Collider Particle Physics
- Chapter 7 -

LEP Physics at the Z pole

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Chapter Summary

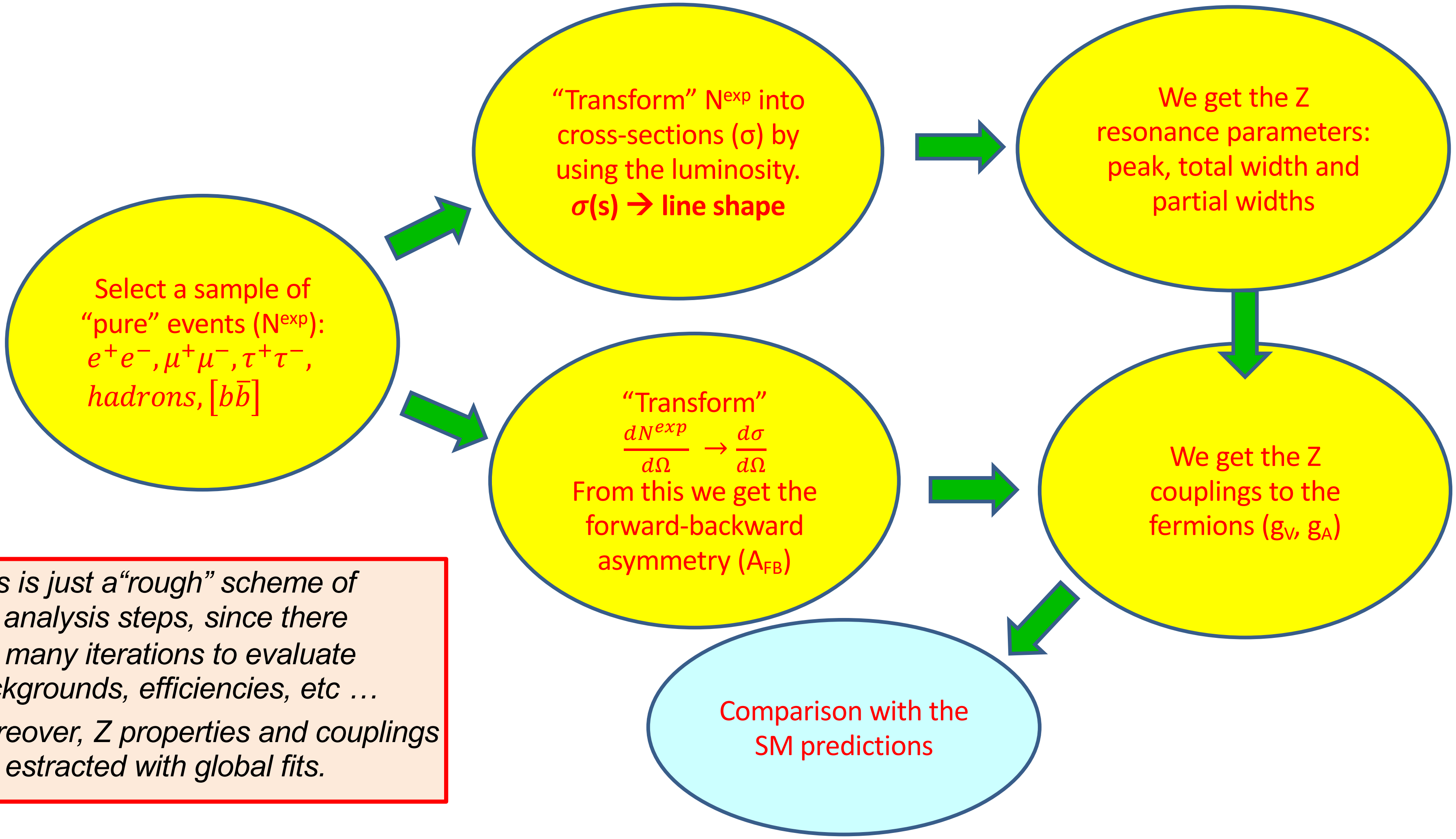
- The line shape
- Forward-Backward asymmetry
- Tau polarization asymmetry and Left-Right asymmetry at SLC
- Radiative corrections
- Measurements: Z mass, Z total and partial widths, Z couplings
- Measurements: Forward-backward asymmetries for heavy quarks
- Measurements: $\sin^2 \theta_W$
- Number of neutrino families

LEP Physics goals (taken from my thesis in 1988)

- Higgs boson discovery;
- Quark top discovery and measurement of the topponium energy levels;
- Supersymmetric particles discovery;
- Measurement of the Z mass with an error of 50 MeV;
  - $\sigma(M_Z)$ about 340 MeV from UA2+CDF in 1989.
  - Hoped to reduce it to about 10 MeV (limited by beam energy precision).
- Precision measurement of the Standard Model parameters;
- Measurement of the number of light neutrino families.
  - 2.5 generations were known in 1989, top quark and ν_τ not yet established.
  - Number of light neutrinos limited by big bang nucleosynthesis to $\lesssim 4$. Expected precision of about $\pm$ 0.2 on the number.
- Lep2: measurement of the W mass and check of the triple gauge boson coupling.

Z precision physics

Analysis experimental steps



This is just a"rough" scheme of the analysis steps, since there are many iterations to evaluate backgrounds, efficiencies, etc …

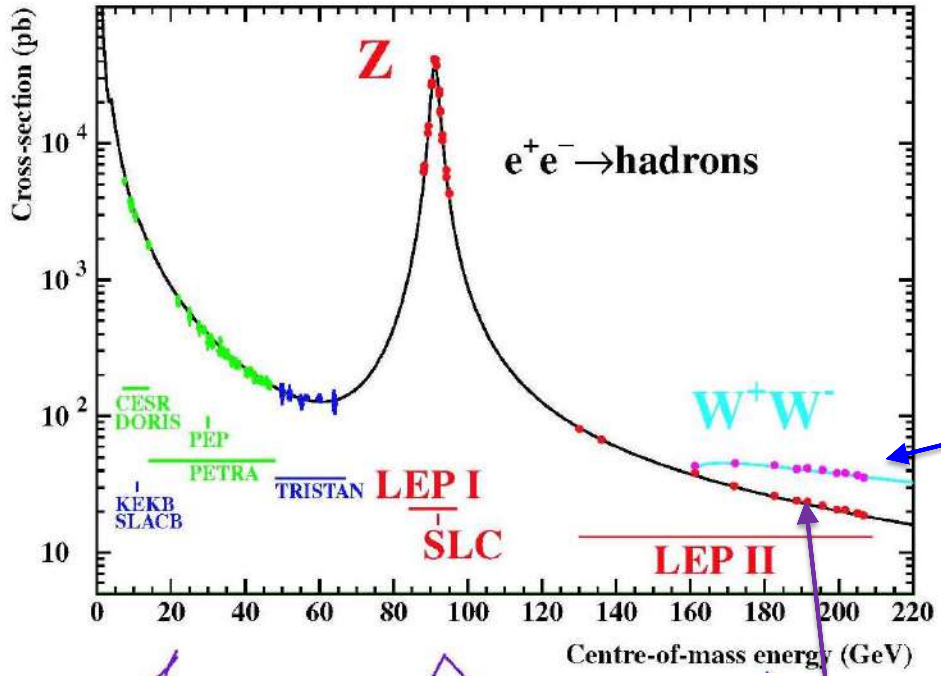
Moreover, Z properties and couplings are estracted with global fits.

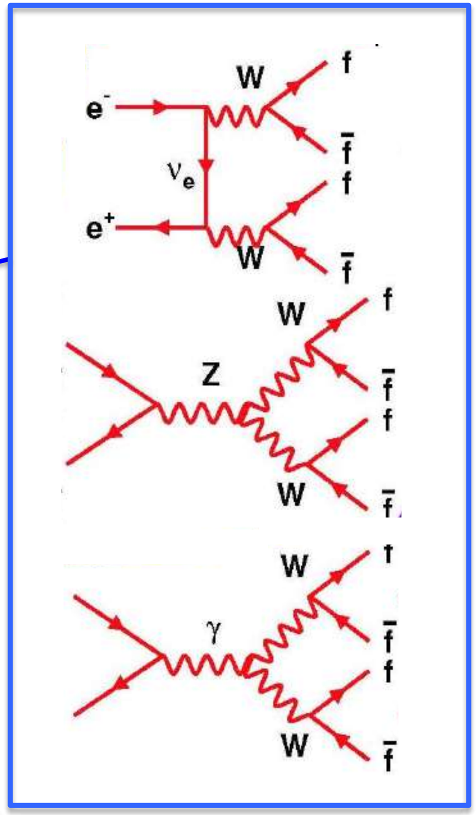
The lineshape

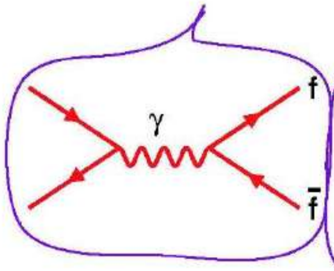
Cross-section as a function of √s

LEP collected 4.5 million Z,

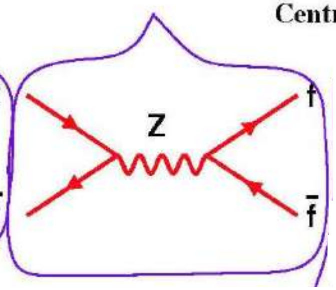
12 thousand WW per experiment



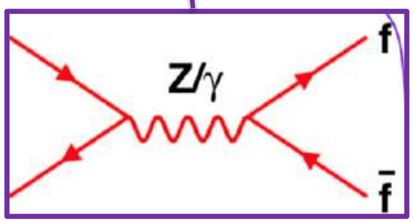




Dominant at low energy



Dominant at Z-pole



Equally important

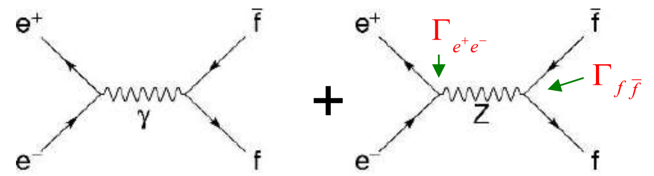
- **in the SM, at the lowest order (Born approximation, no radiative corrections), for $f \neq e^{\pm}$ and $m \ll m_Z$:**

- $\sigma_{Born}(e^+e^- \rightarrow f\bar{f}) = \sigma_{Zs} + \sigma_{\gamma s} + J_f;$
- $\sigma_{Zs} = \frac{s\Gamma_Z^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \times \frac{12\pi\Gamma_e\Gamma_f}{m_Z^2\Gamma_Z^2};$
- $\sigma_{\gamma s} = \frac{4\pi\alpha^2(s)}{3s} c_f Q_f^2; \quad \left[c_f = 1 \text{ (leptons)}, 3 \text{ (quark)}\right];$
- $J_f = -\frac{(s-m_Z^2)m_Z^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \frac{2\sqrt{2}\alpha(s)}{3} c_f Q_f G_F g_V^e g_V^f;$
- $\Gamma_Z = \Gamma_{tot} = \sum_f \Gamma(Z \rightarrow f\bar{f});$
- $\Gamma_f \equiv \Gamma(Z \rightarrow f\bar{f}) = \frac{G_F m_Z^3 c_f}{6\sqrt{2}\pi}\left[g_V^{f\,2} + g_A^{f\,2}\right];$
- for $\sqrt{s} \approx m_z \rightarrow$ interference and γ^* negligible;
- $\sigma_{Born}(e^+e^- \rightarrow f\bar{f}, \sqrt{s} = m_Z) = \frac{12\pi\Gamma_e\Gamma_f}{m_Z^2\Gamma_Z^2}.$

Z (s-channel)

γ^* (s-channel)

interference $Z_s \leftrightarrow \gamma^*_s$



$\sigma \propto |A_\gamma + A_Z|^2 = |A_\gamma|^2 + |A_Z|^2 + 2\,\mathrm{Re}\left(A_\gamma \cdot A_Z^*\right)$

The Z couplings intervene linearly in the interference term (and not quadratically as in the partial widths), so any parity violation effects, like the forward-backward asymmetry, will be due to this term.

In the e^+e^- final state we have to take into account also the photon exchange in the t-channel (the Z contribution in the t-channel is negligeable)

$e^+e^- \to Z \to f\bar{f}$: g_V^f and g_A^f

- In the SM, at the lowest order, the partial width Γ_f (e.g. Γ_μ) has the following expression:

$$\Gamma_f = \frac{G_F m_Z^3 c_f}{6\sqrt{2}\,\pi}\left[g_V^{f\,2} + g_A^{f\,2}\right] \;\to\; (f=\mu^\pm) \;\to\; \Gamma_\mu \approx \frac{1}{4}\frac{G_F m_Z^3}{6\sqrt{2}\,\pi} \approx 83\text{MeV};$$

$$c_V = I_3^f - 2Q^f \sin^2\theta_W \qquad c_A = I_3^f$$

- All partial widths can be summarised in this table:

f	Q_f	g_A^f	g_V^f	Γ_f (MeV)	Γ_f / Γ_μ	R_f (%)
$\nu_e\ \nu_\mu\ \nu_\tau$	0	+½	+½	166	1.99	6.8
$e^-\ \mu^-\ \tau^-$	−1	-½	−.038	83	[1]	3.4
u c [t]	⅔	+½	+.192	286	3.42	11.8
d s b	-⅓	-½	−.346	368	4.41	15.2

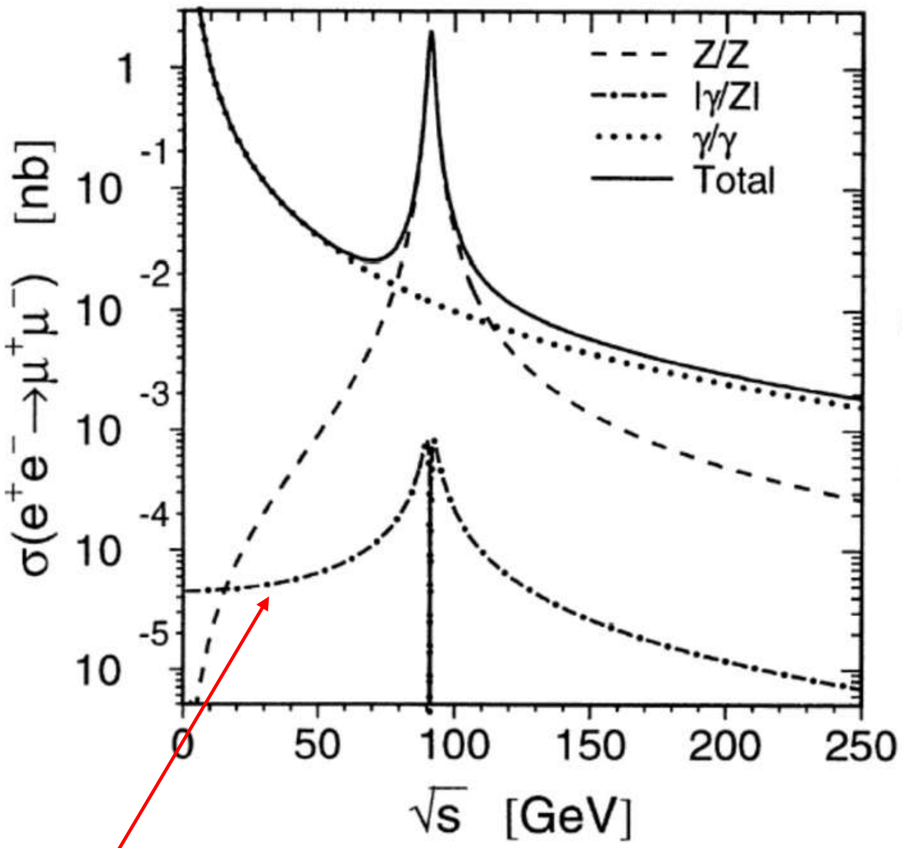
$$R_f = \frac{\Gamma_f}{\Gamma_Z}$$

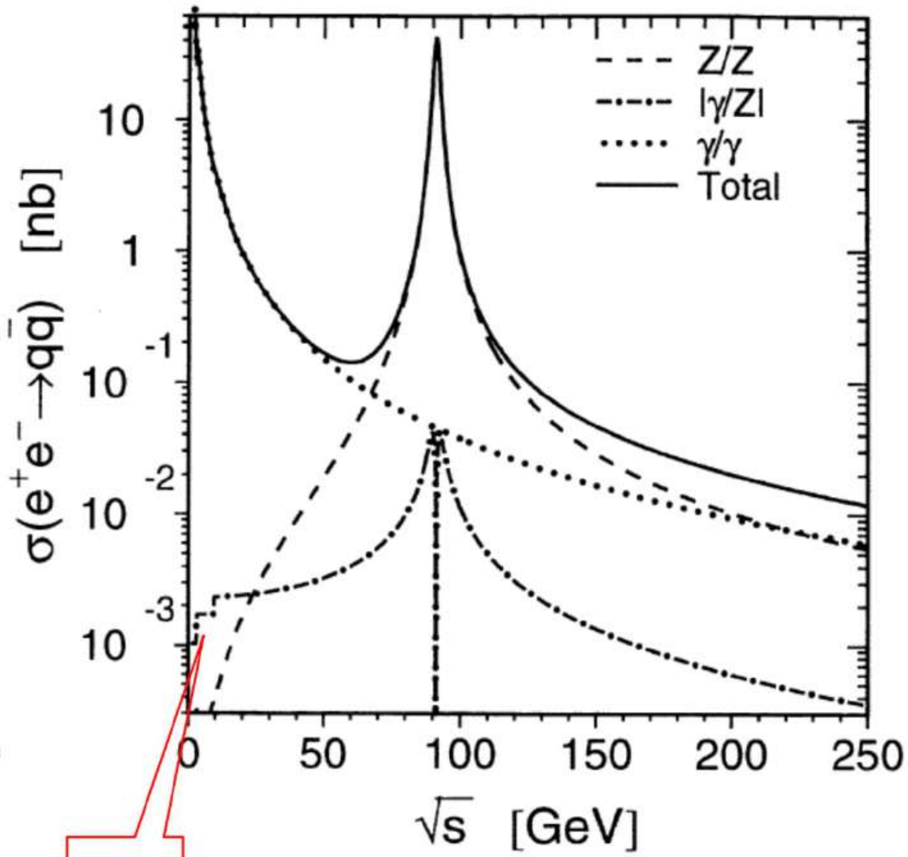
- Then, at the loweste order (Born approximation) we have:
  - $\Gamma_Z^B = 2423$ MeV, $\Gamma_{hadr.}^B = 1675$ MeV, $\Gamma_{invis.}^B = \Gamma_\nu^B = 498$ MeV;
  - $R_{hadr.}^B = 69.1$ %, $R_{lept\pm}^B = 10.2$ %, $R_{invis.}^B = R_{\nu's}^B = 20.5$ %,
  - $R_{hadr.}^B / R_{vis.}^B = 87.0$ %.

$$R_b = \frac{\Gamma_b}{\Gamma_{had}} = \frac{368}{1675} \approx 22\%$$

It was particularly important to measure precisely the b-quark B.R., since a deviation from the SM prediction could have been an indication of new physics.

$e^+e^- \to Z \to f\bar{f}$: predictions





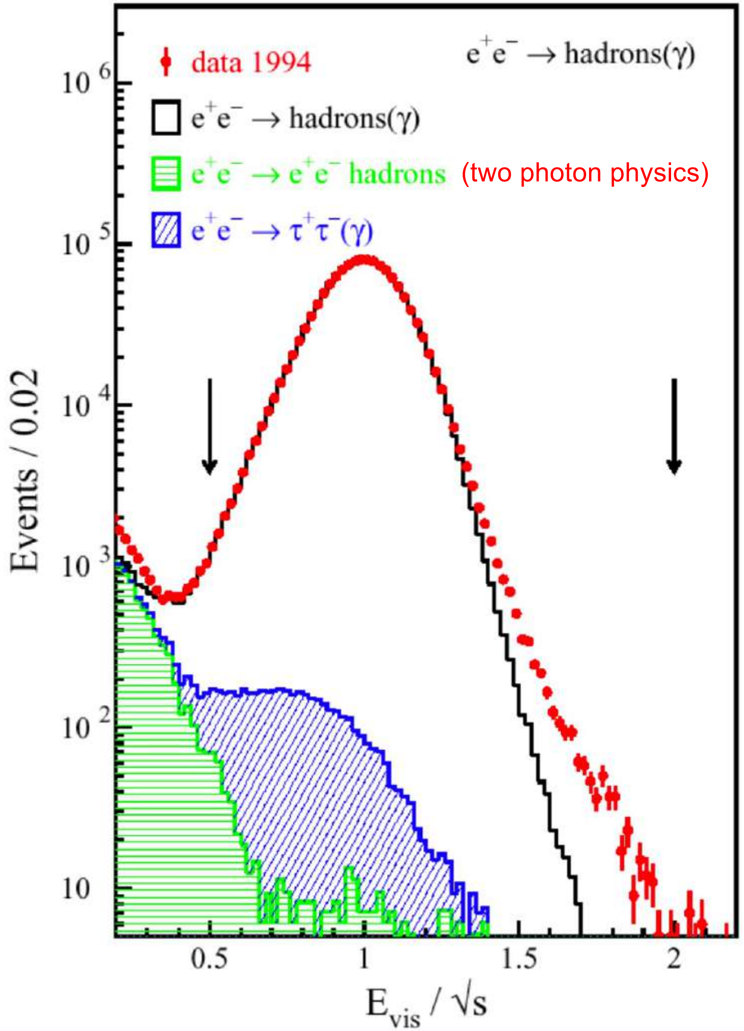
The interferference term is negative below m_Z, therefore here is plotted the module.

!!!

quark pair thresholds

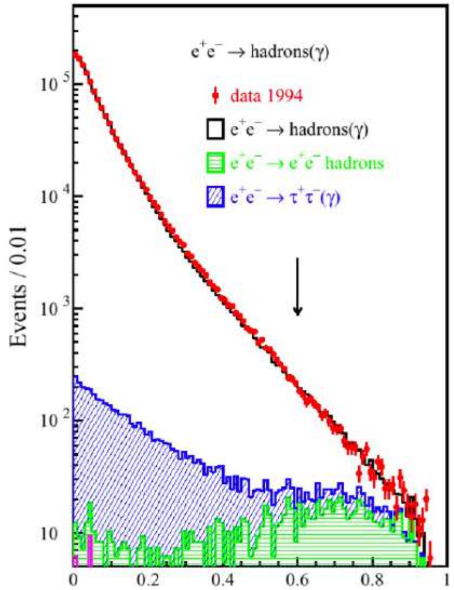
The interference term is zero at the Z-pole

$e^+e^- \to Z \to f\bar{f}$: hadrons (1)

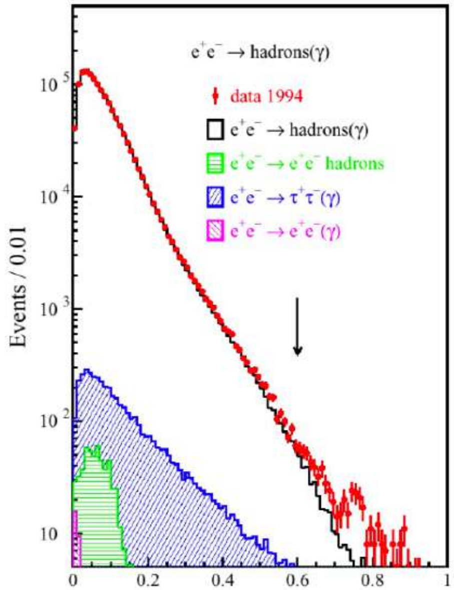


Example (L3): $e^+e^+ \to hadrons$ $(e^+e^+ \to q\bar{q})$

- *A lot of energy;*
- *No missing energy;*
- *Many particles in the final state;*



$|E_{\parallel}|/E_{vis}$



$|E_{\perp}|/E_{vis}$

Event selection

- $0.5 < E_{vis} / \sqrt{s} < 2.0$;
- $|E_{\parallel}| / E_{vis} < 0.6$;
- $E_{\perp} / E_{vis} < 0.6$;
- $N_{clusters} > 13$ (barrel), > 17 (endcap)

$E_{VIS} = \sum_{seen} |\vec{p}_j|$;

$\vec{P} = \sum_{seen} \vec{p}_j$;

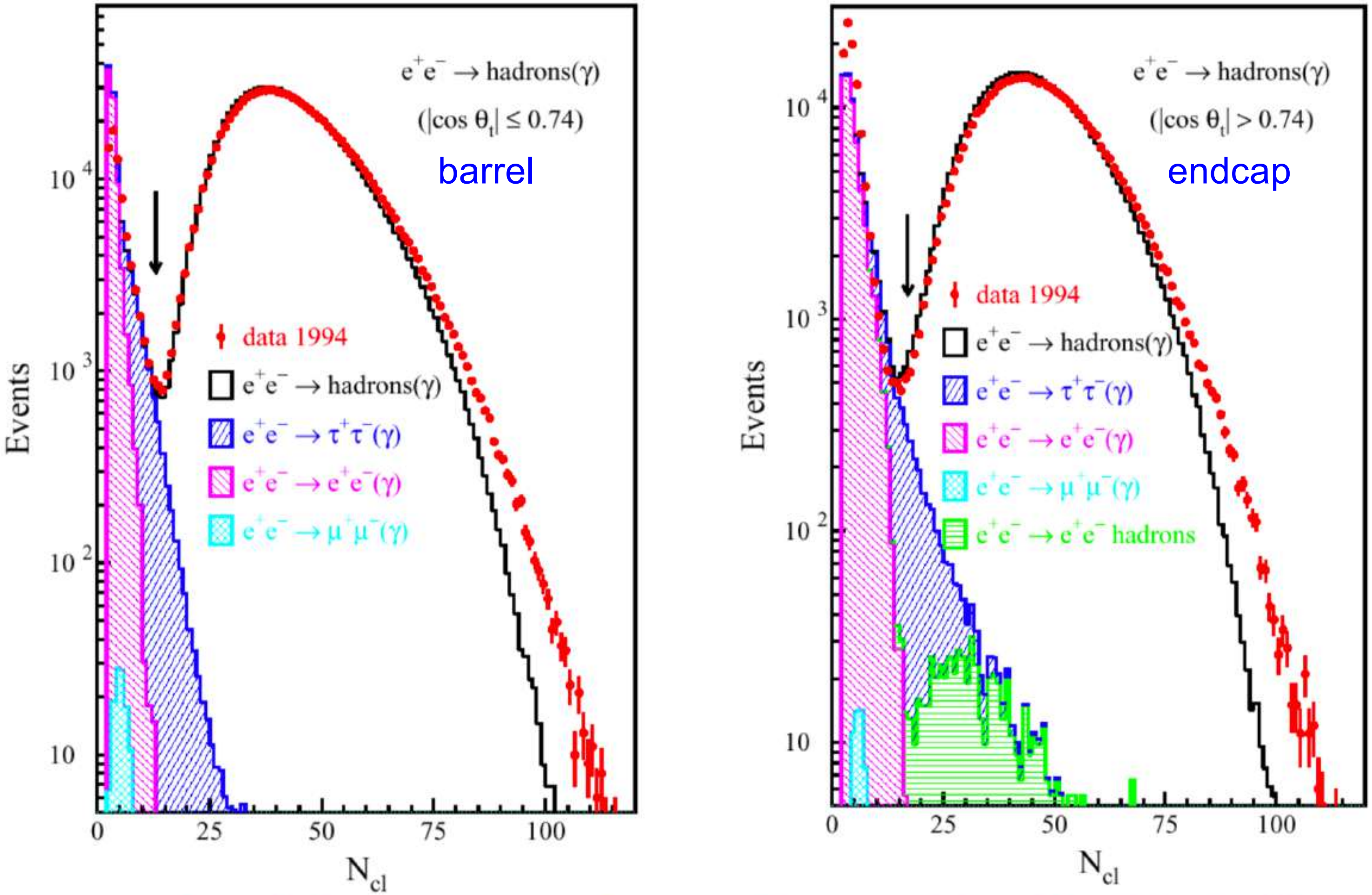
$|E_{\parallel}| = |P_Z|$; $\quad E_{\perp} = P_T$.

$e^+e^- \to Z \to f\bar{f}$: hadrons (2)

A cluster is defined as a continuous group of crystals or hcal cells.

It could very roughly identified with a particle, or at least they are proportional to the number of particles.

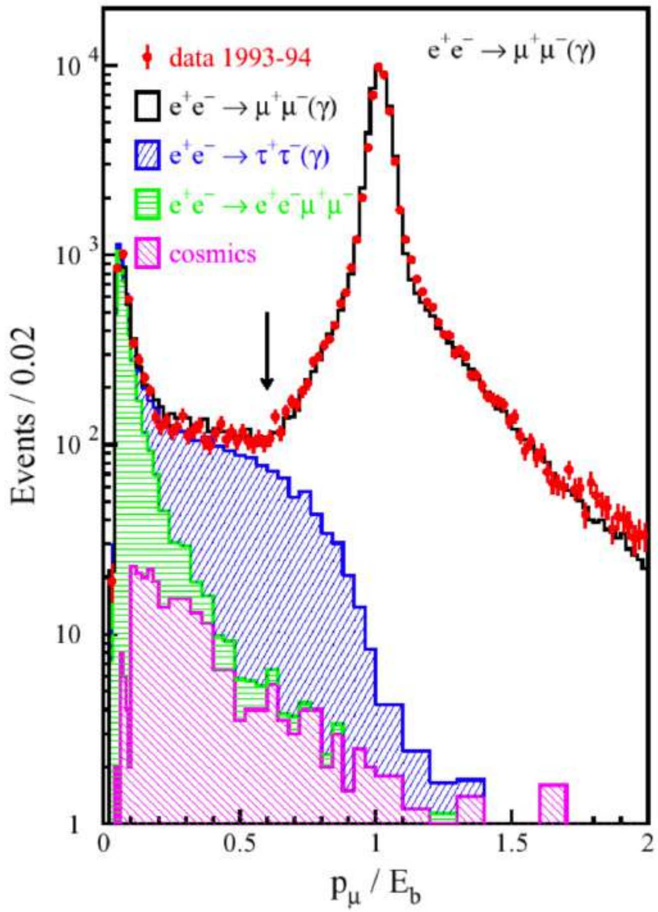
So, requiring a large number of clusters in the event, is equivalent to require a large number of particles in the event, as we should have in the Z hadronic decay

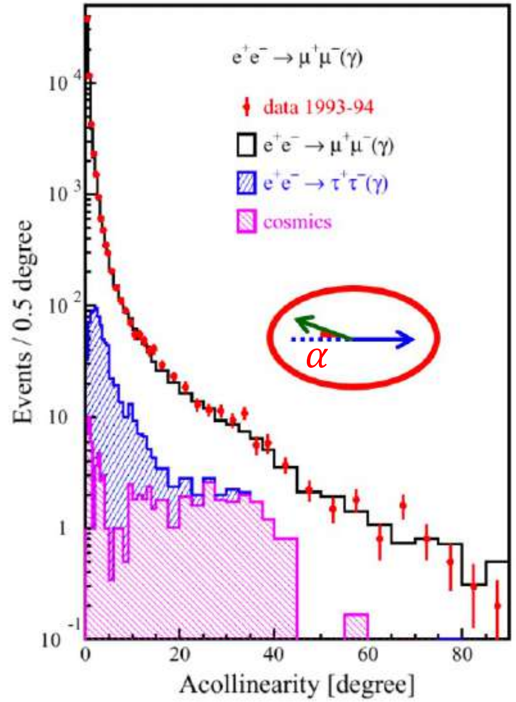


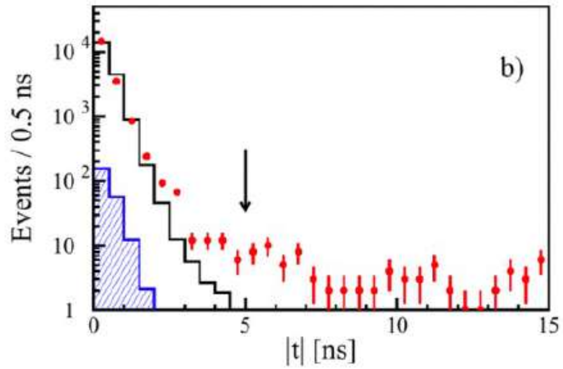
Example : $e^+e^- \to$ hadrons (i.e. e^+e^- $q\bar{q}$) in [$N_{clusters}$ > 13 (barrel), > 17 (endcap)]

$e^+e^- \to Z \to f\bar{f}: \mu^+\mu^-$

Example (L3): $e^+e^+ \to \mu^+\mu^-(\gamma)$

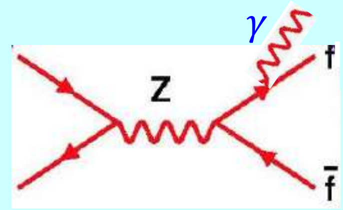






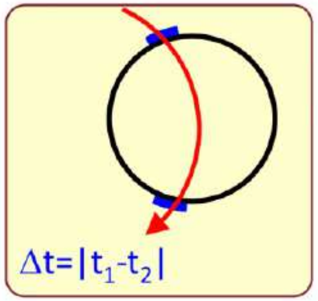
Event selection

- ≥ 1 μ identified;
- $|p_\mu| > 0.6\ (\sqrt{s}/2)$;
- α(□) "small";
- $N_{clusters} < 15$;
- $time_{scintillators}$.



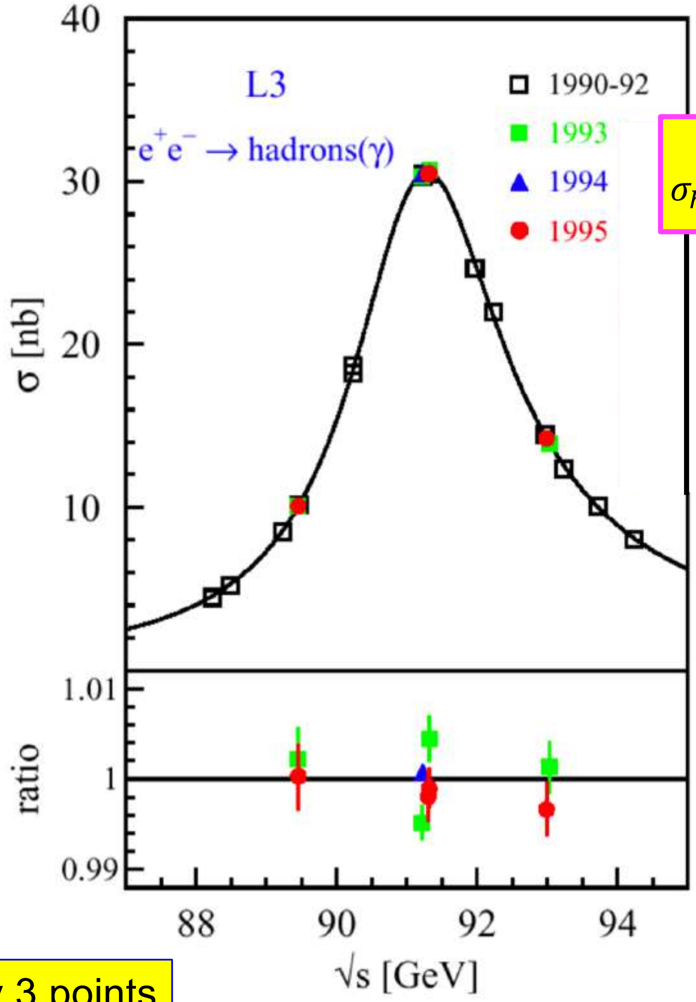
Usually it is not possible to identify the photon emitted from the final legs, therefore the finale state is given as $f\bar{f}(\gamma)$

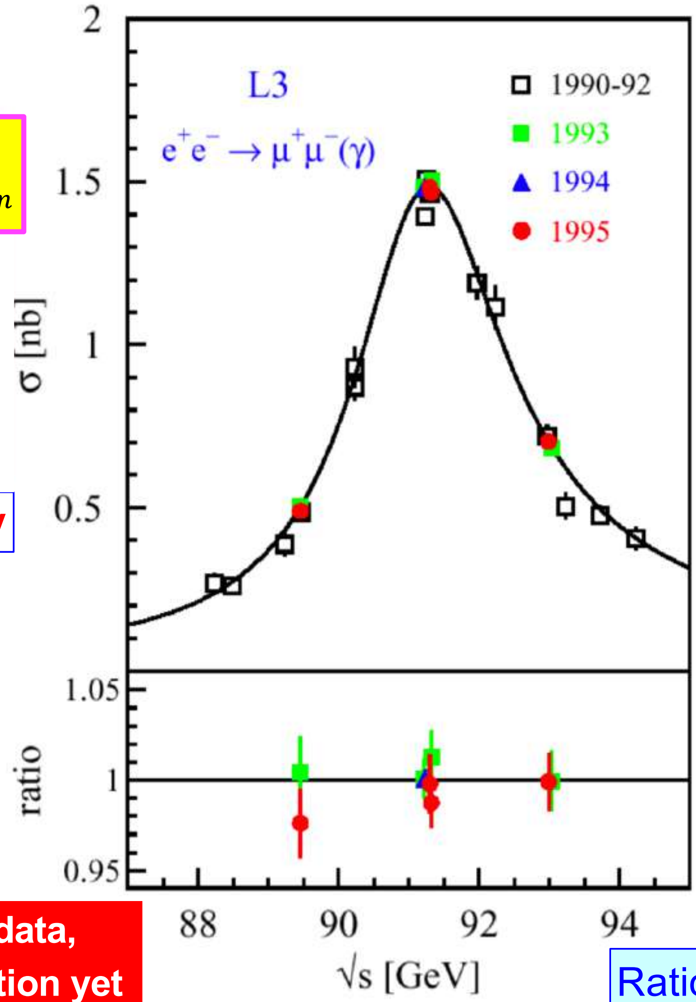
Q. : why μ's have smaller acollinearity than τ's ?



- Distance between 2 scintillator is (at least) 1.8 m
- A cosmic muon takes 6 ns to cover 1.8 m
- A muon pair from Z decay hit the two scintillators at the same time.

$e^+e^- \rightarrow Z \rightarrow f\bar{f}$: lineshape





Notice: $\sigma_{hadron} \gg \sigma_{lepton}$

92: peak only

93-94: only 3 points

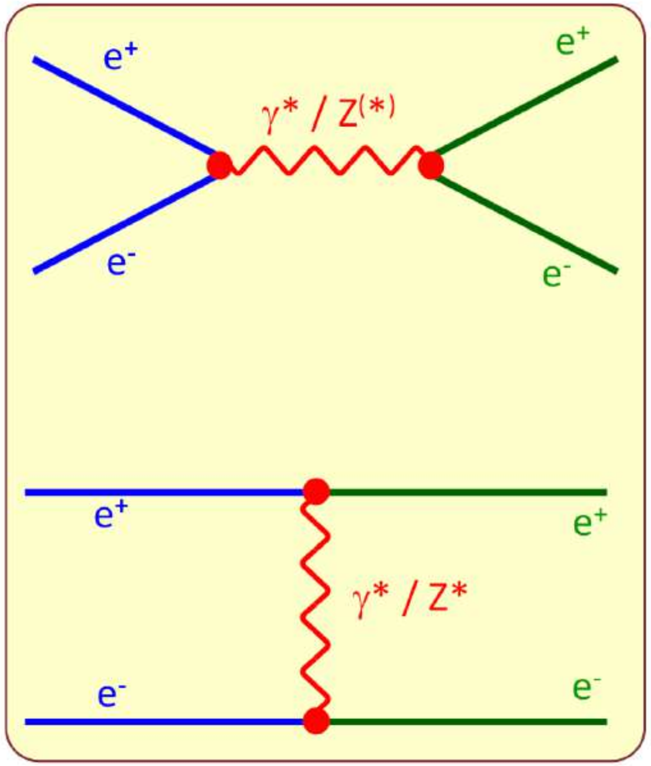
These are data, no interpretation yet

Ratio = data/SM

$e^+e^- \rightarrow Z \rightarrow e^+e^-$

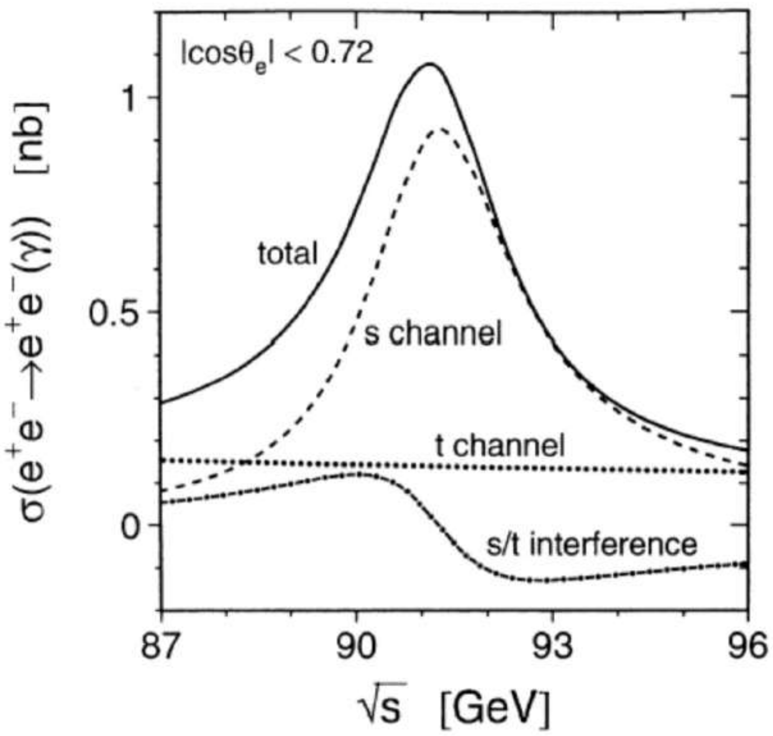
Slide from P. Bagnaia

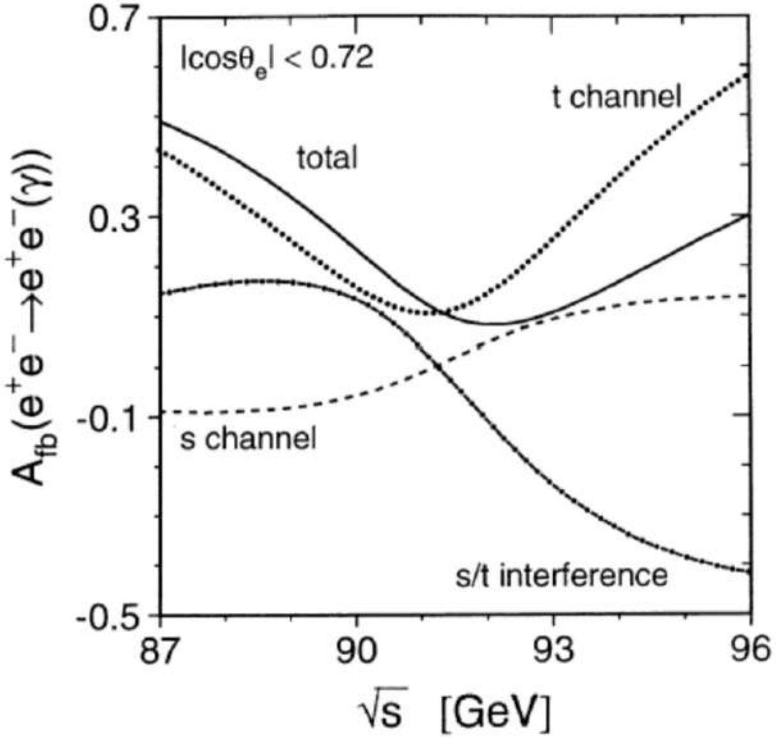
- Bhabha scattering is more difficult, due to the presence of another Feynman diagram: the γ^* / Z exchange in the t-channel;
- 4 Feynman diagrams → 10 terms :
  - Z s-channel (Z_s);
  - γ^* s-channel (γ_s);
  - Z t-channel (Z_t);
  - γ^* t-channel (γ_t);
  - 6 interferences;
- qualitatively :
  - $\mathbf{Z_t}$ negligible;
  - @ $\sqrt{s} \approx m_Z$ and $\theta >> 0°$, $\mathbf{Z_s}$ dominates.
  - @ $\theta \approx 0°$, $\boldsymbol{\gamma_t}$ dominates for all $\sqrt{s}$;
  - @ $\sqrt{s} << m_Z$ and $\theta >> 0°$, $\boldsymbol{\gamma_s}$ and $\boldsymbol{\gamma_t}$ are both important, while $\mathbf{Z_s}$ is negligible.



$e^+e^- \rightarrow Z \rightarrow e^+e^-$: σ_{SM}

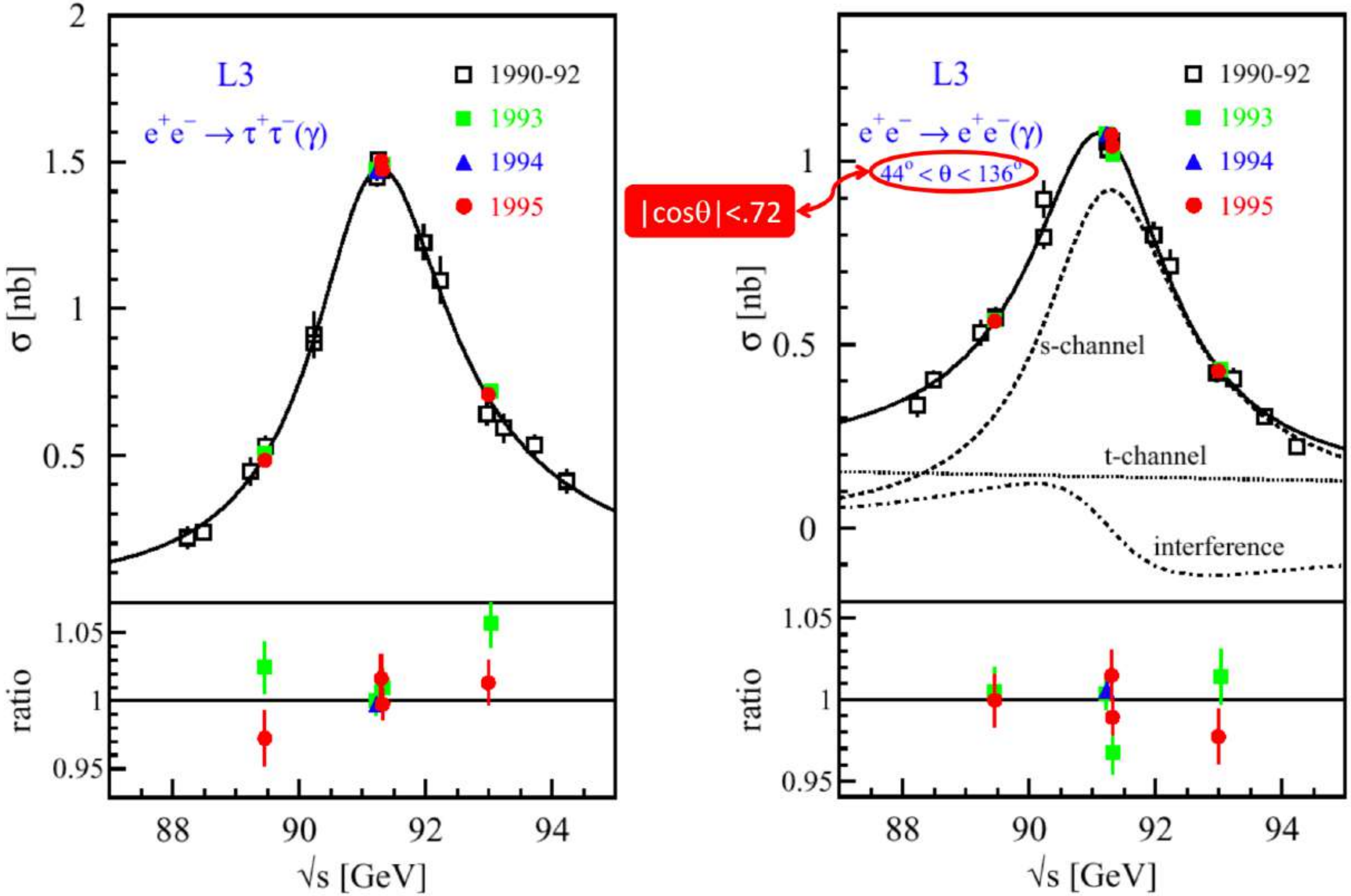
Slide from P. Bagnaia





- s, t, interference s/t vs $\sqrt{s}$, with a θ cut ($|\cos\theta| < 0.72$, i.e. $44° < \theta < 136°$);
- data @ $|\cos\theta| > 0.72$ available, but not used here [used for lumi];
- notice : the cut on $\cos\theta$ is NOT instrumental, but used OFFLINE to enhance Z_s over γ_t, to increase signal/bckgd and decrease stat error.

$e^+e^- \rightarrow Z \rightarrow e^+e^-$: results

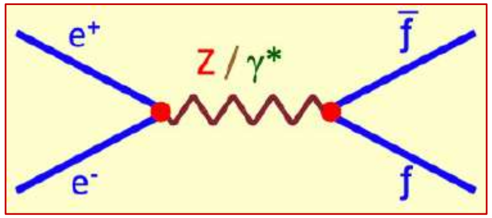


Forward-backward asymmetry

$d\sigma\left(e^+e^- \rightarrow f\bar{f}\right)/d\Omega$

- Differential cross-section at the lowest order:

$$\frac{d\sigma_{Born}\left(e^+e^- \rightarrow f\bar{f}\right)}{d\cos\theta} = \frac{\pi\alpha^2(s)c_f}{2s}\left\{ \begin{array}{l} \left(1+\cos^2\theta\right)\times\left[\begin{array}{l} Q_e^2Q_f^2 - 2\boxed{\chi}Q_fQ_eg_V^eg_V^f\cos\delta_R + \\ +\boxed{\chi^2}\left[\left(g_A^e\right)^2+\left(g_V^e\right)^2\right]\left[\left(g_A^f\right)^2+\left(g_V^f\right)^2\right] \end{array}\right] + \\ +2\cos\theta\times\left[-2\boxed{\chi}Q_eQ_fg_A^eg_A^f\cos\delta_R + 4\boxed{\chi^2}g_A^eg_A^fg_V^eg_V^f\right] \end{array} \right\};$$



$$\chi = \frac{G_F}{2\sqrt{2}\pi\alpha(s)}\times\frac{sm_Z^2}{\sqrt{\left(m_Z^2-s\right)^2+m_Z^2\Gamma_Z^2}}; \quad \tan\delta_R = \frac{m_Z\Gamma_Z}{m_Z^2-s} \quad \left[\rightarrow \cos\delta_R(\sqrt{s}=m_Z)=0\right];$$

Definition

$$A_f^{FB}\left(\sqrt{s}\right) \equiv \frac{\sigma\left(\cos\theta>0,\sqrt{s}\right)-\sigma\left(\cos\theta<0,\sqrt{s}\right)}{\sigma\left(\cos\theta>0,\sqrt{s}\right)+\sigma\left(\cos\theta<0,\right)}$$

$\boxed{A_f^{FB}}$ is the "forward-backward asymmetry" for $e^+e^- \rightarrow f\bar{f}$.

$$A_f^{FB}\left(\sqrt{s}=m_Z, Z_{s\text{-channel}}\ \text{only}\right) = 3\frac{g_V^eg_A^e}{\left(g_V^e\right)^2+\left(g_A^e\right)^2}\times\frac{g_V^fg_A^f}{\left(g_V^f\right)^2+\left(g_A^f\right)^2};$$

$$c_V = I_3^f - 2Q^f\sin^2\theta_W \qquad c_A = I_3^f$$

$d\sigma(e^+e^- \to f\bar{f})/d\Omega$: comments

Slide from P. Bagnaia

$$\frac{d\sigma_{Born}(e^+e^- \to f\bar{f})}{d\cos\theta} = \frac{\pi\alpha^2(s)c_f}{2s}\left\{(1+\cos^2\theta)\times\left[\begin{array}{l} Q_e^2Q_f^2 - 2\chi Q_f Q_e g_V^e g_V^f \cos\delta_R + \\ +\chi^2\left[(g_A^e)^2+(g_V^e)^2\right]\left[(g_A^f)^2+(g_V^f)^2\right]\end{array}\right] + \atop +2\cos\theta\times\left[-2\chi Q_e Q_f g_A^e g_A^f \cos\delta_R + 4\chi^2 g_A^e g_A^f g_V^e g_V^f\right]\right\};$$

$$A_f^{FB}(\sqrt{s}) \equiv \frac{\sigma(\cos\theta>0,\sqrt{s})-\sigma(\cos\theta<0,\sqrt{s})}{\sigma(\cos\theta>0,\sqrt{s})+\sigma(\cos\theta<0,\sqrt{s})} \xrightarrow{\sqrt{s}\to m_Z} 3\frac{g_V^e g_A^e}{(g_V^e)^2+(g_A^e)^2}\times\frac{g_V^f g_A^f}{(g_V^f)^2+(g_A^f)^2}.$$

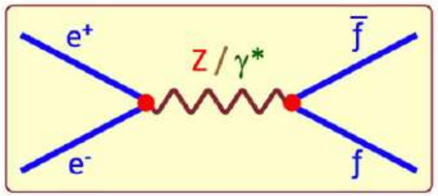
$$\chi = \frac{G_F}{2\sqrt{2}\pi\alpha(s)}\times\frac{s m_Z^2}{\sqrt{(m_Z^2-s)^2+m_Z^2\Gamma_Z^2}}$$

$$\cos\delta_R = m_Z^2 - s$$

mediators : γ, Z [= $Z_A + Z_V$];
$\mathbb{P}$-cons : $\gamma\gamma$, γZ_V, ZZ [= $Z_A^2 + Z_V^2$];
$\mathbb{P}$-viol. : γZ_A, $Z_A Z_V$.

- standard SM computation for $Z_s \oplus \gamma_s$ only (average on initial and sum on final polarization), then sum on φ:
- notice : the term $\propto$ ($\cos\theta$) is anti-symmetric; it does NOT contribute to σ_{tot} ($\int \cos\theta \, d\cos\theta = 0$), but only to the ($\mathbb{P}$-violating) forward-backward asymmetry;
- the $\mathbb{P}$-violation clearly comes from the interference between the vector ($\gamma + Z_V$) and axial (Z_A) terms.

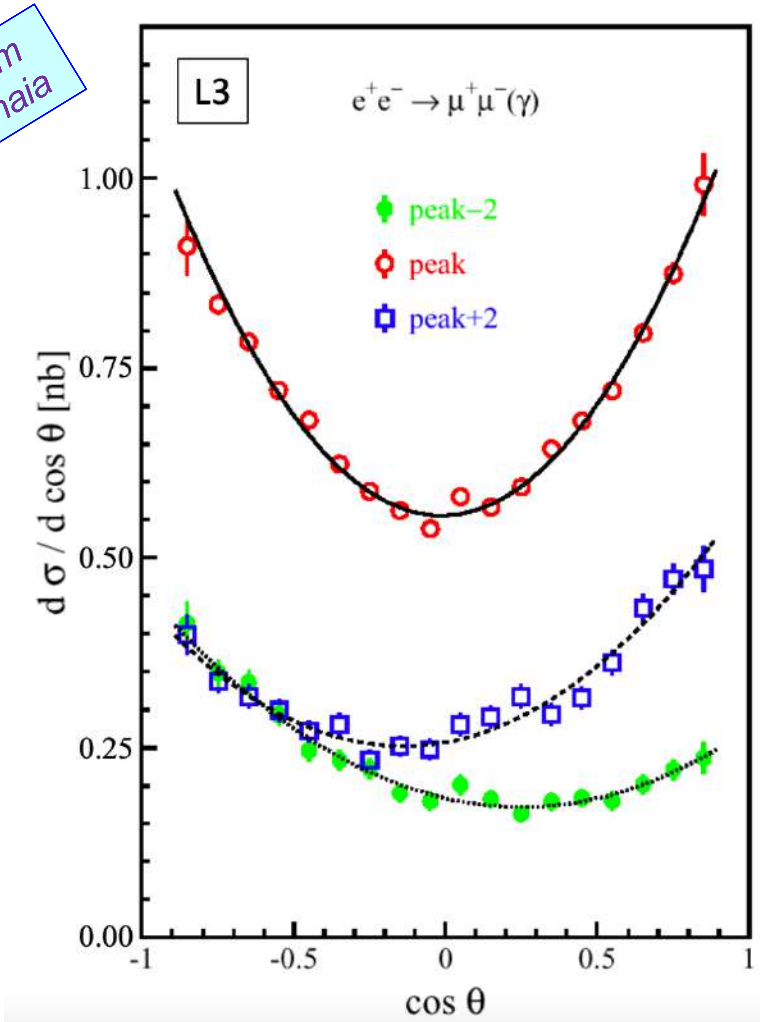
- at the pole ($\sqrt{s}=m_Z$), only few terms : [the photon-Z interferference terms vanish]
  - $\cos\delta_R = 0$;
  - the asymmetry, i.e. the term $\propto \cos\theta$, is $\propto g_V^e$ (very small) for all fermions;
  - for the $\mu^+\mu^-$ case [easily measurable], it is even smaller ($\propto g_V^e g_V^\mu$).



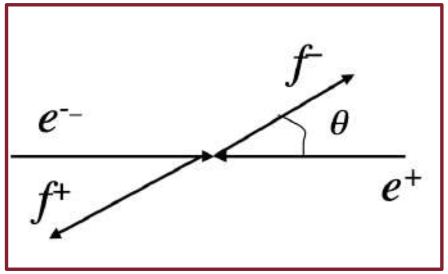
We will see later other asymmetries related to the polarization states. Here, no polarization is taken into account

$d\sigma\left(e^+e^- \to f\bar{f}\right)/d\Omega$: data

Slide from P. Bagnaia



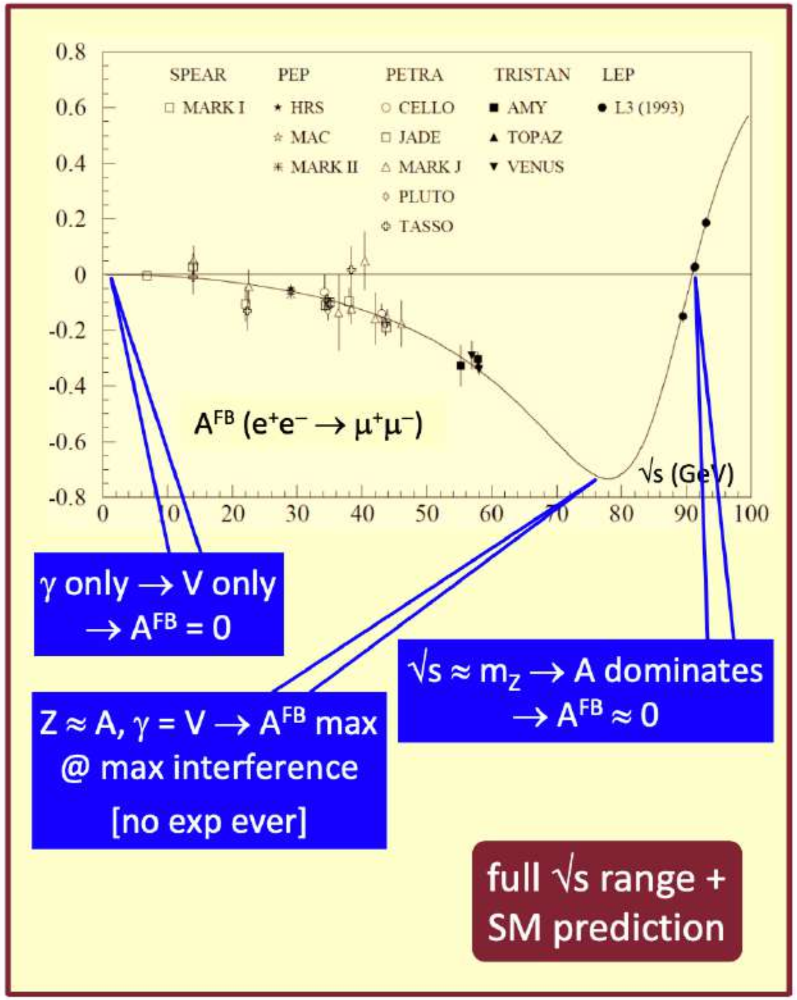
- Experimentally, the main problem is the selection $f \leftrightarrow \bar{f}$ (i.e. $\theta \leftrightarrow -\theta$). This is
  - essentially impossible for light quarks $u \leftrightarrow \bar{u}$, $d \leftrightarrow \bar{d}$ (despite heroic efforts based on charge counting);
  - difficult for heavy quarks c,b (based on lepton charge in semileptonic quark decays, e.g. $c \to s\ell^+\nu$, $\bar{c} \to \bar{s}\ell^-\bar{\nu}$);
  - "simple" for $\mu^\pm$ (only problem: wrong sagitta sign because of high momentum);
  - best channel for $d\sigma/d\cos\theta$ and A_{FB}: $e^+e^- \to \mu^+\mu^-(\gamma)$;
- unfortunately, $A_{FB}(\sqrt{s}=m_Z)$ is very small in the $\ell^+\ell^-$ channels, due to the extra small factor g_V^μ;
- notice the asymmetry change for peak ±2 GeV.

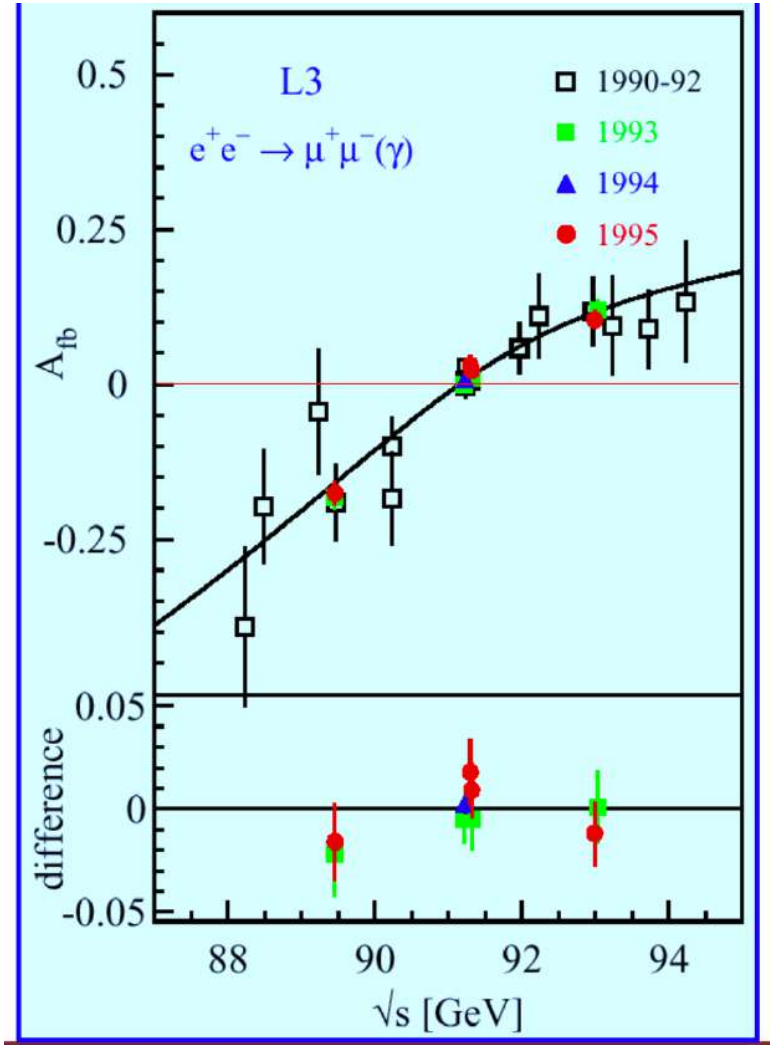


Pay attention: in the plot is shown the differential cross-section, and not the forward-backward asymmetry. Then, from this plot, we build the A_{FB}.

$d\sigma(e^+e^- \rightarrow f\bar{f})/d\Omega$: $A_{FB}(\mu^+\mu^-)$

Slide from P. Bagnaia





dσ/d cosθ: another set of formulae

$$\frac{d\sigma_f}{d\cos\Theta} = N_c^f \frac{\pi\alpha^2}{2s}\Big\{Q_f^2(1+\cos^2\Theta) \qquad (\gamma \text{ exchange})$$

$$- Q_f\Big[2 g_V^e g_V^f (1+\cos^2\Theta) + 4 g_A^e g_A^f \cos\Theta\Big]\Re\{\chi\} \qquad \text{(Interference)}$$

$$+ \Big(\Big[(g_V^e)^2+(g_A^e)^2\Big]\Big[(g_V^f)^2+(g_A^f)^2\Big](1+\cos^2\Theta)$$

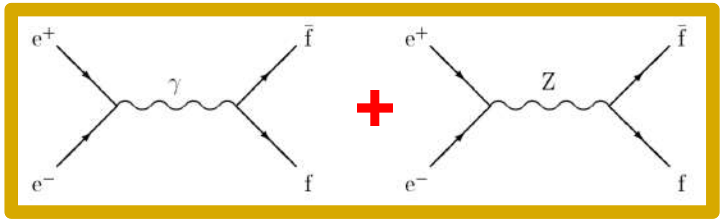
$$+ 8 g_V^e g_A^e g_V^f g_A^f \cos\Theta\Big)|\chi|^2\Big\} \qquad (Z \text{ exchange})$$

$$\chi = \frac{1}{4\sin^2\vartheta_W\cos^2\vartheta_W}\frac{s}{s-m_Z^2+i\Gamma_Z m_Z} \qquad N_c^f = \begin{cases} 1 \text{ for leptons} \\ 3 \text{ for quarks}\end{cases}$$

Z couplings

$$g_V^f = T_3^f - 2Q_f\sin^2\vartheta_W$$
$$g_A^f = T_3^f$$

$$g_L^f = g_V^f + g_A^f = 2T_3^f - 2Q_f\sin^2\vartheta_W$$
$$g_R^f = g_V^f - g_A^f = -2Q_f\sin^2\vartheta_W$$



$$\sigma_f = \frac{4\pi\alpha^2}{3s}N_c^f\Big\{Q_f^2 - 2Q_f g_V^e g_V^f\,\Re\{\chi\} + \Big[(g_V^e)^2+(g_A^e)^2\Big]\Big[(g_V^f)^2+(g_A^f)^2\Big]|\chi|^2\Big\}$$

These are the same formulae we met before, but written in a slightly different manner.
It could be usefull to see them, just in case.

$$\Gamma_Z = \sum_f \Gamma_f \qquad (m_f < m_Z/2)$$

$$\Gamma_f = N_c^f\frac{\alpha\, m_Z}{12\sin^2\vartheta_W\cos^2\vartheta_W}\Big[(g_V^f)^2+(g_A^f)^2\Big]$$

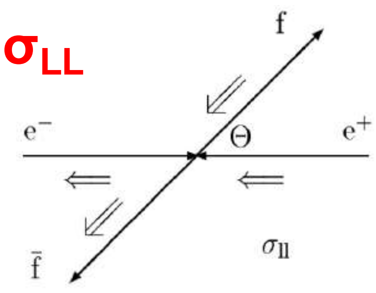
$$\sigma_f(\sqrt{s}=m_Z) \approx \frac{12\pi}{m_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2} = \frac{12\pi}{m_Z^2} BR(Z\to e^+e^-)\cdot BR(Z\to f\bar{f})$$

Polarization asymmetries

Helicities in $e^+e^- \to f\bar{f}$ at the Z pole

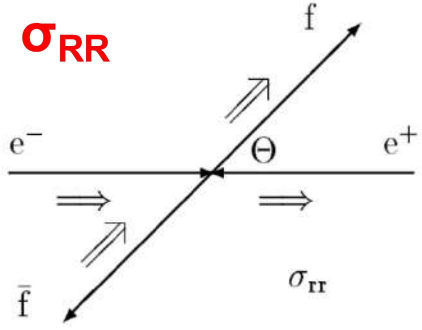
- Let's consider the helicity of the initial and final states.
- Since they are connected by a propagator of spin 1 (Z), the total angular momentum of the initial and final states must be also 1, with J_Z=+-1
- So, we can look only at the helicity state of the electron in the initial state and of the fermion in the final state. **We have four combinations:**
- electron left-handed and fermion left-handed: LL
- electron left-handed and fermion right-handed: LR
- electron right-handed and fermion right-handed: RR
- electron right-handed and fermion left-handed: RL
- **the final states are distinguishable due to the spin, so the cross sections can be calculated separately.**
- To be noticed that we can not have the spin flip along Z (J_Z must be conserved, either 1 or -1), therefore the differential cross-sections are 0 at theta=0° or at theta=180°.

σ_{LL}



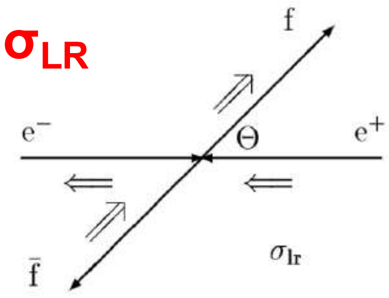
$$\frac{d\sigma_{ll}}{d\cos\Theta} \propto (g_L^e)^2 (g_L^f)^2 (1+\cos\Theta)^2$$

σ_{RR}



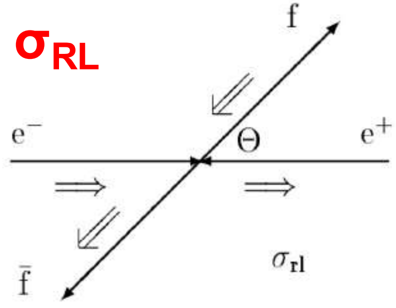
$$\frac{d\sigma_{rr}}{d\cos\Theta} \propto (g_R^e)^2 (g_R^f)^2 (1+\cos\Theta)^2$$

σ_{LR}



$$\frac{d\sigma_{lr}}{d\cos\Theta} \propto (g_L^e)^2 (g_R^f)^2 (1-\cos\Theta)^2$$

σ_{RL}



$$\frac{d\sigma_{rl}}{d\cos\Theta} \propto (g_R^e)^2 (g_L^f)^2 (1-\cos\Theta)^2$$

Helicities in $e^+e^- \rightarrow f\bar{f}$ at the Z pole

- From the four helicity cross-sections ($\sigma_{LL}, \sigma_{LR}, \sigma_{RR}, \sigma_{RL}$), we can get all the asymmetry cross-sections.

- Total cross-section (just the sum of the four):

$$\sigma_{tot} = \sigma_{LL} + \sigma_{LR} + \sigma_{RR} + \sigma_{RL}$$

- Left-Right asymmetry (initial state polarization); *we need to measure the initial state polarization*:

$$\sigma_{LR} = (\sigma_{LL} + \sigma_{LR}) - (\sigma_{RR} + \sigma_{RL})$$

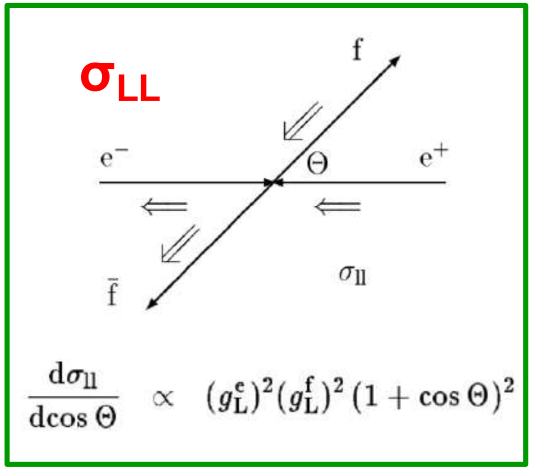
- Polarization asymmetry (final state polarization); *we need to measure the final state polarization*:

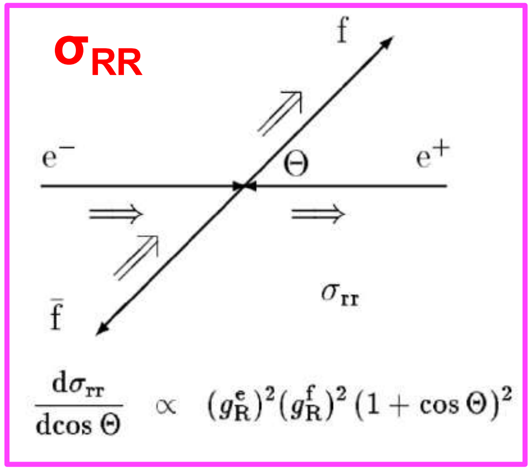
$$\sigma_{pol} = (\sigma_{LL} + \sigma_{RL}) - (\sigma_{LR} + \sigma_{RR})$$

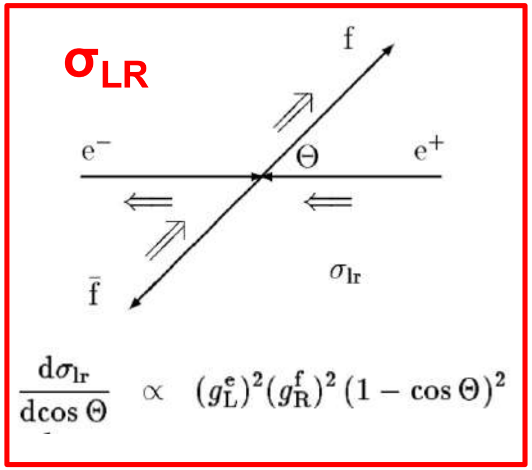
- Forward-Backward asymmetry; *it can be built from the helicity cross-sections since the differential cross-sections have a different theta behaviour*

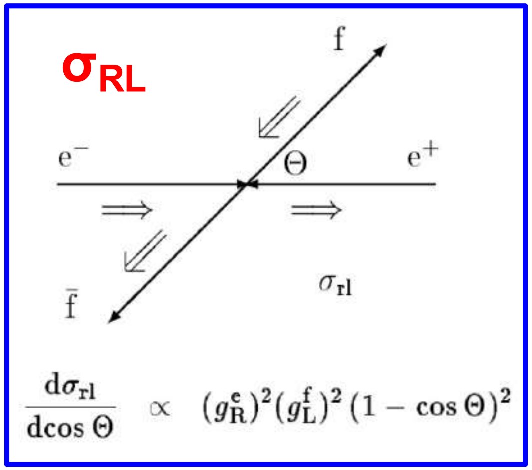
$$\sigma_{FB} = (\sigma_{LL} + \sigma_{RR}) - (\sigma_{LR} + \sigma_{RL})$$

- We get the asymmetry values at the Z pole by normalising the cross-sections with σ_{tot}.









Asymmetry parameter A_f

- Helicity cross-sections are proportional to g couplings, for instance:

$$\sigma_{LL} = \int_{-1}^{+1} \frac{d\sigma_{LL}}{d\cos\theta} d\cos\theta \propto (g_L^e)^2 \left(g_L^f\right)^2 \int_{-1}^{+1} (1+\cos\theta)^2 d\cos\theta \propto \frac{8}{3} (g_L^e)^2 \left(g_L^f\right)^2$$

To notice: we have also

$$\int_{-1}^{+1} (1-\cos\theta)^2 d\cos\theta = \frac{8}{3}$$

- Let's consider, for instance, the polarization asymmetry:

$$A_{pol} \equiv \mathcal{P}_f = \frac{\sigma_{pol}}{\sigma_{tot}} = \frac{(\sigma_{LL} + \sigma_{RL}) - (\sigma_{LR} + \sigma_{RR})}{\sigma_{LL} + \sigma_{LR} + \sigma_{RR} + \sigma_{RL}} = \frac{(g_L^e)^2 \left(g_L^f\right)^2 + (g_R^e)^2 \left(g_L^f\right)^2 - (g_L^e)^2 \left(g_R^f\right)^2 - (g_R^e)^2 \left(g_R^f\right)^2}{(g_L^e)^2 \left(g_L^f\right)^2 + (g_R^e)^2 \left(g_L^f\right)^2 + (g_L^e)^2 \left(g_R^f\right)^2 + (g_R^e)^2 \left(g_R^f\right)^2} =$$

$$= \frac{\left(\left(g_L^f\right)^2 - \left(g_R^f\right)^2\right)\left((g_L^e)^2 + (g_R^e)^2\right)}{\left(\left(g_L^f\right)^2 + \left(g_R^f\right)^2\right)\left((g_L^e)^2 + (g_R^e)^2\right)} = \frac{\left(g_L^f\right)^2 - \left(g_R^f\right)^2}{\left(g_L^f\right)^2 + \left(g_R^f\right)^2}$$

- We can define the fermion asymmetry parameter A_f:

$$A_f = \frac{\left(g_L^f\right)^2 - \left(g_R^f\right)^2}{\left(g_L^f\right)^2 + \left(g_R^f\right)^2}$$

Parity violation is contained in A_f

We can also write it as a function of g_V and g_A

$$g_V^f = T_3^f - 2 Q_f \sin^2\vartheta_W$$
$$g_A^f = T_3^f$$

$$g_L^f = g_V^f + g_A^f = 2T_3^f - 2Q_f \sin^2\vartheta_W$$
$$g_R^f = g_V^f - g_A^f = -2 Q_f \sin^2\vartheta_W$$

$$\mathcal{A}_f = \frac{g_{Lf}^2 - g_{Rf}^2}{g_{Lf}^2 + g_{Rf}^2} = \frac{2 g_{Vf} g_{Af}}{g_{Vf}^2 + g_{Af}^2} = 2\frac{g_{Vf}/g_{Af}}{1+(g_{Vf}/g_{Af})^2}.$$

Asymmetries at the Z pole

- **By using A_f we can define all the asymmetries at the Z pole.**

$$\mathcal{A}_{\mathrm{f}} = \frac{g_{\mathrm{Lf}}^2 - g_{\mathrm{Rf}}^2}{g_{\mathrm{Lf}}^2 + g_{\mathrm{Rf}}^2} = \frac{2 g_{\mathrm{Vf}} g_{\mathrm{Af}}}{g_{\mathrm{Vf}}^2 + g_{\mathrm{Af}}^2} = 2 \frac{g_{\mathrm{Vf}}/g_{\mathrm{Af}}}{1 + (g_{\mathrm{Vf}}/g_{\mathrm{Af}})^2} .$$

- **Polarization asymmetry:**

$$\mathcal{P}_{\mathrm{f}} = -\frac{\sigma_{\mathrm{pol}}}{\sigma_{\mathrm{tot}}} = -\mathcal{A}_{\mathrm{f}}$$

Minus sign is due to historical definition of the asymmetry as Right minus Left

In order to measure the asymmetry, we need to measure the polarization of the final state fermions.
It can not be done, except one case: tau lepton.

- **Left-Right asymmetry at the Z-pole:**

$$A_{\mathrm{LR}} = \frac{\sigma_{\mathrm{LR}}}{\sigma_{\mathrm{tot}}} = \mathcal{A}_{\mathrm{e}}$$

In order to measure this asymmetry, we need to have polarized electrons in the initial state. This was not possible at LEP because any longitudinal polarization would have been destroyed while the electrons were going around the ring, while it was possibile to achieve it at SLC since the electrons went around the circular part only once.

- **Forward-backward asymmetry at the Z-pole; we got it in terms of g_V and g_A:**

$$A_{\mathrm{FB}}^{\mathrm{f}}(\sqrt{s} = m_Z) = 3 \frac{g_{\mathrm{V}}^{\mathrm{e}}\, g_{\mathrm{A}}^{\mathrm{e}}}{(g_{\mathrm{V}}^{\mathrm{e}})^2 + (g_{\mathrm{A}}^{\mathrm{e}})^2} \frac{g_{\mathrm{V}}^{\mathrm{f}}\, g_{\mathrm{A}}^{\mathrm{f}}}{(g_{\mathrm{V}}^{\mathrm{f}})^2 + (g_{\mathrm{A}}^{\mathrm{f}})^2} \Rightarrow A_{\mathrm{FB}} = \frac{3}{4} \frac{\sigma_{\mathrm{FB}}}{\sigma_{\mathrm{tot}}} = \frac{3}{4} \mathcal{A}_{\mathrm{e}} \mathcal{A}_{\mathrm{f}}$$

Polarization Asymmetries at LEP/SLC

- σ_{tot} and A_{FB} can be measured for all charged leptons, heavy quarks and, inclusively, for all five quarks flavours.
- The polarization of the final state fermion is observable only in the reaction $e^+e^- \rightarrow \tau^+\tau^-$ where $\mathcal{P}_\tau$ is inferred from the energy spectra of the decay product of the tau.
- For the measurement of A_{LR} all Z decays into hadrons and charged leptons can be used. However, it requires the longitudinal polarization of the incoming electrons in e^+e^- collisions which was achieved only at the SLC collider.

- The cross section differences of left and right-handed fermions in the initial and final state σ_{LR} and σ_{pol} can be measured as a function of the scattering angle θ. This way we define:

$$A_{FB}^{pol} = \frac{1}{\sigma_{tot}}\left[\int_0^1 \frac{\partial\sigma_{pol}}{\partial\cos\Theta}\,\mathrm{d}\cos\Theta - \int_{-1}^0 \frac{\partial\sigma_{pol}}{\partial\cos\Theta}\,\mathrm{d}\cos\Theta\right]$$

$$A_{FB}^{LR} = \frac{1}{\sigma_{tot}}\left[\int_0^1 \frac{\partial\sigma_{LR}}{\partial\cos\Theta}\,\mathrm{d}\cos\Theta - \int_{-1}^0 \frac{\partial\sigma_{LR}}{\partial\cos\Theta}\,\mathrm{d}\cos\Theta\right].$$

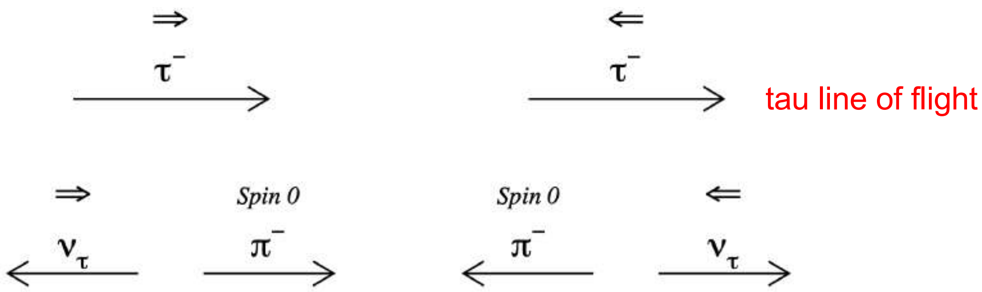
⇒

$$A_{FB}^{pol} = \frac{3}{4}A_e \qquad A_{FB}^{LR} = \frac{3}{4}A_f$$

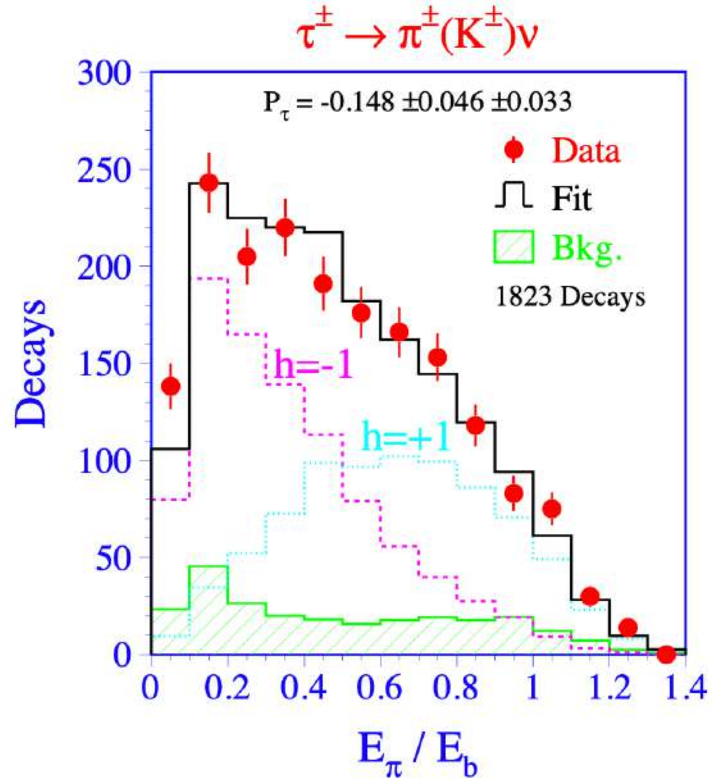
- As a consequence of helicity conservation at the Zff vertices, the measurement of the angular dependence of the final state polarization asymmetry provides information on the couplings of the initial state electrons and viceversa.
- Thus the measurement of the tau polarization as a function of cos θ yields a measurement of the electron coupling.
- Therefore, polarization asymmetry measurements at LEP and SLC are complementary.

Tau polarization

- ❑ **Tau leptons decay close to the interaction point before the decay products reach the detector.**
- ❑ **The energy spectrum of the final state particles in the two body decays depends on the tau polarization.**



Example: $\tau \rightarrow \nu + \pi$



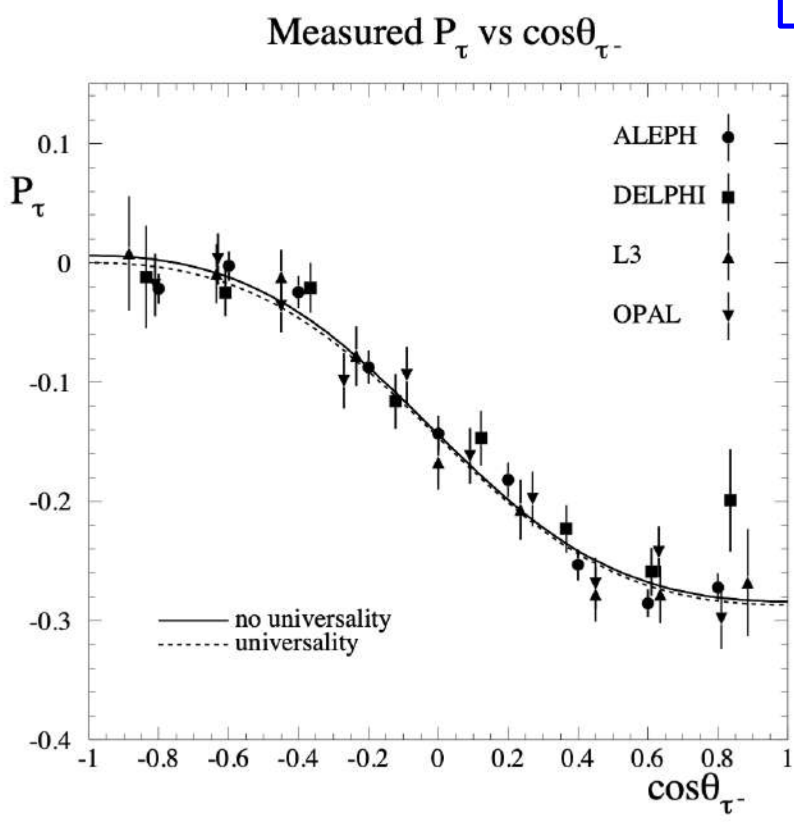
- pion preferentially escapes in the direction of the tau helicity
- Hence, when boosted into the Lab frame, the pion receives on average more energy from the decay of a right-handed tau than from a left-handed tau.
- From the distribution one can fit the contribution from the two helicity states.

$$P_{\tau^-} = \frac{\sigma^R_{\tau^-} - \sigma^L_{\tau^-}}{\sigma^R_{\tau^-} + \sigma^L_{\tau^-}} = -\frac{2\,\bar{g}^\tau_A\,\bar{g}^\tau_V}{(\bar{g}^\tau_A)^2 + (\bar{g}^\tau_V)^2} \equiv -A_\tau$$

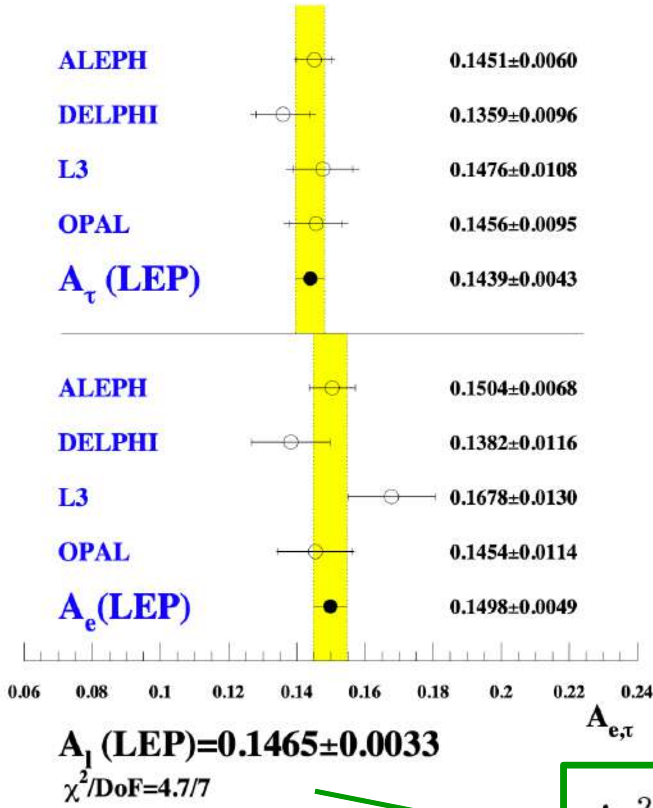
Tau polarization

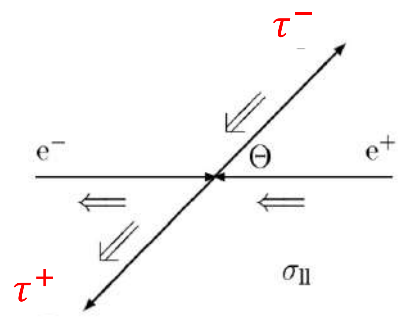
- Polarization as a function of the angle:

$$P_{\tau^-}(\cos\theta) = -\frac{(1+\cos^2\theta)\,A_\tau + 2\cos\theta\,A_e}{(1+\cos^2\theta) + 2\cos\theta\,A_\tau A_e}$$



From this distribution we get A_τ and A_e



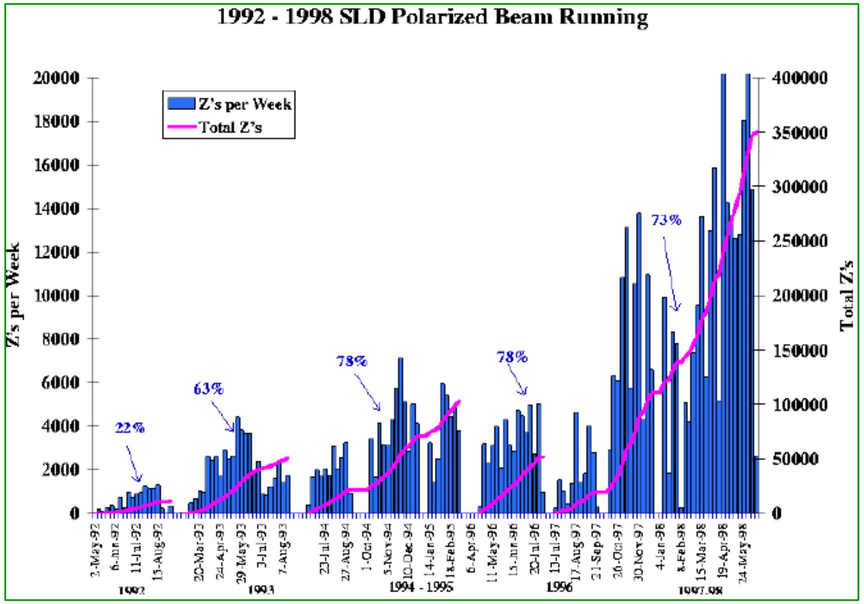


From g_V (contained in A_f) we get $\sin\theta_W$

$$\sin^2\theta_{\rm eff}^{\rm lept} = 0.23159 \pm 0.00041\,.$$

SLC: Initial state polarization

SLD experiment at the Stanford Linear Collider

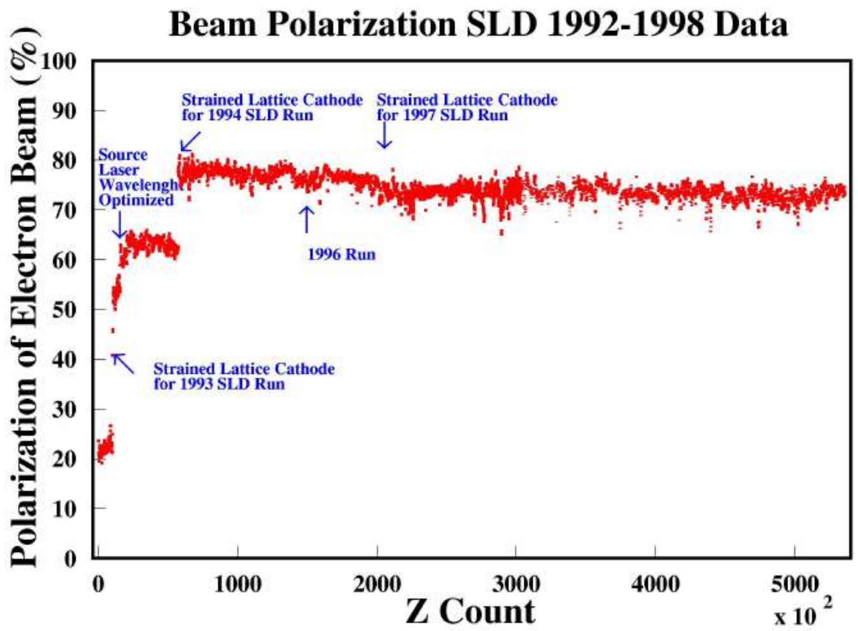


≈ 500 000 Z decays observed

≈ 1/10 of typical LEP experiment

BUT

≈ 77% longitudinal electron polarisation



- Longitudinally polarized electrons were produced at the Polarized Electron Source by illuminating a GaAs photocathode with circular-polarized light from a laser of wavelength 715 nm.
- A system of bending magnets and a superconducting solenoid were used to rotate the spins so that the polarization was preserved while the 1.21 GeV electrons were stored in the damping ring.
- Another set of bending magnets and two superconducting solenoids oriented the spin vectors so that longitudinal polarization of the electrons was achieved at the collision point with the unpolarized positrons.

(with $g_V/g_A = 1 - 4\sin^2\vartheta_W$)

SLD: Left-Right asymmetry

❑ **The Left-Right asymmetry, taking into account the average beam polarization, is:**

$$A_{LR} = \frac{1}{<P_e>} \frac{N_{e_l^-} - N_{e_r^-}}{N_{e_l^-} + N_{e_r^-}} .$$

⇨ $A_e = \langle P_e \rangle A_{LR}$

❑ **Electron asymmetry:** $A_e = 0.1516 \pm 0.0021$ $A_e = 0.1498 \pm 0.0049\ (at\ LEP)$

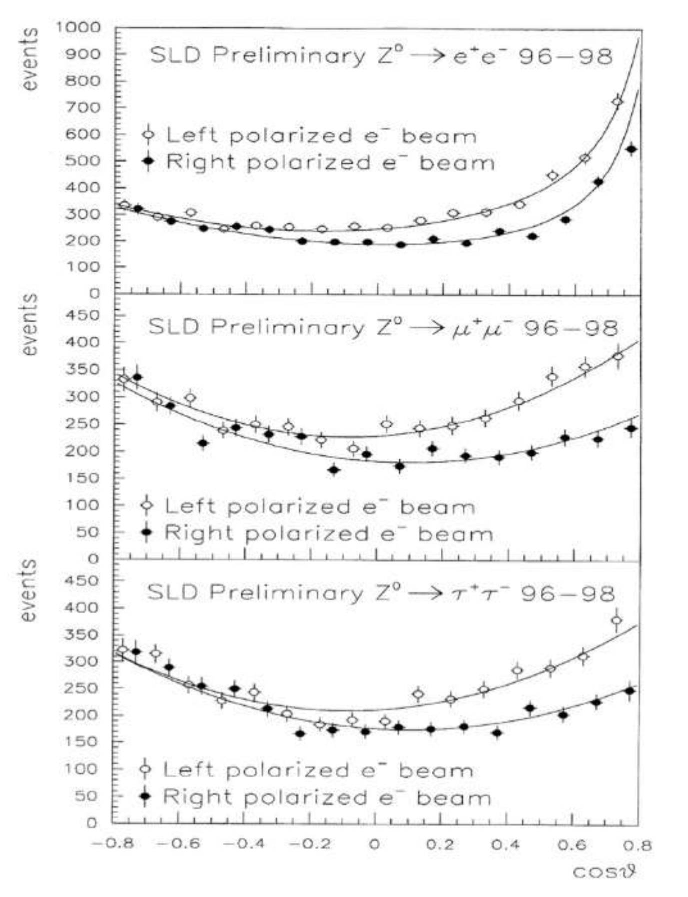
$\sin^2\vartheta_W = 0.23098 \pm 0.00026$ Best single measure of the World

❑ **By measuring the forward-backward asymmetry for a given final state, it was possible to measure the asymmetries for the different fermions:**

Leptons		
A_e	=	0.1516 ± 0.0021
A_μ	=	0.142 ± 0.015
A_τ	=	0.136 ± 0.015

Quarks		
A_s	=	0.85 ± 0.09
A_c	=	0.922 ± 0.020
A_b	=	0.670 ± 0.026

s quarks were identified by tagging high-momentum K and Λ



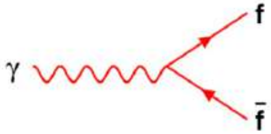
Radiative corrections

Standard Model relationships

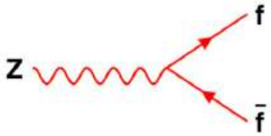
Masses of heavy gauge bosons and their couplings to fermions depend on SAME mixing angle

$$\cos\theta_W = M_W/M_Z$$

$SU(2)\times U(1)$ coupling constants, g, g', proportional to electric charge e: $g = e\sin\theta_W$, $g' = e\cos\theta_W$



$$-ieQ\gamma_\mu$$



$$ie\gamma_\mu(g_v - g_a\gamma_5)\frac{1}{2\sin\theta_W\cos\theta_W}$$

$$ie\gamma_\mu(1-\gamma_5)\frac{1}{2\sqrt{2}\sin\theta_W}$$

where Q, g_a and g_v depend on fermion type, with

$$g_a = \quad T^3 \quad = \pm\frac{1}{2}$$

$$g_v = (T^3 - 2Q\sin^2\theta_W) = \pm\frac{1}{2}(1-4|Q|\sin^2\theta_W)$$

g_v/g_a gives $\sin^2\theta_W$ if you know $|Q|$.

Relate e, $\sin\theta_W$ and M_W to the best measured parameters:

$$\alpha \equiv \frac{e^2}{4\pi} = 1/137.035\,999\,76(50)$$

$$G_F \equiv \frac{\pi\alpha}{\sqrt{2}M_W^2\sin^2\theta_W} = 1.166\,39(1)\times 10^{-5}\ \text{GeV}^{-2}$$

$$M_Z = 91.1875(21)\ \text{GeV}$$

G_F measured from muon decay; M_Z from LEP.

These relations are true at tree level, but to check that they are valid, must take into account radiative corrections, which give sensitivity to virtual heavy particles, and possibly new physics!

Aside: Other SM inputs needed are fermion masses, Higgs mass, CKM matrix (quark mass eigenstates are not weak eigenstates), strong coupling constant, α_s

The need for radiative corrections

- **the mixing angle $s_W = \sin\theta_W$ can be extracted from the ratio of the coupling g_V^f / g_A^f that enters in the cross section measurements. From the observables with leptons in the final states, the latest LEP measurement of s_W^2 from leptonic observables, which we will identify as s^2, gives an average value:**

$$s_W^2 = s_{\mathrm{eff},l}^2 = 0.23159 \pm 0.00018 \quad \text{(i.e., 0.08\% precision)}.$$

- **This angle also describes the ratio M_W^2 / M_Z^2 and therefore could be extracted from a combination of entirely different measurements (LEP1 and LEP2). The latest data give:**

$$M_Z = 91.1875 \pm 0.0021 \text{ GeV } (\Delta M_Z / M_Z \sim 2 \times 10^{-5})$$
$$M_W = 80.451 \pm 0.033 \text{ GeV } \left(\frac{\Delta M_W}{M_W} \sim 5 \times 10^{-4}\right),$$

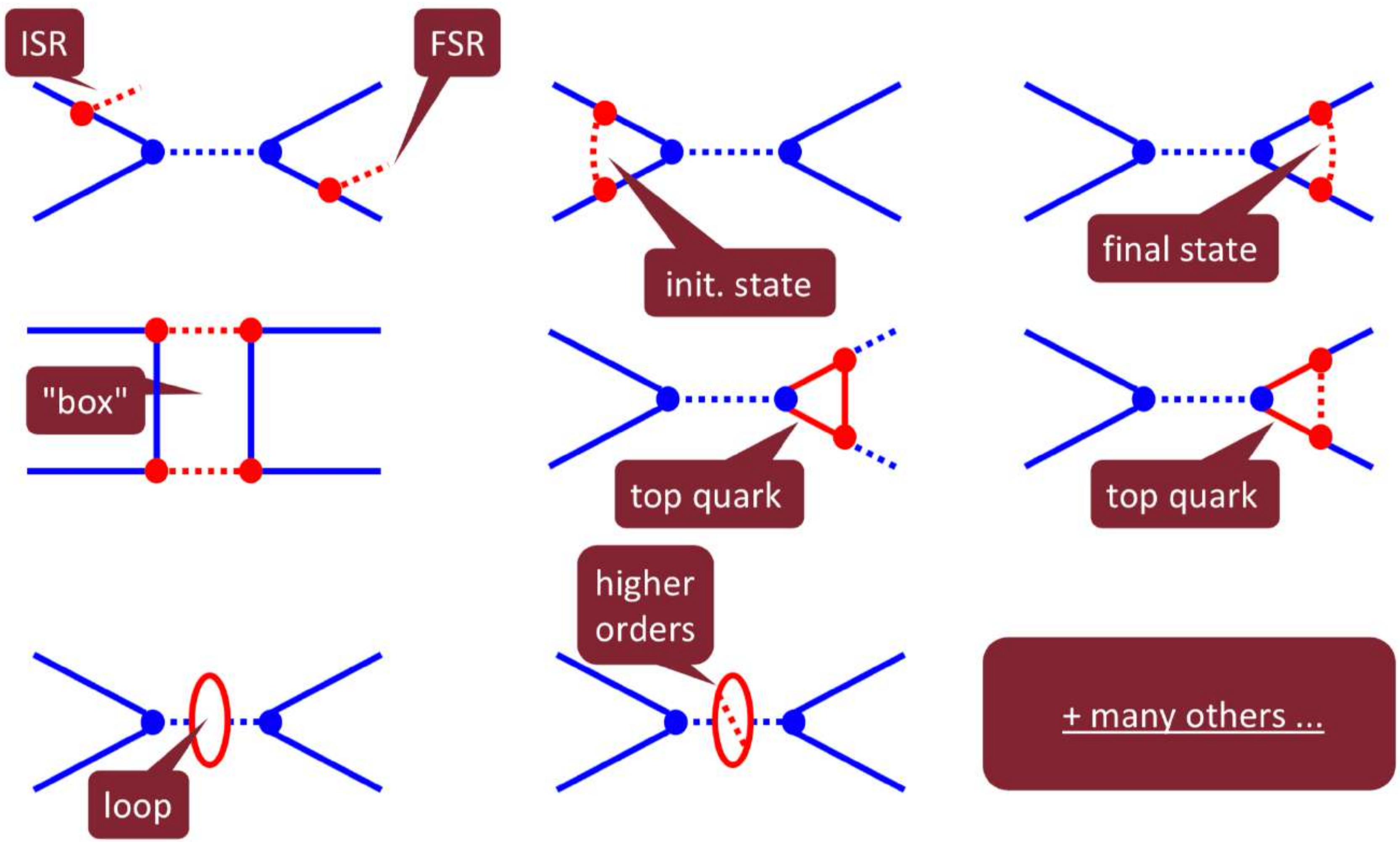
- **leading to:**

$$s_M^2 = 0.22162 \pm 0.00067 \text{ (i.e., 0.3\% precision)}.$$

- **Not only do we get a much better precision on the effective mixing angle extracted from the Z observables, but more telling is that this value is about 14σ away from that extracted from the mass ratio, M_W^2 / M_Z^2. The Born approximation is insufficient to explain the LEP measurements. Improving on the Born approximation necessitates the inclusion of radiative corrections which are contributions from the quantum fluctuations of the vacuum.**

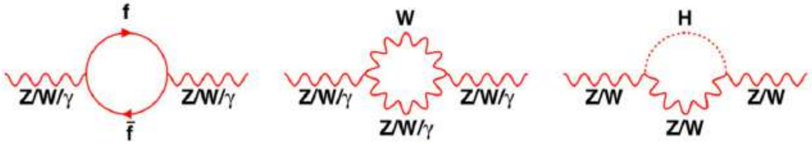
Radiative corrections

Slide from P. Bagnaia

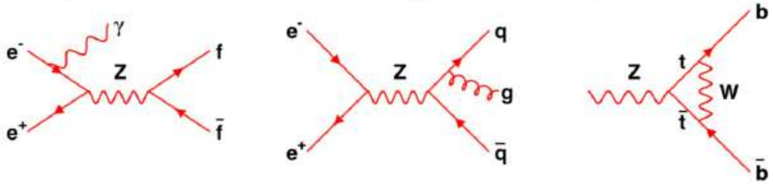


Radiative corrections

Propagator corrections are the same for each fermion type.



QED, QCD and vertex corrections give fermion dependent terms.



Electroweak corrections absorbed into effective couplings:

$$g_V \equiv g_V^{\text{eff}} = \sqrt{(1+\Delta\rho)}(T^3 - 2Q\sin^2\theta_{\text{eff}})$$
$$g_A \equiv g_A^{\text{eff}} = \sqrt{(1+\Delta\rho)}T^3$$
$$\sin^2\theta_{\text{eff}} = (1+\Delta\kappa)\sin^2\theta_W$$

$$\Delta\rho = \frac{3G_F M_W^2}{8\sqrt{2}\pi^2}\left(\frac{M_t^2}{M_W^2} - \tan^2\theta_W\left[\ln\frac{M_H^2}{M_W^2} - \frac{5}{6}\right]\right) + \cdots$$
$$\Delta\kappa = \frac{3G_F M_W^2}{8\sqrt{2}\pi^2}\left(\cot^2\theta_W\frac{M_t^2}{M_W^2} - \frac{11}{9}\left[\ln\frac{M_H^2}{M_W^2} - \frac{5}{6}\right]\right) + \cdots$$

Extra M_t^2/M_W^2 contributions for b quark

The value of G_F is also modified:

$$G_F = \frac{\pi\alpha}{\sqrt{2}M_W^2\sin^2\theta_W}\frac{1}{1-\Delta r}$$

where

$$\Delta r = \Delta\alpha + \Delta r_w = \Delta\alpha - \Delta\kappa + \cdots$$

$\Delta\alpha$ term incorporates the running of the electromagnetic coupling due to fermion loops in the photon propagator. The difficult part of the calculation is to account for all the hadronic states. Use experimental measurement of $e^+e^- \to \text{hadrons}$ at low $\sqrt{s}$.

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha}$$

$\alpha(0) = 1/137.035\,999\,76(50)$; $\alpha(M_Z) = 1/128.936(46)$

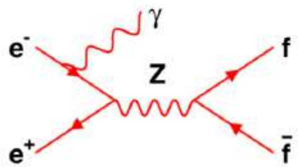
Quadratic dependence on M_t
Logarithmic dependence on M_H
Can **fit** both M_t and M_H

Use programs such as ZFITTER (D Bardin et al.) and TOPAZ0 (G Montagna et al.) for calculations to higher order.

Leading order expressions above are for large M_H.

QED corrections

Dominant QED correction from initial state radiation.

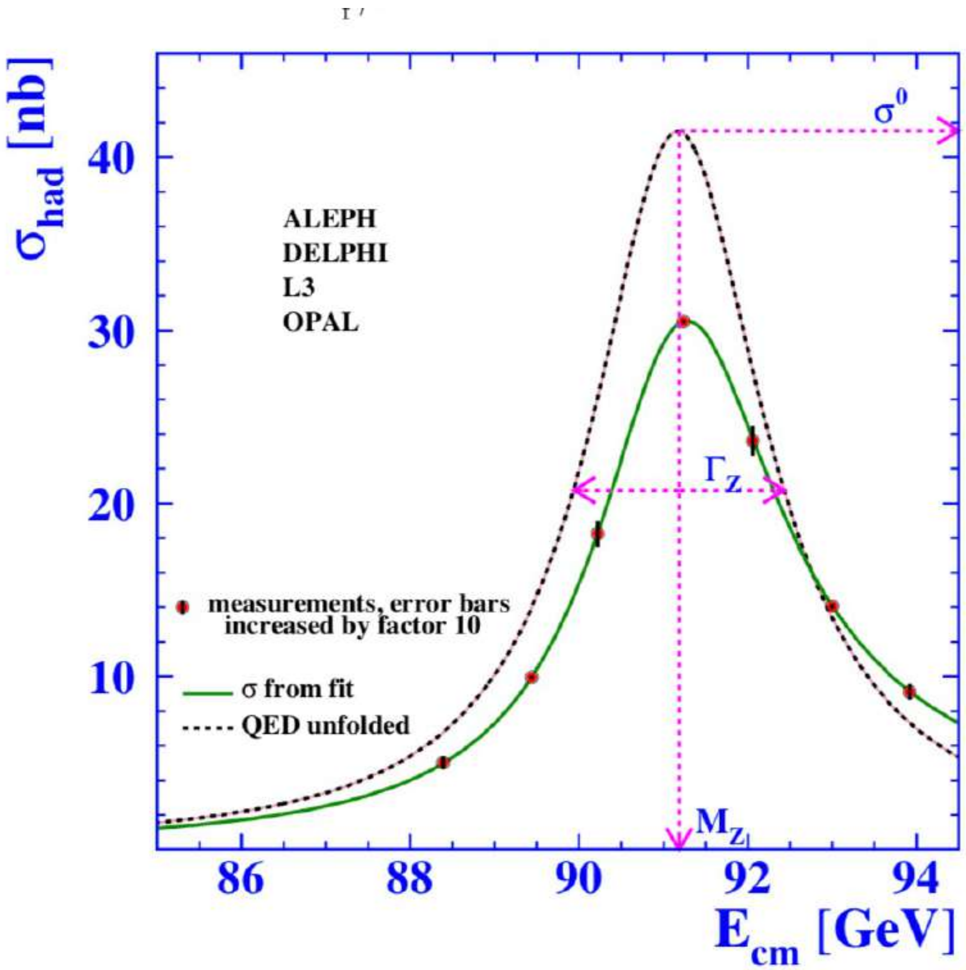


Accounted for by radiator function H. We want $\sigma_{\rm ew}(s)$

$$\sigma(s) = \int_{4m_{\rm f}^2/s}^{1} dz H_{\rm QED}^{\rm tot}(z,s)\sigma_{\rm ew}(zs).$$

- ❑ The emission of the photon from the initial legs modifies the effective $\sqrt{s}$ of the Z interaction
- ❑ We have to take it into account by doing an integral over all the possible center of mass energies.
- ❑ As a consequence the lineshape is heavily modified by the initial state radiation (ISR).

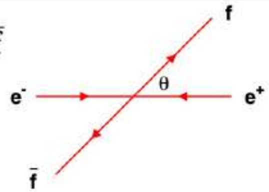
- ❑ Cross-section at the peak is reduced by about 26%
- ❑ The peak is slighlyt shifted at higher energies (112 MeV)
- ❑ The lineshape becomes more asymmetric



Differential cross-section and asymmetries

Improved Born Approximation for $e^+e^- \to f\bar{f}$

(Ignoring fermion masses, QED/QCD ISR/FSR ...)



$$\frac{d\sigma_{ew}}{d\cos\theta} = \frac{\pi N_c^f}{2s} 16|\chi(s)|^2 \times$$
$$[(g_{Ve}^2 + g_{Ae}^2)(g_{Vf}^2 + g_{Af}^2)(1+\cos^2\theta) + 8 g_{Ve} g_{Ae} g_{Vf} g_{Af} \cos\theta]$$
$$+[\gamma \text{ exchange}] + [\gamma Z \text{ interference}]$$

Where
$$\chi(s) = \frac{G_F M_Z^2}{8\pi\sqrt{2}} \frac{s}{s - M_Z^2 + is\Gamma_Z/M_Z}$$

$|\chi(s)|^2$ gives lineshape as a function of s.

Even term in $\cos\theta$ gives total cross-section

$$\sigma_{f\bar{f}} \propto (g_{Ve}^2 + g_{Ae}^2)(g_{Vf}^2 + g_{Af}^2)$$

Odd term in $\cos\theta$ leads to forward-backward asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

where $\sigma_F = \int_0^1 (d\sigma/d\cos\theta) d\cos\theta$. At the Z peak:

$$A_{FB}^{0,f} = \frac{3}{4} \frac{2 g_{Ve} g_{Ae}}{g_{Ve}^2 + g_{Ae}^2} \frac{2 g_{Vf} g_{Af}}{g_{Vf}^2 + g_{Af}^2} \equiv \frac{3}{4} \mathcal{A}_e \mathcal{A}_f$$

A_{FB} depends on g_{Vf}/g_{Af}, i.e. on $\sin^2\theta_{eff}$

Cross-section plus A_{FB} allow g_{Vf} and g_{Af} to be derived.

Final state fermions in $e^+e^- \to f\bar{f}$ are polarised. Polarisation can be measured for τ lepton final states at LEP.

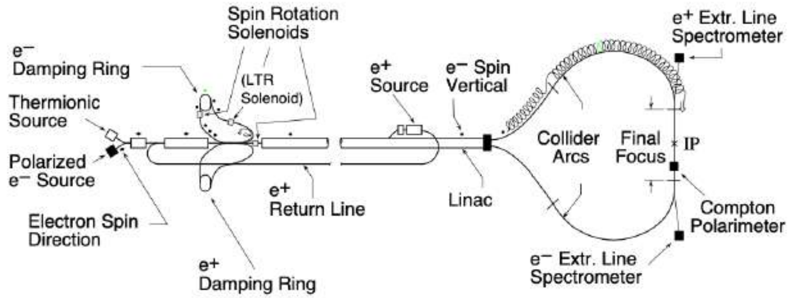
$$\mathcal{P}_\tau \equiv (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-)$$

where $\sigma_{+(-)}$ cross section for producing + (-) helicity τ^- leptons.

Eg. $\tau \to \pi\nu$, momentum of the π depends on the τ helicity

Initial state: LEP beams are unpolarised (except for special energy calibration conditions)

Stanford Linear Collider - longitudinally polarised electron beam to detector SLD. Electron beam ≈ 75% polarised from 1994–1998.



Knowing polarisation of final (τ) or initial (SLD) state, can construct left-right, left-right-forward-backward... asymmetries, and measure $\mathcal{A}_e$ or $\mathcal{A}_f$, eg.

$$A_{LR}(s) = \frac{N_L - N_R}{N_L + N_R} \frac{1}{\langle \mathcal{P}_e \rangle}, \quad A_{LR}^0 \equiv \mathcal{A}_e$$

Cross-section and partial widths

Cross-section as a function of s (from $|\chi(s)|^2$): "Z lineshape"

$$\sigma_{\mathrm{f\bar{f}}}(s) = \sigma^0_{\mathrm{f\bar{f}}} \frac{s\Gamma_Z^2}{(s-M_Z)^2 + s^2\Gamma_Z^2/M_Z^2}$$

where pole cross-section is

$$\sigma^0_{\mathrm{f\bar{f}}} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{\mathrm{ee}}\Gamma_{\mathrm{f\bar{f}}}}{\Gamma_Z^2}.$$

with $\Gamma_{\mathrm{f\bar{f}}}/\Gamma_Z = \mathrm{BR}(Z \to \mathrm{f\bar{f}})$ and partial width is

$$\Gamma_{\mathrm{f\bar{f}}} = N_c^{\mathrm{f}} \frac{G_F M_Z^3}{6\sqrt{2}\pi}\left(g_{\mathrm{Af}}^2 + g_{\mathrm{Vf}}^2\right)$$

+ QED/QCD corrections eg. QCD: $\Gamma_{\mathrm{q\bar{q}}} \to \Gamma_{\mathrm{q\bar{q}}}(1 + \alpha_s/\pi + \cdots)$

Total width of Z

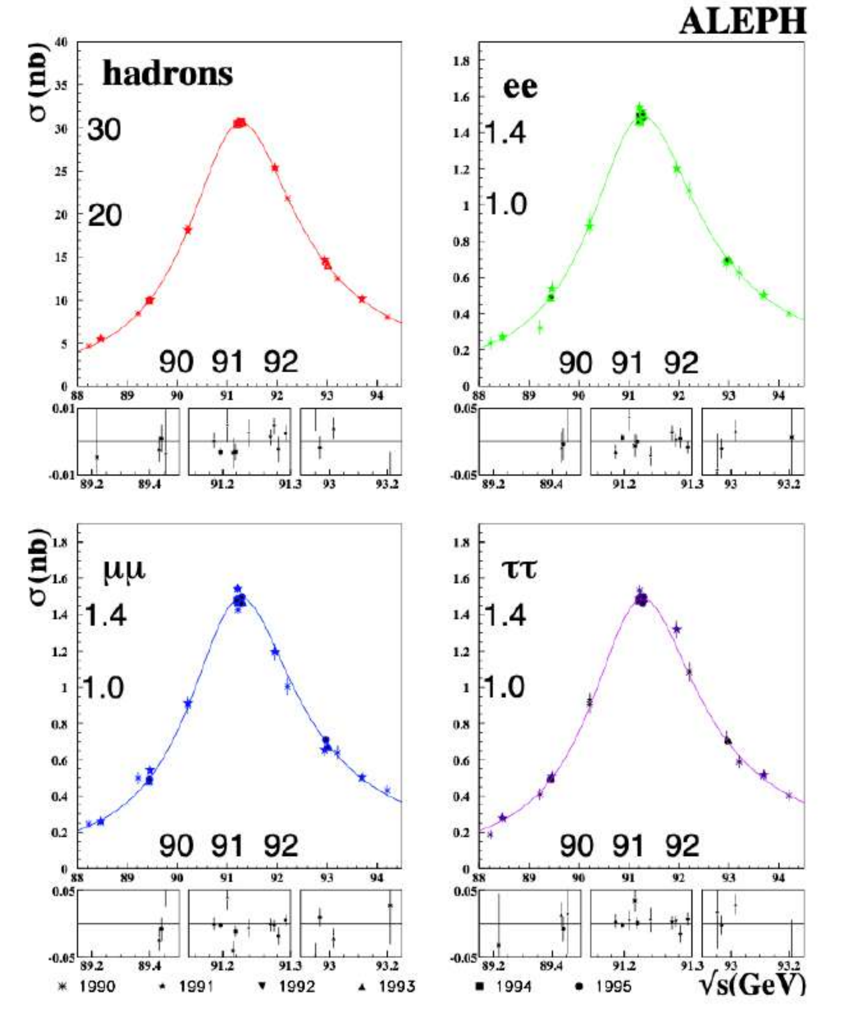
$$\Gamma_Z = \Gamma_{\mathrm{had}} + 3\Gamma_{\ell\ell} + \Gamma_{\mathrm{inv}} = \Sigma\Gamma_{\mathrm{q\bar{q}}} + 3\Gamma_{\ell\ell} + N_\nu\Gamma_{\nu\nu}$$

Comparing total width to partial width gives N_ν

Cross-sections and widths correlated. Choose to fit:

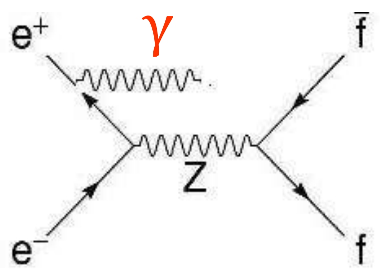
- M_Z, Γ_Z, σ^0_{h}
- Ratios: $R^0_e \equiv \Gamma_{\mathrm{had}}/\Gamma_{\mathrm{ee}}$, $R^0_\mu \equiv \Gamma_{\mathrm{had}}/\Gamma_{\mu\mu}$, $R^0_\tau \equiv \Gamma_{\mathrm{had}}/\Gamma_{\tau\tau}$ or $R^0_\ell \equiv \Gamma_{\mathrm{had}}/\Gamma_{\ell\ell}$
- Asymmetries: $A^{0,e}_{\mathrm{FB}}$, $A^{0,\mu}_{\mathrm{FB}}$ and $A^{0,\tau}_{\mathrm{FB}}$ or $A^{0,\ell}_{\mathrm{FB}}$

Extra information from tagging some quark flavours



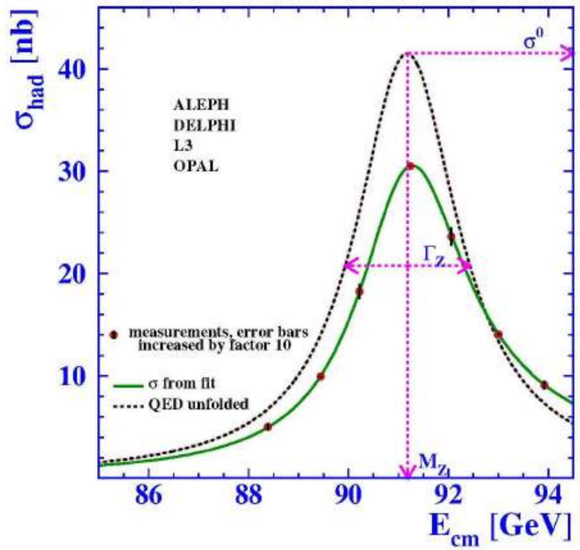
Measurements

Measurement of the Z partial widths



The emission of a photon from the inital state lower the effective center of mass energy. This effect is taken into account in the fit by a "radiator" function (it is a pure QED effect and can be computed with great precision).

$$\sigma_{q\bar{q}} = \frac{12\pi}{M_Z^2} \frac{s\Gamma_{e^+e^-}\Gamma_{q\bar{q}}}{\left(s-M_Z^2\right)^2 + \frac{s^2\Gamma_Z^2}{M_Z^2}} \xrightarrow{s=M_Z^2} \sigma_{q\bar{q}}^0 = \frac{12\pi}{M_Z^2}\frac{\Gamma_{e^+e^-}\Gamma_{q\bar{q}}}{\Gamma_Z^2}$$



- To measure the partial widths of the Z decays in the various fermionic channels, we need to measure the cross-section at the peak:
- We select the following channels:

$Z \to q\bar{q}$

$Z \to \mu^+\mu^-$

$Z \to \tau^+\tau^-$

$Z \to e^+e^-$

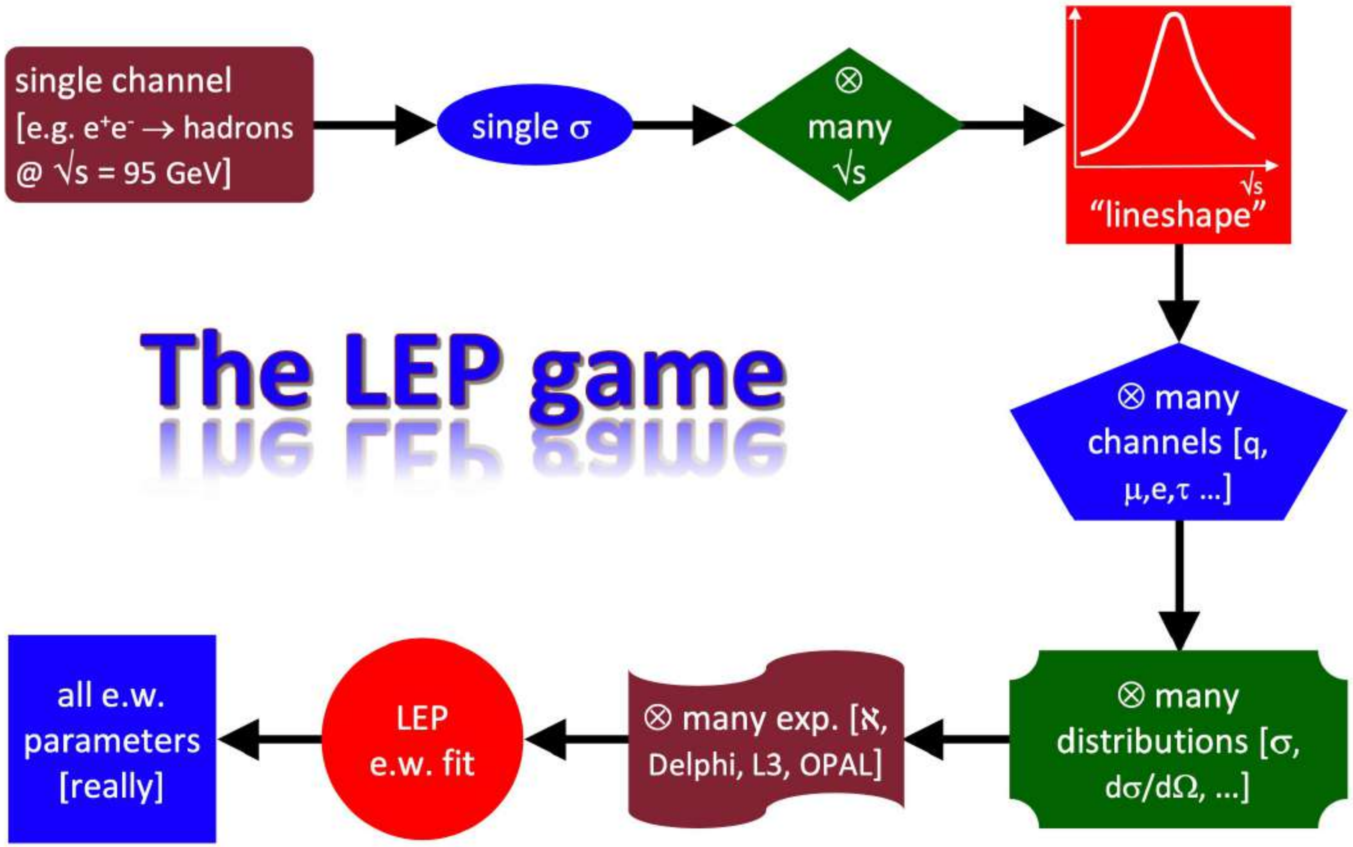
⇒

1. Peak cross-section;
2. Partial width;
3. Z couplings

- N.B. The total width Γ_Z is the same in all channels; from a channel to the other does not change the resonance shape but only the peak value;
- N.B. the electron channel is more complicated because there is also the photon exchange in the t channel;
- N.B. in the hadron channel it is possible to distinguish the b quark from its impact paramenter (B_0 mesons live long enough); therefore we can measure the partial width also in the $b\bar{b}$ channel.

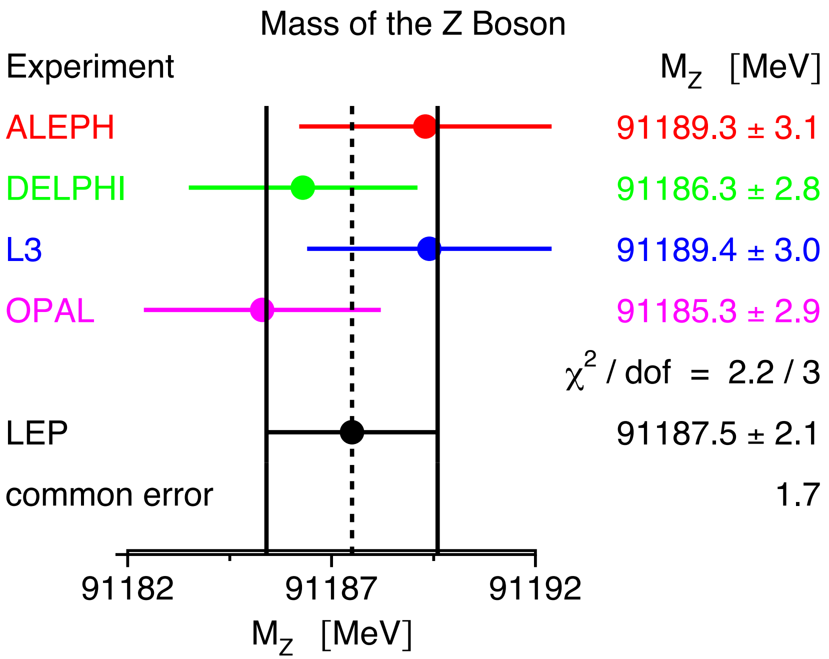
L1 SM Fit

Slide from P. Bagnaia

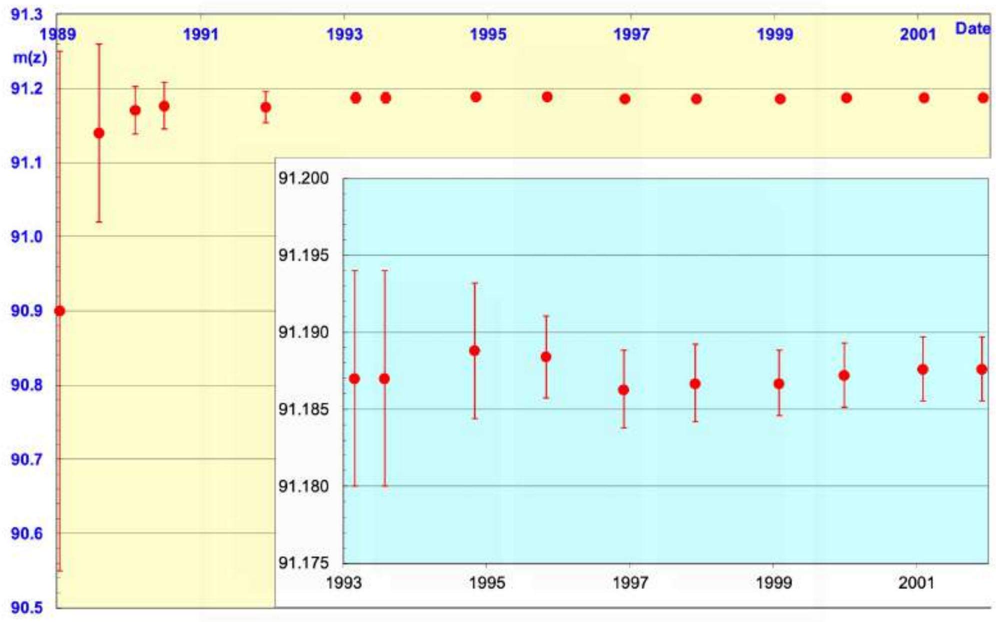


Fits are done using Model Independent programs

Z Mass



M_Z history



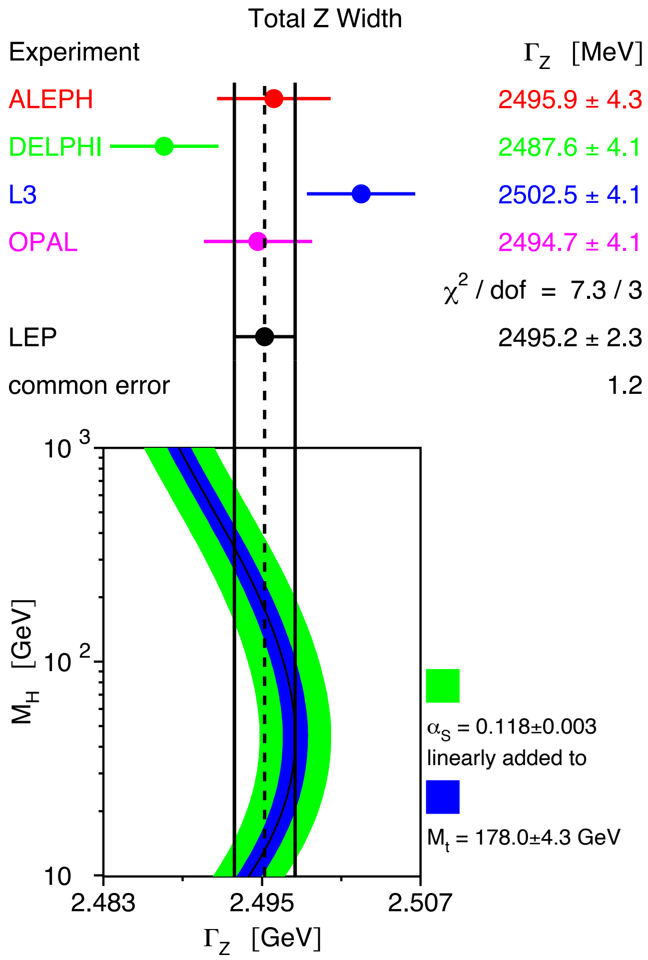
$$M_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

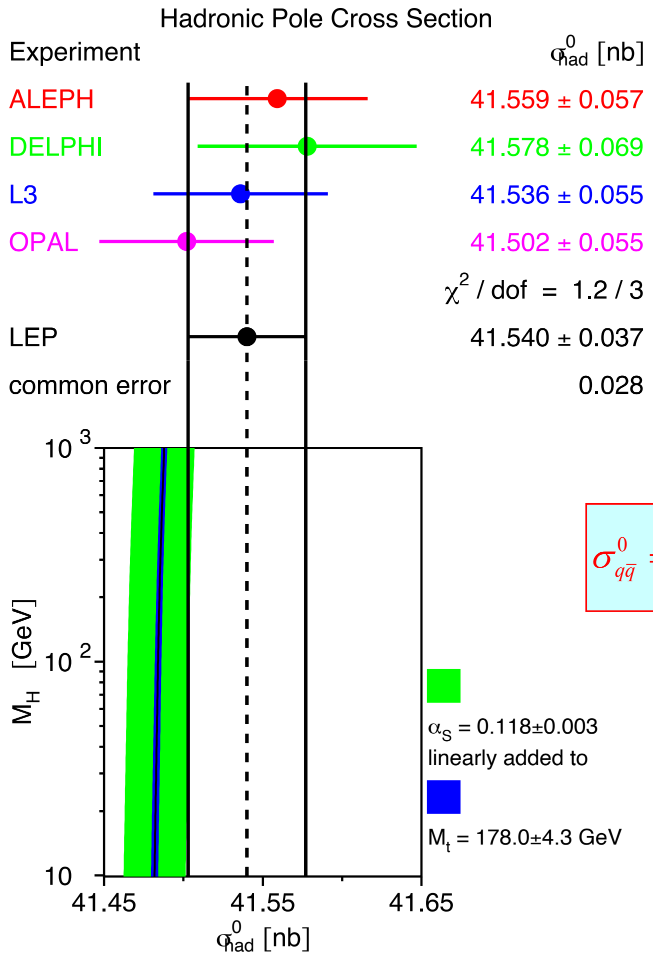
$$\frac{\Delta M_Z}{M_Z} = \pm 2.3 \cdot 10^{-5}$$

N.B. there is no SM prediction for the Z Mass because it is an INPUT parameter of the Model

The γ-Z interference term is taken from the SM. If it is left as a free parameter in the fit, it would add an additional 9 MeV error on the Z mass

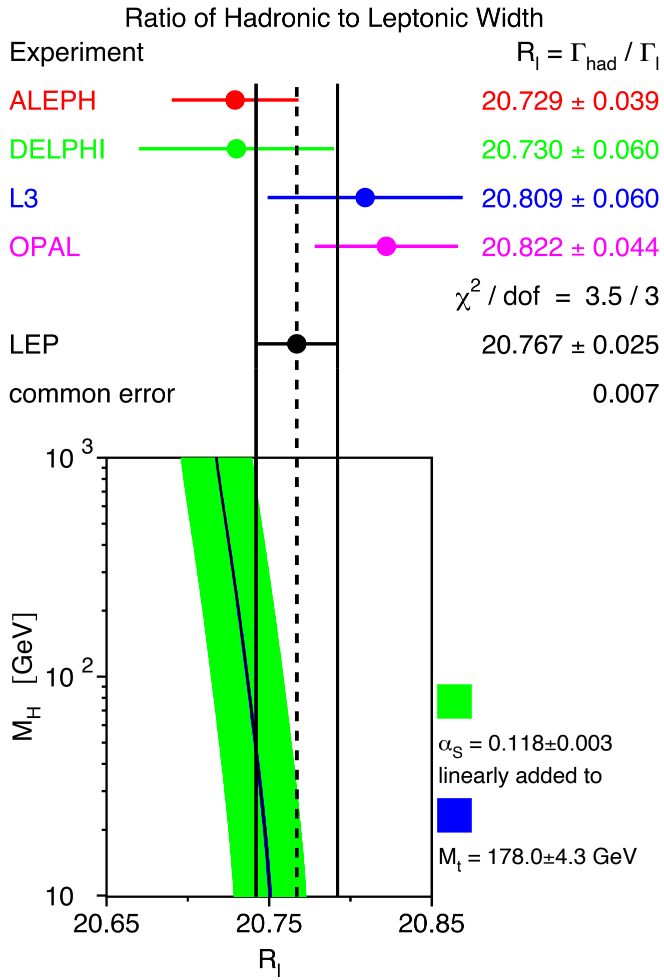
Z total width and Hadronic Pole Cross Section

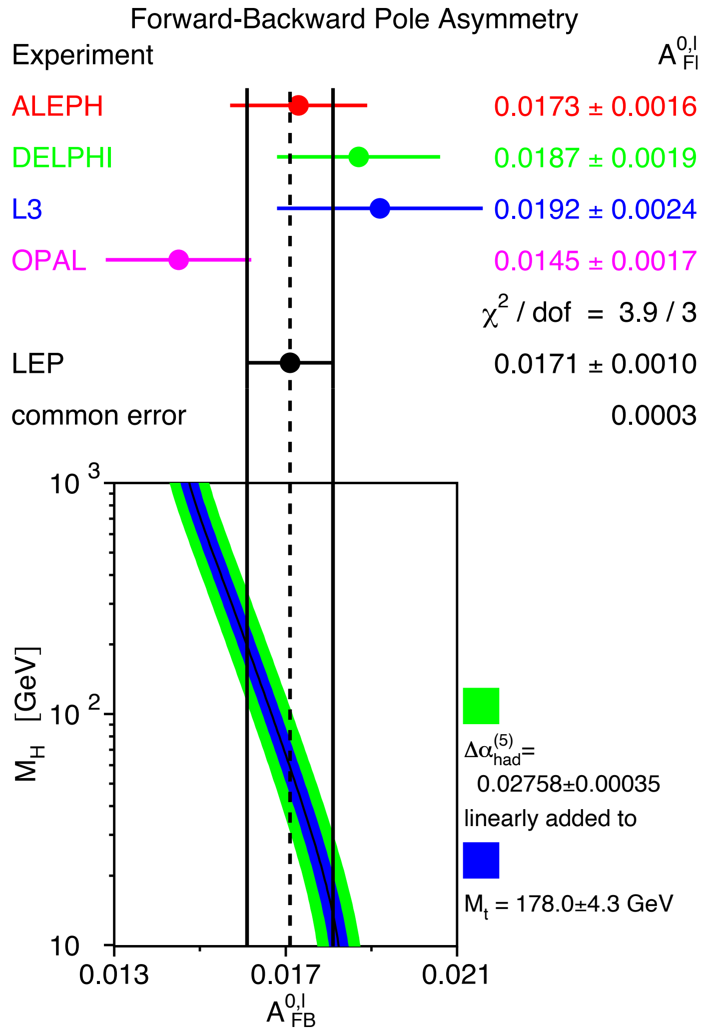




$$\sigma^0_{q\bar{q}} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{e^+e^-}\Gamma_{q\bar{q}}}{\Gamma_Z^2}$$

R_{lepton} and Forward-Backward Asymmetry





Measurement of g_V and g_A

The measurement of the Z couplings before LEP did not have enough precision to make stringent tests of the Standard Model, for instance they could not disantangle the sign of the couplings, we need the asymmetries to do it.

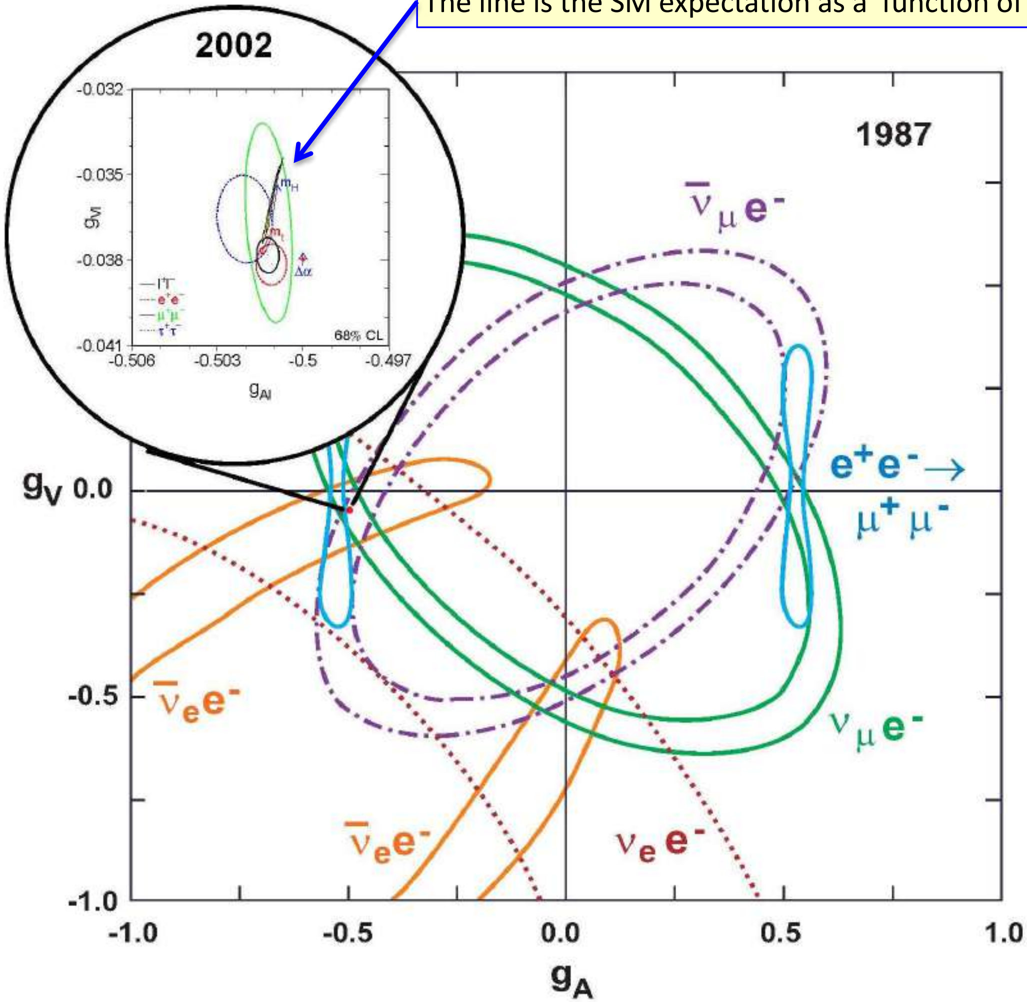
g_V=-0.03783(41)
g_A=-0.50123(26)

- **Lepton coupling to the Z**
  Ratios of coupling constants:

g_A^μ/g_A^e	=	1.0002	±	0.0014
g_A^τ/g_A^e	=	1.0019	±	0.0015
g_V^μ/g_V^e	=	0.962	±	0.063
g_V^τ/g_V^e	=	0.958	±	0.029

Lepton universality is verified at the per mill level.

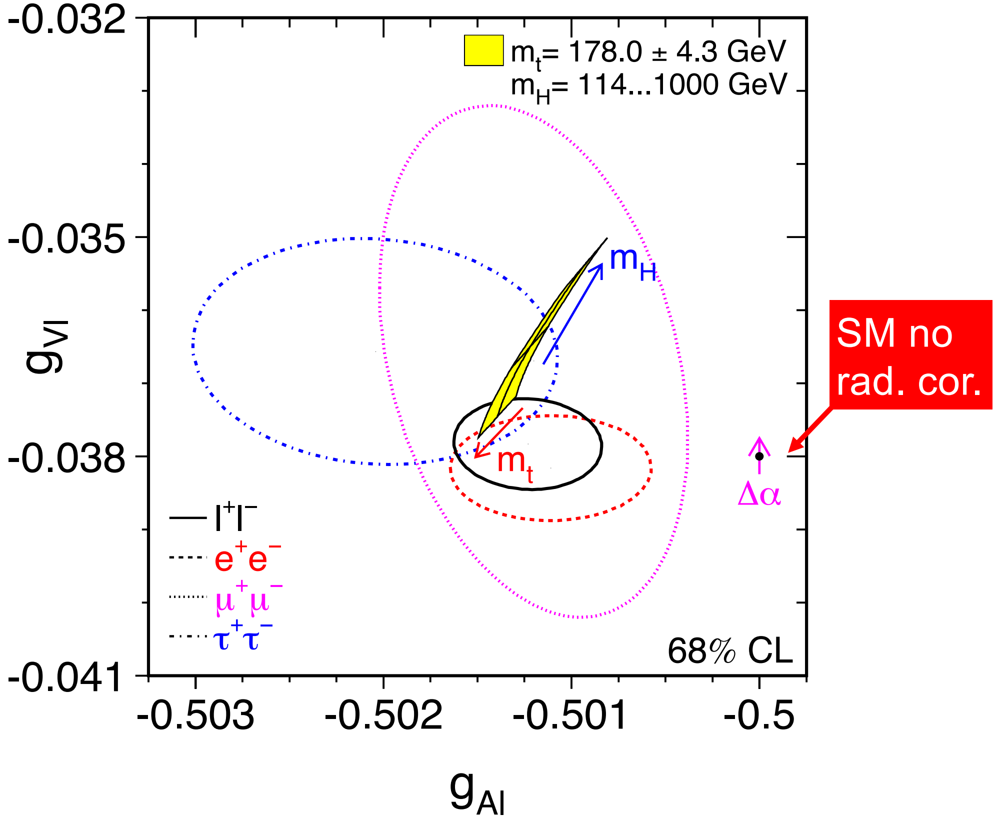
The line is the SM expectation as a function of m_{top} and m_H

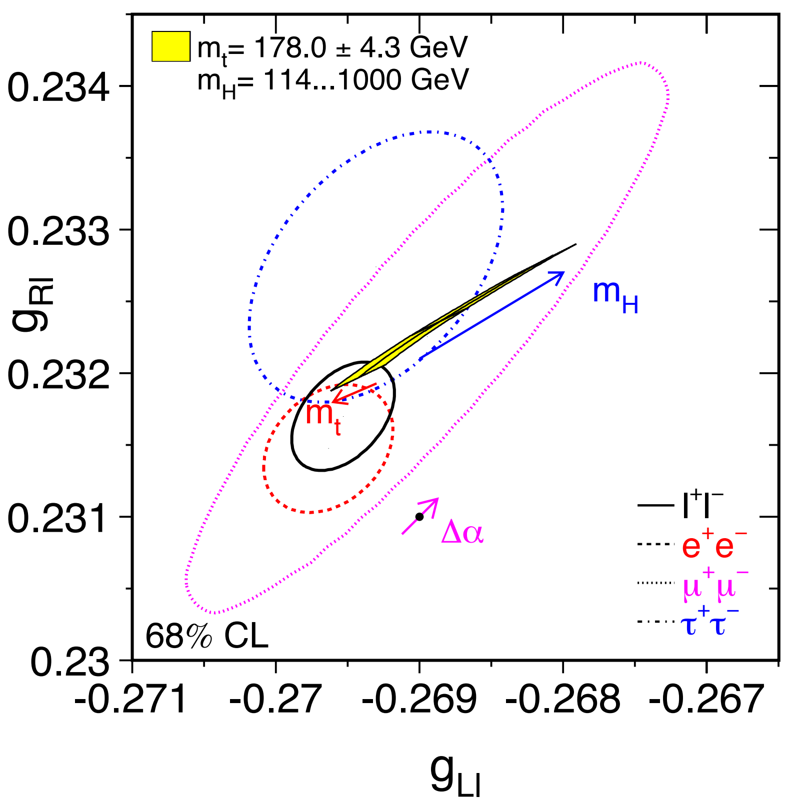


g_V versus g_A and g_R versus g_L

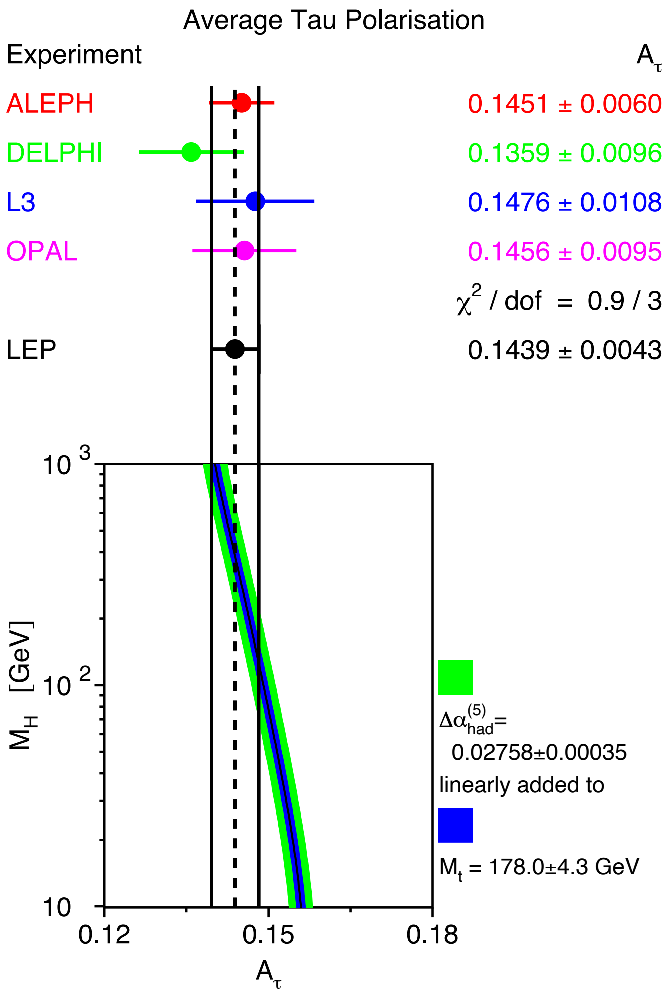
$$g_V^f = T_3^f - 2\,Q_f\,\sin^2\vartheta_W$$
$$g_A^f = T_3^f$$

$$g_L^f = g_V^f + g_A^f = 2\,T_3^f - 2\,Q_f\,\sin^2\vartheta_W$$
$$g_R^f = g_V^f - g_A^f = -2\,Q_f\,\sin^2\vartheta_W$$

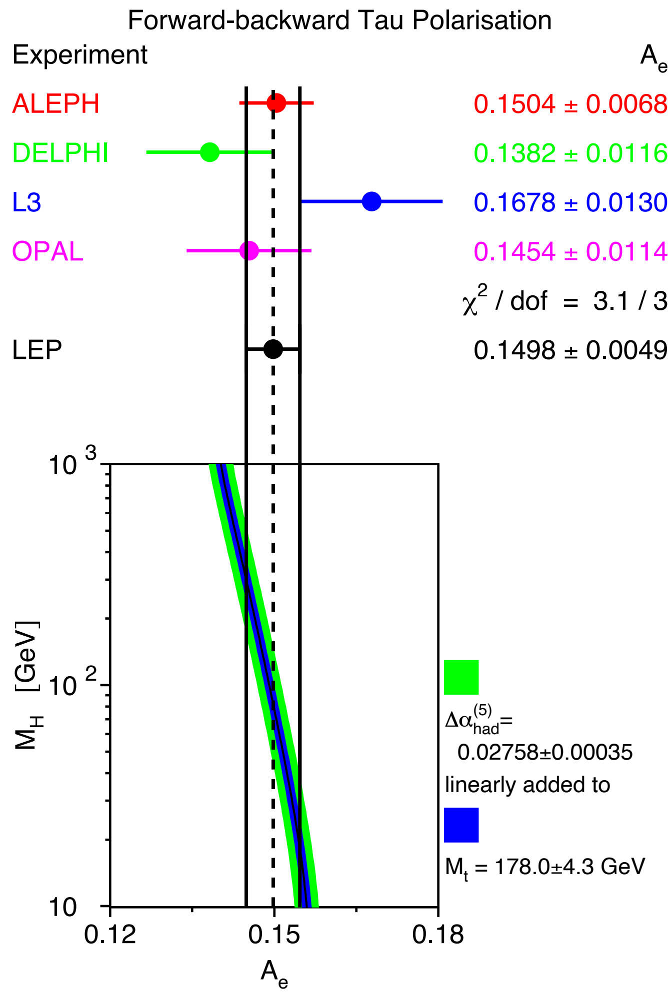




Final result for tau polarization



$$A_f \equiv 2\frac{g_V^f\, g_A^f}{(g_V^f)^2 + (g_A^f)^2}$$



Lepton Universality

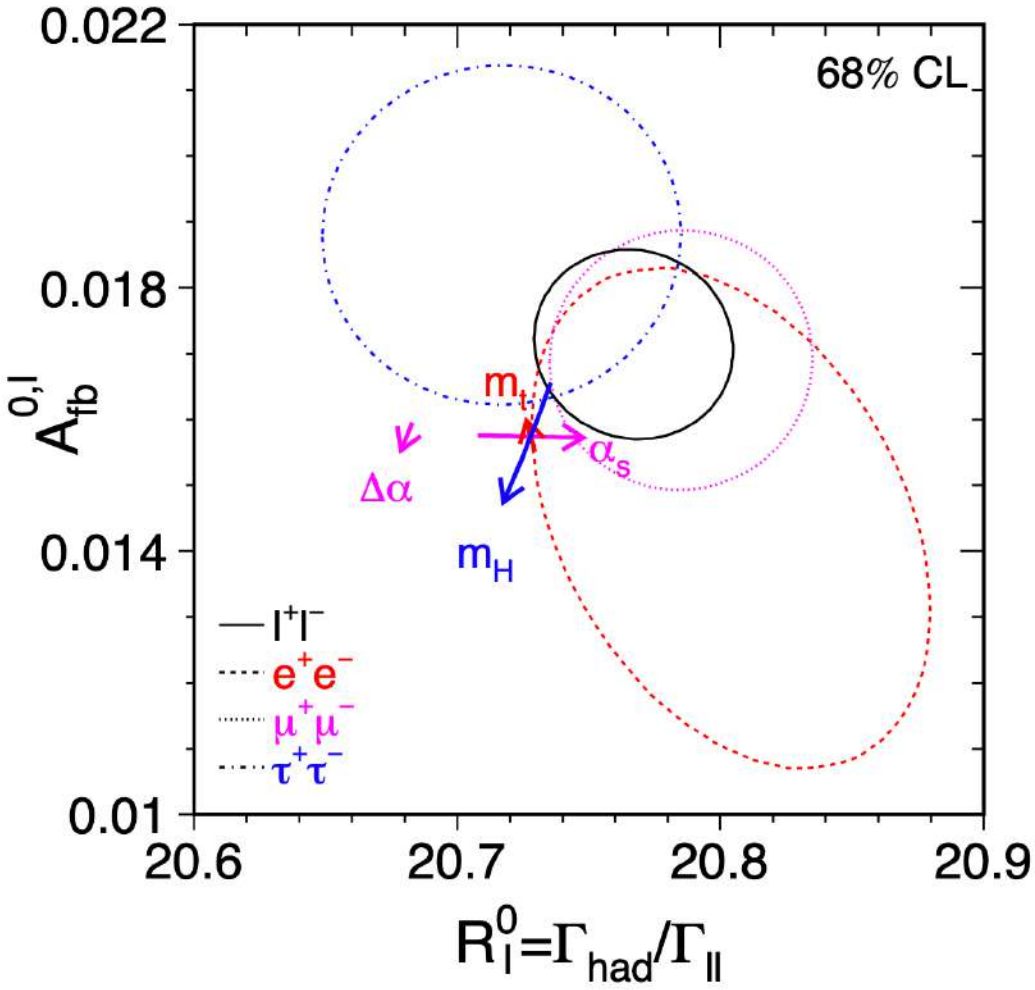
Plot $A_{FB}^{0,\ell}$ vs. $R_\ell^0 = \Gamma_{had}/\Gamma_{\ell\ell}$. Contours contain 68% probability.

Lepton universality OK. Results agree with SM (arrows)

$M_t = 174.3 \pm 5.1$ GeV

$M_H = 300^{+700}_{-186}$ GeV (low M_H preferred)

$\alpha_s(M_Z^2) = 0.118 \pm 0.002$

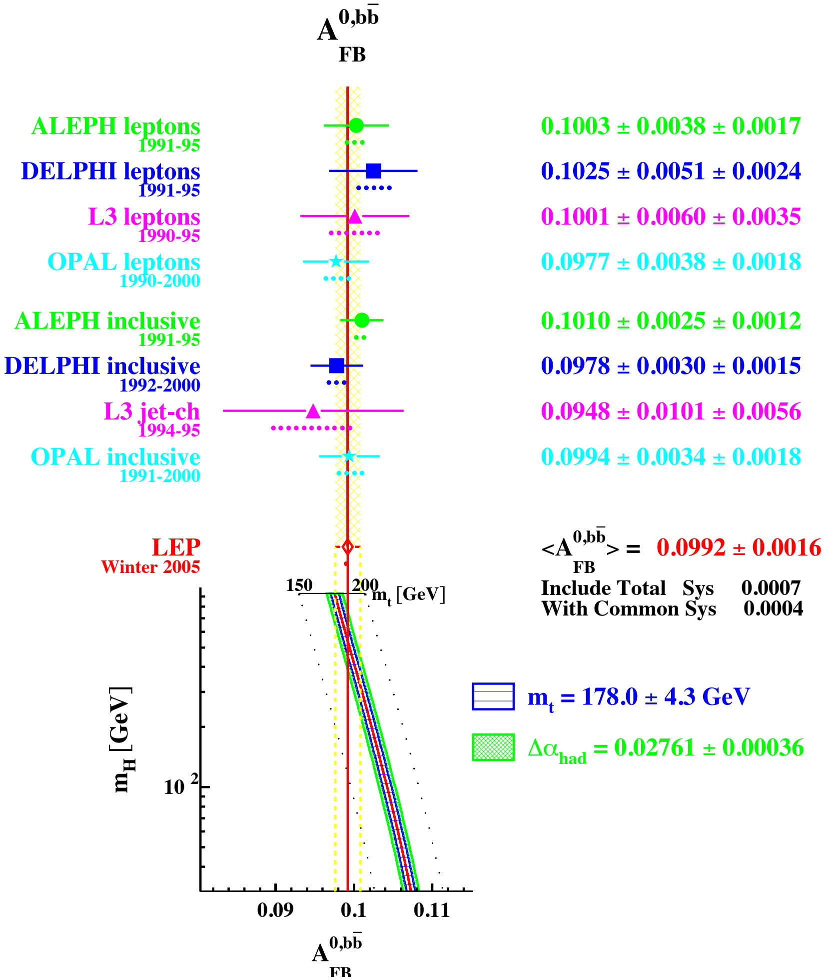


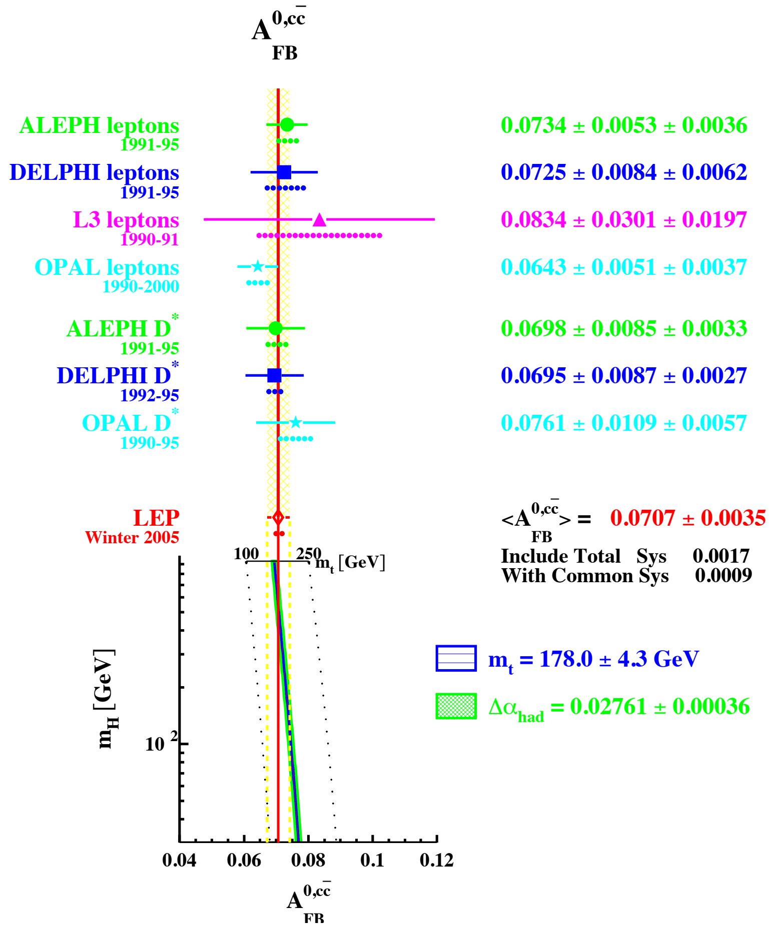
Ratios of couplings:

g_A^μ/g_A^e	=	1.0002	± 0.0014
g_A^τ/g_A^e	=	1.0019	± 0.0015
g_V^μ/g_V^e	=	0.962	± 0.063
g_V^τ/g_V^e	=	0.958	± 0.029

Lepton universality tested to 10^{-3} in g_A

$b\bar{b}$ and $c\bar{c}$ Forward-Backward Asymmetry

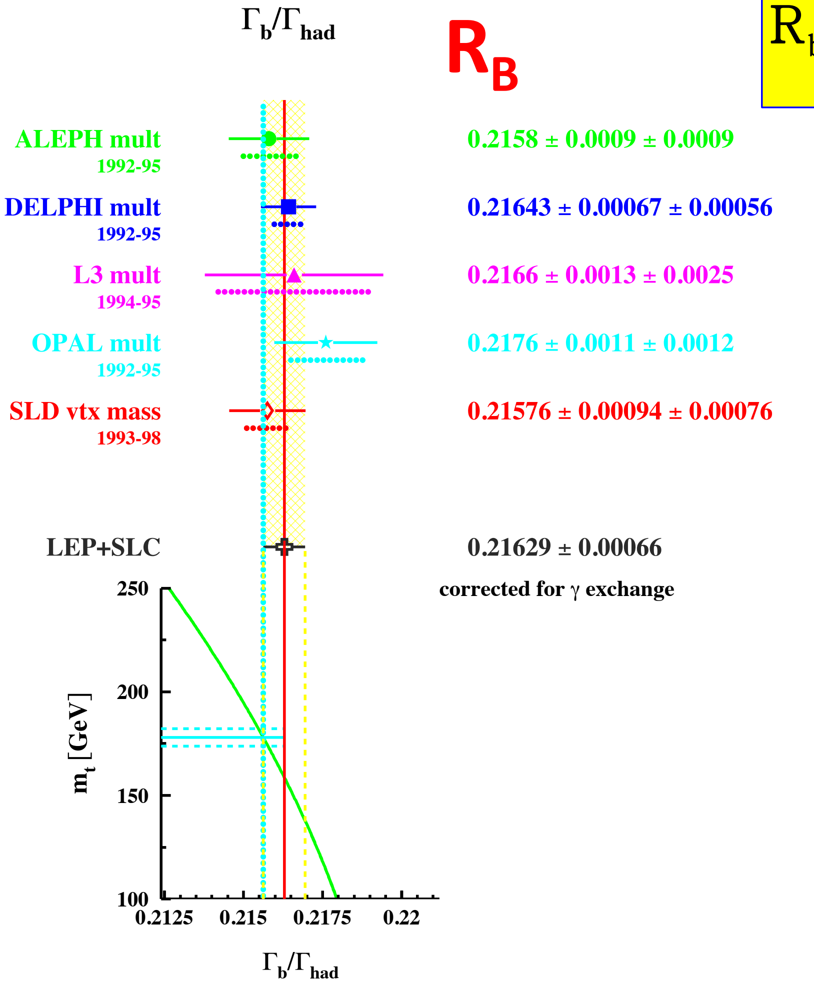




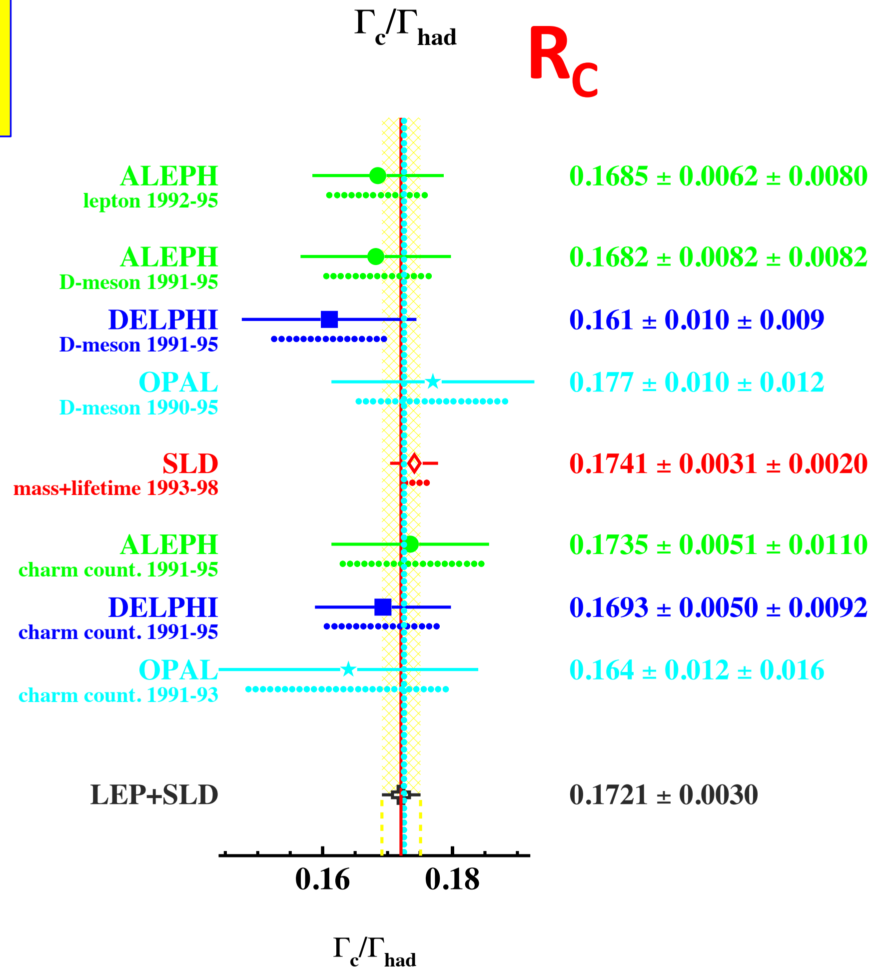
R_B and R_C

$$R_{b,c} = \frac{\Gamma_{b,c}}{\Gamma_{had}}$$

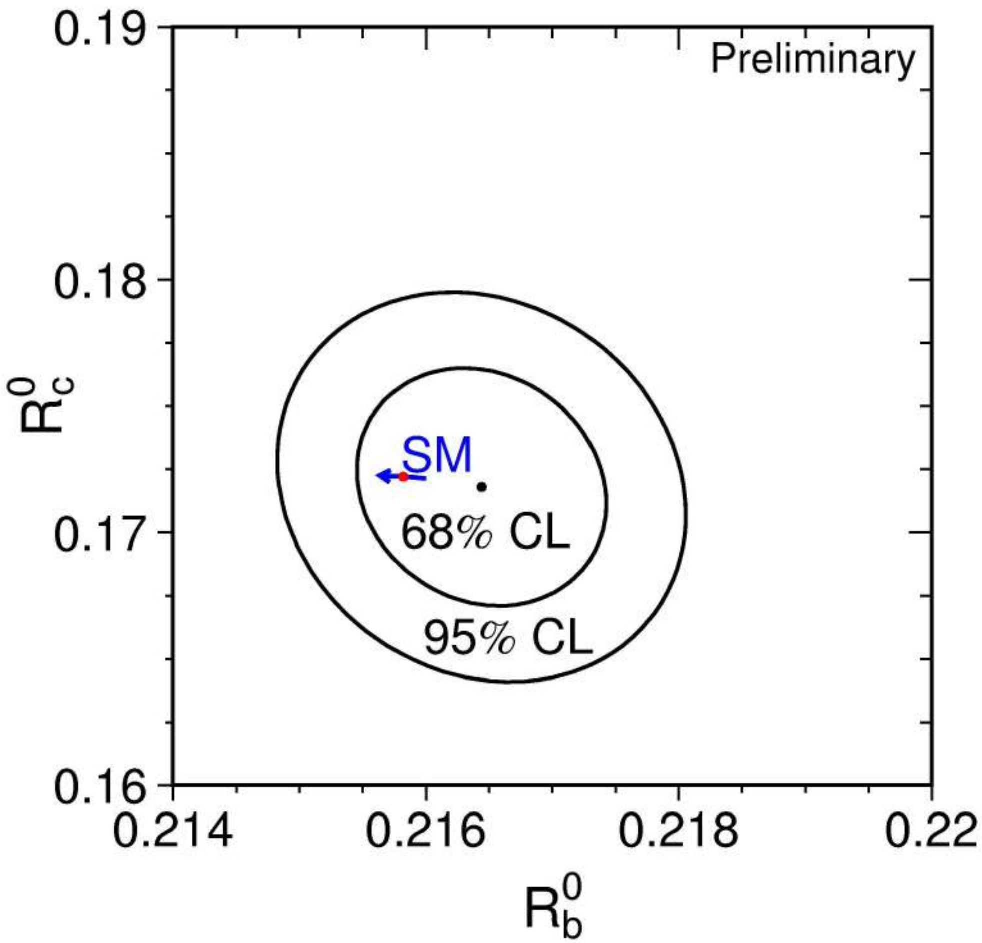
R_B



R_C



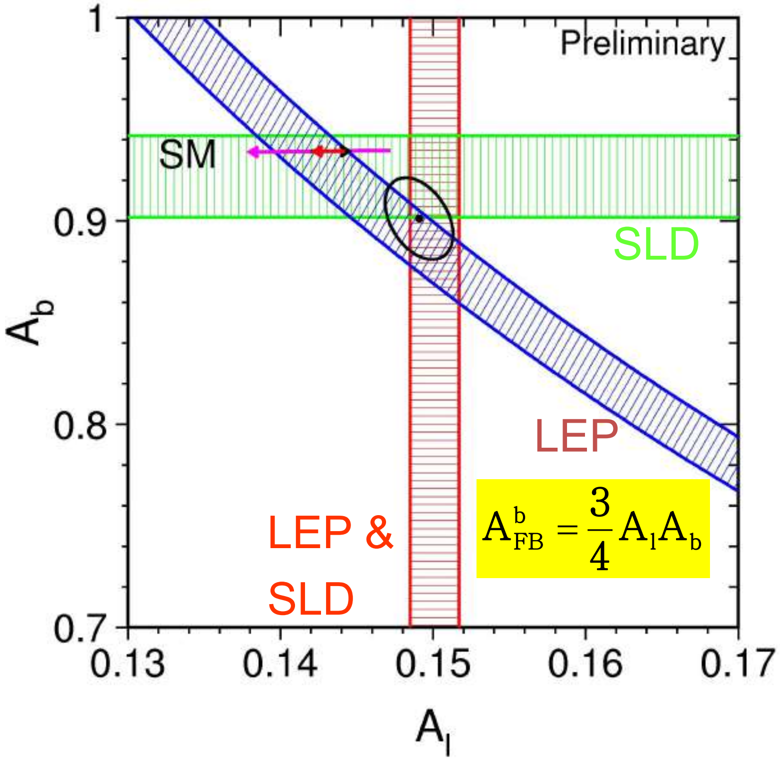
The partial widths $Z \rightarrow b\bar{b}$ and $Z \rightarrow c\bar{c}$



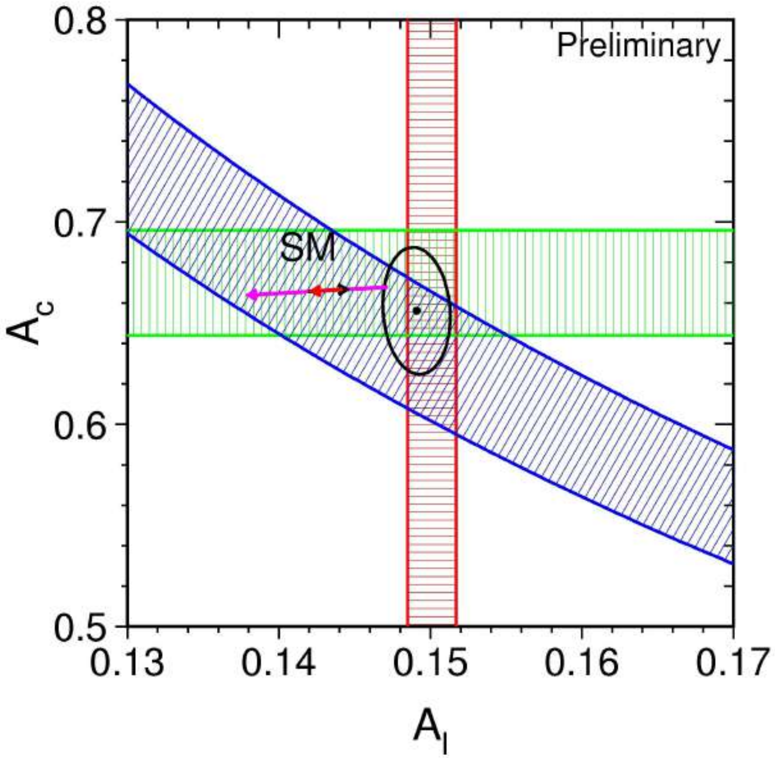
$$R_{b,c} = \frac{\Gamma_{b,c}}{\Gamma_{had}}$$

Neutral current coupling of quarks

b-quark:

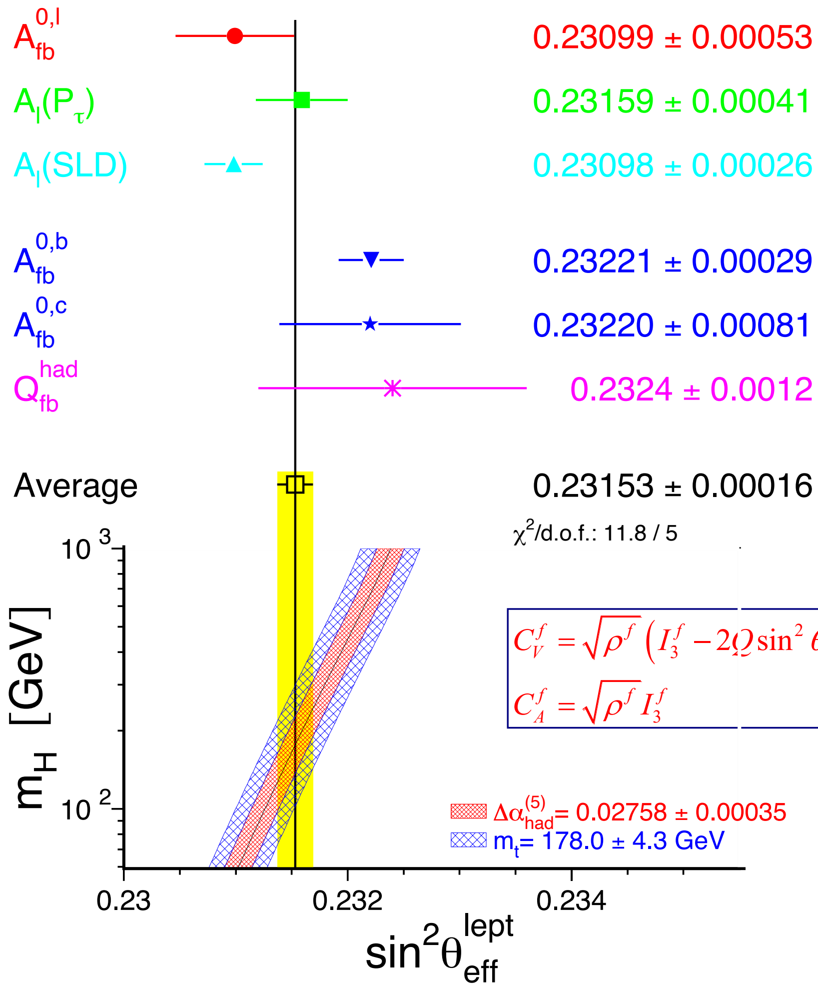


c-quark:



SLD: $A_{FB}^{LR,b} = \langle P_e \rangle \frac{3}{4} A_b$

Measurement of $\sin^2\vartheta_{eff}$



$$C_V^f = \sqrt{\rho^f}\left(I_3^f - 2Q\sin^2\theta_{eff}^f\right)$$

$$C_A^f = \sqrt{\rho^f}\, I_3^f$$

- **Asymmetries at Z pole**

- forward-backward

- left-right (SLD)

- tau polarisation

$\sin^2\vartheta_{eff}$ is a renormalized value of $\sin^2\vartheta_W$. The tree level prediction of the SM is not sufficient to have an agreement with real data.

From the measured values of various asymmetries we can get the value of the Weinberg angle.

The radiative corrections depend of the top mass and Higgs mass, therefore with a comparison with the measured value we can make a prediction on these two parameters.

From this kind of measurements it has been possible to predict the value of the top mass and to put constraints on the Higgs mass.

Lep combined results

Z resonance parameters - recall pre-LEP hopes:

- $\sigma(M_Z) \approx 10$ MeV (limited by beam energy precision)
- Number of generations $\sigma(N_\nu) \approx 0.2$

Fitted	M_Z [GeV]	91.1875 ± 0.0021
	Γ_Z [GeV]	2.4952 ± 0.0023
	σ_h^0 [nb]	41.540 ± 0.037
	R_ℓ^0	20.767 ± 0.025
	$A_{FB}^{0,\ell}$	0.0171 ± 0.0010
Derived	Γ_{inv} [MeV]	499.0 ± 1.5
	Γ_{had} [MeV]	1744.4 ± 2.0
	$\Gamma_{\ell\ell}$ [MeV]	83.984 ± 0.086
	N_ν	2.984 ± 0.008

$$M_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

$$\frac{\Delta M_Z}{M_Z} = \pm 2.3 \cdot 10^{-5}$$

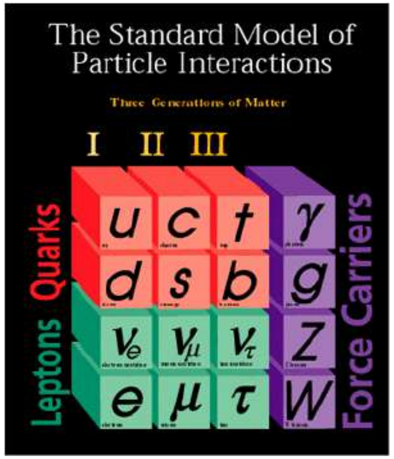
Summary - Very precise measurements of Z mass, width, cross-sections, partial widths and lepton forward-backward asymmetries.

High statistics data samples. Careful control of systematic errors.

Number of neutrino families

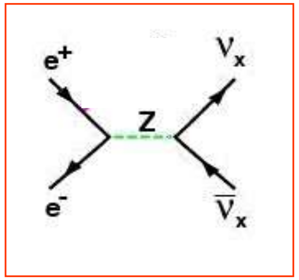
Measurement of the number of light neutrinos

- ❑ The number of lepton families is not foreseen in the Standard Model but it has to be determined experimentally.
- ❑ Before LEP operations a fourth family of leptons was not excluded by the available data.
- ❑ In every family is present a neutrino, massless or in any case with a negligeable mass; therefore the LEP strategy was to look for the presence of a fourth light neutrino (where light means of mass less than half of m_Z).
- ❑ If the forth neutrino were identified it would have been the first hint of a fourth lepton family.

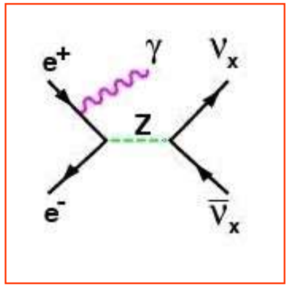


- ❑ Therefore the goal was to measure the Z partial width in the neutrino channel and from this deduce the number of light neutrinos.
- ❑ Let's recall the fact that neutrinos are not "seen" in the LEP detectors, so we need a "trick" to perform the measurement.

$$\Gamma_Z = \Gamma_{charged\ leptons} + \Gamma_{hadrons} + N \cdot \Gamma_{\nu\bar{\nu}}$$



- ❑ There were two kind of measurement of the so called invisible width (Γ_{inv}): an indirect measurement where Γ_{inv} is obtained as a difference by subtracting to Γ_Z the "visible" partial widths, and a direct measurement where it was detected the photon emitted from the initial state; in this case the event signature was a single photon with energy around 1 GeV.

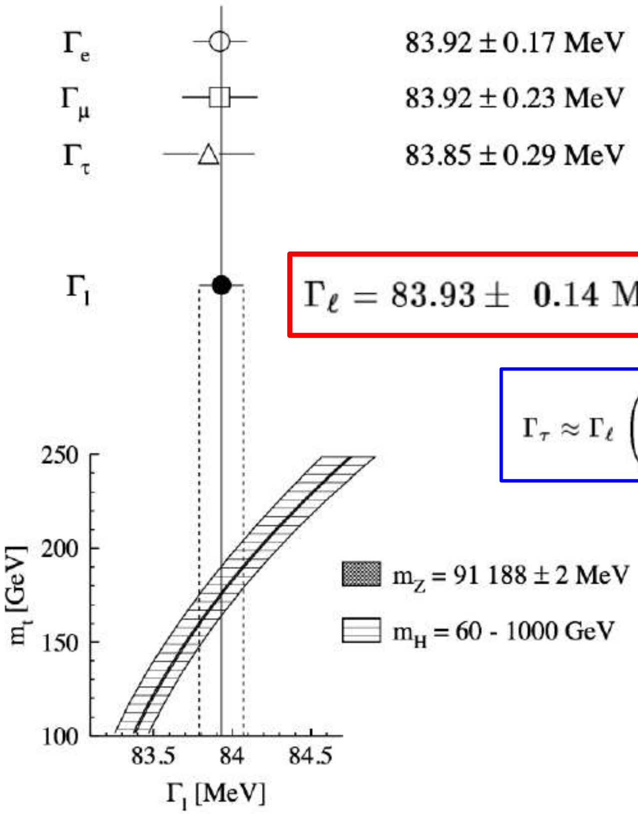


Z partial widths

LEP averages of leptonic widths

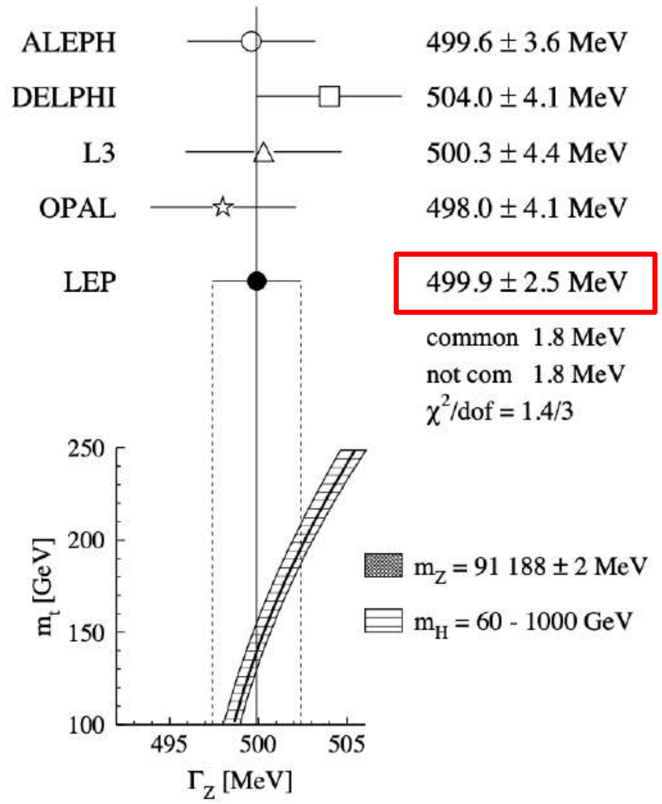
$$\Gamma_{inv} = \Gamma_Z - \Gamma_{had} - 3\,\Gamma_\ell .$$

Invisible width Γ_{inv}



$$\Gamma_\ell = 83.93 \pm \ 0.14 \text{ MeV}.$$

$$\Gamma_\tau \approx \Gamma_\ell \left(1 - 4\,\frac{m_\tau^2}{m_Z^2}\right)^{3/2} = \Gamma_\ell - 0.190 \text{ MeV}.$$



$$\Gamma_{had} = 1744.8 \pm 3.0 \text{ MeV}$$

N_ν with the "indirect" method

- The invisible width is obtained as a difference between the total width and the "visible" width.

$$\Gamma_{inv} = \Gamma_Z - \Gamma_{had} - 3\,\Gamma_\ell .$$

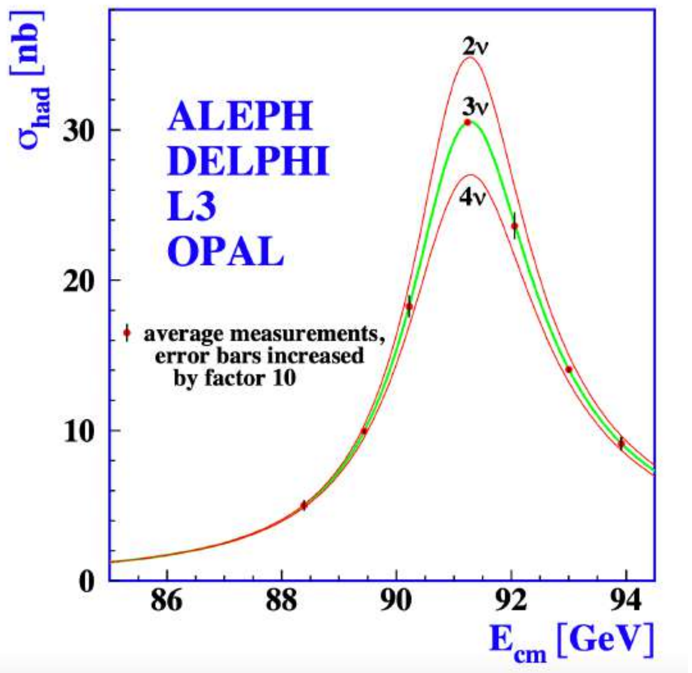
- Using the average LEP results we obtain:

$$\Gamma_{inv} = 499.9 \pm 2.5 \text{ MeV}$$

- If the Z would decay in any new particles not interacting in the detector, their contribution would enter in the invisible width.
- In order to get the number of neutrino families we have to use the neutrino partial width predicted by the SM.
- To minimize systematic errors we use ratios:

$$N_\nu = \frac{\Gamma_{inv}}{\Gamma_\nu} = \frac{\Gamma_{inv}}{\Gamma_\ell}\left(\frac{\Gamma_\ell}{\Gamma_\nu}\right)^{SM}$$

$$\left(\frac{\Gamma_\ell}{\Gamma_\nu}\right)^{SM} = 0.5021\,^{+0.0012}_{-0.0008}\,(m_t, m_H, \alpha(m_Z))$$



$$N_\nu = 2.990 \pm 0.015\,^{+0.008}_{-0.005}\,(m_t, m_H, \alpha(m_Z))$$

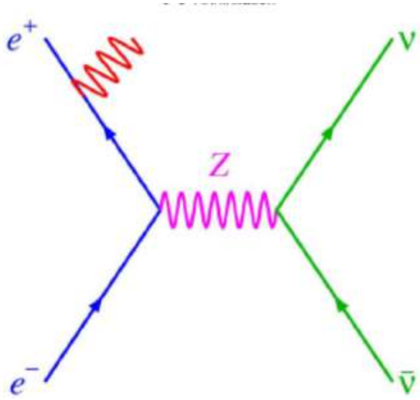
Latest result: $N_\nu = 2.9841 \pm 0.0083$

1.9 σ away from 3

N_ν with the "direct" method

- N_ν is measured very precisely with the "indirect" method, so … why do we need another measurement, with a bigger error?
- A direct measurement, with different systematic errors, is necessary to confirm the indirect measurement and, in case the result would have been different from 3, to understand the origin of the discrepancy.
- For instance, if the Z would decays in new "visible" particles not taken properly into account in Γ_{hadr} and/or in Γ_{lept}, they would appear in Γ_{inv} with the indirect method, but maybe not in the direct measurement.

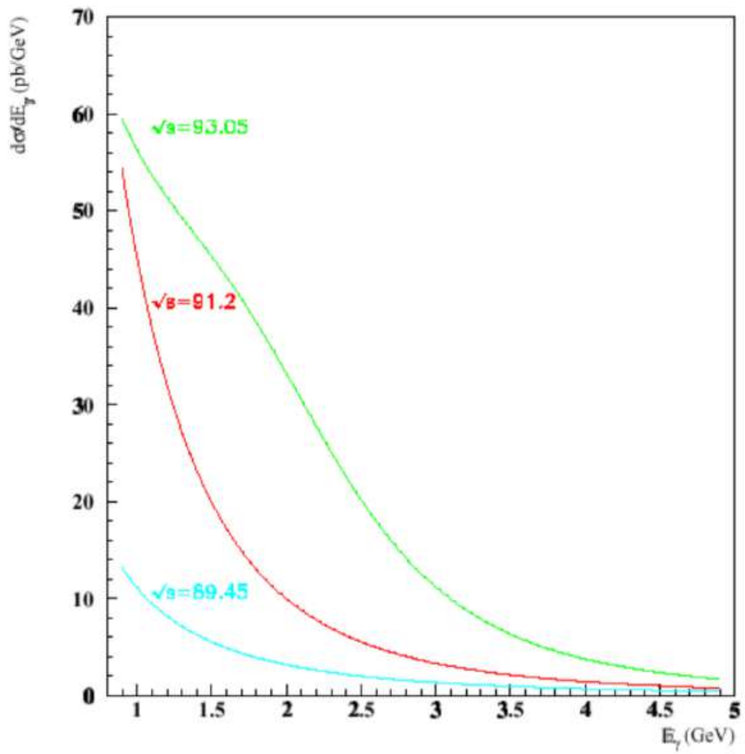
- The direct method exploits the emission of a photon from the initial legs:



- We have only a photon in the detector and nothing else, so we have to be sure that no other particle is present.
- The energy of the photon is very little (around 1 GeV) at the Z-pole
- It is difficult to trigger on such a low energy photon.
- The signal cross-section is very low (around 30 pb)

N_ν : cross-section and experimental issues

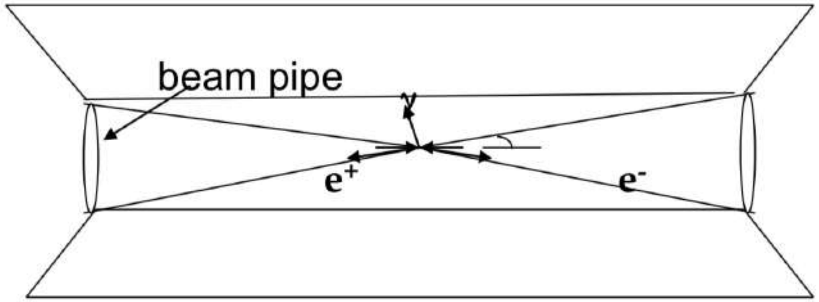
- **The cross section falls rapidly as a function of the photon energy and depends on the theta angle of emission.**



- **Experimental background:**

  - $e^+e^- \longrightarrow \gamma\gamma\gamma$
  - $e^+e^- \longrightarrow e^+e^- X$, with $X = l^+l^-\gamma, \pi^0, \eta, \eta'..$; $X \longrightarrow \gamma\gamma$
  - $e^+e^- \longrightarrow e^+e^-\gamma$

Radiative Bhabha scattering



- **The amount of background depends on the ability to detect particles at small angle and on the overall hermeticity of the electromagnetic calorimeter.**

N_ν: selection cuts (L3 experiment)

❑ **Photon selection**

1) an energy deposit in the BGO greater than 1 GeV and less than 10 GeV, at a polar angle between 45° and 135°, shared amongst at least five crystals;

2) the lateral shape of the energy deposit must be consistent with that expected from a single electromagnetic particle originating from the interaction point.

❑ **Veto cuts to make sure the detector is "empty"**

3) no other energy deposits in the BGO, consisting of 3 or more contiguous crystals and exceeding a total energy of 100 MeV;

4) no tracks in the central tracking chamber (TEC);

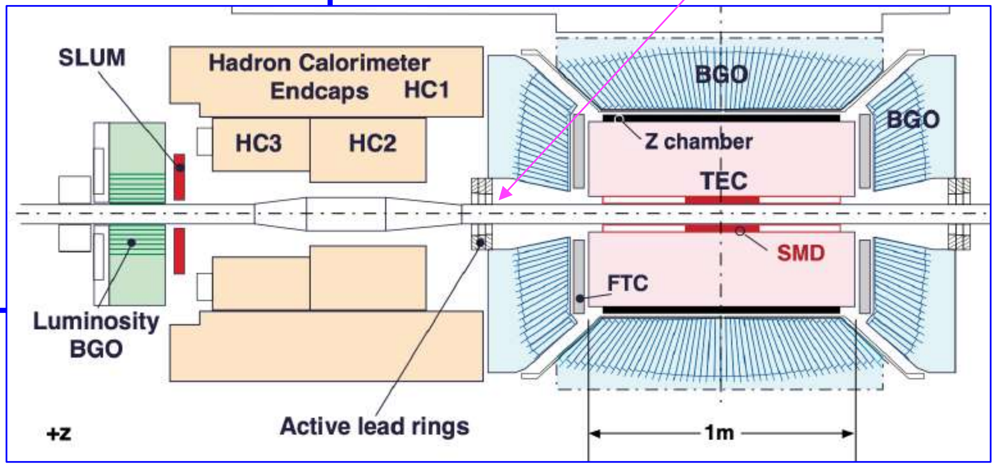
5) less than 1.5 GeV deposited in either luminosity monitor;

6) no signal in the ALR;

7) less than 3 GeV deposited in the HCAL;

8) no tracks measured in the muon spectrometer.

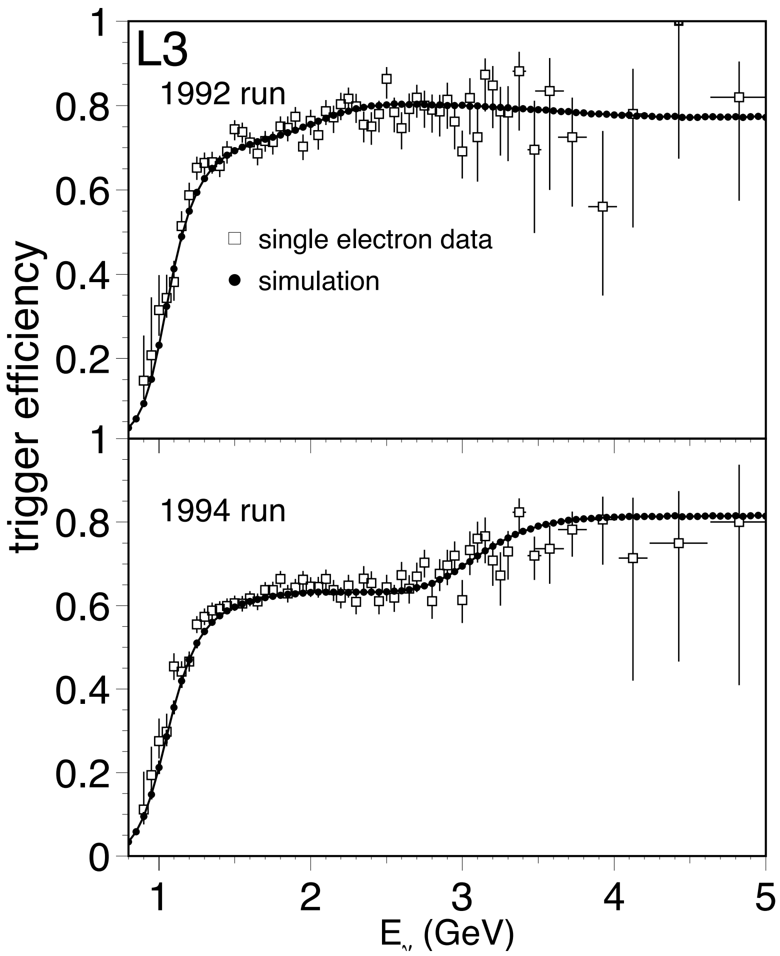
There is a little "hole" between lumi monitor and ALR



N_ν: trigger efficiency (L3 experiment)

- Given the rapidly falling photon spectrum, the L1 trigger should go down in energy as far as possible.
- The L1 trigger had 1 GeV threshold.

- The trigger efficiency was determined in two ways: from data and from a detailed simulation of the single photon trigger.
  - The first method uses a sample of radiative Bhabha events with an isolated electron in the BGO barrel (the single electron control sample), which is triggered by requiring the coincidence of a charged track and an energy exceeding 30 GeV in one of the luminosity monitor.
  - The second one uses unbiased triggers (the so called beam gate) as input of a dedicated simulation program.
- The agreement of the simulation with the single electron data at the level of 1%, justifies the uses of the simulated curve also for periods with limited statistics.



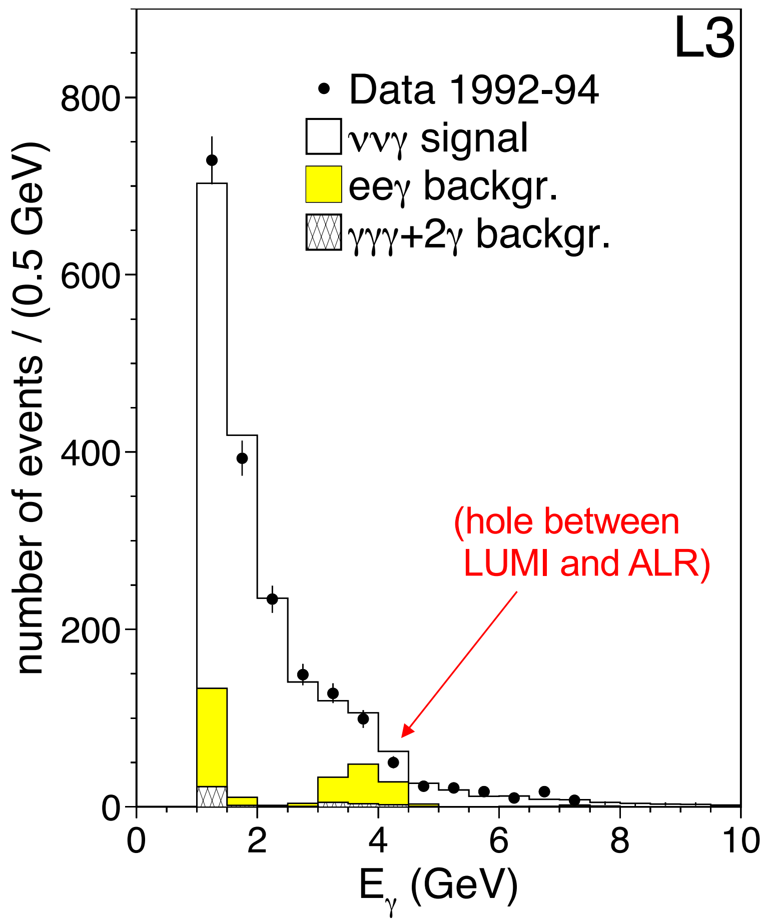
N_ν : event sample (L3 experiment)

Selected data sample and expected number of events

Year	$\sqrt{s}$ (GeV)	$\int \mathcal{L}\,dt$ (pb^{-1})	Observed events	Expected events			
				$N_{\nu\bar{\nu}\gamma}$	$N_{e^+e^-\gamma}$	$N_{other\ back.}$	Total MC
1991	88.56-93.75	9.57	202	169.6	25.0	4.9	199.5
1992	91.34	20.52	456	381.3	60.1	9.0	450.4
1993	91.32	4.12	99	74.8	10.8	2.0	87.6
1993	89.45	8.25	77	46.5	19.6	3.9	70.0
1993	91.21	9.25	180	152.4	23.6	4.3	180.3
1993	93.04	8.30	375	370.7	20.9	3.8	395.4
1994	91.22	39.88	702	596.1	93.8	16.9	706.8
Total		99.89	2091	1791.4	253.8	44.8	2090.0

Total efficiency and corrected cross-section

Year	$\sqrt{s}$ (GeV)	Efficiency	σ (pb)
1992	91.34	0.572	$32.9 \pm 1.8(stat) \pm 0.6(sys)$
1993	91.32	0.594	$35.2 \pm 4.1(stat) \pm 0.6(sys)$
1993	89.45	0.578	$11.2 \pm 1.8(stat) \pm 0.3(sys)$
1993	91.21	0.570	$28.8 \pm 2.5(stat) \pm 0.5(sys)$
1993	93.04	0.602	$70.1 \pm 3.9(stat) \pm 1.1(sys)$
1994	91.22	0.505	$29.4 \pm 1.3(stat) \pm 0.5(sys)$



N_ν: results (L3 experiment)

- **Invisible width:**

$$\Gamma_{\rm inv} = 498 \pm 12\ ({\rm stat}) \pm 12({\rm sys})\quad {\rm MeV}.$$

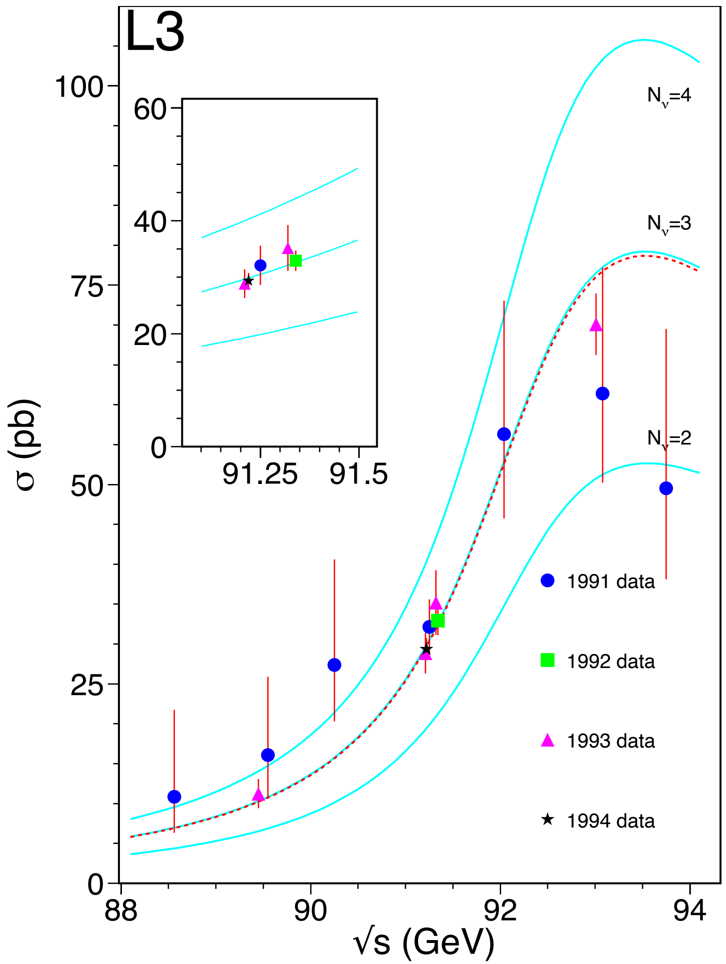
- **Number of neutrino families:**

$$N_\nu = 2.98 \pm 0.07({\rm stat}) \pm 0.07({\rm sys}).$$

- **This result is compatible (and support) the one found with the indirect method.**

- **Breakdown of systematic errors:**

Systematic error source	$\Delta\Gamma_{\rm inv}$ (Mev)	ΔN_ν
Trigger efficiency	8.4	0.050
Background subtraction	4.8	0.029
Selection efficiency	4.0	0.024
Energy scale	4.0	0.024
Monte Carlo generators	3.5	0.021
Cosmic ray background	1.7	0.010
Luminosity error	1.8	0.011
$\Gamma_{\nu\nu}$ theoretical error	–	0.004
Fit procedure	2.5	0.015
Total error	12.3	0.073



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End of chapter 7