

Collider Particle Physics

- Chapter 2 -

ISR – the first hadron collider and the “soft” physics



Claudio Luci

SAPIENZA
UNIVERSITÀ DI ROMA

last update : 070117

Chapter Summary

- ☐ A reminder of resonances and strong interactions in the '60s.
- ☐ A reminder of cross-section and luminosity measurements
- ☐ Partial wave, optical theorem and total cross-section measurement.
- ☐ ISR
- ☐ “Soft” Physics at the ISR: proton-proton total cross section.
- ☐ “Soft” Physics at LHC.

Reminder about Resonances and how strong interactions were dealt with in the '60

(of course ... this is not part of the exam program ... well, resonances yes.)

Reminder: what is a resonance?

❑ We have two kind of resonances:

- **formation resonance:** we have an enhancement of the cross-section because the reaction proceed through a resonant state
- **production resonance:** we have a resonance among the many particles produced in the finale state

❑ **Formation:** the scattering process happens through the “formation” of an intermediate resonant state **R**;

❑ The resonance can decay in:

➤ same particles of the initial state (elastic scattering)



➤ other particles (anelastic scattering)

➤ the sum of elastic plus anelastic gives the total cross section



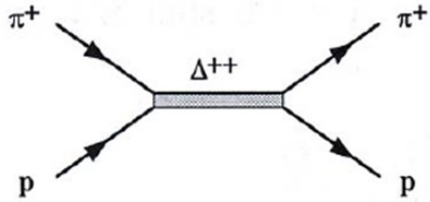
❑ The resonance is described by the Breit-Wigner formula:

$$\sigma(E) = \frac{4\pi\hbar^2}{p_{cm}^2} \frac{2J+1}{(2S_a+1) \cdot (2S_b+1)} \left[\frac{\Gamma_{in} \cdot \Gamma_{fin}}{(E - M_R)^2 + \Gamma^2 / 4} \right]$$

- P_{cm} : beam momentum in the center of mass reference frame
- E : center of mass energy ($\sqrt{s}$)
- M_R : resonance mass

- S_a, S_b : initial state spins (average over the initial spin states)
- J : resonance spin (sum over the final spin states)
- $\Gamma, \Gamma_{in}, \Gamma_{fin}$: resonance total and partial widths. Γ_{in} and Γ_{fin} take into account the coupling of the resonance with the initial and final states.

An example: the Δ resonance



$$\tau = \frac{\hbar}{\Gamma_{\text{tot}}} = \frac{6.58 \cdot 10^{-16} \text{ eV} \cdot \text{s}}{118 \cdot 10^6 \text{ eV}} = 5.6 \cdot 10^{-24} \text{ s}$$

This lifetime is typical of the strong interactions

At $\sqrt{s} < 1.4 \text{ GeV}$ $\sigma_{\text{elast}} = \sigma_{\text{total}}$

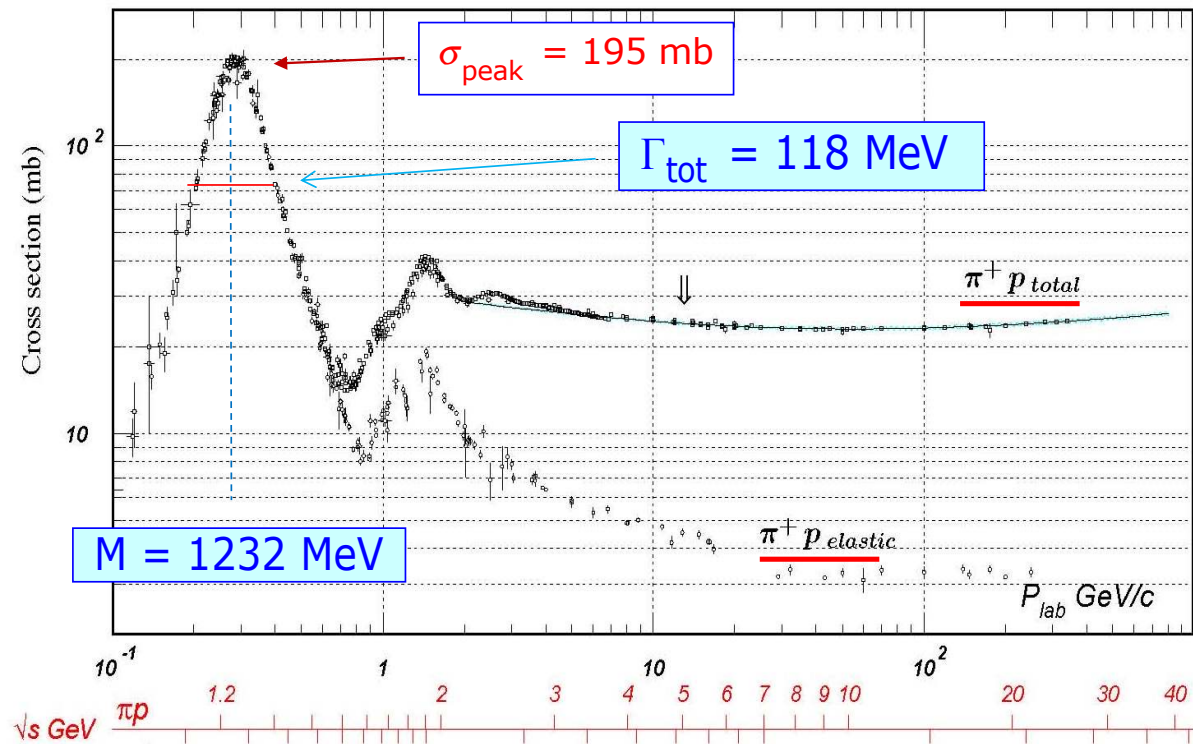
Why?

Reminder:

$$m_p = 938 \text{ MeV}$$

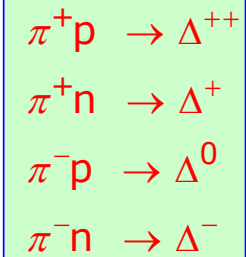
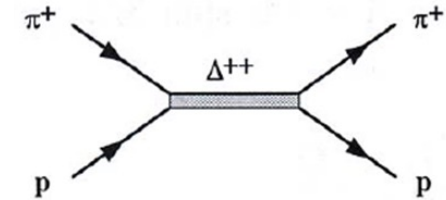
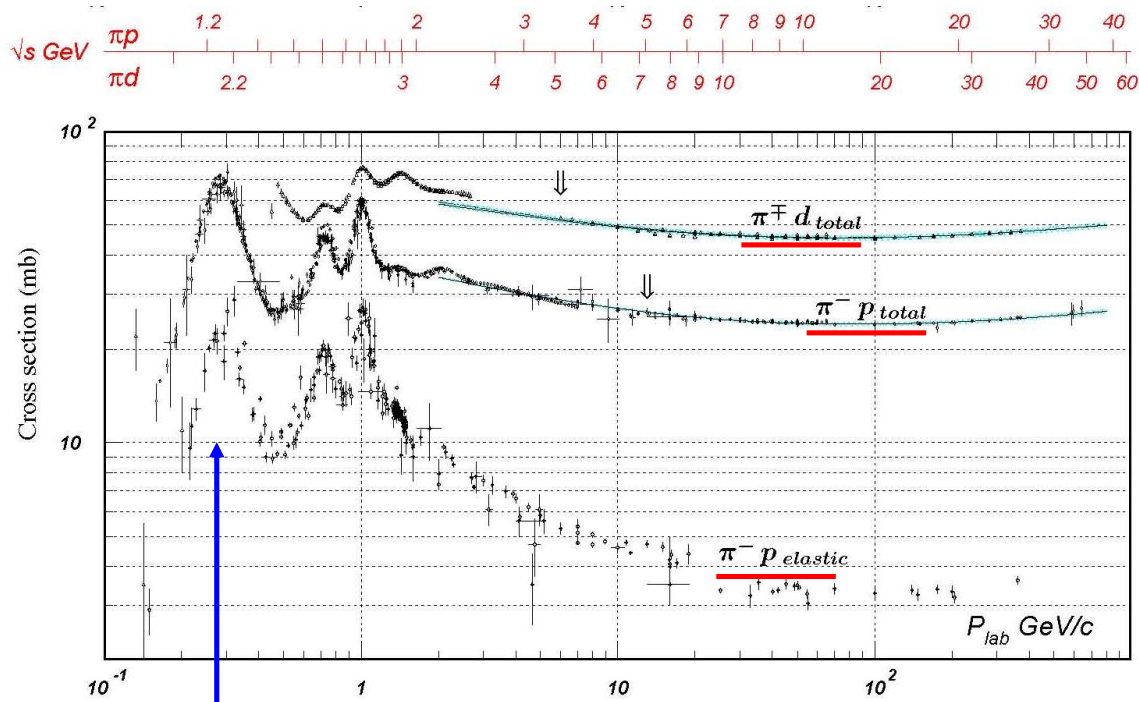
$$m_\pi = 138 \text{ MeV}$$

Peak in the elastic cross section $\pi^+ p$



From angular distribution of the decay products we derive that the spin of the Δ is $3/2$

the Δ resonance: cross-section π^-p , π^-n , π^+p , π^+n



$M = 1232 \text{ MeV}$

peak position is the same

... and Γ_{tot} is the same too

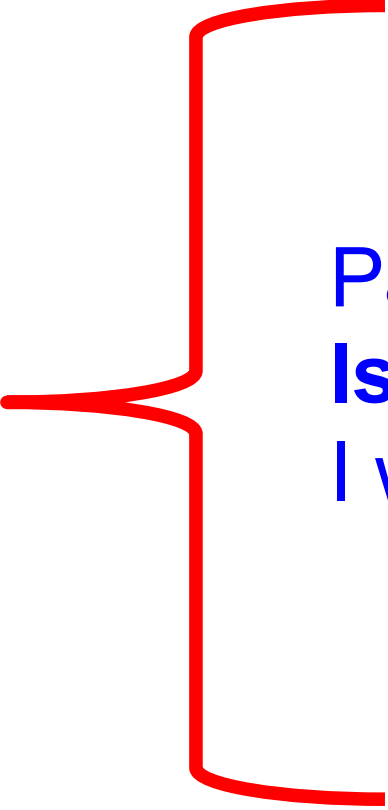
N.B. in the π^-p channel σ_{elastic} and σ_{total} are different

$$\begin{aligned}\sigma_{\text{picco}}(\pi^-p \rightarrow \pi^-p) &= 22 \text{ mb} \\ \sigma_{\text{picco}}(\pi^-p \rightarrow \pi^0n) &= 45 \text{ mb}\end{aligned}$$

$$\sigma_{\text{peak}}(\pi^+p \rightarrow \pi^+p) = 195 \text{ mb}$$

Question: why σ_{elastic} in the channels π^-p e π^+p are different? The answer is in the Δ isospin.

$$\frac{\sigma(\pi^+p \rightarrow \pi^+p)}{\sigma(\pi^-p \rightarrow \pi^-p)} = \frac{195 \text{ mb}}{22 \text{ mb}} = 8.86 \approx 9$$



Parenthesis

Isospin: this is just for your reading,
I will not explain it in the lecture

ISOSPIN: Gell-Mann – Nishijima's formula

The third component of the isospin distinguishes the electrical charge within an isospin multiplet

charge → $Q = I_3 + \frac{1}{2}(B+S)$ ← strangeness
 ← Baryonic number

N.B. $B+S = Y$ (hypercharge)

→ $Q = I_3 + \frac{1}{2} Y$

The electromagnetic interaction breaks the isospin symmetry; as a consequence, the masses within a multiplet are different (m_p different from m_n)

Isospin

Let's see a dynamical consequence of the isospin conservation

- Let's suppose to have two nucleons. From the rule of the addition of angular momentum we know that the total isospin can be **1** or **0**.

Symmetric triplet; $I = 1$

a) $|1,1\rangle = pp$

b) $|1,0\rangle = \frac{1}{\sqrt{2}} (pn + np)$

c) $|1,-1\rangle = nn$

Antisymmetric isosinglet; $I = 0$

$$|0,0\rangle = \frac{1}{\sqrt{2}} (pn - np)$$

- It exists a bound state proton-neutron (deuteron), but do not exist bound states proton-proton or neutron-neutron, hence the deuteron must be an isospin singlet, otherwise they should exist also the other two states that differ by a rotation in the isospin space.

nucleon-nucleon scattering

Let's consider the following processes:

- a) $p + p \rightarrow d + \pi^+$
- b) $p + n \rightarrow d + \pi^0$
- c) $n + n \rightarrow d + \pi^-$

the π has isospin 1 because it exists in three different states

□ Since the deuteron has $I=0$, for the right hand processes we have:

$$d + \pi^+ = |1,1\rangle ; d + \pi^0 = |1,0\rangle ; d + \pi^- = |1,-1\rangle$$

while for the ones on the left we have:

$$p + p = |1,1\rangle ; p + n = \frac{1}{\sqrt{2}}(|1,0\rangle + |0,0\rangle) ; n + n = |1,-1\rangle$$

□ Since the total isospin I must be conserved, only the state with $I=1$ will contribute.
The scattering amplitudes have to be in the ratio:

$$1 : \frac{1}{\sqrt{2}} : 1$$

and the σ

$$2 : 1 : 2$$

□ The processes a) and b) have been measured, and once we take into account the e.m. interaction, they are in the predicted ratio.

pion-nucleon scattering

Let's consider the four reactions:

$$\text{a) } \pi^+ + p \rightarrow \pi^+ + p$$

$$\text{b) } \pi^- + p \rightarrow \pi^0 + n$$

$$\text{c) } \pi^- + p \rightarrow \pi^- + p$$

$$\text{d) } \pi^- + n \rightarrow \pi^- + n$$

The initial states are the composition of $l=1$ and $l=1/2$ that give $l=1/2$ and $l=3/2$

$$1 \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{3}{2}$$

□ Let's express the various states in the base of the total isospin by using the Clebsch-Gordan coefficients

$$\begin{aligned} |\pi^+, p\rangle &= \left| \frac{3}{2}, +\frac{3}{2} \right\rangle \quad ; \quad |\pi^-, p\rangle = \sqrt{\frac{1}{3}} \left| \frac{3}{2}, -\frac{1}{2} \right\rangle - \sqrt{\frac{2}{3}} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \\ |\pi^-, n\rangle &= \left| \frac{3}{2}, -\frac{3}{2} \right\rangle \quad ; \quad |\pi^0, n\rangle = \sqrt{\frac{2}{3}} \left| \frac{3}{2}, -\frac{1}{2} \right\rangle + \sqrt{\frac{1}{3}} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \end{aligned}$$

pion-nucleon scattering

□ Let's write the four processes in the new base:

$$\begin{aligned}
 \text{a) } & \left| \frac{3}{2}, +\frac{3}{2} \right\rangle \rightarrow \left| \frac{3}{2}, +\frac{3}{2} \right\rangle \\
 \text{b) } & \sqrt{\frac{1}{3}} \left| \frac{3}{2}, -\frac{1}{2} \right\rangle - \sqrt{\frac{2}{3}} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \rightarrow \sqrt{\frac{2}{3}} \left| \frac{3}{2}, -\frac{1}{2} \right\rangle + \sqrt{\frac{1}{3}} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \\
 \text{c) } & \sqrt{\frac{1}{3}} \left| \frac{3}{2}, -\frac{1}{2} \right\rangle - \sqrt{\frac{2}{3}} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \rightarrow \sqrt{\frac{1}{3}} \left| \frac{3}{2}, -\frac{1}{2} \right\rangle - \sqrt{\frac{2}{3}} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \\
 \text{d) } & \left| \frac{3}{2}, -\frac{3}{2} \right\rangle \rightarrow \left| \frac{3}{2}, -\frac{3}{2} \right\rangle
 \end{aligned}$$

□ to compute the probability amplitude, we have to perform the scalar product $\langle f|S|i\rangle$

$$\left\langle \frac{3}{2}, I_3 \right| S \left| \frac{3}{2}, I_3 \right\rangle = A_{3/2} ; \quad \left\langle \frac{1}{2}, I_3 \right| S \left| \frac{1}{2}, I_3 \right\rangle = A_{1/2}$$

$$\begin{aligned}
 \text{a) } & A_{\text{tot}} = A_{3/2} \\
 \text{b) } & A_{\text{tot}} = \left(\frac{\sqrt{2}}{3} A_{3/2} - \frac{\sqrt{2}}{3} A_{1/2} \right) \\
 \text{c) } & A_{\text{tot}} = \left(\frac{1}{3} A_{3/2} + \frac{2}{3} A_{1/2} \right) \\
 \text{d) } & A_{\text{tot}} = A_{3/2}
 \end{aligned}$$

N.B. strong interactions do not mix states with different total isospin

pion-nucleon scattering

- ❑ The cross-sections of the four processes are proportional, by means of a factor K equal for all 4 processes (that takes into account the phase space, 2π factors, etc...), to:

$$\begin{aligned} \text{a) } \sigma(\pi^+ + p \rightarrow \pi^+ + p) &= K \left| A_{3/2} \right|^2 \\ \text{b) } \sigma(\pi^- + p \rightarrow \pi^0 + n) &= K \left| \frac{\sqrt{2}}{3} A_{3/2} - \frac{\sqrt{2}}{3} A_{1/2} \right|^2 \\ \text{c) } \sigma(\pi^- + p \rightarrow \pi^- + p) &= K \left| \frac{1}{3} A_{3/2} + \frac{2}{3} A_{1/2} \right|^2 \\ \text{d) } \sigma(\pi^- + n \rightarrow \pi^- + n) &= K \left| A_{3/2} \right|^2 \end{aligned}$$

- ❑ From these relations we infer that the processes a) and d) must have the same cross-section at the same energy. This has been verified experimentally.
- ❑ For the other processes we need to know $A_{1/2}$ and the relative phase between the amplitudes.
- ❑ **N.B. To compute K, $A_{1/2}$ and $A_{3/2}$ we need the underlying theory of the strong interactions (that we didn't have).**

The Δ resonance

❑ The Δ resonance has isospin 3/2 (it exists in 4 states of different charge), therefore all processes in which it appears as formation resonance can proceed only through the channel with $I=3/2$ (strong interactions conserve isospin). As a consequence:

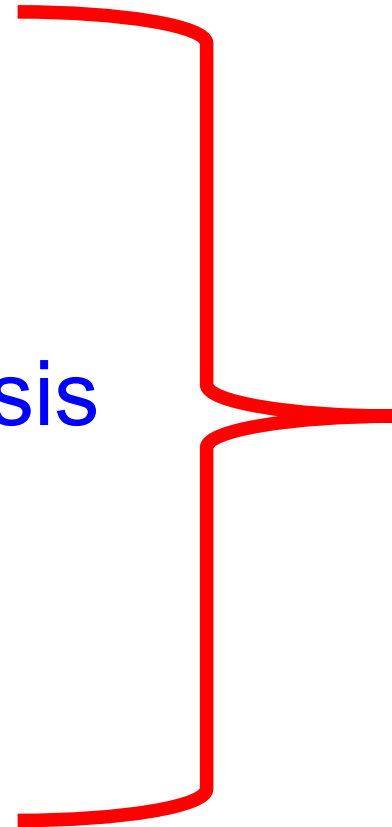
$$\begin{aligned} \text{a) } \sigma(\pi^+ + p \rightarrow \pi^+ + p) &= K \left| A_{3/2} \right|^2 \\ \text{b) } \sigma(\pi^- + p \rightarrow \pi^0 + n) &= K \left| \frac{\sqrt{2}}{3} A_{3/2} \right|^2 = \frac{2}{9} \left| A_{3/2} \right|^2 \\ \text{c) } \sigma(\pi^- + p \rightarrow \pi^- + p) &= K \left| \frac{1}{3} A_{3/2} \right|^2 = \frac{1}{9} \left| A_{3/2} \right|^2 \\ \text{d) } \sigma(\pi^- + n \rightarrow \pi^- + n) &= K \left| A_{3/2} \right|^2 \end{aligned}$$

❑ from these relations we can infer now:

$$\frac{\sigma(\pi^+ + p \rightarrow \pi^+ + p)}{\sigma(\pi^- + p \rightarrow \pi^- + p)} = 9 ; \quad \frac{\sigma(\pi^- + p \rightarrow \pi^0 + n)}{\sigma(\pi^- + p \rightarrow \pi^- + p)} = 2$$

❑ that we know they have been verified experimentally.

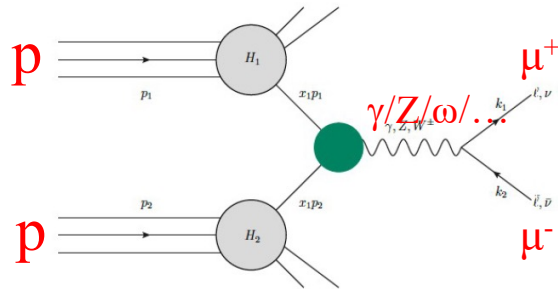
Let's close the parenthesis



Production Resonance: an example

$$p+p \rightarrow \mu^+ + \mu^- + X$$

production resonance



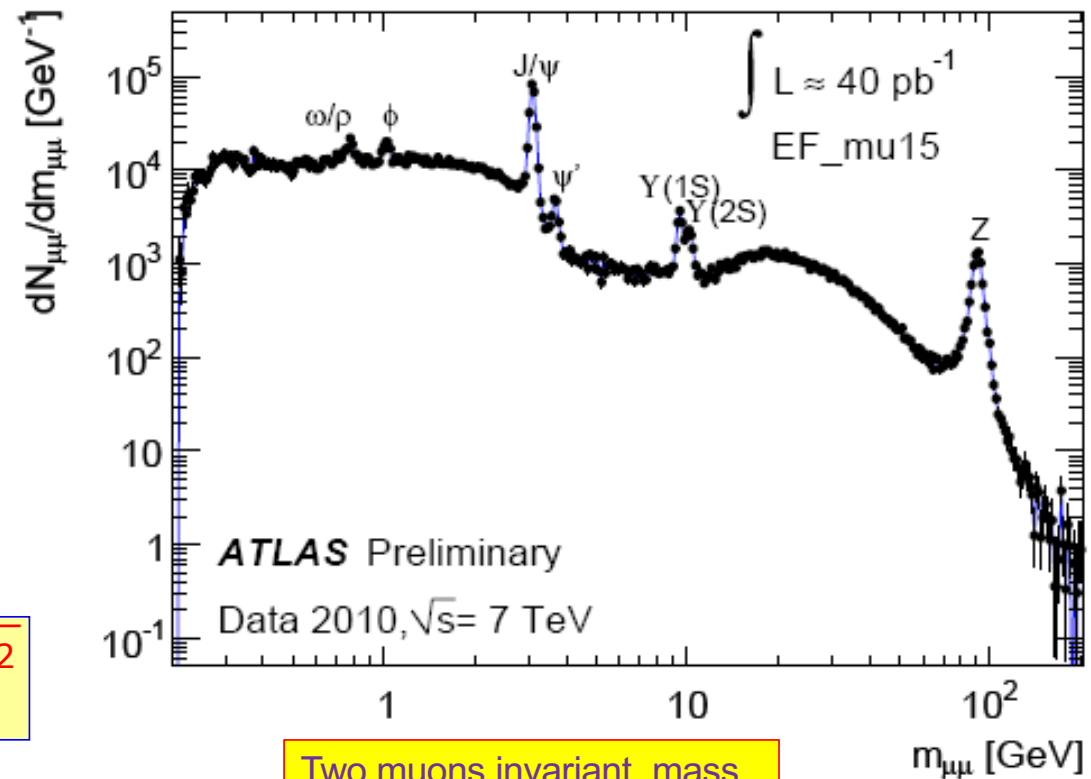
Drell-Yan process
(the resonance must have the same quantum numbers of the photon (1⁻))

$$m_{\mu\mu} = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}$$

It is a relativistic invariant

ATLAS: 50 years of history in one slide

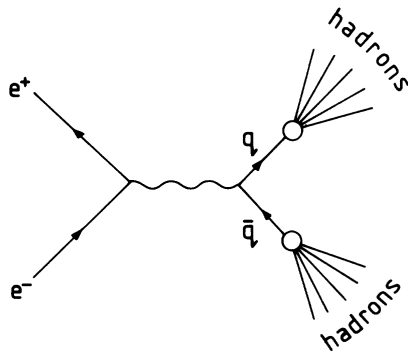
Spin 1 mesons with different quark compositions



Two muons invariant mass

Cross Section $e^+e^- \rightarrow \text{hadrons}$

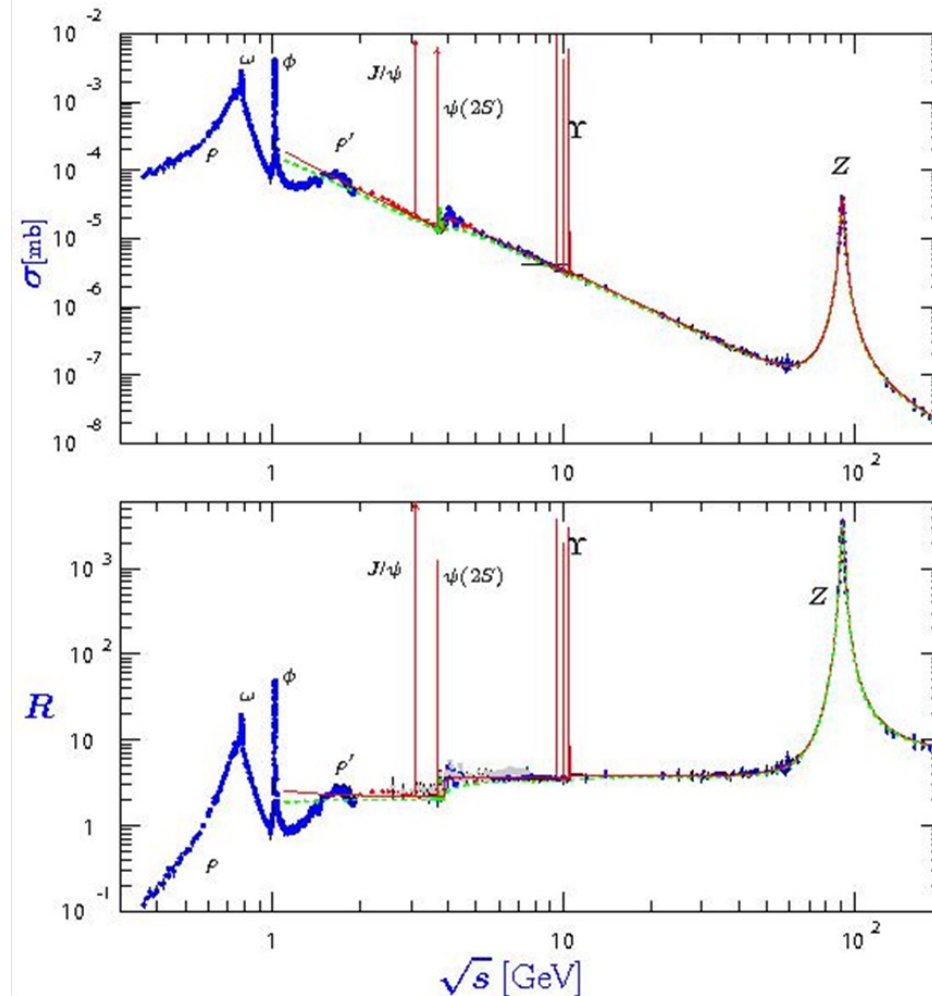
$$\sigma(e^+e^- \rightarrow \text{hadrons})$$



$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

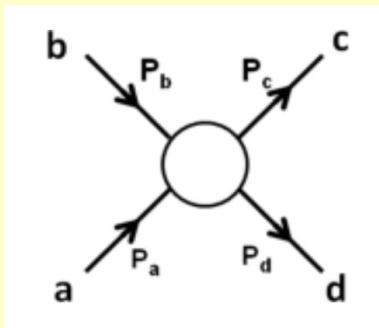
N.B.: the resonances are much narrower than the case of $\mu^+\mu^-$ invariant mass: **why?**

Question: here we are talking about formation resonances or production resonances?



Strong Interaction Processes in the '60s.

- ❑ There was no strong interaction theory in the '60s, so it was not possible to predict the value of a cross-section to be compared with the experimental measurements.
- ❑ A possibility was to compare ratios of cross-sections where the coupling constants cancel (see the Δ resonance).
- ❑ People tried also to exploit symmetry properties, for instance the crossing symmetry, to try to guess a cross-section
 - A process where a particle with a 4-momentum p_μ in the initial (final) state has the same amplitude of the process where it is replaced by its antiparticle in final (initial) state with the same 4-momentum



The graph describes these three processes
(s becomes t if you rotate the graph by 90°)

$a + b \rightarrow c + d$	s - channel
$a + \bar{d} \rightarrow \bar{b} + c$	t - channel
$a + \bar{c} \rightarrow \bar{b} + d$	u - channel

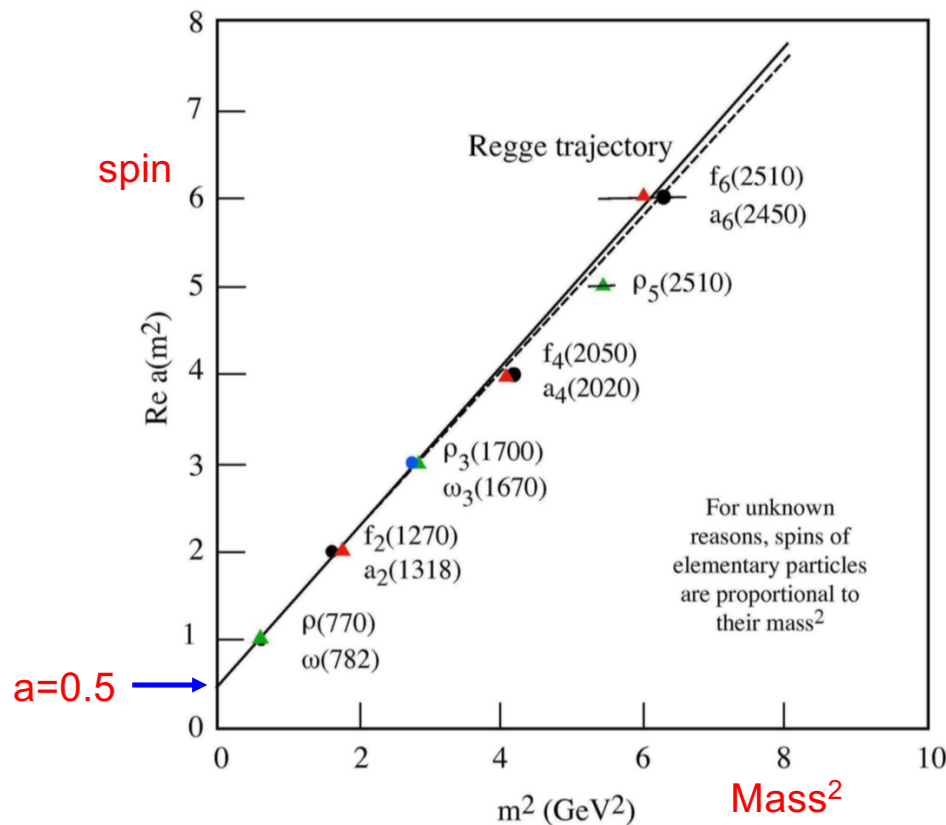
These processes involve different regions of the parameter space; variables s, t, u are the Mandelstam variables

$$s = (p_a + p_b)^2 ; t = (p_a - p_d)^2 ; u = (p_a - p_c)^2$$

- ❑ People tried also to find some patterns among the different particle in order to have an understanding of the underlying theory, for instance
 - $SU(3)$ led to quarks, colour and, eventually, to QCD
- ❑ A less known classification was the one based on Regge poles

Regge theory

- Tullio Regge studied the analytical properties of the scattering amplitude of the collision process between two particles. He considered (in 1959) the angular momentum as a complex variable and derived the singularities of the scattering amplitude that became universally known as Regge poles.



The present situation of the Chew–Frautschi plot shows that the Regge trajectory containing the ρ meson (mass = 770 MeV) is practically linear up to very large masses

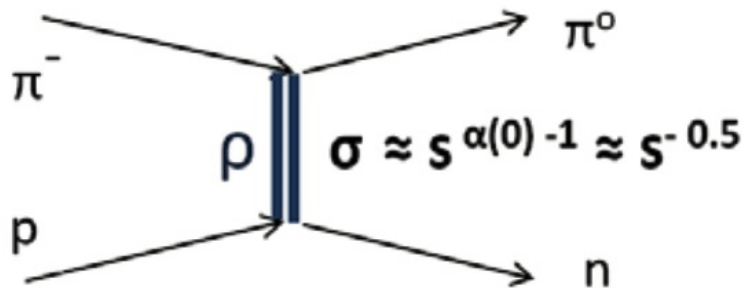
For unknown reasons, spins of elementary particles are proportional to their mass^2

In 1960 Chew and *Frautschi* conjectured that the strongly interacting particles had a very simple dependence of the squared-mass on the angular momentum: the particles fall into families where the Regge trajectory functions were straight lines with the same slope for all the trajectories. *The straight-line Regge trajectories were later understood as arising from massless endpoints on rotating relativistic strings.* Since a Regge description implied that the particles were bound states, *Chew and Frautschi concluded that none of the strongly interacting particles were elementary.*

Regge theory: pion-proton scattering

- The exchange of the ρ trajectory dominates the charge-exchange cross-section of the pion-proton interaction.

According to the Regge theory the cross-section should vary as $s^{\alpha(t=0)-1} = 1/E_{\text{cm}} [\alpha(0) \approx 0.5]$



In the 1960s the experimental confirmation of this prediction was one of the strongest arguments in favour of the Regge description of the scattering of two hadrons. Such a description is still used because these phenomena cannot be computed with quantum chromodynamics

Pion-Proton Charge-Exchange Scattering from 500 to 1300 MeV*

CHARLES B. CHIU, RICHARD D. EANDI, A. CARL HELMHOLZ, ROBERT W. KENNEY,
BURTON J. MOYER, JOHN A. POIRIER,[†] AND W. BRUCE RICHARDS[‡]
Lawrence Radiation Laboratory, University of California, Berkeley, California

AND

ROBERT J. CENCE, VINCENT Z. PETERSON, NARENDER K. SEHGAL, AND VICTOR J. STENGER
University of Hawaii, Honolulu, Hawaii

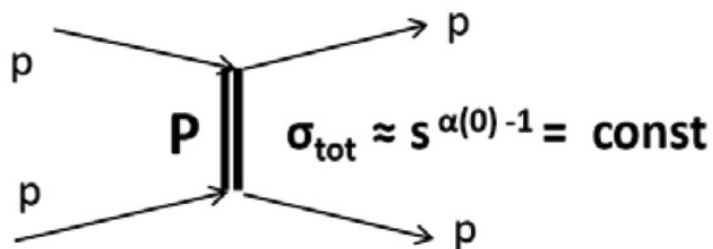
(Received 16 November 1966)

Differential cross sections for the reaction $\pi^- p \rightarrow \pi^0 n$ were measured at nine incident-pion kinetic energies in the interval from 500 to 1300 MeV. The negative pion beam from the bevatron was focused on a liquid-hydrogen target completely surrounded by a cubic array of six steel-plate spark chambers. The spark

One of the many
papers on this
subject

Regge theory: proton-proton scattering

- ❑ In the Regge model, the exchange of a **pomeron** trajectory is the dominant phenomenon in all high-energy elastic collisions.
- ❑ In the “t-channel view” $\alpha(t = 0) = 1 \rightarrow$ energy-independent total cross-section, as confirmed by experiments before ISR results.



The pomeron itself was introduced by V. Gribov and he incorporated the 'Pomeranchuk' theorem into the Regge theory.

*The modern interpretation is that the pomeron has no conserved charges (electric charge or color charge) and the particles on his Regge trajectory **have the quantum numbers of the vacuum.***

S-channel description theorems:

- **Pomeranchuk theorem:** in the the limit $s \rightarrow \infty$, the hadron–hadron and the antihadron–hadron cross-sections become equal.
- **Froissart-Martin theorem:** the total cross-section should satisfy the bound

$$\sigma_{\text{tot}} \leq C \ln^2(s/s_0) \approx 60 \text{ mb} \ln^2(s/s_0)$$

where the numerical value $C = \pi(\hbar/m\pi)^2$ is determined by the mass of the pion, which is the lightest particle that can be exchanged between the two colliding hadrons, and s_0 is usually taken equal to 1 GeV^2 .

One of the tasks of the ISR experiments was the measurement of the proton-proton cross-section

**Just a reminder about what is a
cross-section and how it could
be measured**

Cross-section: geometrical definition

$a + b \rightarrow \text{anything}$

Bullet (beam)

target

The cross section is proportional to the probability of a given process

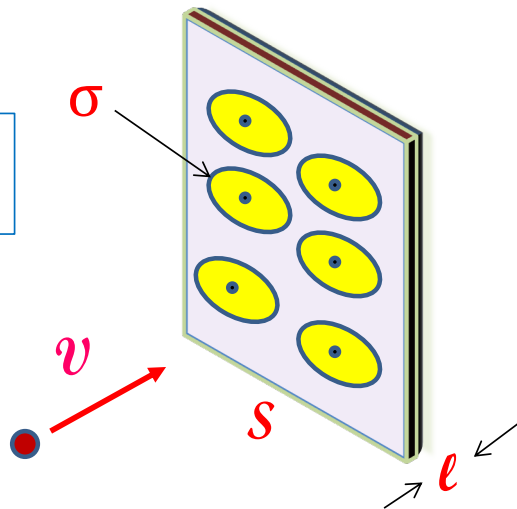
Thin target approximation:
 $\ell \ll \text{attenuation length } \lambda$

σ : effective area of a target particle

N_t : number of target particles

S : total target area

□ a



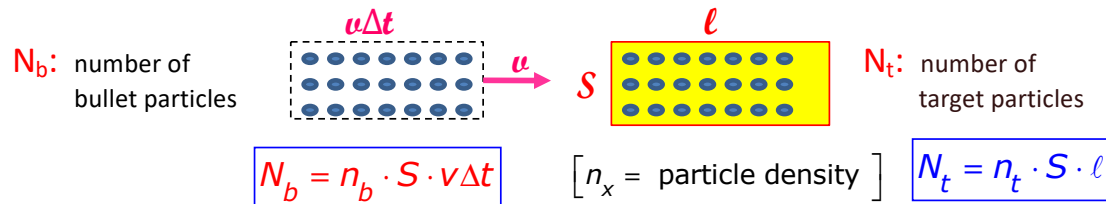
Probability that the bullet hit a target particle

$$p = \frac{\text{effective area}}{\text{total area}} = \frac{N_t \cdot \sigma}{S}$$

Cross-section measurement: fixed target

$$a + b \rightarrow \text{anything}$$

The cross section σ is proportional to the probability p of a given process



$$\left(\text{N.B. } \lambda = \frac{1}{n_t \cdot \sigma} \right)$$

λ is the attenuation length; σ is the cross-section
 n_t is the density of the target

$$\Phi = n_b \cdot v = \text{particles flux} \Rightarrow N_b = \Phi \cdot S \cdot \Delta t$$

□ Number of interactions (Nevents):

$$N_{events} = N_b \cdot p = N_b \cdot \frac{N_t \cdot \sigma}{S} = \Phi \cdot S \cdot \Delta t \cdot \frac{N_t \cdot \sigma}{S} = \Phi \cdot N_t \cdot \sigma \cdot \Delta t$$

□ Number of interactions in the time interval Δt :

$$\frac{N_{eventi}}{\Delta t} = \sigma \cdot \Phi \cdot N_t$$

Φ depends on the properties of the beam extracted from the accelerator; N_t is a characteristic of the target



$$\sigma = \frac{N_{events}}{\Delta t} \cdot \frac{1}{\Phi} \cdot \frac{1}{N_t}$$

N_{events} are the ones you select in your experiment

Interaction probability per unit of time, unit of area and only one target particle (transition probability, that is what you get from theory calculation and you can compare with data):

$$W = \sigma \cdot \Phi$$

Cross-section measurement: colliders

- ❑ We have the following relationship among Number of Event selected, Integrated Luminosity and cross-section of a given process
(for the time being we do not consider efficiency and background contamination, i.e. $\epsilon=1$ and $p=1$)

$$N_{exp} = \mathcal{L}_{int} \cdot \sigma$$

- ❑ If we want to measure the cross section we use the following relationship:

$$\sigma = \frac{N_{exp}}{\mathcal{L}_{int}}$$

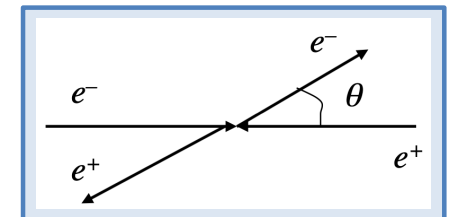
- ❑ ... but if we wanted to measure the Integrated Luminosity we could turn the formula around:

$$\mathcal{L}_{int} = \frac{N_{exp}}{\sigma}$$

The luminosity measurement coming from the collider parameters is not good enough (namely, not precise enough) to be used in the analysis.

- ❑ ... therefore we have to find a process for which we know how to calculate the cross section with great precision.

- for e⁺e⁻ collider we can use the Bhabha scattering at small angle that is a pure QED process:
- We do not have a similar process for a proton-proton collider and we need to use other “tricks”, for instance the optical theorem.



A thought about total cross section measurement

- ❑ In principle, the total cross-section measurement seems to be a straightforward measurement:

$$\sigma = \frac{N_{exp}}{\mathcal{L}_{int}}$$

- ❑ You count **ALL** events and you divide by the integrated luminosity.
- ❑ The point is ... how do we make sure that we really counted all events and we didn't miss any?
 - Maybe there is a final state that we didn't consider or the apparatus is not sensitive about it (for instance, the neutrinos)
 - or the apparatus do not cover the entire solid angle and there are holes (usually, we use extrapolations)
- ❑ So, it is much easier to do an exclusive measurement (namely, just for a given final state) and in a given portion of the solid angle
 - then, the total cross-section is obtained as the sum of the cross-section of the final states.
- ❑ Last but not least, the theoretical calculations are always exclusive

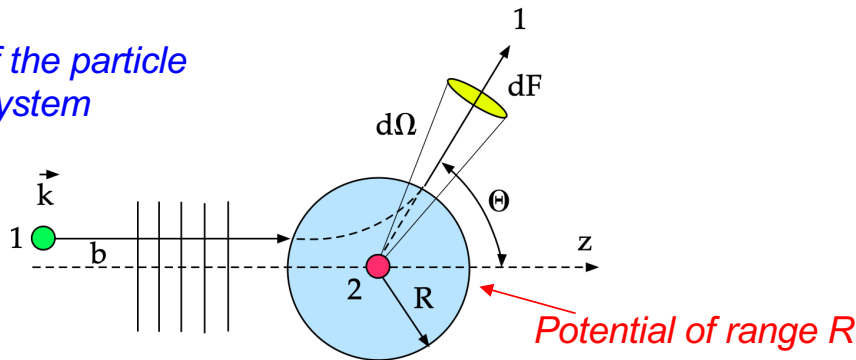
Partial wave analysis, optical theorem and Total Cross-section Measurements

(this should be part of the exam program ... even though it is difficult to remember the formulae)

Partial wave analysis

- elastic scattering between two particles of mass m_1 and m_2

k : momentum of the particle in the CoM system



At $r \gg R$

$$\psi(r, \theta) = e^{ikz} + f(\theta) \frac{e^{ikr}}{r}$$

Incoming plane wave *Scattered radial wave*
scattering amplitude

- $f(\theta)$ can be parameterised in terms of partial waves, that is as a function of angular momentum L .

$$f(\theta) = \frac{1}{k} \sum_{\ell=0}^{\infty} (2\ell + 1) \left[\frac{\eta_{\ell} e^{2i\delta_{\ell}} - 1}{2i} \right] P_{\ell}(\cos \theta)$$

δ_{ℓ} : phase shift; η_{ℓ} : inelasticity parameter
Legendre Polynomials

- The total elastic cross-section is equal to: $\sigma_e = \int |f(\theta)|^2 d\Omega = \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{\max}} (2\ell + 1) |\eta_{\ell} e^{2i\delta_{\ell}} - 1|^2$

- The inelastic cross-section is: $\sigma_r = \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{\max}} (2\ell + 1) (1 - \eta_{\ell}^2)$ $\ell_{\max} = kR$ $\eta_{\ell}=1$ (elastic); $\eta_{\ell}<1$ (inelastic)

Optical theorem

□ The total cross-section (elastic plus inelastic) is: $\sigma_t = \sigma_e + \sigma_r = \frac{2\pi}{k^2} \sum_{\ell=0}^{\ell_{\max}} (2\ell + 1)(1 - \eta_\ell \cos 2\delta_\ell).$

□ From the elastic scattering amplitude we find that the imaginary part at $\theta=0$ is:

$$\text{Im}f(0) = \frac{1}{2k} \sum_{\ell=0}^{\ell_{\max}} (2\ell + 1)(1 - \eta_\ell \cos 2\delta_\ell).$$

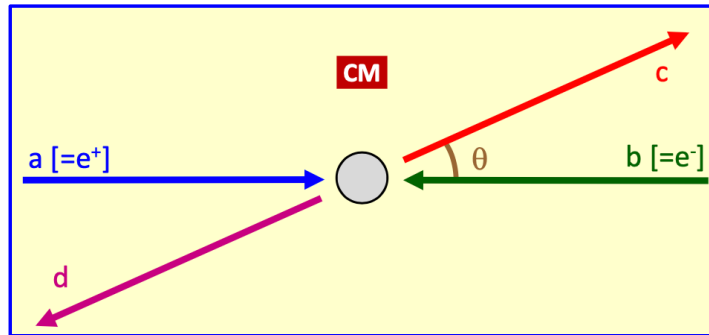
□ If we compare the two expressions we find the **optical theorem**: $\sigma_t = \frac{4\pi}{k} \text{Im}f(0)$

□ This theorem is a wave mechanics relation between two unknown quantities: σ_t and $\text{Im}f(0)$.

The dynamics, carried by the potential scattering $V(r)$, is contained in the scattering amplitude **$f(\theta)$** or, in an analogous way, in the **phase shifts δ_ℓ** and in the **inelasticity parameters η_ℓ**

□ The optical theorem is used to measure the total cross section in the hadron collider such as LHC (or ISR)

Mandelstam variables: s, t, u



CM system

- $p_a = [E, \quad p, \quad 0, \quad 0];$
- $p_b = [E, \quad -p, \quad 0, \quad 0];$
- $p_c = [E, \quad p \cos\theta, p \sin\theta, \quad 0];$
- $p_d = [E, -p \cos\theta, -p \sin\theta, 0];$

Lorentz-invariant variables for 2→2 processes.

Assume $E \gg m_i$, for the masses of all 4 bodies (otherwise, look for the formulæ in [PDG]).

s,t,u L-invariant

- $s \equiv (p_a + p_b)^2 = (p_c + p_d)^2 = 4E^2;$
- $t \equiv (p_a - p_c)^2 = (p_b - p_d)^2 \approx -\frac{1}{2} s (1 - \cos\theta) = -s \sin^2(\theta/2);$
- $u \equiv (p_a - p_d)^2 = (p_b - p_c)^2 \approx -\frac{1}{2} s (1 + \cos\theta) = -s \cos^2(\theta/2);$
- $s + t + u = 0$ (→ 1+1 independent variables, e.g. $[E, \theta]$, $[s, t]$, $[\sqrt{s}, \theta]$)

If $\theta \rightarrow 0 \Rightarrow t \rightarrow 0$

Total cross section determination



$$\sigma_t = \frac{4\pi}{k} \text{Im} f(0)$$

$$k = \frac{\sqrt{s}}{2} = \frac{\sqrt{4E^2}}{2}$$

$$\text{Im} f(t=0) = \frac{\sqrt{s}}{8\pi} \sigma_t$$

Proton momentum in the CoM

□ We need to derive $\text{Im} f(t=0)$ from the elastic scattering at very low angle.

1. Define the differential cross-section in terms of $f_{\text{el}}(\theta)$: $\sigma_{\text{el}} = \int |f_{\text{el}}(\theta)|^2 d\Omega \Rightarrow \frac{d\sigma_{\text{el}}}{d\Omega} \equiv \frac{d^2\sigma_{\text{el}}}{d\phi d\cos\theta} = |f_{\text{el}}(\theta)|^2$

2. We need the relationship between t and $\cos\theta$:

$$t = -\frac{s}{2}(1 - \cos\theta) \Rightarrow \cos\theta = 1 + \frac{2t}{s} \Rightarrow \frac{\partial \cos\theta}{\partial t} = \frac{2}{s} \quad \frac{\partial \sigma}{\partial t} = \frac{\partial \sigma}{\partial \cos\theta} \cdot \frac{\partial \cos\theta}{\partial t}$$

3. We integrate over ϕ , we change variable and we obtain the dependency of the cross section with respect to t :

$$\frac{d\sigma_{\text{el}}}{dt} = \int d\phi \left(\frac{d^2\sigma_{\text{el}}}{d\phi d\cos\theta} \right) \left| \frac{\partial \cos\theta}{\partial t} \right| = 2\pi |f_{\text{el}}(\theta)|^2 \frac{2}{s} = \frac{4\pi}{s} |f_{\text{el}}(s, t)|^2 \Rightarrow |f_{\text{el}}^{t=0}|^2 = \frac{s}{4\pi} \frac{d\sigma_{\text{el}}}{dt} \Big|_{t=0}$$

It is an observable

Total cross section determination



$$\sigma_t = \frac{4\pi}{k} \text{Im} f(0) \quad \text{Im} f(t=0) = \frac{\sqrt{s}}{8\pi} \sigma_t$$

$$\Rightarrow |\text{Im}[f_{el}(0)]|^2 = \frac{s}{64\pi^2} \cdot \sigma_{tot}^2$$

4. Define: $\rho = \text{Re}[f_{el}(0)]/\text{Im}[f_{el}(0)] \Rightarrow |f_{el}(0)|^2 = |\text{Re}[f_{el}(0)]|^2 + |\text{Im}[f_{el}(0)]|^2 = |\text{Im}[f_{el}(0)]|^2 \cdot (1 + \rho^2)$

$$\Rightarrow |f_{el}^{t=0}|^2 = \frac{\sigma_{tot}^2 s}{64\pi^2} (1 + \rho^2)$$

5. In the previous slide we found:

$$|f_{el}^{t=0}|^2 = \frac{s}{4\pi} \frac{d\sigma_{el}}{dt} \Big|_{t=0}$$

6. Combining the the two expression we find:

$$\sigma_{tot} = \sqrt{\frac{16\pi}{1 + \rho^2} \cdot \left(\frac{d\sigma_{el}}{dt} \right)_{t=0}}$$

7. We need the **luminosity** to measure the differential elastic cross section and we need ρ to measure σ_{tot} .

Total cross section determination without the Luminosity

□ define R_{tot} as the total number of events (el. plus inelastic) per second and R_{el} the rate for elastic event:

$$R_{\text{tot}} = \mathcal{L} \sigma_{\text{tot}}, \quad \sigma_{\text{tot}}^2 = \sigma_{\text{tot}} R_{\text{tot}} / \mathcal{L}, \quad R_{\text{el}} = \sigma_{\text{el}} \mathcal{L}, \quad \boxed{d\sigma_{\text{el}}/dt = (dR_{\text{el}}/dt) / \mathcal{L}} \quad \textcolor{red}{L: \textit{luminosity}}$$

□ put together the various pieces:

$$\left| f_{\text{el}}^{t=0} \right|^2 = \frac{\sigma_{\text{tot}}^2 s}{64\pi^2} (1 + \rho^2) = \frac{R_{\text{tot}} \sigma_{\text{tot}}}{\mathcal{L}} \frac{s}{64\pi^2} (1 + \rho^2);$$

$$\left| f_{\text{el}}^{t=0} \right|^2 = \frac{s}{4\pi} \frac{d\sigma_{\text{el}}}{dt} \bigg|_{t=0} \quad \Rightarrow \quad \boxed{\frac{d\sigma_{\text{el}}}{dt} \bigg|_{t=0} = \frac{1}{\mathcal{L}} \frac{dR_{\text{el}}}{dt} \bigg|_{t=0}} = \frac{R_{\text{tot}} \sigma_{\text{tot}}}{16\pi \mathcal{L}} (1 + \rho^2);$$

$$\sigma = \frac{R}{\mathcal{L}}$$

To measure R_{tot} , we have to make sure that, experimentally, we are counting all kind of proton-proton interactions. On top, we have to take into account all the efficiencies to record the events (geometrical acceptance, trigger efficiency, detector efficiency, etc...)

□ We can discard the luminosity in both terms and derive the final formula:

$$\boxed{\sigma_{\text{tot}} = \frac{16\pi(\hbar c)^2}{1 + \rho^2} \frac{1}{R_{\text{tot}}} \frac{dR_{\text{el}}}{dt} \bigg|_{t=0}}.$$

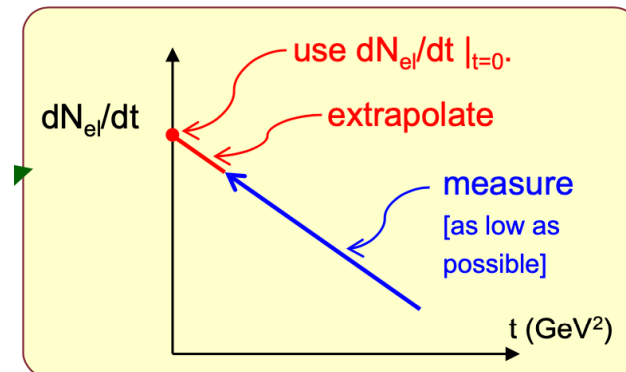
We don't need to know the luminosity

Total cross section determination (without Lum.)

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \Im[f_{\text{el}}(\theta=0)] = \frac{16\pi(\hbar c)^2}{1+\rho^2} \frac{1}{R_{\text{tot}}} \left. \frac{dR_{\text{el}}}{dt} \right|_{t=0}.$$



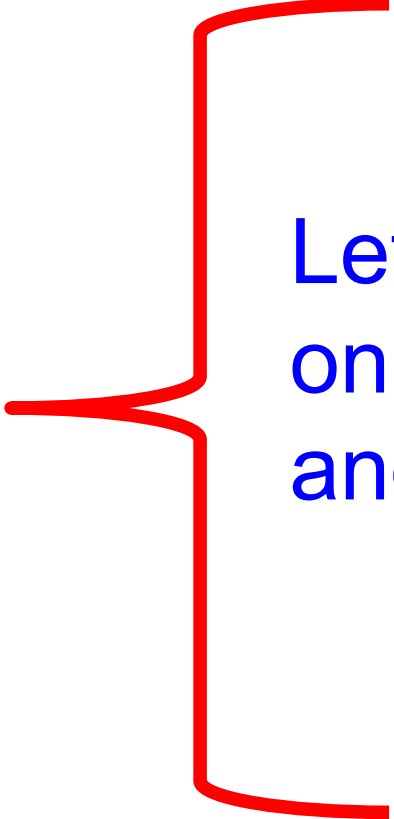
- ❑ Everything (but ρ) is directly measurable $\rightarrow \sigma_{\text{tot}}$ can be measured without knowing the luminosity
- ❑ R_{el} and R_{tot} : only the ratio count $\rightarrow$ do the measurement in the same time interval (N_{el} and N_{tot})
- ❑ $dR_{\text{el}}/dt|_{t=0}$: do the following plot and extrapolate to zero:



$$t = -\frac{s}{2}(1 - \cos\theta)$$

To go to low t , we need to go to small θ , therefore the detectors for this measurement are placed far away from the interaction point and as close as possible to the beam. Moreover, at LHC dedicated runs at high- β are done just for this measurement, to minimize the pile-up

- ❑ The ratio ρ : it can be computed/guessed by first principle; at LHC it is about 0.14 with an error about 0.5%.

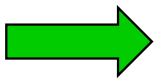


Let's open a parenthesis
on the knowledge of particle physics
and strong interactions in the '60s

(it is not part of the exam program)

Gauge Theories and the Standard Model

- ❑ 1946-49: QED, fully renormalizable gauge theory to describe e.m. interactions, mediated by the photon.
- ❑ 1953: Yang and Mills tried to describe the strong interactions as a gauge field theory based on $SU(2)$ strong isospin symmetry:
 - of course, they failed; QCD came only 20 years later based on $SU(3)$ colour symmetry.
- ❑ 1960: Glashow proposed $SU(2)_L \times U(1)_Y$ as the symmetry group for the electroweak theory.
 - Problem: all particles (fermions and bosons) must be massless
- ❑ 1964: Higgs, Englert & Brout published two independent papers on spontaneous symmetry breaking of a Lagrangian which is invariant under a local gauge transformation.
- ❑ 1967: Weinberg (and later Salam) used the Englert, Brout and Higgs mechanism to give mass to fermions and bosons (actually, electrons/muons ... quarks were not yet there).
 - unintended consequence: a massive scalar boson should also be present: the Higgs boson
 - today is (one of) the most cited paper, but it went unnoticed until middle of '70s.
- ❑ 1971: 't Hooft and Veltman proved that the Weinberg theory is renormalizable



The hunt for the W, Z and H bosons began

1973: first experimental evidence of the Standard Model. Discovery at CERN of "neutral currents" in neutrino-nucleon interactions, which can be explained only by the exchange of a Z.

And then there were ...quarks

- ❑ In the '50s began the "economic boom" for particle physics too. First in cosmic rays and then with the new accelerators a lot of new particles were discovered, too many!
- ❑ The first resonance discovered by Fermi at Chicago in 1953, the Δ , suggested that the proton maybe was not a fundamental particle
- ❑ To put order in the zoo of particles, Gell-Mann and Neeman proposed a classification scheme based on symmetries (SU (3)), which they called: "the eightfold way".
- ❑ The eightfold way predicted a new particle (1962), Ω^- , discovered in 1963 by Samios at the AGS.
- ❑ To explain the symmetry, Gell-Mann and Zweig made the hypothesis that the particles subject to the strong interaction were composed of elementary particles. Gell-Mann called the new particles "quarks" (Zweig called them aces).

"Three quarks for Muster Mark" – James Joyce's Finnegans Wake

quark	charge	strangeness
up	$+2/3 e$	0
down	$-1/3 e$	0
strange	$-1/3 e$	-1

Quarks are very bizarre objects with fractional charge. There was a lot of reluctance to accept them. Zweig was one of the few people firmly believing that the quarks were real particles.

Barions: 3 quark; Mesons: a quark and an antiquark

Greenberg (1964): quarks come in three colours: red, green and blue. Colour charge is the source of strong force



Experimental evidence of the existence of quarks

- ❑ At SLAC, a laboratory near San Francisco, the "monster" came into operation in 1967, a linear accelerator of 20 GeV electrons 2 miles long (*it is still the longest linac in the world*).
- ❑ With an experiment similar to that of Rutherford, but using electrons as projectiles, H.W.Kendal, J.I.Friedman and R.E.Taylor , demonstrated that point-like particles must be present inside protons and neutrons (called partons by Feynman).
- ❑ This result was then confirmed at CERN in the '70s with a neutrino beam, but a lot of things happened in the meanwhile (for instance, GIM).



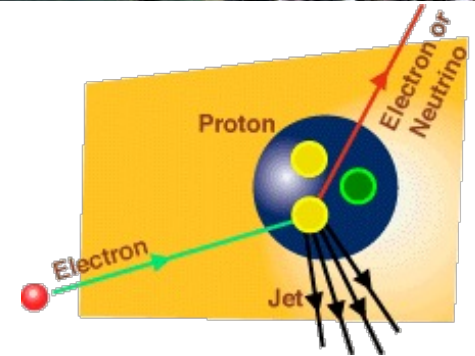
The fundamental particles were (in 1968):

Leptons: e^- , ν_e , μ^- , ν_μ

Quarks: up, down, strange



their antiparticles



The proton is no longer a fundamental particle but there are many things inside (not only the three valence quarks, we have also the sea and the gluons). PDF (parton density functions) were introduced to describe the proton inner content

But ... people really believed in quarks in 1968?

The quest for quarks

Rutherford's Legacy in Particle Physics: Exploring the Proton

Jerome I. Friedman
MIT

Talk by J.I. Friedman at CERN in November 2011 at the conference to celebrate the centenary of the Rutherford's atom.

Prevailing model of the proton in the 1960's

NUCLEAR DEMOCRACY

BOOTSTRAP MODEL

Particles are composites of one another

$$p = \pi^+ + n + \dots$$

$$n = \pi^- + p + \dots$$

Particles have diffuse substructures and no elementary building blocks

Implausibility of Quark Model

“...the idea that mesons and baryons are made primarily of quarks is hard to believe..”

M. Gell-Mann 1966

“Additional data are necessary and very welcome to destroy the picture of elementary constituents.”

J. Bjorken 1967

“I think Professor Bjorken and I constructed the sum rules in the hope of destroying the quark model.”

K. Gottfried 1967

“Of course the whole quark idea is ill founded.”

J.J. Kokkedee 1969

GENERAL POINT OF VIEW IN 1966

*Quarks most likely just mathematical representations
Useful but NOT real !*

Weak interaction in the quark sector

❑ Experimental discrepancy between charged K and charged pion decays violating the weak interaction universality

➤ 1963: Cabibbo: weak eigenstates are not mass eigenstates → Cabibbo angle

❑ Experimental discrepancy between neutral K and charged K decays

➤ 1970: Glashow, Iliopoulos and Maiani (GIM) introduced the quark charm
→ Flavour changing neutral current are suppressed

❑ 1964: Fitch and Cronin discovered CP violation in the K_L decays

➤ 1973: Kobayashi and Maskawa proposed the existence of three quark families in order to introduce a phase in the quark mixing matrix (CKM)

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

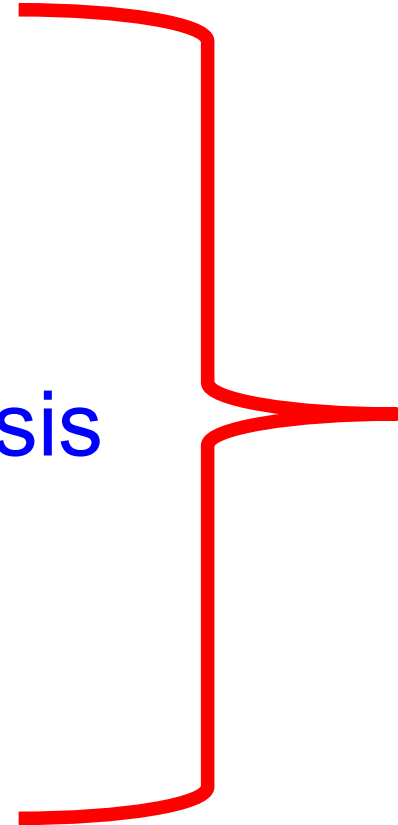
Question: do we have CP violation in other systems besides neutral K?

The amazing years: 1974 ÷ 1977

- ❑ **1970:** Glashow, Iliopoulos and Maiani proposed the existence of a fourth quark, the "charm", charge $+2/3 e$.
- ❑ **1974:** discovery of the quark charm. Ting at BNL and Richter at SLAC. A few weeks later it was also discovered in Frascati pushing Adone beyond its limits (electron-positron collider of 3 GeV) (In life you need (also) luck 😊).
- ❑ **1975:** discovery at SLAC of a third charged lepton, the τ , with a mass about 3500 times greater than that of the electron and an average lifetime of 0.3 ps.
- ❑ **1977:** discovery at FNAL (Chicago) of a fifth quark, the "bottom" or "beauty", charge $-1/3 e$. The bottom was discovered at a new proton accelerator of 500 GeV, 2 km in diameter.

For symmetry reasons, the Standard Model predicts the existence of a third neutrino, the neutrino τ , discovered at FNAL in 2000 and of a sixth quark, the "top" or "truth", discovered at FNAL in 1995, with a mass of about 280 times the proton mass.

Let's close the parenthesis



ISR overview

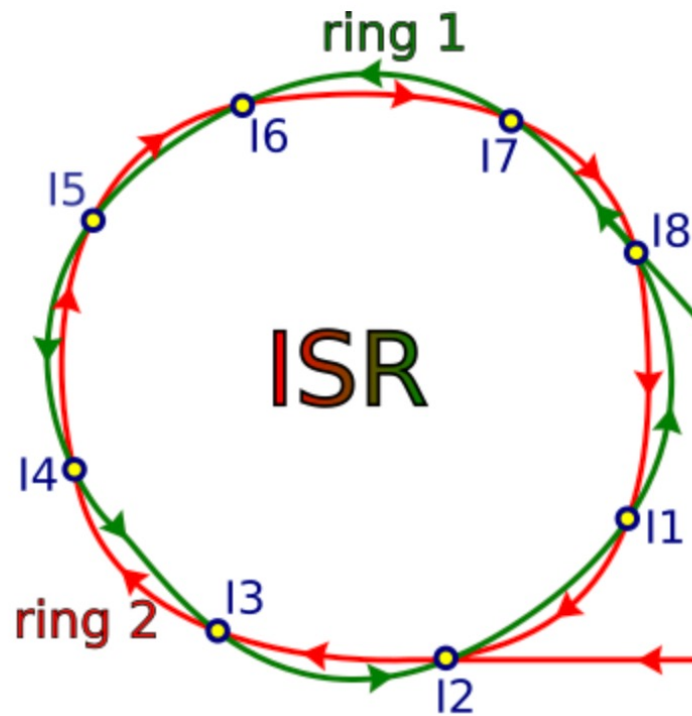
"Blue" slides are taken from Ugo Amaldi presentation "ISR Physics" at The 50th Anniversary of Hadron Colliders at CERN – 14 October 2021-
<https://indico.cern.ch/event/1068633/timetable/>

(This is not part of the exam program, but it is an important step toward the SppS physics)

First hadron collider at CERN ... the ISR

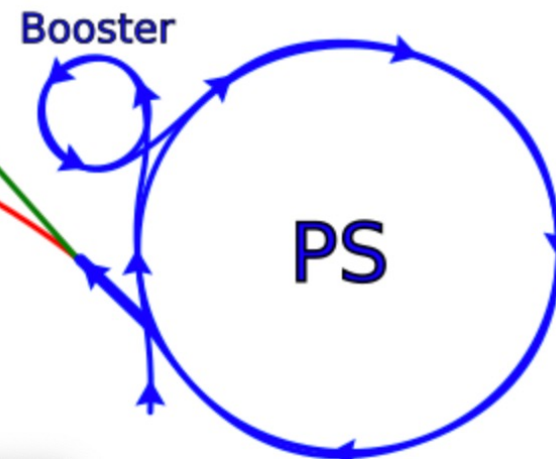
- ❑ In 1956, studies for the second generation of CERN accelerators began and gradually converged towards a proton–proton collider.
- ❑ From 1961 onwards, a study of a 300 GeV proton synchrotron was carried out. It was decided to construct the ISR first.
- ❑ In June 1965 ISR was approved and in December 1965 the construction started.
- ❑ First beams in 1971 and operation for Physics from 1971 to 1983.
- ❑ The ISR was the only CERN collider built without a specific physics goal.
- ❑ The program was shaped by the dominant view at the time:
proton-proton collisions are SOFT processes
- ❑ The ISR Committee favoured the “PS approach”:
many experiments performed by small groups for a short time.

ISR (Intersecting Storage Rings)



ISR - Intersecting Storage Rings
PS - Proton Synchrotron

- 2 Rings with a 300 m diameter.
- 8 interaction points; 12 experiments in 5 points
- Proton beam up to 31 GeV energy (62 GeV CoM energy)
- **It could have discovered:**
 - J/Psi; Upsilon; Jet structures ...
- ... but the detectors were not looking at high-Pt regions
- However machine people learned how to build a hadron collider
- ... and physicists how to build a detector for such a collider



52 GeV ~ 1352 GeV
CoM ISR (26 + 26) Fixed target beam

One of the ISR key performance parameter: vacuum system

- ❑ Usually, the beam is kept inside the accelerator for a “short time”, from a few seconds to a few minutes, while in a collider should be kept for several hours.

- ❑ The integrated luminosity of the ISR was proportional to:

$$\int \frac{I_1 \cdot I_2}{h} dt$$

(I is the beam current and h is the vertical separation at the interaction point)

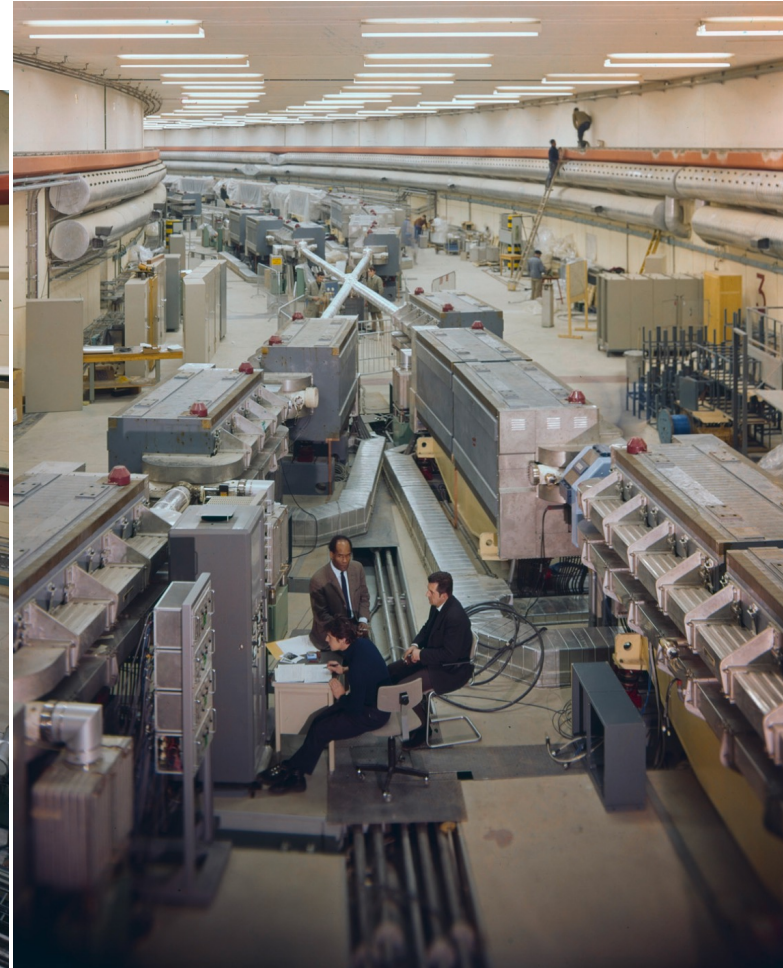
with all three variables depending on time t.

This is just an example of how many new technology challenges had to be overcome to build a collider.

- ❑ Protons in the beams are lost due to nuclear and Coulomb scattering with the residual gas in the beam pipe, and the effective beam height h_{eff} gets blown up by a similar mechanism.
- ❑ Imposing a beam loss of less than 50% and a growth of h_{eff} of less than 40% in 12 h, that will translate in a drop of less than 18% in luminosity after 12 h, the pressure should be less than 10^{-9} Torr over a total length of nearly 2 km (10^{-11} Torr at the interaction points) [1 atm = 760 Torr]
- ❑ Even new methods to measure such a low pressure had to be invented (*they succeeded*)

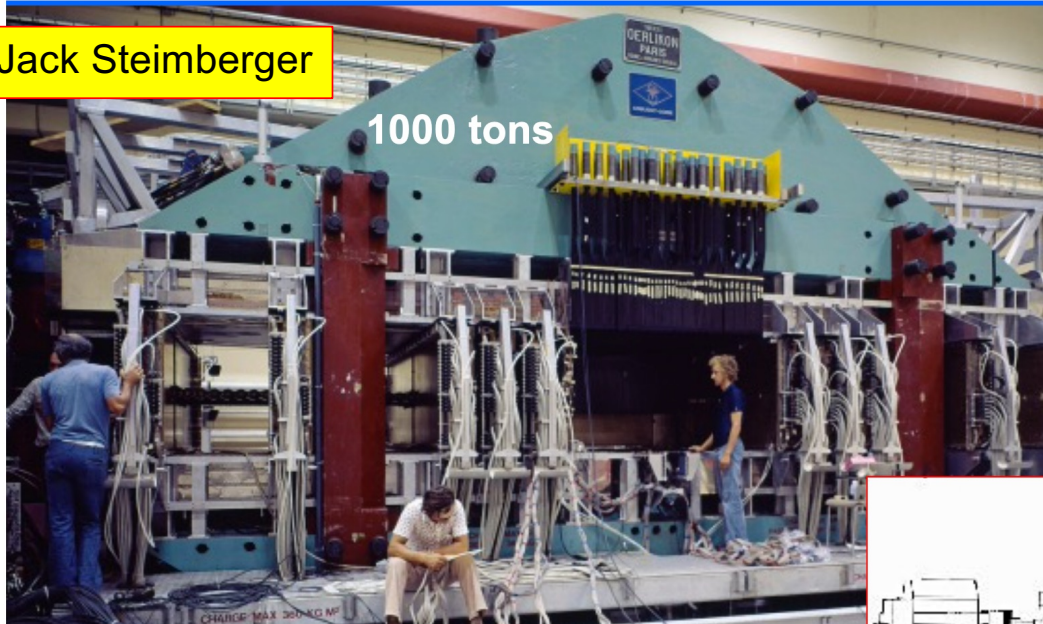
14 intersection point at the ISR

It hosted the split field magnet detector



Split Field Magnet Detector

Team led by Jack Steimberger



1000 tons

FIRST PERIOD
1971-74

Major instrument
for inelastic collisions

Close to the
beam pipe

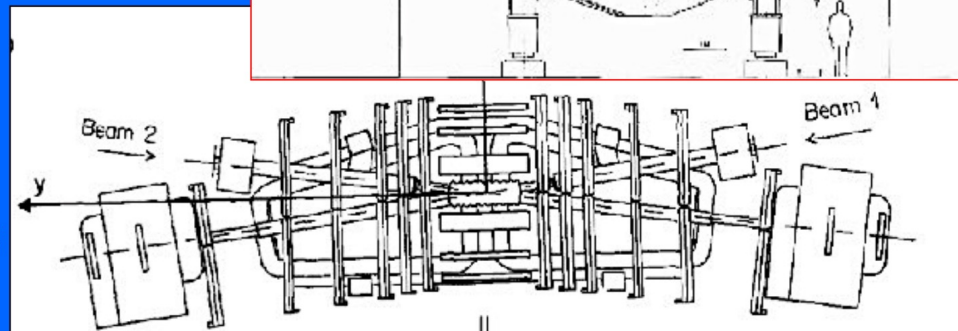
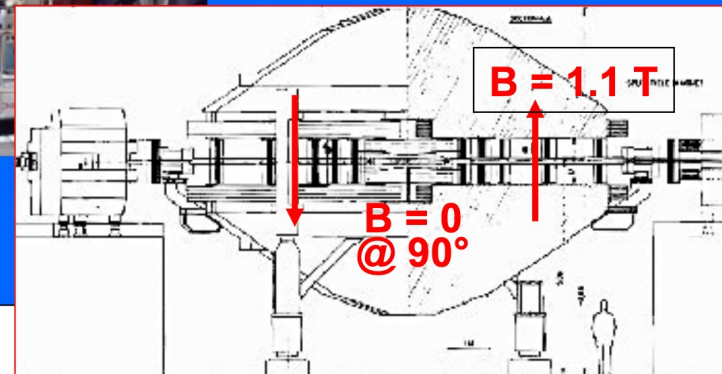
Split Field Magnet
1973

ISRC rejected other proposals

20 MWPC
70,000 wires !

Only 5 years after first MWPC

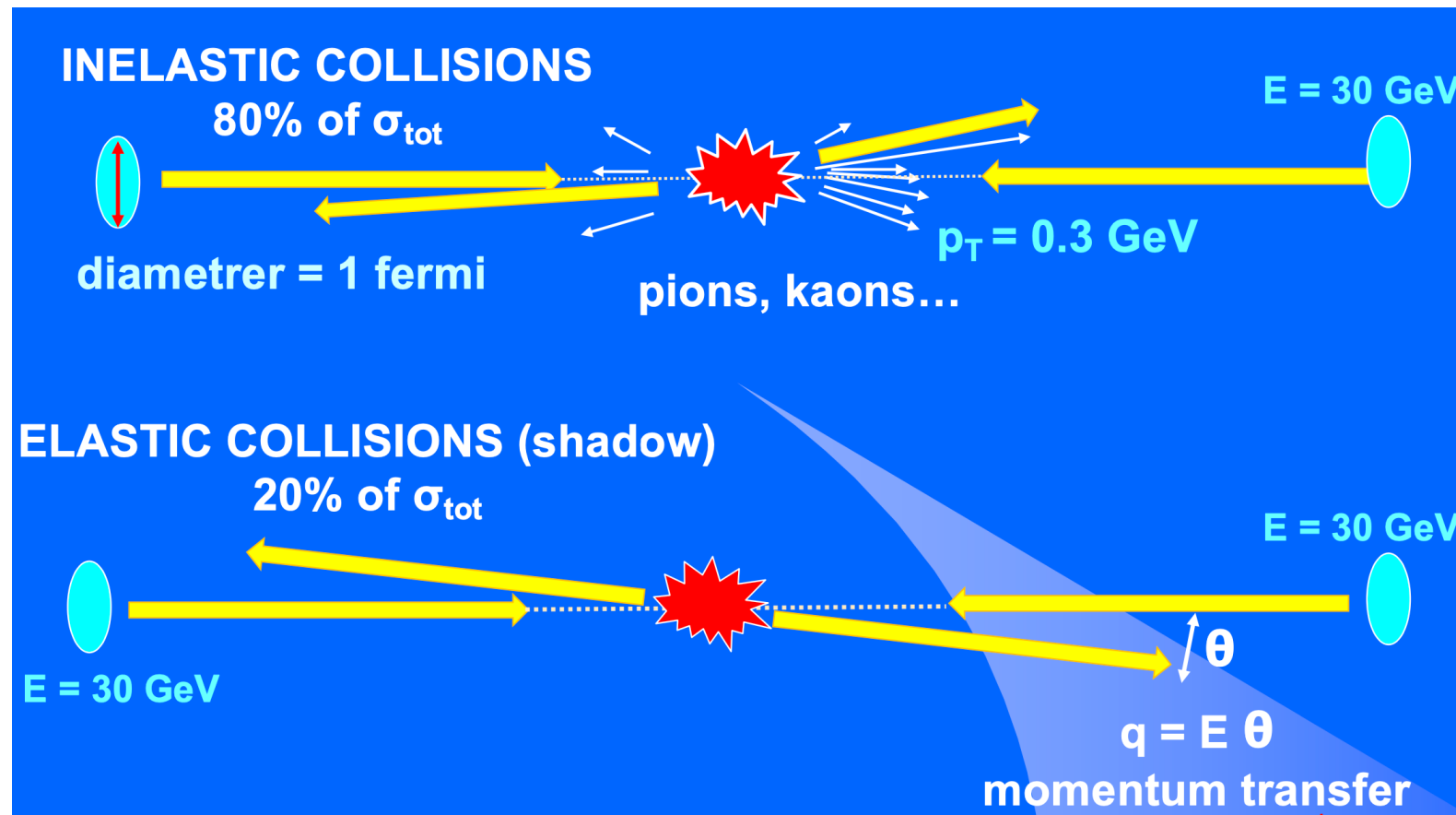
50 years of hadron colliders - UA - 18.10.21



Total cross-section measurement at the ISR

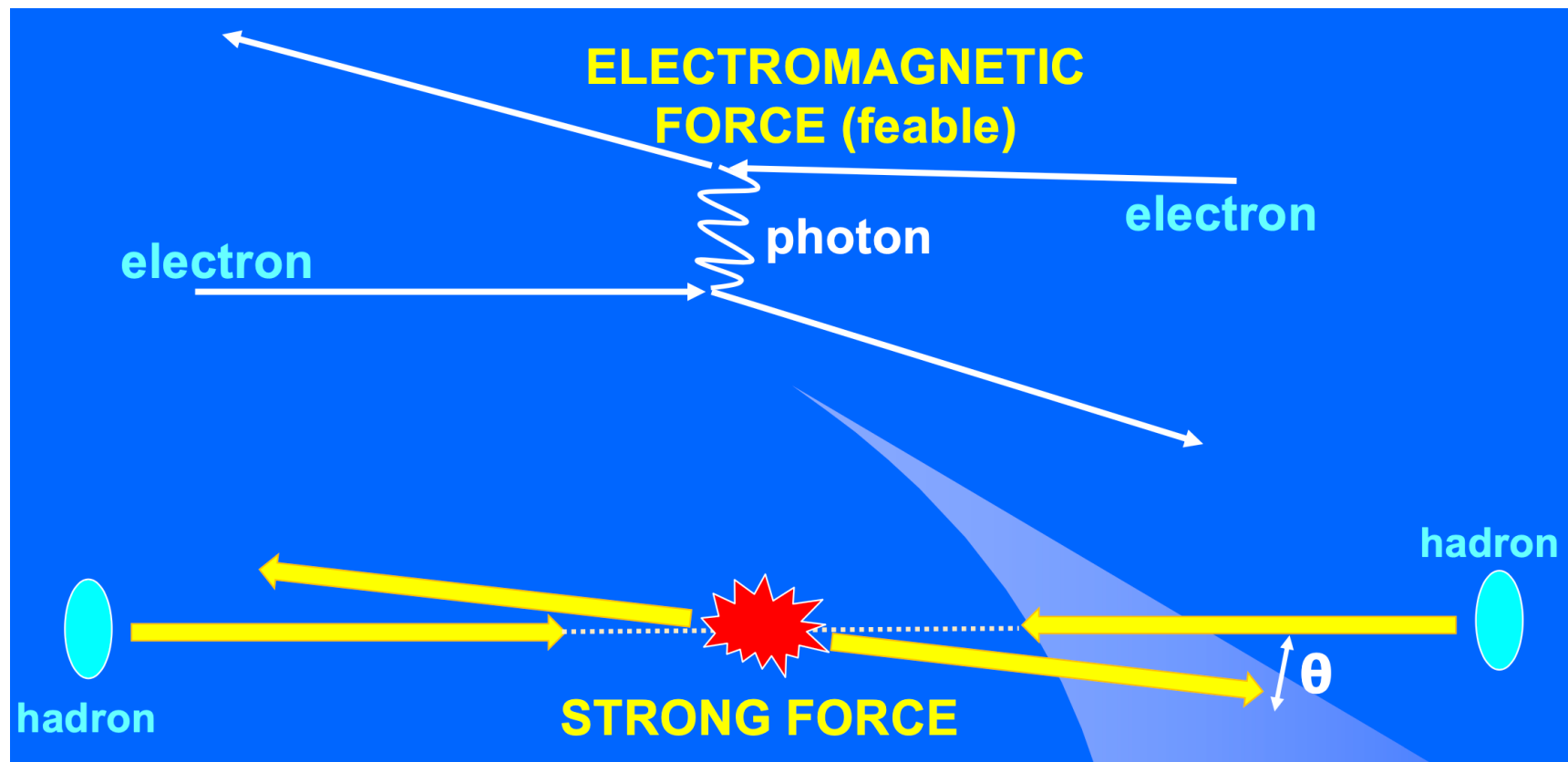
(There were other important topics covered at the ISR, but just look at this one.)
It is not part of the exam program.

Dominant view: particles are created in SOFT processes

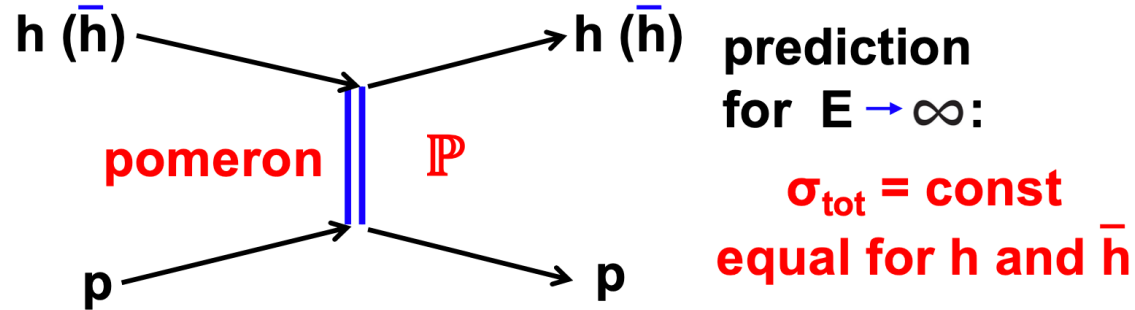
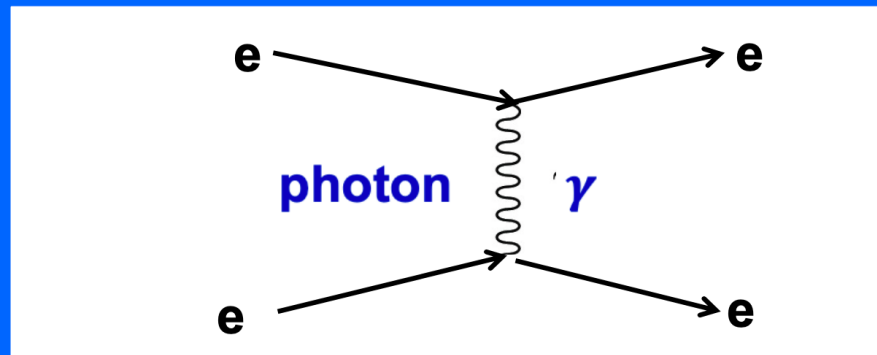


Reminder: elastic collisions: final state particles are equal to initial state particles
inelastic collisions: final state particles are different from initial state particles

Hadron-hadron collisions were described in the framework of the Regge theory



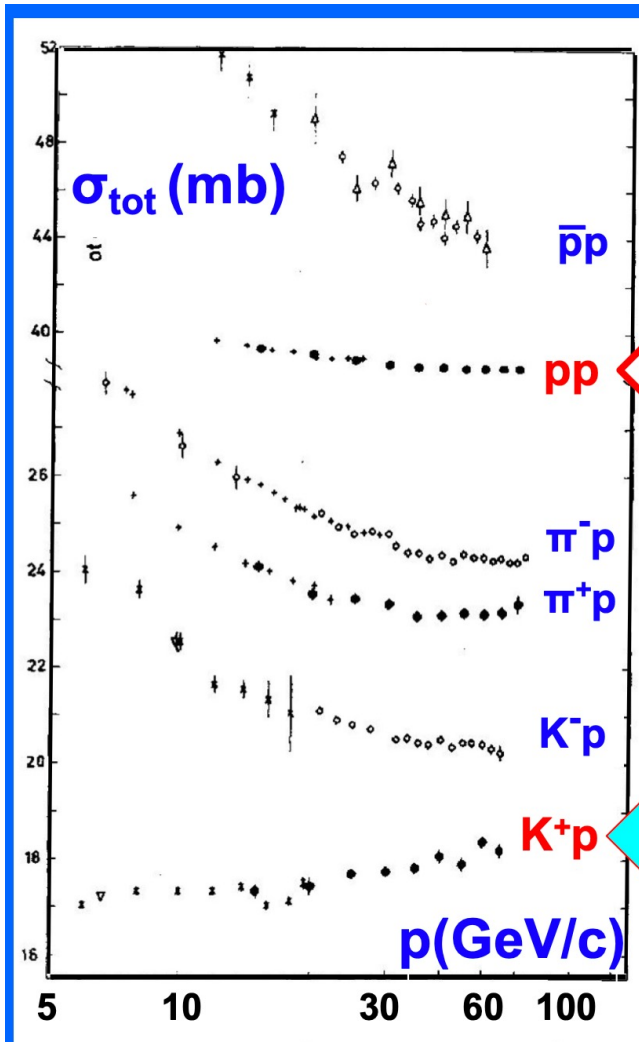
Elastic scattering between two hadrons is due to the exchange of the same Pomeron trajectory



Proton-proton total cross-section should be equal to proton-antiproton total cross-section

(The pomeron is still used in the modern description of the proton-proton elastic scattering)

In July 1971 Serphukov data confirmed the prediction



TOTAL CROSS SECTIONS OF π^+ , K^+ AND p ON PROTONS AND DEUTERONS
IN THE MOMENTUM RANGE 15-60 GeV/c

S. P. DENISOV, S. V. DONSKOV, Yu. P. GORIN, A. I. PETRUKHIN, Yu. D. PROKOSHKIN
D. A. STOYANOVA, J. V. ALLABY* and G. GIACOMELLI**
Institute of High Energy Physics, Serpukhov, U.S.S.R.

Received 30 July 1971

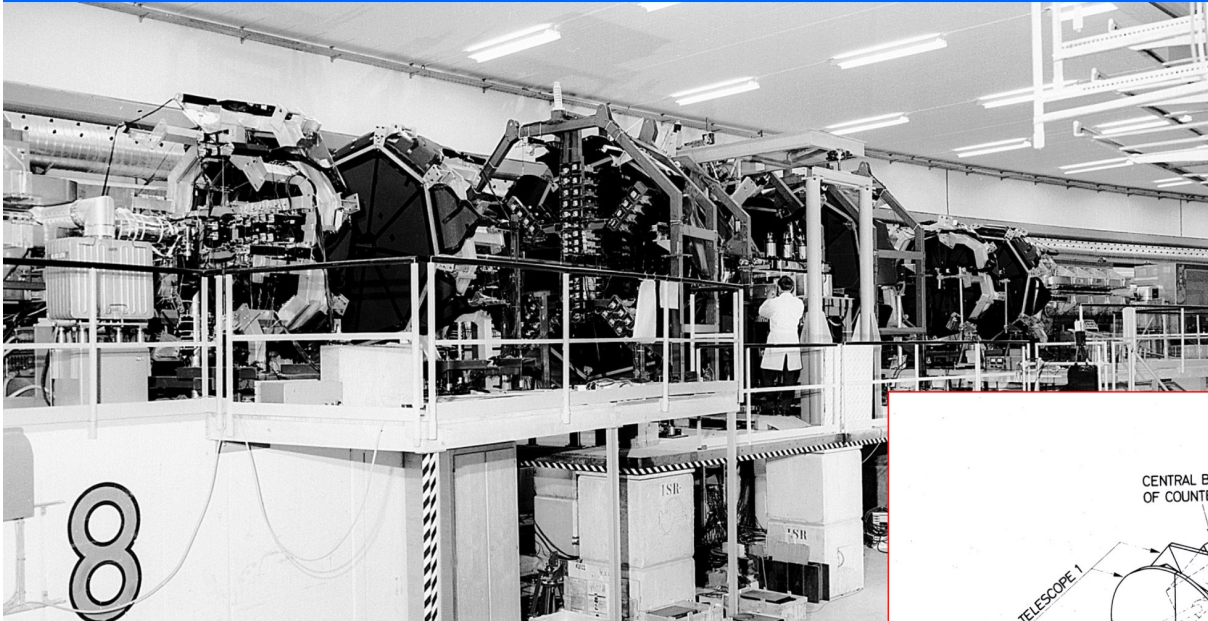
**PROTON-PROTON ASYMPTOTIA IS
ALREADY REACHED AT $E_{\text{beam}} = 100$ GEV**

(total cross-section should remain constant
according to the Regge theory)

This figure suggests that the total cross-section for K^+p will approach the asymptotic value from below

Total cross section measurement

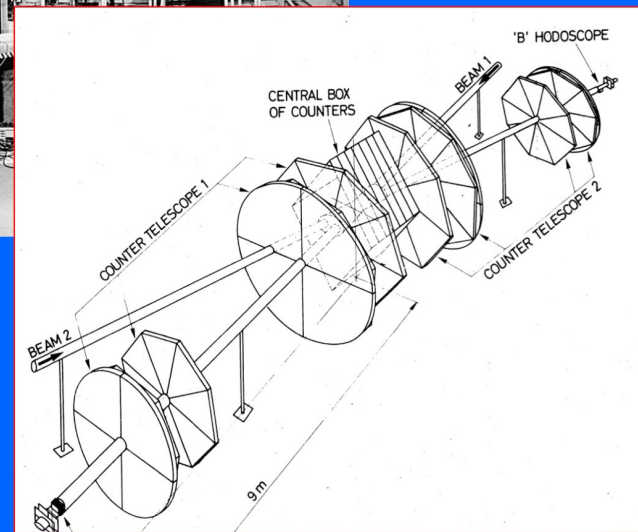
*In IR-8 the total cross section was measured by the
Pisa - Stony Brook Coll.*



Measuring p-p total cross-section was one of the most important measurements in the early phase of the ISR. Then, the knowledge of the luminosity was of a paramount importance. As we said, it can not be measured from a known cross-section, so it must be extracted from the collider itself, but it was not clear how.

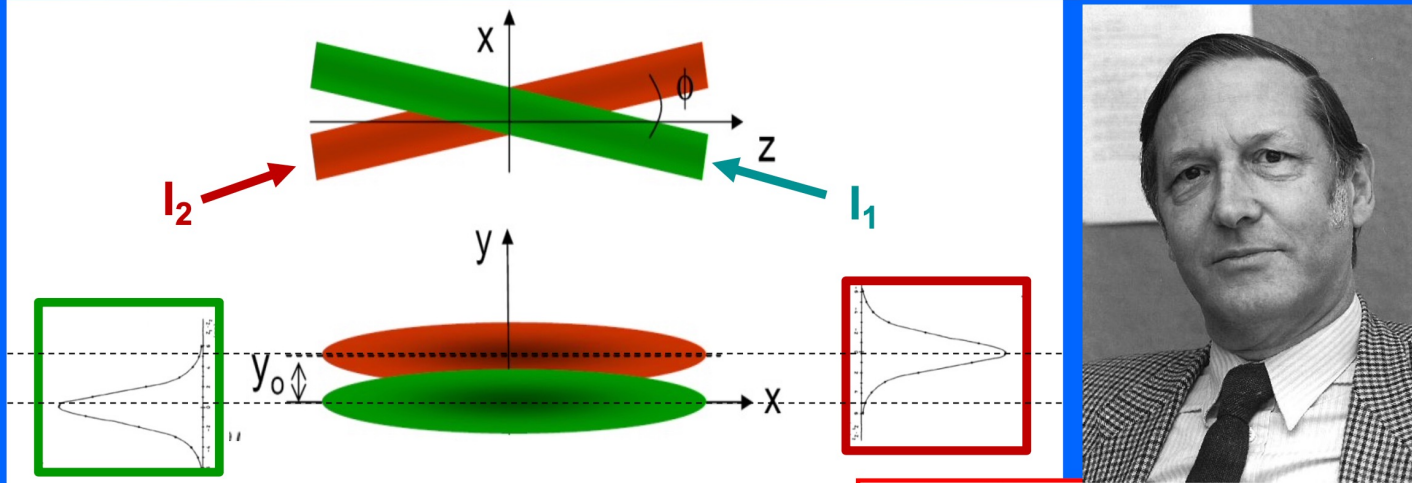
$$\sigma = \frac{N_{events}}{Luminosity}$$

50 years of hadron colliders - UA - 18.10.21



Luminosity measurement

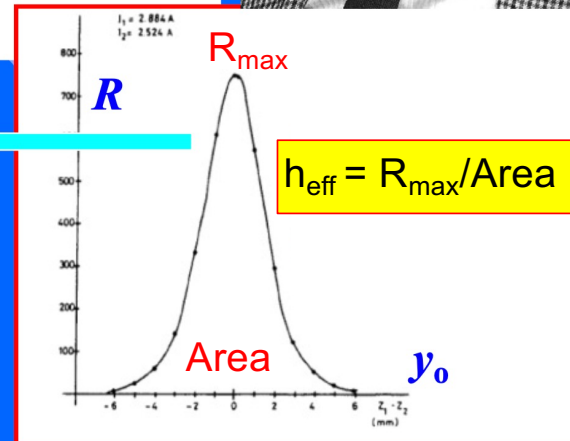
The luminosity was measured with the method invented by Simon van der Meer



- The luminosity is proportional to the overlap of two beams
- If the beams are very narrow, with a little displacement the counting rate goes to zero; on the contrary if the “bell” is large also the beams are large and the luminosity is small.

h_{eff} = effective beam height

$$\text{Luminosity} = K \frac{I_1 I_2}{h_{\text{eff}}}$$
 Beam current



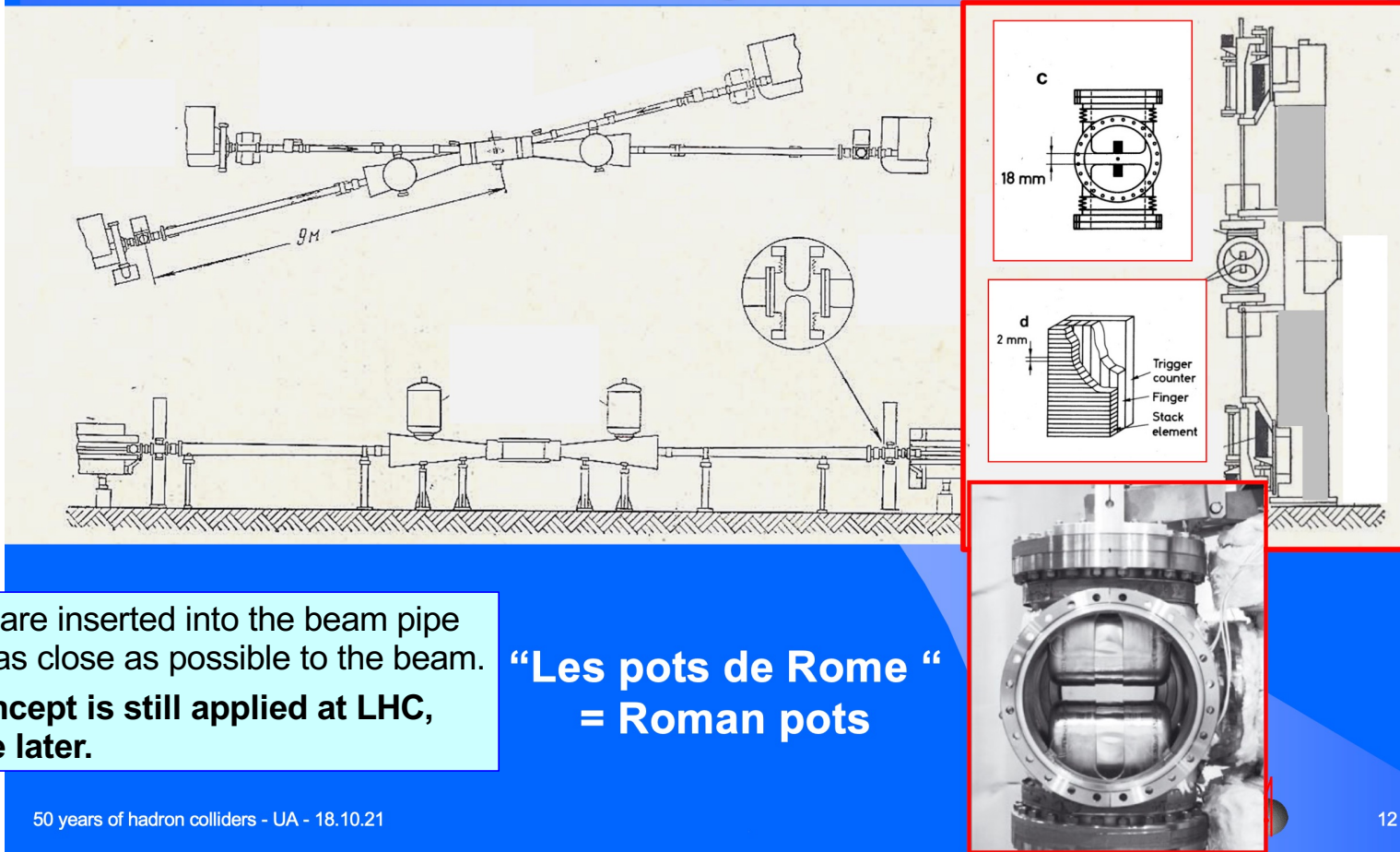
- R is the rate measured by a reference counter
- This is the method still used at LHC to measure the luminosity:
van der Meer scan

50 years of hadron colliders - UA - 18.10.21

The Roman pots

In IR-6 the total cross section was measured by the CERN-Rome Coll. through the forward elastic x-section

(through the optical theorem)



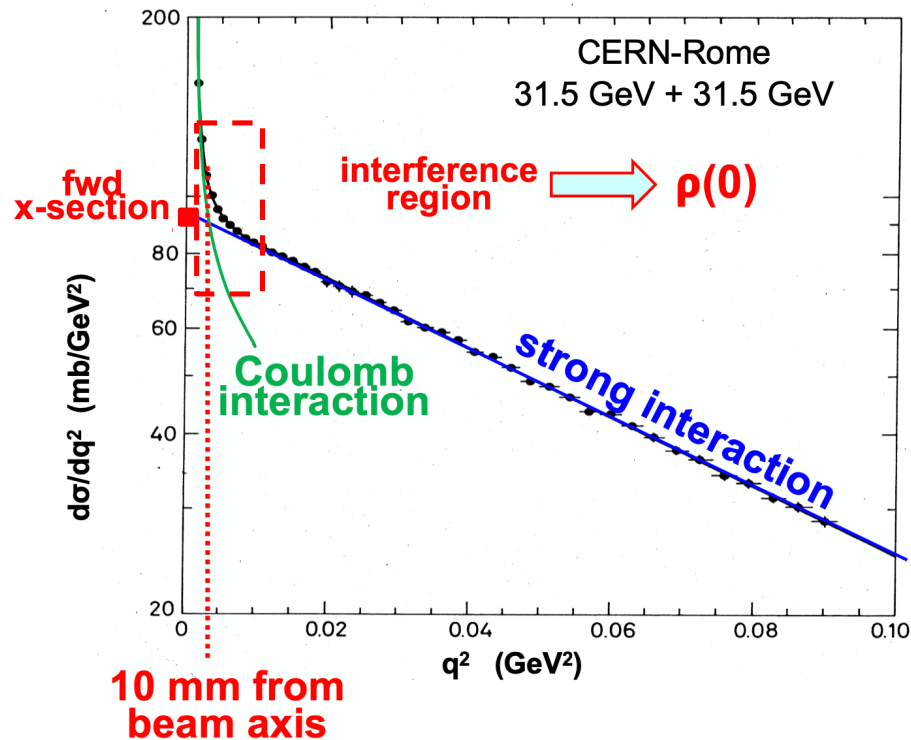
The detectors are inserted into the beam pipe in order to go as close as possible to the beam.
The same concept is still applied at LHC, as we will see later.

**“Les pots de Rome”
= Roman pots**

50 years of hadron colliders - UA - 18.10.21

12

Roman Pots results



Behaviour of the elastic cross-section

S-matrix theory:
Scattering amplitude = $A(q^2) [i + \rho(q^2)]$

A theory assuming only a few general principles of Quantum Field Theory

1. Optical theorem:
 $\sigma_{\text{tot}} = C \sqrt{\text{fwd x-section}}$

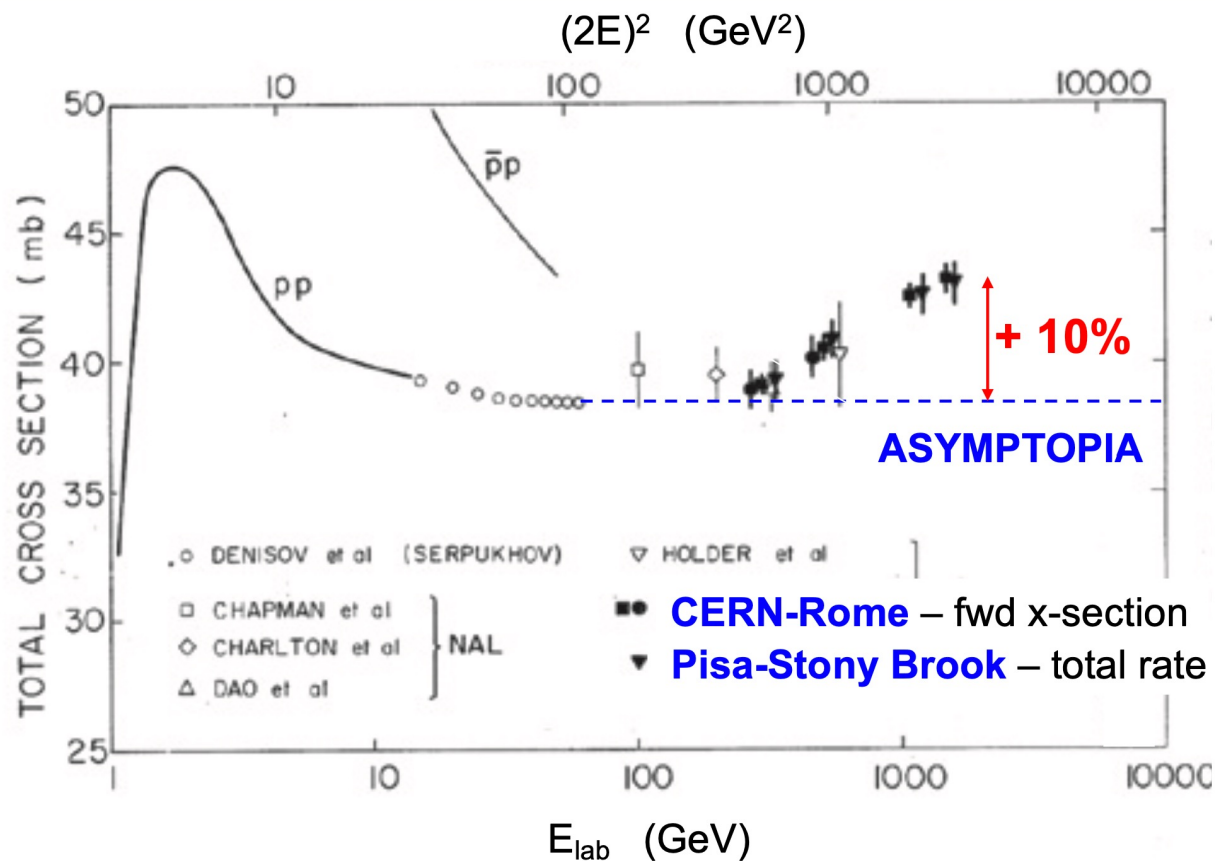
2. Dispersion relation:
 $\rho(0)$ expressed as an integral over all energies of $\sigma_{\text{tot}}(E)$

$$\rho = \text{Re}[f_{el}(0)] / \text{Im}[f_{el}(0)]$$

(N.B. the optical theorem can be deduced also from the S-matrix theory)

First important ISR result on pp total cross-section

In 1973 the two Collaborations found that 1. Asymptopia does not apply to protons; 2. the Pomeron is much more complicated



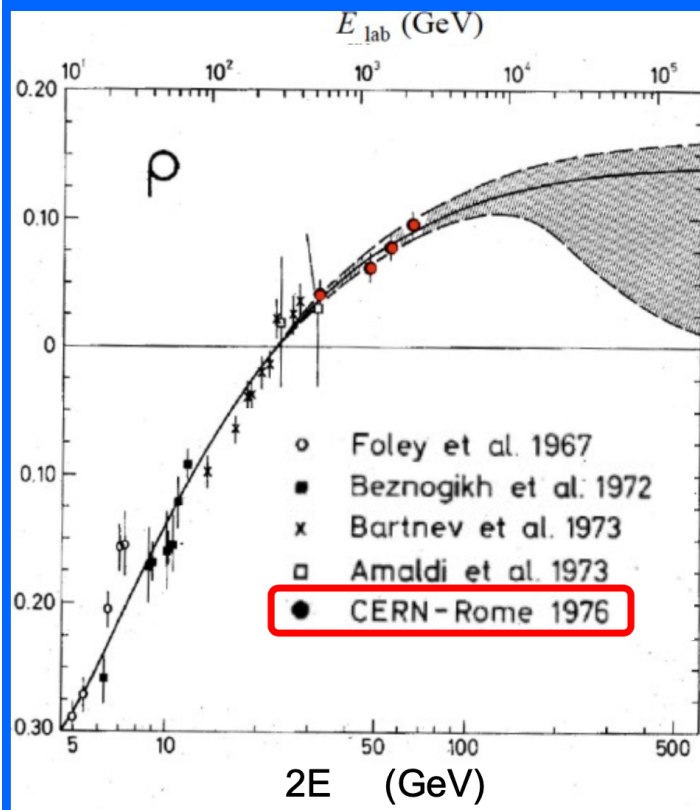
Regge theory predicted that the pp cross-section should be constant for large energy.

Data do not agree with this prediction

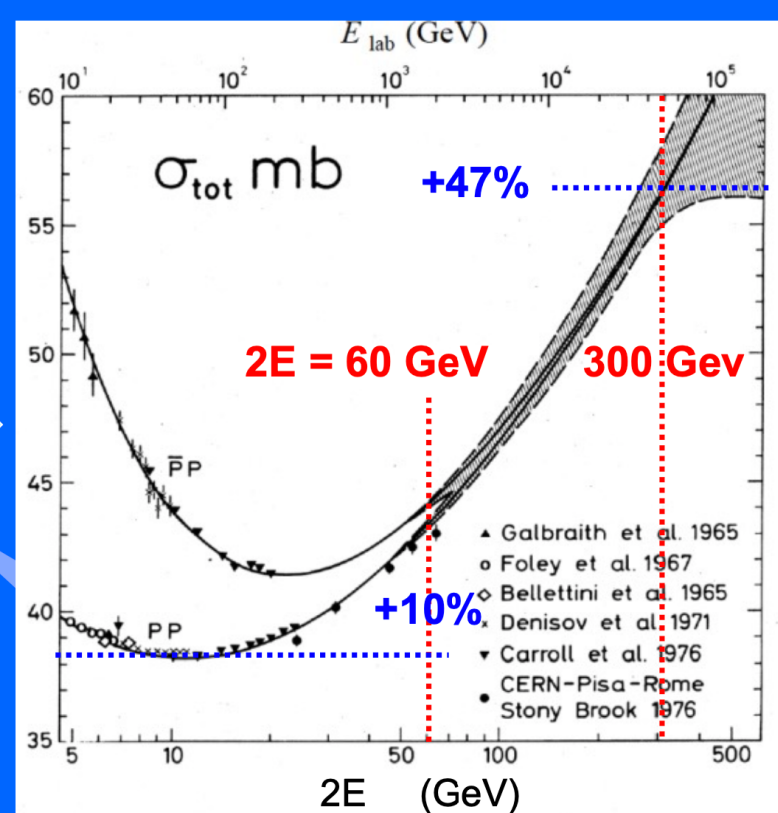
50 years

16

pp cross-section measurements



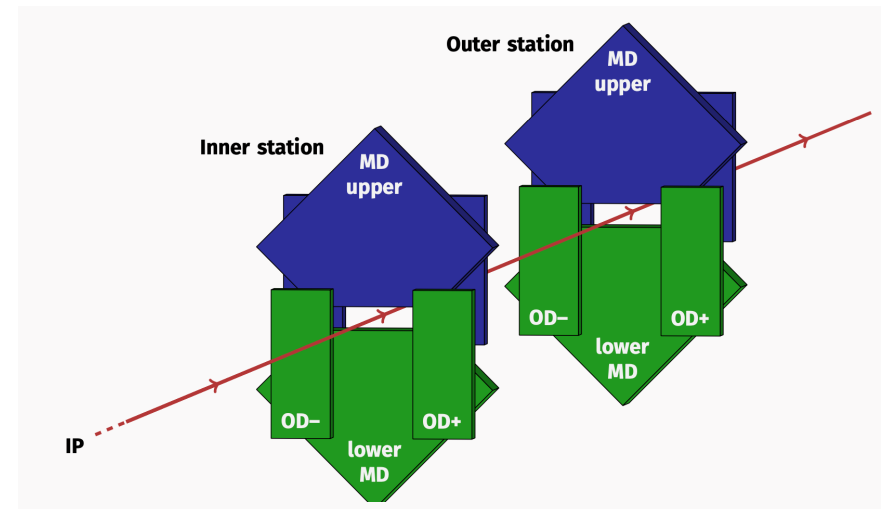
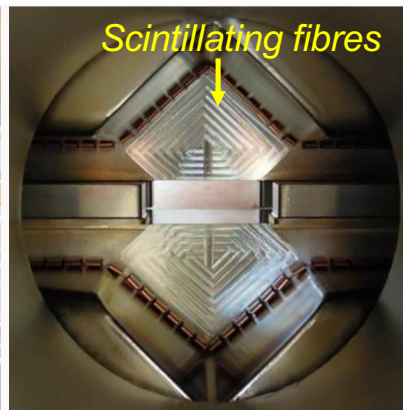
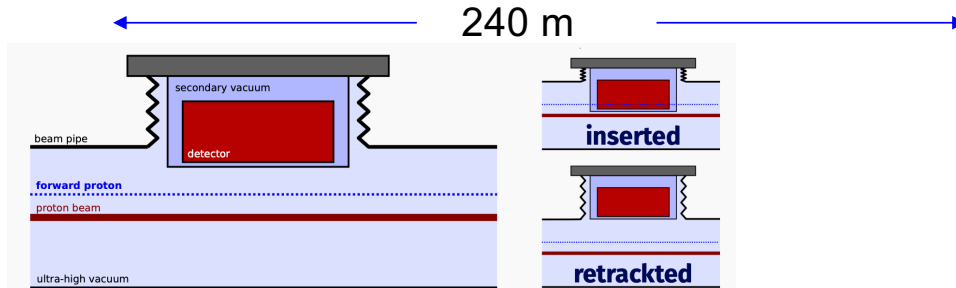
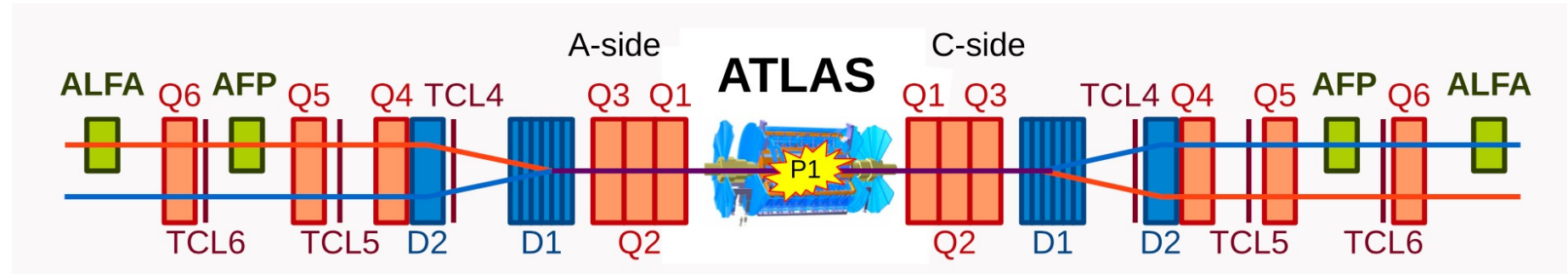
Dispersion
relation



“Soft” Physics: ISR experiments have shown that the proton-proton cross-section increases by 50% when the collision energy increases from 15 GeV + 15 GeV to 150 GeV + 150 GeV

Soft Physics at LHC

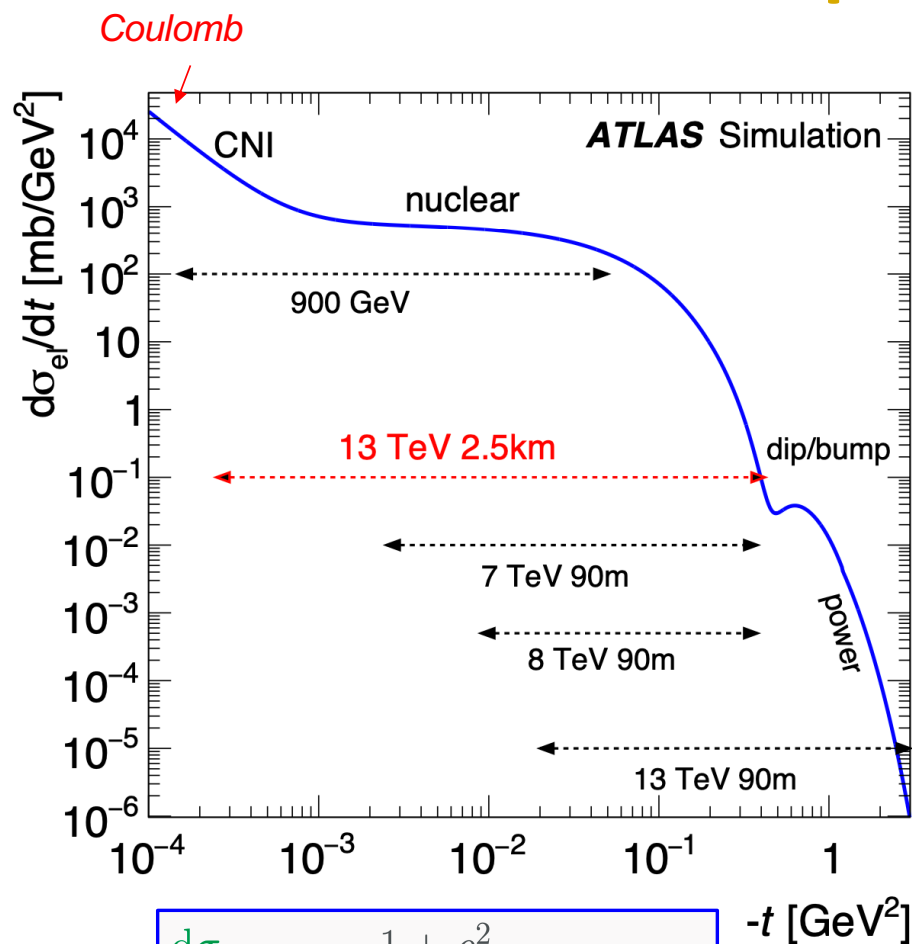
“soft” physics at LHC



Main detectors (MDs) – for physics
Overlap detectors (ODs) – for alignment

Near CMS we have the TOTEM detector

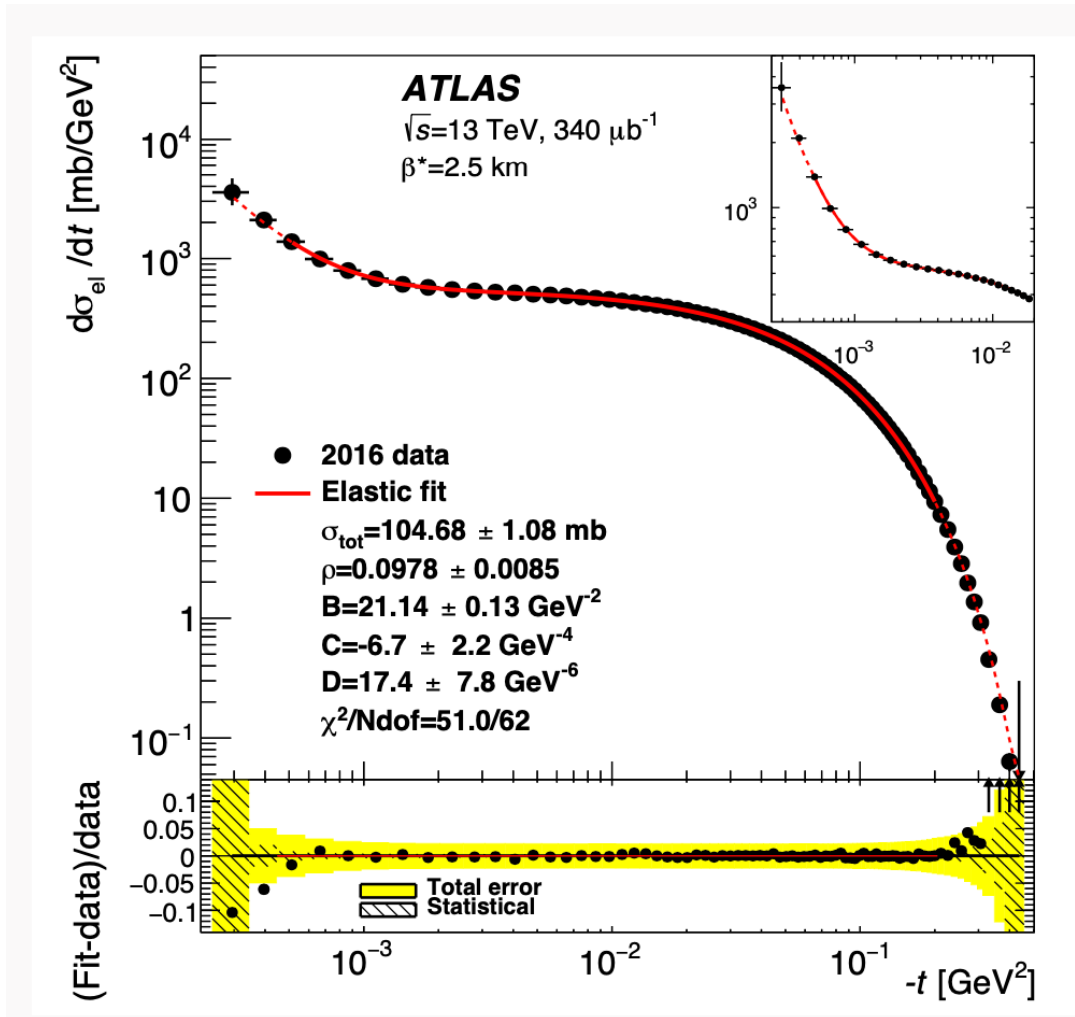
ALFA experimental reach



$$\frac{d\sigma_{\text{el}}}{dt} = \sigma_{\text{tot}}^2 \frac{1 + \rho^2}{16\pi} \exp(-B|t|)$$

Dedicated LHC runs with high beta for ALFA measurements

Differential cross section



Fitted function:

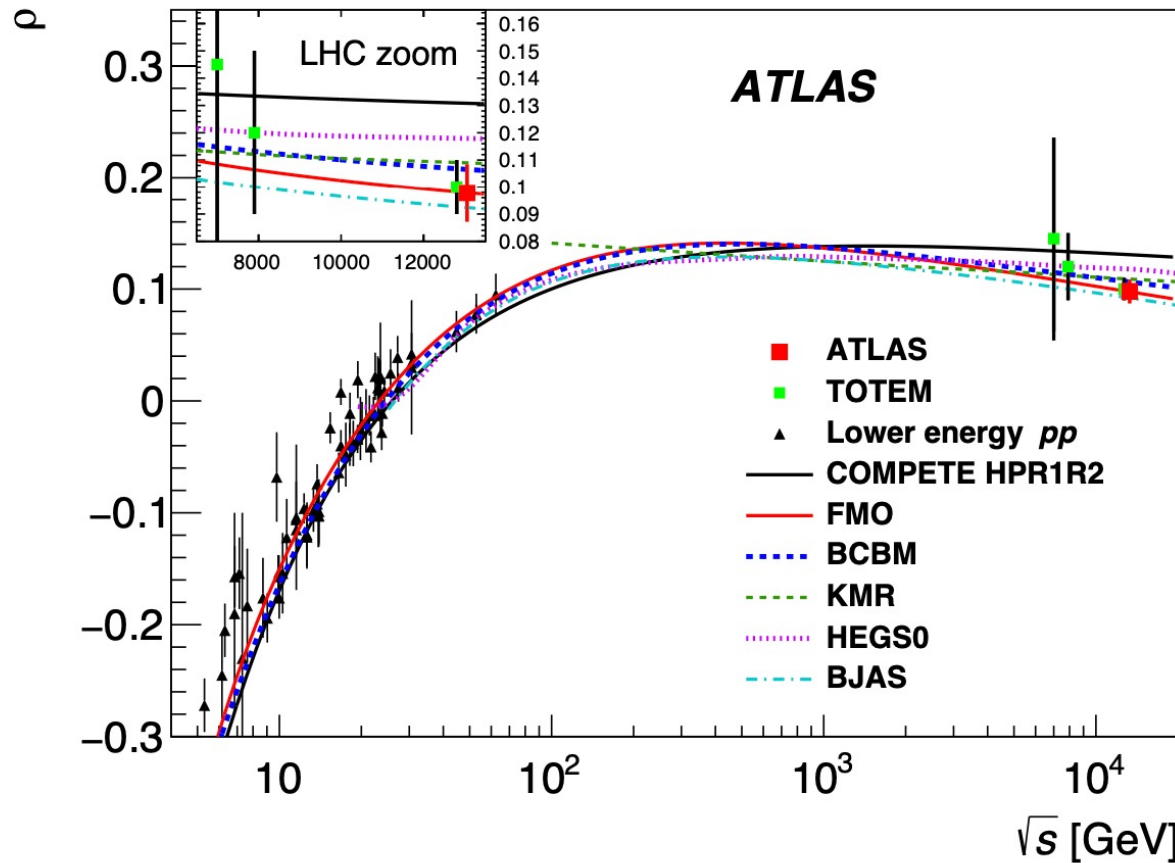
$$\frac{d\sigma}{dt} = \frac{1}{16\pi} \left| f_N(t) + f_C(t) e^{i\alpha\phi(t)} \right|^2$$

$$f_C(t) = -8\pi\alpha\hbar c \frac{G^2(t)}{|t|}$$

$$f_N(t) = (\rho + i) \frac{\sigma_{\text{tot}}}{\hbar c} e^{(-B|t| - C|t|^2 - D|t|^3)/2}$$

$$\rho = \frac{\text{Re } f_N(0)}{\text{Im } f_N(0)}$$

Results in interference region: ρ measurement



*Result incompatible with COMPETE
(community-standard semi-empirical fits)
indicating Odderon exchange or a
slowdown of σ_{tot} rise at high $\sqrt{s}$*

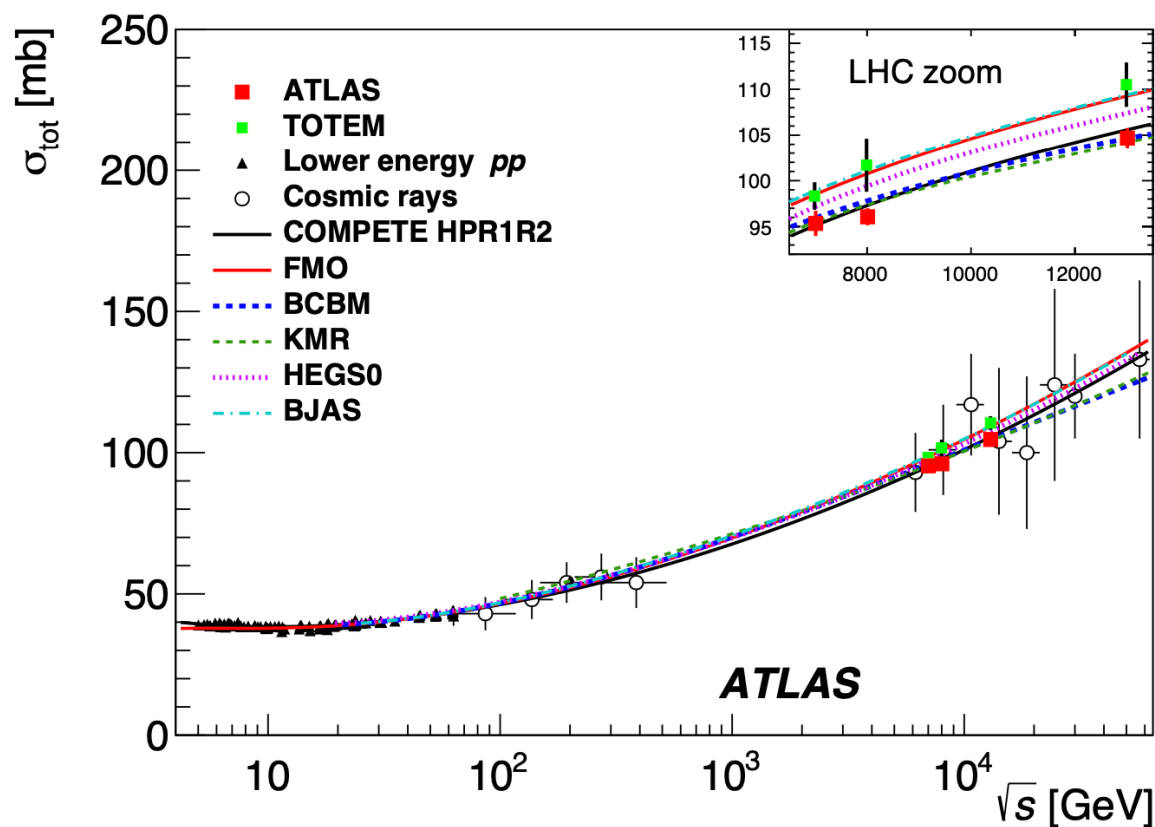
Today view

Pomeron: two gluons exchange

Odderon: three gluons exchange

$$\rho = 0.0978 \pm 0.0043(\text{stat.}) \pm 0.0073(\text{exp.}) \pm 0.0064(\text{th.})$$

Results in nuclear region: σ_{tot}



$$\sigma_{\text{tot}} = 104.68 \pm 0.22(\text{stat.}) \pm 1.06(\text{exp.}) \pm 0.12(\text{th.}) \text{ mb}$$

Most precise σ_{tot} measurement. 2.2σ tension with TOTEM σ_{tot} result.

Method of σ_{tot} measurement

Luminosity-dependent (ATLAS)

$$\sigma_{\text{tot}}^2 = \frac{16\pi}{1 + \rho^2} \frac{1}{L} \frac{dN_{\text{el}}}{dt} \bigg|_{t \rightarrow 0}$$

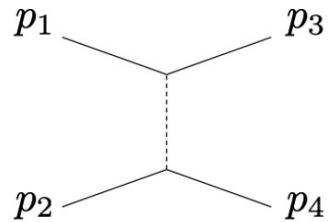
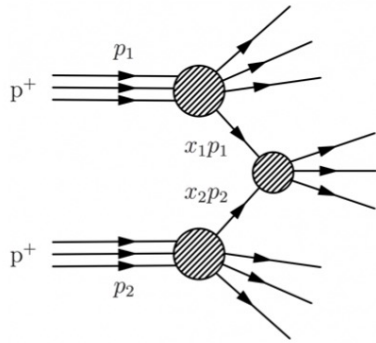
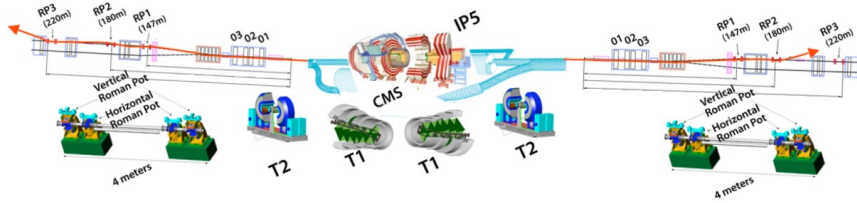
Requires a dedicated luminosity measurement

Luminosity-independent (TOTEM)

$$\sigma_{\text{tot}} = \frac{16\pi}{1 + \rho^2} \frac{1}{N_{\text{el}} + N_{\text{inel}}} \frac{dN_{\text{el}}}{dt} \bigg|_{t \rightarrow 0}$$

Requires correction for not measured small-mass diffraction

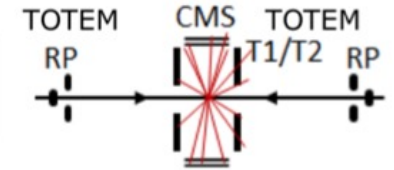
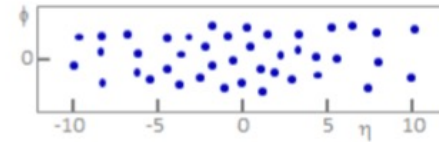
Still on pp total cross section



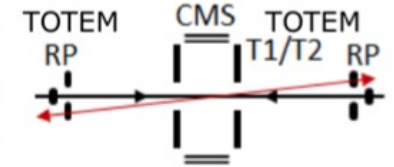
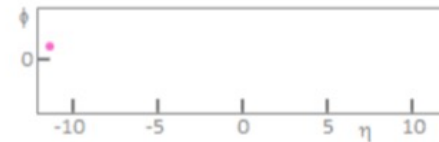
$$t = (p_1 - p_3)^2$$

(very naive view of the pomeron is a colorless pair of gluons)

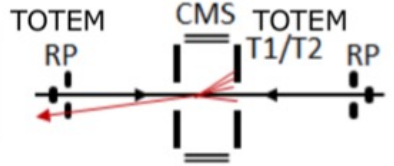
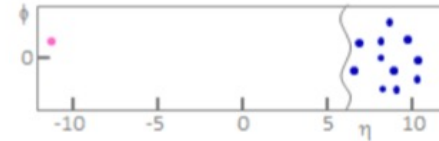
Non-diffr. (~60 mb)



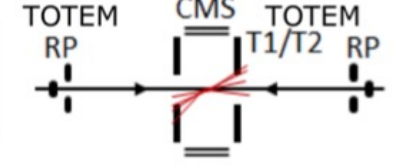
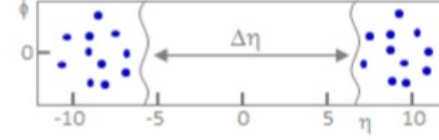
Elastic (~25 mb)



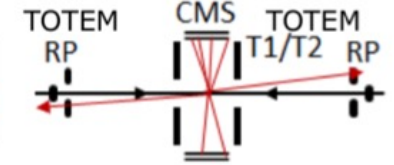
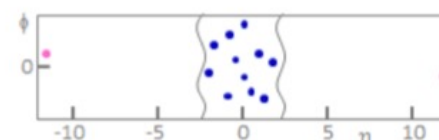
Single-diffr. (~10 mb)



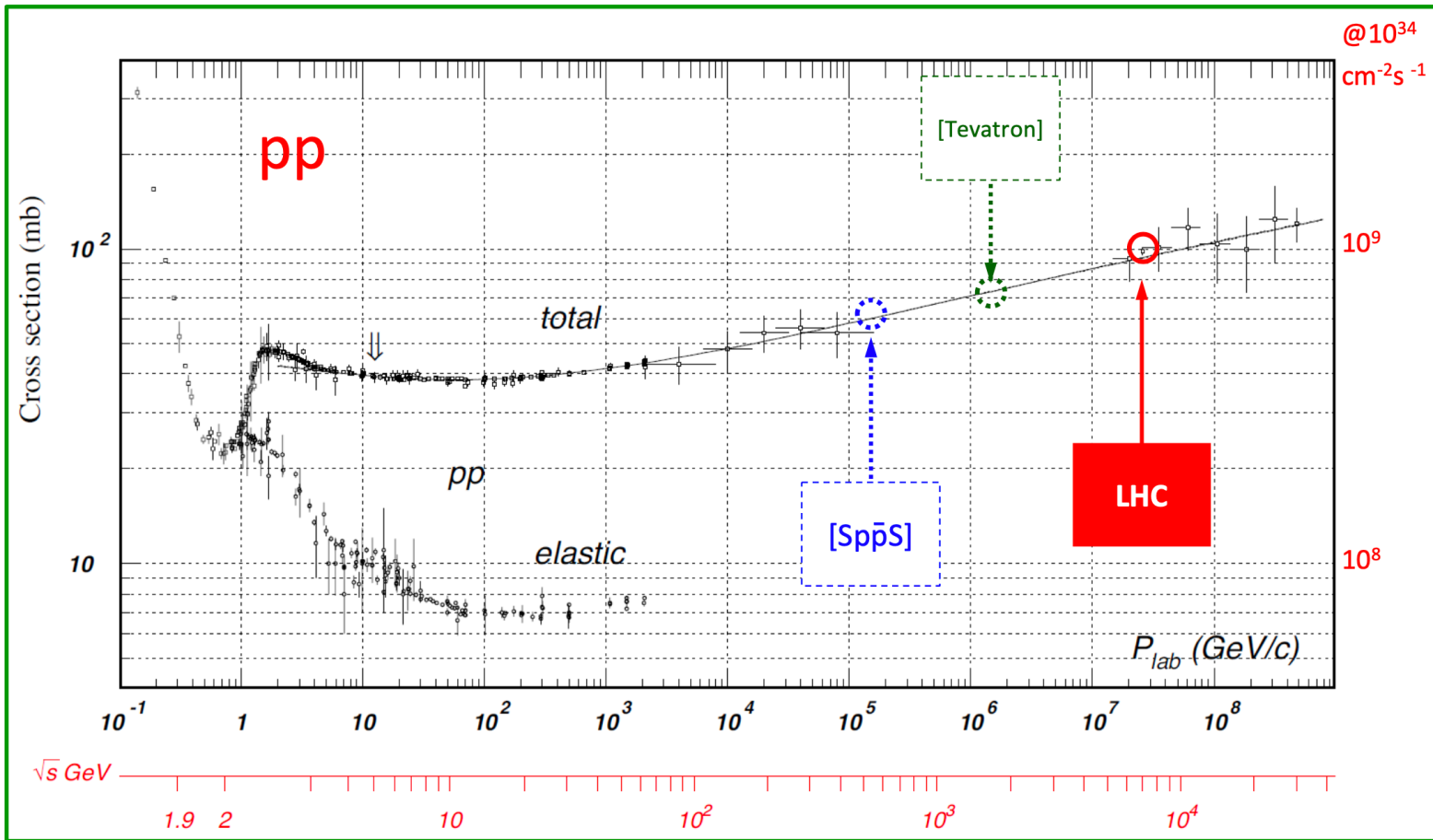
Double-diffr. (~5 mb)



Central-diffr. (~1 mb)



pp σ_{tot} as a function of $\sqrt{s}$

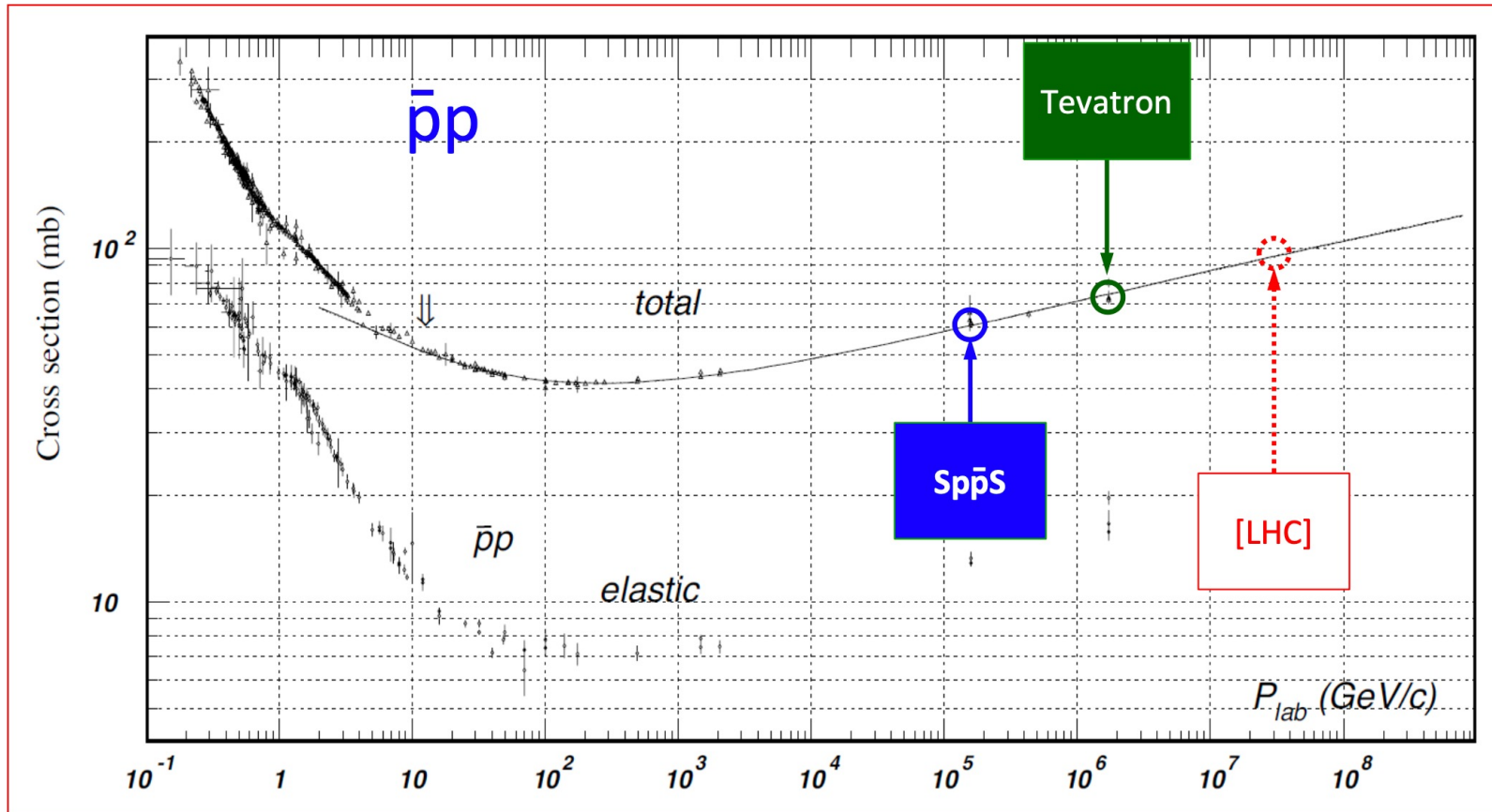


Rise as $\ln^2(s)$

$1 \text{ b} = 10^{-28} \text{ m}^2 = 10^{-24} \text{ cm}^2$
 $1 \text{ mb} = 10^{-31} \text{ m}^2 = 10^{-27} \text{ cm}^2$

The data of $\sigma(\bar{p}p)$, i.e. Sp $\bar{p}$ S and Tevatron, are dashed, to show the similarity of the cross sections.

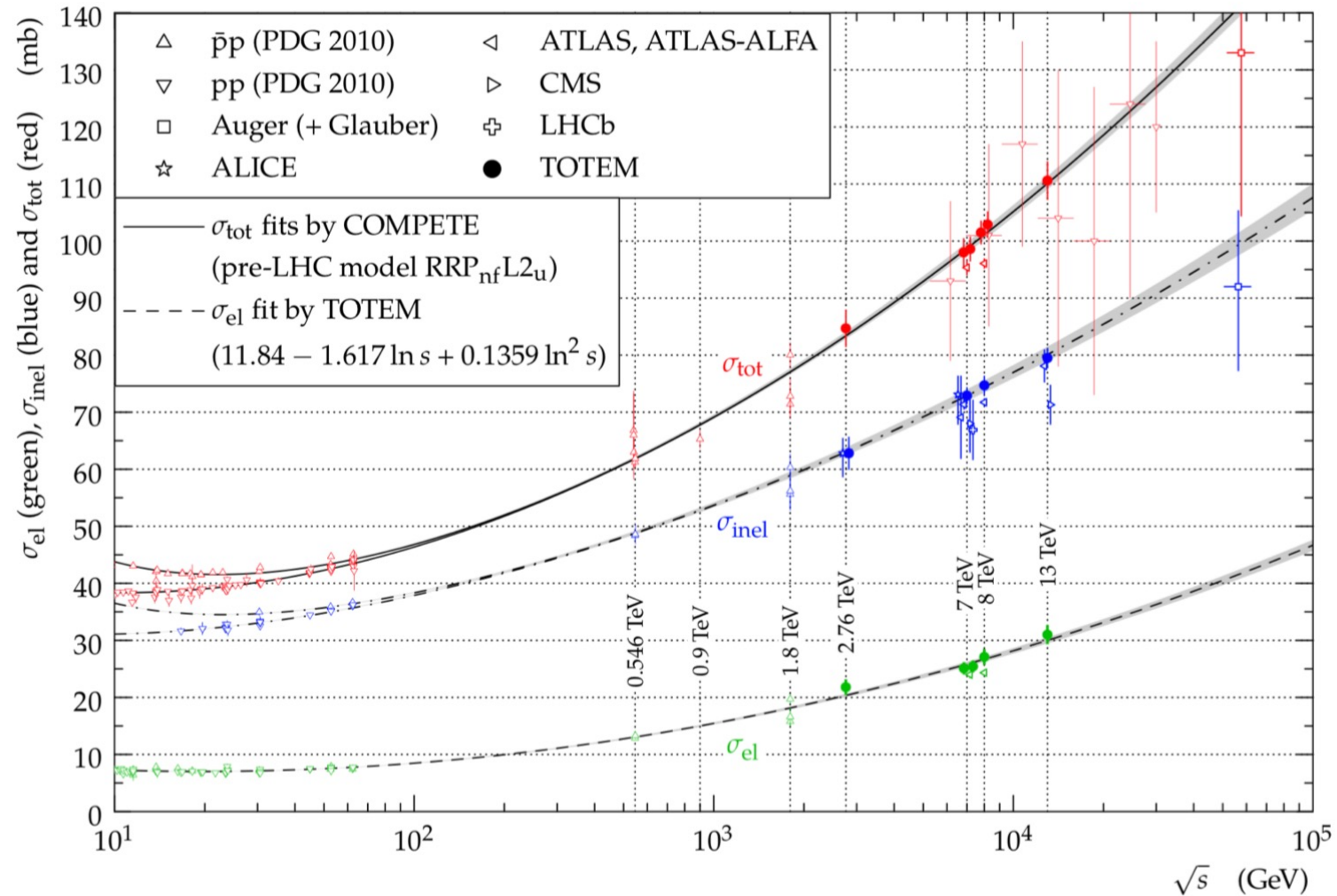
$\bar{p}p$ σ_{tot} as a function of $\sqrt{s}$



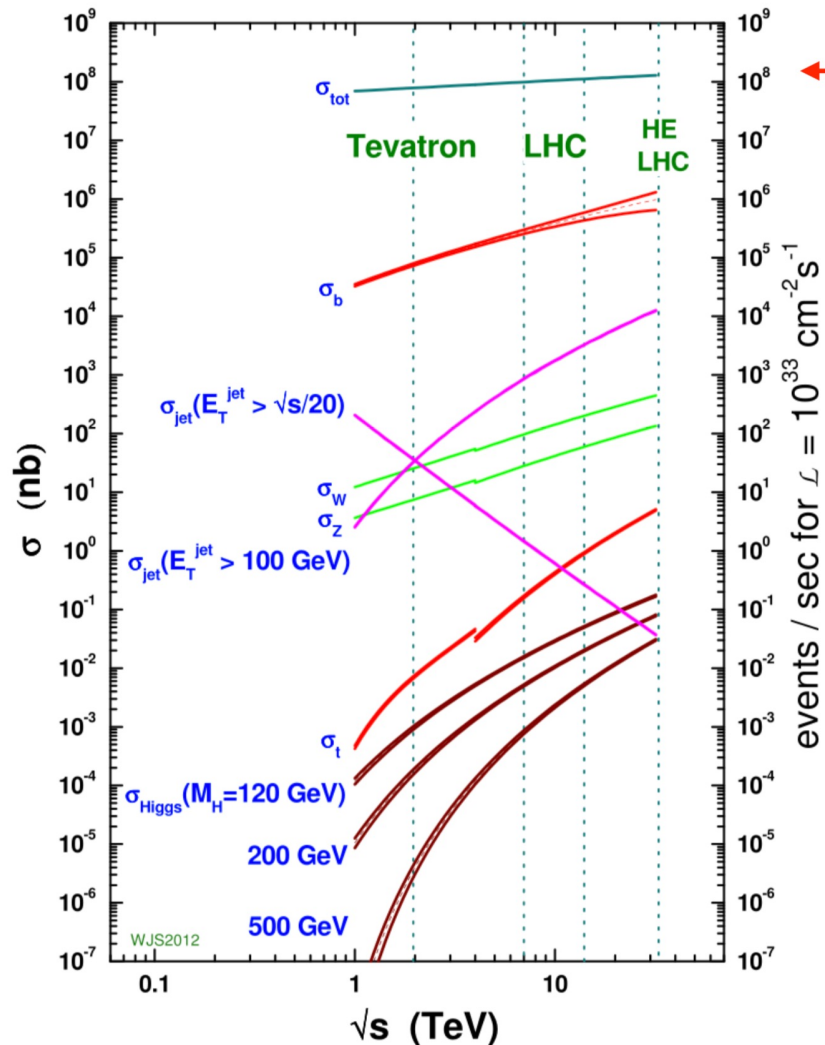
The data of $\sigma(pp)$, i.e. LHC, do NOT belong to this plot; they are plotted dashed, to show the similarity of the cross sections ("Pomeranchuk theorem").

pp cross section: elastic, inelastic and total

Elastic
Inelastic
Total



Closer inspection to the total cross section



100 mb

Total cross section

60 mb

Start seeing events in the detector!
Starting point of everything!!

(we don't see scattered protons in the beam pipe)

From the nominal LHC luminosity:

$$2 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$$

With a total cross section of approximately 100mb:

$$100 \times 10^{-27} (\text{cm}^2) \times 2 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$$

$$\sim 2 \times 10^9 \text{ evts/s}$$

*The protons collide every 25 ns (40 MHz);
what we should conclude?*



SAPIENZA
UNIVERSITÀ DI ROMA

End of chapter 2