Collider Particle Physics - Chapter 7 -

LEP Physics at the Z pole



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Chapter Summary

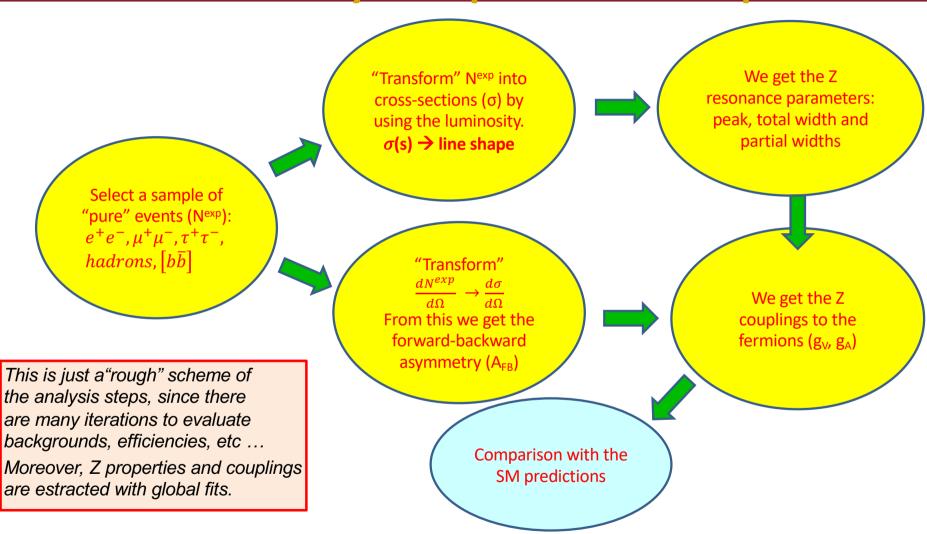
- ☐ The line shape
- ☐ Forward-Backward asymmetry
- ☐ Tau polarization asymmetry and Left-Right asymmetry at SLC
- Radiative corrections
- ☐ Measurements: Z mass, Z total and partial widths, Z couplings
- ☐ Measurements: Forward-backward asymmetries for heavy quarks
- \Box Measurements: $\sin^2 \theta_W$
- Number of neutrino families

LEP Physics goals (taken from my thesis in 1988)

☐ Higgs boson discovery;	
☐ Quark top discovery and measurement of the topponium energy levels;	
☐ Supersymmetric particles discovery;	
☐ Measurement of the Z mass with an error of 50 MeV;	
> σ(M _z) about 340 MeV from UA2+CDF in 1989.	
> Hoped to reduce it to about 10 MeV (limited by beam energy precision).	
☐ Precision measurement of the Standard Model parameters;	
☐ Measurement of the number of light neutrino families.	
wieasurement of the number of light fleutimo families.	
\gt 2.5 generations were known in 1989, top quark and \mathbf{v}_{τ} not yet established.	
	of

Z precision physics

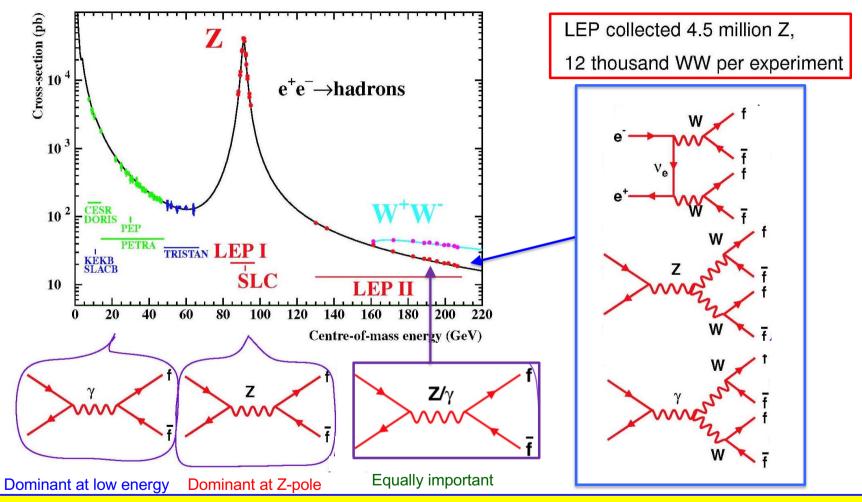
Analysis experimental steps



The lineshape

Namely, the cross-section as a function of \sqrt{s}

Cross-section as a function of Vs



In the slides of this chapter we will consider only the cross-sections and the asymmetries of the Z resonance

$e^+e^- o Z o far f: \ \sigma_{Born}^{SM}$

 $oldsymbol{\Box}$ in the SM, at the lowest order (Born approximation, no radiative corrections), for $f
eq e^\pm$ and $m \ll m_Z$:

Peak cross-section

•
$$\sigma_{Born}(e^+e^- \to f\overline{f}) = \sigma_{Zs} + \sigma_{ys} + J_f;$$

•
$$\sigma_{zs} = \frac{s\Gamma_z^2}{(s-m_z^2)^2 + m_z^2\Gamma_z^2} \times \frac{12\pi\Gamma_e\Gamma_f}{m_z^2\Gamma_z^2};$$

•
$$\sigma_{\gamma s} = \frac{4\pi\alpha^2(s)}{3s}c_fQ_f^2$$
; $[c_f = 1 \text{ (leptons)}, 3 \text{ (quark)}]$;

•
$$J_f = -\frac{(s - m_z^2)m_z^2}{(s - m_z^2)^2 + m_z^2\Gamma_z^2} \frac{2\sqrt{2}\alpha(s)}{3} c_f Q_f G_F g_V^e g_V^f;$$

• $\Gamma_{\rm Z} = \Gamma_{\rm tot} = \sum_{\rm f} \Gamma({\rm Z} \to {\rm f} \overline{\rm f})$; Total width is the sum of partial widths

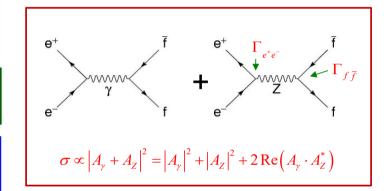
•
$$\Gamma_f \equiv \Gamma(Z \rightarrow f\overline{f}) = \frac{G_F m_Z^3 c_f}{6\sqrt{2} \pi} \left[g_V^{f2} + g_A^{f2} \right];$$

- for $\sqrt{s} \approx m_z \rightarrow \text{interference and } \gamma * \text{negligible};$
- $\sigma_{Born}(e^+e^- \to f\overline{f}, \sqrt{s} = m_z) = \frac{12\pi\Gamma_e\Gamma_f}{m_z^2\Gamma_z^2}$.

Z (s-channel)

 γ^* (s-channel)

 $\begin{array}{c} interference \\ Z_s \longleftrightarrow \gamma^*_s \end{array}$



The Z couplings intervene linearly in the interference term (and not quadratically as in the partial widths), so any parity violation effects, like the forward-backward asymmetry, will be due to this term.

In the e⁺e⁻ final state we have to take into account also the photon exchange in the t-channel (the Z contribution in the t-channel is negligeable)

$e^+e^- o Z \, o far f:\, g_V^f$ and g_A^f

 \square In the SM, at the lowest order, the partial width Γ_f (e.g. Γ_u) has the following expression:

$$\Gamma_{f} = \frac{G_{F} m_{Z}^{3} c_{f}}{6\sqrt{2} \pi} \left[g_{V}^{f2} + g_{A}^{f2} \right] \rightarrow (f = \mu^{\pm}) \rightarrow \Gamma_{\mu} \approx \frac{1}{4} \frac{G_{F} m_{Z}^{3}}{6\sqrt{2} \pi} \approx 83 \text{MeV};$$

$$c_V = I_3^f - 2Q^f \sin^2 \theta_W$$
 $c_A = I_3^f$

☐ All partial widths can be summarised in this table:

f	Q_f	g_A^f	g_V^f	$\Gamma_{\rm f}$ (MeV)	$\Gamma_{\rm f}/\Gamma_{\rm \mu}$	R _f (%)
$\nu_e \nu_\mu \nu_\tau$	0	+½	+½	166	1.99	6.8
e- μ- τ-	-1	-1/2	038	83	[1]	3.4
u c [t]	2/3	+½	+.192	286	3.42	11.8
d s b	-1/3	-1/2	346	368	4.41	15.2

$$R_f = \frac{\Gamma_f}{\Gamma_Z}$$

☐ Then, at the loweste order (Born approximation) we have:

$$ho$$
 $\Gamma_{\rm Z}^{\rm B}$ = 2423 MeV, $\Gamma_{\rm hadr.}^{\rm B}$ = 1675 MeV, $\Gamma_{\rm invis.}^{\rm B}$ = $\Gamma_{\rm v}^{\rm B}$ = 498 MeV;

$$Arr$$
 R_{hadr.} = 69.1 %, R_{lept±} = 10.2 %, R_{invis.} = R_{v's} = 20.5 %,

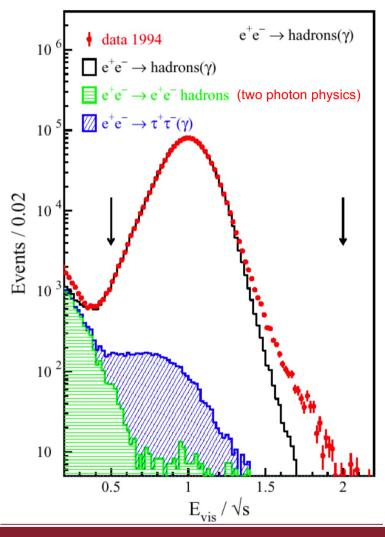
$$R_{\text{hadr.}}^{\text{B}} / R_{\text{vis.}}^{\text{B}} = 87.0 \%.$$

$$R_b = \frac{\Gamma_b}{\Gamma_{had}} = \frac{368}{1675} \approx 22\%$$

It was particularly important to measure precisely the b-quark B.R., since a deviation from the SM prediction could have been an indication of new physics.

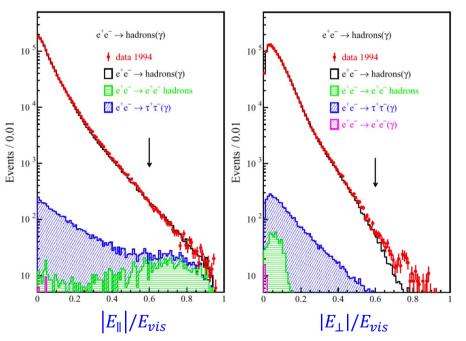
The lineshape: let's look at the data

$e^+e^- \rightarrow Z \rightarrow f\bar{f}$: hadrons (1) [event selection]



Example (L3): $e^+e^+ \rightarrow hadrons \ (e^+e^+ \rightarrow q\bar{q})$

- A lot of energy;
- No missing energy;
- Many particles in the final state;



Event selection

- $0.5 < E_{vis} / \sqrt{s} < 2.0$;
- $|E_{||}| / E_{vis} < 0.6$;
- $E_{\perp} / E_{vis} < 0.6$;
- N_{clusters} > 13 (barrel),
 > 17 (endcap)

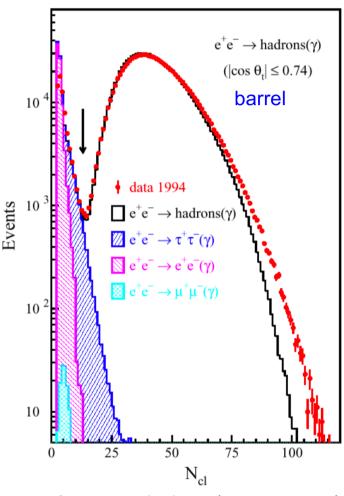
$$\begin{aligned} E_{VIS} &= \sum_{seen} |\vec{p}_{j}|; \\ \vec{P} &= \sum_{seen} \vec{p}_{j}; \\ |E_{\parallel}| &= |P_{Z}|; \quad E_{\perp} = P_{T}. \end{aligned}$$

$e^+e^- \rightarrow Z \rightarrow f\bar{f}$: hadrons (2)[event selection]

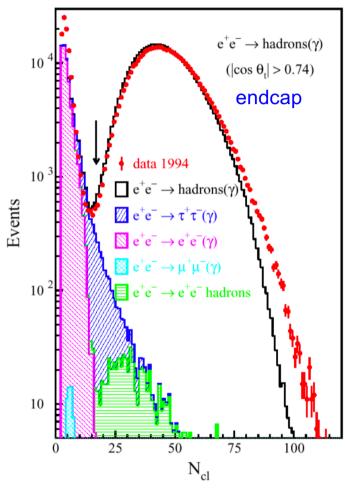
A cluster is defined as a continuous group of crystals or heal cells.

It could very roughly identified with a particle, or at least they are proportional to the number of particles.

So, requiring a large number of clusters in the event, is equivalent to require a large number of particles in the event, as we should have in the Z hadronic decay



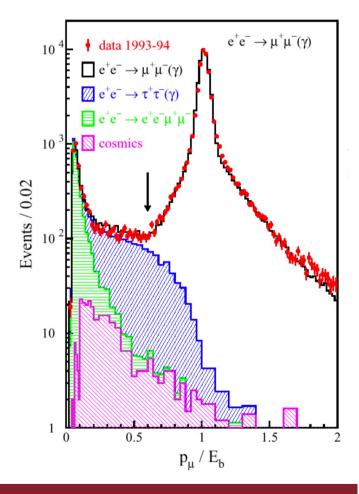


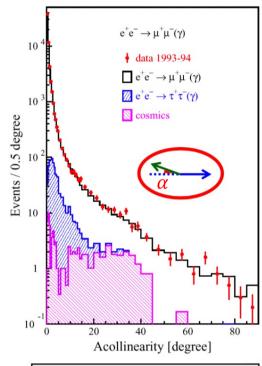


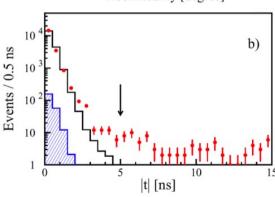
[N_{clusters} > 13 (barrel), > 17 (endcap)]

$e^+e^- o Z o f \overline{f}$: $\mu^+\mu^-$ [event selection]

Example (L3): $e^+e^+ \rightarrow \mu^+\mu^-(\gamma)$

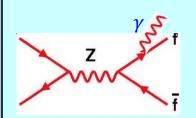






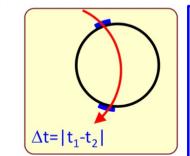
Event selection

- \geq 1 μ identified;
- $|p_{\mu}| > 0.6 (\sqrt{s/2});$
- $\alpha(\Box$) "small";
- N_{clusters} < 15;
- time_{scintillators}.



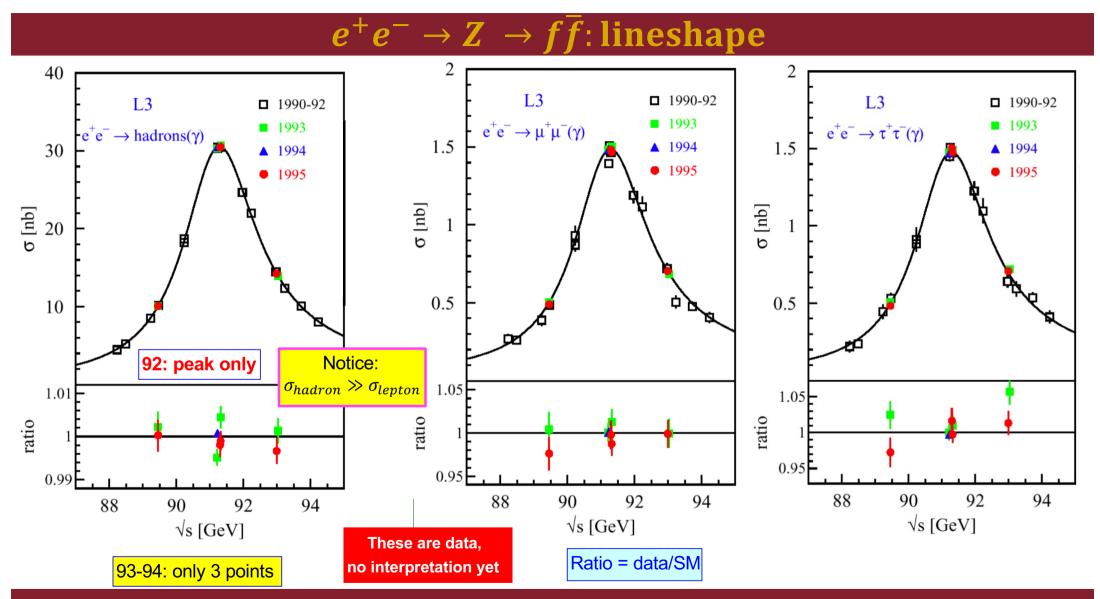
Usually it is not possible to identify the photon emitted from the final legs, therefore the finale state is given as $f \bar{f}(\gamma)$

Q. : why μ 's have smaller acollinearity than τ 's ?



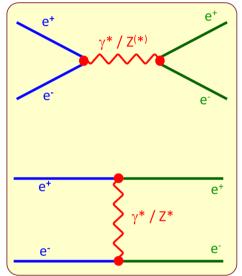
- Distance between 2 scintillator is (at least) 1.8 m
- A cosmic muon takes 6 ns to cover 1.8 m
- A muon pair from Z decay hit the two scintillators at the same time.

Claudio Luci – Collider Particle Physics – Chapter 7

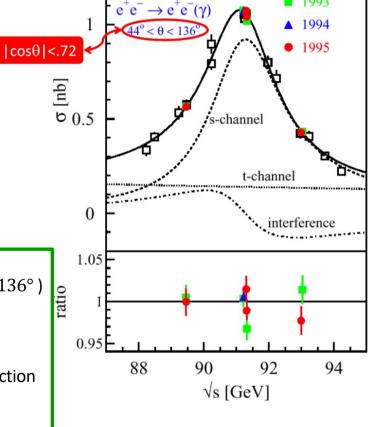


$e^+e^- \rightarrow Z \rightarrow e^+e^-$

- Bhabha scattering is more difficult, due to the presence of another Feynman diagram: the γ^* / Z exchange in the t-channel;
- 4 Feynman diagrams → 10 terms :
 - > Z s-channel (Z_s);
 - $\rightarrow \gamma^*$ s-channel (γ_s);
 - Z t-channel (Z_t);
 - $\rightarrow \gamma^*$ t-channel (γ_t) ;
 - > 6 interferences;
- qualitatively:
 - > Z negligible;
 - \rightarrow @ $\sqrt{s} \approx m_z$ and $\theta >> 0^\circ$, Z_s dominates.
 - \triangleright @ θ ≈ 0°, γ_t dominates for all \sqrt{s} ;
 - both important, while **Z** is negligible.







L3

- The analysis is restricted in the barrel region ($|cos\theta_e| < 0.72$; $44^{\circ} < \theta < 136^{\circ}$) in order to reduce the t-channel contributions.
- The t-channel and s/t interference are subtracted from the total cross-section to extract the "pure" Z resonant cross-section

■ 1990-92

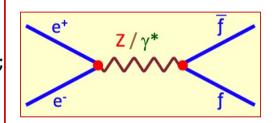
1993

Forward-backward asymmetry

$d\sigma(e^+e^- o far f)/d\Omega$

☐ Differential cross-section at the lowest order:

$$\frac{d\sigma_{Born}\left(e^{+}e^{-}\rightarrow f\overline{f}\right)}{dcos\theta} = \frac{\pi\alpha^{2}(s)c_{f}}{2s} \begin{cases} \left(1+cos^{2}\theta\right)\times \begin{bmatrix}Q_{e}^{2}Q_{f}^{2}-2\boxed{\chi}Q_{f}Q_{e}g_{v}^{e}g_{v}^{f}cos\delta_{R}+\\ +\boxed{\chi^{2}}\left[\left(g_{A}^{e}\right)^{2}+\left(g_{V}^{e}\right)^{2}\right]\left[\left(g_{A}^{f}\right)^{2}+\left(g_{V}^{f}\right)^{2}\right] +\\ +2cos\theta\times \left[-2\boxed{\chi}Q_{e}Q_{f}g_{A}^{e}g_{A}^{f}cos\delta_{R}+4\boxed{\chi^{2}}g_{A}^{e}g_{A}^{f}g_{v}^{e}g_{V}^{f} \right] \end{cases};$$



$$\chi = \frac{G_F}{2\sqrt{2}\pi\alpha(s)} \times \frac{sm_Z^2}{\sqrt{\left(m_Z^2 - s\right)^2 + m_Z^2\Gamma_Z^2}}; \quad \tan\delta_R = \frac{m_Z\Gamma_Z}{m_Z^2 - s} \quad \left[\rightarrow \cos\delta_R(\sqrt{s} = m_Z) = 0 \right];$$

Definition

$$A_{f}^{FB}\left(\sqrt{s}\right) = \frac{\sigma\left(\cos\theta > 0, \sqrt{s}\right) - \sigma\left(\cos\theta < 0, \sqrt{s}\right)}{\sigma\left(\cos\theta > 0, \sqrt{s}\right) + \sigma\left(\cos\theta < 0, 0\right)}$$

 A_f^{FB} is the "forward-backward asymmetry" for $e^+e^- \rightarrow f\overline{f}$.

$$\begin{split} A_f^{FB} \Big(\sqrt{s} = m_Z, Z_{s-channel} \text{ only} \Big) = \\ = 3 \frac{g_V^e g_A^e}{\left(g_V^e\right)^2 + \left(g_A^e\right)^2} \times \frac{g_V^f g_A^f}{\left(g_V^f\right)^2 + \left(g_A^f\right)^2}; \end{split}$$

$$c_V = I_3^f - 2Q^f \sin^2 \theta_W \qquad c_A = I_3^f$$

Asymmetry is caused by the parity violation of weak interactions

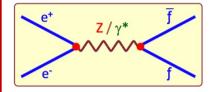
$d\sigma(e^+e^- o f\overline{f})/d\Omega$: comments

$$\frac{d\sigma_{Born}\left(e^{+}e^{-}\rightarrow f\overline{f}\right)}{dcos\theta} = \frac{\pi\alpha^{2}(s)c_{f}}{2s} \left\{ (1+cos^{2}\theta)\times \begin{bmatrix} Q_{e}^{2}Q_{f}^{2}-2\chi Q_{f}Q_{e}g_{v}^{e}g_{v}^{f}cos\delta_{R} + \\ +\chi^{2}\left[\left(g_{A}^{e}\right)^{2}+\left(g_{v}^{e}\right)^{2}\right]\left[\left(g_{A}^{f}\right)^{2}+\left(g_{v}^{f}\right)^{2}\right] + \\ +2cos\theta\times \left[-2\chi Q_{e}Q_{f}g_{A}^{e}g_{A}^{f}cos\delta_{R} + 4\chi^{2}g_{A}^{e}g_{A}^{f}g_{v}^{e}g_{v}^{f}\right] + \right\};$$

Mediator

photon: $\propto Q$ Z vect.: $\propto g_V^f$

Z axial: $\propto g_A^J$



- $\sum_{z=2}^{k-1} 2\sqrt{2\pi\alpha(s)} \sqrt{(m_z^2 s)^2 + m_z^2 \Gamma_z^2}$
- **Notice**: the term prop. to $\cos \theta$ is antisymmetric; therefore it does NOT contribute to the total cross-section $(\int \cos \theta \cdot d \cos \theta = 0)$, but it is the only contributing term to the forward-backward asymmetry.
- The parity violation comes from the interference between the vector current (photon or Z-vector) with the axial current (Z-axial), that are the only terms proportional to $\cos\theta$ and not to $(1+\cos^2\theta)$

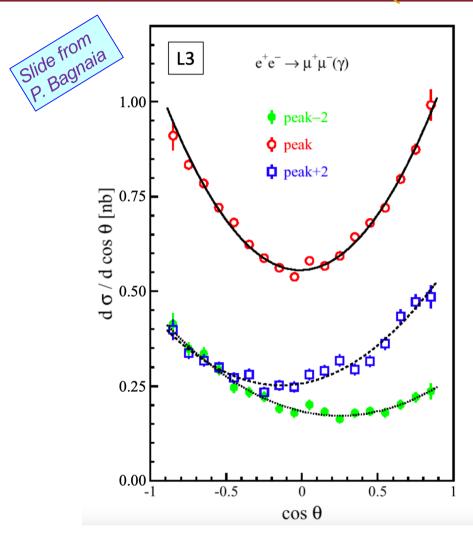
We will see later other asymmetries related to the polarization states. Here, no polarization is taken into account

• **Notice**: at the Z pole ($\sqrt{s} = m_Z$) we have $\cos \delta_R = 0$, therefore the photon/Z-vector interference terms vanish and we are left only with the interference between Z-vector and Z-axial:

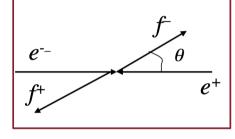
$$\frac{d\sigma}{d\cos\theta} \propto \begin{cases} \left(1+\cos^2\theta\right) \times \left[Q_e^2 Q_f^2 + \left(g_v^e\right)^2 + \left(g_v^e\right)^2\right] \left[\left(g_A^f\right)^2 + \left(g_v^f\right)^2\right] + \left(g_V^f\right)^2 \right] \\ + 2\cos\theta \times \left[4 \frac{\chi^2}{\chi^2} g_A^e g_A^f g_V^e g_V^f\right] \end{cases}$$

- The asymmetry, i.e. the term $\propto \cos \theta$, is $\propto g_V^e$ (which is very small) for all fermions;
- for the channel $\mu^+\mu^-$, where the asymmetry can be measured more easily with respect to other channels, it is even smaller because it is $\propto g_V^e g_V^\mu$

$d\sigma(e^+e^- o f\overline{f})/d\Omega$: data



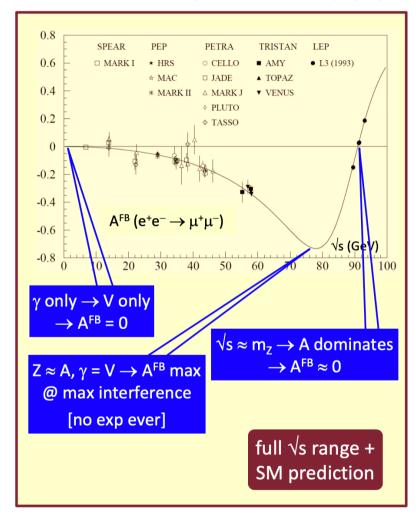
- Experimentally, the main problem is the selection $f \leftrightarrow f$ (i.e. $\theta \leftrightarrow -\theta$). This is
 - → essentially impossible for light quarks
 u ↔ ū, d ↔ đ (despite heroic efforts
 based on charge counting);
 - > difficult for heavy quarks c,b (based on lepton charge in semileptonic quark decays, e.g. c → s $\ell^+\nu$, \bar{c} → $\bar{s}\ell^-\bar{\nu}$);
 - > "simple" for μ^{\pm} (only problem: wrong sagitta sign because of high momentum);
 - > best channel for d σ /dcos θ and A_{FB}: e⁺e⁻ → μ ⁺ μ ⁻(γ);
- unfortunately, $A_{FB}(\sqrt{s}=m_Z)$ is very small in the $\ell^+\ell^-$ channels, due to the extra small factor g^{μ}_{V} ;
- notice the asymmetry change for peak ±2 GeV.

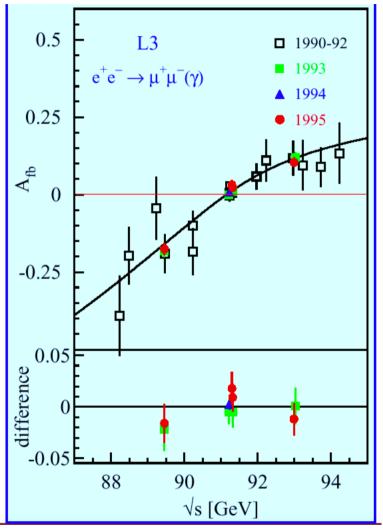


Pay attention: in the plot is shown the differential cross-section, and not the forward-backward asymmetry. Then, from this plot, we build the A_{FB}.

$d\sigma(e^+e^- o far{f})/d\Omega$: $A_{FB}(\mu^+\mu^-)$

Slide from P. Bagnaia





$d\sigma/d \cos\theta$: another set of formulae

$$\frac{\mathrm{d}\sigma_{\mathrm{f}}}{\mathrm{d}\cos\Theta} \ = \ N_c^{\mathrm{f}} \frac{\pi\alpha^2}{2\,s} \left\{ Q_{\mathrm{f}}^2 \left(1 + \cos^2\Theta\right) \right. \qquad \left(\gamma \, \mathrm{exchange}\right)$$

$$- Q_{\mathrm{f}} \left[2\,g_{\mathrm{V}}^{\mathrm{e}} g_{\mathrm{V}}^{\mathrm{f}} (1 + \cos^2\Theta) + 4\,g_{\mathrm{A}}^{\mathrm{e}} g_{\mathrm{A}}^{\mathrm{f}} \cos\Theta \right] \Re\{\chi\} \qquad \mathrm{(Interference)}$$

$$+ \left(\left[\left(g_{\mathrm{V}}^{\mathrm{e}}\right)^2 + \left(g_{\mathrm{A}}^{\mathrm{e}}\right)^2\right] \left[\left(g_{\mathrm{V}}^{\mathrm{f}}\right)^2 + \left(g_{\mathrm{A}}^{\mathrm{f}}\right)^2\right] \left(1 + \cos^2\Theta\right)$$

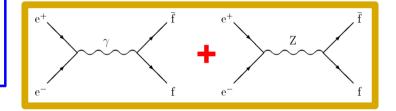
$$+ 8\,g_{\mathrm{V}}^{\mathrm{e}} g_{\mathrm{A}}^{\mathrm{e}} g_{\mathrm{V}}^{\mathrm{f}} g_{\mathrm{A}}^{\mathrm{f}} \cos\Theta \right) |\chi|^2 \right\} \qquad \mathrm{(Z\,\,exchange)}$$

$$\chi \ = \ \frac{1}{4\,\sin^2\!\vartheta_{\mathrm{W}} \cos^2\!\vartheta_{\mathrm{W}}} \frac{s}{s - m_{\mathrm{Z}}^2 + i\,\Gamma_{\mathrm{Z}}\,m_{\mathrm{Z}}} \qquad N_c^{\mathrm{f}} \ = \ \left\{ \begin{array}{l} 1\,\,\mathrm{for\,\,leptons} \\ 3\,\,\mathrm{for\,\,quarks} \end{array} \right.$$

☐ Z couplings

$$egin{array}{lcl} g_{
m V}^{
m f} & = & T_3^{
m f} - 2\,Q_{
m f}\,\sin^2\!artheta_{
m W} \ g_{
m A}^{
m f} & = & T_3^{
m f} \end{array}$$

$$egin{array}{lll} g_{
m L}^{
m f} &=& g_{
m V}^{
m f} + g_{
m A}^{
m f} = 2\,T_{
m 3}^{
m f} - 2\,Q_{
m f}\sin^2\!artheta_{
m W} \ g_{
m R}^{
m f} &=& g_{
m V}^{
m f} - g_{
m A}^{
m f} = -2\,Q_{
m f}\sin^2\!artheta_{
m W} \end{array}$$



 $\sigma_{
m f} \;\; = \;\; rac{4\,\pilpha^2}{3\,s} N_c^{
m f} iggl\{ Q_{
m f}^2 \, - 2\,Q_{
m f}\,g_{
m V}^{
m e}\,g_{
m V}^{
m f}\,\Re\{\chi\} \, + \, iggl[(g_{
m V}^{
m e})^2 + (g_{
m A}^{
m e})^2 iggr] iggl[(g_{
m V}^{
m f})^2 + (g_{
m A}^{
m f})^2 iggr] |\chi|^2 iggr\}$

These are the same formulae we met before, but written in a slightly different manner. It could be usefull to see them, just in case.

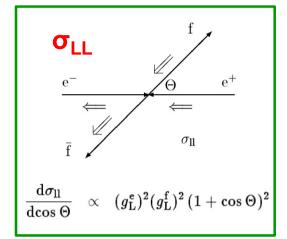
$$egin{array}{lcl} \Gamma_{
m Z} & = & \sum_{
m f} \Gamma_{
m f} & \left(m_{
m f} < m_{
m Z}/2
ight) \ & \ \Gamma_{
m f} & = & N_c^{
m f} rac{lpha \, m_{
m Z}}{12 \, \sin^2\!artheta_{
m W} \cos^2\!artheta_{
m W}} \left[(g_{
m V}^{
m f})^2 + (g_{
m A}^{
m f})^2
ight] \end{array}$$

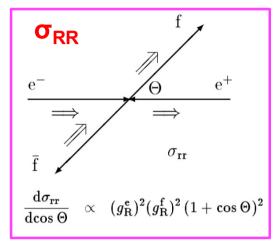
$$egin{aligned} \sigma_{ extbf{f}}(\sqrt{s} = m_{ extbf{Z}}) &pprox & rac{12\,\pi}{m_{ extbf{Z}}^2}\,rac{\Gamma_{ extbf{e}}\Gamma_{ extbf{f}}}{\Gamma_{ extbf{Z}}^2} \ &=& rac{12\,\pi}{m_{ extbf{Z}}^2}\,BR\left(ext{Z}
ightarrow ext{e}^+ ext{e}^-
ight)\cdot BR\left(ext{Z}
ightarrow ext{f}ar{ ext{f}}
ight) \end{aligned}$$

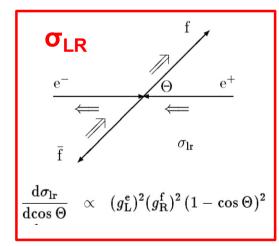
Polarization asymmetries

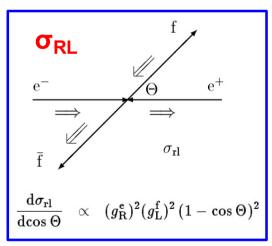
Helicities in $e^+e^- \rightarrow f\bar{f}$ at the Z pole

- ☐ Let's consider the helicity of the initial and final states.
- ☐ Since they are connected by a propagator of spin 1 (Z), the total angular momentum of the initial and final states must be also 1, with J_7 =+-1
- ☐ So, we can look only at the helicity state of the electron in the initial state and of the fermion in the final state. We have four combinations:
- ☐ electron left-handed and fermion left-handed: LL
- ☐ electron left-handed and fermion right-handed: LR
- ☐ electron right-handed and fermion right-handed: RR
- ☐ electron right-handed and fermion left-handed: RL
- ☐ the final states are distinguishable due to the spin, so the cross sections can be calculated separately.
- ☐ To be noticed that we can not have the spin flip along Z $(J_z \text{ must be conserved, either 1 or -1})$, therefore the differential cross-sections are 0 at theta=0° or at theta=180°.









Helicities in $e^+e^- \rightarrow f\bar{f}$ at the Z pole

- \square From the four helicity cross-sections (σ_{LL} , σ_{LR} , σ_{RR} , σ_{RL}), we can get all the asymmetry cross-sections.
- ☐ Total cross-section (just the sum of the four):

$$\sigma_{tot} = \sigma_{LL} + \sigma_{LR} + \sigma_{RR} + \sigma_{RL}$$

■ Left-Right asymmetry (initial state polarization); we need to measure the initial state polarization:

$$\sigma_{LR} = (\sigma_{LL} + \sigma_{LR}) - (\sigma_{RR} + \sigma_{RL})$$

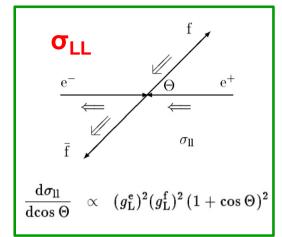
□ Polarization asymmetry (final state polarization); we need to measure the final state polarization:

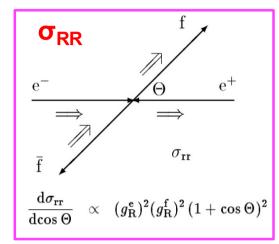
$$\sigma_{pol} = (\sigma_{LL} + \sigma_{RL}) - (\sigma_{LR} + \sigma_{RR})$$

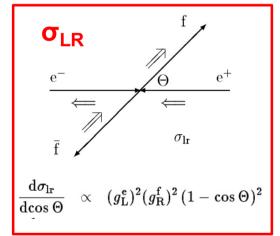
☐ Forward-Backward asymmetry; it can be built from the helicity cross-sections since the differential crosssections have a different theta behaviour

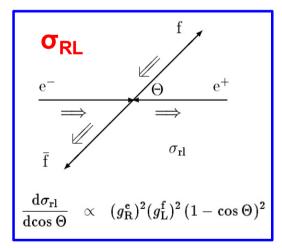
$$\sigma_{FB} = (\sigma_{LL} + \sigma_{RR}) - (\sigma_{LR} + \sigma_{RL})$$

We get the asymmetry values at the Z pole by normalising the cross-sections with σ_{tot} .









Asymmetry parameter A_f

☐ Helicity cross-sections are proportional to g couplings, for instance:

$$\sigma_{LL} = \int_{-1}^{+1} \frac{d\sigma_{LL}}{d\cos\theta} d\cos\theta \propto (g_L^e)^2 (g_L^f)^2 \int_{-1}^{+1} (1+\cos\theta)^2 d\cos\theta \propto \frac{8}{3} (g_L^e)^2 (g_L^f)^2$$

$$\int_{-1}^{+1} (1-\cos\theta)^2 d\cos\theta = \frac{8}{3}$$

☐ Let's consider, for instance, the polarization asymmetry:

$$A_{pol} \equiv \mathcal{P}_{f} = \frac{\sigma_{pol}}{\sigma_{tot}} = \frac{(\sigma_{LL} + \sigma_{RL}) - (\sigma_{LR} + \sigma_{RR})}{\sigma_{LL} + \sigma_{LR} + \sigma_{RR} + \sigma_{RL}} = \frac{(g_{L}^{e})^{2} (g_{L}^{f})^{2} + (g_{R}^{e})^{2} (g_{L}^{f})^{2} - (g_{L}^{e})^{2} (g_{R}^{f})^{2} - (g_{R}^{e})^{2} (g_{R}^{f})^{2}}{(g_{L}^{e})^{2} + (g_{R}^{e})^{2} (g_{L}^{f})^{2} + (g_{R}^{e})^{2} (g_{L}^{f})^{2} + (g_{R}^{e})^{2} (g_{R}^{f})^{2} + (g_{R}^{e})^{2} (g_{R}^{f})^{2}} = \frac{(g_{L}^{f})^{2} - (g_{R}^{f})^{2}}{(g_{L}^{f})^{2} + (g_{R}^{f})^{2} (g_{L}^{f})^{2} + (g_{R}^{e})^{2} (g_{L}^{f})^{2} + (g_{R}^{e})^{2} (g_{L}^{f})^{2} + (g_{R}^{e})^{2} (g_{L}^{f})^{2}} = \frac{(g_{L}^{f})^{2} - (g_{R}^{f})^{2}}{(g_{L}^{f})^{2} + (g_{R}^{f})^{2} (g_{L}^{f})^{2} + (g_{R}^{f})^{2}} = \frac{(g_{L}^{f})^{2} - (g_{R}^{f})^{2}}{(g_{L}^{f})^{2} + (g_{R}^{f})^{2}} = \frac{(g_{L}^{f})^{2} - (g_{R}^{f})^{2}}{(g_{L}^{f})^{2}} = \frac{(g_{L}^{f})^{2}$$

 \square We can define the fermion asymmetry parameter A_f :

$$A_{f} = \frac{(g_{L}^{f})^{2} - (g_{R}^{f})^{2}}{(g_{L}^{f})^{2} + (g_{R}^{f})^{2}}$$

Parity violation is contained in A_f

We can also write it as a function of g_V and g_A

$$egin{array}{lcl} g_{
m V}^{
m f} &=& T_3^{
m f} - 2\,Q_{
m f}\,\sin^2\!artheta_{
m W} \ g_{
m A}^{
m f} &=& T_3^{
m f} \end{array}$$

$$egin{array}{lcl} g_{
m V}^{
m f} &=& T_3^{
m f} - 2\,Q_{
m f}\,\sin^2\!artheta_{
m W} \ g_{
m A}^{
m f} &=& T_3^{
m f} \ \end{array} \ g_{
m L}^{
m f} &=& g_{
m V}^{
m f} + g_{
m A}^{
m f} = 2\,T_3^{
m f} - 2\,Q_{
m f}\,\sin^2\!artheta_{
m W} \ g_{
m R}^{
m f} &=& g_{
m V}^{
m f} - g_{
m A}^{
m f} = -2\,Q_{
m f}\,\sin^2\!artheta_{
m W} \end{array}$$

$$\mathcal{A}_{\mathrm{f}} = \frac{g_{\mathrm{Lf}}^2 - g_{\mathrm{Rf}}^2}{g_{\mathrm{Lf}}^2 + g_{\mathrm{Rf}}^2} = \frac{2g_{\mathrm{Vf}}g_{\mathrm{Af}}}{g_{\mathrm{Vf}}^2 + g_{\mathrm{Af}}^2} = 2\frac{g_{\mathrm{Vf}}/g_{\mathrm{Af}}}{1 + (g_{\mathrm{Vf}}/g_{\mathrm{Af}})^2}.$$

Asymmetries at the Z pole

 \square By using A_f we can define all the asymmetries at the Z pole.

$${\cal A}_{
m f} \;\; = \;\; rac{g_{
m Lf}^2 - g_{
m Rf}^2}{g_{
m Lf}^2 + g_{
m Rf}^2} = rac{2g_{
m Vf}g_{
m Af}}{g_{
m Vf}^2 + g_{
m Af}^2} = 2rac{g_{
m Vf}/g_{
m Af}}{1 + (g_{
m Vf}/g_{
m Af})^2} \,.$$

□ Polarization asymmetry:

$$\mathcal{P}_{
m f} \;\; = \;\; -rac{\sigma_{
m pol}}{\sigma_{
m tot}} = -A_{
m f}$$

Minus sign is due to historical definition of the asymmetry as Right minus Left

In order to measure the asymmetry, we need to measure the polarization of the final state fermions. It can not be done, except one case: tau lepton.

☐ Left-Right asymmetry at the Z-pole:

$$A_{
m LR} \;\; = \;\;\; rac{\sigma_{
m LR}}{\sigma_{
m tot}} = A_{
m e}$$

In order to measure this asymmetry, we need to have polarized electrons in the initial state. This was not possible at LEP because any longitudinal polarization would have been destroyed while the electrons were going around the ring, while it was possibile to achieve it at SLC since the electrons went around the circular part only once.

 \Box Forward-backward asymmetry at the Z-pole; we got it in terms of g_V and g_A :

$$A_{
m FB}^{
m f}(\sqrt{s}=m_{
m Z}) \;\; = \;\; 3 \, rac{g_{
m V}^{
m e} \, g_{
m A}^{
m e}}{(g_{
m V}^{
m e})^2 + (g_{
m A}^{
m e})^2} \, rac{g_{
m V}^{
m f} \, g_{
m A}^{
m f}}{(g_{
m V}^{
m f})^2 + (g_{
m A}^{
m f})^2} \;\;\; lacksquare$$



$$A_{
m FB} \;\; = \;\; rac{3}{4} \, rac{\sigma_{
m FB}}{\sigma_{
m tot}} = rac{3}{4} \, A_{
m e} A_{
m f}$$

Polarization Asymmetries at LEP/SLC

- \Box σ_{tot} and A_{FB} can be measured for all charged leptons, heavy quarks and, inclusively, for all five quarks flavours.
- □ The polarization of the final state fermion is observable only in the reaction $e^+e^- \to \tau^+\tau^-$ where \mathcal{P}_{τ} is inferred from the energy spectra of the decay product of the tau.
- ☐ For the measurement of A_{LR} all Z decays into hadrons and charged leptons can be used. However, it requires the longitudinal polarization of the incoming electrons in e⁺e⁻ collisions which was achieved only at the SLC collider.
- The cross section differences of left and right-handed fermions in the initial and final state σ_{LR} and σ_{pol} can be measured as a function of the scattering angle θ. This way we define:

$$A_{
m FB}^{
m pol} = rac{1}{\sigma_{
m tot}} \left[\int_0^1 rac{\partial \sigma_{
m pol}}{\partial \cos \Theta} \, {
m d} \cos \Theta - \int_{-1}^0 rac{\partial \sigma_{
m pol}}{\partial \cos \Theta} \, {
m d} \cos \Theta
ight]$$

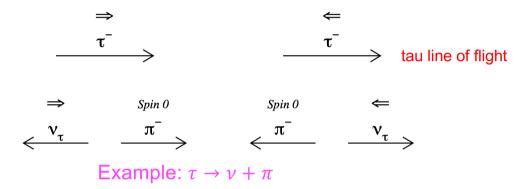


$$A_{
m FB}^{
m LR} = rac{1}{\sigma_{
m tot}} \left[\int_0^1 rac{\partial \sigma_{
m LR}}{\partial \cos \Theta} \, {
m d} \cos \Theta - \int_{-1}^0 rac{\partial \sigma_{
m LR}}{\partial \cos \Theta} \, {
m d} \cos \Theta
ight].$$

- ☐ As a consequence of helicity conservation at the Zff vertices, the measurement of the angular dependence of the final state polarization asymmetry provides information on the couplings of the initial state electrons and viceversa.
- \Box Thus the measurement of the tau polarization as a function of cos θ yields a measurement of the electron coupling.
- ☐ Therefore, polarization asymmetry measurements at LEP and SLC are complementary.

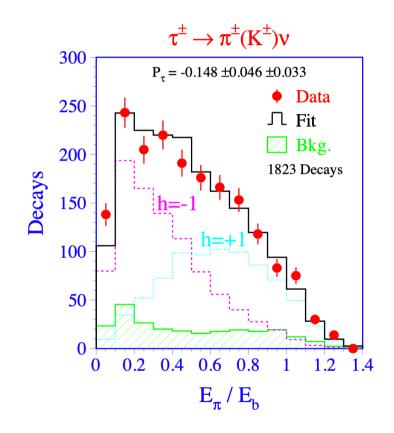
Tau polarization

- ☐ Tau leptons decay close to the interaction point before the decay products reach the detector.
- ☐ The energy spectrum of the final state particles in the two body decays depends on the tau polarization.



- pion preferentially escapes in the direction of the tau helicity
- Hence, when boosted into the Lab frame, the pion receives on average more energy from the decay of a right-handed tau than from a left-handed tau.
- From the distribution one can fit the contribution from the two helicity states.

$$P_{ au^-} \;\; = \;\; rac{\sigma_{ au^-}^R - \sigma_{ au^-}^L}{\sigma_{ au^-}^R + \sigma_{ au^-}^L} \; = \;\; -rac{2\,ar{g}_{
m A}^ au\,ar{g}_{
m V}^ au}{\left(ar{g}_{
m A}^ au
ight)^2 + \left(ar{g}_{
m V}^ au
ight)^2} \; \equiv -A_ au$$

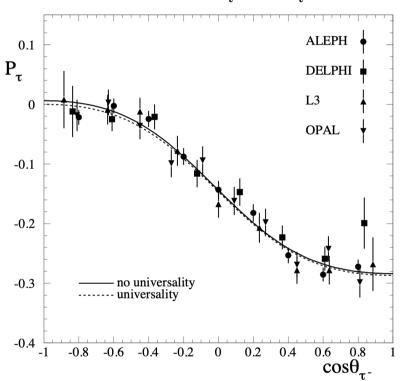


Tau polarization

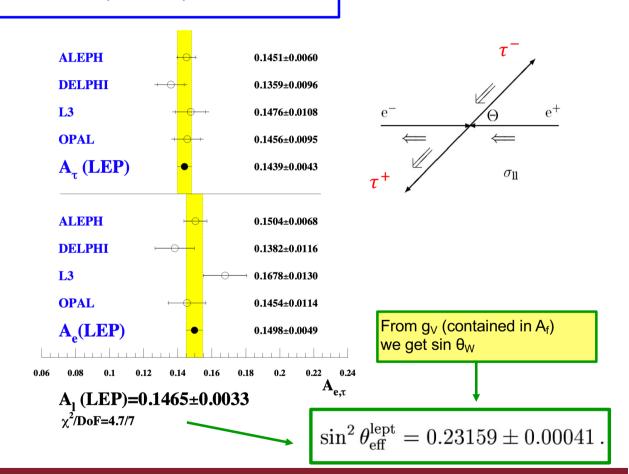
☐ Polarization as a function of the angle:

$$P_{ au^{-}}(\cos heta) = -rac{\left(1+\cos^{2} heta
ight)A_{ au}+2\cos heta\,A_{e}}{\left(1+\cos^{2} heta
ight)+2\cos heta\,A_{ au}A_{e}}$$

Measured P_{τ} vs $\cos \theta_{\tau}$ -

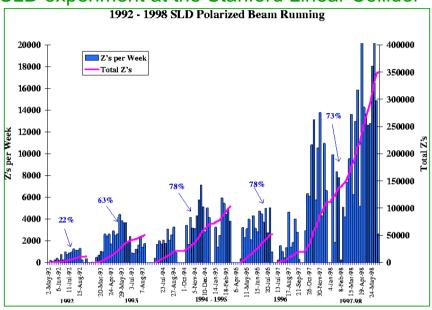


From this distribution we get A_τ and A_e



SLC: Initial state polarization

SLD experiment at the Stanford Linear Collider



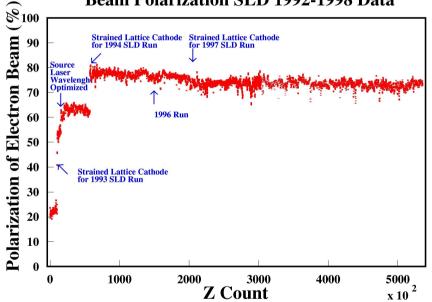
- Longitudinally polarized electrons were produced at the Polarized Electron Source by illuminating a GaAs photocathode with circular-polarized light from a laser of wavelength 715 nm.
- A system of bending magnets and a superconducting solenoid were used to rotate the spins so that the polarization was preserved while the 1.21 GeV electrons were stored in the damping ring.
- Another set of bending magnets and two superconducting solenoids oriented the spin vectors so that longitudinal polarization of the electrons was achieved at the collision point with the unpolarized positrons.

- $\approx 500~000~Z$ decays observed
- ≈ 1/10 of typical LEP experiment

BUT

≈ 77% longitudinal electron polarisation





SLD: Left-Right asymmetry

☐ The Left-Right asymmetry, taking into account the average beam polarization, is:

$$A_{
m LR} = rac{1}{< P_{
m e}>} rac{N_{
m e_{
m l}^-} - N_{
m e_{
m r}^-}}{N_{
m e_{
m l}^-} + N_{
m e_{
m r}^-}} \, .$$

$$A_e = \langle P_e \rangle A_{LR}$$

□ Electron asymmetry:
$$A_e = 0.1516 \pm 0.0021$$
 $A_e = 0.1498 \pm 0.0049$ (at LEP)

$$A_e = 0.1498 \pm 0.0049 (at LEP)$$

$$\sin^2 \theta_W = 0.23098 \pm 0.00026$$

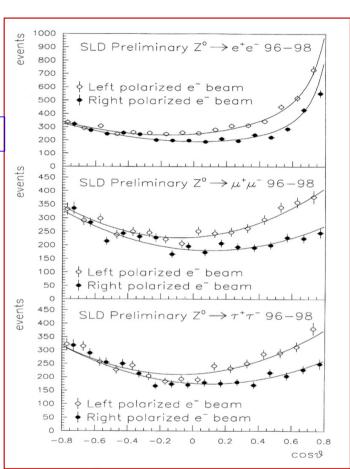
Best single measure of the World

☐ By measuring the forward-backward asymmetry for a given final state, it was possible to measure the asymmetries for the different fermions:

Leptons		
A_e	=	0.1516 ± 0.0021
A_{μ}	=	0.142 ± 0.015
$A_{ au}$	=	0.136 ± 0.015

Quarks		
A_{S}	=	0.85 ± 0.09
A_c	=	0.922 ± 0.020
A_b	=	0.670 ± 0.026

s quarks were identified by tagging high-momentum K and Λ



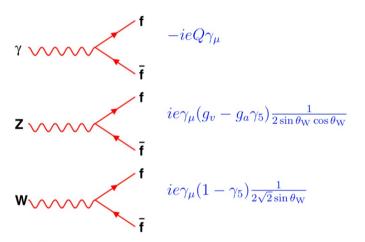
Radiative corrections

Standard Model relationships

Masses of heavy gauge bosons and their couplings to fermions depend on SAME mixing angle

$$\cos \theta_{\rm W} = M_{\rm W}/M_{\rm Z}$$

 $SU(2) \times U(1)$ coupling constants, g,g', proportional to electric charge e: $g=e\sin\theta_{\rm W},\,g'=e\cos\theta_{\rm W}$



where Q, g_a and g_v depend on fermion type, with

$$g_a = T^3 = \pm \frac{1}{2}$$

 $g_v = (T^3 - 2Q\sin^2\theta_W) = \pm \frac{1}{2}(1 - 4|Q|\sin^2\theta_W)$

 g_v/g_a gives $\sin^2\theta_W$ if you know |Q|.

Relate $e, \sin \theta_{\mathrm{W}}$ and M_{W} to the best measured parameters:

$$\alpha \equiv \frac{e^2}{4\pi} = 1/137.035\,999\,76(50)$$

$$G_{\rm F} \equiv \frac{\pi\alpha}{\sqrt{2}M_{\rm W}^2\sin^2\theta_{\rm W}} = 1.166\,39(1)\times10^{-5}\,{\rm GeV}^{-2}$$

$$M_{\rm Z} = 91.1875(21)\,{\rm GeV}$$

 $G_{\rm F}$ measured from muon decay; $M_{\rm Z}$ from LEP.

These relations are true at tree level, but to check that they are valid, must take into account radiative corrections, which give sensitivity to virtual heavy particles, and possibly new physics!

Aside: Other SM inputs needed are fermion masses, Higgs mass, CKM matrix (quark mass eigenstates are not weak eigenstates), strong coupling constant, α_s

The need for radiative corrections

 \Box the mixing angle $s_W = \sin \theta_W$ can be extracted from the ratio of the coupling g_V^f/g_A^f that enters in the cross section measurements. From the observables with leptons in the final states, the latest LEP measurement of $s_{\rm w}^2$ from leptonic observables, which we will identify as s^2 , gives an average value:

$$s_W^2 = s_{\text{eff},l}^2 = 0.23159 \pm 0.00018$$
 (i.e., 0.08% precision).

 \Box This angle also describes the ratio M_W^2/M_z^2 and therefore could be extracted from a combination of entirely different measurements (LEP1 and LEP2). The latest data give:

$$M_Z = 91.1875 \pm 0.0021 \text{ GeV } \left(\Delta M_Z / M_Z \sim 2 \times 10^{-5}\right)$$

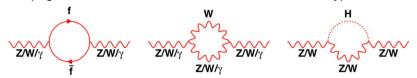
 $M_W = 80.451 \pm 0.033 \text{ GeV } \left(\frac{\Delta M_W}{M_W} \sim 5 \times 10^{-4}\right),$

□ leading to:
$$s_M^2 = 0.22162 \pm 0.00067$$
 (i.e., 0.3% precision).

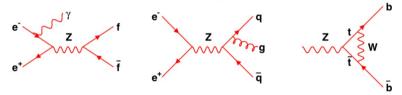
□ Not only we get a much better precision on the effective mixing angle extracted from the Z observables, but more telling is that this value is about 14σ away from that extracted from the mass ratio, M_W^2/M_Z^2 . The Born approximation is insufficient to explain the LEP measurements. Improving on the Born approximation necessitates the inclusion of radiative corrections which are contributions from the quantum fluctuations of the vacuum.

Radiative corrections

Propagator corrections are the same for each fermion type.



QED, QCD and vertex corrections give fermion dependent terms.



Electroweak corrections absorbed into effective couplings:

$$g_{\rm V} \equiv g_{\rm V}^{\rm eff} = \sqrt{(1 + \Delta \rho)} (T^3 - 2Q \sin^2 \theta_{\rm eff})$$

$$g_{\rm A} \equiv g_{\rm A}^{\rm eff} = \sqrt{(1 + \Delta \rho)} T^3$$

$$\sin^2 \theta_{\rm eff} = (1 + \Delta \kappa) \sin^2 \theta_{\rm W}$$

$$\Delta \rho = \frac{3G_{\rm F} M_{\rm W}^2}{8\sqrt{2}\pi^2} \left(\frac{M_{\rm t}^2}{M_{\rm W}^2} - \tan^2 \theta_{\rm W} \left[\ln \frac{M_{\rm H}^2}{M_{\rm W}^2} - \frac{5}{6} \right] \right) + \cdots$$

$$\Delta \kappa = \frac{3G_{\rm F} M_{\rm W}^2}{8\sqrt{2}\pi^2} \left(\cot^2 \theta_{\rm W} \frac{M_{\rm t}^2}{M_{\rm W}^2} - \frac{11}{9} \left[\ln \frac{M_{\rm H}^2}{M_{\rm W}^2} - \frac{5}{6} \right] \right) + \cdots$$

Extra $M_{
m t}^2/M_{
m W}^2$ contributions for b quark

The value of $G_{\rm F}$ is also modified:

$$G_{\rm F} = \frac{\pi \alpha}{\sqrt{2} M_{\rm W}^2 \sin^2 \theta_{\rm W}} \frac{1}{1 - \Delta r}$$

where

$$\Delta r = \Delta \alpha + \Delta r_{\rm w} = \Delta \alpha - \Delta \kappa + \cdots$$

 $\Delta \alpha$ term incorporates the running of the electromagnetic coupling due to fermion loops in the photon propagator. The difficult part of the calculation is to account for all the hadronic states. Use experimental measurement of $e^+e^- \to hadrons$ at low \sqrt{s} .

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha}$$

$$\alpha(0) = 1/137.03599976(50)$$
; $\alpha(M_Z) = 1/128.936(46)$

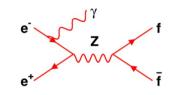
Quadratic dependence on $M_{
m t}$ Logarithmic dependence on $M_{
m H}$ Can fit both $M_{
m t}$ and $M_{
m H}$

Use programs such as ZFITTER (D Bardin et al.) and TOPAZ0 (G Montagna et al.) for calculations to higher order.

Leading order expressions above are for large $M_{\rm H}$.

QED corrections

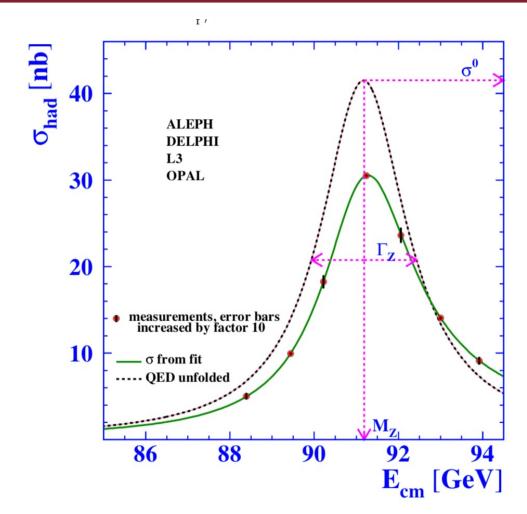
Dominant QED correction from initial state radiation.



Accounted for by radiator function H. We want $\sigma_{\rm ew}(s)$

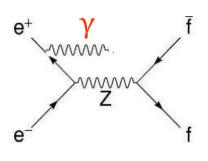
$$\sigma(s) = \int_{4m_{\rm f}^2/s}^1 dz H_{\rm QED}^{\rm tot}(z,s) \sigma_{\rm ew}(zs).$$

- ☐ The emission of the photon from the initial legs modifies the effective \sqrt{s} of the Z interaction
- ☐ We have to take it into account by doing an integral over all the possible center of mass energies.
- ☐ As a consequence the lineshape is heavily modified by the initial state radiation (ISR).
- ☐ Cross-section at the peak is reduced by about 26%
- ☐ The peak is slighlyt shifted at higher energies (112 MeV)
- ☐ The lineshape becomes more asymmetric

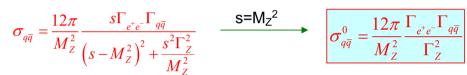


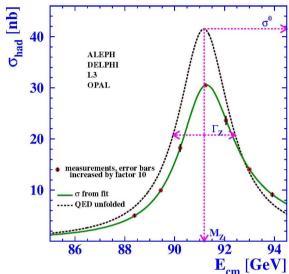
Measurements

Measurement of the Z partial widths



The emission of a photon from the inital state lower the effective center of mass energy. This effect is taken into account in the fit by a "radiator" function (it is a pure QED effect and can be computed with great precision).





- To measure the partial widths of the Z decays in the various fermionic channels, we need to measure the cross-section at the peak:
- We select the following channels:

$$Z \to q\overline{q}$$

$$Z \to \mu^{+}\mu^{-}$$

$$Z \to \tau^{+}\tau^{-}$$

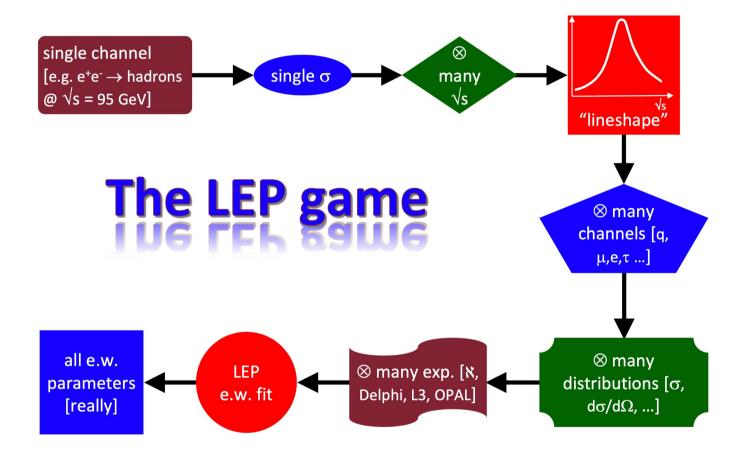
- 1. Peak cross-section;
- 2. Partial width;
- 3. Z couplings
- N.B. The total width Γ_Z is the same in all channels; from a channel to the other does not change the resonance shape but only the peak value;

 $Z \rightarrow e^+e^-$

- N.B. the electron channel is more complicated because there is also the photon exchange in the t channel;
- N.B. in the hadron channel it is possible to distinguish the b quark from its impact parameter (B_0 mesons live long enough); therefore we can measure the partial width also in the $b\bar{b}$ channel.

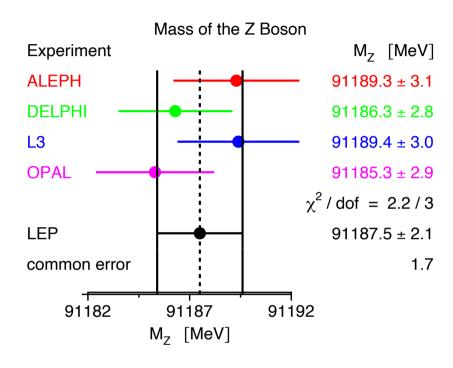
L1 SM Fit





Fits are done using Model Independent programs

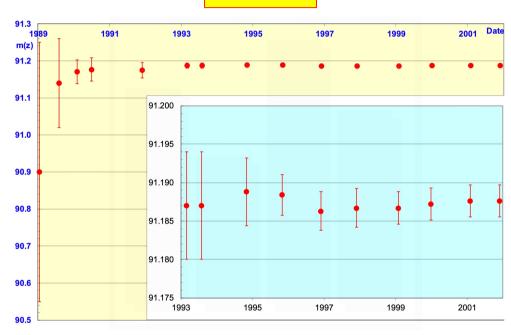
Z Mass



$M_Z = 91.1875 \pm 0.0021 \text{ GeV}$

$$\frac{\Delta M_Z}{M_Z} = \pm 2.3 \cdot 10^{-5}$$

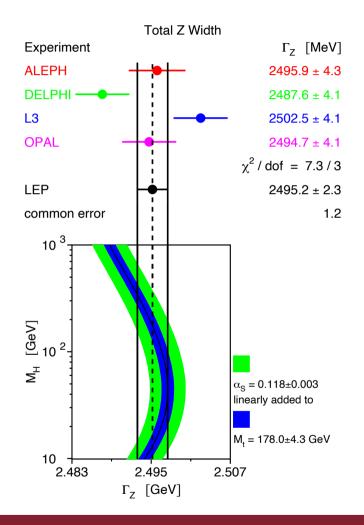
M_Z history

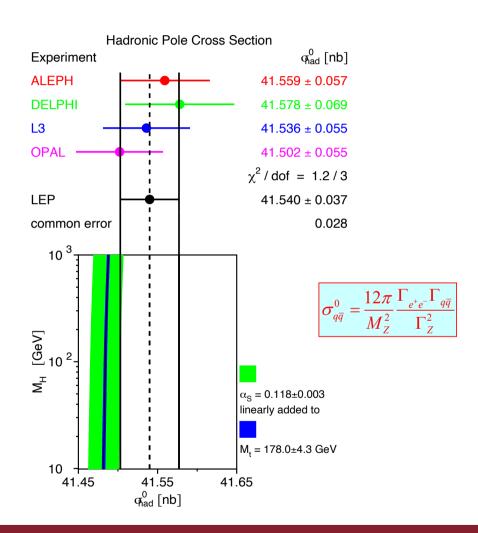


N.B. there is no SM prediction for the Z Mass because it is an INPUT parameter of the Model

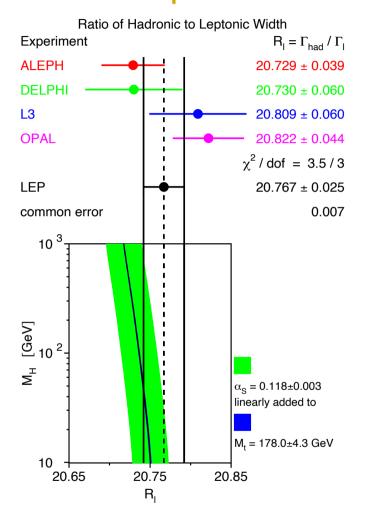
The γ-Z interference term is taken from the SM. If it is left as a free parameter in the fit, it would add an additional 9 MeV error on the Z mass

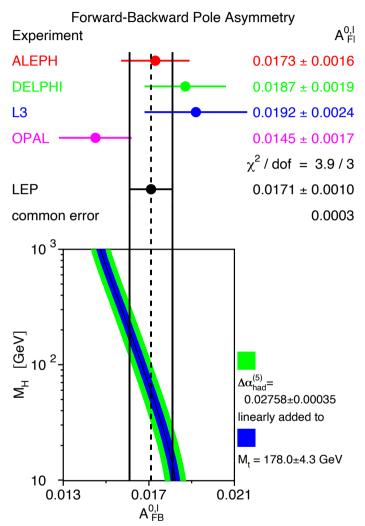
Z total width and Hadronic Pole Cross Section





R_{lepton} and Forward-Backward Asymmetry





Measurement of g_V and g_A

The measurement of the Z couplings before LEP did not have enough precision to make stringent tests of the Standard Model, for instance they could not disantangle the sign of the couplings, we need the asymmetries to do it.

$$g_V = -0.03783(41)$$

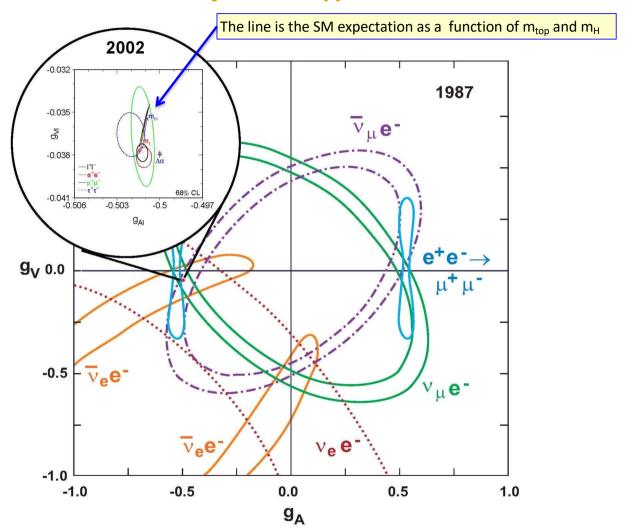
 $g_A = -0.50123(26)$

Lepton coupling to the Z Ratios of coupling constants:

$$g_A^{\mu}/g_A^e = 1.0002 \pm 0.0014$$

 $g_A^{\tau}/g_A^e = 1.0019 \pm 0.0015$
 $g_V^{\mu}/g_V^e = 0.962 \pm 0.063$
 $g_V^{\tau}/g_V^e = 0.958 \pm 0.029$

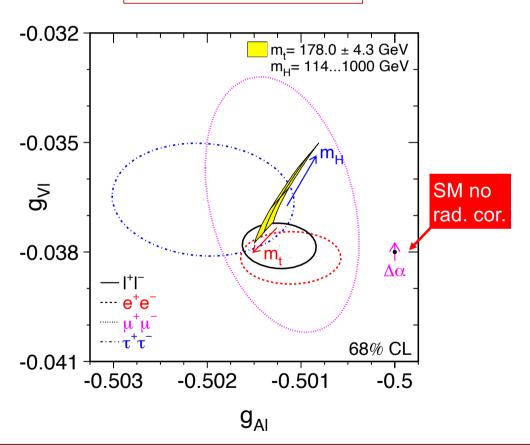
Lepton universality is verified at the per mill level.

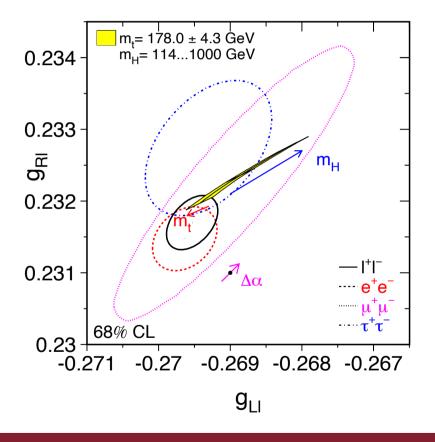


g_V versus g_A and g_R versus g_L

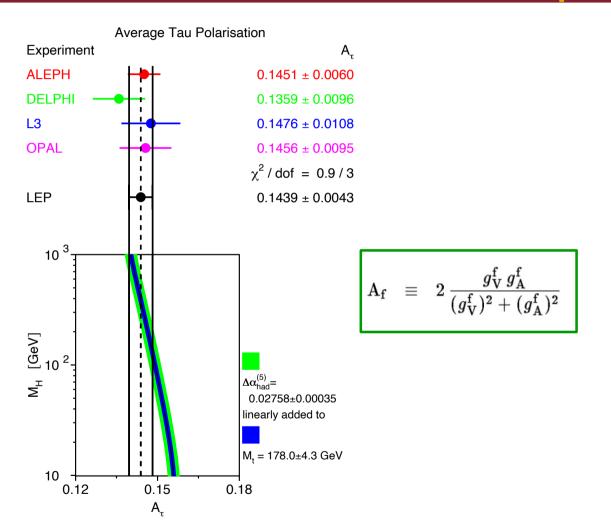
$$egin{array}{lcl} g_{
m V}^{
m f} & = & T_3^{
m f} - 2\,Q_{
m f}\,\sin^2\!artheta_{
m W} \ g_{
m A}^{
m f} & = & T_3^{
m f} \end{array}$$

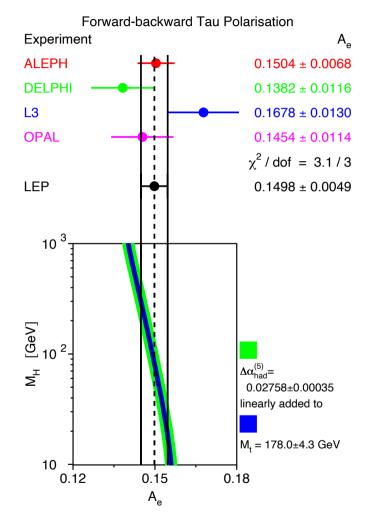
$$egin{array}{lll} g_{
m L}^{
m f} &=& g_{
m V}^{
m f} + g_{
m A}^{
m f} = 2\,T_{
m 3}^{
m f} - 2\,Q_{
m f}\sin^2\!artheta_{
m W} \ g_{
m R}^{
m f} &=& g_{
m V}^{
m f} - g_{
m A}^{
m f} = -2\,Q_{
m f}\sin^2\!artheta_{
m W} \end{array}$$





Final result for tau polarization





Lepton Universality

Plot $A_{\mathrm{FB}}^{0,\,\ell}$ vs. $R_\ell^0 = \Gamma_{\mathrm{had}}/\Gamma_{\ell\ell}$. Contours contain 68% probability.

Lepton universality OK. Results agree with SM (arrows)

$$M_{
m t} = 174.3 \pm 5.1~{
m GeV}$$
 $M_{
m H} = 300^{+700}_{-186}~{
m GeV}$ (low $M_{
m H}$ preferred) $lpha_s(M_{
m Z}^2) = 0.118 \pm 0.002$

Ratios of couplings:

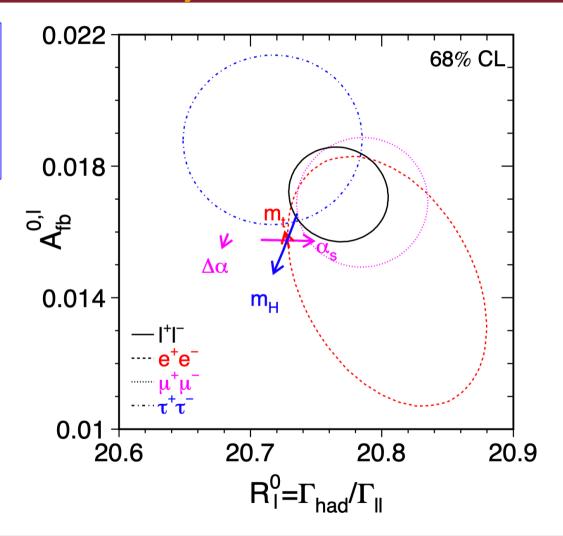
$$g_A^{\mu}/g_A^e = 1.0002 \pm 0.0014$$

$$g_{A^{T}}/g_{A^{e}} = 1.0019 \pm 0.0015$$

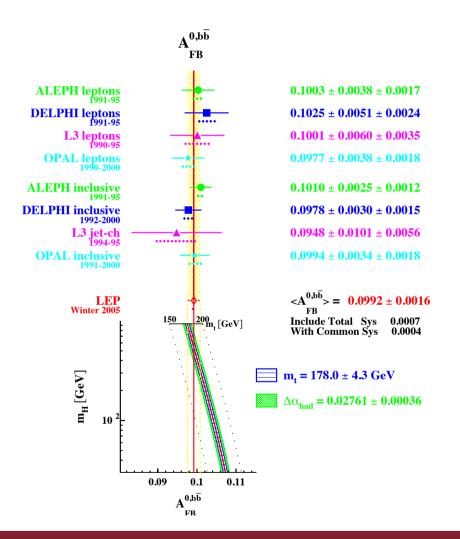
$$g_V^{\mu}/g_V^e = 0.962 \pm 0.063$$

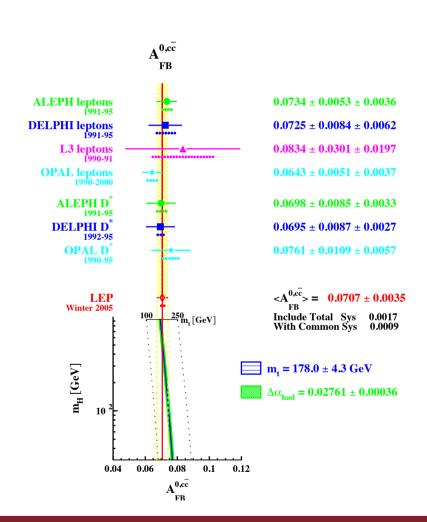
$$g_{V}^{\tau}/g_{V}^{e} = 0.958 \pm 0.029$$

Lepton universality tested to 10⁻³ in g_A

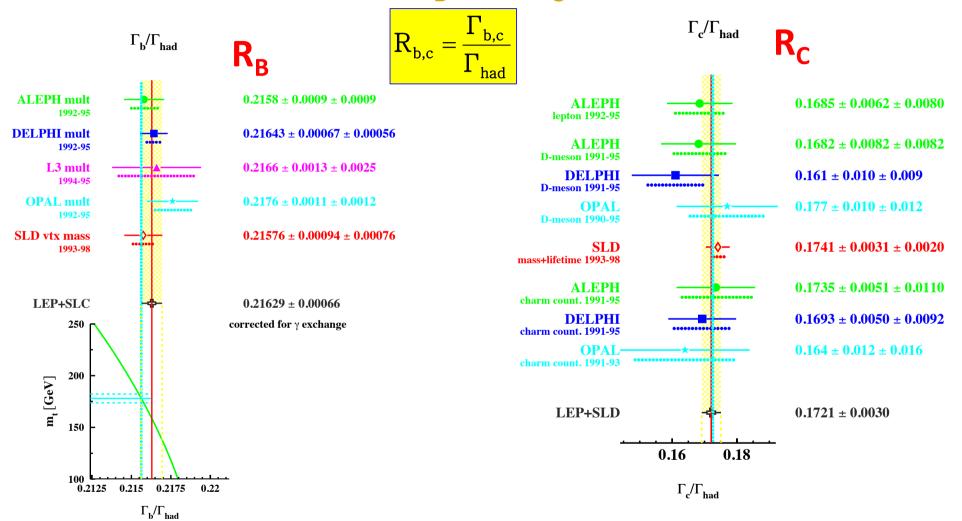


bb and cc Forward-Backward Asymmetry

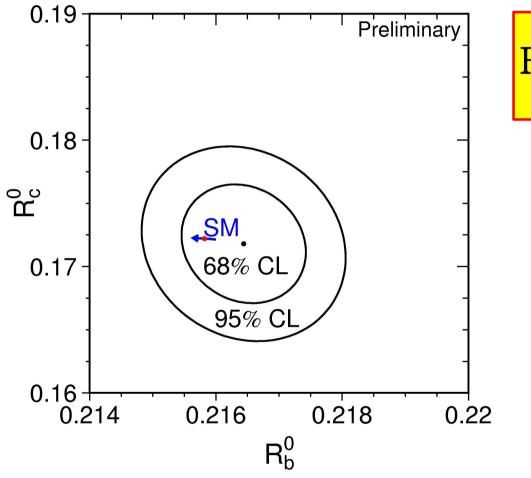




R_B and R_C



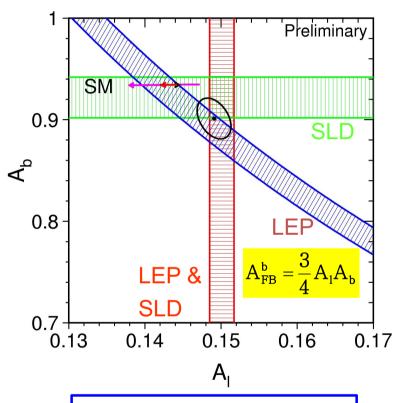
The partial widths $Z \rightarrow b\bar{b}$ and $Z \rightarrow c\bar{c}$



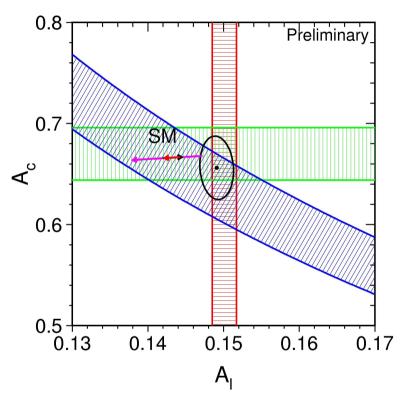
$$R_{b,c} = \frac{\Gamma_{b,c}}{\Gamma_{had}}$$

Neutral current coupling of quarks



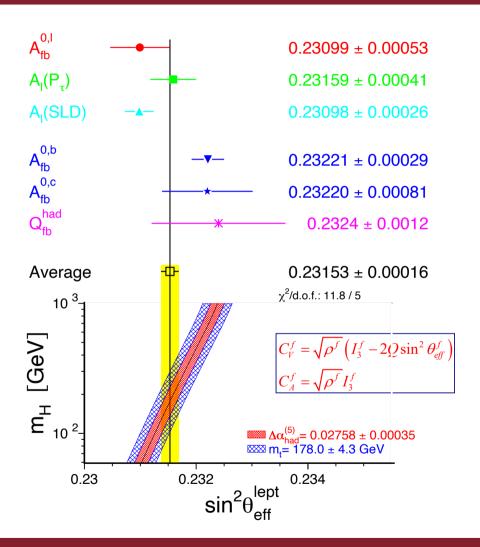


c-quark:



SLD:
$$A_{\mathrm{FB}}^{\mathrm{LR,\,b}} = \, <\! P_{\mathrm{e}} \!> rac{3}{4}\,A_{\mathrm{b}}$$

Measurement of sin² ϑ eff



- Asymmetries at Z pole
- forward-backward
- left-right (SLD)
- tau polarisation

 $\sin^2\vartheta_{eff}$ is a renormalized value of $\sin^2\vartheta_W$. The tree level prediction of the SM is not sufficient to have an agreement with real data.

From the measured values of various asymmetries we can get the value of the Weinberg angle.

The radiative corrections depend of the top mass and Higgs mass, therefore with a comparison with the measured value we can make a prediction on these two parameters.

From this kind of measurements it has been possible to predict the value of the top mass and to put constraints on the Higgs mass.

Lep combined results

Z resonance parameters - recall pre-LEP hopes:

- ullet $\sigma(M_{
 m Z})pprox 10$ MeV (limited by beam energy precision)
- ullet Number of generations $\sigma(N_
 u)pprox 0.2$

Fitted	$M_{ m Z}$ [GeV]	91.1875 ± 0.0021
	Γ_{Z} [GeV]	2.4952 ± 0.0023
	$\sigma_{ m h}^0$ [nb]	41.540 ± 0.037
	R_ℓ^0	20.767 ± 0.025
	$A_{ m FB}^{0,\ell}$	0.0171 \pm 0.0010
Derived	Γ_{inv} [MeV]	499.0 ± 1.5
	Γ_{had} [MeV]	1744.4 ± 2.0
	$\Gamma_{\ell\ell}$ [MeV]	83.984 \pm 0.086
	$N_{ u}$	2.984 ± 0.008
		·

$$M_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

$$\frac{\Delta M_Z}{M_Z} = \pm 2.3 \cdot 10^{-5}$$

Summary - Very precise measurements of Z mass, width, cross-sections, partial widths and lepton forward-backward asymmetries.

High statistics data samples. Careful control of systematic errors.

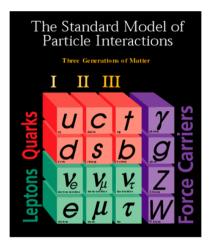
Number of neutrino families

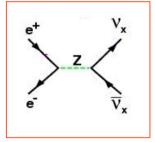
Measurement of the number of light neutrinos

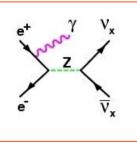
- ☐ The number of lepton families is not foreseen in the Standard Model but it has to be determined experimentally.
- ☐ Before LEP operations a fourth family of leptons was not excluded by the available data.
- \square In every family is present a neutrino, massless or in any case with a negligeable mass; therefore the LEP strategy was to look for the presence of a fourth light neutrino (where light means of mass less than half of m_z).
- ☐ If the forth neutrino were identified it would have been the first hint of a fourth lepton family.
- Therefore the goal was to measure the Z partial width in the neutrino channel and from this deduce the number of light neutrinos.
- Let's recall the fact that neutrinos are not "seen" in the LEP detectors, so we need a "trick" to perform the measurement.

$$\Gamma_Z = \Gamma_{charged\ leptons} + \Gamma_{hadrons} + N \cdot \Gamma_{\nu\nu}$$

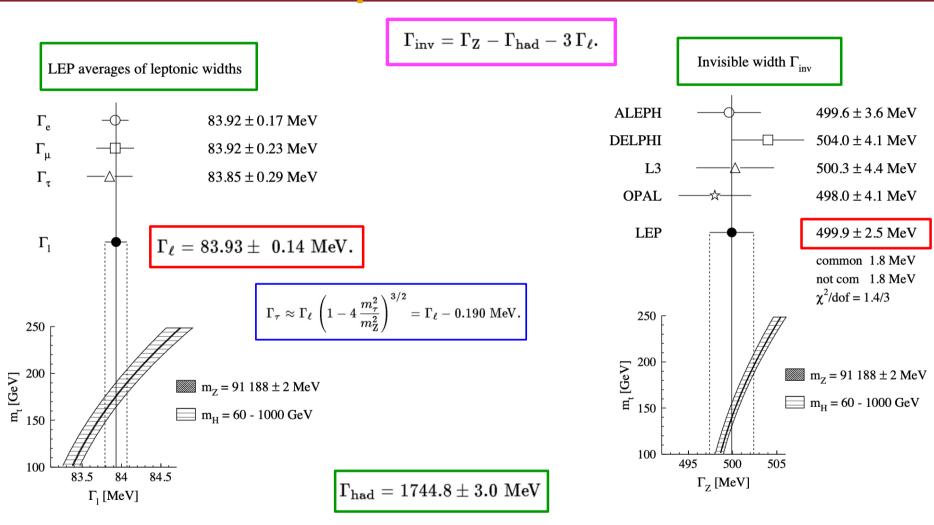
There were two kind of measurement of the so called invisible width (Γ_{inv}): an indirect measurement where Γ_{inv} is obtained as a difference by subtracting to Γ_Z the "visible" partial widths, and a direct measurement where it was detected the photon emitted from the initial state; in this case the event signature was a single photon with energy around 1 GeV.







Z partial widths



N, with the "indirect" method

☐ The invisible width is obtained as a difference between the total width and the "visible" width.

$$\Gamma_{\mathrm{inv}} = \Gamma_{\mathrm{Z}} - \Gamma_{\mathrm{had}} - 3 \, \Gamma_{\ell}$$
.

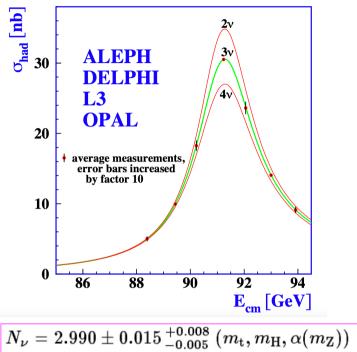
☐ Using the average LEP results we obtain:

$$\Gamma_{\rm inv} = 499.9 \pm 2.5 \ {
m MeV}$$

- ☐ If the Z would decay in any new particles not interacting in the detector, their contribution would enter in the invisible width.
- ☐ In order to get the number of neutrino families we have to use the neutrino partial width predicted by the SM.
- ☐ To minimize systematic errors we use ratios:

$$N_{
u} = rac{\Gamma_{
m inv}}{\Gamma_{
u}} = rac{\Gamma_{
m inv}}{\Gamma_{\ell}} \left(rac{\Gamma_{\ell}}{\Gamma_{
u}}
ight)^{
m SM}$$

$$\left(rac{\Gamma_{\ell}}{\Gamma_{
u}}
ight)^{
m SM} \;\; = \;\; 0.5021 \, {}^{+0.0012}_{-0.0008} \left(m_{
m t}, m_{
m H}, lpha(m_{
m Z})
ight)$$



$$N_{
u} = 2.990 \pm 0.015 ^{+0.008}_{-0.005} \ (m_{
m t}, m_{
m H}, lpha(m_{
m Z}))$$

Latest EW group result:
$$N_v = 2.9841 \pm 0.0083$$

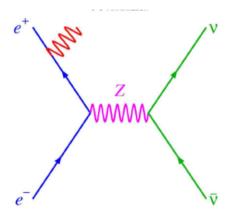
 1.9σ away from 3

2020: new result after an update of the Bhabha cross section used in the luminosity measurement

$$N_{v} = 2.9963 \pm 0.0074$$

N_v with the "direct" method

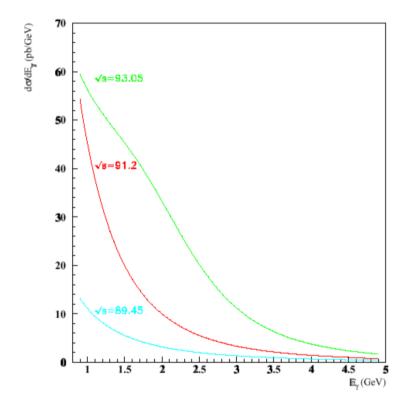
- \square N_{ν} is measured very precisely with the "indirect" method, so ... why do we need another measurement, with a bigger error?
- ☐ A direct measurement, with different systematic errors, is necessary to confirm the indirect measurement and, in case the result would have been different from 3, to understand the origin of the discrepancy.
- \Box For instance, if the Z would decays in new "visible" particles not taken properly into account in Γ_{hadr} and/or in Γ_{lept} , they would appear in Γ_{inv} with the indirect method, but maybe not in the direct measurement.
- ☐ The direct method exploits the emission of a photon from the initial legs:



- We have only a photon in the detector and nothing else, so we have to be sure that no other particle is present.
- The energy of the photon is very little (around 1 GeV) at the Z-pole
- It is difficult to trigger on such a low energy photon.
- The signal cross-section is very low (around 30 pb)

N_v: cross-section and experimental issues

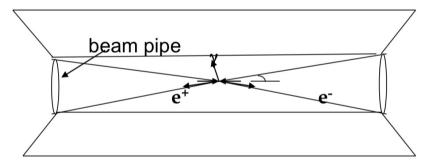
☐ The cross section falls rapidly as a function of the photon energy and depends on the theta angle of emission.



☐ Experimental background:

>
$$e^+e^- \longrightarrow \gamma \gamma \gamma$$
 with $X = I^+I^-\gamma$, π^0 , η , η' ...
> $e^+e^- \longrightarrow e^+e^- X$ $\longrightarrow \gamma \gamma$
> $e^+e^- \longrightarrow e^+e^- \gamma$

Radiative Bhabha scattering



☐ The amount of background depends on the ability to detect particles at small angle and on the overall hermeticity of the electromagnetic calorimeter.

N_v: selection cuts (L3 experiment)

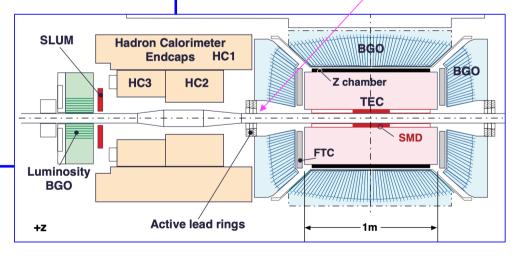
☐ Photon selection

- 1) an energy deposit in the BGO greater than 1 GeV and less than 10 GeV, at a polar angle between 45° and 135°, shared amongst at least five crystals;
- 2) the lateral shape of the energy deposit must be consistent with that expected from a single electromagnetic particle originating from the interaction point.

☐ Veto cuts to make sure the detector is "empty"

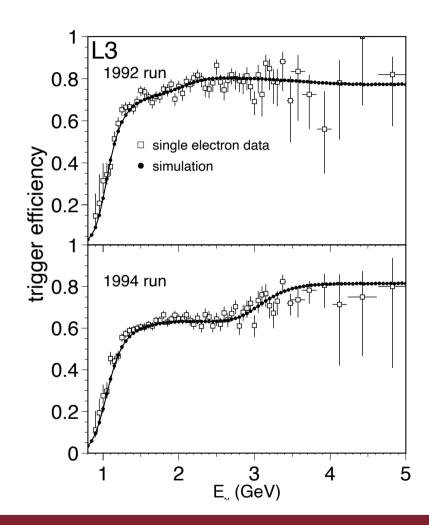
- 3) no other energy deposits in the BGO, consisting of 3 or more contiguous crystals and exceeding a total energy of 100 MeV;
- 4) no tracks in the central tracking chamber (TEC);
- 5) less than 1.5 GeV deposited in either luminosity monitor;
- 6) no signal in the ALR;
- 7) less than 3 GeV deposited in the HCAL;
- 8) no tracks measured in the muon spectrometer.

There is a little "hole" between lumi monitor and ALR



N_v: trigger efficiency (L3 experiment)

- ☐ Given the rapidly falling photon spectrum, the L1 trigger should go down in energy as far as possible.
- ☐ The L1 trigger had 1 GeV threshold.
- ☐ The trigger efficiency was determined in two ways: from data and from a detailed simulation of the single photon trigger.
 - The first method uses a sample of radiative Bhabha events with an isolated electron in the BGO barrel (the single electron control sample), which is triggered by requiring the coincidence of a charged track and an energy exceeding 30 GeV in one of the luminosity monitor.
 - > The second one uses unbiased triggers (the so called beam gate) as input of a dedicated simulation program.
- ☐ The agreement of the simulation with the single electron data at the level of 1%, justifies the uses of the simulated curve also for periods with limited statistics.



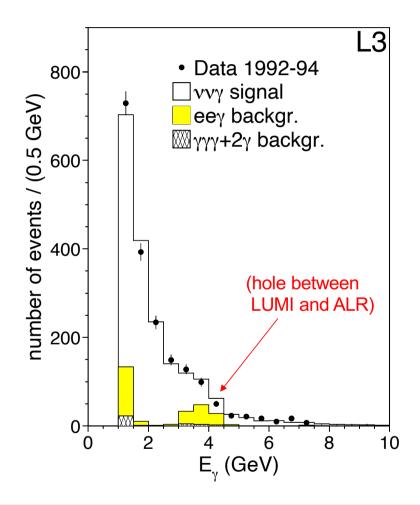
N_v: event sample (L3 experiment)

Selected data sample and expected number of events

Year	$\sqrt{s} \; (\text{GeV})$	$\int \mathcal{L} dt$	Observed	Expected events			
		(pb^{-1})	events	$N_{\nu\bar{\nu}\gamma}$	$N_{e^+e^-\gamma}$	Nother back.	Total MC
1991	88.56-93.75	9.57	202	169.6	25.0	4.9	199.5
1992	91.34	20.52	456	381.3	60.1	9.0	450.4
1993	91.32	4.12	99	74.8	10.8	2.0	87.6
1993	89.45	8.25	77	46.5	19.6	3.9	70.0
1993	91.21	9.25	180	152.4	23.6	4.3	180.3
1993	93.04	8.30	375	370.7	20.9	3.8	395.4
1994	91.22	39.88	702	596.1	93.8	16.9	706.8
Total		99.89	2091	1791.4	253.8	44.8	2090.0

Total efficiency and corrected cross-section

Year	$\sqrt{s} \; (\text{GeV})$	Efficiency	σ (pb)
1992	91.34	0.572	$32.9 \pm 1.8(\text{stat}) \pm 0.6(\text{sys})$
1993	91.32	0.594	$35.2 \pm 4.1(\text{stat}) \pm 0.6(\text{sys})$
1993	89.45	0.578	$11.2 \pm 1.8(\text{stat}) \pm 0.3(\text{sys})$
1993	91.21	0.570	$28.8 \pm 2.5(\text{stat}) \pm 0.5(\text{sys})$
1993	93.04	0.602	$70.1 \pm 3.9(\text{stat}) \pm 1.1(\text{sys})$
1994	91.22	0.505	$29.4 \pm 1.3(\text{stat}) \pm 0.5(\text{sys})$



N_v: results (L3 experiment)

☐ Invisible width:

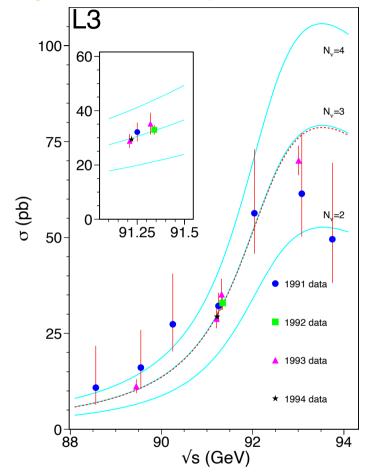
$$\Gamma_{\rm inv} = 498 \pm 12 \; ({\rm stat}) \pm 12 ({\rm sys}) \; {\rm MeV}.$$

☐ Number of neutrino families:

$$N_{\nu} = 2.98 \pm 0.07 \text{(stat)} \pm 0.07 \text{(sys)}.$$

- ☐ This result is compatible (and support) the one found with the indirect method.
- **☐** Breakdown of systematic errors:

Systematic error source	$\Delta\Gamma_{\rm inv}$ (Mev)	ΔN_{ν}
Trigger efficiency	8.4	0.050
Background subtraction	4.8	0.029
Selection efficiency	4.0	0.024
Energy scale	4.0	0.024
Monte Carlo generators	3.5	0.021
Cosmic ray background	1.7	0.010
Luminosity error	1.8	0.011
$\Gamma_{\nu\nu}$ theoretical error		0.004
Fit procedure	2.5	0.015
Total error	12.3	0.073



This is the line shape. The peak of the resonance is much higher than M_z due to the very energetic photons emitted by the initial legs



End of chapter 7