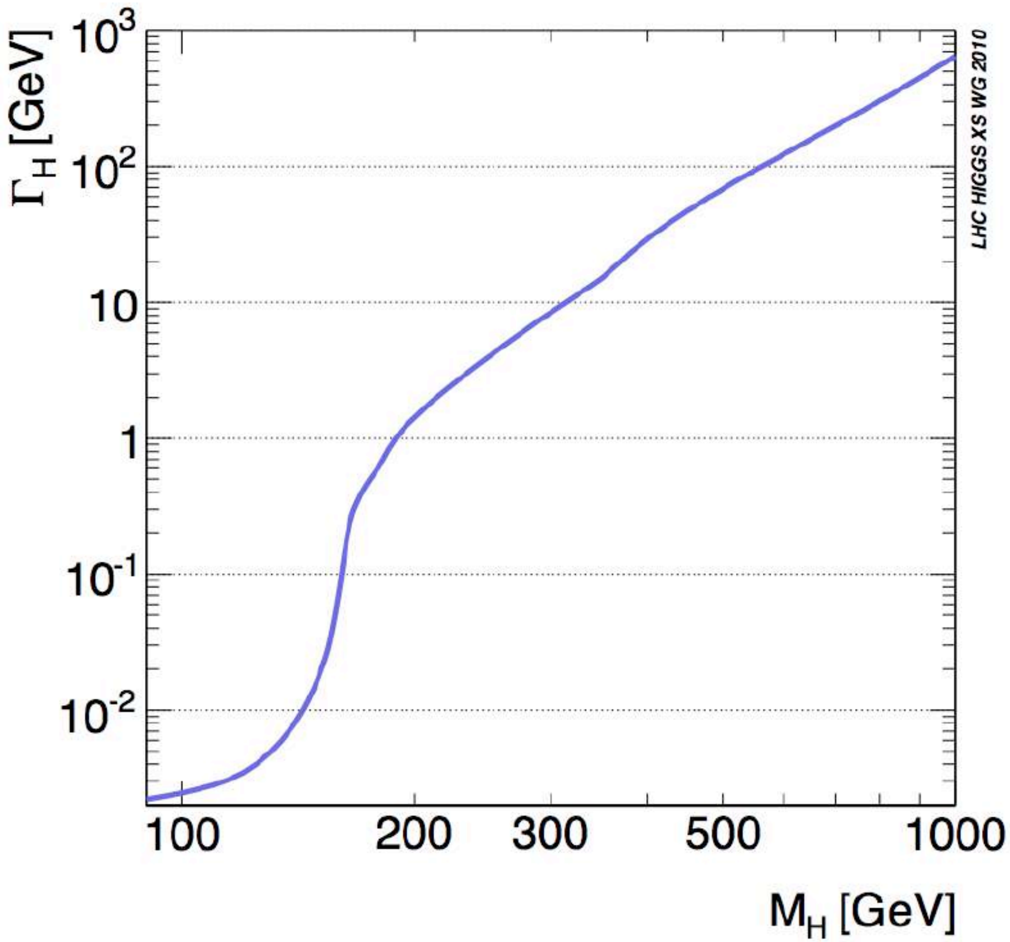
m_H=125.32±0.19(stat)±0.29(syst)

Width measurement

- In the SM, the width of the Higgs at 125 GeV is only 4 MeV

- We have seen that the experimental resolutions on the Higgs mass are of the order of the GeV, so three order of magnitudes
  - ➢ No way to directly measure the width
  - ➢ only limits very far from the SM value can be set

- But, there is a way to indirectly measure the width, looking at the off-shell signal strength



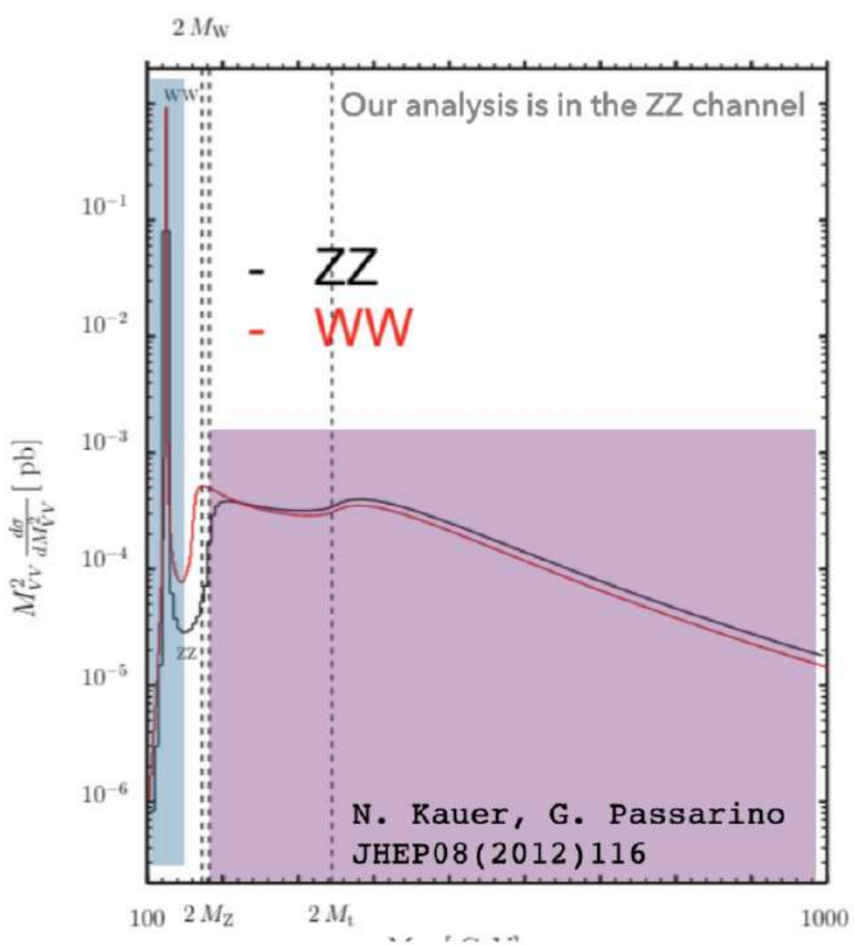
Width measurement

- ❑ Better limits on the width can be set using the off-shell Higgs signal strength
- ❑ The two most sensitive channels are H→ZZ*→4l and H→WW→lvlv
- ❑ Some assumptions need to be done:
  - ➢ Same couplings for the on-shell and off-shell Higgs
  - ➢ No new resonance exists in the region below ~2 TeV (as confirmed by direct searches)

$$\sigma_{onshell}^{pp\to H\to ZZ} \sim \frac{g_{Hgg}^2\, g_{HZZ}^2}{m_H \Gamma_H}$$

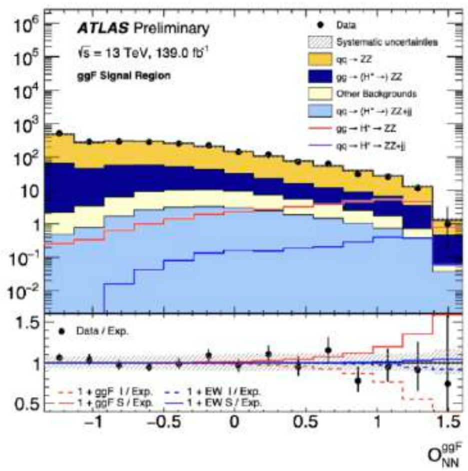
$$\sigma_{offshell}^{pp\to H\to ZZ} \sim \frac{g_{Hgg}^2\, g_{HZZ}^2}{M_{ZZ} - m_H}$$

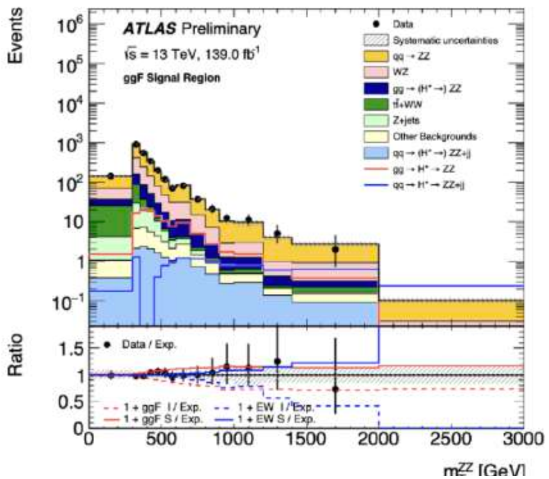
$$\frac{\sigma_{offshell}}{\sigma_{onshell}} \propto \Gamma_H$$

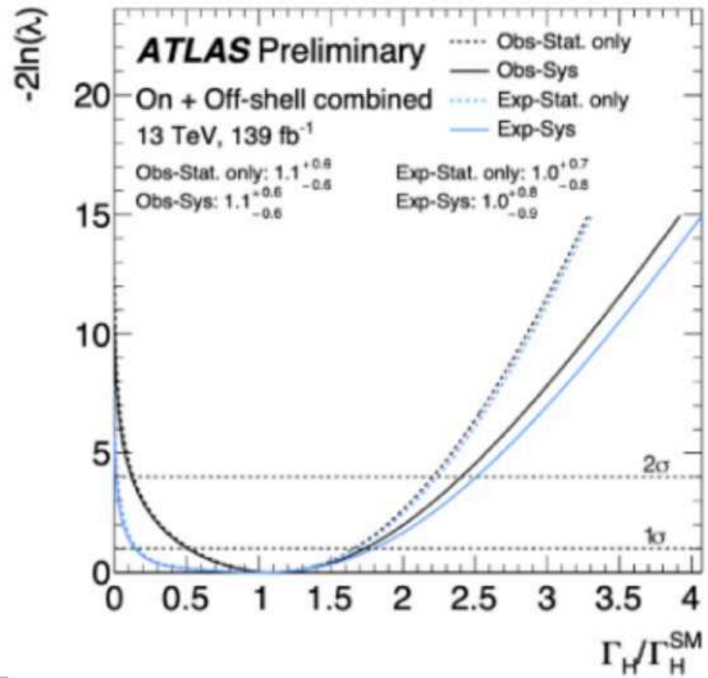


Example of width results

- Combining the 4l and the llνν channels from ZZ decays
- DNN discriminant and transverse mass to discriminate the signal from the backgrounds



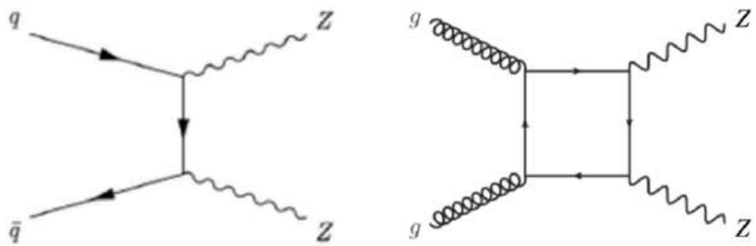




$$m_T^{ZZ} \equiv \sqrt{\left[\sqrt{m_Z^2 + (p_T^{\ell\ell})^2} + \sqrt{m_Z^2 + (E_T^{miss})^2}\right]^2 - \left|\vec{p}_T^{\,\ell\ell} + \vec{E}_T^{miss}\right|^2}$$

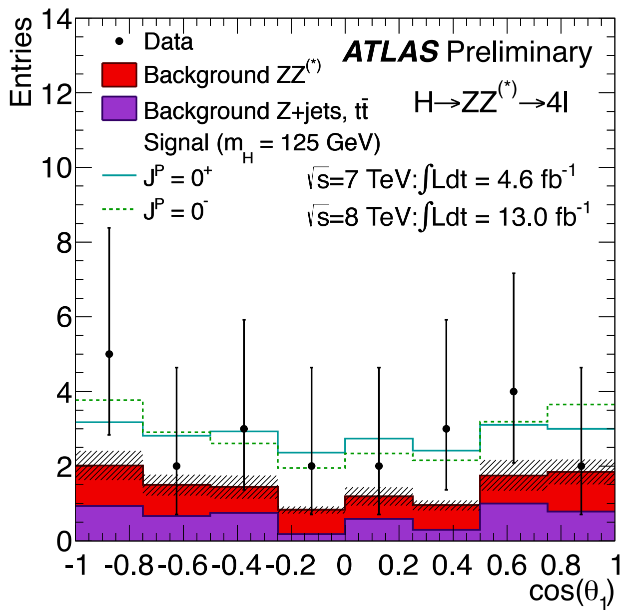
$$\Gamma_H = 4.6^{+2.6}_{-2.5}\ \mathrm{MeV}$$

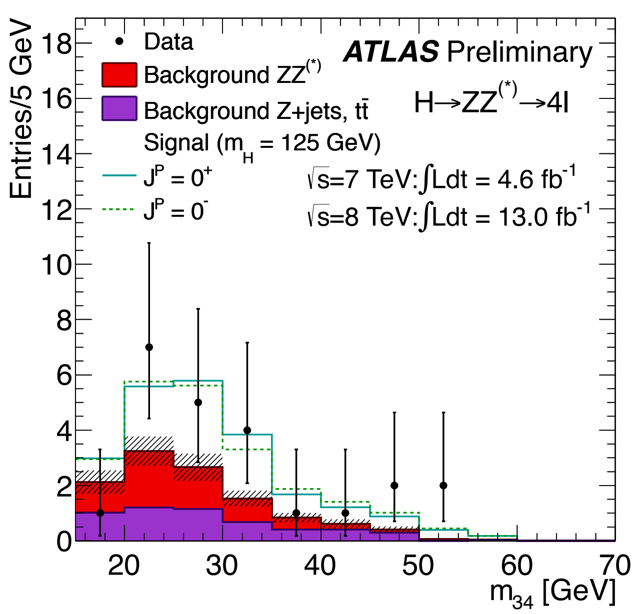
Main backgrounds

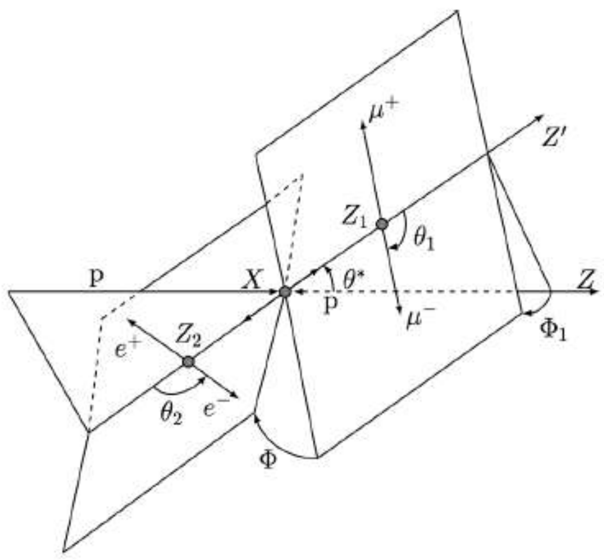


Spin and parity

- Just after the discovery, the spin and parity quantum numbers had to be measured to verify the scalar hypothesis (0+)
- For this, the angular distributions of the decay products in H→ZZ, H→WW and H→$\gamma\gamma$ have been used





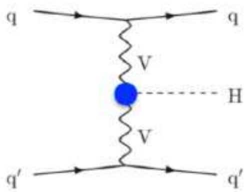


Higgs Couplings

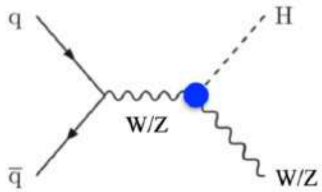
Higgs couplings

- Gauge couplings to the vector bosons

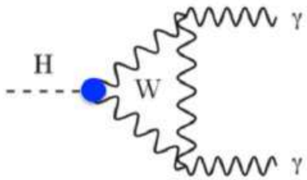
VBF Production



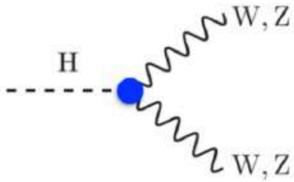
Associated Production



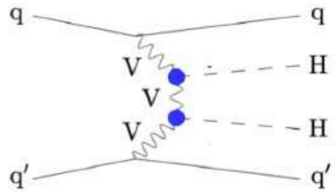
γγ decay



VV decay

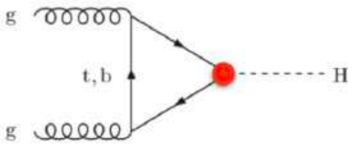


VBF pair production

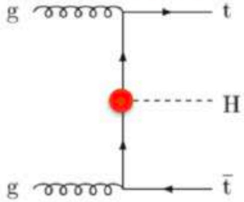


- Yukawa couplings to fermions

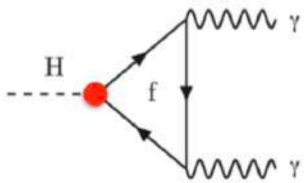
ggF production



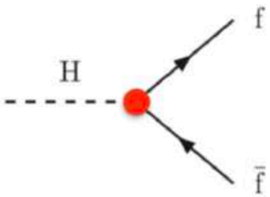
ttH production



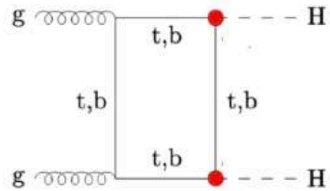
γγ decay



fermion-antifermion decay

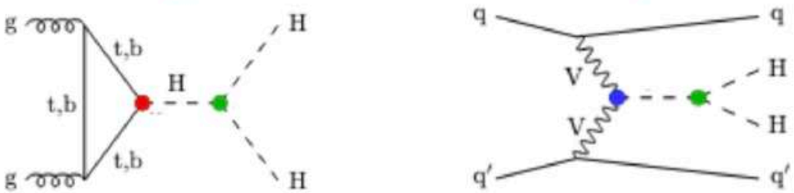


ggF pair production

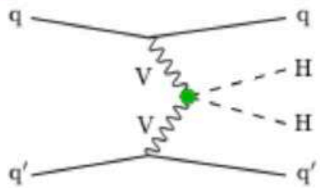


- Higgs self coupling and VVHH coupling

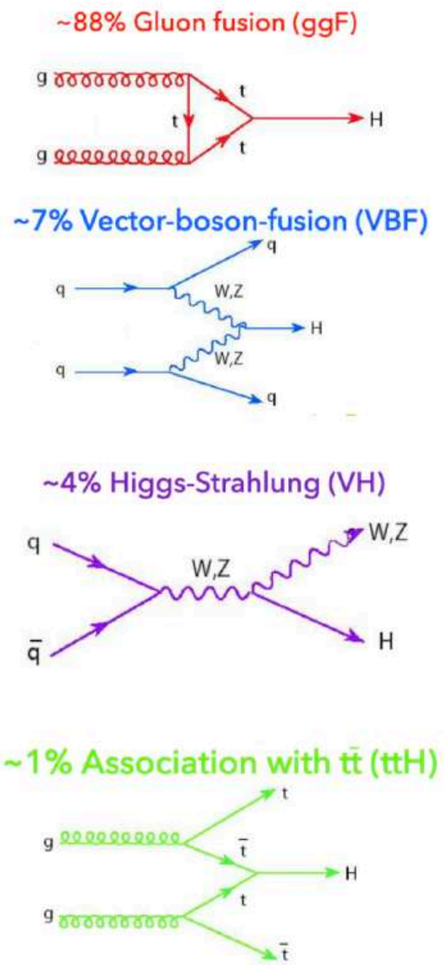
Higgs-pair production via self-coupling



Higgs-pair production via VVHH coupling



Identifying the production modes



- ❑ ggF is the dominant production mechanism
  - ➢ use carachteristics of the other production modes can be used to disentangle them
- ❑ VBF
  - ➢ Select events with two jets in the forward regions, and with large eta separation
- ❑ VH Associated production (or Higgs-Strahlung)
  - ➢ look for the decay products of the W,Z boson
  - ➢ typically leptons (e,mu) to avoid large background contaminations
- ❑ ttH is a complex final state
  - ➢ can be caracterized by the presence of b-jets from top decays
  - ➢ also, high multplicity events
- ❑ None of these selections is perfect→ always some level of contamination

Signal strength

- An observable that is very important for the couplings measurements is the signal strength
  - Defined as the ratio between the measured and the SM cross section times BR

$$\mu_i^f = \frac{\sigma_i \times \mathrm{BR}_f}{(\sigma_i \times \mathrm{BR}_f)_{\mathrm{SM}}} \equiv \mu_i \times \mu_f, \quad \text{with} \quad \mu_i = \frac{\sigma_i}{(\sigma_i)_{\mathrm{SM}}} \quad \text{and} \quad \mu_f = \frac{\mathrm{BR}_f}{(\mathrm{BR}_f)_{\mathrm{SM}}}$$

- Thus the number of events per channel can be defined as:

$$n_s^c = \sum_i \sum_f \mu_i (\sigma_i)_{\mathrm{SM}} \times \mu_f (\mathrm{BR}_f)_{\mathrm{SM}} \times A_{if}^c \times \varepsilon_{if}^c \times \mathcal{L}^c$$

- Measuring the event yields per production mode and decay channel allows to look for possible deviations from the SM (μ's different from 1)

Signal strength and κ-framework

Signal strength

- The first characterisation of the Higgs couplings properties happens via the measurement of the signal strength
  - Can fit a global μ or fit μ_i assuming SM for the decay, or fit μ_f assuming SM for the production cross section

$$\mu_i^f = \frac{\sigma_i \times \mathrm{BR}_f}{(\sigma_i \times \mathrm{BR}_f)_{\mathrm{SM}}}$$

  - Global μ fit results:
    - ATLAS **μ = 1.05 ± 0.06 = 1.05 ± 0.03(stat) ± 0.03(exp) ± 0.02(bkg th) ± 0.04(sig th)**
    - CMS **μ = 1.002 ± 0.057 = 1.002 ± 0.029(stat) ± 0.033(syst) ± 0.036(sig th)**

κ-framework

- In this framework a set of coupling modifiers κ alter the signal strength, without affecting the kinematic distributions
- Each σ x BR can be parametrized in terms of these couplings strength modifiers
- Can also accomodate any non-SM invisible or undetected component

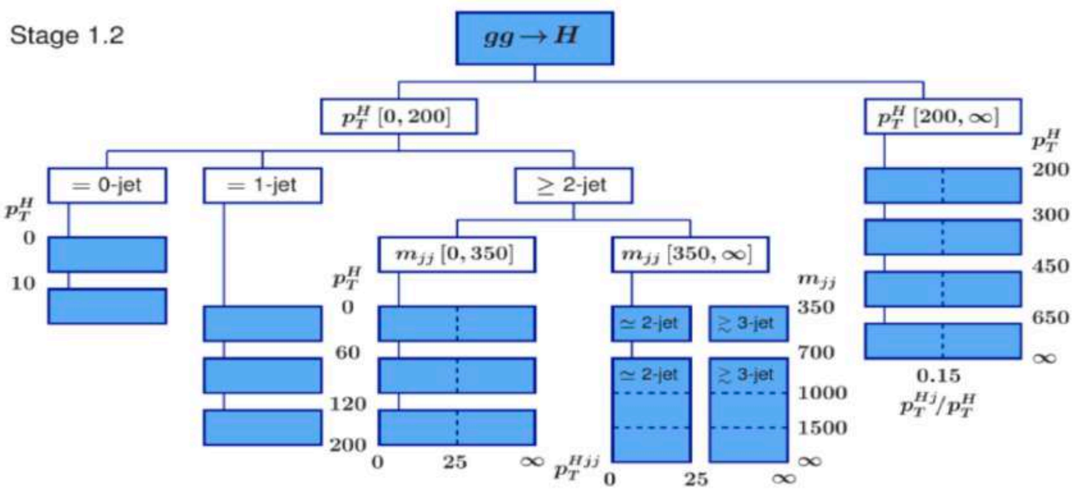
$$(\sigma_i \times B_f) = k_i^2 \sigma_i^{SM} \frac{k_f^2 \Gamma_f^{SM}}{k_H^2 \Gamma_H^{SM}}$$

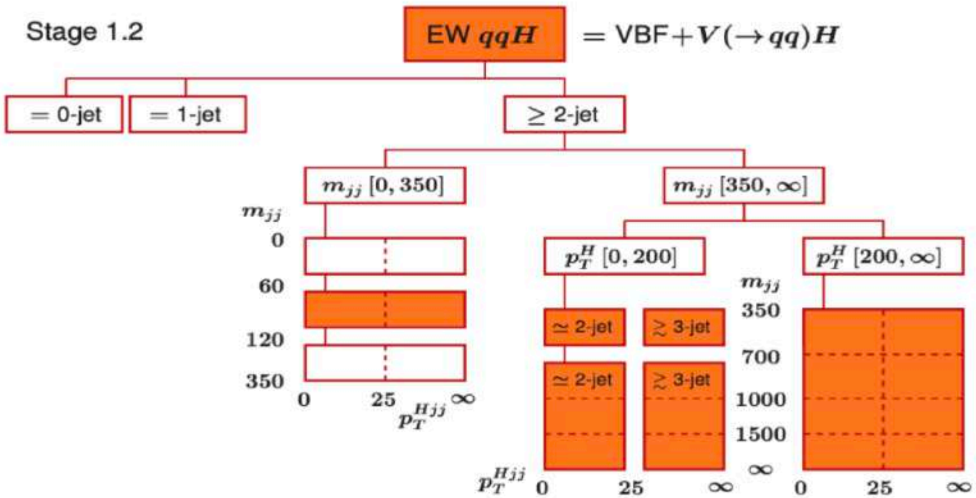
Simplified template cross sections (STXS)

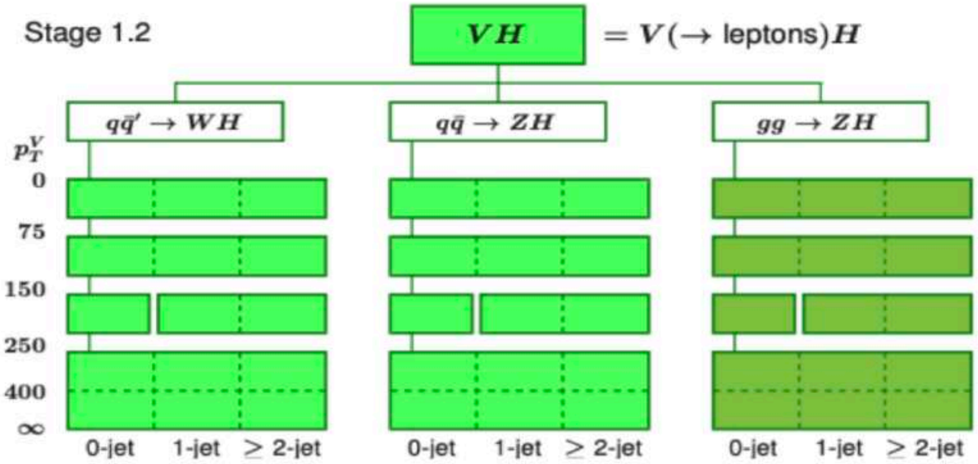
- Including this topic for completeness (but no need to go into the details…)

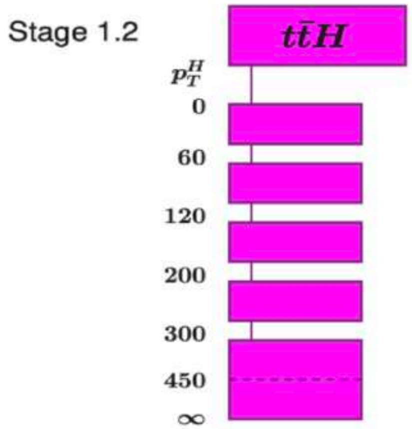
- The κ-framework introduces modification factors that do not alter the kinematic distribution with respect to the Standard Model
  - Only an overall normalization of each coupling contribution is left free to vary
- To allow looking for deviations of the differential distributions, the so-called STXS framework is used
- Measure signal strengths in bin of the truth quantities
  - Within each bin the SM behavior is assumed
- Use the same binning definition for all channels simplifies a lot the final combination

STXS bins









The analyses are designed trying to match the bins of truth quantities

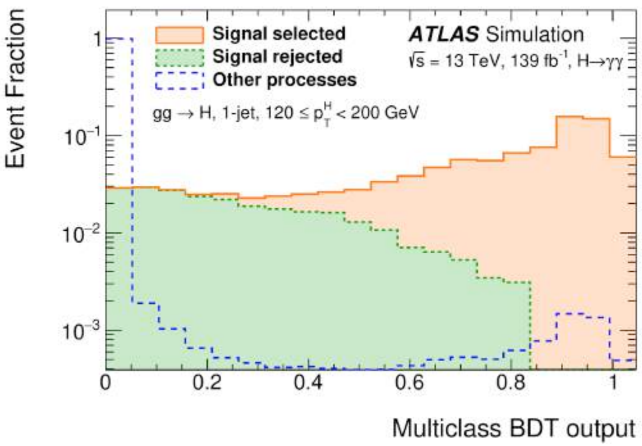
Different granularity --> can be chosen depending on the channel's statistics

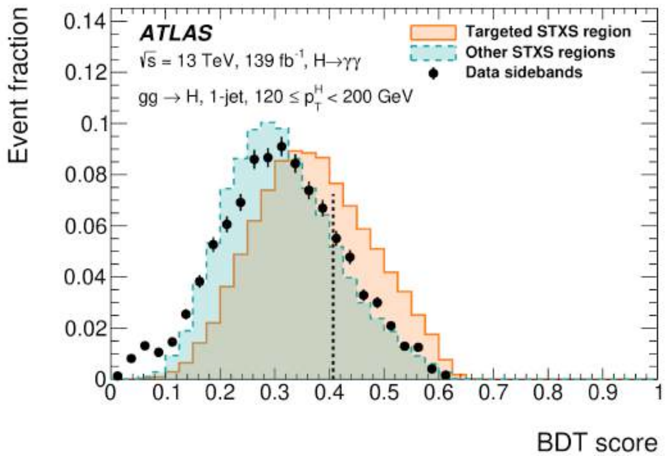
Main results with run-2 data

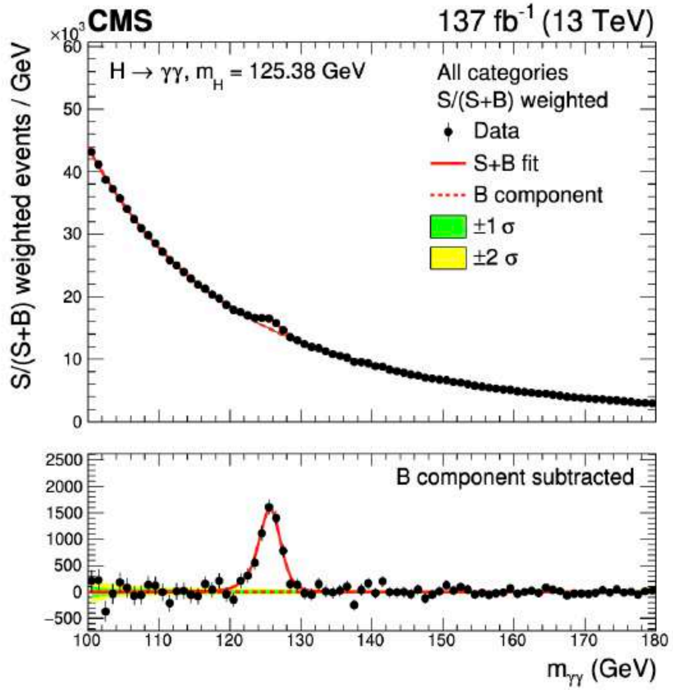
H→γγ

CERN-EP-2022-094
JHEP07 (2021) 027

- Analysis requiring two high momentum photons, background fit from data
- Machine Learning -based algorithms for event categorization
- Multi-class discriminant to categorize the events aiming at STXS bins
  - ➢ Better performance than just using $P_T(\gamma\gamma)$ and Njets
- Dedicated discriminants to separate signal from background in each STXS bin
  - ➢ Training on signal MC and background sidebands
  - ➢ Choosing variables in such a way that no bias on the $m_{\gamma\gamma}$ distribution can be caused
    - e.g. use $pT(\gamma)/m_{\gamma\gamma}$ instead of just $pT(\gamma)$





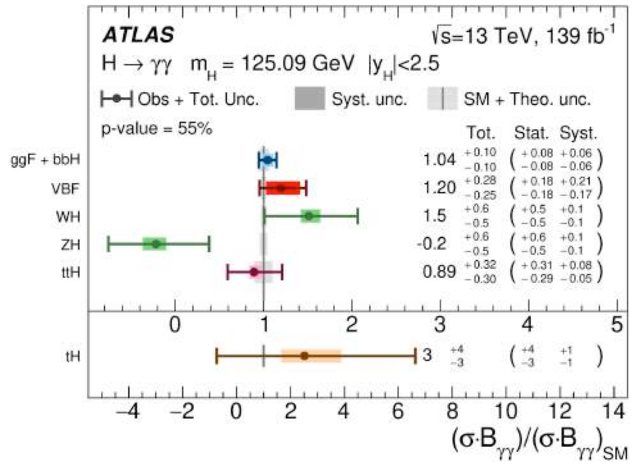


H→γγ signal strengths and STXS

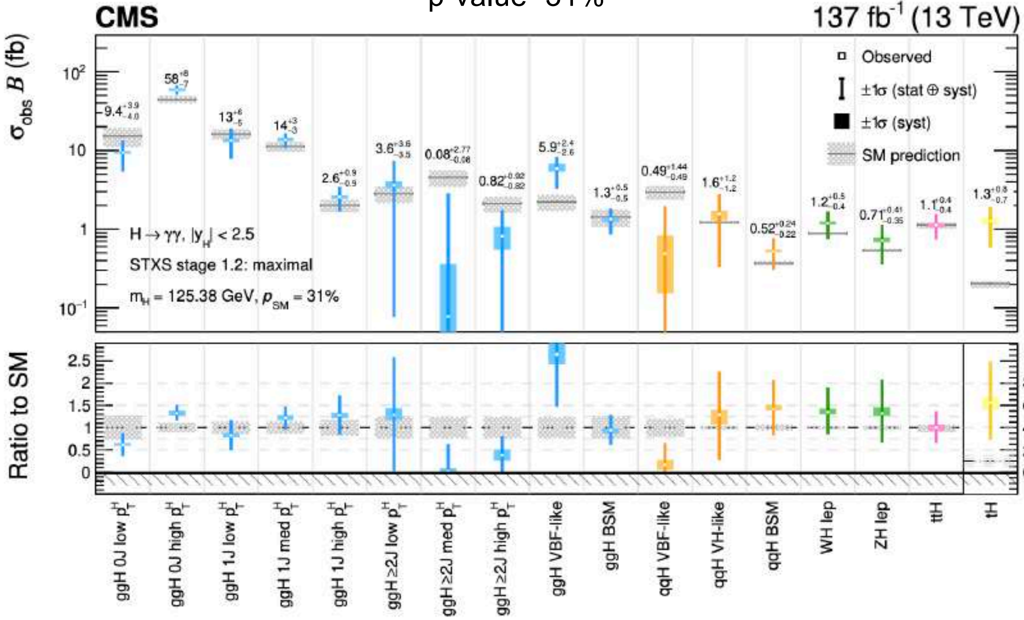
CERN-EP-2022-094
JHEP07 (2021) 027

- STXS number of parameters of interest can change depending on the merging scheme adopted,
  - Min merging scheme 28 bins, results from both ATLAS and CMS

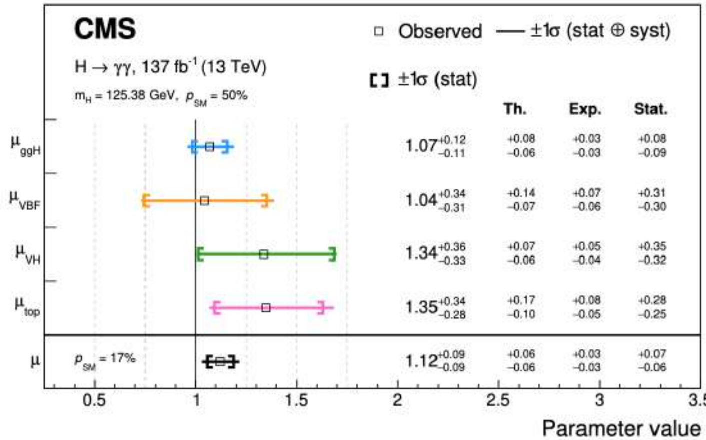
p-value=55%



p-value=31%



p-value=50%

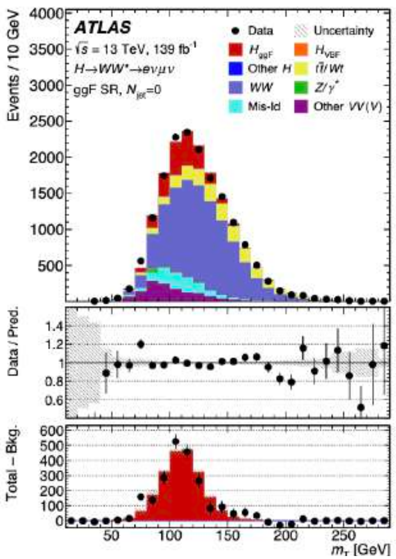


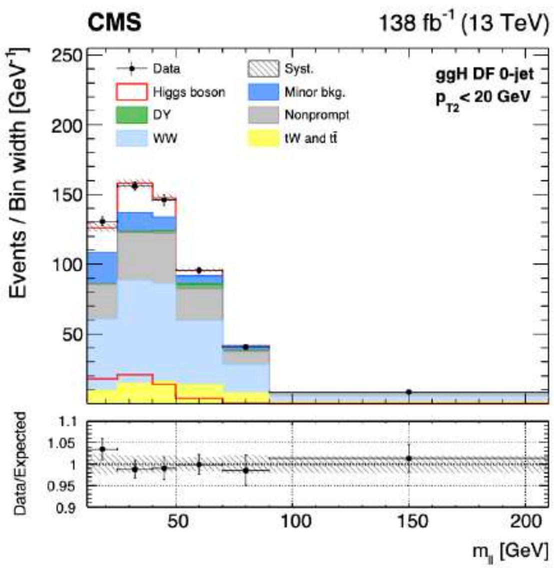
H→WW→lνlν

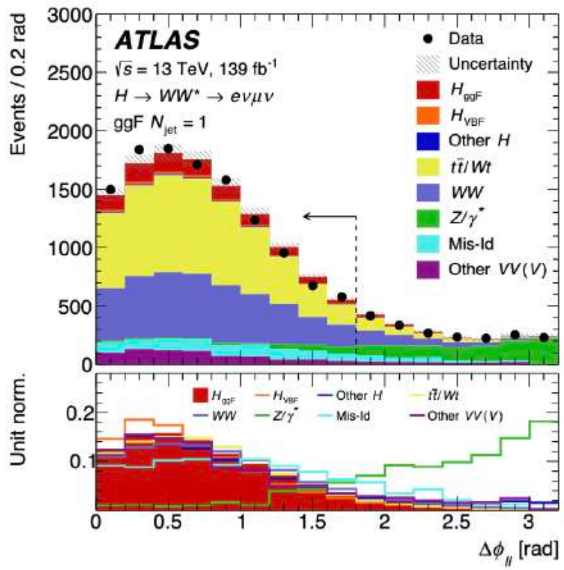
CERN-EP-2022-078
CERN-EP-2022-120

- Given the large BR, this channel is one of the most relevant in the couplings measurement
  - ➢ Analyses either Different-Flavor only (ATLAS) or Different+Same-Flavor (CMS)
  - ➢ Further categorization based on kinematic properties
  - ➢ Backgrounds mainly from control regions normalizations
- Signal strength extraction from the transverse mass, mll, or from the fit to ML-based discriminants (in particular for VBF and associated production channels)
  - ➢ Dedicated discriminant against DY in SF channels

$$m_T = \sqrt{\left(E_T^{\ell\ell} + E_T^{\text{miss}}\right)^2 + |\mathbf{p}_T^{\ell\ell} + \mathbf{E}_T^{\text{miss}}|^2}$$

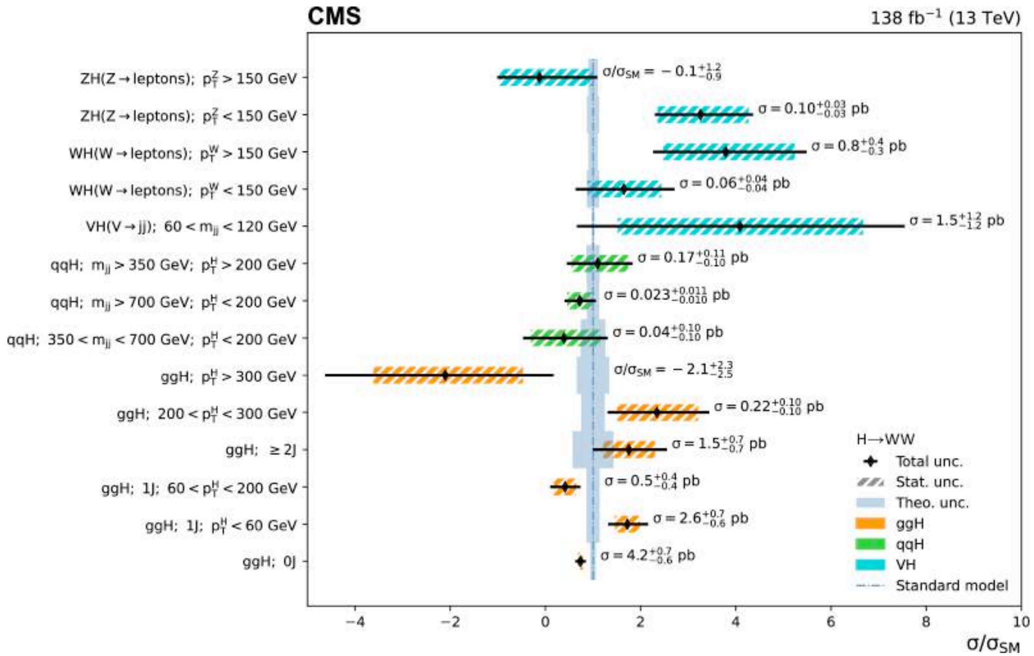




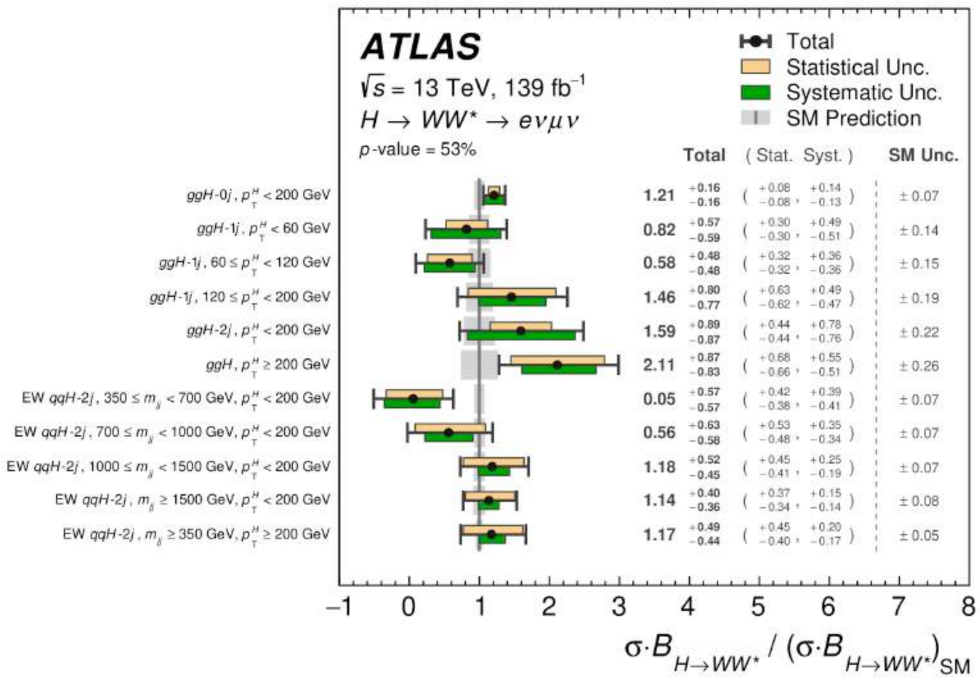


H→WW→lνlν STXS results

CERN-EP-2022-078
CERN-EP-2022-120



- STXS measurement via analysis categories targeting STXS bins

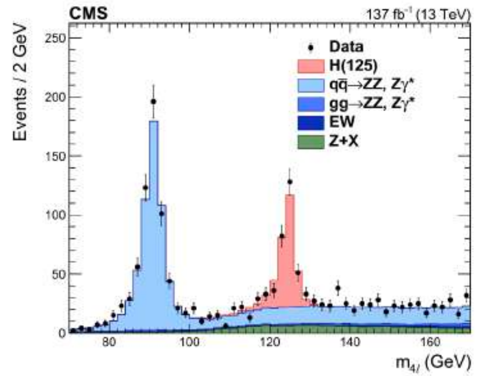


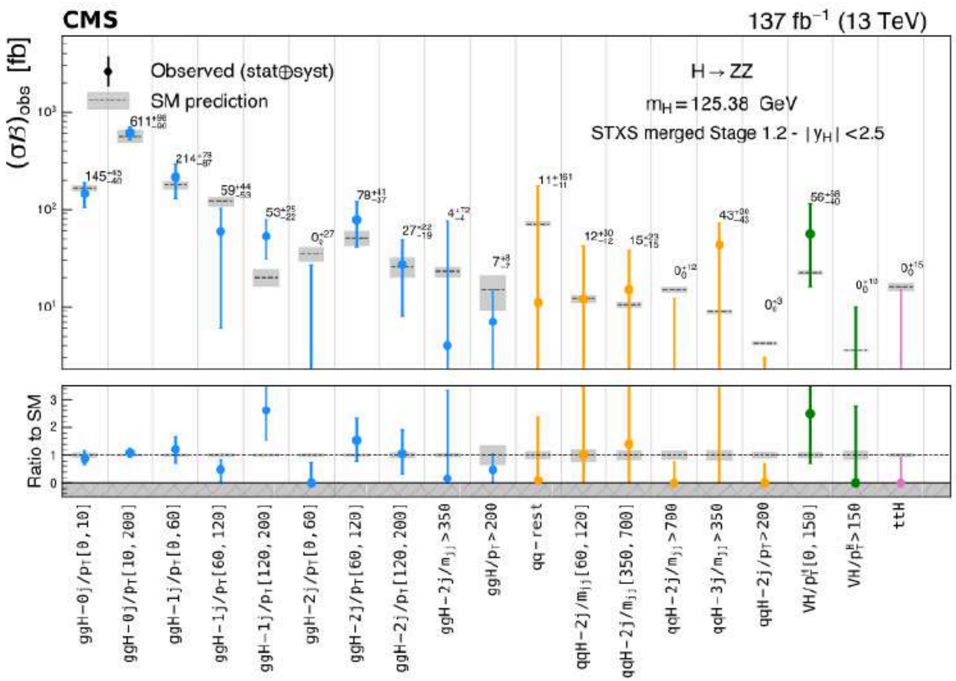
The p-value of the compatibility with the SM is 53% (ATLAS) and 1% (CMS)

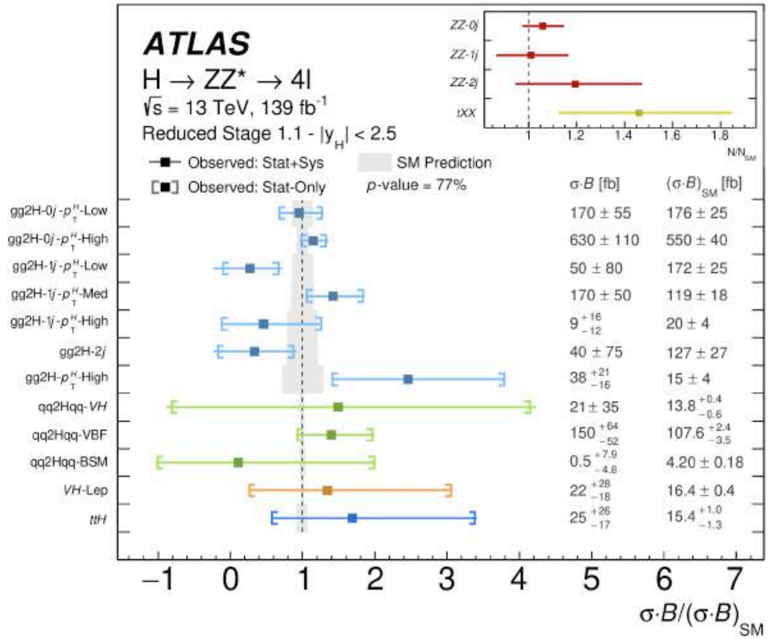
H→ZZ*→4l

Eur. Phys. J. C 80 (2020) 957
Eur. Phys. J. C 81 (2021) 488

- Very clean signature, limited by the small BR
- Mass peak clearly reconstructed on top of the background, ideal channel for many precision measurement of Higgs properties





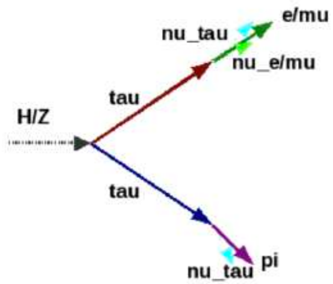


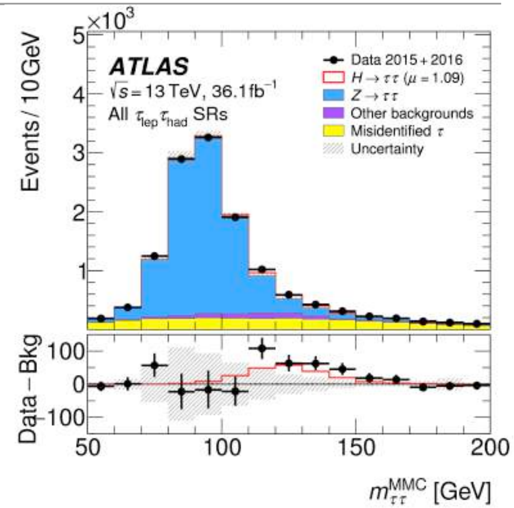
022

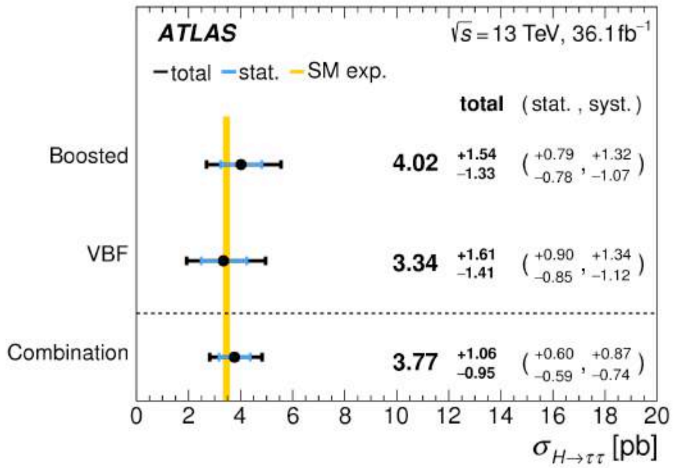
H→ττ

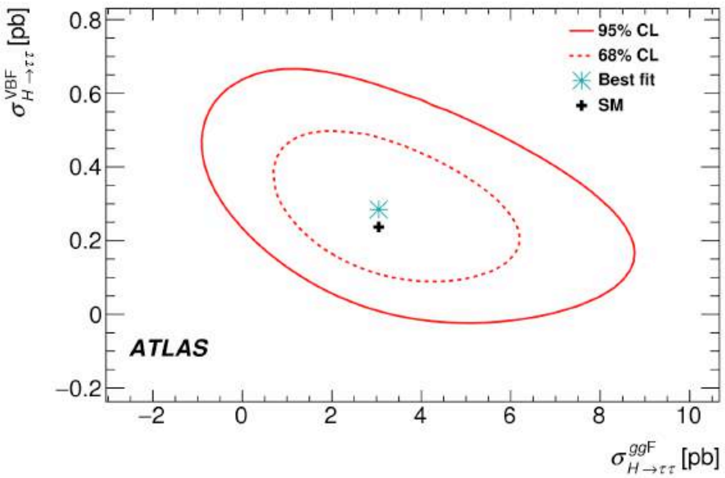
Phys. Rev. D 99 (2019) 072001

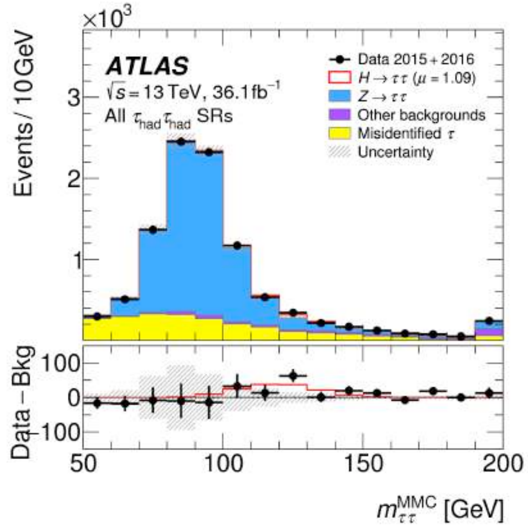
- First direct decay to fermions observed
  - Combining 13, 8 and 7 TeV results
- Select the three combination of decay channels
  - lep-lep, lep-had, had-had
- Reconstruct the invariant mass in the collinear approximation
- Main background is Z-->ττ, mostly determined from data
- Events categorized in VBF-like and "boosted", i.e. with large H pT, due to either ggF+jets or VH production
  - Mainly to remove Z background







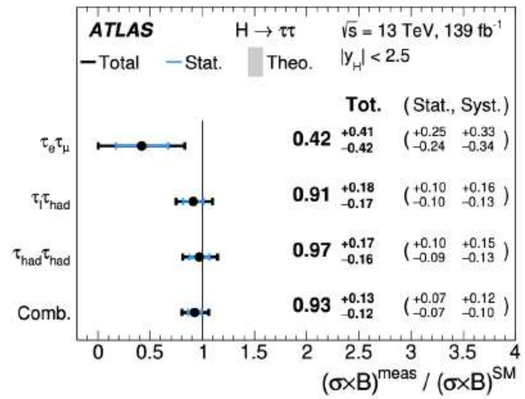


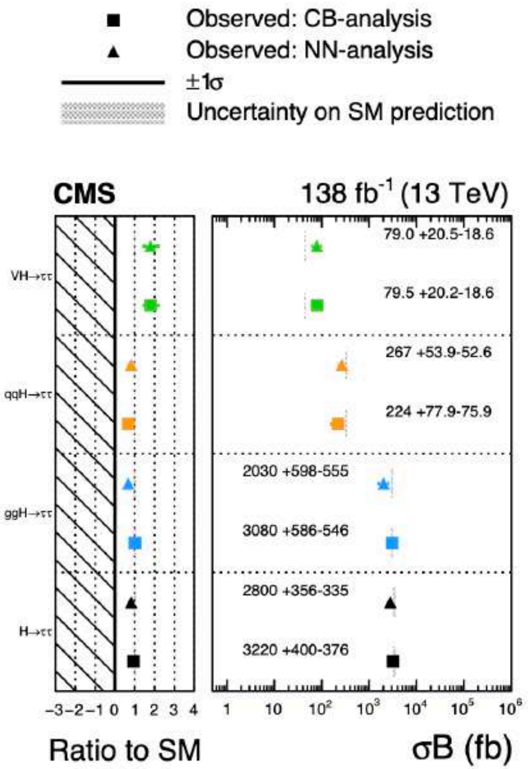


H→ττ updated results

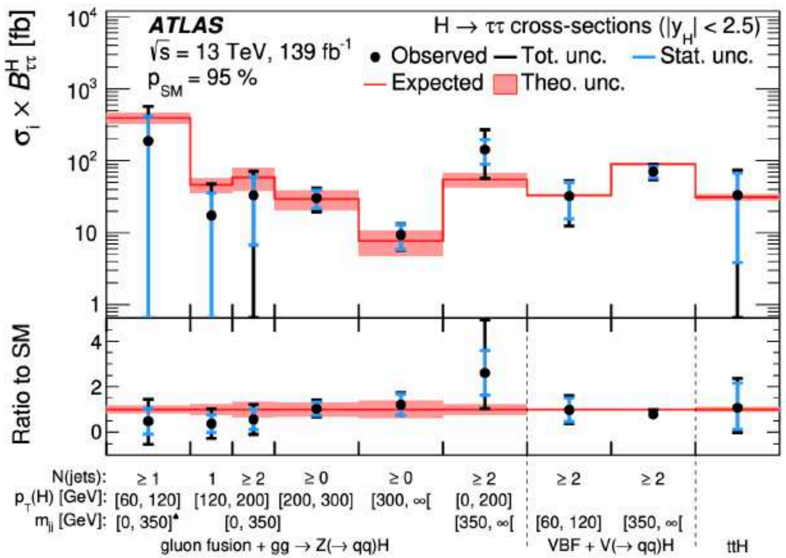
CERN-EP-2022-027
JHEP 08 2022 175

- Channel with the highest BR to leptons (first observation of lepton coupling), but with up to 4 neutrinos in the final state. Large background from Z→ττ.
- All three decay channels ll lh hh entering in the analysis
- Dedicated categories for each production mode (ggF, VBF, VH,ttH)





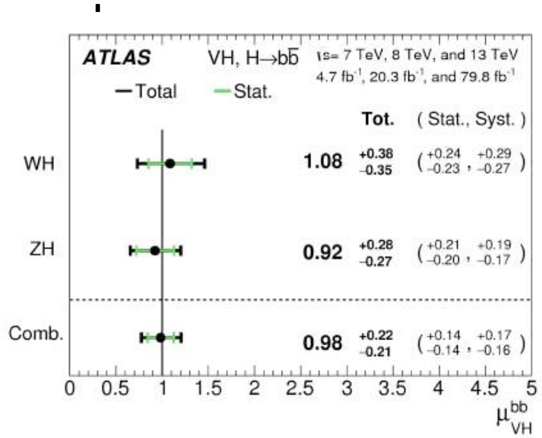
Error on the inclusive signal strength at ~13% for each experiment

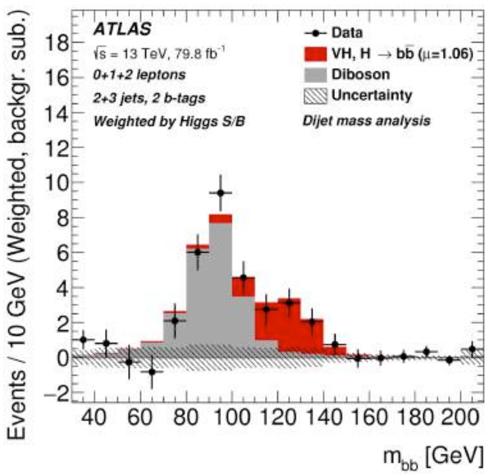


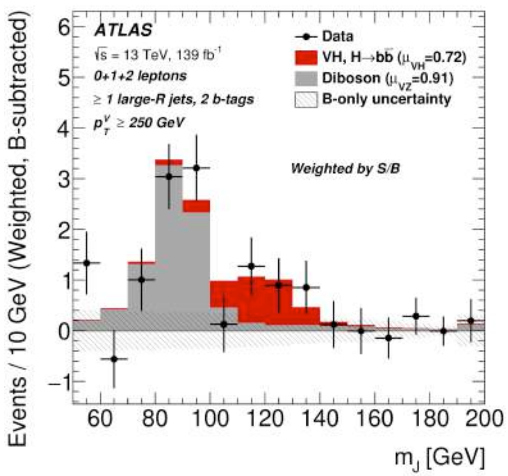
H→bb

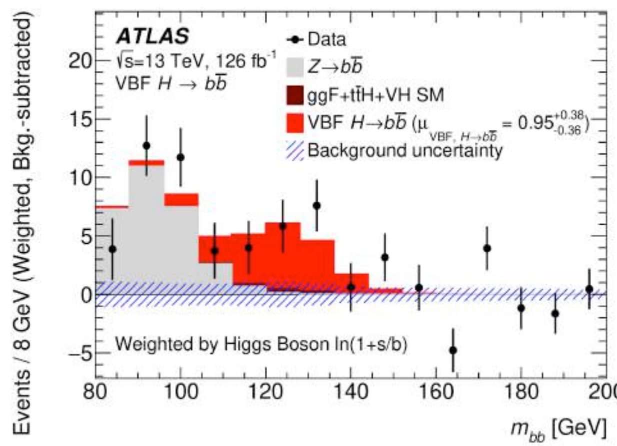
[Eur. Phys. J. C. 81 (2021) 537](#)
[Phys. Lett. B 816 (2021) 136204](#)
[Eur. Phys. J. C 81 (2021) 178](#)

- Channel with the largest BR, but complicated due to the large backg at hadron colliders
- Observed exploiting the associated production signature
- Many exclusive channels added later
  - ➢ VBF, VH and VH boosted
- Boosted topologies looking for "fat" jets with sub-jet structures (2 b tagged subjets)





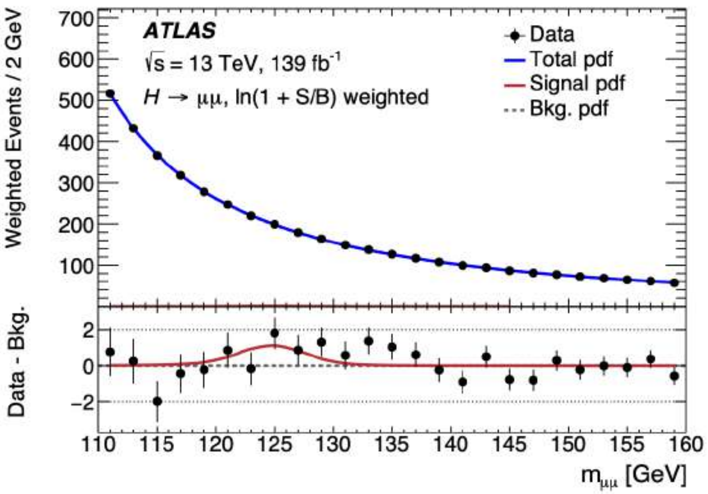


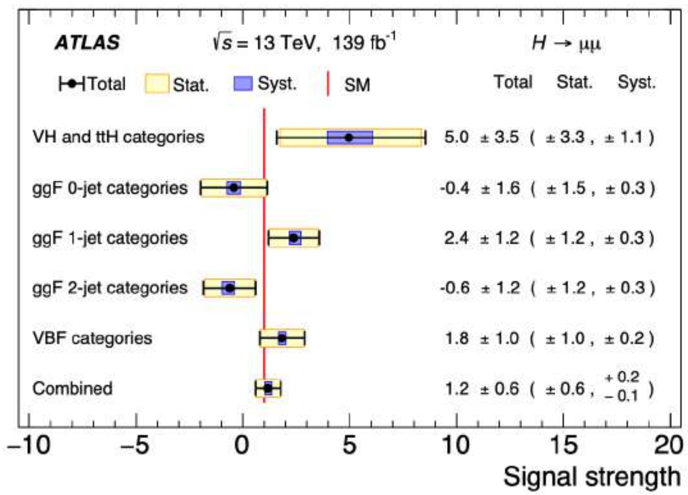


Search for the H→μμ decay

Phys. Lett. B 812 (2021) 135980

- The H→μμ decay offers the best opportunity to measure the Higgs interactions with a second-generation fermion
- Events are categorized in 20 mutually exclusive categories
  - Di-lepton mass fit in the 110-160 GeV region, correct for muon QED FSR
  - Background modeling from core function (LO DY mass shape) convolved with di-lepton mass dependent resolution x empirical functions, category dependent
- Result improve by a factor ~2.5 previous ATLAS result, with a factor ~2 coming from the increased int. luminosity an additional ~25% from analysis improvements
  - Signal strength **μ=1.2±0.6** corresponding to an observed(expected) significance of **2.0σ (1.7σ)**

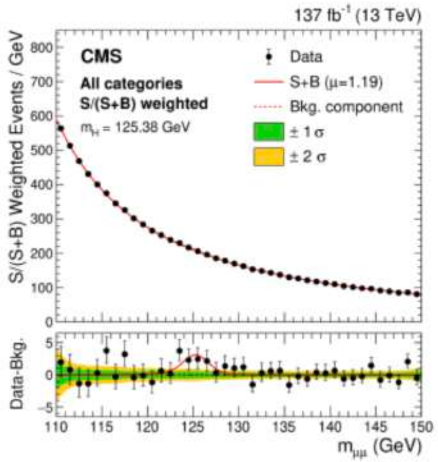


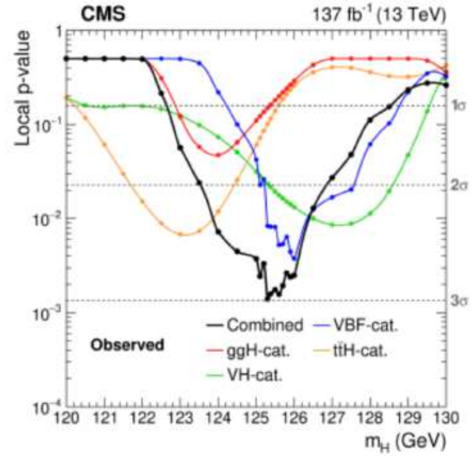


H→μμ decay

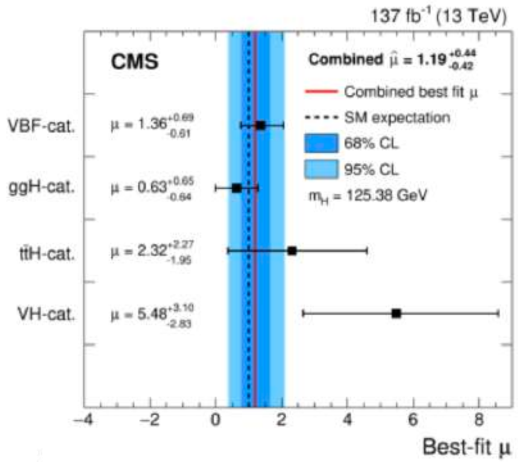
JHEP 01 (2021) 148
Phys. Lett. B812 (2021) 135980

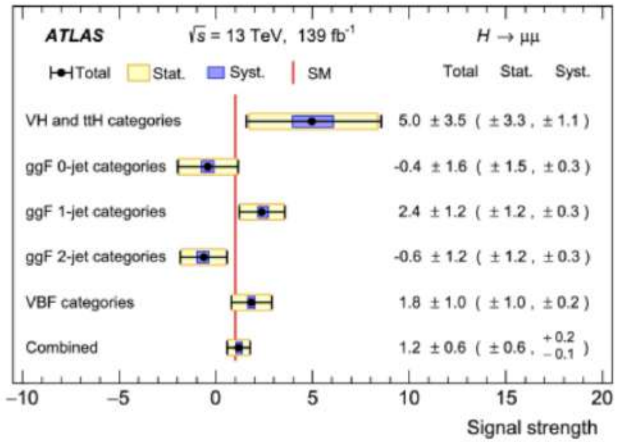
- Rare decay channel (BR=0.02%) that offers the possibility of observing the direct coupling to second generation fermions
- Check of the non-universality of the Higgs-leptons coupling
- Selection optimized exploiting signature of the different production modes and categorization, plus multi-variate discriminants
- Drell-Yan Background fit from data in each category





- CMS obs (exp) significance
  - **3.0σ (2.5σ)**
  - Evidence of μμ decay
- ATLAS obs (exp) significance
  - **2.0σ (1.7σ)**

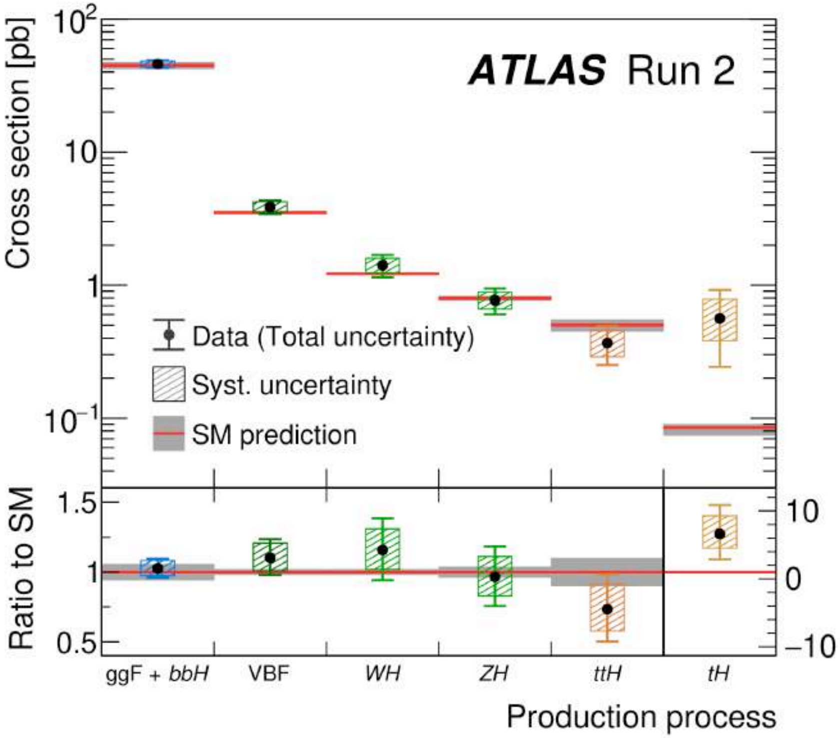


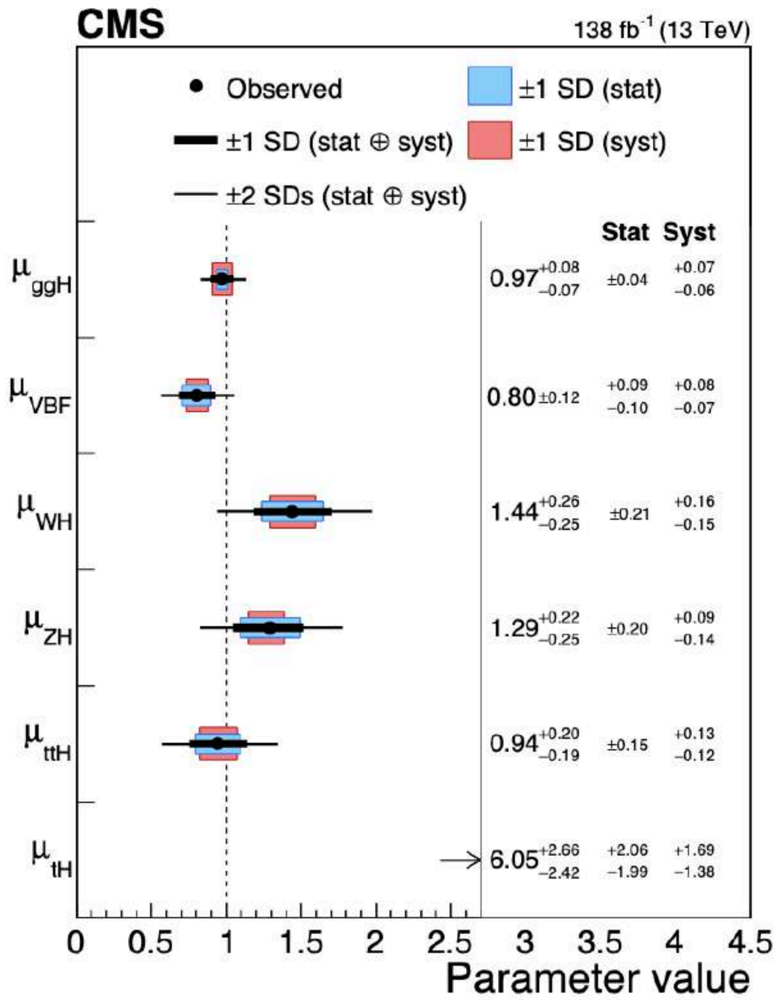


Combined couplings fits

Signal strength: production modes

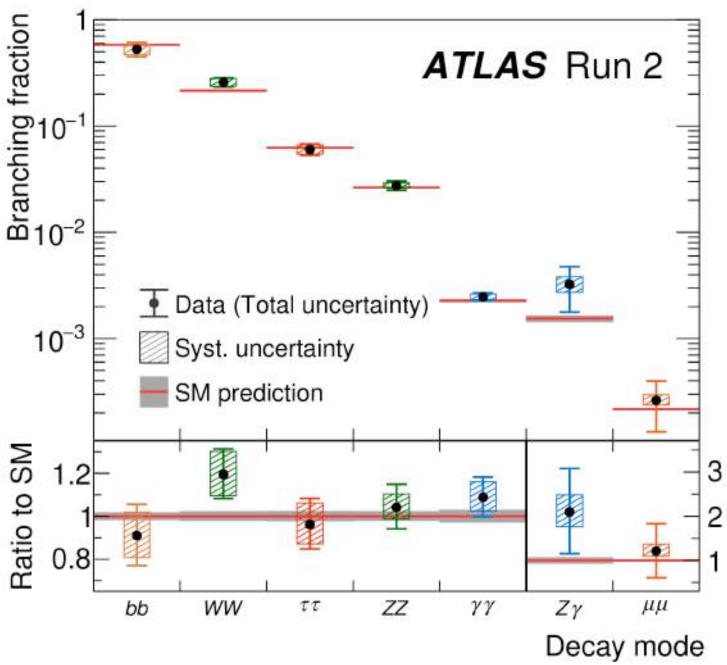
- ❑ By production mode, assuming SM decay BR
- ❑ All the main production modes have been observed with a significance of 5 σ or more
  - ➢ Uncertainties from ~7% to ~25%

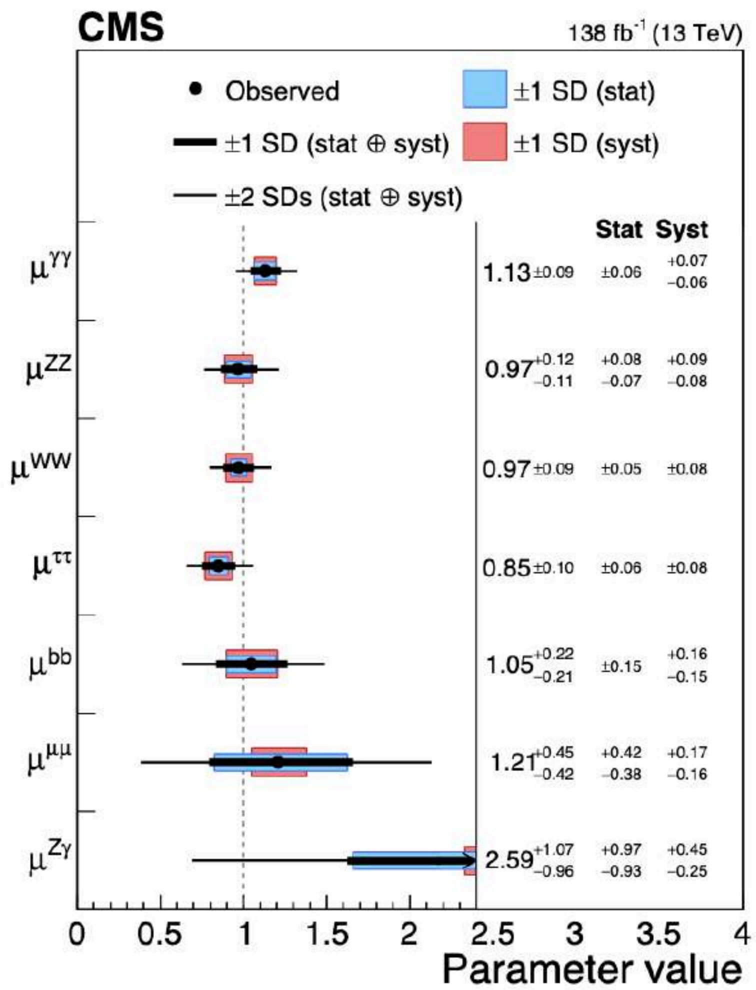




Signal strength: decay channels

- Signal strengths per decay channel, assuming SM values for production cross sections
- Rare decay channels Zγ and μμ not observed yet, all other decay modes observed with more than 5σ significance

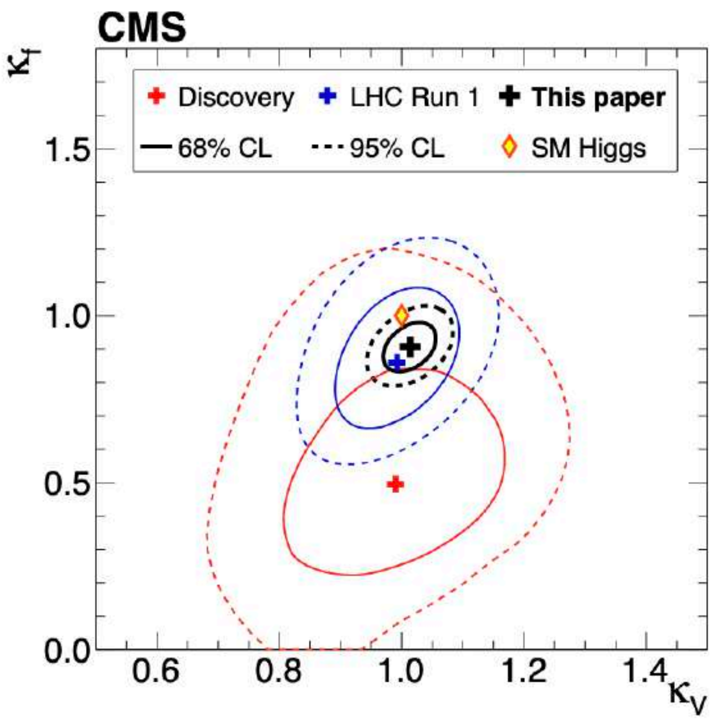


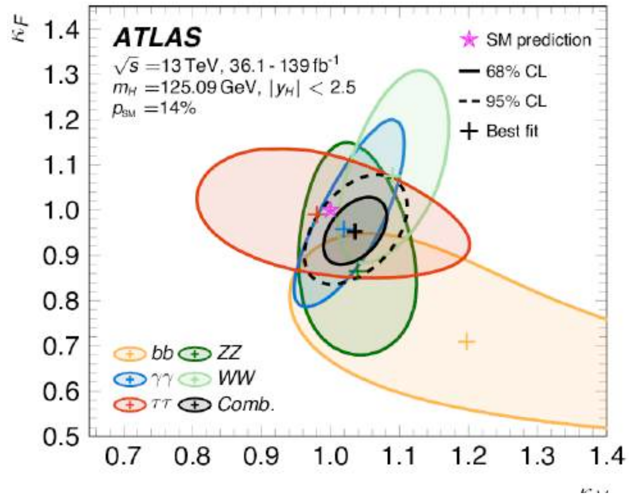


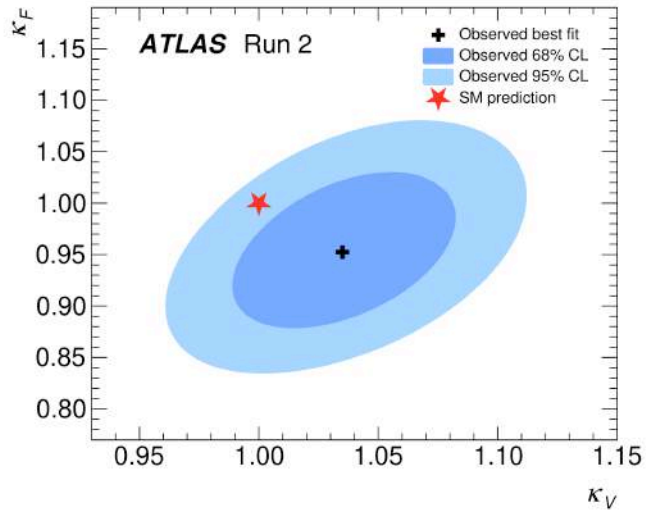
		Stat	Syst
$\mu^{\gamma\gamma}$	1.13 ± 0.09	± 0.06	$^{+0.07}_{-0.06}$
μ^{ZZ}	$0.97^{+0.12}_{-0.11}$	$^{+0.08}_{-0.07}$	$^{+0.09}_{-0.08}$
μ^{WW}	0.97 ± 0.09	± 0.05	± 0.08
$\mu^{\tau\tau}$	0.85 ± 0.10	± 0.06	± 0.08
μ^{bb}	$1.05^{+0.22}_{-0.21}$	± 0.15	$^{+0.16}_{-0.15}$
$\mu^{\mu\mu}$	$1.21^{+0.45}_{-0.42}$	$^{+0.42}_{-0.38}$	$^{+0.17}_{-0.16}$
$\mu^{Z\gamma}$	$2.59^{+1.07}_{-0.96}$	$^{+0.97}_{-0.93}$	$^{+0.45}_{-0.25}$

κ-framework fits

- ❑Fitting only two coupling modifiers, one for all vector bosons and one for all fermions
- ❑Assuming no BSM invisible or undetected decays

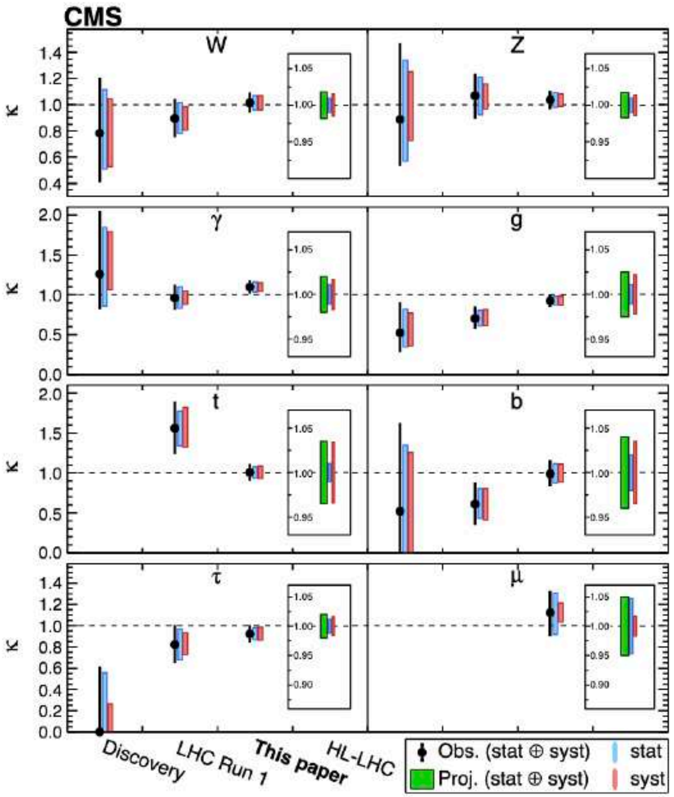


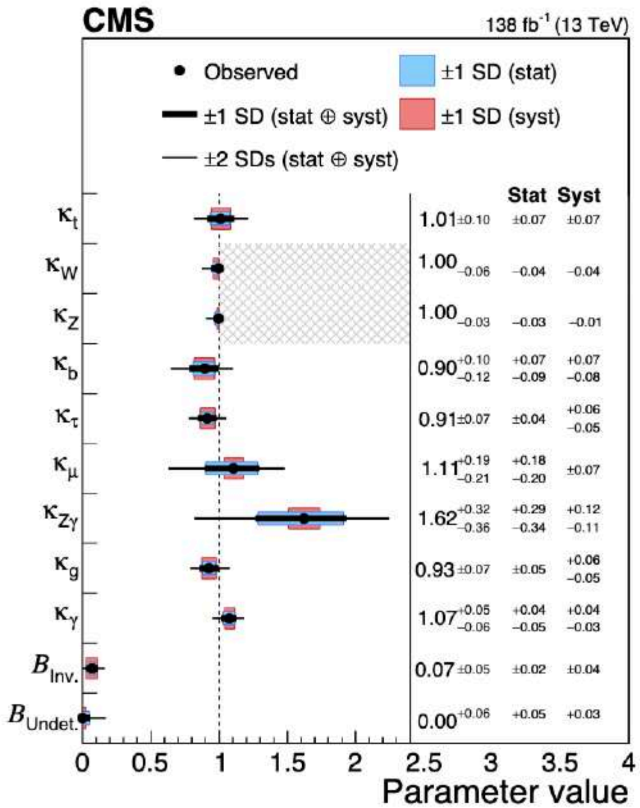


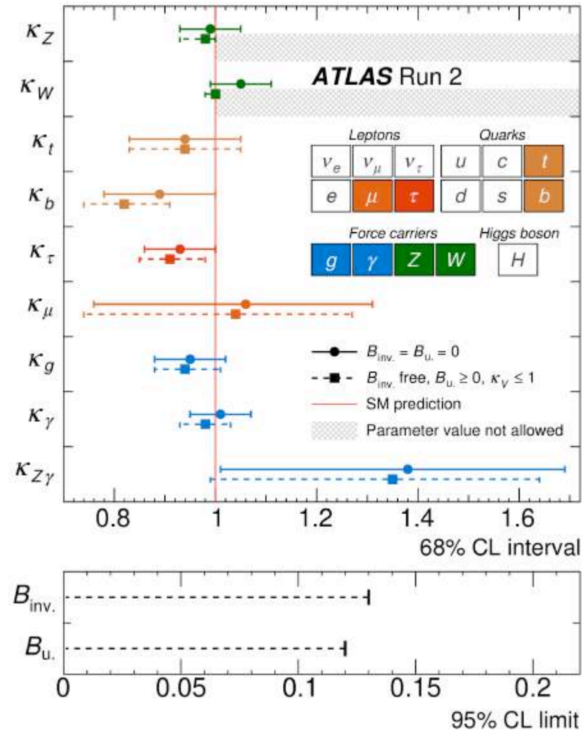


κ-framework fits

- ❑ Couplings modifiers per interaction vertex, left free in the combination of all production and decay modes
  - ➢ Including effective couplings for loop-mediated processes
- ❑ Also fit in a scenario in which invisible (SM+BSM) and undetected decays can contribute to the total width

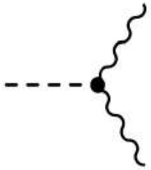




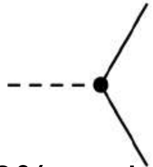


Couplings vs mass

- ❑Cross check of the linear behaviour of the vector bosons and Yukawa couplings



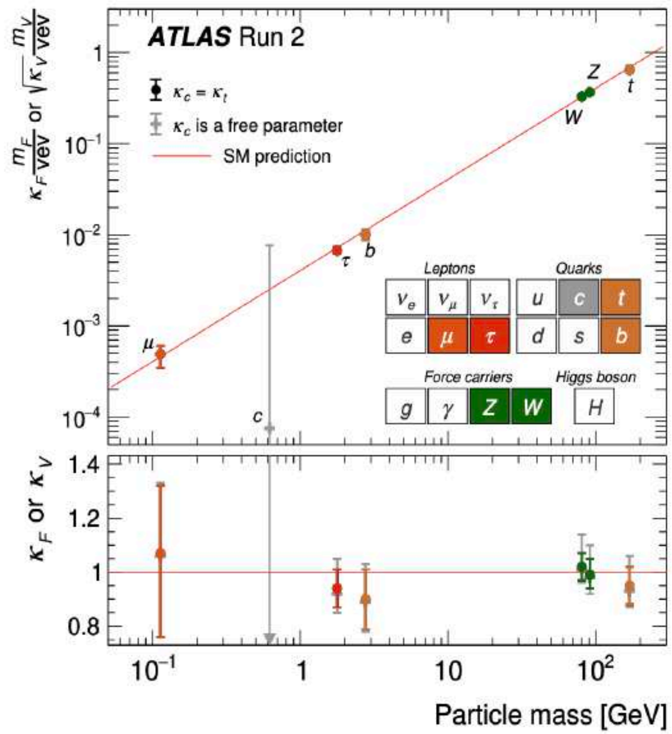
$$g_V = 2\frac{m_V^2}{v}$$



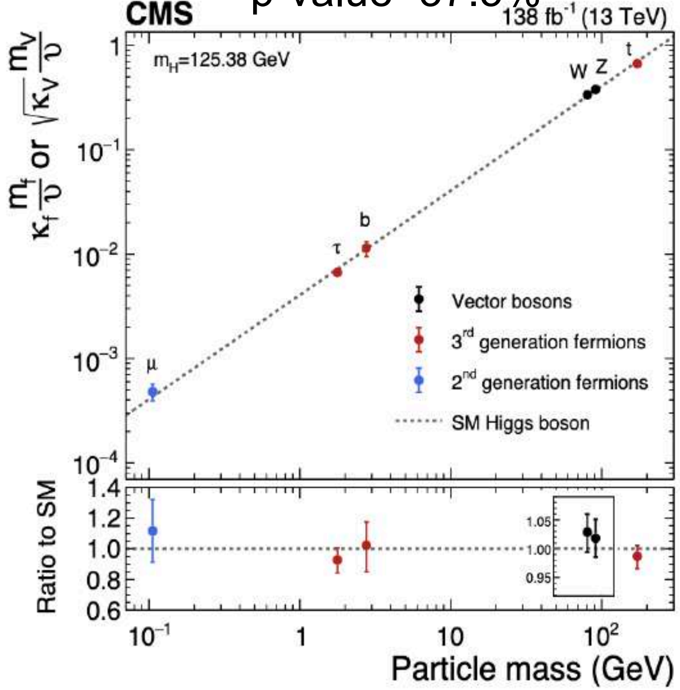
$$g_F = \sqrt{2}\frac{m_f}{v}$$

$$\kappa_F \frac{m_F}{V} \text{ or } \sqrt{\kappa_V}\frac{m_V}{V}$$

p-value=56% and 65%



p-value=37.5%



Fiducial and differential cross sections

Cross sections

- ❑ Use the events selected, in a window around the mass peak, to measure the production cross sections
  - ➢ Another important check of consistency with SM predictions
- ❑ To be completely model independent, the “fiducial” cross section can be measured
  - ➢ Define the acceptance A as the fraction of signal events falling in the detector volume
    - ▪ This can be a model-dependent quantity
  - ➢ Efficiency C as the fraction of signal events within the acceptance, that are reconstructed and selected

$$A = \frac{N(Fiducial)}{N(Generated)} \qquad C = \frac{N(Reco)}{N(Fiducial)} \qquad \sigma_{Tot} \cdot BR = \frac{N_{sig}}{A \cdot C \cdot \mathcal{L}_{int}}$$

The fiducial cross section is simply given by:

$$\sigma_{Fid} \cdot BR = \frac{N_{sig}}{C \cdot \mathcal{L}_{int}}$$

Differential cross sections

$$\frac{d\sigma_{\text{fid},i}}{dx_i} = \frac{n_i^{\text{sig}}}{c_i \cdot \mathcal{L}_{\text{int}} \cdot \Delta x_i}$$

Nsig is calculated as the number of events in data, minus the number of expected events

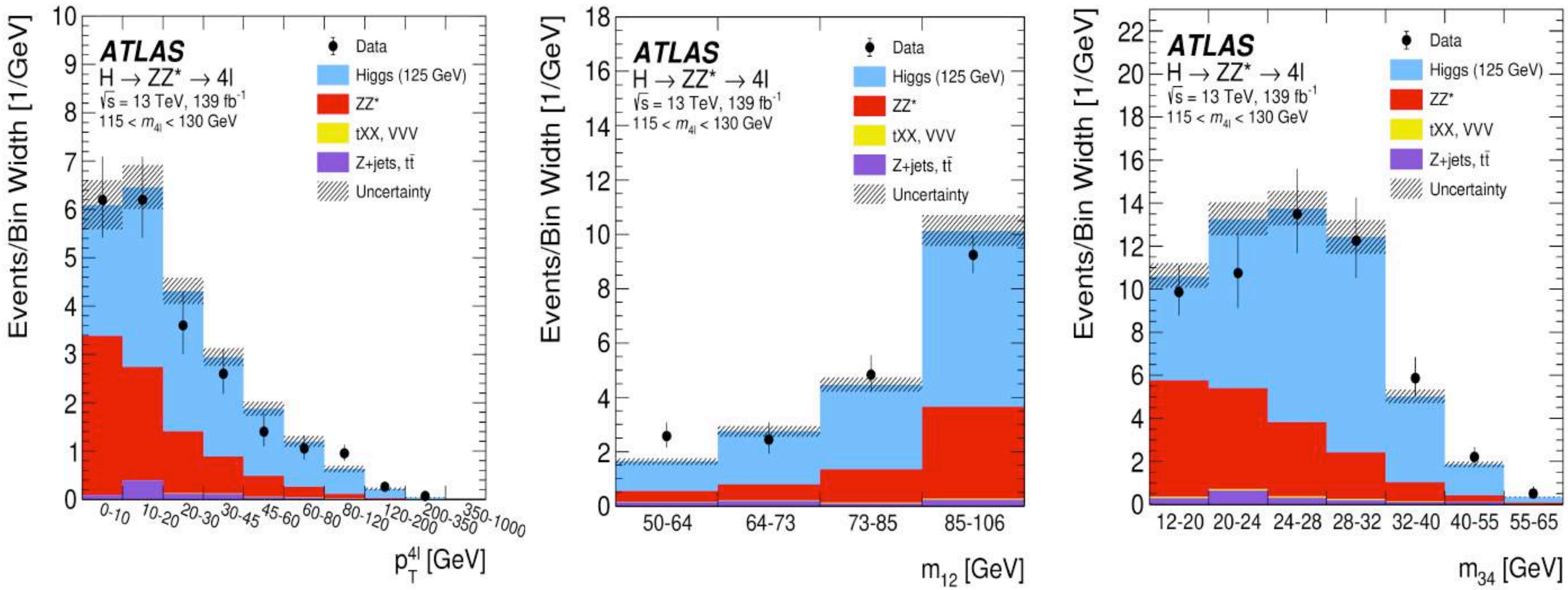
$$c_i = \frac{N_i^{\text{reco}}}{N_i^{\text{fid}}}$$

Correction factors calculated for each distribution bin

- In fact, the correction factors need to take into account the detector effects
- Typical variables for which the differential cross sections are calculated are those most sensitive to possible New Physics
  - pT and rapidity of the Higgs
  - number of jets, and their transverse energy
  - for channels like $H \rightarrow ZZ^* \rightarrow 4l$, the invariant masses of the 2 lepton pairs

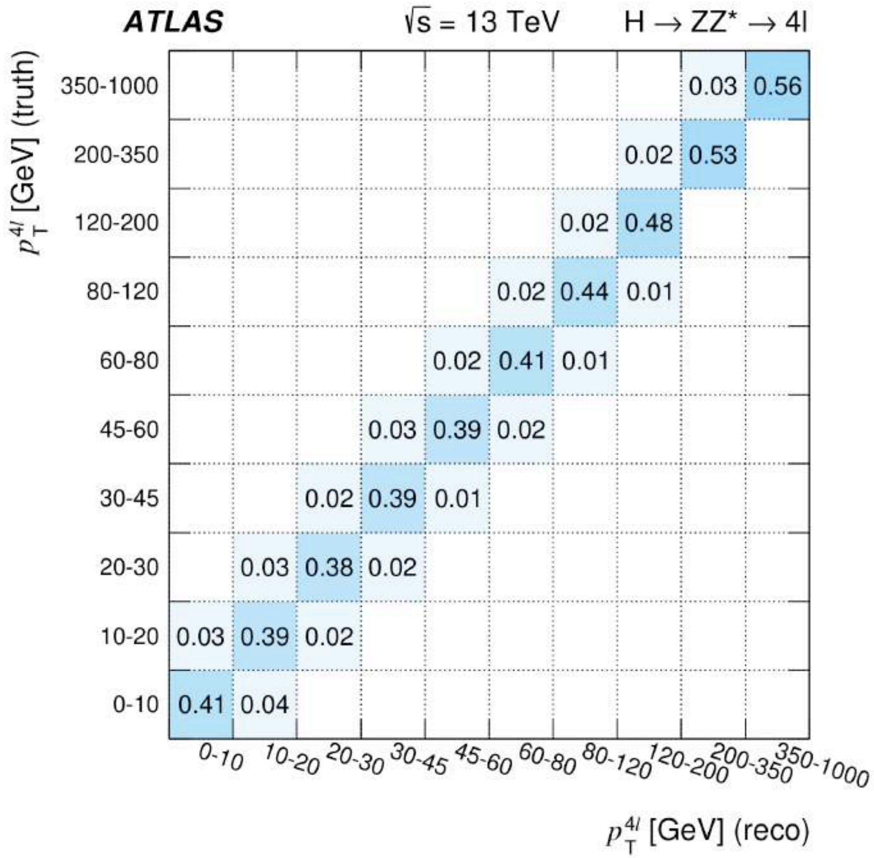
Differential cross sections in H→ZZ*→4l

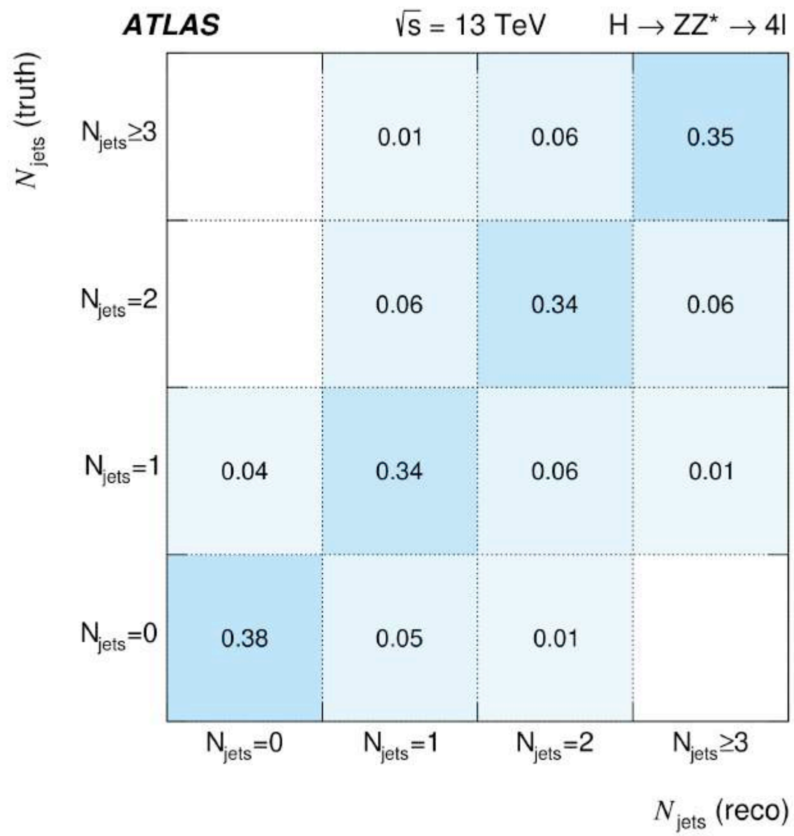
- General method to extract the cross section:
  - Consider the signal region (via e.g. a 4l mass window)
  - Subtract the background, unfold to truth level quantities with bin-by-bin correction factors



Unfolding matrices

- Get (from simulation) the probability P(truth bin|reco bin) with the binning required for the differential measurement





Differential fiducial cross sections in 4l

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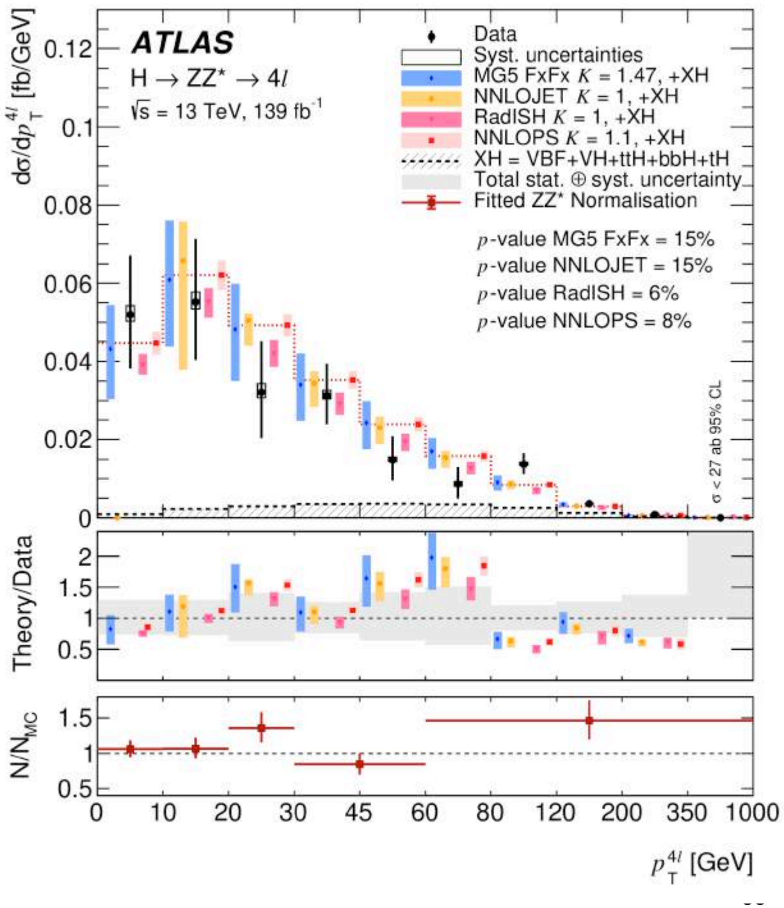
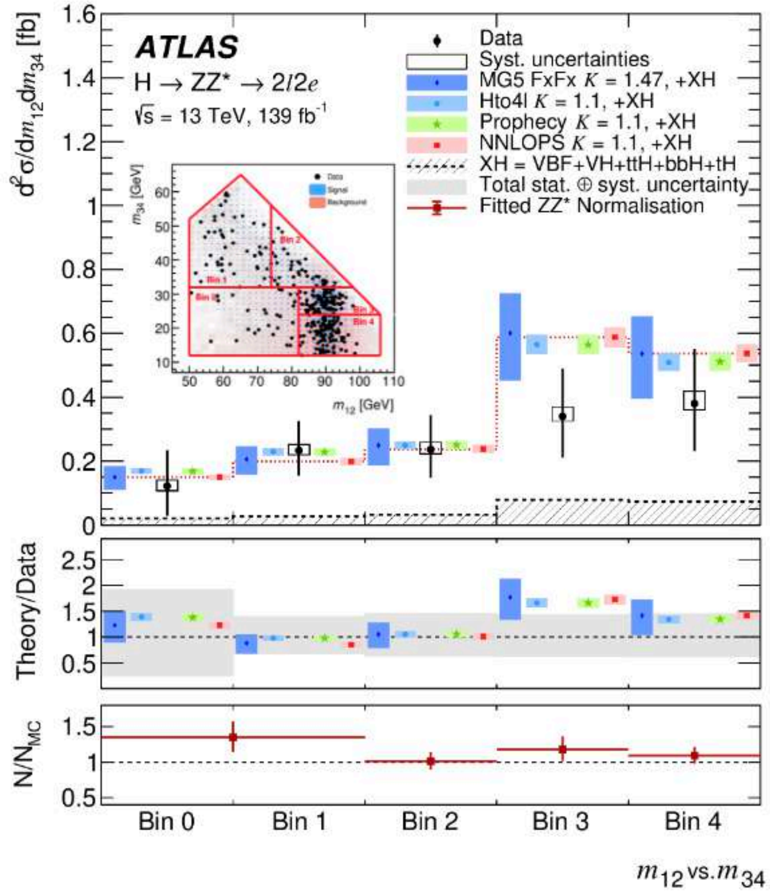
- Some results and comparison to models / generators





Differential fiducial cross sections in γγ

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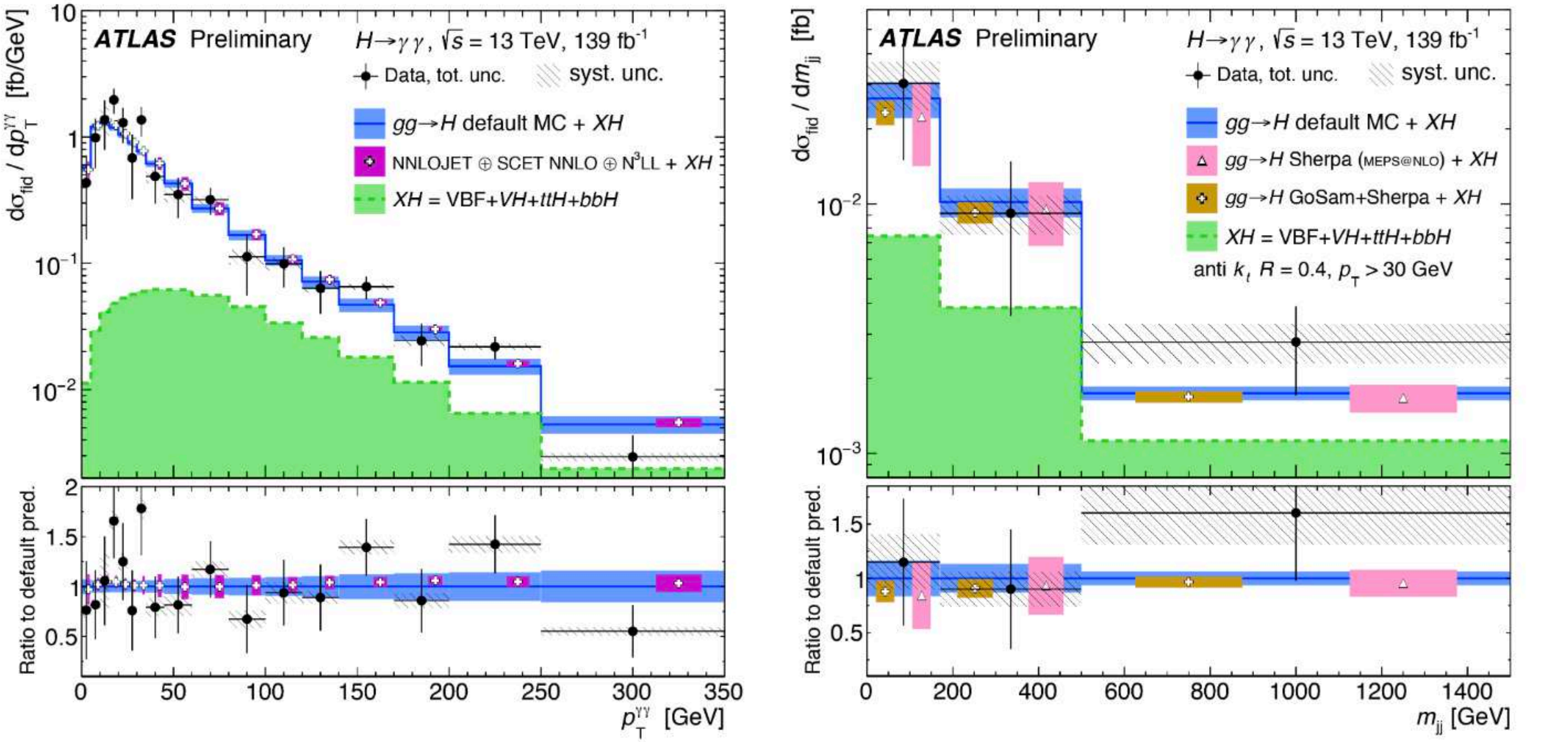
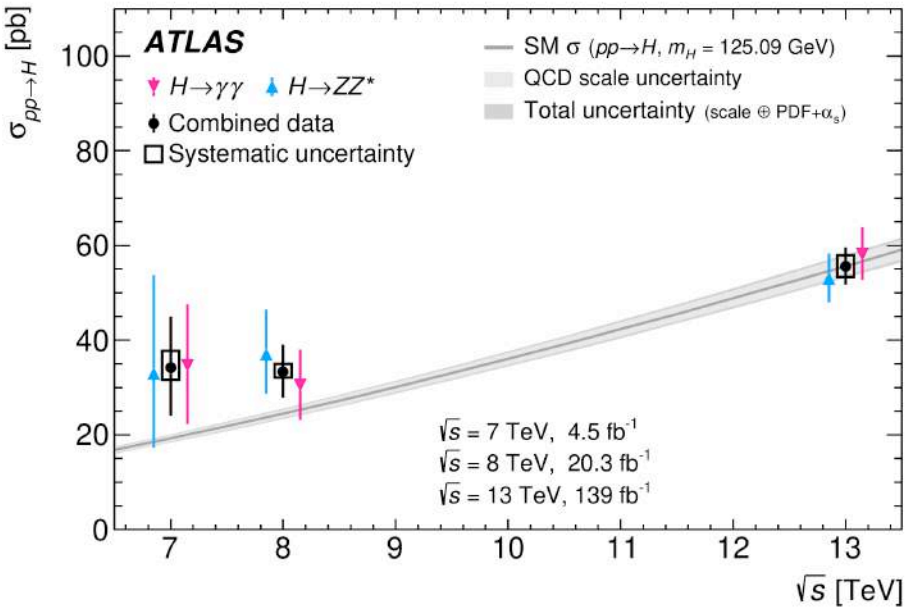
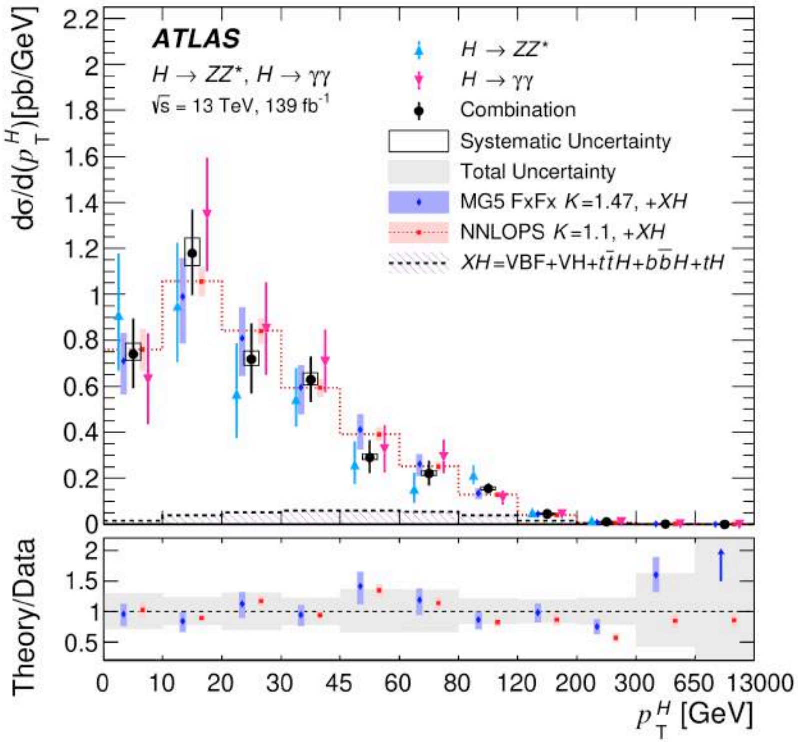


Combination of 4l and γγ

arXiv:2207.08615

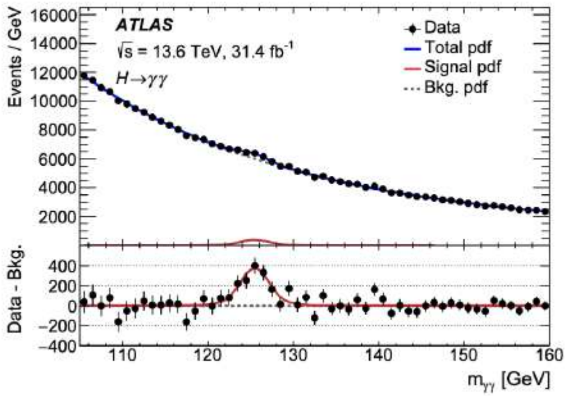
- Total and differential cross sections
- Typical variables that can be combined are the Higgs pT and rapidity
  - ➢ Sensitive to possible new physics contributions

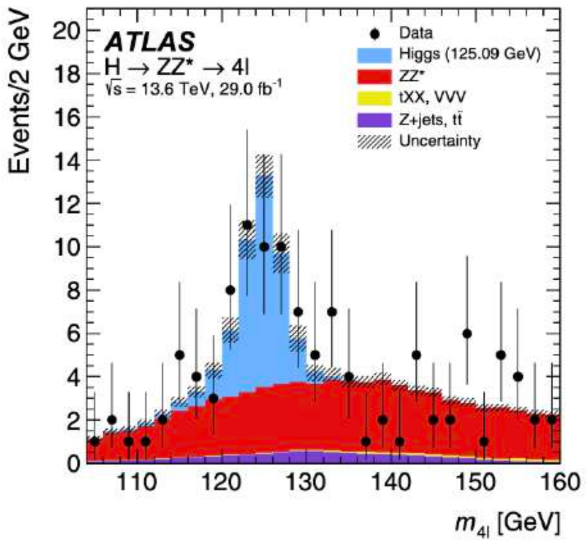


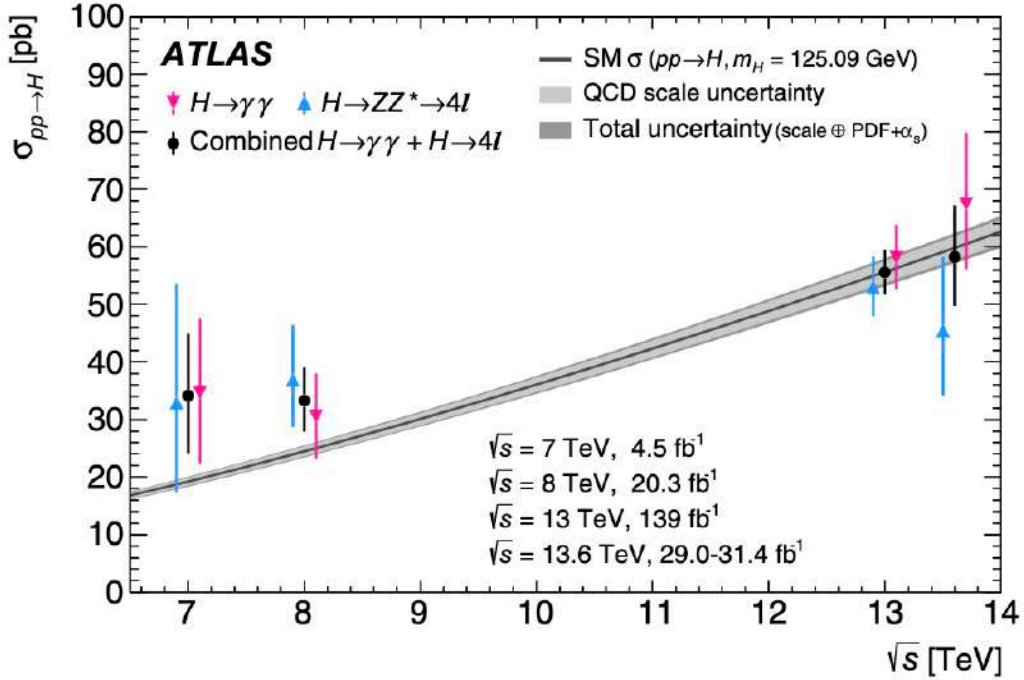


Run-3 update

- ❑ Adding the point at 13.6 TeV with the first run-3 data







Effective field theory

- Effective Field Theory (EFT) is a tool to study possible effects beyond the SM, whose direct observation, for example via a new resonance, would be possible only at scales higher than those actually reachable

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_{i}^{N_{d6}} \frac{c_i}{\Lambda^2} O_i^{(6)} + \sum_{j}^{N_{d8}} \frac{b_j}{\Lambda^4} O_j^{(8)} + \dots$$

- Limiting to O(6) operators, a given matrix element can be expressed as:

$$\mathcal{M}_{\mathrm{SMEFT}} = \left| \mathcal{M}_{\mathrm{SM}} + \sum_i \frac{c_i}{\Lambda^2} \mathcal{M}_i \right|^2$$

$$\begin{aligned} \sigma \propto |\mathcal{M}_{\mathrm{SMEFT}}|^2 &= \left| \mathcal{M}_{\mathrm{SM}} + \sum_i \frac{C_i}{\Lambda^2} \mathcal{M}_i \right|^2 \\ &= |\mathcal{M}_{\mathrm{SM}}|^2 + \sum_i 2Re\left(\mathcal{M}_{\mathrm{SM}}^* \mathcal{M}_i\right) \frac{C_i}{\Lambda^2} \\ &\quad + \sum_{ij} 2Re\left(\mathcal{M}_i^* \mathcal{M}_j\right) \frac{C_i C_j}{\Lambda^4}, \end{aligned}$$

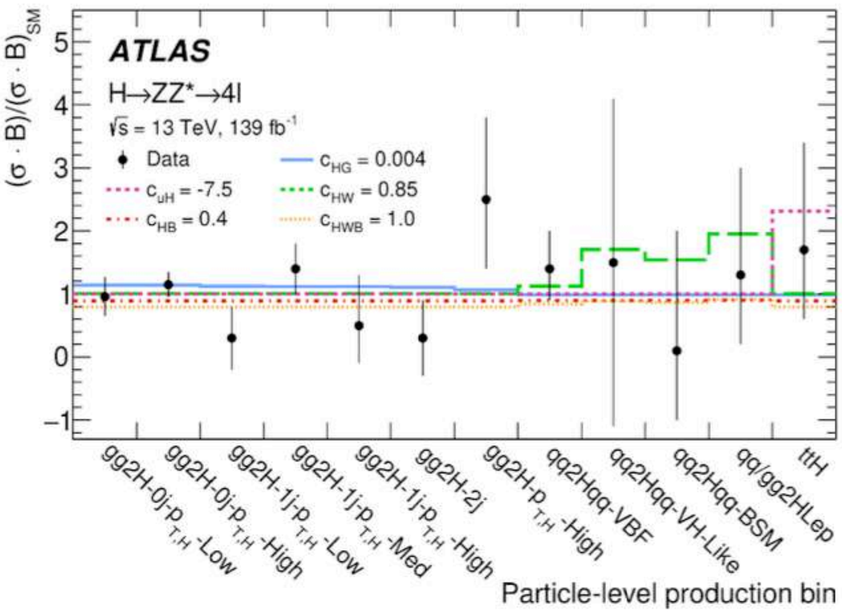
$$\frac{\sigma^p(\vec{c})}{\sigma^p_{\mathrm{SM}}} = 1 + \sum_i A_i^p c_i + \sum_{ij} B_{ij}^p c_i c_j,$$

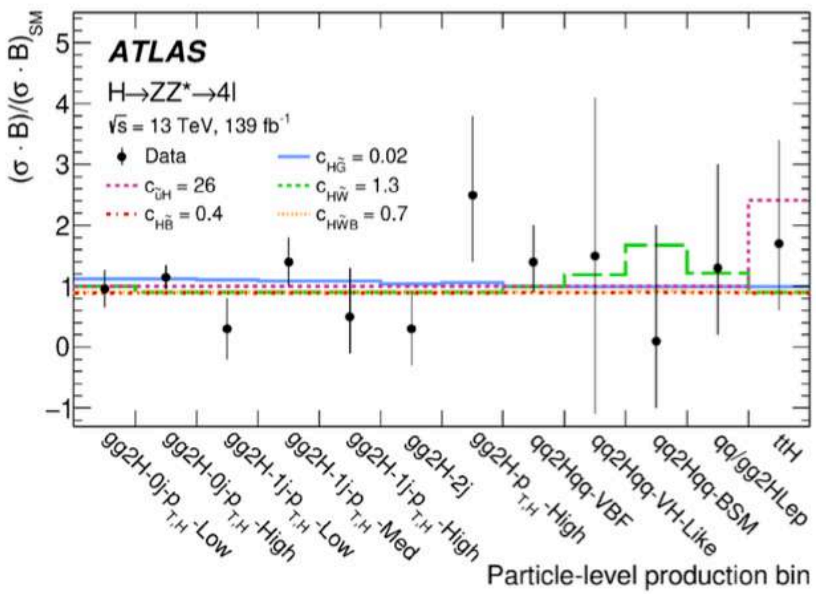
Interference terms can contain CP-even Beyond Standard Model contributions, while CP-odd can only be BSM

EFT couplings fit in H→ZZ*→4l

- ❑ Each production mode is more sensitive to a different set of couplings

CP-even			CP-odd			Impact on	
Operator	Structure	Coeff.	Operator	Structure	Coeff.	production	decay
O_{uH}	$HH^\dagger \bar{q}_p u_r \tilde{H}$	c_{uH}	O_{uH}	$HH^\dagger \bar{q}_p u_r \tilde{H}$	$c_{\tilde{u}H}$	ttH	-
O_{HG}	$HH^\dagger G^A_{\mu\nu} G^{\mu\nu A}$	c_{HG}	$O_{H\tilde{G}}$	$HH^\dagger \tilde{G}^A_{\mu\nu} G^{\mu\nu A}$	$c_{H\tilde{G}}$	ggF	Yes
O_{HW}	$HH^\dagger W^l_{\mu\nu} W^{\mu\nu l}$	c_{HW}	$O_{H\widetilde{W}}$	$HH^\dagger \widetilde{W}^l_{\mu\nu} W^{\mu\nu l}$	$c_{H\widetilde{W}}$	VBF, VH	Yes
O_{HB}	$HH^\dagger B_{\mu\nu} B^{\mu\nu}$	c_{HB}	$O_{H\tilde{B}}$	$HH^\dagger \tilde{B}_{\mu\nu} B^{\mu\nu}$	$c_{H\tilde{B}}$	VBF, VH	Yes
O_{HWB}	$HH^\dagger \tau^l W^l_{\mu\nu} B^{\mu\nu}$	c_{HWB}	$O_{H\widetilde{W}B}$	$HH^\dagger \tau^l \widetilde{W}^l_{\mu\nu} B^{\mu\nu}$	$c_{H\widetilde{W}B}$	VBF, VH	Yes



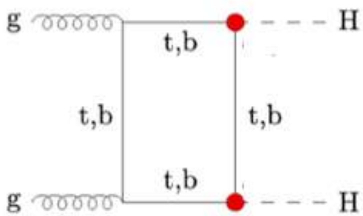


Double Higgs production

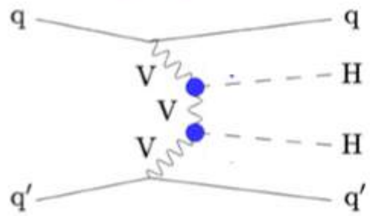
Double Higgs production

- ❑ Double Higgs production can happen via ggF and VBF, with fermion and vector boson couplings:

ggF pair production

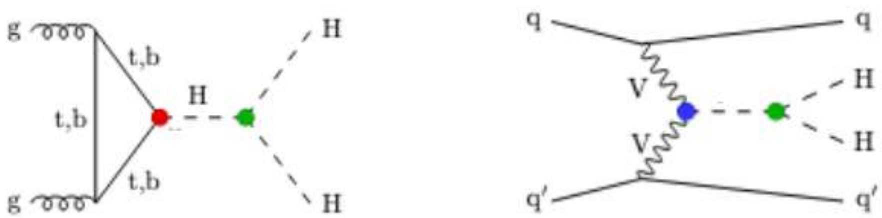


VBF pair production

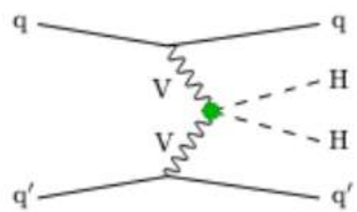


- ❑Or, via Higgs self-coupling and VVHH quartic coupling:

Higgs-pair production via self-coupling



Higgs-pair production via VVHH coupling

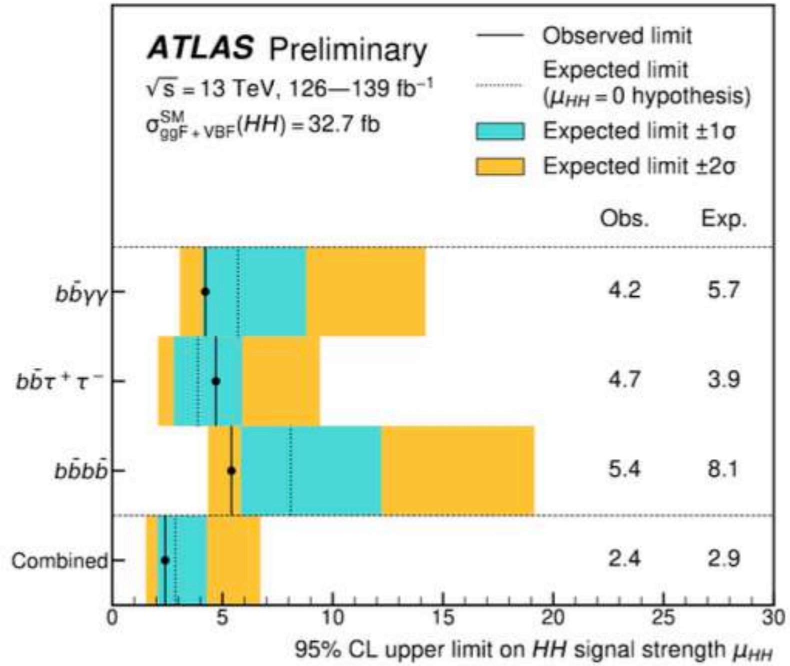


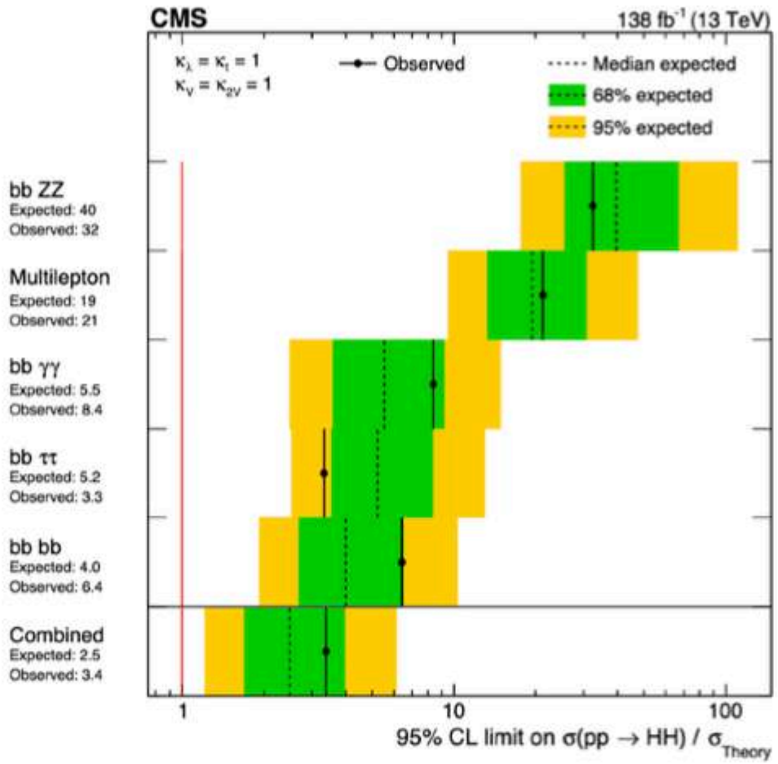
- ❑But, the total cross section is three orders of magnitude smaller than the one for the single Higgs

Di-Higgs analyses

- Given the very small cross section, in this case one has to rely on the highest-BR channels
- Main analyses are bb+(bb, $\tau\tau$, $\gamma\gamma$), although also multi-lepton and ZZ are considered
- Current observed (expected) limits on the HH signal strength are:
  - CMS: **μ_{HH}<3.4 (2.5)**
  - ATLAS: **μ_{HH}<2.4 (2.9)**

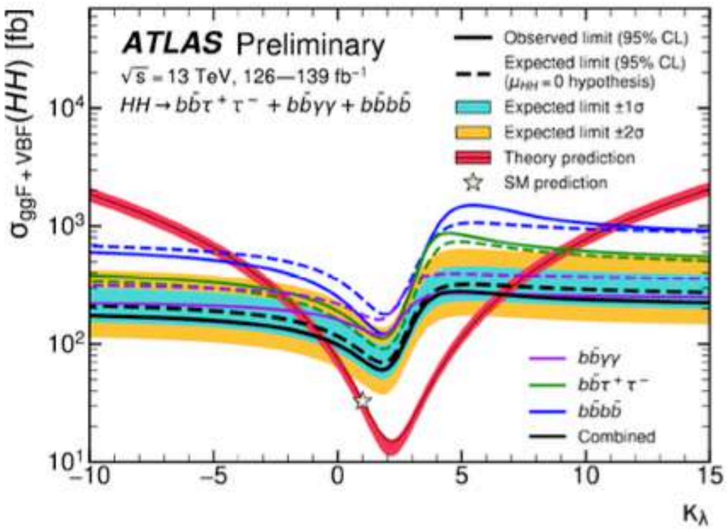
ATLAS-CONF-2022-050
Nature 607 (2022) 60-68

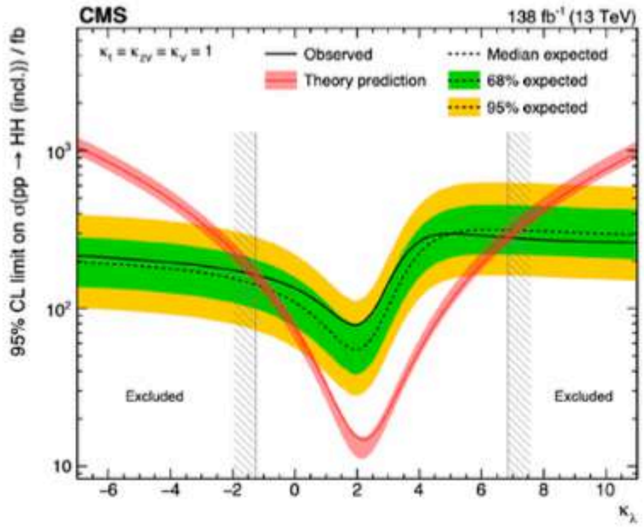


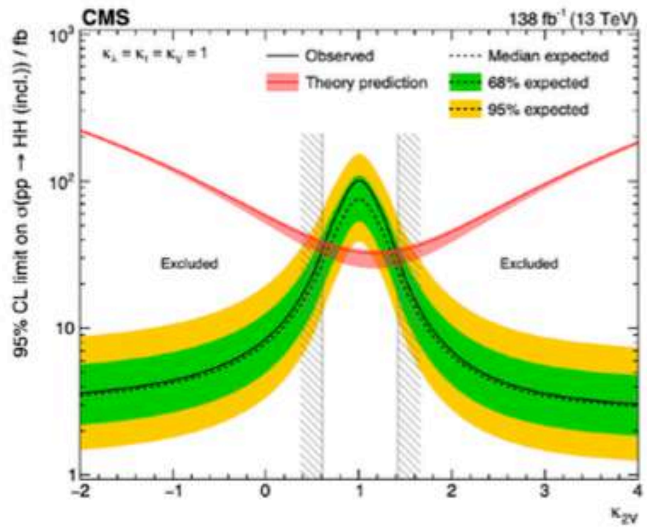


Self and quartic couplings modifiers

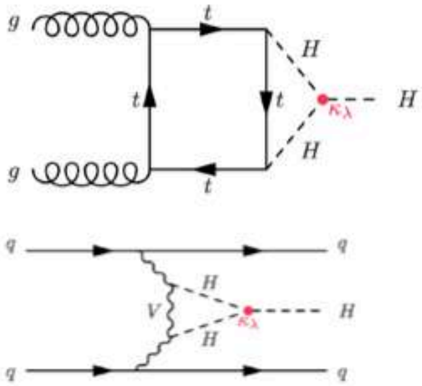
- Limits on the signal strengths can be used to derive limits on coupling modifiers κ_λ κ_{2V}, applied to the HHH and VVHH vertices respectively
- The cross-section dependency on the modifiers comes from interference
- κ_{2V} =0 excluded by CMS with 6.6σ significance



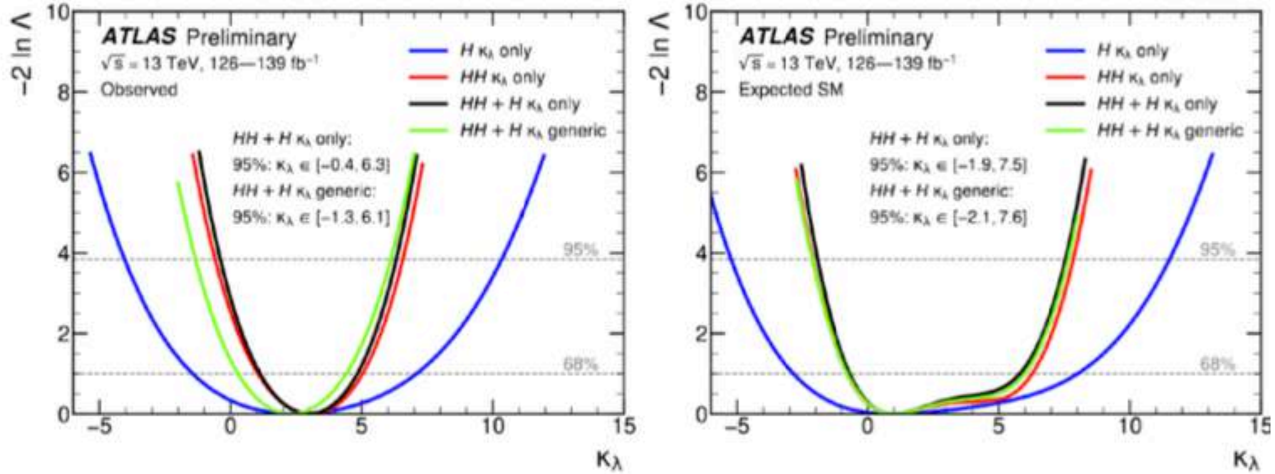




Self and quartic couplings modifiers

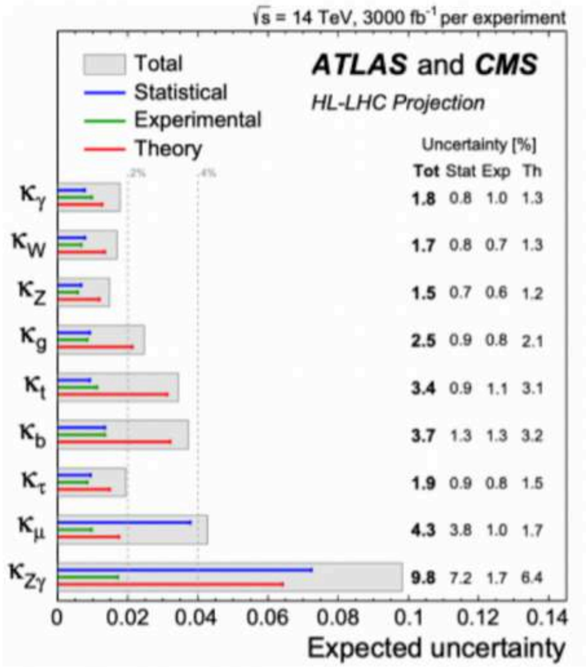


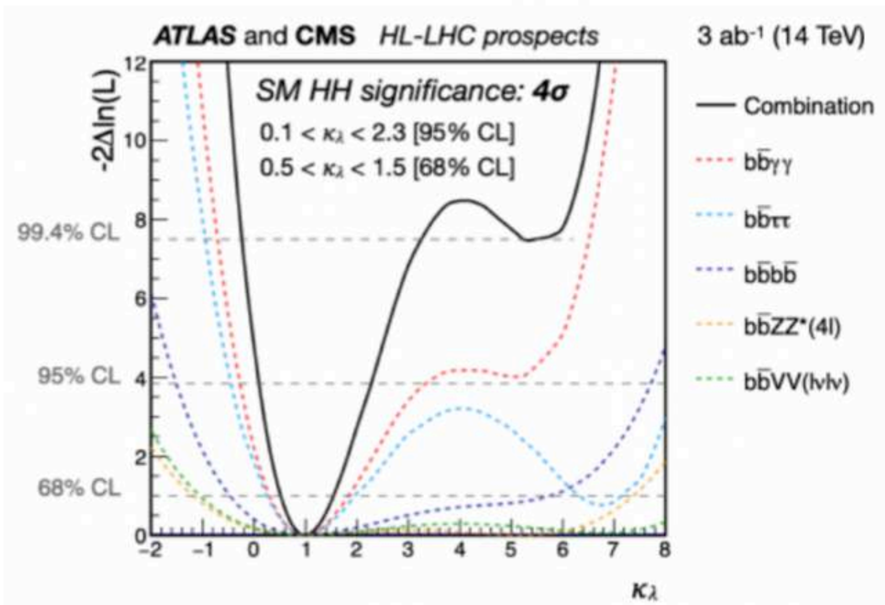
One-loop single Higgs production depending on κ_λ, included in the ATLAS combination



Conclusions

- Run-3 has jus started, providing new data at 13.6 TeV with upgraded detectors
  - Plan is reach ~300 fb-1
- A lot of new results on Higgs properties measurements will be based on these new data
- Hi-Lumi LHC with phase-2 upgraded detectors and further improved theoretical calculations will bring even more the Higgs measurements in the era of the precision measurements





What you can mostly focus on

- ❑ Higgs production mechanisms and their main characteristics
- ❑ Higgs decays and their main characteristics and signatures
- ❑ Main discovery channels
  - ➢ Experimental signatures and main backgrounds
- ❑ Exclusion and discovery significance (only in the case of a simple counting experiment)
- ❑ Higgs mass and width
- ❑ Methods to extract information on the Higgs couplings and look for possible deviations from the Standard Model (you don't have to know the results in detail, just understand the methods)
  - ➢ Signal strengths
  - ➢ κ-factor method
  - ➢ Fiducial and differential cross sections
- ❑ Double Higgs production

- ❑ In case of questions or curiosities just pass by the office 229

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End of chapter 12