Collider Particle Physics
- Chapter 3 -

A brief overview of the Standard Model

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last update : 070117

Chapter Summary

- Gauge theories
- Goldstone theorem
- Higgs mechanism
- Gauge bosons mass and Weinberg angle
- fermions mass
- W coupling and weak charged current
- Z coupling and weak neutral current
- Feynman vertex in the SM
- QCD Lagrangian
- Running coupling constants

Gauge theories

- **Global gauge invariance:** $\Psi(x) \to \Psi'(x) = e^{iQ\Lambda} \cdot \Psi(x)$ ➡ Charge Q is conserved

- **Let's do a transformation where Λ is a function of the space-time point x:**

$$\Lambda = \Lambda(x)$$

$$\Psi(x) \to \Psi'(x) = e^{iq\Lambda(x)} \cdot \Psi(x)$$
$$\bar{\Psi}(x) \to \bar{\Psi}'(x) = e^{-iq\Lambda(x)} \cdot \bar{\Psi}(x)$$

- **Dirac Lagrangian of a free particle:** $L = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi - m\bar{\Psi}\Psi$

- **This Lagrangian is not invariant for a local gauge transformation:**
  - **mass term:** $m\bar{\Psi}\Psi = m\bar{\Psi}(x) \cdot e^{-iq\Lambda(x)} \cdot e^{iq\Lambda(x)}\Psi(x) = m\bar{\Psi}\Psi$ ➡ OK
  - **Kinetic term:**

$$\partial_\mu \Psi \to \partial_\mu \Psi' = \partial_\mu \left(e^{iq\Lambda(x)} \cdot \Psi(x)\right) =$$
$$= e^{iq\Lambda(x)} \cdot \partial_\mu \Psi(x) + iq \cdot e^{iq\Lambda(x)}\Psi(x) \cdot \partial_\mu \Lambda(x)$$

➡ $\partial_\mu \Psi \neq \partial_\mu \Psi'$ Local gauge invariance is not preserved

Covariant derivative

- To preserve the local invariance we introduce the covariant derivative:

$$D_\mu \equiv \partial_\mu + iqA_\mu(x)$$ (minimal substitution)

- A_μ is a vector field (the photon field) which, under the gauge transformation, becomes:

$$A_\mu(x) \rightarrow A_\mu(x) - \partial_\mu \Lambda(x)$$

- The covariant derivative is invariant under a gauge transformation: $D_\mu \Psi \rightarrow D_\mu \Psi' = e^{iq\Lambda(x)} D_\mu \Psi$

- **Proof:** $D_\mu \Psi = \left(\partial_\mu + iqA_\mu(x)\right)\Psi(x) \rightarrow \left(\partial_\mu + iqA_\mu(x) - iq\partial_\mu \Lambda(x)\right) e^{iq\Lambda(x)}\Psi(x) =$

$$= e^{iq\Lambda(x)}\partial_\mu \Psi(x) + \cancel{iq\partial_\mu \Lambda(x) \cdot e^{iq\Lambda(x)}\Psi(x)} +$$
$$+ iqA_\mu(x) \cdot e^{iq\Lambda(x)}\Psi(x) - \cancel{iq\partial_\mu \Lambda(x) \cdot e^{iq\Lambda(x)}\Psi(x)} =$$

$$= e^{iq\Lambda(x)}\left(\partial_\mu + iqA_\mu(x)\right)\Psi(x) = e^{iq\Lambda(x)} D_\mu \Psi(x)$$

QED Lagrangian

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu D_\mu \Psi - m\bar{\Psi}\Psi$$

- This is invariant for a local gauge transformation:

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu\left(\partial_\mu + iqA_\mu\right)\Psi - m\bar{\Psi}\Psi = $$
$$= i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi - qA_\mu\bar{\Psi}\gamma^\mu\Psi = \mathcal{L}_{free} - J^\mu A_\mu$$

e.m. interaction

$\left(\begin{array}{l} J^\mu = q\bar{\Psi}\gamma^\mu\Psi : \\ \text{corrente e.m.} \end{array}\right)$

- For completeness we have to add to the Lagrangian the kinetc term for A_μ:

$$\mathcal{L}_{free}(fotone) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad \left[F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\right]$$

- If the photon were massive, we should add to the Lagrangian a mass term like this one:

$$\frac{1}{2}m_\gamma^2 A_\mu A^\mu$$

which would violate the local gauge invariance.

$$A_\mu A^\mu \rightarrow \left(A_\mu - \partial_\mu\Lambda\right)\left(A^\mu - \partial^\mu\Lambda\right) \neq A_\mu A^\mu$$

SU(2) symmetry and Yang-Mills field

- **Let's take the following doublet:** $\Psi \equiv \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$ Ψ_1 e Ψ_2 Dirac spinors

- **We can write the Lagrangian as:** $L = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi$ $\bar{\Psi} \equiv \begin{pmatrix} \bar{\Psi}_1 & \bar{\Psi}_2 \end{pmatrix}$

- **Let the Lagrangian be invariant for a (infinitesimal) logal gauge transformation:**

$\Psi(x) \to \left[1 - ig\vec{\Lambda}(x)\cdot\vec{I}\right]\Psi(x)$ $\vec{I} = \left(I_1\ ,\ I_2\ ,\ I_3\right)$ Isospin operators $\left[I_i\ ,\ I_j\right] = \varepsilon_{1jk} I_k$

- **Let's introduce the covariant derivative:** $D_\mu \equiv \partial_\mu + ig\vec{I}\cdot\vec{W}_\mu(x)$ (g=coupling constant)

- **The vector fields W_μ transform as:** $\vec{W}_\mu(x) \to \vec{W}_\mu(x) - \partial_\mu\vec{\Lambda}(x) + g\vec{\Lambda}(x)\times\vec{W}_\mu(x)$

- **The kinetic term is:** $L_{free}(W) = -\frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu}$ $\left[\vec{W}_{\mu\nu} = \partial_\mu\vec{W}_\nu - \partial_\nu\vec{W}_\mu - g\vec{W}_\mu\times\vec{W}_\nu\right]$

- **Also here the W must be massless**

Gauge bosons self-coupling

Glashow-Weinberg-Salam Model

- In the SM the particles are classified as: Weak Isospin doublet → $\begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L \; ; \; e^-_R \; ; \; \begin{pmatrix}u \\ d'\end{pmatrix}_L \; ; \; u_R \; ; \; d'_R$

- Glashow introduced the weak hypercharge: $Q = I_3 + \frac{1}{2}Y$

- The weak isospin doublet can be rotated in the space $SU(2)_L$ and the Lagrangian must stay unchanged.
- Moreover the Lagrangian must be invariant under $U(1)_Y$ transformation.

	I	I_3	Q	Y
ν_e	$\frac{1}{2}$	$\frac{1}{2}$	0	-1
e^-_L	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
e^-_R	0	0	-1	-2
u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$
d'_L	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$
u_R	0	0	$\frac{2}{3}$	$\frac{4}{3}$
d'_R	0	0	$-\frac{1}{3}$	$-\frac{2}{3}$

Symmetry Group of the Model ⇒ $SU(2)_L \otimes U(1)_Y$

- Free Lagrangian of the Model: $\mathcal{L}_{free} = i\bar{\Psi}_L \gamma^\mu \partial_\mu \Psi_L + i\bar{\Psi}_R \gamma^\mu \partial_\mu \Psi_R$

- Infinitesimal gauge transformations:

$SU(2)_L$

$$\Psi_L(x) \to \left[1 - ig\vec{\Lambda}(x)\cdot\vec{I}\,\right]\Psi_L(x)$$
$$\Psi_R(x) \to \Psi_R(x)$$

$U(1)_Y$

$$\Psi_L(x) \to \left[1 - i\frac{g'}{2}\lambda(x)\cdot Y\right]\Psi_L(x)$$
$$\Psi_R(x) \to \left[1 - i\frac{g'}{2}\lambda(x)\cdot Y\right]\Psi_R(x)$$

$\vec{\Lambda}(x)$: vector in the weak isospin space

GWS Lagrangian

- **Covariant derivative:**

$$D_\mu \equiv \partial_\mu + ig\vec{I}\cdot\vec{W}_\mu(x) + i\frac{g'}{2}\mathrm{Y}\cdot \mathrm{B}_\mu$$

- **gauge bosons must transform accordingly:**

$SU(2)_L$

$$\vec{W}_\mu \to \vec{W}_\mu + \partial_\mu \vec{\Lambda}(x) + g\vec{\Lambda}(x)\times\vec{W}_\mu$$
$$B_\mu \to B_\mu$$

$U(1)_Y$

$$\vec{W}_\mu \to \vec{W}_\mu$$
$$B_\mu \to B_\mu + \partial_\mu\lambda(x)$$

- **kinetic term of the vector boson:**

$$L_{free}(\vec{W},B) = -\frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}\cdot B^{\mu\nu}$$

- **The complete Lagrangian is:**

$$L = \bar{\Psi}_L\gamma^\mu\left[i\partial_\mu - g\vec{I}\cdot\vec{W}_\mu(x) - \frac{g'}{2}Y\cdot B_\mu\right]\Psi_L + \bar{\Psi}_R\gamma^\mu\left[i\partial_\mu - \frac{g'}{2}Y\cdot B_\mu\right]\Psi_R + L_{free}(\vec{W},B)$$

N.B. we don't have mass terms for the gauge bosons because they break the local gauge symmetry

We don't have $m\bar{\Psi}\Psi$ because $\bar{\Psi}\Psi = \bar{\Psi}_R\Psi_L + \bar{\Psi}_L\Psi_R$

$\lambda\varphi^4$ Lagrangian

- **Scalar field Lagrangian:** $$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{1}{2}m^2\varphi^2 \Rightarrow \partial_\mu\partial^\mu\varphi + m^2\varphi = 0$$ [Eq. of motion]

  (spin 0 particle of mass m)

- **Let's add "something" to the Lagrangian:** $$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{1}{2}\mu^2\varphi^2 - \frac{1}{4}\lambda\varphi^4$$

  μ and λ are constant, with $\mu^2 < 0$; $\lambda > 0$

  N.B. if $\mu^2 < 0$, then $-\frac{1}{2}\mu^2\varphi^2$ can not be the mass term

- **to note: the Lagrangian has reflection symmetry ($\varphi \rightarrow -\varphi$):**

to note: The calculation of scattering amplitudes with the technique of Feynman diagrams is a perturbative method where the fields are treated as fluctuations around a state of minimum energy: the ground state (vacuum, $\varphi = 0$).

In the present case $\varphi = 0$ is not the ground state.

Spontaneous breaking of a discrete symmetry

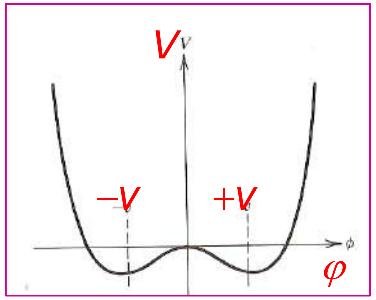
- We consider the Lagrangian as a kinetic term $T = \frac{1}{2}\left(\partial_\mu \varphi\right)\left(\partial^\mu \varphi\right)$ minus a potential energy term V equal to:

$$V(\varphi) = \frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4$$

- The minimum of the potential correspond to: $\frac{\partial V}{\partial \varphi} = \varphi\left(\mu^2 + \lambda\varphi^2\right) = 0$

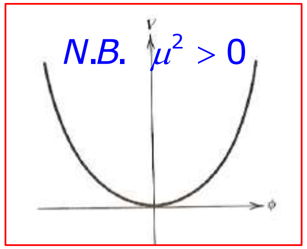
$$\varphi = 0;$$

$$\varphi = \pm v \quad ; \quad v = \sqrt{-\frac{\mu^2}{\lambda}}$$



We choose the minimum φ = v as the ground state and introduce the field $\chi(x)$

$$\varphi(x) = v + \chi(x)$$



$$L = \frac{1}{2}\left(\partial_\mu \chi\right)\left(\partial^\mu \chi\right) - \lambda v^2 \chi^2 - \lambda v \chi^3 - \frac{1}{4}\lambda\chi^4 + \frac{1}{4}\lambda v^4$$

mass term self-interaction it is a constant

$$m_\chi = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$$

N.B. One could also choose to develop φ (x) around -v

Although the Lagrangian has reflection symmetry, the ground state does not have this symmetry, and when we choose one we break the symmetry. **This is the spontaneous symmetry breaking.**

Spontaneous breaking of a continuous symmetry

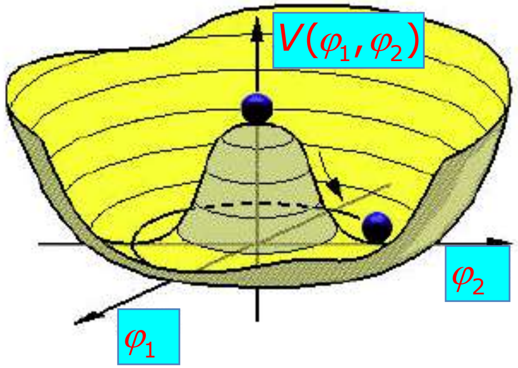
- **Complex scalar field:** $\varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$

$$\mathcal{L} = \left(\partial_\mu \varphi\right)^* \left(\partial^\mu \varphi\right) - \mu^2 \varphi^* \varphi - \lambda \left(\varphi^* \varphi\right)^2$$

$$\Rightarrow \mathcal{L} = \frac{1}{2}\left(\partial_\mu \varphi_1\right)^2 + \frac{1}{2}\left(\partial_\mu \varphi_2\right)^2 - \frac{1}{2}\mu^2\left(\varphi_1^2 + \varphi_2^2\right) - \frac{1}{4}\lambda\left(\varphi_1^2 + \varphi_2^2\right)^2$$

$$V(\varphi_1, \varphi_2) = \frac{1}{2}\mu^2\left(\varphi_1^2 + \varphi_2^2\right) + \frac{1}{4}\lambda\left(\varphi_1^2 + \varphi_2^2\right)^2$$

- **The Lagrangian is invariant under U(1):** $\varphi \to \varphi' = e^{i\alpha}\varphi$



- **The minimum condition occurs on the circle:** $\varphi_1^2 + \varphi_2^2 = v^2 \quad ; \quad v = \sqrt{-\frac{\mu^2}{\lambda}}$

- **We choose the following minimum around which do the perturbative expansion:** $\varphi_1 = v \; ; \; \varphi_2 = 0$

$$\begin{aligned} \varphi_1(x) &= v + \chi_1(x) \\ \varphi_2(x) &= \chi_2(x) \end{aligned} \quad \Rightarrow \quad \varphi(x) = \frac{1}{\sqrt{2}}\left(v + \chi_1(x) + i\chi_2(x)\right)$$

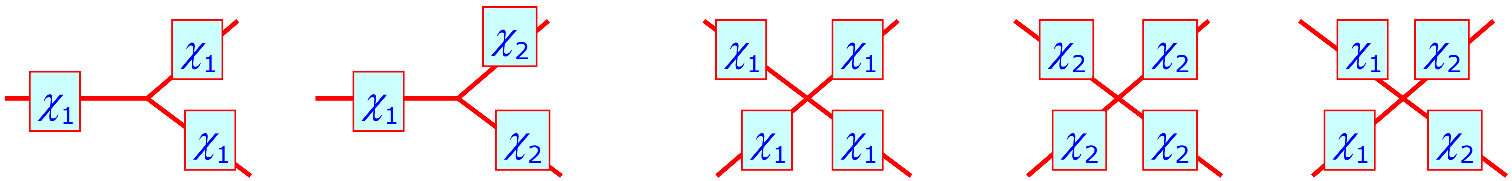
Goldstone theorem

- After the choice of the minimum, the Lagrangian becomes:

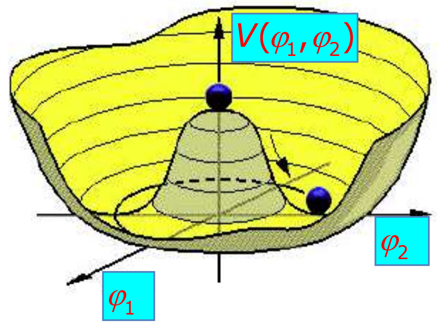
$$L = \left[\frac{1}{2}\left(\partial_\mu \chi_1\right)\left(\partial^\mu \chi_1\right) - \lambda v^2 \chi_1^2\right] + \boxed{\left[\frac{1}{2}\left(\partial_\mu \chi_2\right)\left(\partial^\mu \chi_2\right)\right]} - \boxed{\left[\lambda v\left(\chi_1^3 + \chi_1 \chi_2^2\right) + \frac{1}{4}\lambda\left(\chi_1^4 + \chi_2^4 + 2\chi_1^2\chi_2^2\right)\right]} + \frac{1}{4}\lambda v^4$$

$$m_{\chi_1} = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} > 0$$
$$m_{\chi_2} = 0$$

- The third term represents self-interactions:



- The second term represents a scalar field with zero mass (**Goldstone boson**):
- You can "move" along the minimum without "wasting" energy.



Goldstone's theorem: the spontaneous breaking of a continuous symmetry generates one (or more) scalar bosons with zero mass.

Brout-Englert-Higgs mechanism

- The Higgs mechanism (for short) corresponds to the spontaneous symmetry breaking of a Lagrangian which is invariant under a local gauge transformation.

$\Rightarrow$ Goldstone's theorem + gauge bosons

- Let us consider the Lagrangian $\lambda\phi^4$ with the covariant derivative:

$$\mathcal{L} = \left(\partial_\mu - iqA_\mu\right)\varphi^*\left(\partial^\mu + iqA^\mu\right)\varphi - \mu^2\varphi^*\varphi - \lambda\left(\varphi^*\varphi\right)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\varphi = \frac{1}{\sqrt{2}}\left(\varphi_1 + i\varphi_2\right)$$

- which is invariant under the gauge transformation U (1): $\varphi(x) \rightarrow \varphi'(x) = e^{iq\Lambda(x)} \cdot \varphi(x)$

- If $\mu^2 < 0$ the field φ must be developed around a minimum different from $\varphi = 0$, for example:

$$\begin{aligned} \varphi_1(x) &= v + \chi_1(x) \\ \varphi_2(x) &= \chi_2(x) \end{aligned} \quad \Rightarrow \quad \varphi(x) = \frac{1}{\sqrt{2}}\left(v + \chi_1 + i\chi_2\right)$$

Actually the mechanism could be also called ***Englert–Brout–Higgs–Guralnik–Hagen–Kibble mechanism*** *since there were three independent papers in 1964. In what follows we will call it Higgs mechanism for short.*

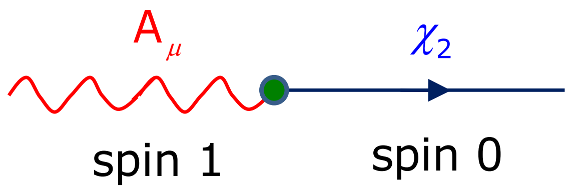
The Higgs mechanism

- The Lagrangian becomes:

$$L = \left[\frac{1}{2}(\partial_\mu \chi_1)(\partial^\mu \chi_1) - \lambda v^2 \chi_1^2\right] + \left[\frac{1}{2}(\partial_\mu \chi_2)(\partial^\mu \chi_2)\right] + $$
$$+\frac{1}{2} q^2 v^2 A_\mu A^\nu - q v A_\mu \partial^\mu \chi_2 + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \text{ interaction terms}$$

- Let's analyze the Lagrangian:
  - scalar field χ_1 with mass $m_{\chi_1} = \sqrt{2\lambda v^2}$
  - a massless Goldstone boson χ_2
  - the gauge boson A_μ has got a mass term $m_A = qv$

- However, the term $A_\mu \partial^\mu \chi_2$, which seems to allow the gauge boson A_μ to transform into χ_2 as it propagates, casts doubt on this interpretation:



Degree of freedom of the Lagrangian

- **Before spontaneous symmetry breaking:**
  - **2 real scalar fields ϕ1 and ϕ2,**
  - **2 helicity states of Aμ (spin 1, zero mass)**

  → **4 degree of freedom .**

- **After spontaneous symmetry breaking:**
  - **2 real scalar fields ϕ1 and ϕ2,**
  - **3 helicity states of Aμ (spin 1, with mass)**

  → **5 degree of freedom .**

IT DOESN'T WORK.

To find the way out of this problem, it must be remembered that it is always possible to do a local gauge transformation

Local gauge transformation

- Let's change the parameterization of φ(x) using the "module" and the "phase":

$$\varphi(x) = \frac{1}{\sqrt{2}}\left[v + H(x)\right]e^{i\frac{\theta(x)}{v}}$$

$H(x)$ and $\theta(x)$ are real fields

- We make a gauge transformation in order to eliminate the field θ(x):

$$\varphi'(x) = e^{iq\Lambda(x)}\varphi(x)$$
$$\Rightarrow \left[v + H'(x)\right]e^{i\frac{\theta'(x)}{v}} = e^{iq\Lambda(x)}\left[v + H(x)\right]e^{i\frac{\theta(x)}{v}}$$

$$H'(x) = H(x)$$
$$\theta'(x) = \theta(x) + qv \cdot \Lambda(x)$$

- If we choose: $\Lambda(x) = -\frac{1}{qv}\theta(x)$ $\Rightarrow$ $\theta'(x) = 0$ (unitarity gauge)

$$\varphi'(x) = \frac{1}{\sqrt{2}}\left[v + H(x)\right]$$

- The Goldstone boson connects the various vacuum states that are degenerate in energy. With the gauge transformation we have "removed" this unwanted degree of freedom and the field ϕ has become real.

With the new parameterization the field θ should not appear explicitly in the Lagrangian.

The Higgs boson

- Let's check the degree of freedom of the transformed Lagragian:

- In the new unitarity gauge we have: $\varphi'(x) = \frac{1}{\sqrt{2}}\left[v + H(x)\right]$; $A'_\mu(x) = A_\mu(x) + \frac{1}{qv}\partial_\mu\theta(x)$

$$\Rightarrow \quad L = \left[\frac{1}{2}\left(\partial_\mu H\right)^2 - \lambda v^2 H^2\right] + \frac{1}{2}q^2 v^2 A_\mu A^\nu + \frac{1}{2}q^2 A_\mu A^\nu H^2$$
$$+ q^2 v A_\mu A^\nu H - \lambda v H^3 - \frac{1}{4}\lambda H^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}\lambda v^4$$

- The Lagrangian does not depend on θ as we expected: the Goldstone boson has disappeared. It was "eaten" by the gauge boson which gained weight and gained mass:

- The Lagrangian now describes a scalar boson H (Higgs) and a vector gauge boson A_μ, of mass respectively:

$$\boxed{m_H = \sqrt{2\lambda v^2}} \qquad \boxed{m_A = qv}$$

- The other terms of the Lagrangian describe the interactions between fields and self-interactions:

N.B. this is the Abelian Higgs mechanism, ie valid for a commutative symmetry group.

Higgs mechanism and Yang-Mills fields

- We study the spontaneous symmetry breaking for the (non-Abelian) SU(2) X U(1) group. We start from the following Lagrangian and study SU(2):

$$L = \left(\partial_\mu \varphi\right)^+ \left(\partial^\mu \varphi\right) - \mu^2 \varphi^+ \varphi - \lambda \left(\varphi^+ \varphi\right)^2$$

$$\varphi = \begin{pmatrix} \varphi_a \\ \varphi_b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$$

- The Lagrangian is invariant under a global transformation of SU(2):

$$\varphi(x) \rightarrow \varphi'(x) = e^{i\vec{\Lambda}\cdot\vec{I}} \varphi(x)$$

- In order for it to be so also for a local transformation, the covariant derivative must be introduced:

$$\varphi(x) \rightarrow \varphi'(x) = \left[1 + i\vec{\Lambda}(x)\cdot\vec{I}\right]\varphi(x)$$

$$\Rightarrow \quad D_\mu \equiv \partial_\mu + ig\vec{I}\cdot\vec{W}_\mu(x) \qquad \vec{W}_\mu(x) \rightarrow \vec{W}_\mu(x) - \partial_\mu \vec{\Lambda}(x) + g\vec{\Lambda}(x)\times\vec{W}_\mu(x)$$

- The Lagrangian can be written as:

$$L = \left(\partial_\mu \varphi + ig\vec{I}\cdot\vec{W}_\mu \varphi\right)^+ \left(\partial_\mu \varphi + ig\vec{I}\cdot\vec{W}_\mu \varphi\right) - \left(\mu^2 \varphi^+ \varphi - \lambda \left(\varphi^+ \varphi\right)^2\right) - \frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu}$$

Higgs mechanism and Yang-Mills fields

$$L = \left(\partial_\mu \varphi + ig\vec{I}\cdot\vec{W}_\mu \varphi\right)^+ \left(\partial_\mu \varphi + ig\vec{I}\cdot\vec{W}_\mu \varphi\right) - \left(\mu^2 \varphi^+ \varphi - \lambda\left(\varphi^+\varphi\right)^2\right) - \frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu}$$

- **Let us consider the case $\mu^2 < 0$ and $\lambda > 0$. The minimum of the potential is for:**

$$\varphi^+\varphi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$$

$$\varphi^+\varphi = \left(\varphi_a^* \ \ \varphi_b^*\right)\begin{pmatrix}\varphi_a \\ \varphi_b\end{pmatrix} = \varphi_a^*\varphi_a + \varphi_b^*\varphi_b = \frac{1}{2}\left(\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2\right) = \frac{v^2}{2}$$

- **We choose a minimum thus breaking the symmetry of the ground state:** $\boxed{\varphi_1 = \varphi_2 = \varphi_4 = 0 \ ; \quad \varphi_3 = v^2}$

- **The vacuum ground state we have chosen is:** $\boxed{\varphi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v\end{pmatrix}}$

- **We make the perturbative expansion around this state, choosing an appropriate gauge in order to have:**

$$\boxed{\varphi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v + H(x)\end{pmatrix}}$$

N.B. in this way three scalar fields have been eliminated from the gauge transformation leaving only one field: H(x)

Higgs mechanism and Yang-Mills fields

- **We can rewrite the Lagrangian in terms of the Higgs field H:**

$$L = \left[\frac{1}{2}\left(\partial_\mu H\right)^2 - \lambda v^2 H^2\right] + \frac{g^2 v^2}{8}\left[\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2 + \left(W_\mu^3\right)^2\right]$$

$+$ higher order terms $+$ kinetic term for the $\vec{W}$

- **This Lagrangian describes a mass scalar Higgs field:** $m_H = \sqrt{2\lambda v^2} = \sqrt{\left(-2\mu^2\right)} = ????\ \text{GeV}$

- **and three massive gauge bosons of mass:** $m_W = \frac{1}{2} g v$

- **The three gauge bosons ”swallowed" the three Goldstone fields, gaining mass.**

It is necessary to extend these concepts to the entire SU(2) X U(1) symmetry

SU(2)$_L$ x U(1)$_Y$ and Higgs field

- Electroweak Lagrangian invariant under gauge transformation:

$$L = \bar{\Psi}_L \gamma^\mu \left[i\partial_\mu - g\vec{I}\cdot\vec{W}_\mu(x) - \frac{g'}{2} Y \cdot B_\mu \right] \Psi_L + \bar{\Psi}_R \gamma^\mu \left[i\partial_\mu - \frac{g'}{2} Y \cdot B_\mu \right] \Psi_R + L_{free}(\vec{W}, B)$$

- We introduce four real scalar fields φ_i into the Lagrangian:

$$\mathrm{L} = \left(D_\mu\varphi\right)^+ \left(D^\mu\varphi\right) - \mu^2\varphi^+\varphi - \lambda\left(\varphi^+\varphi\right)^2$$

$$D_\mu \equiv \partial_\mu + ig\vec{I}\cdot\vec{W}_\mu(x) + i\frac{g'}{2}\mathrm{Y}\cdot\mathrm{B}_\mu$$

- We are interested in the case where $\mu^2 < 0$ and $\lambda > 0$.

- We follow Weinberg and arrange the four φ_i fields in a weak isospin doublet with weak hypercharge Y=1

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$$

φ^+ ha carica elettrica Q=1
e φ^0 ha Q=0

- We choose the minimum of the potential such that $\varphi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$ and develop $\varphi(x)$ around this point.

- With an appropriate choice of the gauge we have:

$$\varphi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

SU(2)$_L$ x U(1)$_Y$ and Higgs field

- **The Lagrangian becomes:**

$$L = \left[\frac{1}{2}\left(\partial_\mu H\right)^2 - \lambda v^2 H^2\right] + \frac{g^2 v^2}{8}\left[W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu}\right] + \frac{v^2}{8}\left(gW_\mu^3 - g' B_\mu\right)\left(gW^{3\mu} - g' B^\mu\right)$$

$$+ \text{ higher order terms } + \text{ kinetic terms for the } \vec{W} \text{ and B}$$

- **From here we see that the W_μ^1 and W_μ^2 fields have a "conventional" mass term** $m_W = \frac{1}{2} g v$ **while the W_μ^3 and B_μ fields are mixed.**

- **We need to rotate these two fields so that the mass term is diagonal in the new two fields A_μ and Z_μ:**

$$\frac{v^2}{8}\begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}$$

Mass matrix. It must be diagonalized. One of the two eigenvalues is zero.

$$\frac{v^2}{8}\left(g^2\left(W_\mu^3\right)^2 - 2gg' W_\mu^3 B^\mu + g'^2 B_\mu^2\right) = \frac{v^2}{8}\cdot\left(gW_\mu^3 - g' B_\mu\right)^2 + 0\cdot\left(g' W_\mu^3 + gB_\mu\right)^2$$

$$A_\mu = \frac{\left(g' W_\mu^3 + gB_\mu\right)}{\sqrt{g^2 + g'^2}} \qquad m_A = 0$$

$$Z_\mu = \frac{\left(gW_\mu^3 - g' B_\mu\right)}{\sqrt{g^2 + g'^2}} \qquad m_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

Mass of bosons and Weinberg angle

- **We introduce the Weinberg angle (i.e. Weak angle) defined as:**

$$\frac{g'}{g} = \tan\theta_W \quad ; \quad \frac{g}{\sqrt{g^2 + g'^2}} = \cos\theta_W \quad ; \quad \frac{g'}{\sqrt{g^2 + g'^2}} = \sin\theta_W$$

$$\Rightarrow \quad \begin{aligned} A_\mu &= \cos\theta_W B_\mu + \sin\theta_W W^3_\mu \\ Z_\mu &= -\sin\theta_W B_\mu + \cos\theta_W W^3_\mu \end{aligned}$$

- **Remembering that:** $m_W = \frac{1}{2} g v$ and $m_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$ $\Rightarrow$ $\boxed{\frac{m_W}{m_Z} = \cos\theta_W}$

- **The spontaneous breaking of the $SU(2)_L \times U(1)_Y$ symmetry gave rise to the following mass spectrum:**

1 Higgs boson, $m_H = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$

2 charged boson $W^\pm$, $m_W = \frac{1}{2} g v$

1 neutral boson Z, $m_Z = \frac{m_W}{\cos\theta_W}$

1 massless neutral boson (photon)

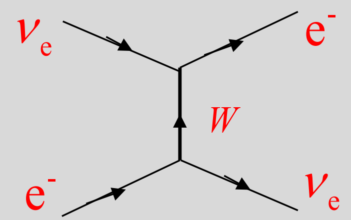
N.B. $Q\varphi_0 = \left(I_3 + \frac{1}{2}Y\right)\begin{pmatrix} 0 \\ v \end{pmatrix} = 0$

The charge of the minimum we have chosen is zero, therefore the symmetry $U(1)^{em}$ is not broken and the photon remains massless

Gauge Bosons Mass

- **From the analysis of the neutrino-electron scattering we obtain the relationship:**

$$\frac{g^2}{8m_W^2} = \frac{G}{\sqrt{2}}$$



$$\mathcal{M} = \left[\frac{g}{\sqrt{2}}\bar{u}_e\gamma^\mu \frac{1-\gamma^5}{2}u_{\nu_e}\right]\frac{1}{M_W^2 - q^2}\left[\frac{g}{\sqrt{2}}\bar{u}_{\nu_e}\gamma_\mu \frac{1-\gamma^5}{2}u_e\right]$$

$$\mathcal{M} = \frac{G}{\sqrt{2}}\left[\bar{u}_p\gamma^\mu(1-\gamma^5)u_n\right]\left[\bar{u}_e\gamma_\mu(1-\gamma^5)u_\nu\right]$$

- **Given that:** $m_W = \frac{1}{2}gv$ $\Rightarrow$ $v = \frac{1}{\sqrt{\left(G\sqrt{2}\right)}}$

- **The vacuum expectation value depends only on the Fermi constant:**

$$v = \frac{1}{\sqrt{\left(G\sqrt{2}\right)}} = \frac{1}{\sqrt{\left(\sqrt{2}\cdot 1.17\cdot 10^{-5}\ \text{GeV}^{-2}\right)}} \approx 246\ \text{GeV}$$

- **As we will see, this relation holds:** $g\cdot\sin\theta_W = e$ $\quad \left[\alpha = \frac{e^2}{4\pi}\right]$

$\Rightarrow$ $m_W = \left(\frac{\pi\alpha}{G\sqrt{2}}\right)^{\frac{1}{2}}\frac{1}{\sin\theta_W}$

$\sin\vartheta_W$ must be determined experimentally. The first measurement was made with the deep inelastic scattering of neutrinos in the 1970s.

$$\sin^2\theta_W \approx 0.23 \quad\Rightarrow\quad m_W \approx 80\ \text{GeV} \ ;\ m_Z \approx 90\ \text{GeV}$$

N.B. The mass of the Higgs boson is not predicted by the Standard Model because it depends on the unknown parameter λ which appears in the potential V(φ).

Fermions mass

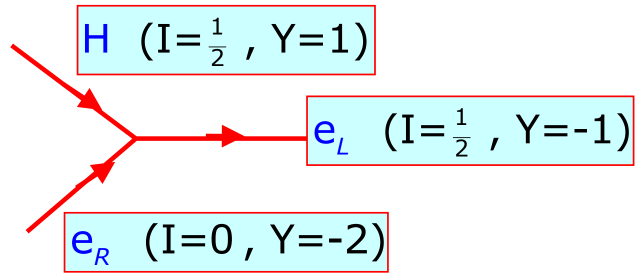
- The fermion mass term -mee cannot be explicitly put in the Lagrangian because it breaks the $SU(2)_L X U(1)_Y$ symmetry (it mixes lefthanded and righthanded components):

$$-m\bar{e}e = -m\bar{e}\left[\frac{1}{2}\left(1-\gamma^5\right)+\frac{1}{2}\left(1+\gamma^5\right)\right]e = -m\left(\bar{e}_R e_L + \bar{e}_L e_R\right)$$

- **We remind that:**

	I	I_3	Y
ν_e	$\frac{1}{2}$	$\frac{1}{2}$	-1
e^-_L	$\frac{1}{2}$	$-\frac{1}{2}$	-1
e^-_R	0	0	-2

$$\begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L \qquad e^-_R$$



The Higgs boson has the right quantum numbers to couple to e_L and e_R.

- **We add to the Lagrangian the ("Yukawa") term invariant under gauge transformations:**

$$\mathrm{L} = -g_e\left[\bar{L}\varphi e_R + \bar{e}_R\bar{\varphi}L\right]$$

where

$$L = \begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L \quad ; \quad \varphi = \begin{pmatrix}\varphi^+ \\ \varphi^0\end{pmatrix}$$

$$\left(\bar{\nu}_e \quad \bar{e}^-_L\right)\begin{pmatrix}\varphi^+ \\ \varphi^0\end{pmatrix}e^-_R = \left(\bar{\nu}_e\varphi^+ e^-_R + \bar{e}^-_L\varphi^0 e^-_R\right)$$

g_e = coupling constant

Fermion mass

- After spontaneous symmetry breaking, the Lagrangian becomes:

$$\varphi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$$\mathcal{L} = -\frac{g_e v}{\sqrt{2}}\left[\bar{e}_R e_L + \bar{e}_L e_R\right] - \frac{g_e}{\sqrt{2}}\left[\bar{e}_R e_L + \bar{e}_L e_R\right]H$$

$$m_e = \frac{g_e \cdot v}{\sqrt{2}}$$

mass term

Coupling of the electron with the Higgs boson

$$\Rightarrow \quad \mathcal{L} = -m_e \bar{e}e - \left(\frac{m_e}{v}\right)\bar{e}eH$$

N.B. The coupling constant is proportional to the mass of the fermion

- To generate the masses of the "up" quarks, a conjugated Higgs doublet is introduced:

$$\tilde{\varphi} = i\sigma_2\varphi^* = \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{\varphi}^0 \\ -\varphi^- \end{pmatrix} \quad \Rightarrow \quad \frac{1}{\sqrt{2}}\begin{pmatrix} v+H \\ 0 \end{pmatrix}$$

$$L = -g_d \bar{L}_q \varphi d_R - g_u \bar{L}_q \tilde{\varphi} u_R \quad + \text{hermitian conjugate}$$

$$L_q = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$\Rightarrow \quad \mathcal{L} = -m_d \bar{d}d - m_u \bar{u}u - \left(\frac{m_d}{v}\right)\bar{d}dH - \left(\frac{m_u}{v}\right)\bar{u}uH$$

Complete Electroweak Lagrangian

- Electroweak Lagrangian invariant under gauge transformation:

$$L = \bar{\Psi}_L \gamma^\mu \left[i\partial_\mu - g\vec{I} \cdot \vec{W}_\mu(x) - \frac{g'}{2} Y \cdot B_\mu \right] \Psi_L + \bar{\Psi}_R \gamma^\mu \left[i\partial_\mu - \frac{g'}{2} Y \cdot B_\mu \right] \Psi_R + L_{free}(\vec{W}, B)$$

- We add to the Lagrangian four real scalar fields φ_i to give mass to the gauge bosons through the mechanism of spontaneous symmetry breaking:

$$\mathrm{L} = \left(D_\mu \varphi\right)^+ \left(D^\mu \varphi\right) - \mu^2 \varphi^+ \varphi - \lambda \left(\varphi^+ \varphi\right)^2$$

- We add to the Lagrangian an interaction between the fermions and the φ field to give mass to the fermions:

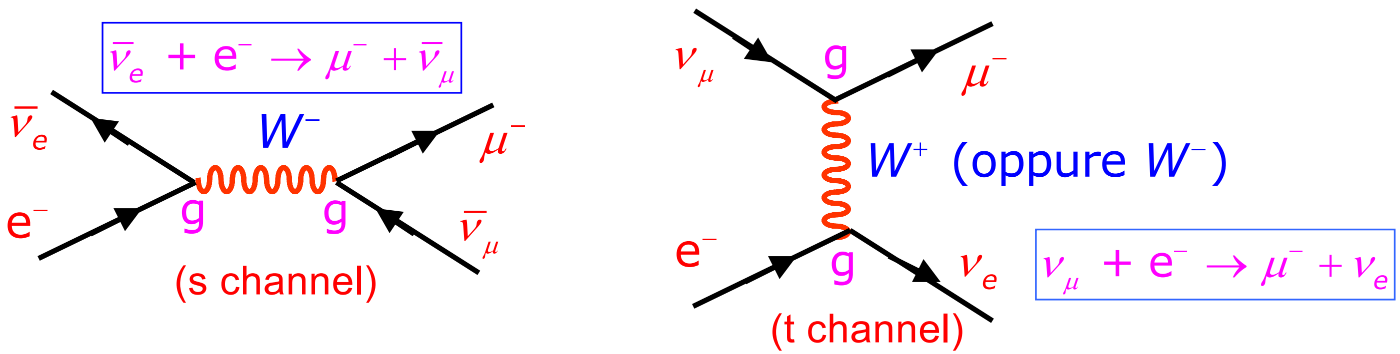
$$L = -g_e \left[\bar{L} \varphi e_R + \bar{e}_R \bar{\varphi} L \right] \qquad L = -g_d \bar{L}_q \varphi d_R - g_u \bar{L}_q \tilde{\varphi} u_R \ + \text{herm. con.}$$

What is this φ field? I DON'T KNOW!

Gauge Bosons Couplings

W coupling

- The (charged) W couples with particles of the doublet producing both of them (channel s) or inducing a transition in the other particle (channel t).



$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$
$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

- N.B. In the s channel the charge of the W boson is unique because the two vertices are temporally separated, while in the t channel they are not (the time order product automatically takes this into account), so you can have the exchange of a W^+ or of W^-. For the purposes of the calculation, the thing is perfectly analogous.

- **The W is coupled to a charged current because there is a transition between the two states of the weak isospin doublet, the electric charge of which differs by one.**

- The matrix element can be written as:

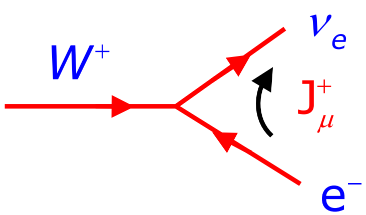
$$M = \frac{g}{\sqrt{2}}\left(J^\mu\right)^+ \frac{1}{M_W^2 - q^2}\frac{g}{\sqrt{2}}\left(J^\mu\right)$$

Weak Charged Current

- Charge-raising weak current of electrons and quarks:

$$J_e^\mu = \bar{u}(\nu)\gamma^\mu \frac{1-\gamma^5}{2} u(e) \qquad J_q^\mu = \bar{u}(u)\gamma^\mu \frac{1-\gamma^5}{2} u(d')$$

- As we can see, the charge-raising weak current has the form:



$$J_\mu^+ = \bar{u}(\nu_e)\gamma_\mu \frac{1-\gamma^5}{2} u(e)$$

- The operator ½ $(1-\gamma^5)$ is the projector of the left-handed chiral state for particles and of the right-handed chiral state for antiparticles, which coincide with the states having negative and positive helicity for particles of zero mass:

$$\frac{1-\gamma^5}{2} u \equiv u_L \quad ; \quad \bar{u}_L = \bar{u}\frac{1+\gamma^5}{2} \quad ; \quad \frac{1-\gamma^5}{2} v \equiv v_R \quad ; \quad \bar{v}_R = \bar{v}\frac{1+\gamma^5}{2}$$

- We also remind you that :

$$\gamma_\mu \frac{1-\gamma^5}{2} = \frac{1+\gamma^5}{2}\gamma_\mu \frac{1-\gamma^5}{2}$$

Weak Charged Current

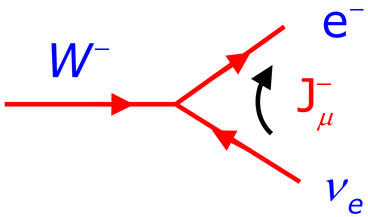
- **The charge-raising weak current can also be written as:**

$$J_\mu^+ = \bar{\nu}\,\frac{1+\gamma^5}{2}\,\gamma_\mu\,\frac{1-\gamma^5}{2}\,e = \bar{\nu}_L \gamma_\mu e_L$$

N.B. Indichiamo con $\bar{\nu}$ ed e gli spinori.

we have thus obtained a purely vector current which only couples to the left-handed components of the particles.

- **We now write the charge-lowering weak current:**



$$J_\mu^- = \bar{e}\gamma_\mu\,\frac{1-\gamma^5}{2}\,\nu = \bar{e}\,\frac{1+\gamma^5}{2}\,\gamma_\mu\,\frac{1-\gamma^5}{2}\,\nu = \bar{e}_L \gamma_\mu \nu_L$$

N.B. we denote the spinors by the name of the particle without distinguishing between u and v

- **We recall the electromagnetic current:** $J_\mu^{e.m.} = -\bar{e}\gamma_\mu e = -\left(\bar{e}_R \gamma_\mu e_R + \bar{e}_L \gamma_\mu e_L\right)$

A vector current does not mix left-handed and right-handed states

Weak Charged Current

- In a compact way, the two raising and lowering weak charged currents can be written as follows:

$$\chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad ; \quad \bar{\chi}_L = \begin{pmatrix} \bar{\nu}_e & e^- \end{pmatrix} \quad ; \quad \sigma^{\pm} = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$$

$$\sigma^{+} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad ; \quad \sigma^{-} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \Rightarrow J_\mu^{\pm} = \bar{\chi}_L \gamma_\mu \sigma^{\pm} \chi_L$$

- If we now require that the weak interactions be invariant under rotations in the space of the weak isospin, it is necessary to introduce a third current of isospin that conserves the charge:

$$J_\mu^3 = \bar{\chi}_L \gamma_\mu \frac{1}{2} \sigma^3 \chi_L = \frac{1}{2} \bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma_\mu e_L$$

$$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- This current cannot be directly associated with the weak neutral current (exchange of the Z) because J_μ^3 couples only to the left-handed components, while the Z also couples to the right-handed ones.

- To try to solve the problem Glashow proposed to deal simultaneously with electromagnetic interactions (which are described by a neutral current) and weak interactions.

Weak Neutral Current

- In 1961 Glashow suggested the introduction of a weak hypercharge current:

$$J_\mu^Y = \bar{\Psi}\gamma^\mu Y \Psi$$

where the weak hypercharge Y is connected to the third component of the weak isospin through a relationship similar to that of Gell-Mann Nishijima:

$$Q = I_3 + \frac{1}{2}Y \quad \Rightarrow \quad J_\mu^{em} = J_\mu^3 + \frac{1}{2}J_\mu^Y$$

- The e.m. current is a combination of the weak hypercharge current and the third component of the weak isospin current.

- The weak hypercharge Y is the generator of the symmetry of the $U(1)_Y$ group, therefore the unification of the weak interactions and the electromagnetic interactions revealed the existence of a larger symmetry group:

$$SU(2)_L \otimes U(1)_Y$$

Quantum numbers

- The quantum numbers of the first family of particles are:

$$\begin{array}{c} \\ \nu_e \\ e_L^- \\ e_R^- \\ u_L \\ d'_L \\ u_R \\ d'_R \end{array}\begin{pmatrix} I & I_3 & Q & Y \\ \frac{1}{2} & \frac{1}{2} & 0 & -1 \\ \frac{1}{2} & -\frac{1}{2} & -1 & -1 \\ 0 & 0 & -1 & -2 \\ \frac{1}{2} & \frac{1}{2} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{2}{3} & \frac{4}{3} \\ 0 & 0 & -\frac{1}{3} & -\frac{2}{3} \end{pmatrix}$$

$$Y=2\left(Q-I_3\right)$$

N.B. Members of the same doublet have the same hypercharge.

The hypercharge current can be written as:

$$J_\mu^Y=2J_\mu^{em}-2J_\mu^3 =$$

$$= -2\left(\bar{e}_R\gamma_\mu e_R + \bar{e}_L\gamma_\mu e_L\right) - \left(\bar{\nu}_L\gamma_\mu\nu_L - \bar{e}_L\gamma_\mu e_L\right) =$$

$$= \underbrace{-2}_{\text{hypercharge}} \cdot \left(\bar{e}_R\gamma_\mu e_R\right) \underbrace{-1}_{\text{hypercharge}} \cdot \left(\bar{\chi}_L\gamma_\mu\chi_L\right)$$

- For quarks we have:

$$J_\mu^Y = \frac{4}{3}\cdot\left(\bar{u}_R\gamma_\mu u_R\right) - \frac{2}{3}\cdot\left(\bar{d}'_R\gamma_\mu d'_R\right) + \frac{1}{3}\left(\bar{u}_L\gamma_\mu u_L + \bar{d}'_L\gamma_\mu d'_L\right)$$

The interaction in the Standard Model

- **To preserve the gauge invariance of the $SU(2)_L$ x $U(1)_Y$ symmetry of the GWS model, it is necessary to introduce 3 vector bosons W associated with the weak isospin and a vector boson B associated with hypercharge.**

- **The interaction has the form:** $$-i\left(g\vec{J}_\mu \cdot \vec{W}^\mu + \frac{1}{2} g' J_\mu^Y \cdot B^\mu\right)$$

g and g 'are two coupling constants

$\vec{J}_\mu$ e $\vec{W}_\mu$: vettori nello spazio dell'isospin debole

- **In terms of charged currents** $\vec{J}_\mu^\pm = J_\mu^1 \pm iJ_\mu^2$ **we have:**

$$\vec{J}_\mu \cdot \vec{W}_\mu = J_\mu^1 W^{\mu 1} + J_\mu^2 W^{\mu 2} + J_\mu^3 W^{\mu 3} \Rightarrow \vec{J}_\mu \cdot \vec{W}_\mu = \frac{1}{\sqrt{2}} J_\mu^+ W^{\mu +} + \frac{1}{\sqrt{2}} J_\mu^- W^{\mu -} + J_\mu^3 W^{\mu 3}$$

- **where:** $$W^{\mu\pm} = \frac{1}{\sqrt{2}}\left(W^{\mu 1} \mp iW^{\mu 2}\right)$$

$W^{\mu\pm}$ descrivono bosoni carichi massivi $W^\pm$, mentre $W^{\mu 3}$ e B^μ sono campi neutri

The interaction in the Standard Model

- In the GWS model the $SU(2)_L \times U(1)_Y$ symmetry is "broken" and the neutral fields mix to give rise to a massless combination (the photon) and a massive combination (the Z)

$$\begin{aligned} A_\mu &= \cos\theta_W B_\mu + \sin\theta_W W_\mu^3 \\ Z_\mu &= -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3 \end{aligned} \quad \Longleftrightarrow \quad \begin{aligned} W_\mu^3 &= \sin\theta_W A_\mu + \cos\theta_W Z_\mu \\ B_\mu &= \cos\theta_W A_\mu - \sin\theta_W Z_\mu \end{aligned}$$

θ_W : angolo di Weinberg (angolo weak)

- In terms of the A_μ and Z_μ fields, the neutral current interaction becomes:

$$-i\left(gJ_\mu^3 \cdot W^{\mu 3} + \frac{1}{2}g' J_\mu^Y \cdot B^\mu\right) = -i\left(g\sin\theta_W J_\mu^3 + g'\cos\theta_W \frac{J_\mu^Y}{2}\right)A^\mu - i\left(g\cos\theta_W J_\mu^3 - g'\sin\theta_W \frac{J_\mu^Y}{2}\right)Z^\mu$$

- The first term can be identified with electromagnetic interaction:

$$-ieJ_\mu^{em} \cdot A^\mu$$

We also remind you that:

$$J_\mu^{em} = J_\mu^3 + \frac{1}{2}J_\mu^Y$$

- The two expressions are consistent if:

$$g\sin\theta_W = g'\cos\theta_W = e \quad \Rightarrow \quad e = \frac{gg'}{\sqrt{g'^2 + g^2}}$$

Weinberg Angle

- The weak mixing angle directly depends on the coupling constants of $SU(2)_L \times U(1)_Y$

$$g \sin\theta_W = g' \cos\theta_W = e \quad \Rightarrow \quad \tan\theta_W = \frac{g'}{g}$$

- The GWS model does not predict the value of θ_W to be measured.
- Of course, for the model to be valid, all electroweak phenomena must be described from a single angle θ_W.
- Many of the experimental tests of the model consisted of measuring the angle θ_W and comparing these values.

But … BE CAREFULL

- There are two definitions of the Weinberg angle:

$$\text{masse:} \quad m_W = m_Z \cos\theta_W \qquad \text{accoppiamenti:} \quad g \sin\theta_W = g' \cos\theta_W = e$$

- At the “tree” level (fundamental level) the two definitions coincide, but the radiative corrections modify the two expressions in a different way, therefore it is necessary to specify the renormalization scheme adopted. (This caused a few additional minor problems in Lep's time).

Z interaction: neutral current

- Let's go back to the interaction term of the Z:

$$\boxed{-i\left(g\cos\theta_W J_\mu^3 - g'\sin\theta_W \frac{J_\mu^Y}{2}\right)Z^\mu}$$ furthermore $$\boxed{J_\mu^Y = 2J_\mu^{em} - 2J_\mu^3}$$

$$= -i\left[g\cos\theta_W J_\mu^3 - g'\sin\theta_W\left(J_\mu^{em} - J_\mu^3\right)\right]Z^\mu = -i\left[g\cos\theta_W J_\mu^3 - g'\sin\theta_W J_\mu^{em} + g'\sin\theta_W J_\mu^3\right]Z^\mu =$$

$$= -i\left[g\frac{\cos^2\theta_W}{\cos\theta_W}J_\mu^3 - g\frac{\sin^2\theta_W}{\cos\theta_W}J_\mu^{em} + g\frac{\sin^2\theta_W}{\cos\theta_W}J_\mu^3\right]Z^\mu =$$

$$\boxed{\left\langle \text{ricorda:}\quad g' = g\frac{\sin\theta_W}{\cos\theta_W}\right\rangle}$$

$$= -i\frac{g}{\cos\theta_W}\left[J_\mu^3 - \sin^2\theta_W J_\mu^{em}\right]Z^\mu = -i\frac{g}{\cos\theta_W}J_\mu^{N.C.}Z^\mu$$

- We have obtained a neutral current that couples with the Z:

$$\boxed{J_\mu^{N.C.} = J_\mu^3 - \sin^2\theta_W J_\mu^{em}}$$

It couples to both lefthanded and righthanded (charged) states

It couples to lefthanded states only

- The Z couples to both left-handed and right-handed states. The coupling depends on the quantum numbers of the particles involved.

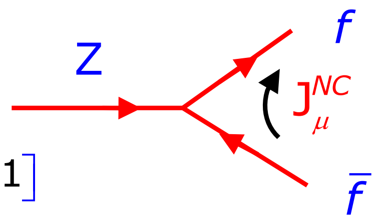
N.B. The Z couples only to lefthanded neutrinos

C_V and C_A determination

- The weak current can be written in terms of the axial and vector couplings:

$$\boxed{J_\mu^{N.C.}(f) = \bar{u}_f \gamma_\mu \frac{1}{2}\left(c_V^f - c_A^f \gamma^5\right) u_f}$$

$\left[\text{per le correnti cariche } c_V = c_A = 1\right]$



- The coupling of Z with $f\bar{f}$ can be written:

$$\boxed{-i\frac{g}{\cos\theta_W}\left[J_\mu^3 - \sin^2\theta_W J_\mu^{em}\right] Z^\mu = -i\frac{g}{\cos\theta_W}\bar{u}_f\gamma_\mu\left[\frac{1-\gamma^5}{2}I^3 - Q\sin^2\theta_W\right]u_f \cdot Z^\mu}$$

- The vectorial and axial couplings are given by the coefficients of the terms:

$$\boxed{\bar{u}_f\gamma_\mu u_f \quad \text{e} \quad \bar{u}_f\gamma_\mu\gamma^5 u_f}$$

then we have: $\boxed{c_V = I_3^f - 2Q^f\sin^2\theta_W \qquad \text{e} \qquad c_A = I_3^f}$

N.B. Neutral current is not of the V-A type, so the Z couples with both left-handed and right-handed particles.

C_V and C_A couplings

$$\begin{array}{c} \\ \nu_e \\ e^-_L \\ u_L \\ d'_L \\ e^-_R \\ u_R \\ d'_R \end{array}
\begin{pmatrix}
I_3^f & Q^f & c_A^f & c_V^f \\
\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} \\
-\frac{1}{2} & -1 & -\frac{1}{2} & -\frac{1}{2} + 2\sin^2\theta_W \\
\frac{1}{2} & \frac{2}{3} & \frac{1}{2} & \frac{1}{2} - \frac{4}{3}\sin^2\theta_W \\
-\frac{1}{2} & -\frac{1}{3} & -\frac{1}{2} & -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W \\
0 & -1 & 0 & 2\sin^2\theta_W \\
0 & \frac{2}{3} & 0 & -\frac{4}{3}\sin^2\theta_W \\
0 & -\frac{1}{3} & 0 & \frac{2}{3}\sin^2\theta_W
\end{pmatrix}$$

$$c_V = I_3^f - 2Q^f \sin^2\theta_W$$

$$c_A = I_3^f$$

the righthanded neutrino has both c_V and c_A equal to zero, so it does not appear in the table.

In the C_V^f coupling we have $\sin^2\theta_W$, which is the quantity that is measured experimentally

- **In the couplings of right-handed particles there is no axial term because these particles interact only through the electromagnetic interaction which is vectorial.**

- **Radiative corrections modify these couplings at the percent level. At Lep the Z couplings were measured with an error of this order of magnitude and it was therefore possible to verify the precision of the radiative corrections of the Standard Model.**

Z coupling to lefthanded and righthanded states

- The W couples only to the left-handed states due to the factor **$(1-\gamma^5)/2$**
- The Z couples to both left-handed and right-handed states because its coupling is of the type **$(c_V-c_A\gamma^5)/2$**

$$J_\mu^{N.C.}(f) = \bar{u}_f \gamma_\mu \frac{1}{2}\left(c_V^f - c_A^f \gamma^5\right) u_f$$

- **Neutral current can also be expressed in terms of coupling with left-handed and right-handed states:**

$$J_\mu^{N.C.}(f) = g_L \bar{u}_L^f \gamma_\mu u_L^f + g_R \bar{u}_R^f \gamma_\mu u_R^f$$

Let's assume: $c_V = g_L + g_R \;\; ; \;\; c_A = g_L - g_R$

$$\frac{1}{2}\left(c_V^f - c_A^f \gamma^5\right) = \frac{1}{2}\left[g_L + g_R - \left(g_L - g_R\right)\gamma^5\right] = g_L \frac{1-\gamma^5}{2} + g_R \frac{1+\gamma^5}{2}$$

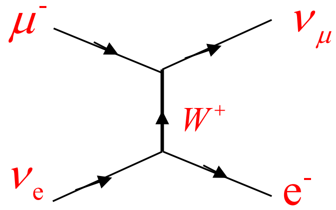
$$g_L = \frac{c_V + c_A}{2} = I_3^f - Q^f \sin^2\theta_W \qquad g_R = \frac{c_V - c_A}{2} = -Q^f \sin^2\theta_W$$

- From here we see again that the neutrino has no right-handed coupling since its charge Q is zero.

Relationship between G_F and neutral current

❑ **From the comparison of Fermi's theory with the GWS model for charged currents (see muon decay) we find the relation:**

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$



$$q^2 \ll M_W^2 \Rightarrow \mathcal{M}^{CC} = \left(\frac{g}{\sqrt{2}} J^\mu\right)\frac{1}{M_W^2}\left(\frac{g}{\sqrt{2}} J_\mu^+\right)$$

❑ **In a process with neutral current where $q^2 \ll M_Z^2$, we can write:**

$$\mathcal{M}^{NC} = \left(\frac{g}{\cos\theta_W} J_\mu^{NC}\right)\frac{1}{M_Z^2}\left(\frac{g}{\cos\theta_W} J^{NC\mu}\right) \Rightarrow \mathcal{M}^{NC} = \frac{g^2}{\cos^2\theta_W}\frac{1}{M_Z^2} J_\mu^{NC} J^{NC\mu}$$

❑ **A parameter ρ is introduced which takes into account the relative intensity of weak neutral and charged currents, linked to the mass of the bosons:**

$$\rho = \frac{M_W^2}{M_Z^2\cos^2\theta_W}$$

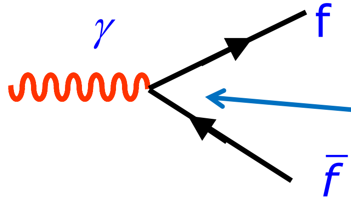
In the SM, at the tree level (fundamental level), ρ = 1. Radiative corrections, or the presence of new physics, change this relationship.

❑ **Therefore, usually the amplitude of neutral currents is written as follows, where the Fermi constant is used:**

$$\mathcal{M}^{NC} = \frac{4G}{\sqrt{2}} 2\rho J_\mu^{NC} J^{NC\mu}$$

Feynman rules for the verteces in the SM

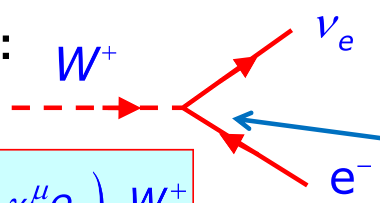
- **Electromagnetic interactions:**

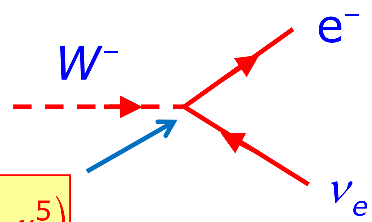


$$-ie\left(J^{em}\right)^{\mu}\cdot A_{\mu}=-ie\left(\bar{\psi}\gamma^{\mu}Q_f\psi\right)\cdot A_{\mu}$$

$-ieQ_f\gamma^{\mu}$ vertex

- **Weak charged interactions:**





$$g=\frac{e}{\sin\theta_W}$$

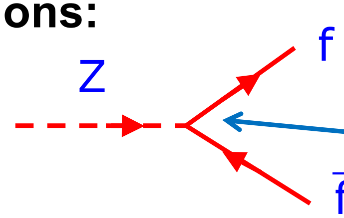
$$-i\frac{g}{\sqrt{2}}\left(\bar{\nu}_L\gamma^{\mu}e_L\right)\cdot W_{\mu}^{+}$$

vertex

$$-i\frac{g}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}\left(1-\gamma^5\right)$$

$$-i\frac{g}{\sqrt{2}}\left(\bar{e}_L\gamma^{\mu}\nu_L\right)\cdot W_{\mu}^{-}$$

- **Weak neutral interactions:**



$$-i\frac{g}{\cos\theta_W}\left[J_{\mu}^{3}-\sin^2\theta_W J_{\mu}^{em}\right]Z^{\mu}=-i\frac{g}{\cos\theta_W}\left[\bar{u}_f\gamma_{\mu}\frac{1}{2}\left(c_V^f-c_A^f\gamma^5\right)u_f\right]Z^{\mu}$$

$$-i\frac{g}{\cos\theta_W}\gamma^{\mu}\frac{1}{2}\left(c_V^f-c_A^f\gamma^5\right)$$ vertex

c_V and c_A determine the intensity of the coupling of the Z with the fermions.

QCD and the Standard Model

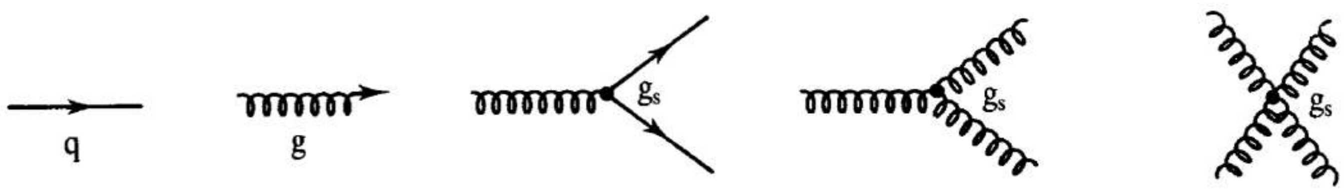
- **The QCD is analogous to QED but with the $U(1)_{em}$ group replaced by $SU(3)_C$**
- **The main difference between QED and QCD comes from the fact that the former is Abelian while the latter is not: the generators of $SU(3)_C$ do not commute and this leads to self-interactions between the gluons.**
- **The Lagrangian for free quarks may be written as:**

$$\mathscr{L} = \sum_q \bar{\psi}_q^j \mathrm{i}\gamma^\mu \partial_\mu \psi_q^k - \sum_q m_q \bar{\psi}_q^j \psi_q^j.$$

The indices j and k refer to colour (j,k:1,2,3)

- **We proceed as we did for QED: we require the Lagrangian to be invariant under a local gauge transformation, we introduce a covariant derivative with 8 gauge bosons (gluons) and we add to the Lagrangian the kinetic term for the gluons**

- **Here are the diagrams of the quark-gluon interaction and gluons self interactions:**



g_s = strong coupling constant

Complete Lagrangian of the Standard Model

$$\mathscr{L} = -\tfrac{1}{4}\boldsymbol{W}_{\mu\nu}\cdot\boldsymbol{W}^{\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{4}G^{a}_{\mu\nu}G_{a}^{\mu\nu}$$

Gluon fields

$\left\{\text{W}^{\pm}, \text{Z}^{0}, \gamma \text{ and gluon kinetic energies and self-interactions}\right.$

$$+ \bar{L}\gamma^{\mu}\left(i\partial_{\mu} - \tfrac{1}{2}g\boldsymbol{\tau}\cdot\boldsymbol{W}_{\mu} - \tfrac{1}{2}g'YB_{\mu}\right)L$$

$$+ \bar{R}\gamma^{\mu}\left(i\partial_{\mu} - \tfrac{1}{2}g'YB_{\mu}\right)R$$

$\left\{\text{fermion kinetic energies and their interactions with W}^{\pm}, \text{Z}^{0} \text{ and } \gamma\right.$

$$+ \left|\left(i\partial_{\mu} - \tfrac{1}{2}g\boldsymbol{\tau}\cdot\boldsymbol{W}_{\mu} - \tfrac{1}{2}g'YB_{\mu}\right)\varphi\right|^{2} - V(\varphi)$$

$\left\{\text{masses and couplings of the W}^{\pm}, \text{Z}^{0}, \gamma \text{ and Higgs boson}\right.$

$$- \left(g_{1}\bar{L}\varphi R + g_{2}\bar{L}\tilde{\varphi}R + \text{Hermitian conjugate}\right)$$

$\left\{\text{fermion masses and couplings to the Higgs boson}\right.$

$$+ \tfrac{1}{2}g_{s}\left(\bar{\psi}^{j}_{q}\gamma^{\mu}\lambda^{a}_{jk}\psi^{k}_{q}\right)G^{a}_{\mu}.$$

$\left\{\text{quark–gluon couplings}\right.$

- **L= left-handed fermion doublet**, **R=right-handed singlet; φ the Higgs doublet and its conjugate**; **ψ= quark colour field**.

Standard Model couplings

- We collect together some relations between the parameters of the Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8m_W^2} \qquad g = e(\sin\theta_W)^{-1} \qquad g' = e(\cos\theta_W)^{-1}$$

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137} \qquad \alpha_1 = \frac{g'^2}{4\pi} \approx \frac{1}{100} \qquad \alpha_2 = \frac{g^2}{4\pi} \approx \frac{1}{30}$$

$$\alpha_3 = \frac{g_s^2}{4\pi} \approx 0.4 \rightarrow 0.1.$$

- These 'constants' depend on a characteristic momentum Q (or, equivalently, a distance 1/Q) of the interaction.

- The values quoted for α, α_1, and α_2 are for Q of the order of a few GeV while for α_3 we give the variation over the range 1-100 GeV.

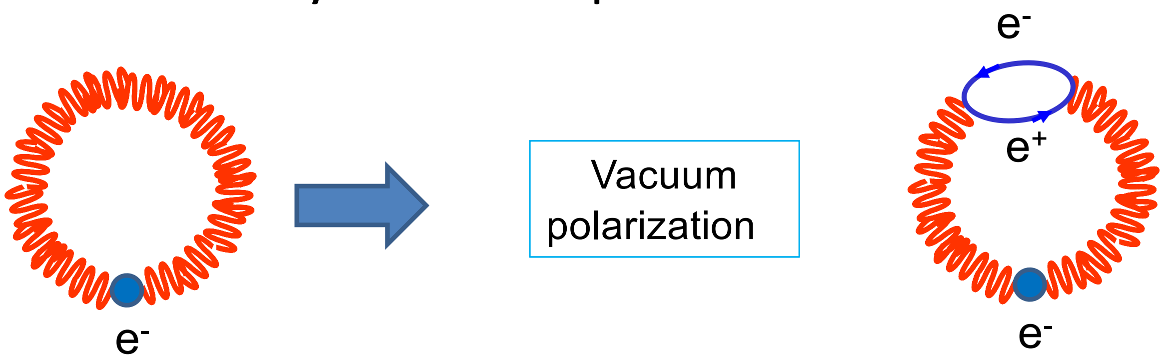
Running of α_{QED}

- In classical electromagnetism the potential energy of an electron in the field generated by the same electron (self-energy) is equal to:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

The potential energy goes to infinity when r goes to zero.

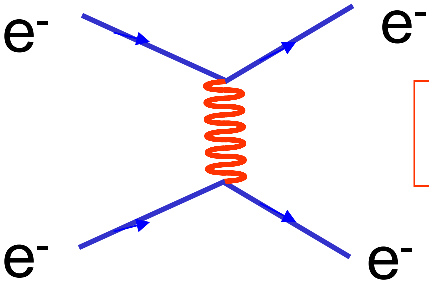
- The self-interaction in the field theory is described as photons that are emitted and then are absorbed again by the same charge:



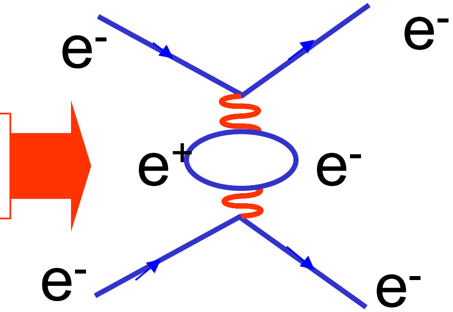
- The positron is "attracted" by the electron and it will "screen" the charge of the electron in a such a way that its effective value diminishes.
- The more you go into the positron "cloud" the lesser will be the shielding effect, so the electron effective charge increases.

Running of α_{QED}

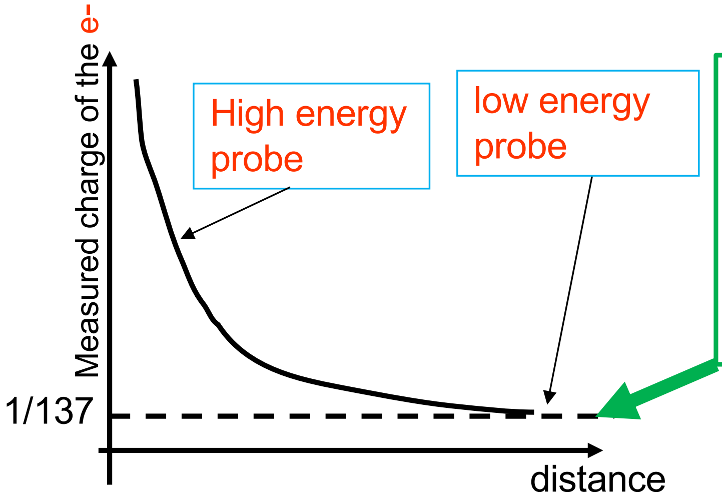
- ❑ **Let's consider the interaction between two electrons:**

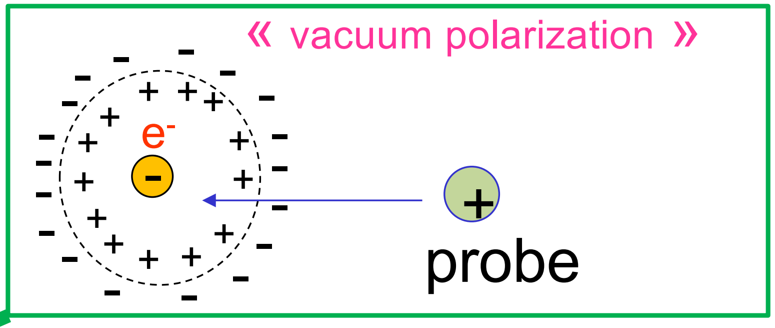


The interaction is modified by the vacuum polarization



- ❑ **A consequence of the vacuum polarization is that the charge of the electron becomes a function of the energy of the "probe" (that is of the other electron). The positrons "screen" the charge e⁻; the nearer we get to the charge the lesser the "screening" is and the effective charge "increase".**





$$\alpha(m_Z) = \frac{1}{128}$$

$$\alpha\left(Q^2\right) = \frac{\alpha\left(\mu^2\right)}{1 - \frac{\alpha\left(\mu^2\right)}{3\pi}\log\frac{Q^2}{\mu^2}}$$

μ: scale factor

Discovery of the asymptotic freedom in QCD (1973)

The Nobel Prize in Physics 2004

"for the discovery of asymptotic freedom in the theory of the strong interaction"

David J. Gross

1/3 of the prize

USA

Kavli Institute for Theoretical Physics, University of California
Santa Barbara, CA, USA

b. 1941

H. David Politzer

1/3 of the prize

USA

California Institute of Technology
Pasadena, CA, USA

b. 1949

Frank Wilczek

1/3 of the prize

USA

Massachusetts Institute of Technology (MIT)
Cambridge, MA, USA

b. 1951

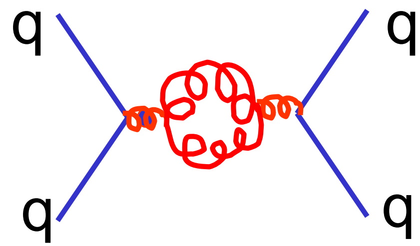
Actually it was found before by 't Hooft and also by Parisi but unfortunately (for them) they didn't publish it.

Running of α_s

❑ **Let's consider the strong interaction between two quarks:**

The production of virtual qq pair in the gluon propagator produces the same screening effect of the colour charge as in QED, hence the charge should diminish at the increase of the distance (that is at low momentum transfer).

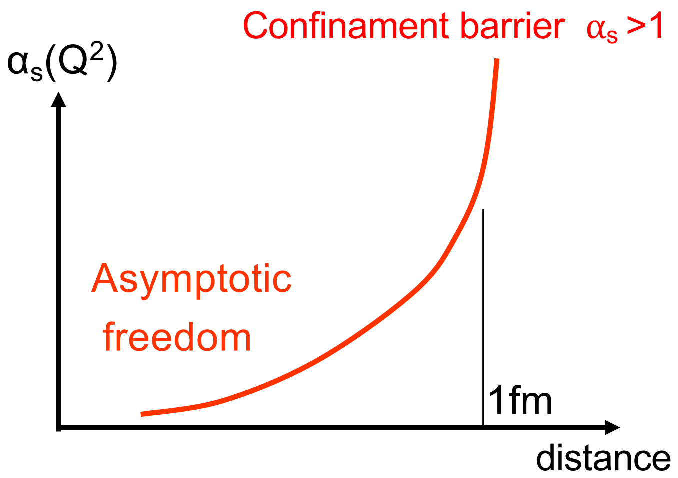
But since the gluons are "coloured" they exist also diagrams like this one that modify the interaction and produce an effect of "antiscreening"



[a fermions loop has opposite sign with respect to a bosons loop]

Running of α_s

❑ **The effect of the gluon self-interaction is such that:**



$$\alpha_s\left(Q^2\right) = \frac{12\pi}{(33-2f)\log\left(\frac{Q^2}{\Lambda^2}\right)}$$

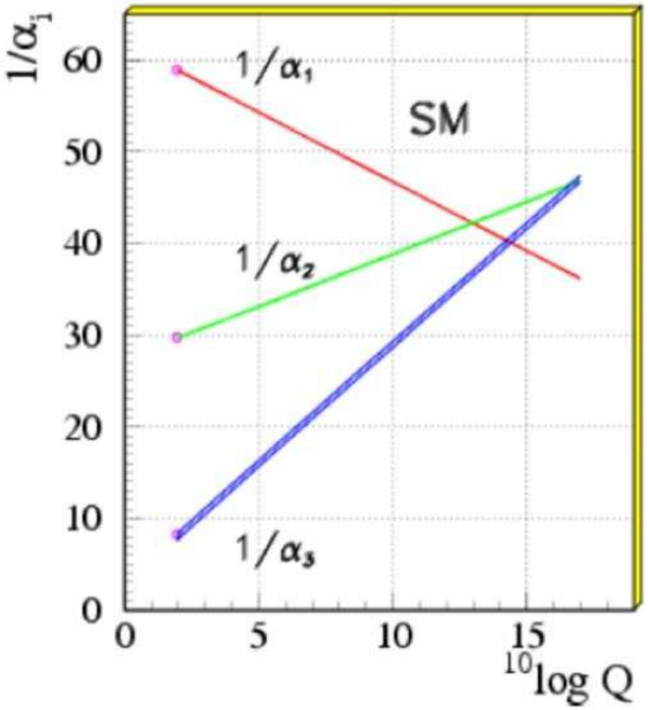
f = number of quarks with $4m^2 < Q^2$
Λ= scale (~200 MeV with f=4)

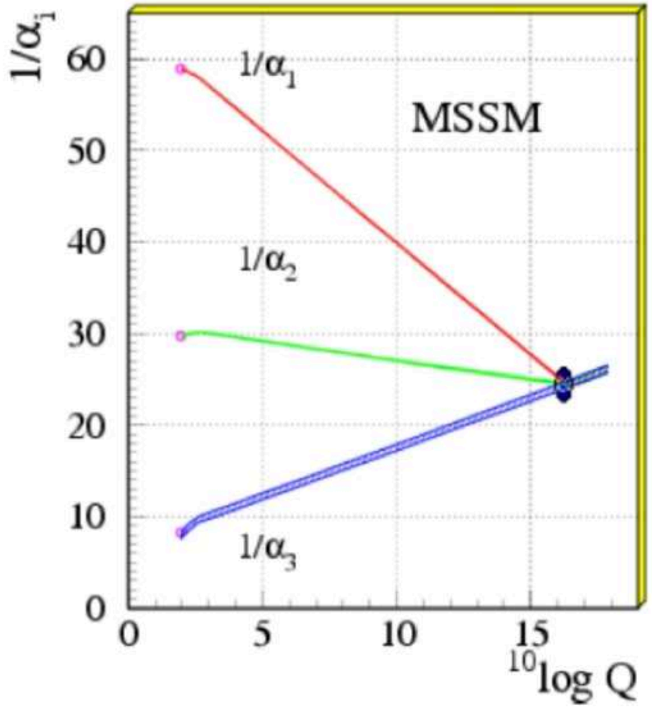
The anti-screening effect is also present in the $SU(2)_L$ since Z and W have self-interactions too.

$Q^2 \sim \Lambda^2$ strong coupling	→	~~perturbation approach~~
$Q^2 \gg \Lambda^2$ weak coupling	→	perturbation approach

At high momentum transfer (that is at small distances) α_s is small and we can do QCD calculation with the perturbative method. At low momentum transfer the constant is big and we can not use the perturbative method.

Running coupling constants





- *The running of the coupling constants has been experimentally confirmed in the accessible energy range, but the more interesting thing here is that one can extrapolate the curves far beyond where we can test them experimentally. One sees then that these couplings form a triangle somewhere around 10^{16} GeV.*

- *This plot shows the running of the gauge couplings within the MSSM. Since the particle content with Supersymmetry (SUSY) is different, the slope of the curves changes. Interestingly, the result is that the gauge couplings meet almost exactly (within the errorbars) in one point, somewhere around 10^{16} GeV, usually referred to as the GUT scale (which isn't too far off the Plank Scale (10^{19} GeV).*

End of chapter 3