Collider Particle Physics
- Chapter 5 -

Standard Model tests at the SppS

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Chapter Summary

- ☐ Parton-parton interactions
- ☐ W and Z properties
- ☐ Kinematics at the hadron colliders
- ☐ SM tests at the SppS: measurement of the W and Z mass

parton-parton interactions

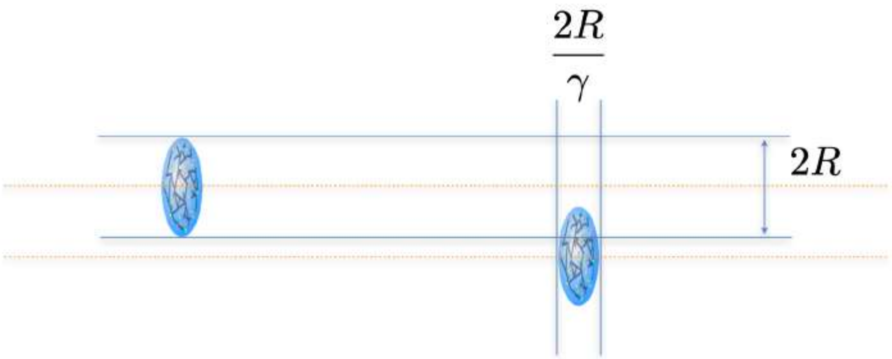
proton-(anti) proton scattering

Naïve (but accurate) model to compute the proton-proton total cross-section.
The protons are treated as two billiards balls that interact only if they ``*get in contact”*

The proton

uud (valence quarks) and a sea of interacting quarks and gluons

$$R = 0.8 \text{ fm}$$



Strong interaction total cross section

$$\pi(2R)^2 \sim 80\text{mb}$$

$$1\text{b (barn)} = 10^{-28}\, m^2$$

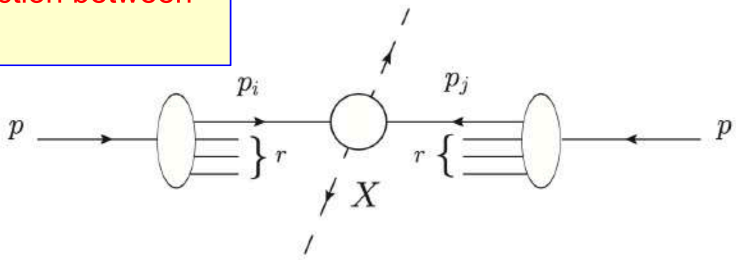
This naive description depicts fairly accurately the strong interaction cross section of proton-proton collisions.

But proton is composite. How to describe the interaction between quarks and gluons occuring at high energies?

The parton model

At high energies protons can be seen as an ensemble (gas) of quarks and gluons (or partons) **non interacting**.

A hard scattering collision, can be viewed at first order as the interaction between two partons of each proton each carrying a fraction x_1 and x_2 of the incoming protons.

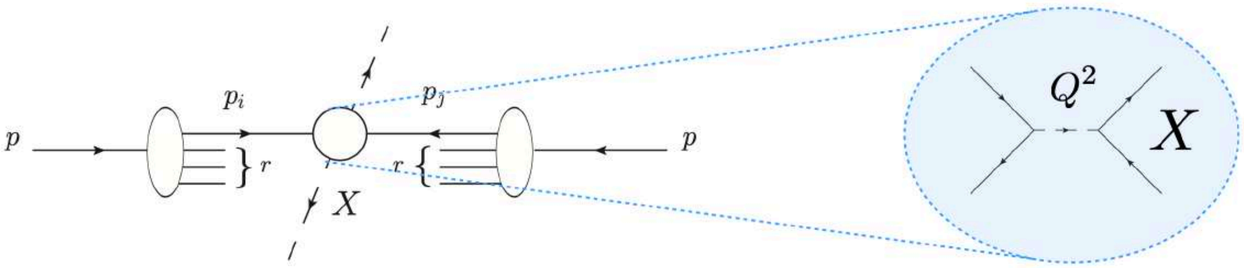


$$\hat{s} = x_1 x_2 s$$

The centre-of-mass energy of the interaction is not known a priori (and essentially impossible to reconstruct due to limited resolution and part of the event being undetected)

Collinear Factorization

The QCD factorization theorem permits to represent the cross section of a given process as a convolution in partonic Momenta of a perturbatively calculable part which involves the hard scale of the process with non-perturbative (soft) distributions of active partons inside the hadrons.



$$\sigma(pp \to X) = \boxed{\sum_{i,j}} \int_0^1 dx_i dx_j \boxed{f_i(x_i, Q^2) f_j(x_j, Q^2)} \boxed{d\hat{\sigma}(q_i q_j \to X, \hat{s}, Q^2)}$$

Sum over initial partonic states i and j

Parton Density Function

Hard scattering cross-section

Q^2 *'Resolution scale'*

In the case depicted above M_X^2

- q_1 and q_2 are the initial partons
- x_1 and x_2 are the momentum fraction of each parton.

Important messages

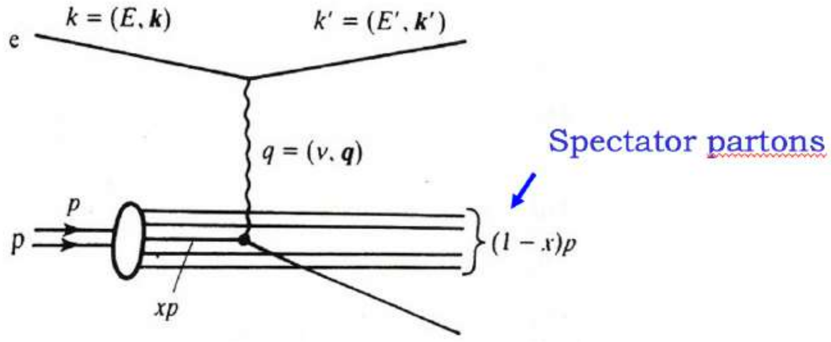
(1) The centre-of-mass energy of the interaction is not known a priori (and essentially impossible to reconstruct due to limited resolution and part of the event being undetected)

(2) At LHC making predictions that are:
- Exact is not possible.
- Accurate and precise is however possible… but difficult.
- At the SppS the prediction of the cross-sections were certainly much less accurate that the ones at LHC since the pdf had big uncertainties

(3) Predictions rely on the knowledge of the number and types of partons and the distributions of their momenta in the protons. (pdf)

Parton Density Function (pdf)

- **Every partons carries a fraction x of the proton quadrimomentum, where x can be different from parton to parton. Let's call $f_i(x)$ the probability that the parton of type i has the fraction x of the proton quadrimomentum (actually $f_i(x)$ is a density probability).**



Sum rules

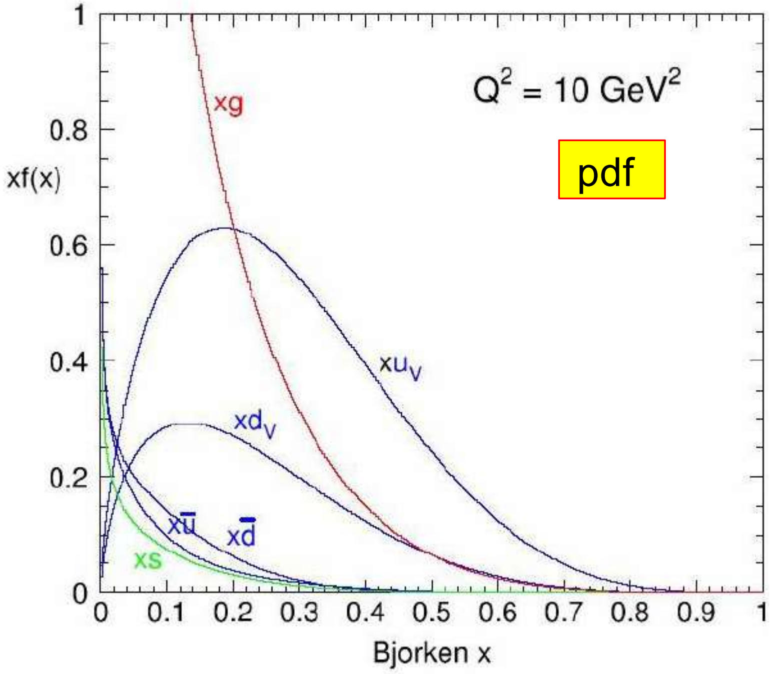
The quark probability density functions (pdf) must satisfy some sum rules. For instance in the proton:

$$\sum_i \int_0^1 dx\, x f_i(x, Q^2) = 1 \longleftarrow$$
momentum conservation

$$\int_0^1 (f_u(x,Q^2) - f_{\bar{u}}(x,Q^2))dx = 2 \longleftarrow$$
2 u valence quarks

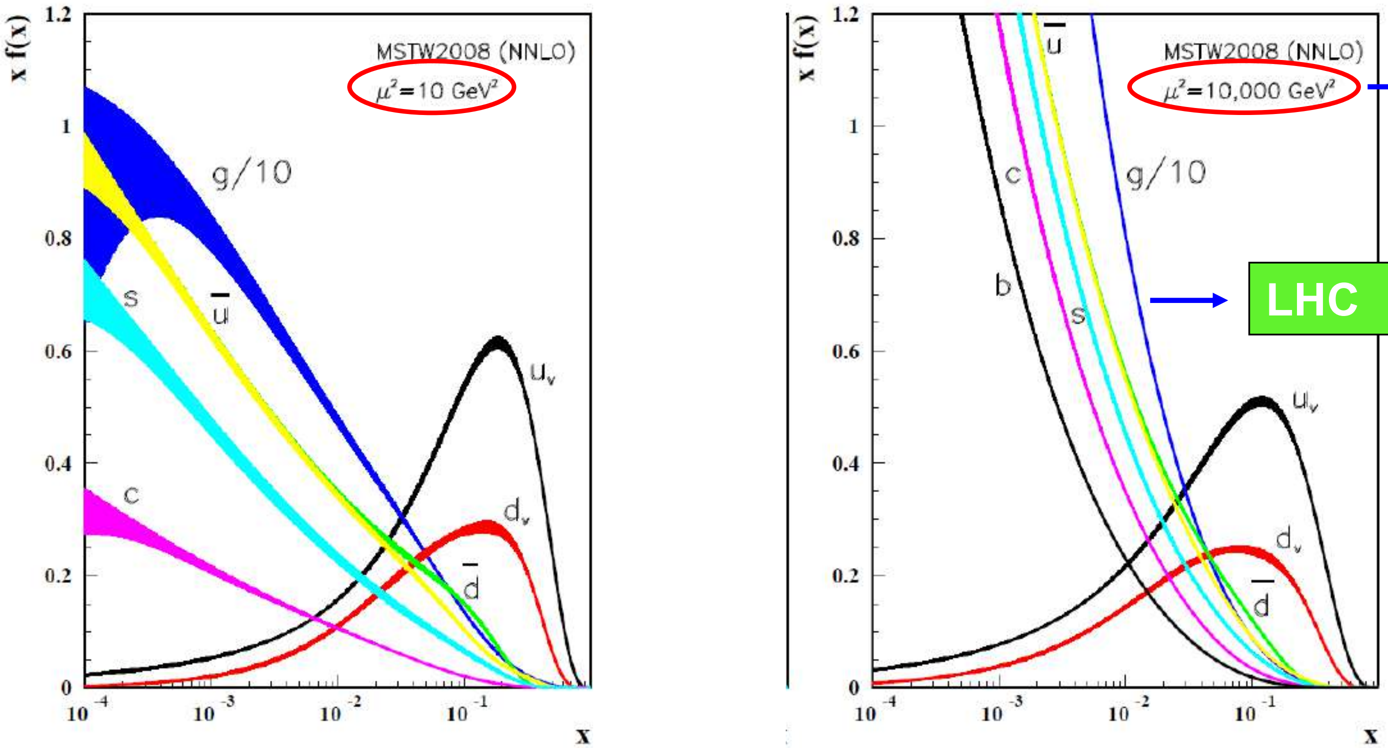
$$\int_0^1 (f_d(x,Q^2) - f_{\bar{d}}(x,Q^2))dx = 1 \longleftarrow$$
1 d valence quark

$$\int_0^1 (f_s(x,Q^2) - f_{\bar{s}}(x,Q^2))dx = 0 \longleftarrow$$
No strangeness



DGLAP evolution equation

- ❑ PDF are not calculable, but measured in DIS experiments (with electron and neutrino scattering on nucleons)
- ❑ DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) are the authors who first wrote the QCD evolution equation.
- ❑ QCD Evolution Equations for Parton Densities valid in the theory of the strong interactions, determine the rate of change of parton densities (probability densities to find a quark or a gluon in the proton) when the energy scale chosen for their definition is varied.



$(100\ GeV)^2$

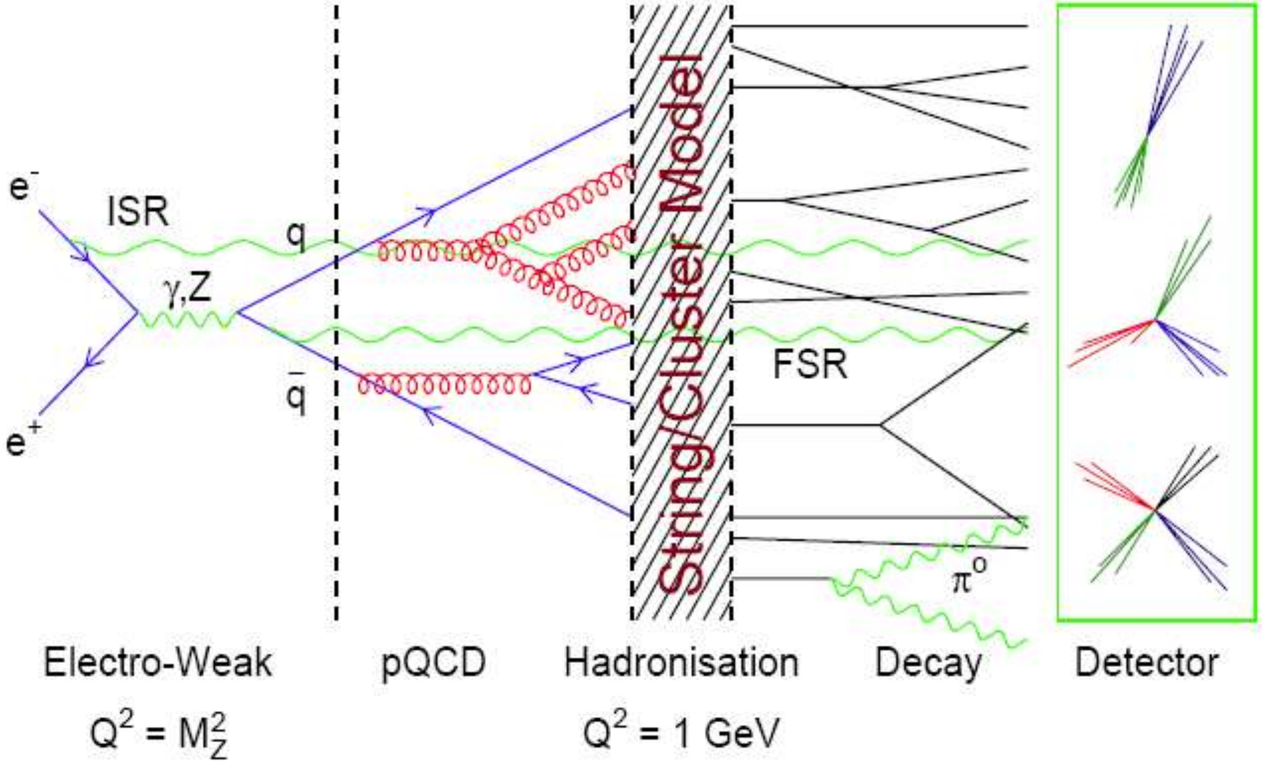
LHC is a gluon machine

hadronisation

- ☐ **The initial quarks are coloured. The final hadrons are white.**
- ☐ **The formation process of the hadrons is called hadronization. It happens for energies "around" 1 GeV and the process is not perturbative, so it can be described only by phenomenological models.**

This example is with an e+e- collision.

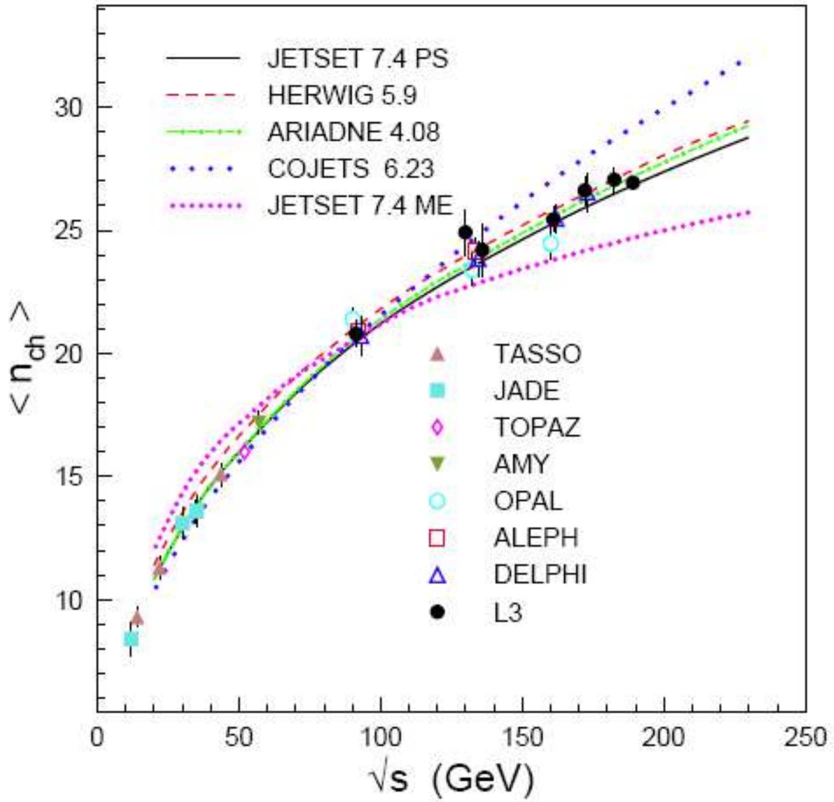
With the hadron collision is even more messy because of the spectator partons and the pile-up



Comparison between hadronisation models

- **The degree of "goodness" of the various hadronization models can be deduced from the comparison of Montecarlo predictions with experimental data for several quantities that characterize a hadronic event. For instance:**

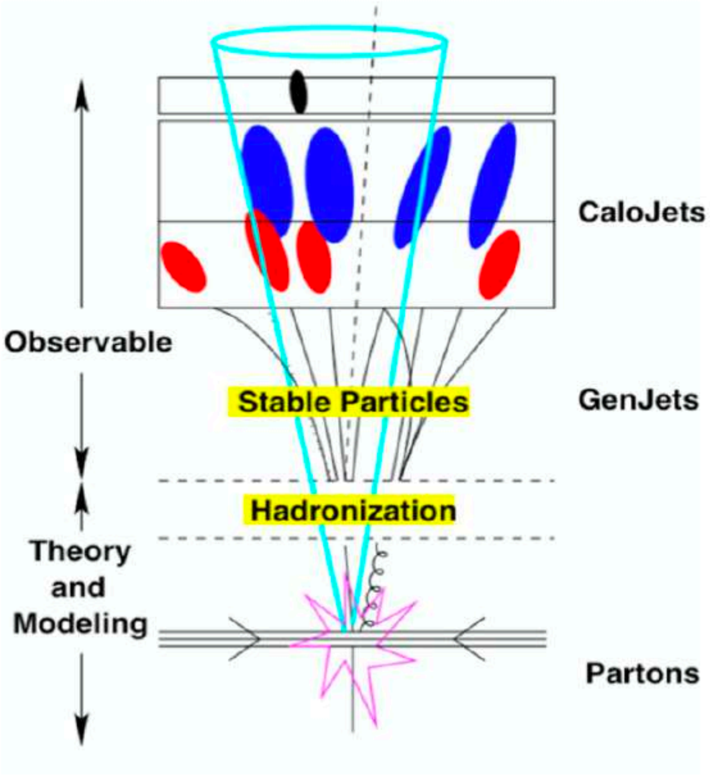
Average number of charged particle in a jet as a function of the center of mass energy of the system e^+e^-.



For instance the MC Jetset 7.4 ME and Cojets 6.23 do not reproduce the data well enough at high energy.

Jet reconstruction algorithm

- ❑ A jet can be defined as a collimated spray of stable particles arising from the fragmentation and hadronisation of a parton(quark or gluon) after a collision.
- ❑ Jet reconstruction algorithms are used to combine the calorimetry and tracking information to define jets.
- ❑ The jets provide a link between the observed colourless stable particles and the underlying physics at the partonic level.
- ❑ An accurate jet algorithm will also be able to calculate the correct amount of missing energy in the detector
- ❑ Some aspects of an algorithm that need to be considered are the jet size and whether the algorithm is infra-red and collinear(IRC) safe. The jet size and area determine the susceptibility of a jet to soft radiation. A larger jet radius is important as it allows the jet to capture enough of the hadronised particles for the accurate calculation of the jets mass and energy. However a smaller jet radius is useful in reducing the amount of the underlying event(UE) and pile-up(PU) captured by the jet, preventing the overestimation of the jets mass and energy.

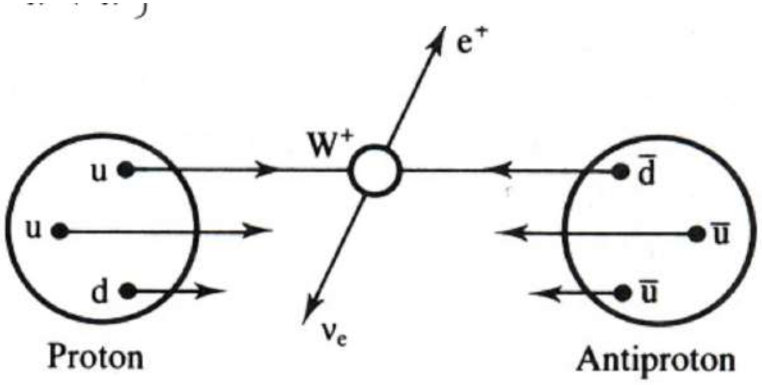


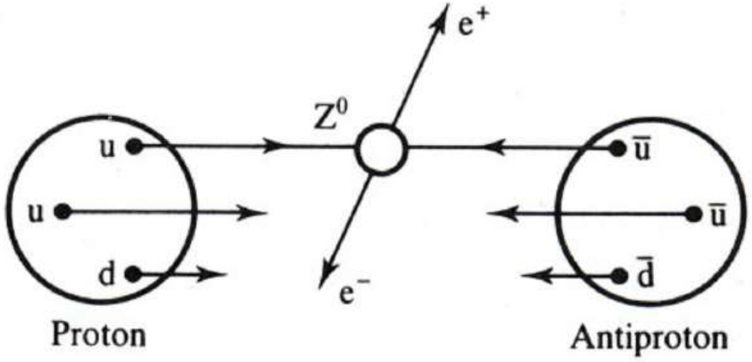
Calojets are those jets created using the calorimeter output whereas Genjets are jets created using stable simulated particles.

W and Z properties

How W and Z are produced

- **Production mechanism of W and Z at the SppS (and Tevatron):**





$$u+\overline{d} \to W^{+} \to e^{+}+\nu_{e},\ \mu^{+}+\nu_{\mu}$$

$$\overline{u}+d \to W^{-} \to e^{-}+\overline{\nu}_{e},\ \mu^{-}+\overline{\nu}_{\mu}$$

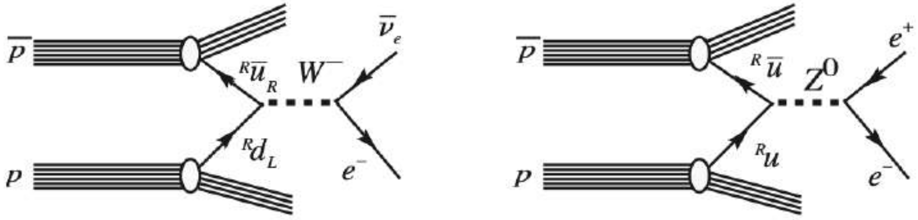
$$\left.\begin{matrix} u+\overline{u} \\ d+\overline{d} \end{matrix}\right\} \to Z^{0} \to e^{+}e^{-},\ \mu^{+}\mu^{-}$$

- **At the SppS the antiquarks are the valence quark of the antiproton. At LHC (proton-proton collider) they come from the sea.**

- ***The center of mass energy of the quark-antiquark interactions is on average 1/6 of the proton-antiproton CoM energy (quarks carry 50% of the proton momentum and we have three valence quarks)***

W and Z production cross sections

- The calculation of the proton–antiproton cross sections at the SppS starts from those at the quark level and takes into account the quark distribution functions (pdf).



Drell-Yan diagram/process

- The evaluation made in the design phase gave the values: $\sigma(\bar{p}p \to W \to e\nu_e) \approx 530\,\text{pb}$ $\sigma(\bar{p}p \to Z \to ee) \approx 35\,\text{pb}.$

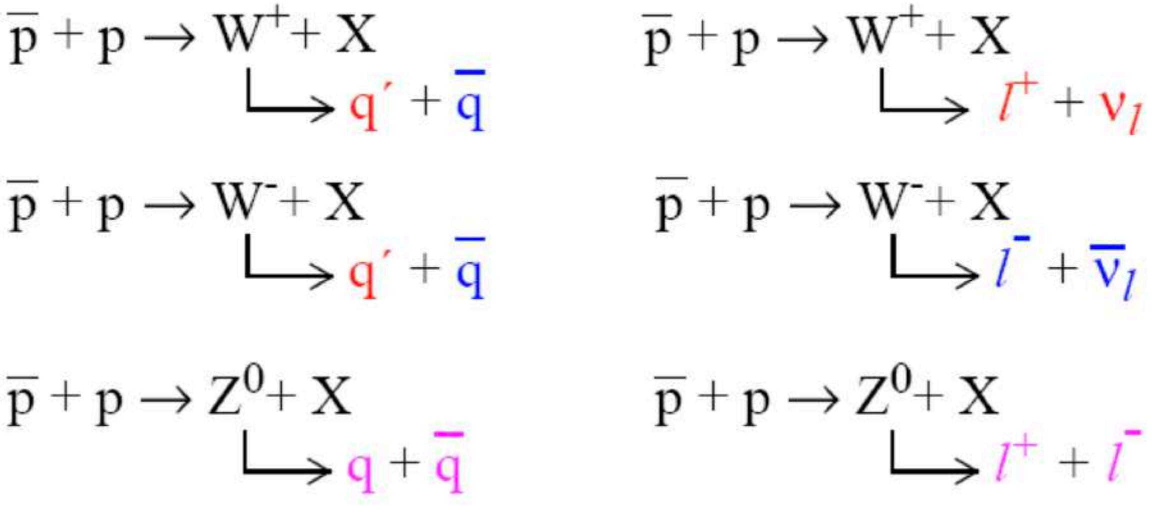
- To be precise, both the valence and the sea quarks contribute to the process, however at $\sqrt{s} = 540$ GeV the average momentum fraction at the W and Z resonances is $< x >_W/\sqrt{s} = 0.15$. Therefore, the process is dominated by the valence quarks, while the sea quarks have momentum fractions that are too small.

- We thus know that the annihilating quark is in the proton, the antiquark in the antiproton. This information is lost at higher collision energies.

- At LHC, with a proton-proton collision, the antiquark is coming from the sea.

W and Z decays

- W and Z decay in the 70% of the cases in hadrons (quark-antiquark channels); however this final state is not easily distinguishable from hadrons coming from QCD quark interactions and could not be used in the analysis.



$$\begin{array}{ll} \bar{p}+p \to W^{+}+X & \bar{p}+p \to W^{+}+X \\ \quad\quad \hookrightarrow q' + \bar{q} & \quad\quad \hookrightarrow l^{+} + \nu_{l} \\ \bar{p}+p \to W^{-}+X & \bar{p}+p \to W^{-}+X \\ \quad\quad \hookrightarrow q' + \bar{q} & \quad\quad \hookrightarrow l^{-} + \bar{\nu}_{l} \\ \bar{p}+p \to Z^{0}+X & \bar{p}+p \to Z^{0}+X \\ \quad\quad \hookrightarrow q + \bar{q} & \quad\quad \hookrightarrow l^{+} + l^{-} \end{array}$$

- W decay B.R. in the lepton channel is about 10% per flavour
- Z decay B.R. in charged leptons is about 3.3%. per flavour
- Z decay B.R. in neutrinos (invisible width) is about 20%.

W partial widths

- **Lepton decay partial width (from the SM):**

$$\Gamma_{e\nu} = \Gamma_{\mu\nu} = \Gamma_{\tau\nu} = \left(\frac{g}{\sqrt{2}}\right)^2 \frac{M_W}{24\pi} = \frac{1}{2}\frac{G_F M_W^3}{3\sqrt{2}\pi} \approx 225\,\text{MeV}.$$

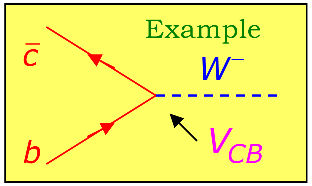
- **If we assume that the phase spaces are all equal due to the large W mass, we can deduce the quark W partial widths.**
- **We must take into account the CKM matrix elements in the quark couplings:**

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Eigenstates of the weak interactions

CKM matrix of the quark mixing

Mass eigenstates



$$s' = V_{cd}\cdot d + V_{cs}\cdot s + V_{cb}\cdot b$$

- **Partial widths for the “wrong” pair of quarks (us) and (cd):**

$$\Gamma_{us} \equiv \Gamma(W \to \bar{u}s) = 3\times|V_{us}|^2\Gamma_{e\nu} = 3\times 0.224^2\times\Gamma_{e\nu} \approx 35\,\text{MeV}$$

$$\Gamma_{cd} \equiv \Gamma(W \to \bar{c}d) = \underset{\uparrow}{3}\times|V_{cd}|^2\Gamma_{e\nu} = 3\times 0.22^2\times\Gamma_{e\nu} \approx 33\,\text{MeV}.$$

Colour factor

- **Partial widths for the “good” pair of quarks (ud) and (cs):**

$$\begin{aligned}\Gamma_{ud} &\equiv \Gamma(W \to \bar{u}d) = 3\times|V_{ud}|^2\Gamma_{e\nu} \\ &= 3\times 0.974^2\times\Gamma_{e\nu} = 2.84\times\Gamma_{e\nu} \approx 640\,\text{MeV}\end{aligned}$$

$$\Gamma_{cs} \equiv \Gamma(W \to \bar{c}s) = 3\times|V_{cs}|^2\Gamma_{e\nu} = 3\times 0.99^2\times\Gamma_{e\nu} \approx 660\,\text{MeV}.$$

- **Total W width:**

$$\Gamma_W = 3\times\Gamma_{l\nu} + \Gamma_{us} + \Gamma_{cd} + \Gamma_{ud} + \Gamma_{cs}$$

$$\boxed{\Gamma_W \approx 2.04\,\text{GeV}.}$$

Z partial widths

- The Z vertex is more complicated than the W vertex. The Z coupling can be written as:

$$g_Z \equiv \frac{g}{\cos\theta_W}\left(I_3^W - Q\sin^2\theta_W\right) = \frac{g}{\cos\theta_W}c_Z.$$

- The neutrino partial width (from the SM) is:

$$\Gamma_\nu \equiv \Gamma(Z \to \nu_l\bar{\nu}_l) = \left(\frac{g}{\cos\theta_W}\right)^2 \frac{M_Z}{24\pi}\left(\frac{1}{2}\right)^2 = \frac{G_F M_W^2 M_Z}{\cos^2\theta_W 3\sqrt{2}\pi}\left(\frac{1}{2}\right)^2.$$

- We use this relation to get rid of M_W:

$$M_W = \left(\frac{g^2\sqrt{2}}{8G_F}\right)^{1/2} = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_F}}\frac{1}{\sin\theta_W} = \frac{37.3}{\sin\theta_W}\,\text{GeV}.$$

- and we get:

$$\Gamma_\nu = \frac{G_F M_Z^3}{3\sqrt{2}\pi}\left(\frac{1}{2}\right)^2 \approx 660 \times \frac{1}{4}\,\text{MeV} = 165\,\text{MeV}.$$

- Neutrinos can not be seen. The “invisible” width is:

$$\boxed{\Gamma_{\text{inv}} = 3\Gamma_\nu \approx 495\,\text{MeV}.}$$

- Lepton width: *[$s^2 = \sin^2\vartheta_W$]*

$$\Gamma_l = \Gamma_e = \Gamma_\mu = \Gamma_\tau = \frac{G_F M_Z^3}{3\sqrt{2}\pi}\left[\left(-\frac{1}{2}+s^2\right)^2 + s^4\right] \approx 660\times 0.125 \approx 83\,\text{MeV}.$$

- Hadron width for up-type quark:

$$\Gamma_u = \Gamma_c = 3\frac{G_F M_Z^3}{3\sqrt{2}\pi}\left[\left(\frac{1}{2}-\frac{2}{3}s^2\right)^2 + \left(-\frac{2}{3}s^2\right)^2\right] \approx 660\times 0.42 \approx 280\,\text{MeV}.$$

- Hadron width for down-type quark:

$$\Gamma_d = \Gamma_s = \Gamma_b = 3\frac{G_F M_Z^3}{3\sqrt{2}\pi}\left[\left(-\frac{1}{2}+\frac{1}{3}s^2\right)^2 + \left(\frac{1}{3}s^2\right)^2\right] \approx 660\times 0.555 \approx 370\,\text{MeV}.$$

- Finally we have the hadronic width:

$$\Gamma_h = 2\Gamma_u + 3\Gamma_d \approx 1.67\,\text{GeV}.$$

- Summing up we have the Z total width:

$$\boxed{\Gamma_Z = \Gamma_{\text{inv}} + 3\Gamma_l + \Gamma_h \approx 2.42\,\text{GeV}.}$$

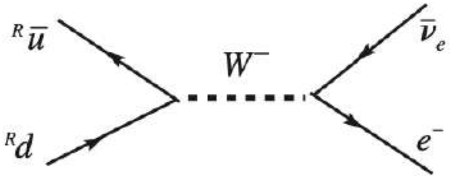
W and Z production cross sections

- **CoM energy of the quark-antiquark collision:**

$$\sqrt{\hat{s}} = x_q x_{\bar{q}} \sqrt{s}.$$

- **Let's consider the process at an energy near the resonance ($\sqrt{\hat{s}} \approx M_W.$):**

$$\bar{u} + d \to e^- + \bar{\nu}_e$$



The quarks must have the same colour

- **Since we are near a resonance we can use the Breit-Wigner formula for 2 spin ½ particles:**

$$\sigma(\bar{u}d \to e^-\bar{\nu}_e) = \frac{1}{9}\frac{3\pi}{\hat{s}}\frac{\Gamma_{ud}\Gamma_{e\nu}}{(\sqrt{\hat{s}} - M_W)^2 + (\Gamma_W/2)^2}.$$

The factor 3 for the colour is already included in the hadron partial width

- **At the resonance peak we have:**

$$\sigma_{\max}(\bar{u}d \to e^-\bar{\nu}_e) = \frac{4\pi}{3}\frac{1}{M_W^2}\frac{\Gamma_{ud}\Gamma_{e\nu}}{\Gamma_W^2} = \frac{4\pi}{3}\frac{1}{81^2}\frac{0.64\times0.225}{2.04^2}$$

$$\times 388\left[\mu\text{b}/\text{GeV}^{-2}\right] \approx 8.8\,\text{nb}.$$

Conversion factor

- **The charge conjugated process** $u + \bar{d} \to e^+ + \nu_e$ **has the same cross-section for the W⁺ production**

- **Let's consider now the Z production and its decay in the e⁺e⁻ pair**

$$\bar{u} + u \to e^- + e^+ \qquad \bar{d} + d \to e^- + e^+$$

- **Their cross-sections at the resonance are:**

$$\sigma_{\max}(\bar{u}u \to e^-e^+) = \frac{4\pi}{3}\frac{1}{M_Z^2}\frac{\Gamma_u\Gamma_e}{\Gamma_Z^2} = \frac{4\pi}{3}\frac{1}{91^2}\frac{0.280\times0.083}{2.42^2} \times 388\,\mu\text{b} \approx 0.8\,\text{nb}$$

$$\sigma_{\max}(\bar{d}d \to e^-e^+) = \frac{4\pi}{3}\frac{1}{M_Z^2}\frac{\Gamma_d\Gamma_e}{\Gamma_Z^2} \approx 1\,\text{nb}.$$

- **Notice that the cross sections for the Z are almost an order of magnitude smaller than the ones of the W.**

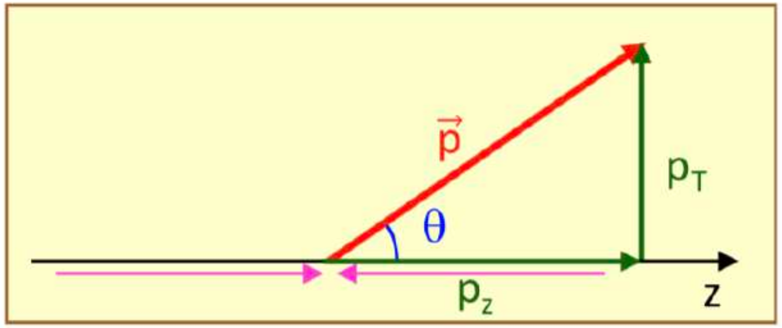
Kinematics at the hadron colliders

Rapidity

- **Definition:**

rapidity

$$y = \frac{1}{2} \ln \left(\frac{E + p_z c}{E - p_z c} \right).$$



If a very energetic particle has a little p_z, then y =0

If it is moving close to the beam, y goes to plus or minus infinite

- **We can write the rapidity in several ways:**

$$y = \ln \sqrt{\frac{E + p_z c}{E - p_z c}} = \ln \left(\frac{E + p_z c}{\sqrt{E - p_z c}\sqrt{E + p_z c}} \right) = \ln \left(\frac{E + p_z c}{\sqrt{E^2 - p_z^2 c^2}} \right) = \ln \left(\frac{E + p_z c}{M_T c^2} \right)$$

Invariant transverse mass

$$E^2 - p_z^2 = M_T^2$$

- **It could also be expressed like this:**

$$y = \tanh^{-1} \left(\frac{p_z c}{E} \right)$$

Rapidity

❑ **Proof of the formula:**

rapidity

$$y = \ln\left(\frac{E+p_z c}{M_T c^2}\right)$$

$$y = \tanh^{-1}\left(\frac{p_z c}{E}\right)$$

The next neat expression for rapidity is found by using hyperbolic tangents. Recall that $\tanh\theta = (e^{\theta} - e^{-\theta})/(e^{\theta} + e^{-\theta})$. We write

$$\begin{aligned}
y &= \tanh^{-1}\left(\tanh\left(\ln\left(\tfrac{E+p_z c}{M_T c^2}\right)\right)\right). \\
&= \tanh^{-1}\left(\frac{\exp\left(\ln\frac{E+p_z c}{M_T c^2}\right)-\exp\left(-\ln\frac{E+p_z c}{M_T c^2}\right)}{\exp\left(\ln\frac{E+p_z c}{M_T c^2}\right)+\exp\left(-\ln\frac{E+p_z c}{M_T c^2}\right)}\right) \\
&= \tanh^{-1}\left(\frac{\frac{E+p_z c}{M_T c^2}-\frac{M_T c^2}{E+p_z c}}{\frac{E+p_z c}{M_T c^2}+\frac{M_T c^2}{E+p_z c}}\right) \\
&= \tanh^{-1}\left(\frac{\frac{(E+p_z c)^2-M_T^2 c^4}{M_T c^2(E+p_z c)}}{\frac{(E+p_z c)^2+M_T^2 c^4}{M_T c^2(E+p_z c)}}\right) \\
&= \tanh^{-1}\left(\frac{(E+p_z c)^2-M_T^2 c^4}{(E+p_z c)^2+M_T^2 c^4}\right) \\
&= \tanh^{-1}\left(\frac{E^2+2Ep_z c+p_z^2 c^2-M_T^2 c^4}{E^2+2Ep_z c+p_z^2 c^2+M_T^2 c^4}\right) \\
&= \tanh^{-1}\left(\frac{2Ep_z c+2p_z^2 c^2}{2E^2+2Ep_z c}\right) \\
&= \tanh^{-1}\left(\frac{p_z c(E+p_z c)}{E(E+p_z c)}\right) \\
y &= \tanh^{-1}\left(\tfrac{p_z c}{E}\right).
\end{aligned}$$

Rapidity

- ❑ **Let's do a boost along z:** $y = \frac{1}{2}\ln\left(\frac{E+p_z c}{E-p_z c}\right)$ ➡ $\boxed{E'/c \;=\; \gamma(E/c - \beta p_z)}$ $\boxed{p'_z \;=\; \gamma(p_z - \beta E/c).}$

$$y' \;=\; \tfrac{1}{2}\ln\left(\frac{\gamma E/c-\beta\gamma p_z+\gamma p_z-\beta\gamma E/c}{\gamma E/c-\beta\gamma p_z-\gamma p_z+\beta\gamma E/c}\right) = \tfrac{1}{2}\ln\left(\frac{\gamma(E/c+p_z)-\beta\gamma(E/c+p_z)}{\gamma(E/c-p_z)+\beta\gamma(E/c-p_z}\right) = \tfrac{1}{2}\ln\left(\frac{E/c+p_z}{E/c-p_z}\frac{\gamma-\beta\gamma}{\gamma+\beta\gamma}\right) = \tfrac{1}{2}\ln\left(\frac{E+p_z c}{E-p_z c}\right) + \ln\sqrt{\frac{1-\beta}{1+\beta}}$$

$$\boxed{y' \;=\; y + \ln\sqrt{\frac{1-\beta}{1+\beta}}}$$

- ❑ **It can be simplified by noting that:** $\ln\sqrt{\frac{1-\beta}{1+\beta}} \;=\; \tanh^{-1}\left(\tanh\ln\sqrt{\frac{1-\beta}{1+\beta}}\right) = \cdots\cdot\cdot \;=\; -\tanh^{-1}\beta.$

- ❑ **This means that upon a Lorentz transformation parallel to the beam axis with velocity v=βc, the equation for the transformation on rapidity is a particularly simple one :**

$$\boxed{y' = y - \tanh^{-1}\beta.}$$

Rapidity

- ☐ **This simple tansformation law for *y* has an important consequence:**
  - Suppose we have two particles produced in a collision and they have rapidities y_1 and y_2 in the CoM of the parton-parton collision, that is moving with respect to the Lab frame.
  - In the Lab we measure the rapidities y'_1 and y'_2:

$$y_1' - y_2' = (y_1 - \tanh^{-1}\beta - (y_2 - \tanh^{-1}\beta)) = y_1 - y_2.$$

The difference between the rapidities of two particles is invariant with respect to Lorentz boosts along the z-axis.

- ☐ **Rapidity is often paired with the azimuthal angle ϕ at which a particle is emitted: (y, ϕ)**
  - In this way the angular separation of two events:

$$(y_2 - y_1, \phi_2 - \phi_1)$$

Is invariant with respect to boosts along the beam axis.

- ☐ **Histograms binned in either the angular separation of events or the rapidity separation of events can be contributed to by events whose centre of mass frames are boosted by arbitrary velocities with respect to the rest frame of the detector (the Lab frame).**

Pseudo-rapidity

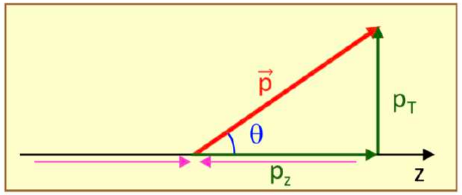
- The only problem with rapidity is that it can be hard to measure for highly relativistic particle. You need both the energy and the total momentum, which is not always easy to measure, in particular for high y.

- However we can define a quantity that is almost the same thing as the rapidity which is much easier to measure than y for highly energetic particles: the pseudo-rapidity η.

$$y = \frac{1}{2}\ln\left(\frac{E+p_zc}{E-p_zc}\right) = \frac{1}{2}\ln\left(\frac{(p^2c^2+m^2c^4)^{\frac{1}{2}}+p_zc}{(p^2c^2+m^2c^4)^{\frac{1}{2}}-p_zc}\right)$$

- For highly relativistic particles the momentum p is much bigger than the mass m; we also do a binomial expansion of the square root:

$$y = \frac{1}{2}\ln\left(\frac{pc\left(1+\frac{m^2c^4}{p^2c^2}\right)^{\frac{1}{2}}+p_zc}{pc\left(1+\frac{m^2c^4}{p^2c^2}\right)^{\frac{1}{2}}-p_zc}\right) \simeq \frac{1}{2}\ln\left(\frac{pc+p_zc+\frac{m^2c^4}{2pc}+\cdots}{pc-p_zc+\frac{m^2c^4}{2pc}+\cdots}\right) \simeq \frac{1}{2}\ln\left(\frac{1+\frac{p_z}{p}+\frac{m^2c^4}{2p^2c^2}+\cdots}{1-\frac{p_z}{p}+\frac{m^2c^4}{2p^2c^2}+\cdots}\right)$$



$$\frac{p_z}{p} = \cos\theta$$

Pseudorapidity

$$1+\frac{p_z}{p}=1+\cos\theta=1+\left(\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\right)=2\cos^2\frac{\theta}{2}.$$

Similarly

$$1-\frac{p_z}{p}=1-\cos\theta=1-\left(\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\right)=2\sin^2\frac{\theta}{2}.$$

⇨

$$y\simeq\ \tfrac{1}{2}\ln\left(\frac{1+\frac{p_z}{p}}{1-\frac{p_z}{p}}\right)=\frac{1}{2}\ln\frac{\cos^2\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}=-\ln\tan\frac{\theta}{2}.$$

- **We define the pseudorapidity η as:**

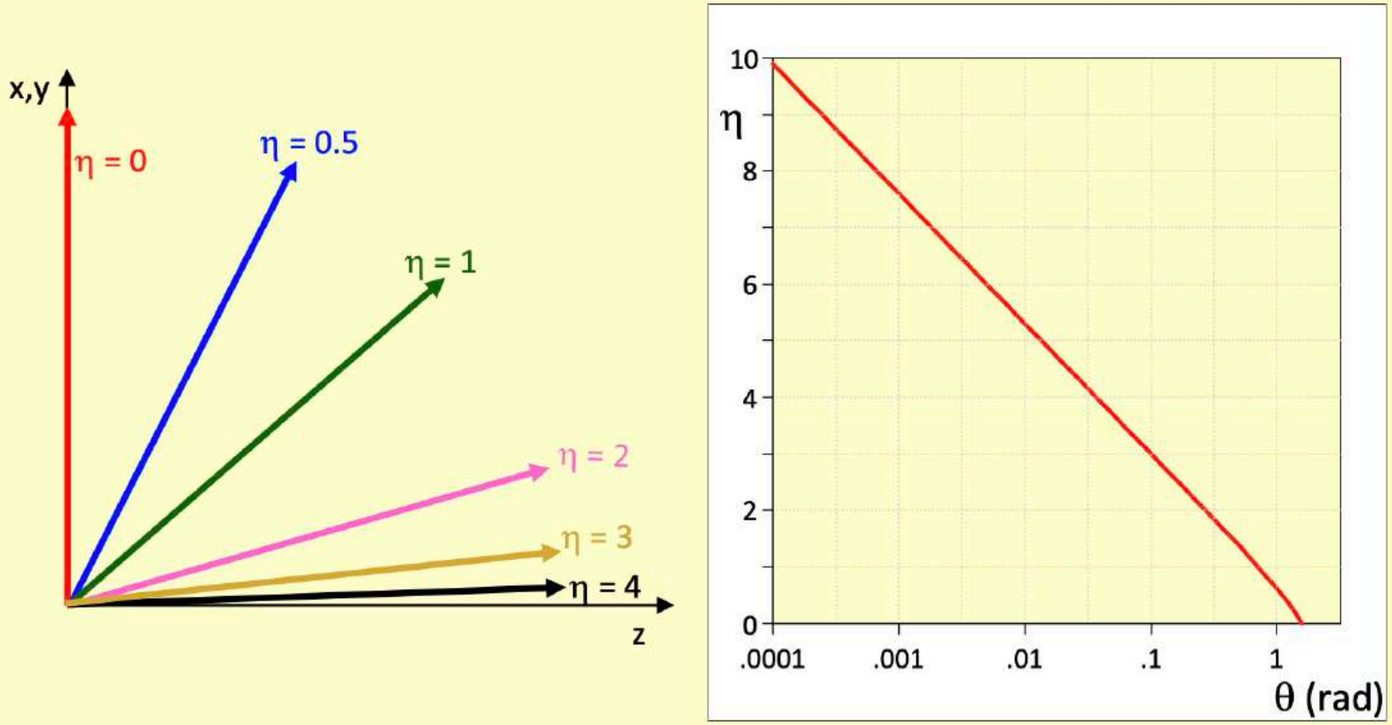
$$\boxed{\eta=-\ln\tan\frac{\theta}{2},}$$

- **For highly relativistic particles, or for massless particles, $y \approx \eta$**

Pseudorapidity: plot

$$\eta = -\ln \tan \frac{\theta}{2},$$

Slide from P. Bagnaia



$\theta = 2\tan^{-1}(e^{-\eta})$

η	θ (°)
0.0	90.000
0.5	62.476
1.0	40.395
1.5	25.157
2.0	15.415
2.5	9.385
3.0	5.700
3.5	3.459
4.0	2.099
4.5	1.273
5.0	0.772
5.5	0.468

Close to the beam pipe a small change in angle corresponds to a large change in eta.

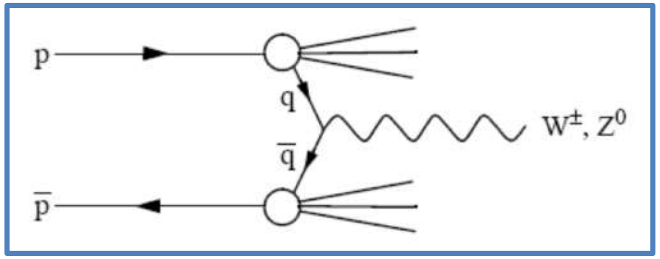
SM tests at the S$\bar{p}$pS:
W and Z mass measurements

W/Z mass determination

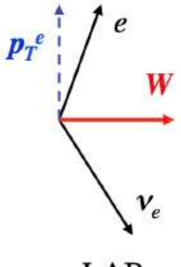
- ☐ **W mass**
  - ➢ **Jacobian peak in the lepton transverse momentum distribution**
  - ➢ **invariant transverse mass**
- ☐ **Z mass**
  - ➢ **invariant mass**

N.B. we can not use the invariant mass for the W because there is a neutrino in the leptonic decays.
In the hadronic decays there are no neutrinos, but the jet energy resolutions is not good enough.

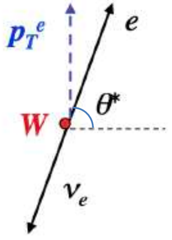
W mass measurement: lepton p_T



Let's consider, in a first approssimation, that the W does not have a transverse momentum



The W is moving with an unknown velocity along the Z axis, and then decays



In the W CM the decays is back to back: $p_e = p_\nu = \frac{m_W}{2}$

☐ **The electron p_T is the same in the two frames:** $\boxed{p_T = \frac{M_W}{2} \sin\theta^*}$

☐ **The angular momentum distribution is:** $dn/d\theta^*$

☐ **The transverse momentum distribution is given by** $\frac{dn}{dp_T} = \frac{dn}{d\theta^*}\frac{d\theta^*}{dp_T}.$

☐ $d\theta^*/dp_T$ is the jacobian of the transformation.

$$p_T = p \sin\theta \implies \theta = arc\sin\frac{p_T}{p}$$

$$\frac{d\theta}{dp_T} = \frac{1}{p}\cdot\frac{1}{\sqrt{1-\left(\frac{p_T}{p}\right)^2}}$$

☐ **The distribution in pT is equal to:**

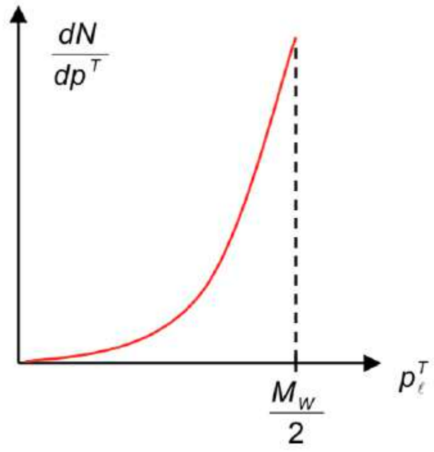
$$\boxed{\frac{dn}{dp_T} = \frac{1}{\sqrt{(M_W/2)^2 - p_T^2}}\frac{dn}{d\theta^*}.}$$

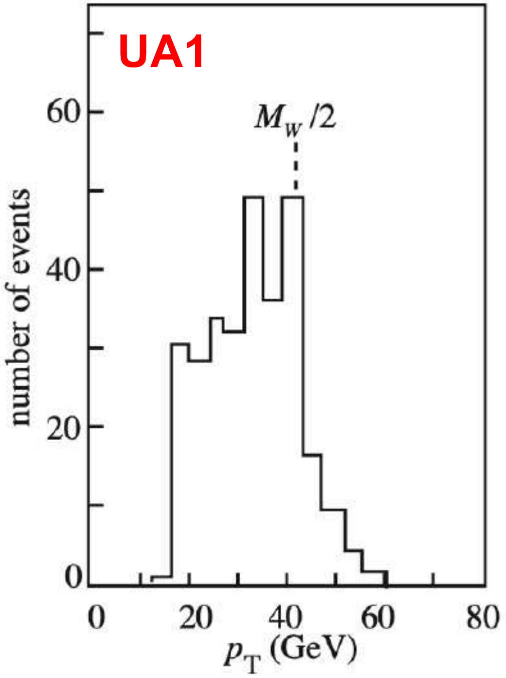
☐ **The essential point is that the jacobian diverges for** $\boxed{p_T = M_W/2.}$

W mass measurement: lepton p_T

- ☐ Therefore the p_T distribution has a sharp maximum at $M_W/2$. This conclusion does not depend on the longitudinal momentum of the W, which may be large.
- ☐ The position of the maximum, on the other hand, depend on the tranverse momentum of the W, which is small but not completely negligible. Its effect is a certain broadening of the peak.
- ☐ W finite width is also contributing to smooth the maximum of the p_T distribution

$$\boxed{\frac{dn}{dp_T} = \frac{1}{\sqrt{(M_W/2)^2 - p_T^2}}}$$





From this distribution UA1 measured:

$$\boxed{M_W = 83 \pm 3\ GeV}$$

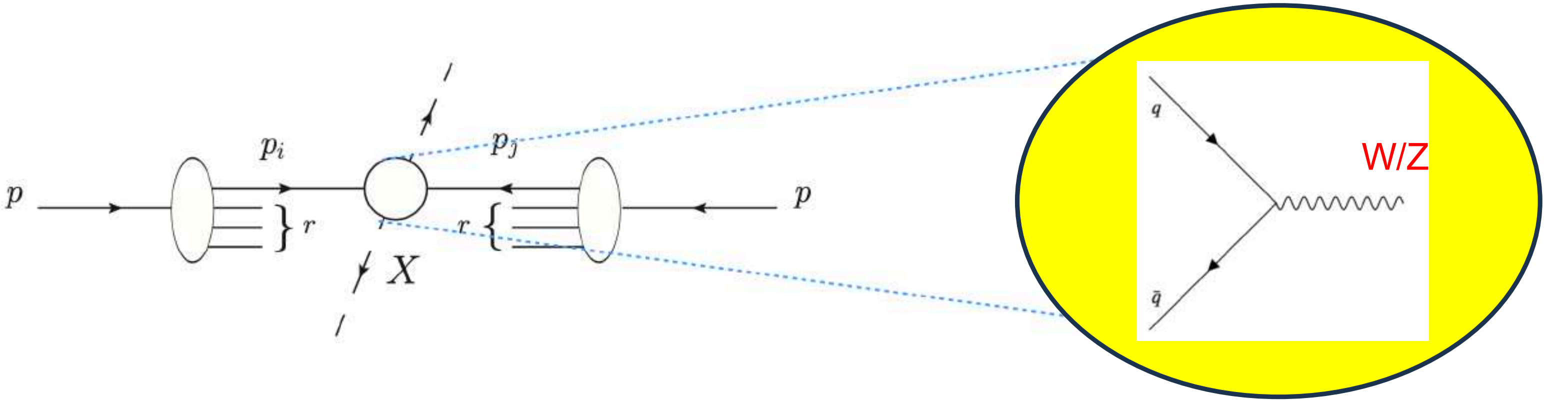
The error is mainly due to to the systematic uncertainty on the energy scale calibration.

From a similar distribution UA2 measured:

$$\boxed{M_W = 80 \pm 1.5\ GeV}$$

The errors was so large that they didn't have to worry about gluons emission from the initial state.
Today, at LHC and Tevatron, it is one of the major source of error.

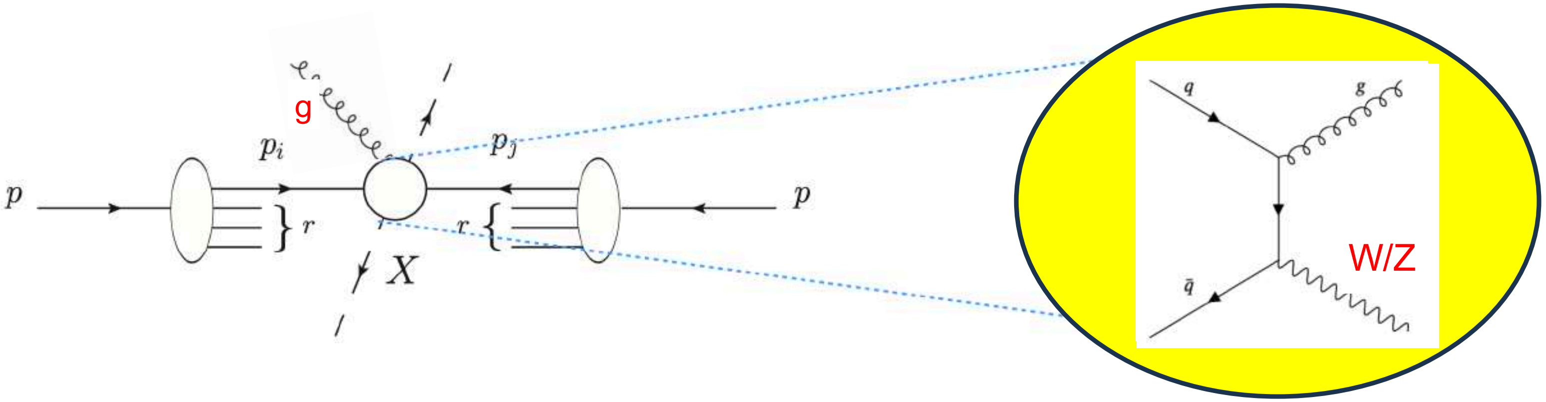
Gluon radiation from the initial state



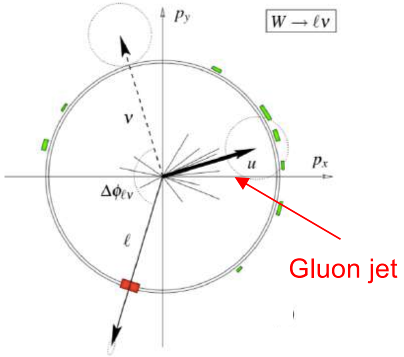
Ferynman Assumption: infinite momentum frame.

Partons have only longitudinal momentum, therefore the W/Z does not have a transverse momentum

BUT … we have to take into account the QCD higher order corrections, namely the emission of gluons from the initial state.



Now the W/Z has got a transverse momentum



Reminder: invariant mass

- ☐ **The total quadrimomentum squared is a relativistic invariant (for Lorentz transformation).**

Lab Frame

$$P_M^{\mu} \cdot P_{M,\mu} = (E_1 + E_2)^2 - \ (\vec{p}_1 + \vec{p}_2)^2$$

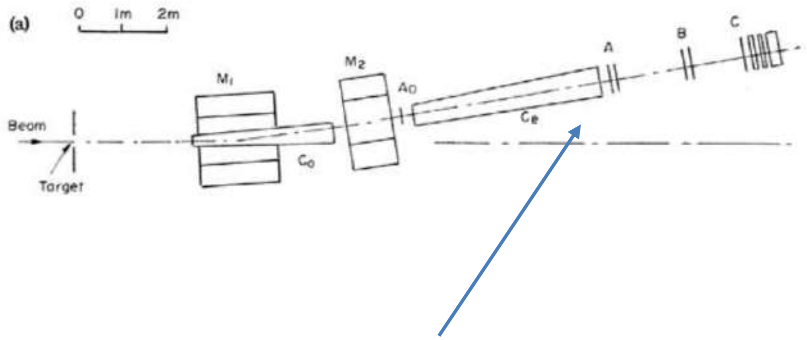
CoM Frame

$$P_M^{*,\mu} \cdot P_{M,\cdot}^{*} = M^2$$

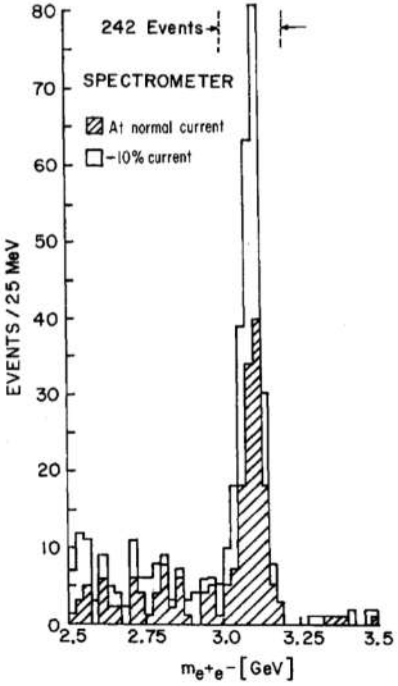
$$M_{inv} = \sqrt{(E_1 + E_2)^2 - \ (\vec{p}_1 + \vec{p}_2)^2}$$

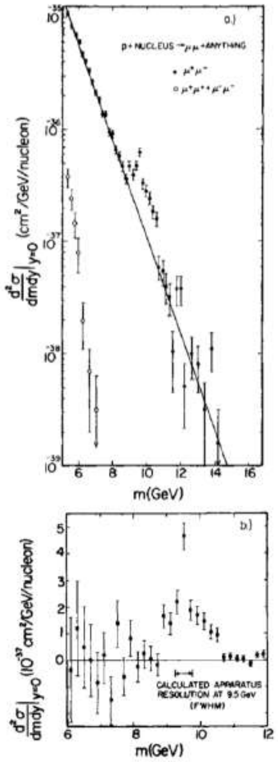
J discovery at BNL

$p + nucleus \ \longrightarrow \ e^+e^- + \text{anything}$



(to be noticed: only a very small fraction of the solid angle was equipped with the electron spectrometer)





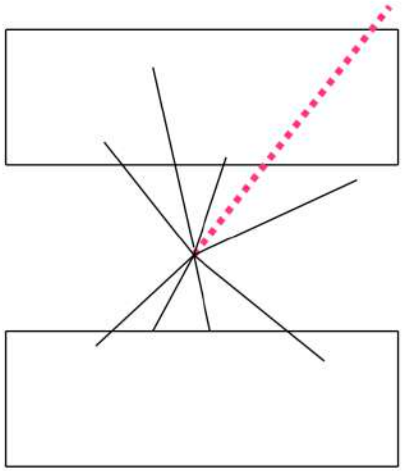
ϒ discovery at Fermilab

$p + nucleus \ \longrightarrow$
$\mu^+\mu^- + \text{anything}$

Detection and measurement of neutrinos

- **Neutrinos traverse the detector without interacting**
  → they can not be measured directly

- **They can be measured indirectly by requiring the total momentum conservation between the initial state and the final state:**



$$E_f, \vec{P}_f \quad = \quad E_i, \vec{P}_i$$

total energy, momentum reconstructed in final state

total energy, momentum of initial state

- **e$^+$e$^-$ colliders:** $E_i = \sqrt{s}, \quad \vec{P}_i = 0$

  If a neutrino is produced, then:

  $E_f < E_i$ ($\rightarrow$ missing energy) and $\vec{P}_f \neq 0$

  $$\Rightarrow \quad \vec{P}_\nu = -\vec{P}_f \qquad E_\nu = |\vec{P}_\nu|$$

- **hadron colliders:**

  Energy and momentum of the initial state (partons) are not known, however: transverse momentum is conserved

  $$\vec{P}_{Ti} = 0$$

  If a neutrino is produced, we have missing transverse momentum:

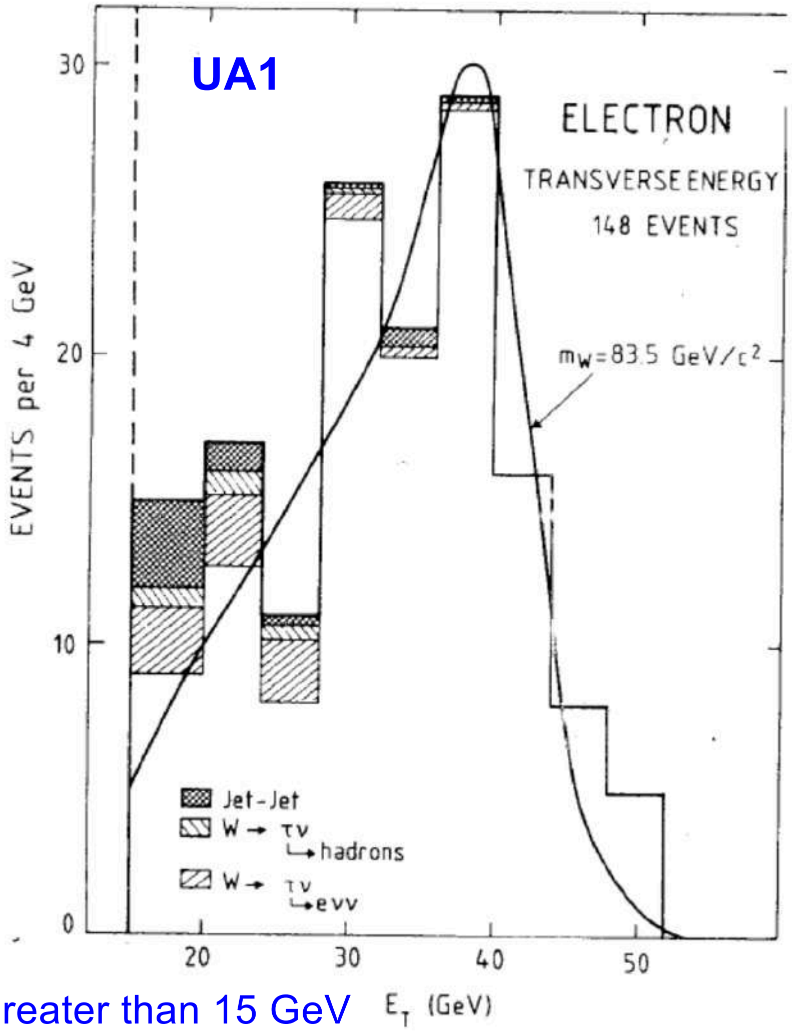
  $$\vec{P}_{Tf} \neq 0$$

  $$\Rightarrow \quad |\vec{P}_{T\nu}| = |\vec{P}_{Tf}| = E_T^{miss}$$

Missing E_T or missing p_T

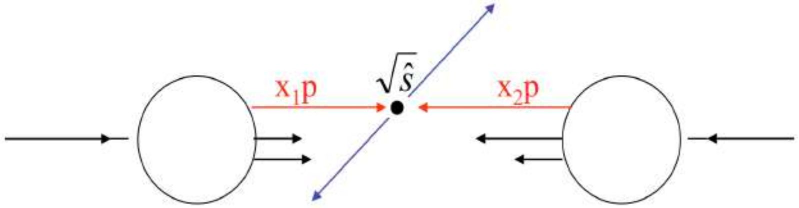
- If the mass is small compared to its energy the missing p_T is equivalent to missing E_T.
- If the momentum of all particles (in the transerve plane) in a collision is added up the results should be zero (momentum conservation). Neutrinos can, however, not be detected and if the total momentum is different from zero, the event is said to have missing [transverse] momentum (or missing [transverse] energy).

- In the SM only neutrinos contribute to missing energy but in other models, for instance SUSY, other particles can contribute. So the missing energy is one of the typical signatures of new Physics.



Distribution of events with missing E_T greater than 15 GeV

Invariant transverse mass



Big problem in the hadron colliders

the Center of Mass frame does not coincide with the Laboratory frame.

- ☐ **As a first approssimation we can assume that the CoM is moving only along the beam axis.**
- ☐ **All quantities in the tranverse plane (with respect to the beam) are conserved in the two frames; we need to find quantities that are invariant with respect to a Lorentz boost allong the beam axis.**

Lorentz boost along z

$$\begin{aligned} ct' &= \gamma(ct - \beta z) \\ x' &= x \\ y' &= y \\ z' &= \gamma(z - \beta ct). \end{aligned}$$

displacement

$$\begin{aligned} E'/c &= \gamma(E/c - \beta p_z) \\ p'_x &= p_x \\ p'_y &= p_y \\ p'_z &= \gamma(p_z - \beta E/c). \end{aligned}$$

4-momentum

$$E^2 - p_x^2 - p_y^2 - p_z^2 = M^2$$

$$E^2 - p_z^2 = M^2 + p_x^2 + p_y^2$$

Invariant transverse mass

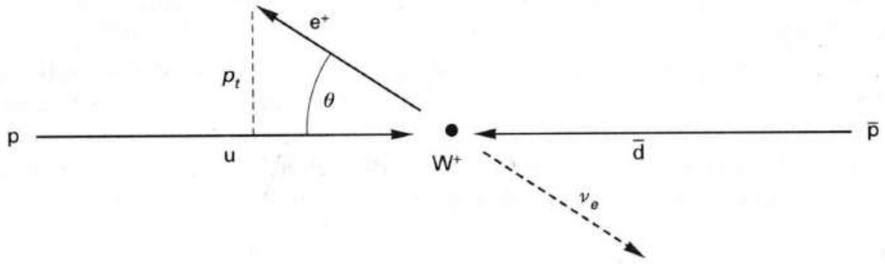
$$M_T^2 = M^2 + p_x^2 + p_y^2$$

- ☐ **In case of a neutrino or when the transverse momentum is much higher than the mass, the definition becomes:**

Invariant transverse mass

$$M_T^2 = p_x^2 + p_y^2$$

Invariant transverse mass



With the W transverse momentum

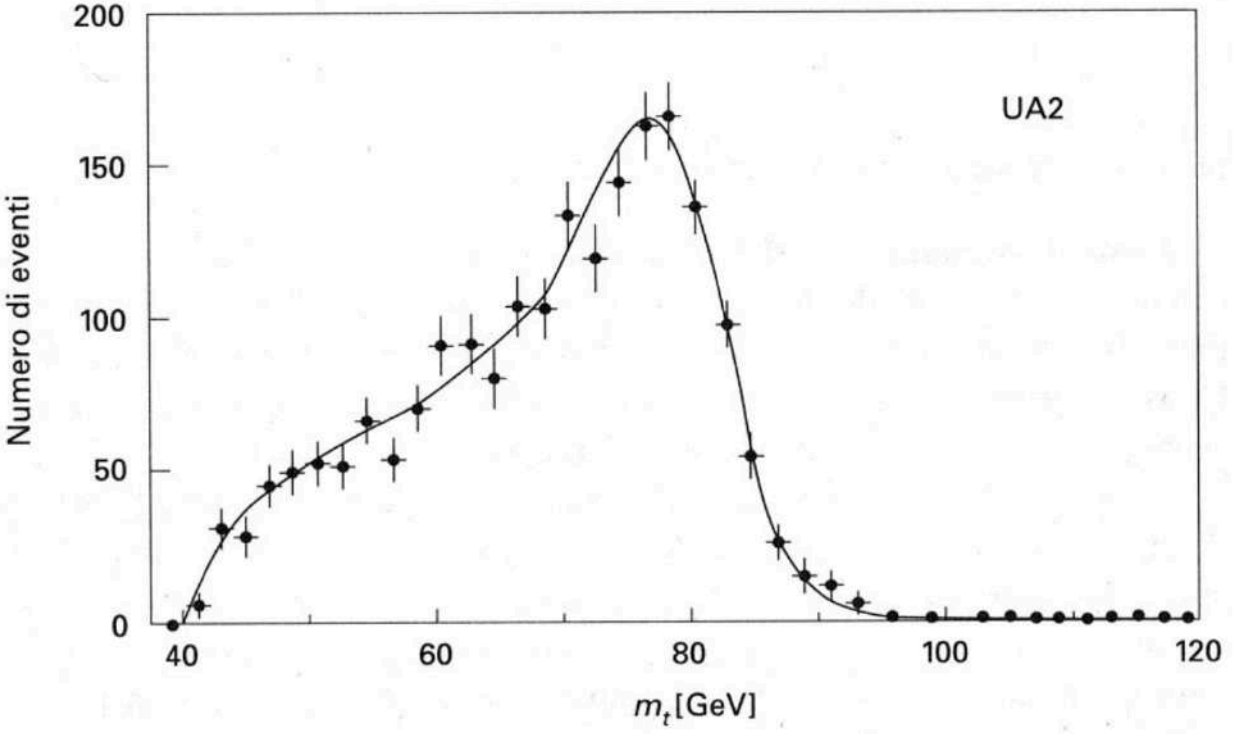
$$m_T^W \equiv \sqrt{2\vec{p}_T^{\,\ell}\vec{p}_T^{\,miss}\left(1-\cos\Delta\phi\right)}$$

Here we neglet the W transverse momentum

$$\boxed{M_T^2 = p_{eT}^2 + p_{\upsilon T}^2 = 2p_{eT}^2}$$

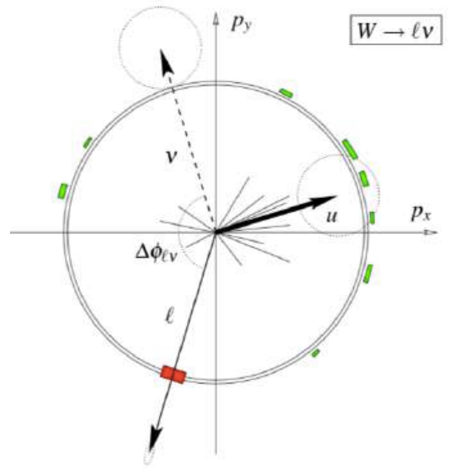
- ❑ **We have a maximum at M_W**
- ❑ **The maximum is broadened by**:
  - ➢ W width
  - ➢ Calorimeter resolution
  - ➢ W transverse momentum

$$E^2 - p_z^2 = M_T^2$$



[*histogram with background subtracted*]

M_W invariant transverse mass

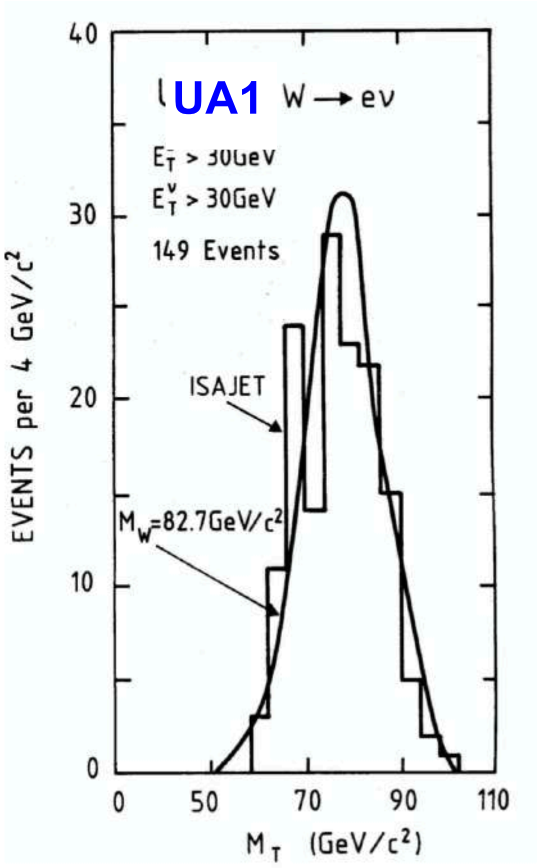


$$m_T^W \equiv \sqrt{2\vec{p}_T^{\,\ell}\vec{p}_T^{\,miss}\left(1-\cos\Delta\phi\right)}$$

$$\vec{p}_T^{\,miss} = -\left(\vec{p}_T^{\,\ell} + \vec{u}_T\right)$$

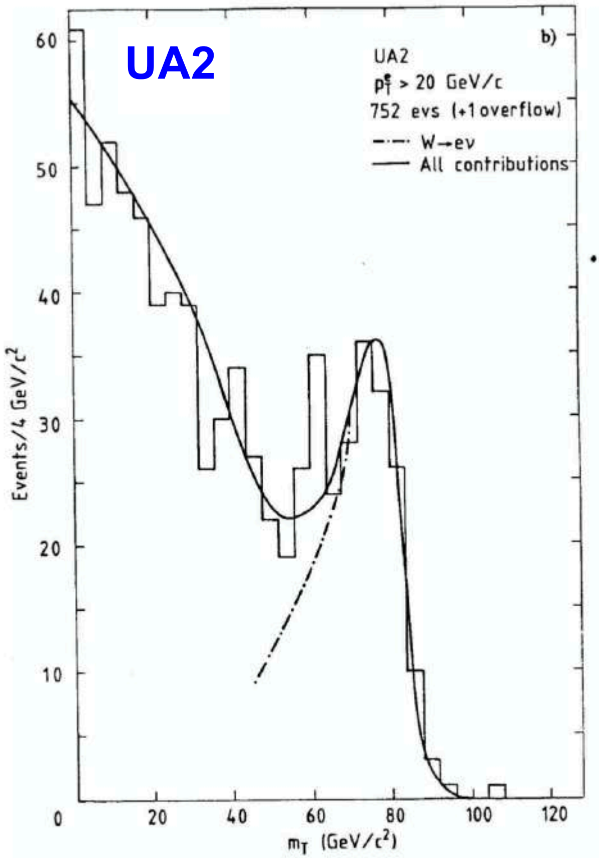
u_T comes from the calorimeter energy cells

- ❑ **UA2 had a better control of the energy calibration of the calorimeter.**



$$M_W = 82.7 \pm 1.0(stat) \pm 2.7(syst)\ GeV$$

$$\Gamma_W < 5.4\ GeV$$

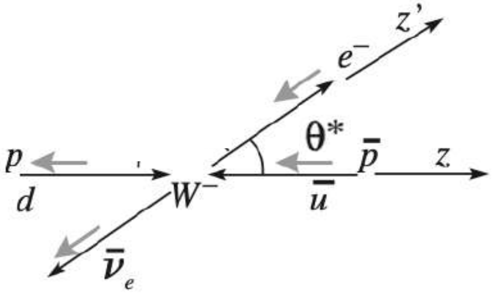


$$M_W = 80.2 \pm 0.8(stat) \pm 1.3(syst)\ GeV$$

$$\Gamma_W < 7\ GeV$$

Electron helicity in the W decay

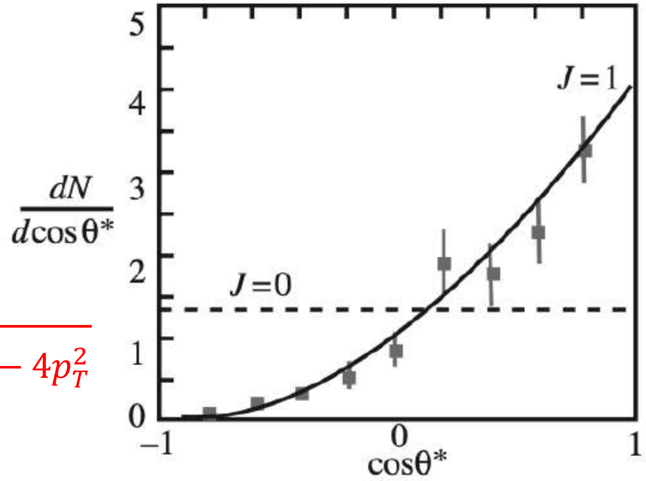
- **Measurement of the electron helicity in the decays $W \rightarrow e\nu$ in the W rest frame:**



- Because of the V-A structure of the weak CC, the antineutrino must be righthanded and the electron lefthanded (neglecting its mass)
- The W^{--} must be created by annihilation of anti-u contained in the antiproton and the d of the proton.
- If we choose the Z-axis along the proton line of flight, the total angular momentum is J=1 while J_Z=-1
- θ* is the angle between the electron and the z-axis; θ*=0 → J_Z'=-1; θ*=180 → J_Z'=1
- The differential cross-section of the scattering depends on the angular momentum of the initial and final state. In this case, with J=1, we have:

$$\frac{d\sigma}{d\Omega} \propto \left[d^1_{-1,-1}\right]^2 = \left[\frac{1}{2}(1+\cos\theta^*)\right]^2.$$

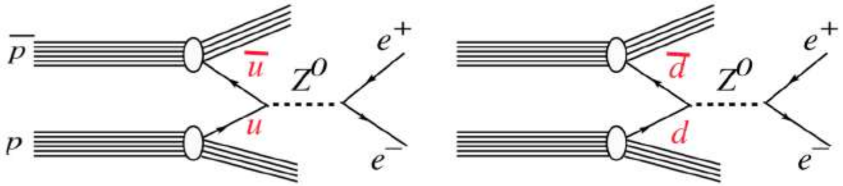
- These are the UA1 data:



$$\cos\theta^* = \frac{1}{M_W}\sqrt{M_W^2 - 4p_T^2}$$

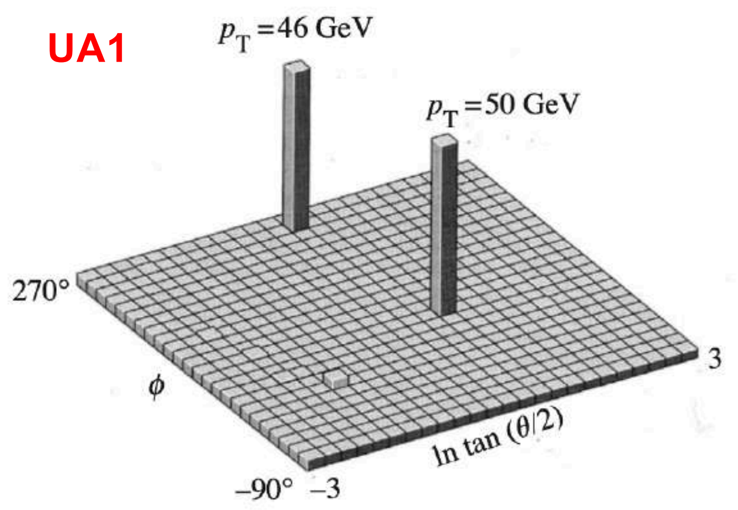
- **The plot is consistent with a W with spin 1.**
- N.B. The plot can not distinguish between V-A and V+A theory, because V+A simply reverse the sign of all helicity.

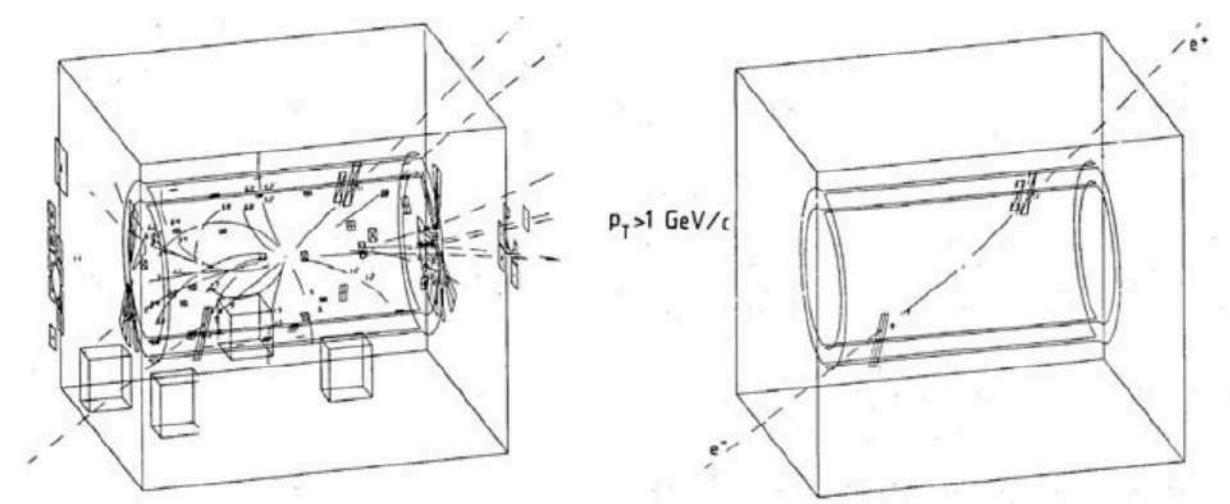
Z boson event selection



A very clean signatures

- ❑Two energetic clusters with no missing energy in the event





Selecting only tracks and cells with $p_T > 1$ GeV/c

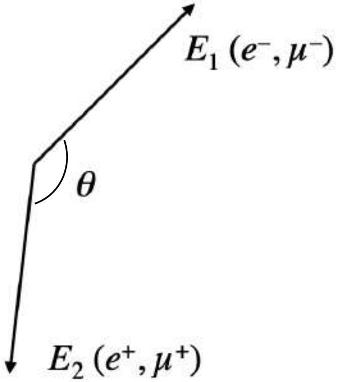
In UA1 the two electrons (muons) must have opposite charge

Z boson mass measurement

- **invariant mass:**

$$m^2 = \left(E_1 + E_2\right)^2 - \left(\vec{p}_1 + \vec{p}_2\right)^2 =$$
$$= E_1^2 + E_2^2 + 2E_1E_2 - p_1^2 - p_2^2 - 2p_1p_2\cos\theta$$
$$\cong 2E_1E_2(1-\cos\theta)$$

$$\boxed{m^2 \cong 4E_1E_2\sin^2\theta/2}$$



- **invariant mass resolution:**

$$\frac{\sigma_m}{m} = \sqrt{\left(\frac{\sigma(E_1)}{E_1}\right)^2 + \left(\frac{\sigma(E_2)}{E_2}\right)^2 + \left(\frac{\sigma(\theta)}{\tan\theta/2}\right)^2}$$

In this case:

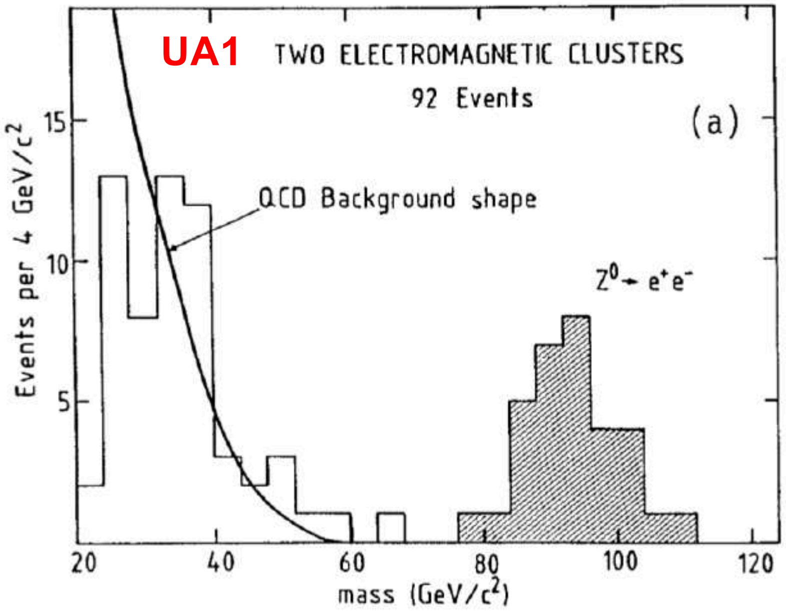
$$\theta \geq 100^\circ \Rightarrow \tan\frac{\theta}{2} \approx O(1)$$

$\sigma(\theta) \approx 10^{-2}$
Evaluated from tracks

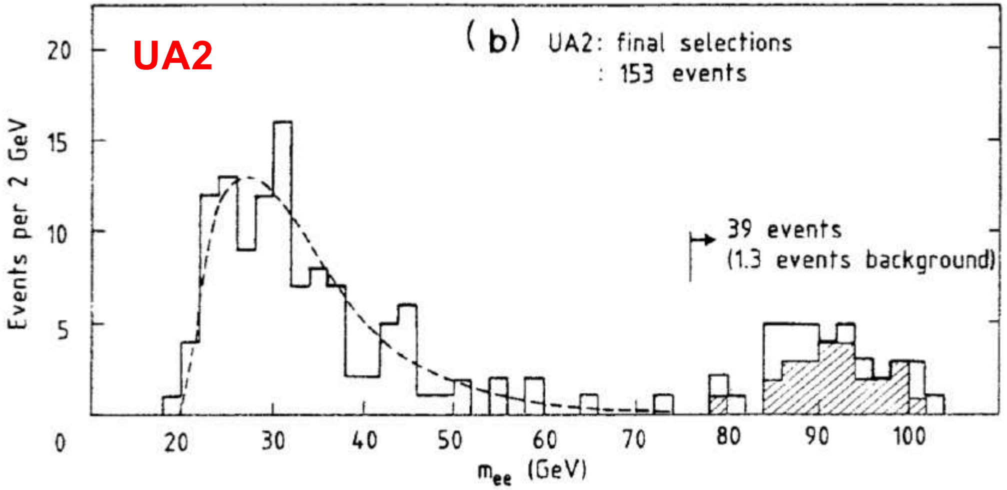
- **the error on the angle is negligeable with respect to the calorimeter energy resolution (~ 2-3%).**

Invariant mass distribution (1983 data)

- ❑ **The signal peak is well separeted from the QCD background (combinatorial background)**



$M_Z = 93 \pm 3\ GeV$



$M_Z = 91.5 \pm 1.7\ GeV$

$\mathrm{Sin}^2\theta_W$ determination

- One of the most important parameter of the Standard Model is the weak (Weinberg) angle. Its measurement at the SppS collider was an important test of the theory.
- At the tree level we have:

$$\cos\theta_W = \frac{M_W}{M_Z} \quad \Rightarrow \quad \boxed{\sin^2\theta_W = 1 - \left(\frac{M_W}{M_Z}\right)^2}$$

- Using the W and Z boson masses measured in 1983, it was obtained:

$$\boxed{\mathrm{UA1}:\ \sin^2\theta_W = 0.211 \pm 0.025 \qquad \mathrm{UA2}:\ \sin^2\theta_W = 0.232 \pm 0.027.}$$

- These values were in agreement with what was measured in the neutrino scattering :

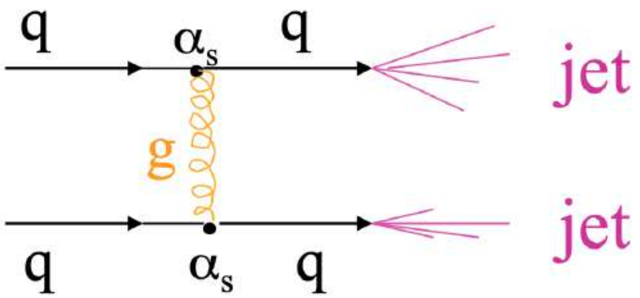
$$\boxed{\sin^2\theta_W = 0.2324 \pm 0.0083.}$$

(CHARM2 result published in 1994)

- **In conclusion, by 1983 the UA1 and UA2 experiments had confirmed that the vector mesons predicted by the electroweak theory exist and have exactly the predicted characteristics.**

QCD background

- ❑ High-p_T events are dominated by QCD jet production



- ❑ Strong interaction → large cross-section
- ❑ Many diagrams contribute: qq → qq ; qg → qg ; gg → gg; etc …
- ❑ They are called “QCD background “

- ❑ Most interesting processes are rare processes:
  - ➢ involve heavy particles
  - ➢ have weak cross-sections (e.g. W cross-sections)
  - ➢ to extract signal over QCD jet background must look at decays to photons and leptons → pay a prize in branching ratio

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End of chapter 5